

Fuzzy Dark (matter) Imprints in Galaxies

Quantum effects on galactic scales...

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Subject matter:

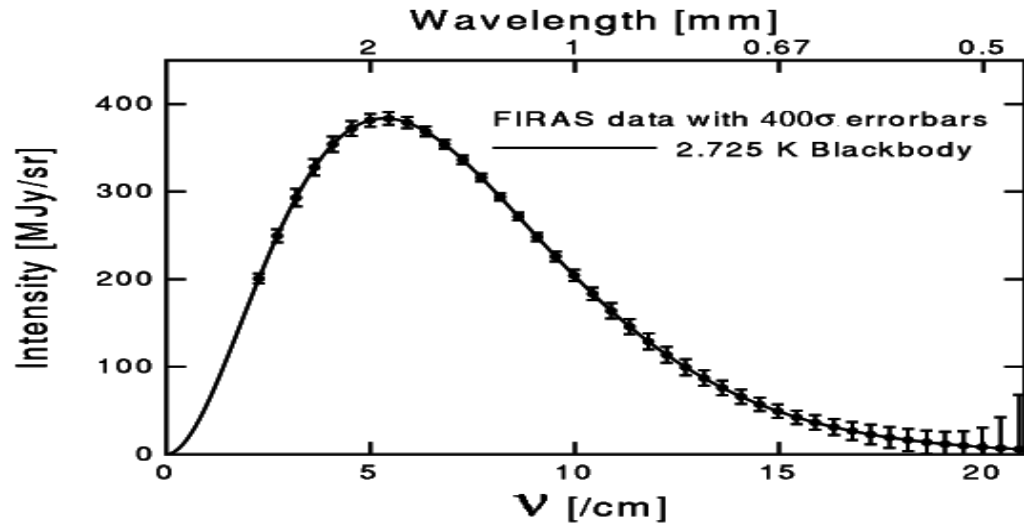
For typical galactic speeds FDM has

De Broglie wavelength $\frac{h}{m v} \sim 100 \text{ pc}$ or more at galactic speeds $\rightarrow m \sim 10^{-22} \text{ eV}$

Outline:

- **Why** ultra light axions? (from a galactic perspective)
- Characterization of Fuzzy Dark Matter fluctuations
- **Effect** on stellar dynamics, central supermassive BH and associated constraints

Hot Big Bang and Cold Dark Matter Driven Structure Formation

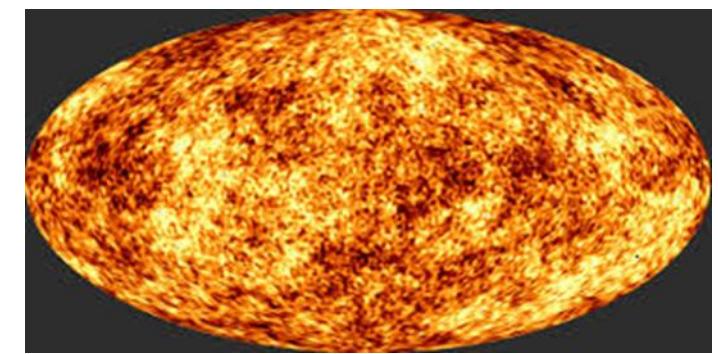


→ Early thermal equilibrium

Baryons coupled to photon bath
→ cannot collapse early enough

But weakly interacting, massive particles (WIMPs)

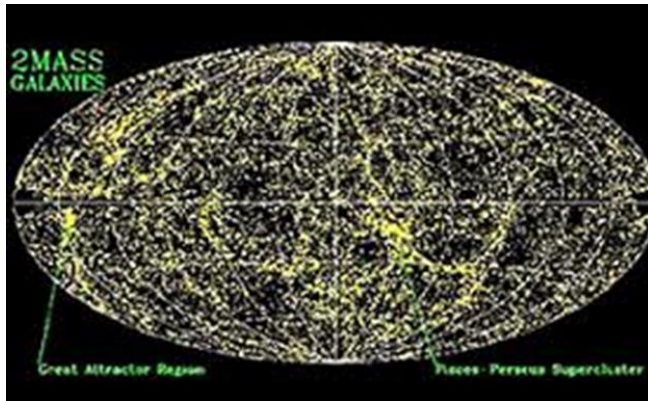
$$mv^2 \sim k_B T \quad \rightarrow \quad m \uparrow \rightarrow v \downarrow$$



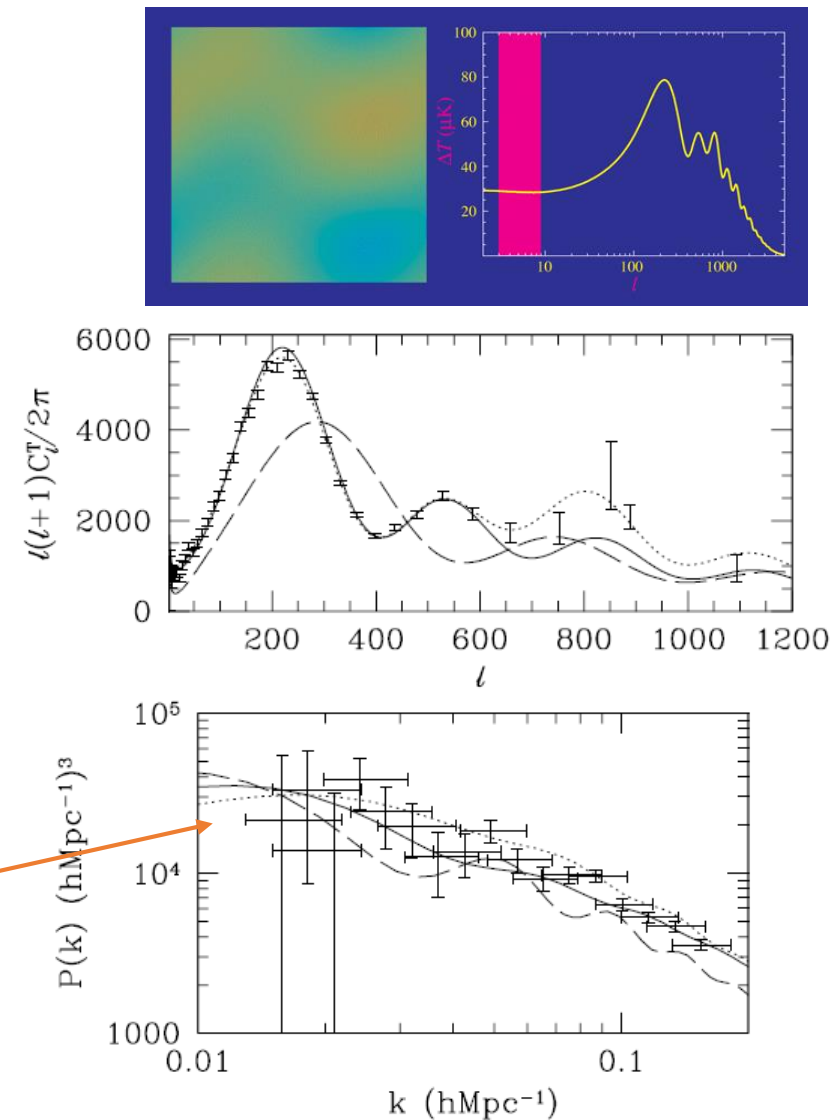
Decouple early with small speed → → Drives galaxy formation

The Case for CDM

- DM needed for structure formation and CMB
- Not easily explainable by modified gravity
- Cold dark matter → Fits cosmological data



Power spectrum of
cosmic structure



CMB, LSS in MOND
(Skordis et. al. 2006)

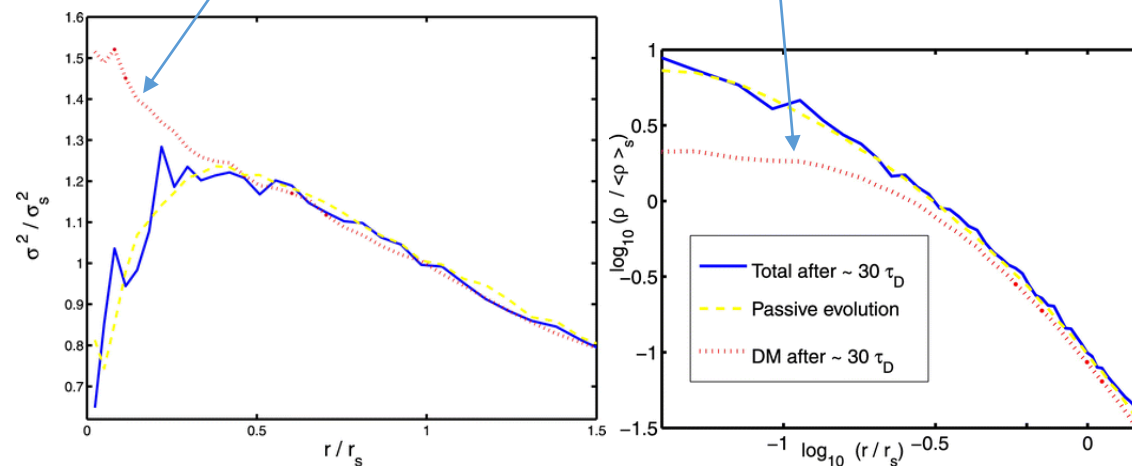
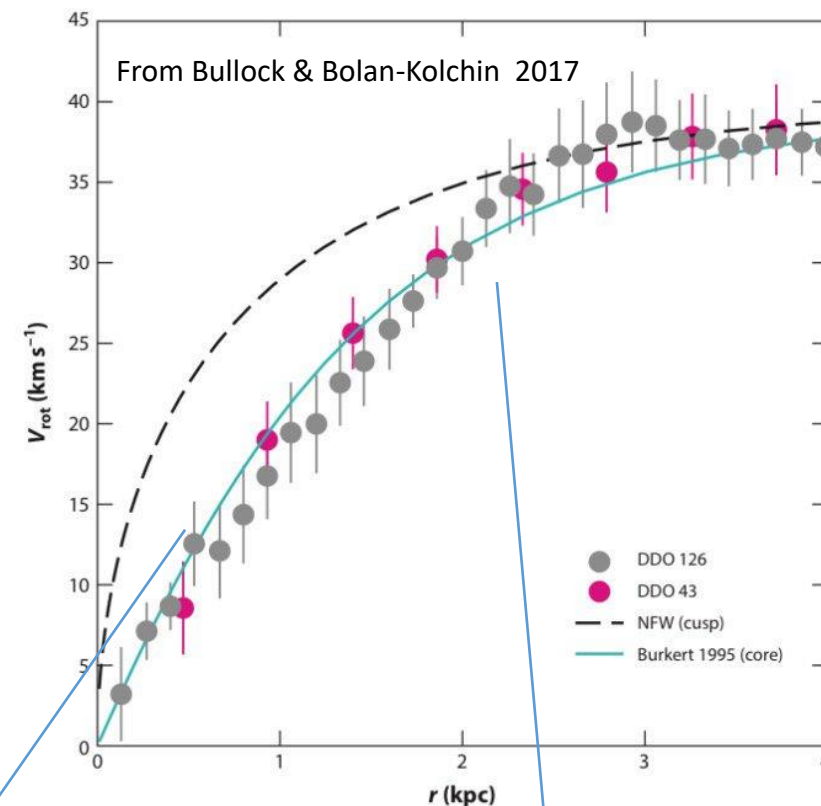
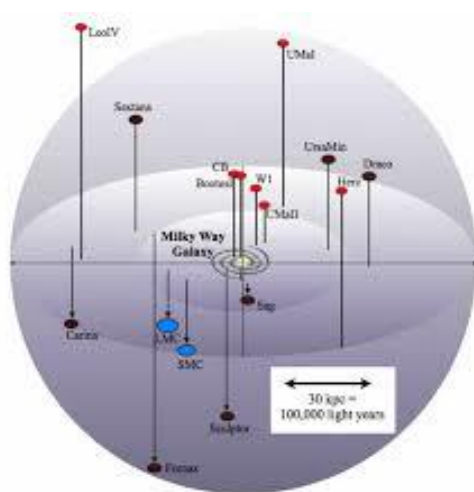
- Predicted by natural Susy theories (until recently a big plus!)
- Right abundance from weak interaction freeze out from thermal equilibrium

Galactic Scale Problems with CDM

DM compensates for mass deficit in outer parts

BUT contributes too much mass to central parts

****Probably related problems: Excess of small haloes and wrong dynamics**



Simulation M.Y. size halo .vs. Dwarf galx. pop.

→ constraints

• Need smaller density

+ more random motion in centre of halo:

Heating of central cusp by dynamical friction; El-Zant (2008)

Some Proposed Solutions

'Heat' DM → decrease DM density:

**** Baryonic solutions:** baryons pump energy into DM

(e.g., El-Zant et. al 2001, 2004; Pontzen & Governato 2014; El-Zant et. al. 2016, Hashim et. al. 2022)

**** Self interacting DM → Conduction**

**** Warm DM → preheat!**

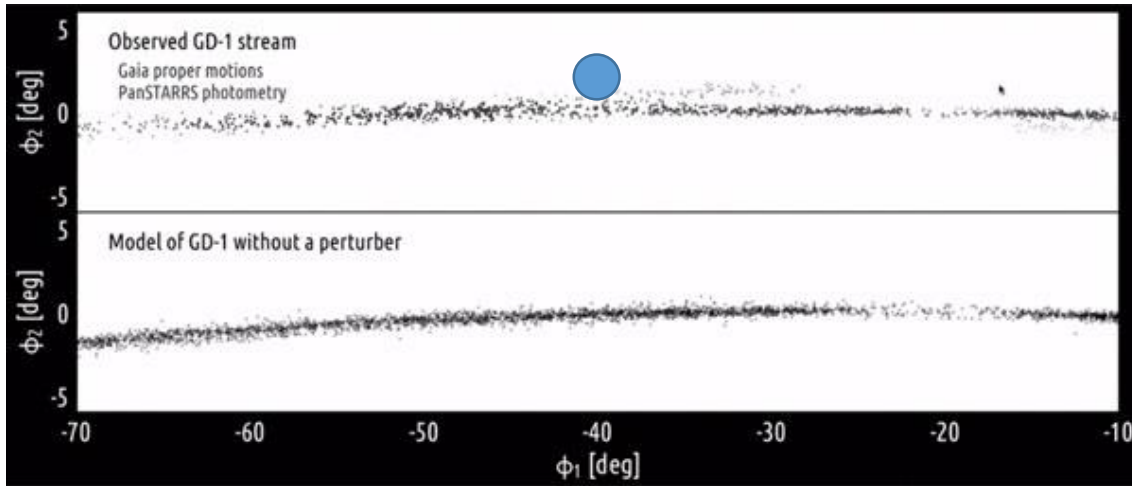
**** Quantum fluctuations** ('Fuzzy Dark Matter' of ultra light bosons)

e.g., Hu et al. (2000), Peebles (2000), Hui et. al. (2017), El-Zant et. al. (2019, 2020)

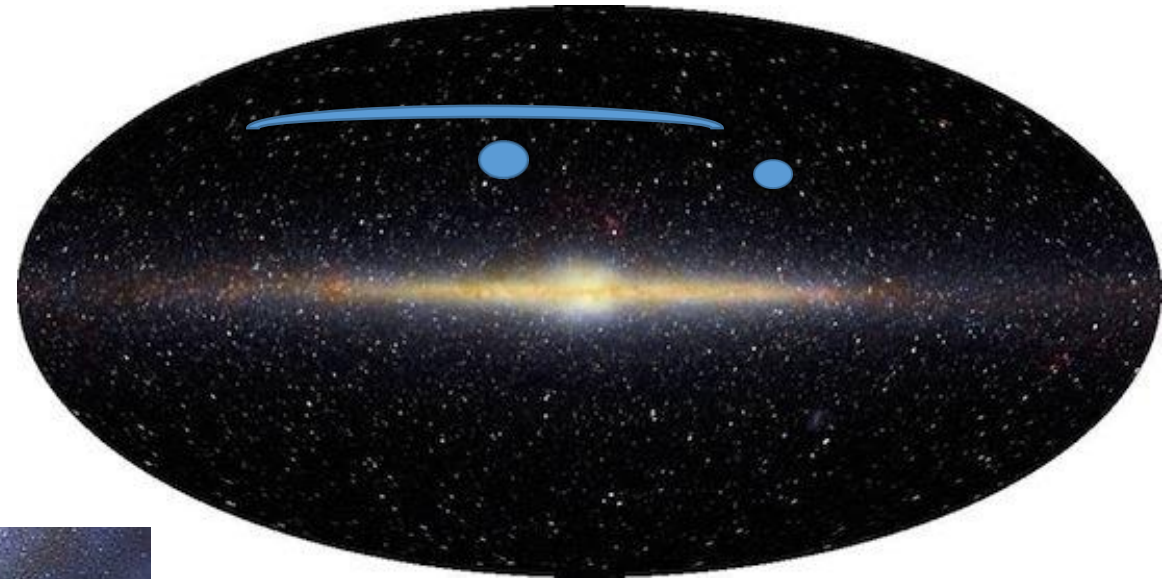


Couple of **effects** of DM **fluctuations**

Tidal streams: thickness and gaps (Bonaca et. al. 2019)



Thickness and Dyn. of Galactic Disk



GAIA



Worked Example: Ultra-light Axion → “Fuzzy DM”

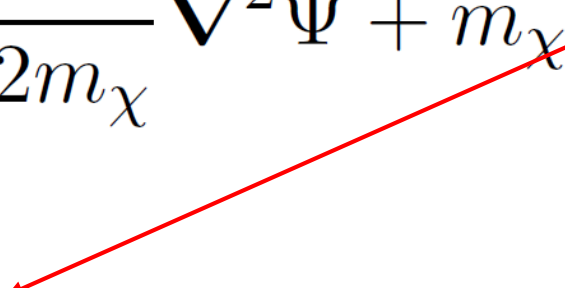
Tiny Mass \sim → **Astrophysical de Broglie wavelength**

$$\frac{\lambda}{2\pi} = \frac{\hbar}{mv} = 1.92 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m} \right) \left(\frac{10 \text{ km s}^{-1}}{v} \right)$$

- Large number of particles in same state and **non-relativistic on galactic scales**

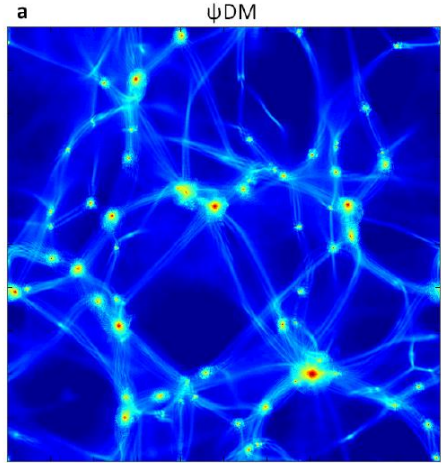
→ Schrodinger-Poisson system

$$i\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m_\chi} \nabla^2 \Psi + m_\chi V \Psi,$$

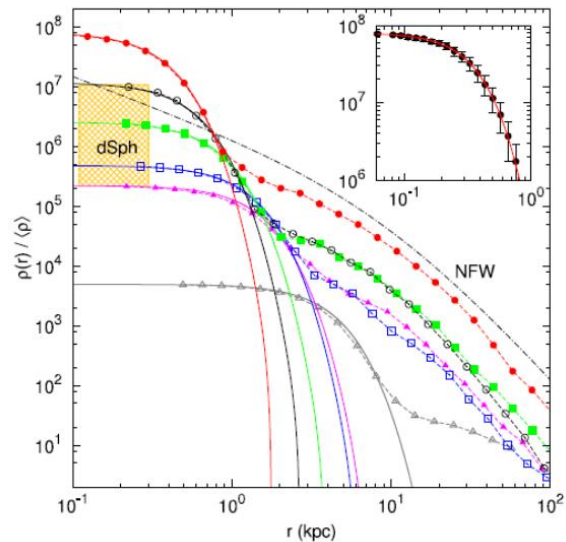
$$\nabla^2 V = 4\pi G m_\chi |\Psi|^2.$$


Structure Formation and fluctuations with fuzzy DM

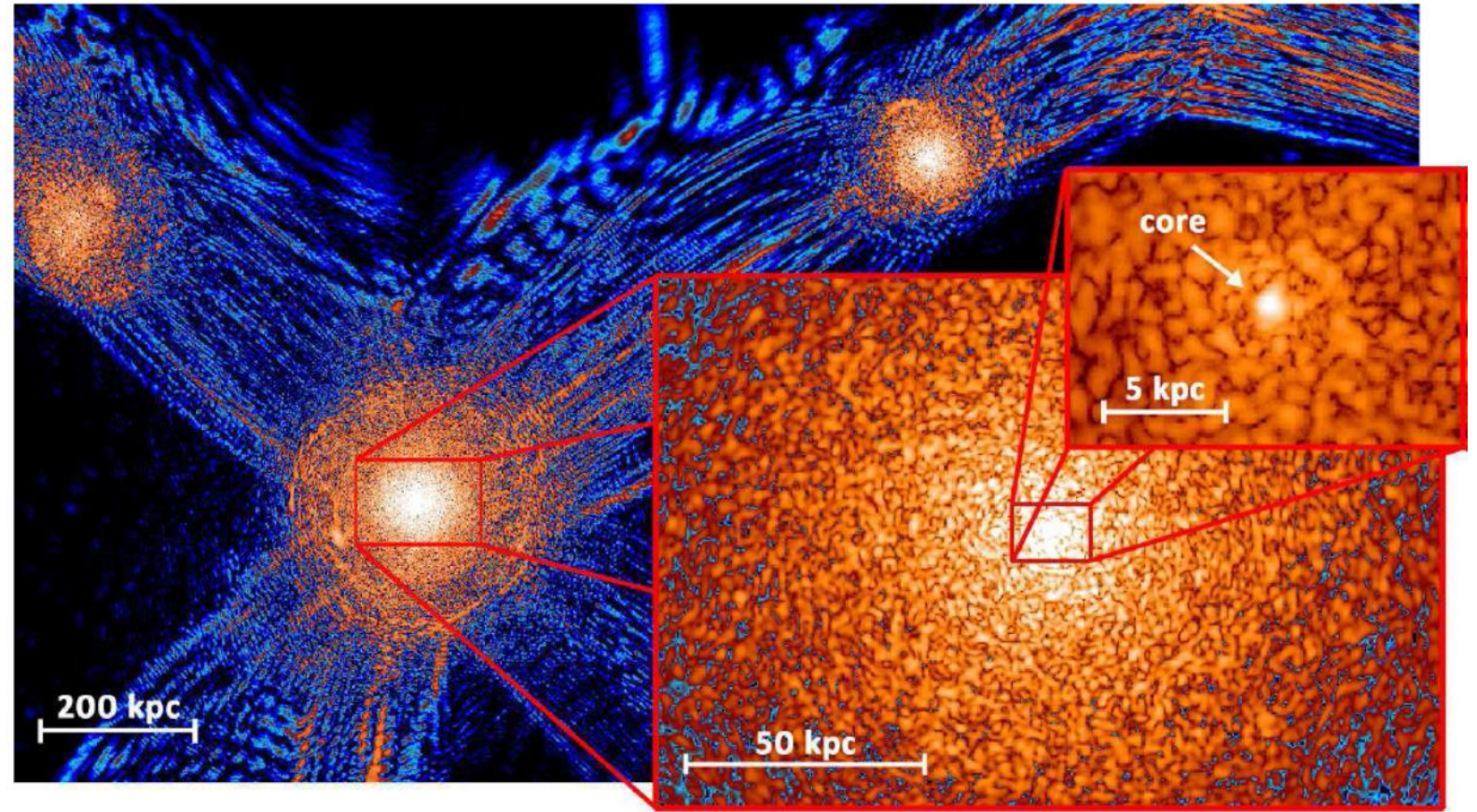
(Schive et. al. 2014)



~as CDM on large scales



~Constant density cores



Few smaller halos (note however interference pattern and fluctuations!)

Axion Fluctuations as Random Gaussian Field

Expand *fluctuations* in modes $\rho_{\mathbf{k}}$ moving at phase velocity \mathbf{v} such that $\mathbf{k} \cdot \mathbf{v} = \omega$

This is the case if

$$\phi_{\mathbf{k}}(t) = \phi_{\mathbf{k}}(0)e^{-i\mathbf{k} \cdot \mathbf{v}t} \quad \text{and} \quad \psi(\mathbf{r}, t) = \int \phi_{\mathbf{k}}(t)e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k}$$

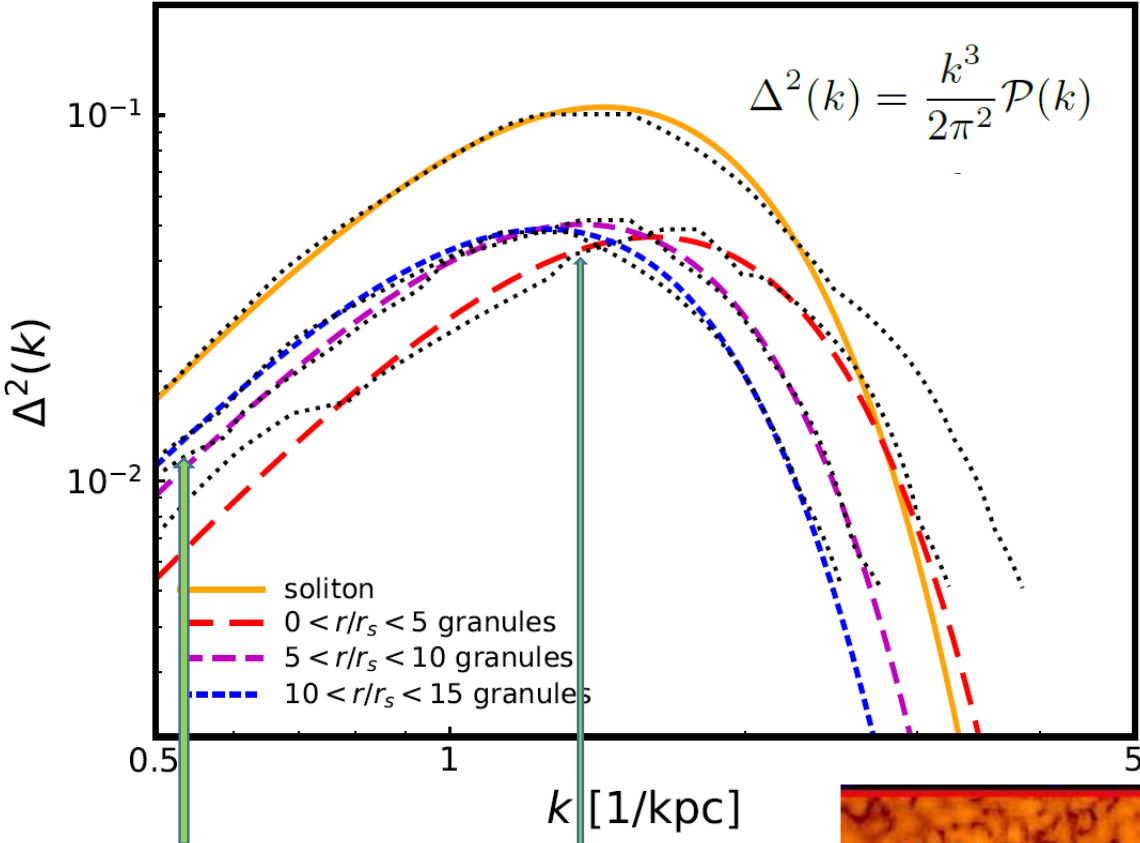
Wave function power spectrum \rightarrow k-space density $\rightarrow \langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'}^* \rangle = f_{\mathbf{k}}(\mathbf{k}) \delta_{\text{D}}(\mathbf{k} - \mathbf{k}')$

**Power spectrum of
density fluctuations
interference pattern \rightarrow**

$$\mathcal{P}(\mathbf{k}, t) = \frac{(2\pi)^3}{\rho_0^2} \times \iint f_{\mathbf{k}}(\mathbf{k}_1) f_{\mathbf{k}}(\mathbf{k}_2) e^{-i[\omega(\mathbf{k}_1) - \omega(\mathbf{k}_2)]t} \delta_{\text{D}}(\mathbf{k} - \mathbf{k}_1 + \mathbf{k}_2) d\mathbf{k}_1 d\mathbf{k}_2$$

Power Spectrum of Density Fluctuations

Interpretation and Comparison with simulations (of Chan et. al. 2018)



- Conservation of probability (number density)
- Correspondence of wavenumber and FDM vely distn function

$$f_{\mathbf{k}}(\mathbf{k})d\mathbf{k} = f(\mathbf{v})d\mathbf{v}$$

Maxwellian velys

$$f(v) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} e^{-\frac{v^2}{2\sigma^2}}$$

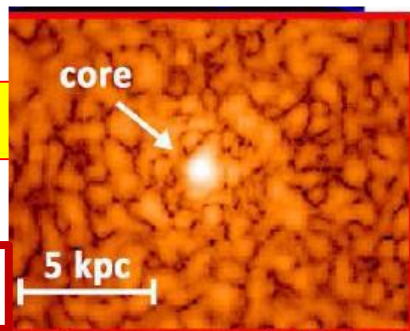
→ Power spectrum

$$\mathcal{P}(\mathbf{k}, 0) = \left(\frac{2\sqrt{\pi}}{m_{\hbar}\sigma} \right)^3 e^{-\frac{k^2}{\sigma^2 m_{\hbar}^2}}$$

$m_{\hbar} = 2m/\hbar$

White noise Effective fluctuation scale

~ Randomly scattered masses ~ m_{eff}



From Density to Force fluctuations

- Use Poisson equation

$$\nabla^2 \Phi = 4\pi G \rho_0 \delta$$

- Homogeneous process \rightarrow

$$\phi_{\mathbf{k}} = -4\pi G \rho_0 \delta_{\mathbf{k}} k^{-2}$$

- Force fluctuation power \rightarrow

$$\mathcal{P}_F(k) = V k^2 \langle |\phi_k|^2 \rangle$$

Fourier Transform \rightarrow Force Correlation Function

$$\langle \mathbf{F}(0, 0) \cdot \mathbf{F}(r, t) \rangle = \frac{1}{(2\pi)^3} \int \mathcal{P}_F(k, t) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k}$$

Stochastic equation \rightarrow Random velocity from fluctuations

$$d\mathbf{v}/dt = \mathbf{F} \quad \longrightarrow \quad \langle (\Delta v_p)^2 \rangle = 2 \int_0^T (T - t) \langle \mathbf{F}(0) \cdot \mathbf{F}(t) \rangle dt$$

Maxwellian \rightarrow

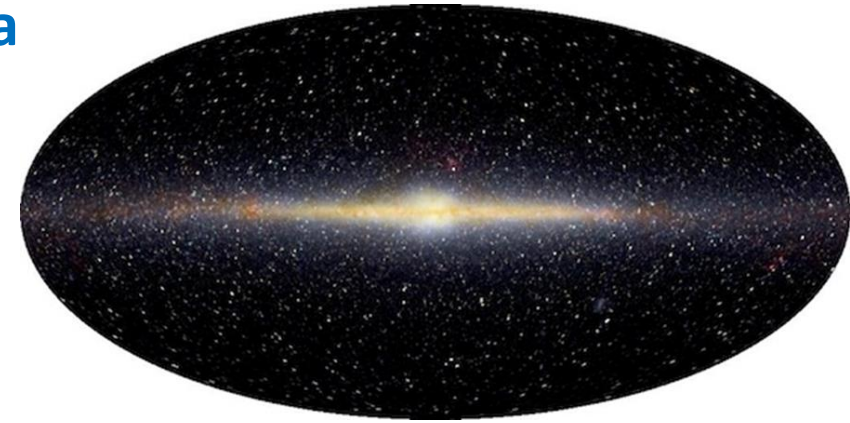
$$\langle (\Delta v_p)^2 \rangle = T \frac{8\pi G^2 \rho_0 m_{\text{eff}} \ln \Lambda}{v_p} \text{erf}(X_{\text{eff}}) \quad m_{\text{eff}} = \frac{8\pi^{3/2} \rho_0}{m_{\hbar}^3 \sigma^3}$$

Observable Effect: Galactic Disk Velocity Dispersion

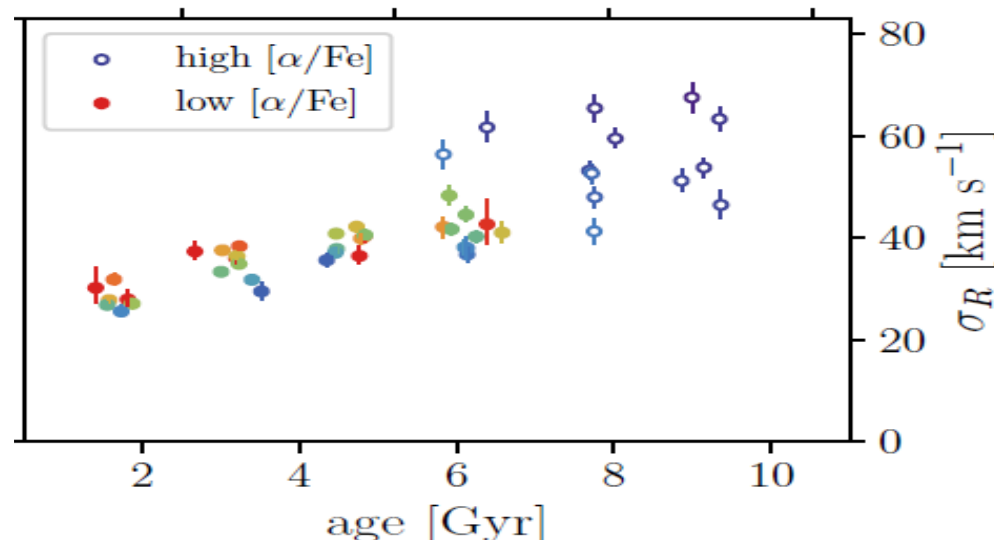
-- Decompose energy input to disk via fluctuations into vertical and radial components

→ **Prediction:** radial velocity dispersion of disk stars increases as

$$\sigma_R = 4.5 \text{ km/s} \left(\frac{10^{-22} \text{ eV}}{m} \right)^{3/2} \left(\frac{8 \text{ kpc}}{r} \right)^2 \left(\frac{T}{10 \text{ Gyr}} \right)^{1/2} \ln \Lambda^{1/2}$$



Observed dispersion does increase BUT as $\sigma_R \sim t^{1/3}$ → Axion fluctuation contribution

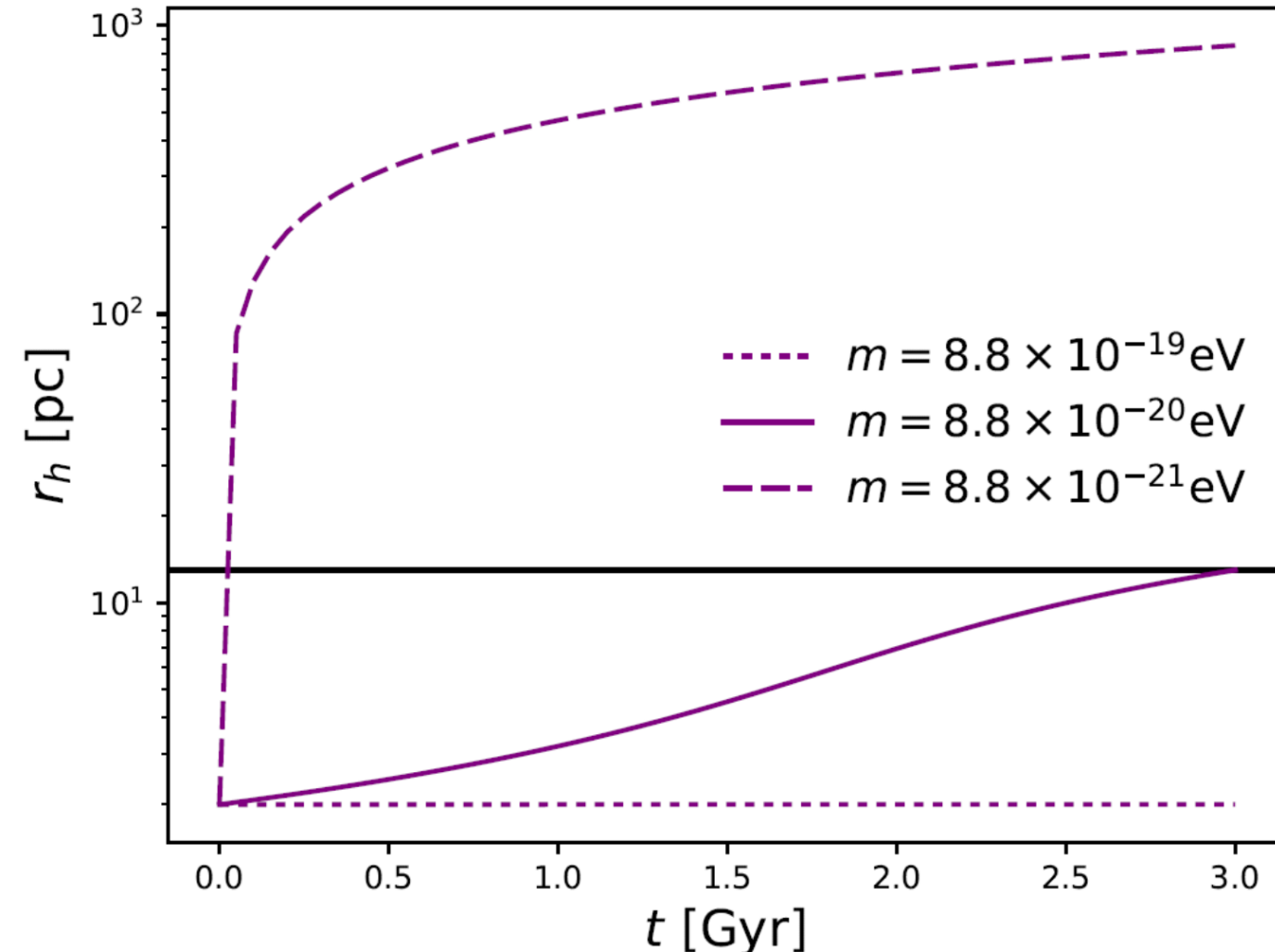


$$\sigma_R \sim 3 \text{ km/s}$$

→ **Limit on axion mass**

$$m \gtrsim 2 \times 10^{-22} \text{ eV}$$

Expansion of the Central Cluster of Dwarf Eridanus II



Basic idea (Marsh & Niemeyer 2019):

-- Fluctuations cause central cluster to expand

-- If axion mass too small \rightarrow cluster too large

\rightarrow quite severe constraints on axion mass

BUT does cluster expand or gets displaced?

$$\frac{dr_h}{dt} = \frac{D}{G} \left(\frac{\alpha M_\star}{r_h^2} + 2\beta\rho_0 r_h \right)^{-1}$$

$$D [(\Delta v)^2] = \frac{4\sqrt{2}\pi G^2 \rho_0 m_{\text{eff}}}{\sigma_{\text{eff}}} \ln \Lambda \left[\frac{\text{erf}(X_{\text{eff}})}{X_{\text{eff}}} \right]$$

Observable Effect: Central Black Hole Displacement

Equipartition of SMBH KE with FDM heat bath →

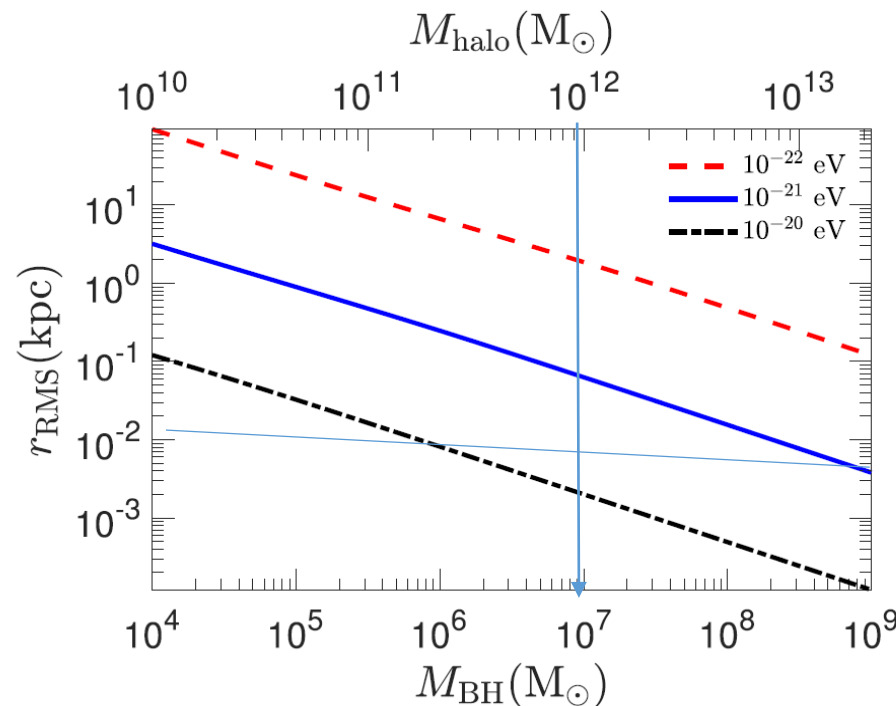
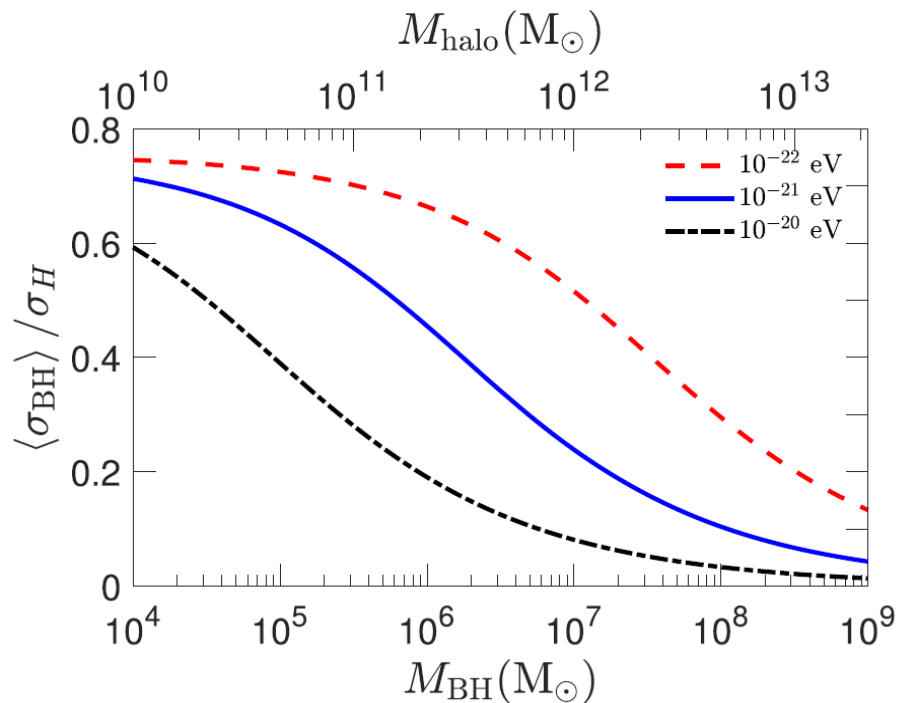
$$M_B \sigma_B^2 = m_{\text{eff}} \sigma_{\text{eff}}^2$$



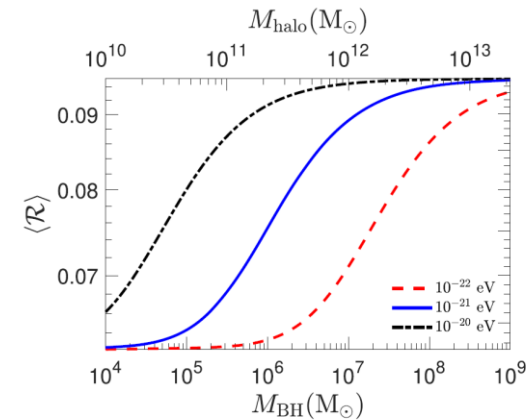
Virial theorem at low masses →

$$\sqrt{\langle r^2 \rangle} \approx 2.6 \text{ kpc} \left(\frac{m_{\text{ax}}}{10^{-22} \text{ eV}} \right)^{-\frac{3}{2}} \left(\frac{M_B}{10^7 M_\odot} \right)^{-\frac{11}{18}}$$

Averaging over thermal distribution



Reduction in merger rate

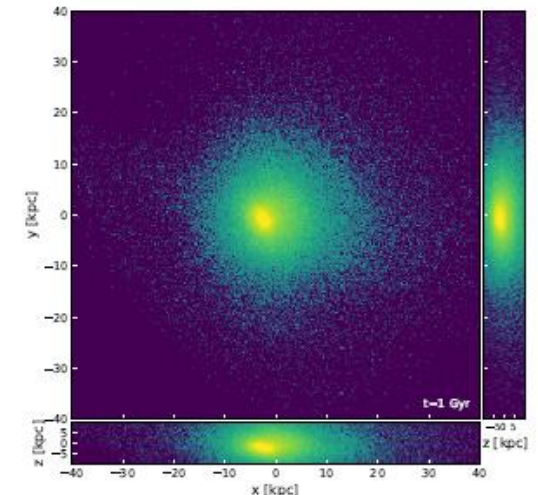
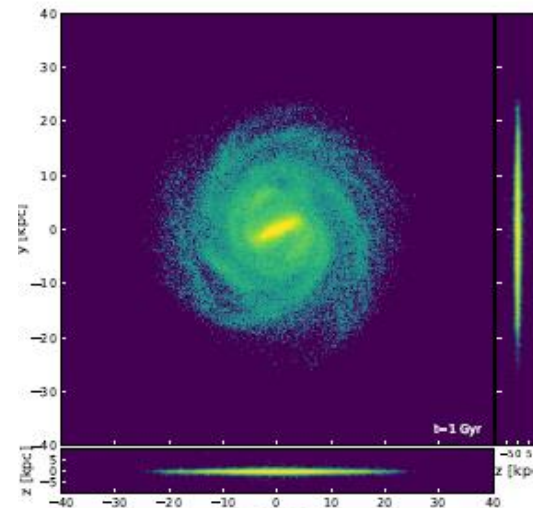


Conclusions and Prospects:

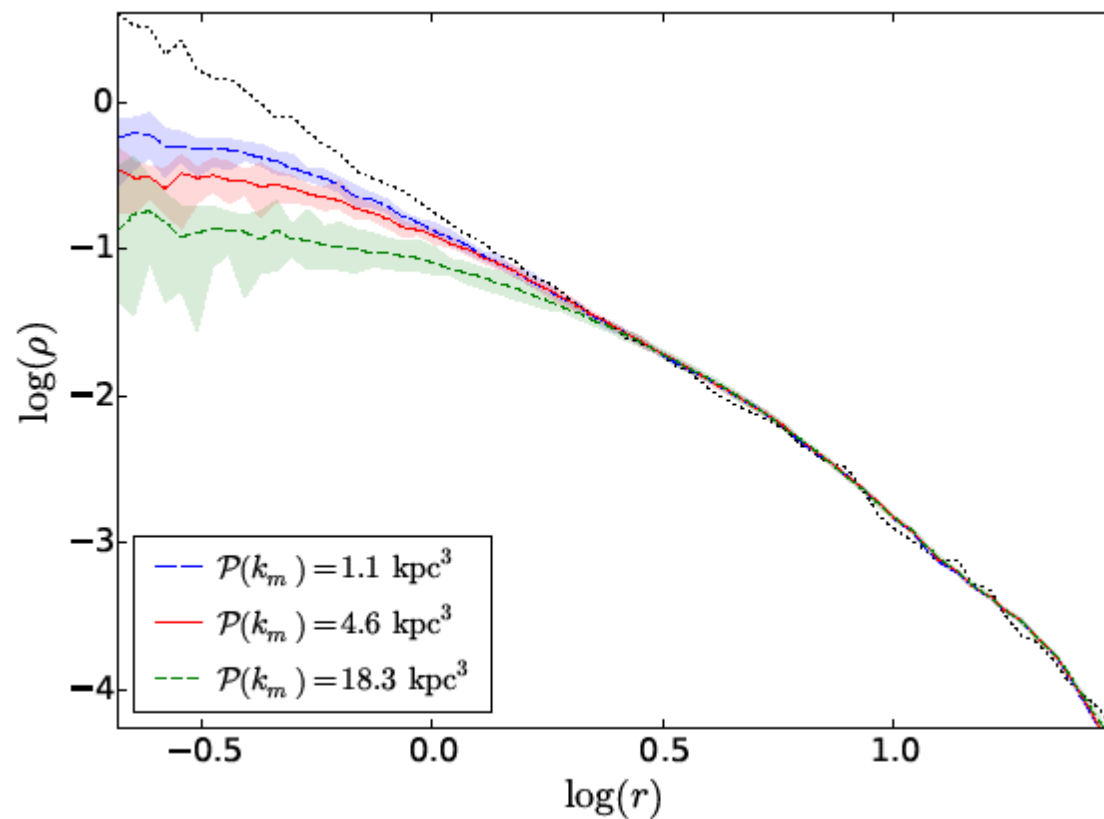
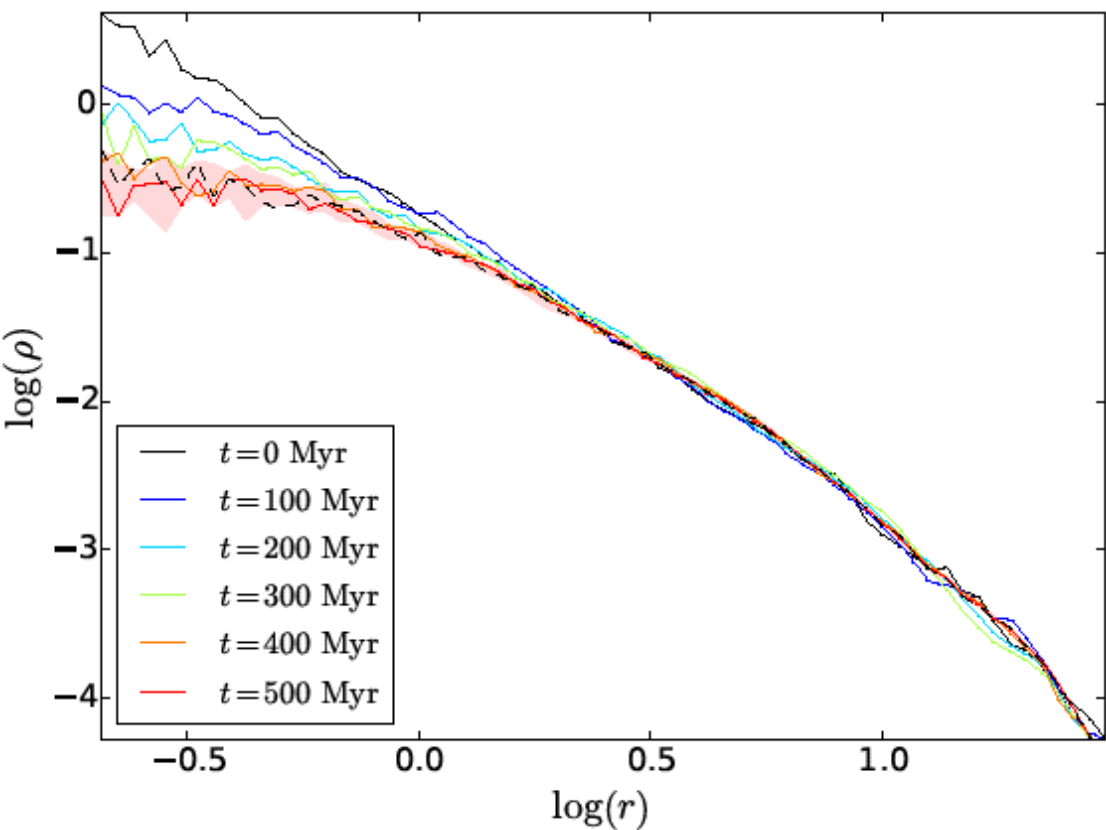
- **CDM Threatened: small scale problems part of a parcel of problems**
- **Alternatives can have observable consequences on galaxies**
- **FDM Alternative: Fluctuations from uncertainty principle \rightarrow 'hotter' DM**
- **But are fluctuations needed to solve core-cusp problem etc., too large?**

Ongoing and prospective work:

- ** **FDM Simulations of disks with FDM noise**
++ **Full S-P simulations**
- **Effect on tidal stream (already much work)**
- **FDM self interaction... baryons etc...**

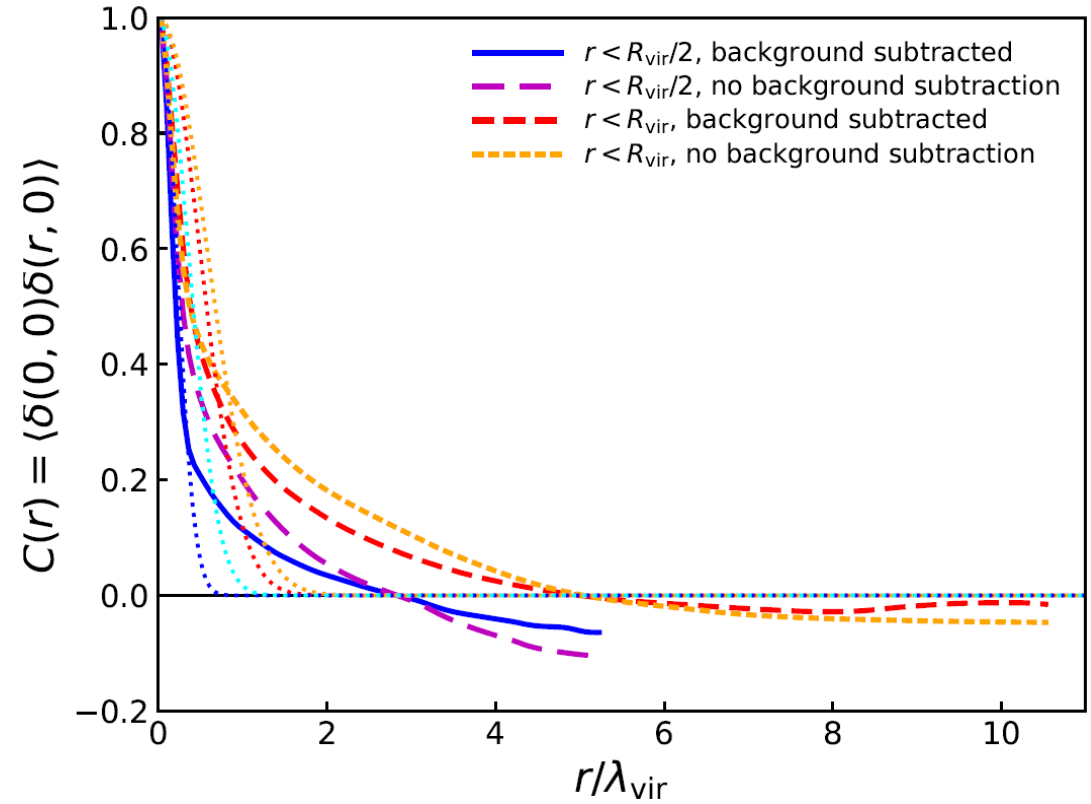
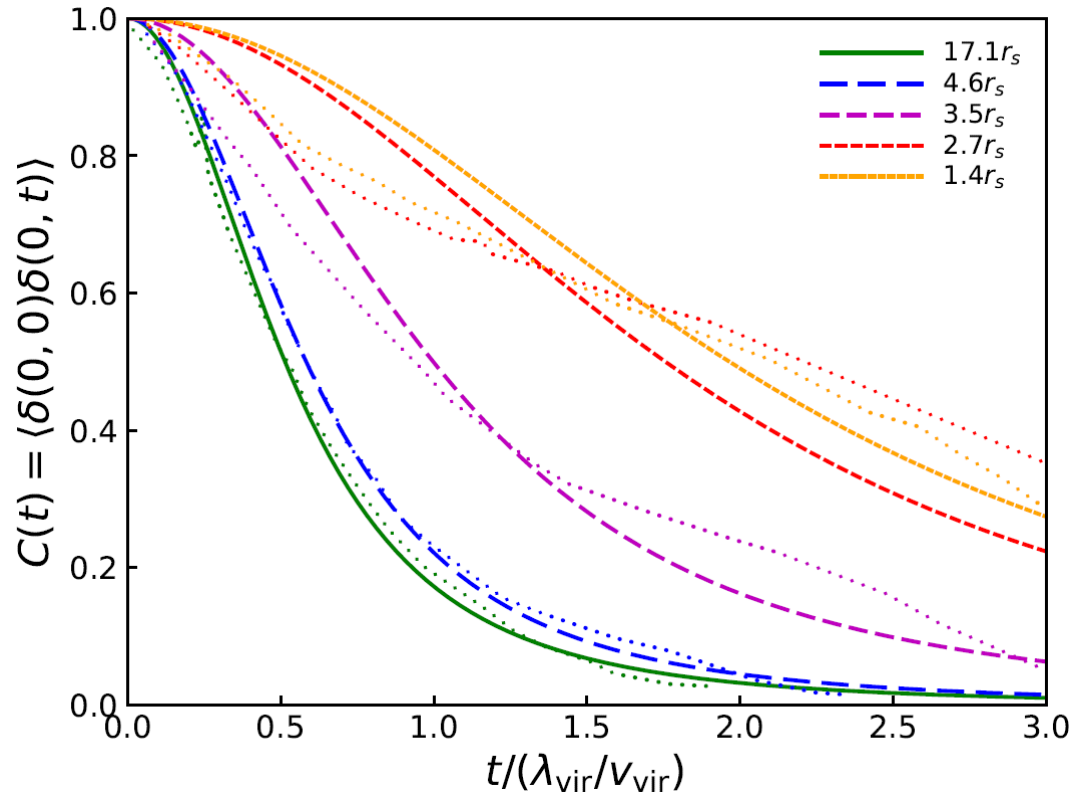


Effect of Non-radial Modes and Power Input



Same power input as previous case but evolution on a tenth of the timescale

Space and Time Correlations



$$\langle \delta(0, 0) \delta(r, t) \rangle = \frac{1}{(1 + \sigma^2 t^2 / \lambda_\sigma^2)^{3/2}} e^{-\frac{r^2 / \lambda_\sigma^2}{1 + \sigma^2 t^2 / \lambda_\sigma^2}}$$