Analytical evaluation of room temperature non-Fourier heat pulse experiments

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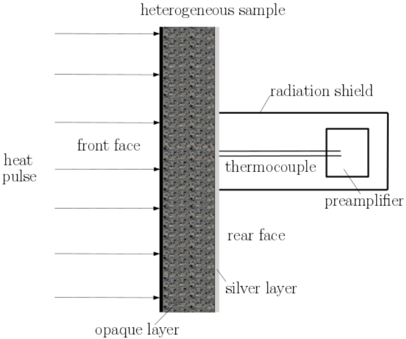
Motivation

- Fourier thermal conduction model with limited validity:
  - Low temperature wave phenomena.
  - Nanostructures.
  - **Heterogeneous materials**.
- Thermal conduction $\Longleftrightarrow$ material structure $\rightarrow$ heterogeneities?
- Technologically controlled structure $\rightarrow$ engineering practice.

Heat pulse (“flash”) measurement method

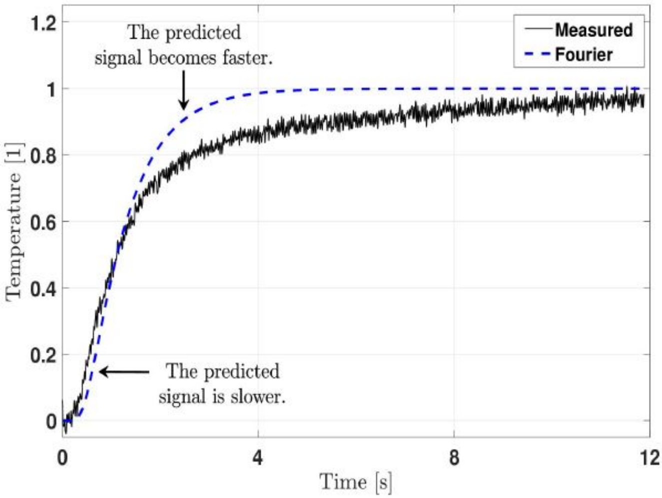
- Thermal diffusivity.
- Excitation pulse → flashing lamp, 0.01 s.
- Photovoltaic sensor.
- PC oscilloscope.
- Sample preparation.



Single flash

Periodic flash

Motivation



The predicted signal becomes faster.
The predicted signal is slower.
Measured
Fourier
Temperature [1]
Time [s]
0
0.2
0.4
0.6
0.8
1
1.2
0
4
8
12

Theoretical background I.

Fourier equation

$$\rho c \partial_t T + \nabla \cdot \boldsymbol{q} = 0, \quad \boldsymbol{q} = -\lambda \nabla T. \tag{1}$$

1D:

$$\partial_t T = \alpha \partial_{xx} T$$

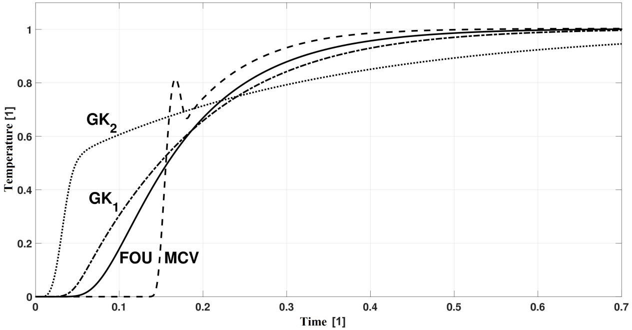
- Constant material parameters, rigid medium.
- Thermal diffusivity: $\alpha = \frac{\lambda}{\rho c}$.
- Based on evaluations: heat transfer coefficient, estimation T_{max} and α.

Theoretical background II.

Guyer-Krumhansl equation (1D):

$$\tau_q \partial_{tt} T + \partial_t T = \alpha \partial_{xx} T + \kappa^2 \partial_{txx} T$$

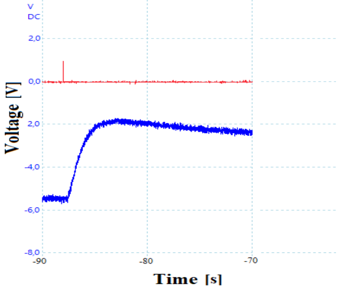
- Fourier resonance: $\kappa^2 / \tau_q = \alpha$.

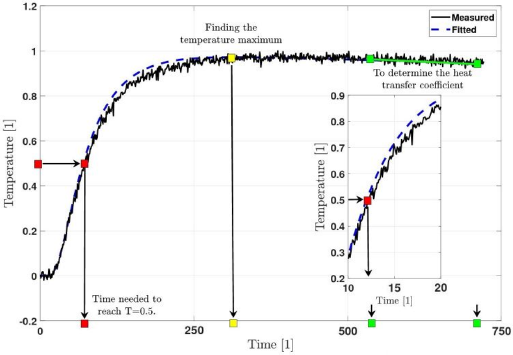


Fourier: standard evaluation

- Analytical solution of the Fourier thermal conduction equation

$$\alpha_F = 1.38 \cdot \frac{L^2}{\pi^2 \cdot t_{0.5}}, \quad \text{and} \quad \hat{h} = -\frac{\ln(\hat{T}_2/\hat{T}_1)}{\hat{t}_2 - \hat{t}_1}, \Rightarrow T_{max} \quad (2)$$

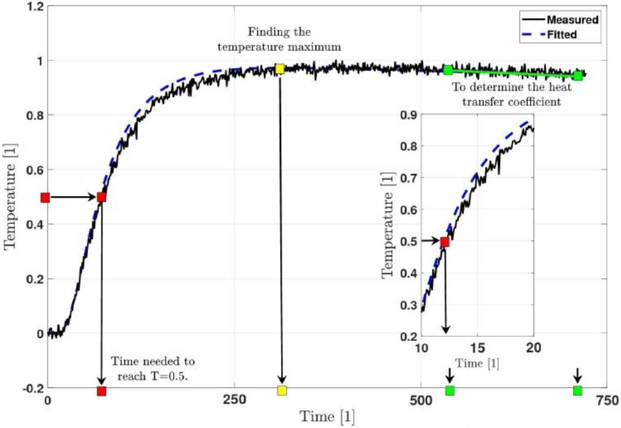




GK evaluation procedure.

- Rear side temperature:
  $T(x = 1, t > t_I) = b_0 - b_1 = Y_0 e^{-ht} - Z_1 e^{x_1 t} - Z_2 e^{x_2 t}$.

1. Fourier fitting: heat transfer coefficient, T_{max}, α_F estimates.

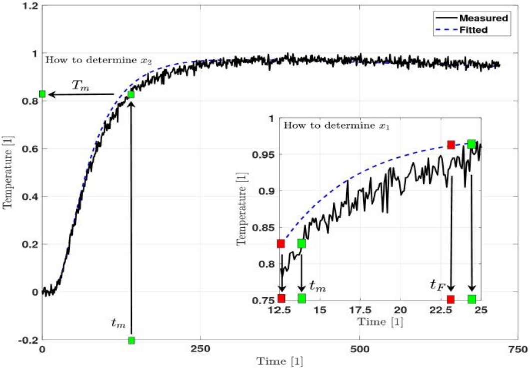


GK evaluation procedure.

2. Estimation of the 'primary' time constant: by deviation from Fourier,

$$x_{1,m} \approx x_F \frac{t_{F1} - t_{F2}}{t_{m1} - t_{m2}}, \qquad (3)$$

m is the number of measurement points; $\{x_{1,m}\}$ set average $\rightarrow x_1$.



GK evaluation procedure.

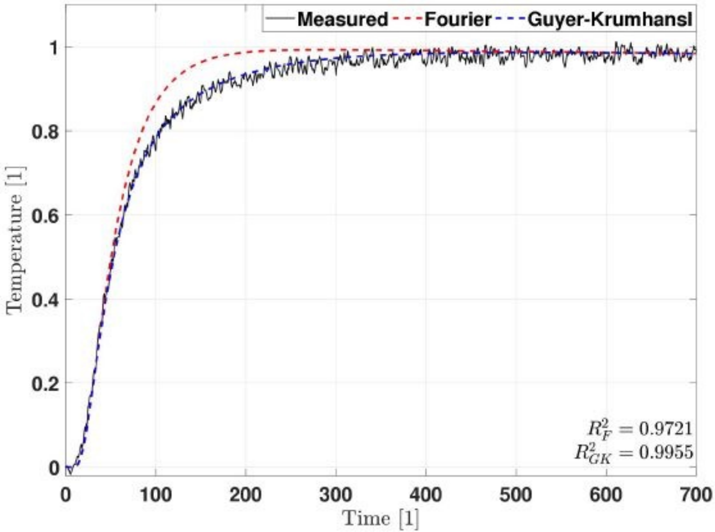
3. Determination of the coefficient Z_1:

$$Z_{1,m} = e^{-x_{1,m} t_m}(T_m - Y_0 e^{-h t_m})$$

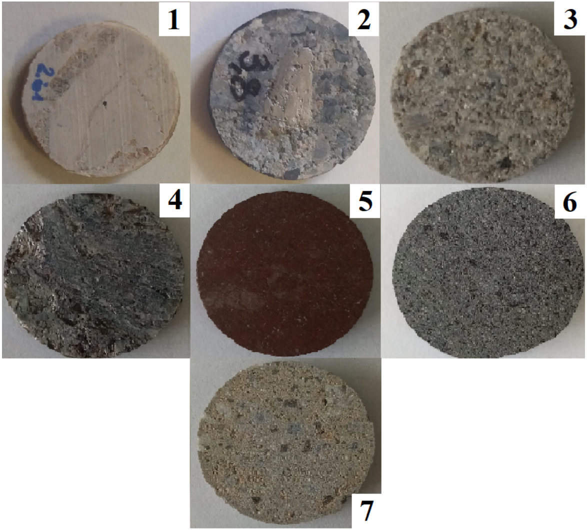
$\{Z_{1,m}\}$ set average $\rightarrow Z_1$.

4. Determination of the 'secondary' time constant, recalculation of material parameters. (arXiv:2102.11744)

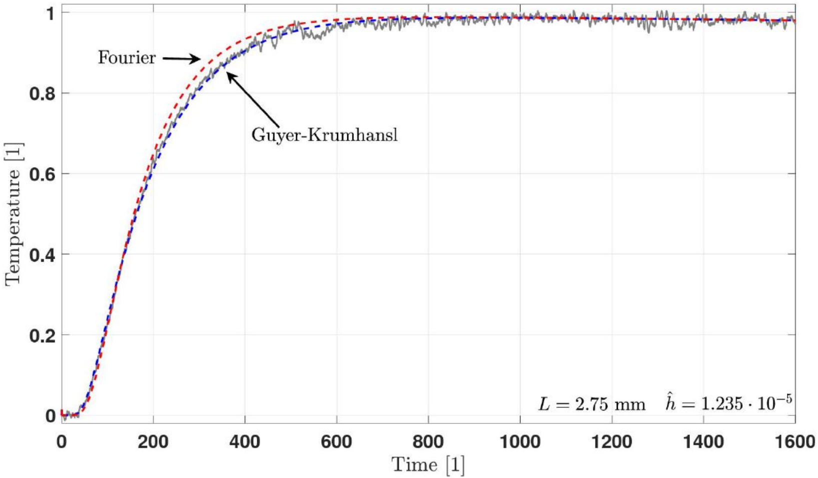
Evaluation - GK fitting



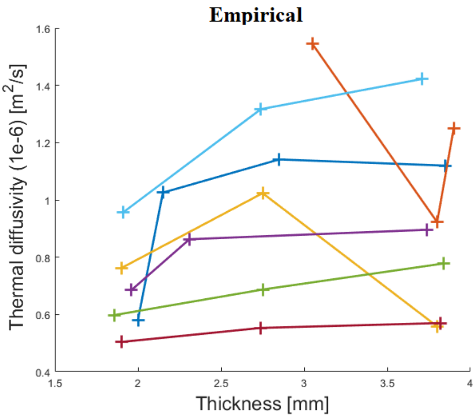
Experimental results - rock samples

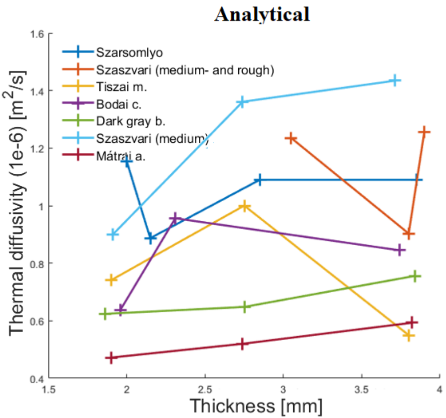


Rock sample evaluation



Size dependence of thermal diffusivity

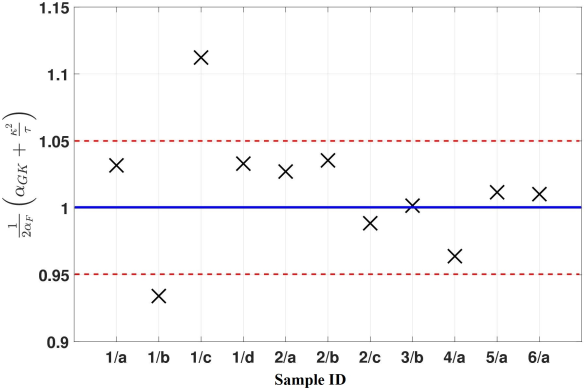




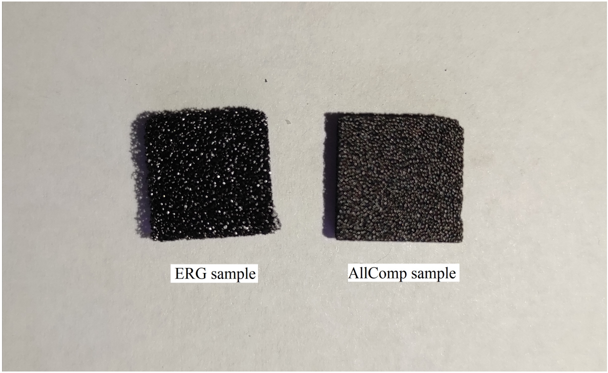
Experimental results - material parameters

Fourier vs. GK-parameters:

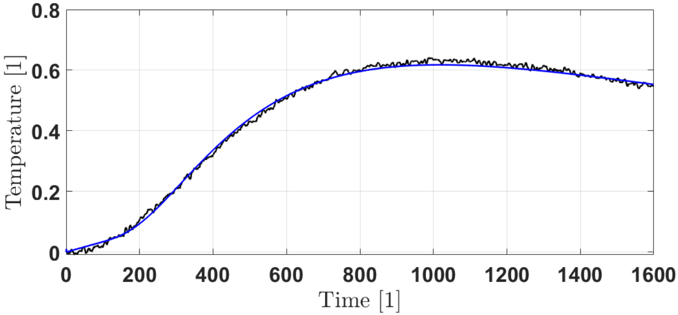
$$\alpha_F \approx \frac{1}{2}\left(\alpha_{GK} + \frac{\kappa^2}{\tau}\right). \quad (4)$$



Carbon foam evaluation

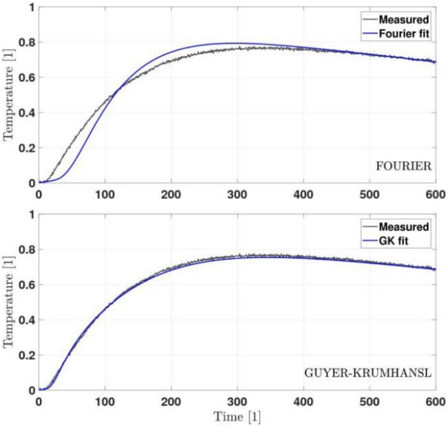


Carbon foam evaluation - ERG sample - Fourier



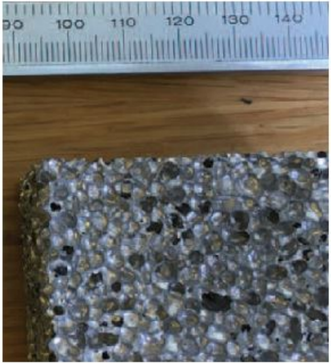
- Fourier thermal diffusivity: 6.0578E-7 $\frac{m^2}{s}$.

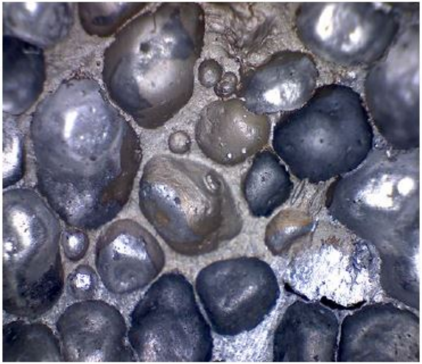
Carbon foam evaluation - AllComp sample - GK



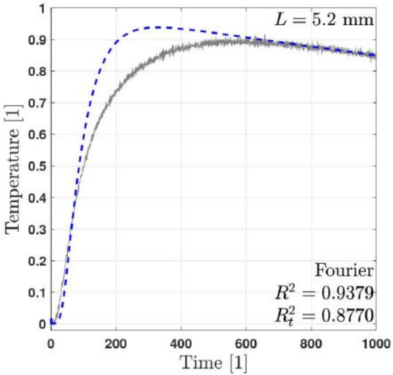
- Fourier thermal diffusivity: 3.427E-6 $\frac{m^2}{s}$.
- GK thermal diffusivity: 2.855E-6 $\frac{m^2}{s}$.

Metal foam

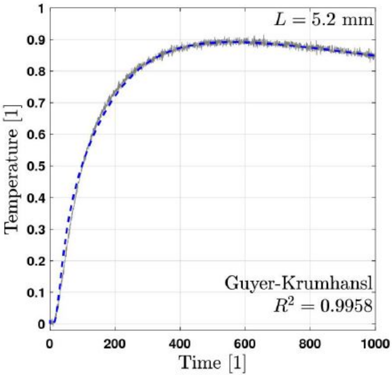




Metal foam

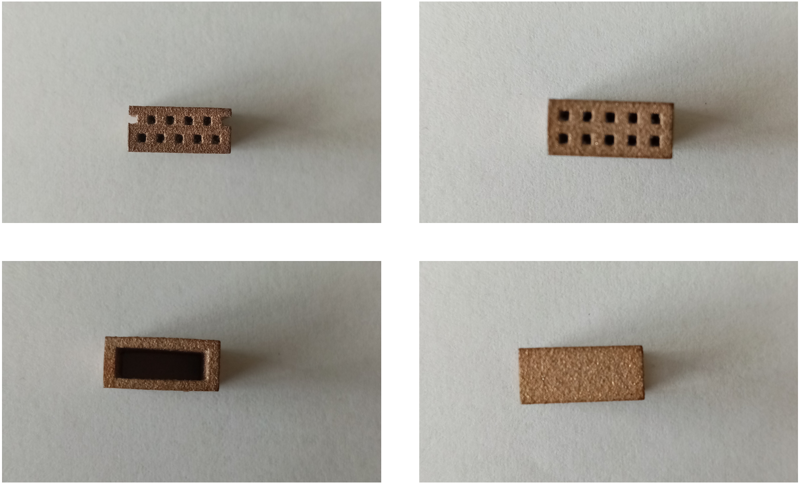


L = 5.2 mm
Fourier
R^2 = 0.9379
R_t^2 = 0.8770
Temperature [1]
Time [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
200
400
600
800
1000



L = 5.2 mm
Guyer-Krumhansl
R^2 = 0.9958
Temperature [1]
Time [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
200
400
600
800
1000

3D printed sample



Summary

- Various samples for the experiments: rocks, carbon foam, metal foam, fibre, insulation.
- Observing size dependencies.
- Relationship between Fourier and GK parameters.
- Practical benefit: simpler description of materials with complex structure.

Thank you for your attention!