Black Holes in teleparallel gravity

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September 2022, Workshop on Astro-particles and Gravity at Cairo University

Gefördert durch
Deutsche
Forschungsgemeinschaft



CENTER OF APPLIED SPACE TECHNOLOGY AND MICROGRAVITY

- Teleparallelism 1.
 - Teleparallel Geometry
 - Symmetry
 - Teleparallale Gravity
- 2. Black Holes in $f(T,B,\phi)$ teleparallel gravity
 - Born-Infeld f(T)-gravity
 - Teleparallel perturbations of GR
 - Scalar-Torsion gravity
- Conclusion and Outlook 3.













Geometric fields

Tetrad components $\theta^a_{\ \mu}(x)$





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- technically: basis 1-forms $\theta^a = \theta^a_{\ \mu} dx^{\mu}$
- practically: 16 field components $\theta^a_{\ \mu}$ with inverse $e_a^{\ \mu} = \theta^a_{\ \mu} e_a^{\ \nu} = \delta^{\nu}_{\mu}$, $\theta^a_{\ \nu} e_b^{\ \nu} = \delta^a_b$
- the metric is a derived object $g_{\mu\nu} = \eta_{ab} \theta^a{}_{\mu} \theta^b{}_{\nu}$, $\eta_{ab} = \text{diag}(-, +, +, +)$





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- tetrad basis representation $\Gamma^{\mu}_{\ \nu\rho} = e_a^{\ \mu} (\partial_{\rho} \theta^a_{\ \nu} + \omega^a_{\ b\rho} \theta^b_{\ \nu})$
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• Levi-Civita $R^{\rho}_{\sigma\mu\nu} \neq 0, \ Q_{\rho\mu\nu} = 0, \ T^{\rho}_{\mu\nu} = 0 \Rightarrow \Gamma^{\sigma}_{\mu\nu} = \frac{1}{2}g^{\rho\lambda}\left(\partial_{\mu}g_{\lambda\nu} + \partial_{\nu}g_{\lambda\mu} - \partial_{\lambda}g_{\mu\nu}\right)$

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Torsion (non closing) $T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$ $T^{\sigma}_{\ \mu\nu} = 0$





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$$Q_{\rho\mu\nu} = \nabla_{\rho\sigma}$$

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• Teleparallel

Non-Metrici



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$$\delta^{\nu}_{\mu}, \, \theta^{a}_{\ \nu} e_{b}^{\ \nu} = \delta^{a}_{b}$$
$$+ \, , + \,)$$

$$R^{\rho}_{\sigma\mu\nu} = 0, \ Q_{\rho\mu\nu} = 0, \ T^{\rho}_{\mu\nu} \neq 0 \Rightarrow \omega^{a}_{\ b\mu} = \Lambda^{a}_{\ b}\partial_{\mu}(\Lambda^{-1})^{c}_{\ b} \quad \eta_{ab} = \eta_{cd}\Lambda^{c}_{\ a}\Lambda^{d}_{\ b}$$

Ity (changing lengths)

$$T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\ \nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

$$\int_{\nu} T^{\sigma}_{\mu\nu} \neq 0$$

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Teleparallel Geometry - Symmetry - Teleparallel Gravity [Ferraro 2007; Pereira 2013; Krššák 2016; MH, CP 2018]

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Geometric fields

Lorentz transformations are gauge transformations



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• Affine connection and metric invariant under transformations







Geometric fields

Lorentz transformations are gauge transformations

• Affine connection and metric invariant under transformations

$$\theta^{a}{}_{\mu} \mapsto \hat{\theta}^{a}{}_{\mu} = \theta^{b}{}_{\mu} (\Lambda^{-1})^{a}{}_{b}, \quad e_{a}{}^{\mu} \mapsto \hat{e}_{a}{}^{\mu} = e_{b}{}^{\mu}\Lambda^{b}{}_{a}, \quad \hat{\Lambda}^{a}{}_{b} \mapsto \tilde{\Lambda}^{a}{}_{b} = \Lambda^{a}{}_{c}\hat{\Lambda}^{c}{}_{b}, \quad \Lambda^{a}{}_{b}, \hat{\Lambda}^{a}{}_{b} \in SO(1,3)$$





Geometric fields

Lorentz transformations are gauge transformations

• Affine connection and metric invariant under transformations

$$\begin{aligned} \theta^{a}{}_{\mu} &\mapsto \hat{\theta}^{a}{}_{\mu} = \theta^{b}{}_{\mu} (\Lambda^{-1})^{a}{}_{b}, \quad e_{a}{}^{\mu} \mapsto \hat{e}_{a}{}^{\mu} = e_{b}{}^{\mu} \Lambda^{b}{}_{a}, \quad \hat{\Lambda}^{a}{}_{b} \mapsto \tilde{\Lambda}^{a}{}_{b} = \Lambda^{a}{}_{c} \hat{\Lambda}^{c}{}_{b}, \quad \Lambda^{a}{}_{b}, \hat{\Lambda}^{a}{}_{b} \in SO(1,3) \\ g_{\mu\nu}[\hat{\theta}] = g_{\mu\nu}[\theta], \quad \Gamma^{\mu}{}_{\nu\rho}[\hat{\theta}, \hat{\Lambda}] = \Gamma^{\mu}{}_{\nu\rho}[\theta, \tilde{\Lambda}], \quad T^{\rho}{}_{\mu\nu}[\hat{\theta}, \hat{\Lambda}] = T^{\rho}{}_{\mu\nu}[\theta, \tilde{\Lambda}] \end{aligned}$$





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• Choose $\Lambda^a{}_b = (\hat{\Lambda}^{-1})^a{}_b \Rightarrow \tilde{\Lambda}^a{}_b = \delta^a_b$ $\Rightarrow \text{Affine connection: } \Gamma^{\mu}{}_{\nu\rho}[\theta^{a}{}_{\mu},\delta^{a}_{b}] = e_{a}{}^{\mu}\partial_{\rho}\theta^{a}{}_{\nu}, \quad \text{Torsion: } T^{\rho}{}_{\mu\nu}[\theta^{a}{}_{\mu},\delta^{a}_{b}] = 2e_{c}{}^{\rho}\partial_{[\mu}\theta^{c}{}_{\nu]}.$





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 \Rightarrow Affine connection: $\Gamma^{\mu}{}_{\nu\rho}[\theta^{a}{}_{\mu},\delta^{a}_{b}] = e_{a}{}^{\mu}\partial_{\rho}\theta^{a}{}_{\nu},$ Torsion: $T^{\rho}{}_{\mu\nu}[\theta^{a}{}_{\mu},\delta^{a}_{b}] = 2e_{a}$

Without loss of generality one can always work with the pair $(\theta^a{}_{\mu}, \delta^a_{h})$, called **Weitzenböck** gauge.

$$\partial_c^{\rho}\partial_{[\mu}\theta^c_{\nu]}.$$





$$\theta^{a}_{\ \mu}(x), \Lambda^{a}_{\ b}(x) \Rightarrow \omega^{a}_{\ b\mu} \text{ and } \Gamma^{\rho}_{\ \mu\nu}, \text{ Torsion } T^{\sigma}_{\ \mu\nu} = \Gamma^{\sigma}_{\ \nu\mu} - \Gamma^{\sigma}_{\ \mu\nu}$$

Weitz
$$(\theta^a)$$

Symmetries on a manifold M

Weitzenböck gauge $(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$





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- ϕ is an action of a Lie Group G as diffeomorphism from M to M
- ϕ is a symmetry of $(M, g, \Gamma) \Leftrightarrow g = \phi^* g, \Gamma = \phi^* \Gamma$
- infinitesimally ϕ is encoded in a vector field $X = X^{\sigma} \partial_{\sigma}$





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Killing equations: φ is a symmetry iff

$$\mathscr{L}_X g)_{\mu\nu} = \nabla_\mu X_\nu + \nabla_\nu X_\mu = 0$$

$$\mathscr{L}_X \Gamma)^{\mu}{}_{\nu\rho} = \nabla_{\rho} \nabla_{\nu} X^{\mu} - X^{\sigma} R^{\mu}{}_{\nu\rho\sigma} - \nabla_{\rho} (X^{\sigma} T^{\mu}{}_{\nu\sigma}) = 0$$

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Teleparallel Killing equations:

 $(\mathscr{L}_X\theta)^a{}_\mu = -\lambda^a{}_b\theta^b{}_\mu$ $(\mathscr{L}_X T)^{\nu}{}_{\mu\nu} = 0$ $(\mathscr{L}_X \omega)^a{}_{b\mu} = \partial_\mu \lambda^a{}_b = 0$









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 (θ^{\prime})

Weitzenböck gauge

$$(a_{\mu}^{a}, \delta_{b}^{a}) \Rightarrow \omega^{a}{}_{b\mu} = 0, \ \Gamma^{\mu}{}_{\nu\rho} = e_{a}{}^{\mu}\partial_{\rho}\theta^{a}{}_{\nu}$$

Teleparallel Killing equations:

$$(\mathscr{L}_X \theta)^a{}_{\mu} = -\lambda^a{}_b \theta^b{}_{\mu}, \quad (\mathscr{L}_X \omega)^a{}_{b\mu} = \partial_{\mu} \lambda^a$$





$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

 (θ^{\prime})

Symmetry algebra

•
$$G = SO(3), \mathfrak{g} = \mathfrak{so}(3)$$

•
$$X_z = \partial_{\phi}$$
,

•
$$X_y = -\cos \varphi \partial_{\vartheta} + \frac{\sin \varphi}{\tan \vartheta} \partial_{\varphi}$$

• $X_x = \sin \varphi \partial_{\vartheta} + \frac{\cos \varphi}{\tan \vartheta} \partial_{\varphi}$

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Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

Symmetry algebra

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$$G = SO(3), \mathfrak{g} = \mathfrak{so}(3)$$

• $X_z = \partial_{\phi},$
• $X_y = -\cos \varphi \partial_{\vartheta} + \frac{\sin \varphi}{\tan \vartheta} \partial_{\varphi}$
• $\lambda : X_z \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
• $X_x = \sin \varphi \partial_{\vartheta} + \frac{\cos \varphi}{\tan \vartheta} \partial_{\varphi}$

Weitzenböck gauge $(\theta^{a}_{\mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu}$

Teleparallel Killing equations:

 $(\mathscr{L}_X \theta)^a_{\ \mu} = -\lambda^a_{\ b} \theta^b_{\ \mu}, \quad (\mathscr{L}_X \omega)^a_{\ b\mu} = \partial_\mu \lambda^a_{\ b} = 0$







Geometric fields

$$\theta^{a}_{\ \mu}(x), \Lambda^{a}_{\ b}(x) \Rightarrow \omega^{a}_{\ b\mu} \text{ and } \Gamma^{\rho}_{\ \mu\nu}, \text{ Torsion } T^{\sigma}_{\ \mu\nu} = \Gamma^{\sigma}_{\ \nu\mu} - \Gamma^{\sigma}_{\ \mu\nu}$$

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• $X_x = \sin\varphi\partial_{\vartheta} + \frac{\cos\varphi}{\tan\vartheta}\partial_{\varphi}$

The general spherically symmetric tetrad, $C_i = C_i(t, r)$

$$\theta^{a}{}_{\mu} = \begin{pmatrix} C_{1} & C_{2} \\ C_{3}\sin\vartheta\cos\varphi & C_{4}\sin\vartheta\cos\varphi & C_{5}\cos\varphi \\ C_{3}\sin\vartheta\sin\varphi & C_{4}\sin\vartheta\sin\varphi & C_{5}\cos\varphi \\ C_{3}\cos\vartheta & C_{4}\cos\vartheta \\ \end{pmatrix}$$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu}$$

Teleparallel Killing equations: $(\mathscr{L}_X \theta)^a_{\ \mu} = -\lambda^a_{\ b} \theta^b_{\ \mu}, \quad (\mathscr{L}_X \omega)^a_{\ b\mu} = \partial_\mu \lambda^a_{\ b} = 0$

0 0 $s \vartheta \cos \varphi - C_6 \sin \varphi - \sin \vartheta (C_5 \sin \varphi + C_6 \cos \vartheta \cos \varphi)$ $s \vartheta \sin \varphi + C_6 \cos \varphi = \sin \vartheta (C_5 \cos \varphi - C_6 \cos \vartheta \sin \varphi)$ $-C_5 \sin \vartheta$ $C_6 \sin^2 \vartheta$







Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

Symmetry algebra

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The general spherically symmetric tetrad, $C_i = C_i(t, r)$

$$\theta^{a}{}_{\mu} = \begin{pmatrix} C_{1} & C_{2} & 0 & 0 \\ C_{3}\sin\vartheta\cos\varphi & C_{4}\sin\vartheta\cos\varphi & C_{5}\cos\vartheta\cos\varphi - C_{6}\sin\varphi & -\sin\vartheta(C_{5}\sin\varphi + C_{6}\cos\vartheta\cos\varphi) \\ C_{3}\sin\vartheta\sin\varphi & C_{4}\sin\vartheta\sin\varphi & C_{5}\cos\vartheta\sin\varphi + C_{6}\cos\varphi & \sin\vartheta(C_{5}\cos\varphi - C_{6}\cos\vartheta\sin\varphi) \\ C_{3}\cos\vartheta & C_{4}\cos\vartheta & -C_{5}\sin\vartheta & C_{6}\sin^{2}\vartheta \end{pmatrix}$$

The standard spherically symmetric metric

$$g = (C_1^2 - C_3^2)dt^2 - (C_4^2 - C_4^2)dt^2 - C_4^2 - C_4^$$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu}$$

Teleparallel Killing equations: $(\mathscr{L}_X \theta)^a_{\ \mu} = -\lambda^a_{\ b} \theta^b_{\ \mu}, \quad (\mathscr{L}_X \omega)^a_{\ b\mu} = \partial_\mu \lambda^a_{\ b} = 0$

 $C_2^2)dr^2 - (C_5^2 + C_6^2)(d\vartheta^2 + sin^2\vartheta d\varphi^2) - (C_3C_4 - C_1C_2)dtdr$







Geometric fields

 $\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$

Weitze
$$(\theta^a_{\ \mu}, \delta$$

enböck gauge

$$\delta^a_b) \Rightarrow \omega^a{}_{b\mu} = 0, \ \Gamma^\mu{}_{\nu\rho} = e_a{}^\mu \partial_\rho \theta^a{}_{\nu},$$

Teleparallel Killing equations:

$$(\mathscr{L}_X \theta)^a{}_{\mu} = -\lambda^a{}_b \theta^b{}_{\mu}, \quad (\mathscr{L}_X \omega)^a{}_{b\mu} = \partial_{\mu} \lambda^a$$






Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

$$\mathbf{\mathfrak{v}}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathbf{\mathfrak{a}}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathbf{\mathfrak{t}}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu}\mathbf{\mathfrak{v}}_{\nu)} - g_{\nu\mu}\mathbf{\mathfrak{v}}_{\rho})$$

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Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$



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$$T_{\text{vec}} = \mathbf{\mathfrak{v}}_{\mu} \mathbf{\mathfrak{v}}^{\mu}, \quad T_{\text{ax}} = \mathbf{\mathfrak{a}}_{\mu} \mathbf{\mathfrak{a}}^{\mu}, \quad T_{\text{ten}} = \mathbf{t}_{\mu\nu\rho} \mathbf{t}^{\mu\nu\rho}$$

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$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

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Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$



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$$T = -\frac{2}{3}T_{\text{vec}} + \frac{3}{2}T_{\text{ax}} + \frac{2}{3}T_{\text{ten}} = -\mathring{R} + \mathring{\nabla}_{\mu}(2T_{\sigma}^{\ \sigma\mu})$$







Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$



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Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

The torsion scalars

$$\mathbf{\mathfrak{b}}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathbf{\mathfrak{a}}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathbf{\mathfrak{t}}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu} \mathbf{\mathfrak{b}}_{\nu)} - g_{\nu\mu} \mathbf{\mathfrak{b}}_{\rho})$$
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Teleparallel theories of gravity $S[\theta] = \int d^4x |\theta| f(T^{\sigma}_{\mu\nu}, \partial T^{\sigma}_{\mu\nu}, \dots)$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

Teleparallel Killing equations:

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Geometric fields

$$\theta^{a}_{\ \mu}(x), \Lambda^{a}_{\ b}(x) \Rightarrow \omega^{a}_{\ b\mu} \text{ and } \Gamma^{\rho}_{\ \mu\nu}, \text{ Torsion } T^{\sigma}_{\ \mu\nu} = \Gamma^{\sigma}_{\ \nu\mu} - \Gamma^{\sigma}_{\ \mu\nu}$$

The torsion scalars

$$\mathfrak{v}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathfrak{a}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathfrak{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu} \mathfrak{v}_{\nu)} - g_{\nu\mu} \mathfrak{v}_{\rho})$$
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Teleparallel theories of gravity $S[\theta] = \int d^4x |\theta| f(T^{\sigma}_{\mu\nu}, \partial T^{\sigma}_{\mu\nu}, \dots)$

• No extra derivative terms \Rightarrow Field eq. second order

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Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

The torsion scalars

$$\mathbf{\mathfrak{b}}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathbf{\mathfrak{a}}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathbf{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu} \mathbf{\mathfrak{b}}_{\nu)} - g_{\nu\mu} \mathbf{\mathfrak{b}}_{\rho})$$
$$T_{\text{vec}} = \mathbf{\mathfrak{b}}_{\mu} \mathbf{\mathfrak{b}}^{\mu}, \quad T_{\text{ax}} = \mathbf{\mathfrak{a}}_{\mu} \mathbf{\mathfrak{a}}^{\mu}, \quad T_{\text{ten}} = \mathbf{t}_{\mu\nu\rho} \mathbf{t}^{\mu\nu\rho}$$

Teleparallel theories of gravity $S[\theta] = \int d^4x |\theta| f(T^{\sigma}_{\mu\nu}, \partial T^{\sigma}_{\mu\nu}, \dots)$

- No extra derivative terms \Rightarrow Field eq. second order
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Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

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Geometric fields

$$\vartheta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

The torsion scalars

$$\mathbf{\mathfrak{v}}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathbf{\mathfrak{a}}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathbf{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu} \mathbf{\mathfrak{v}}_{\nu)} - g_{\nu\mu} \mathbf{\mathfrak{v}}_{\rho})$$
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$$E_{\mu\nu} = e^a{}_\nu g_{\mu\sigma} E_a{}^\sigma = \kappa \theta \Theta_{\mu\nu}$$

$$E_{(\mu\nu)} = e^a{}_{(\nu}g_{\mu)\sigma}E_a{}^{\sigma} = \kappa\Theta_{(\mu\nu)} \qquad E_{[\mu\nu]} = e^a{}_{[\nu}g_{\mu]\sigma}E_a{}^{\sigma} = \kappa\Theta_{[\mu\nu]}$$

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Geometric fields

$$\vartheta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

The torsion scalars

$$\mathbf{\mathfrak{v}}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathbf{\mathfrak{a}}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathbf{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu} \mathbf{\mathfrak{v}}_{\nu)} - g_{\nu\mu} \mathbf{\mathfrak{v}}_{\rho})$$
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$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

Teleparallel Killing equations:

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and the torsion scalar

$$T = -\frac{2}{3}T_{\text{vec}} + \frac{3}{2}T_{\text{ax}} + \frac{2}{3}T_{\text{ten}} = -\mathring{R} + \underbrace{\mathring{\nabla}_{\mu}(2T_{\sigma}^{\ \sigma\mu})}_{B}$$

Most prominent theories in the literature







Geometric fields

$$\vartheta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

The torsion scalars

$$\mathbf{\mathfrak{v}}_{\mu} = T^{\rho}{}_{\rho\mu}, \quad \mathbf{\mathfrak{a}}_{\mu} = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}, \quad \mathbf{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3} (g_{\rho(\mu} \mathbf{\mathfrak{v}}_{\nu)} - g_{\nu\mu} \mathbf{\mathfrak{v}}_{\rho})$$
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Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

Teleparallel Killing equations:

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$$\mathbb{T} = -\frac{2}{3}T_{\text{vec}} + \frac{3}{2}T_{\text{ax}} + \frac{2}{3}T_{\text{ten}} = -\overset{\circ}{R} + \underbrace{\overset{\circ}{\nabla}_{\mu}(2T_{\sigma}^{\sigma\mu})}_{B}$$

Most prominent theories in the literature

• Teleparallel equivalent of general relativity TEGR $S[\theta] = \int d^4x \, |\theta| \, T = \int d^4x \, |\theta| \, (-\mathring{R} + B)$







Geometric fields

$$\vartheta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

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Teleparallel theories of gravity $S[\theta] = \begin{bmatrix} d^4x | \theta | f(T^{\sigma}_{\mu\nu}, \partial T^{\sigma}_{\mu\nu}, \dots) \end{bmatrix}$

- No extra derivative terms \Rightarrow Field eq. second order
- Field equations decay into symmetric and antisymmetric parts

$$E_{\mu\nu} = e^a{}_\nu g_{\mu\sigma} E_a{}^\sigma = \kappa \theta \Theta_{\mu\nu}$$

$$E_{(\mu\nu)} = e^a{}_{(\nu}g_{\mu)\sigma}E_a{}^{\sigma} = \kappa\Theta_{(\mu\nu)} \qquad E_{[\mu\nu]} = e^a{}_{[\nu}g_{\mu]\sigma}E_a{}^{\sigma} = \kappa\Theta_{[\mu\nu]}$$

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Teleparallel Killing equations:

$$(\mathscr{L}_X \theta)^a{}_{\mu} = -\lambda^a{}_b \theta^b{}_{\mu}, \quad (\mathscr{L}_X \omega)^a{}_{b\mu} = \partial_{\mu} \lambda^a{}_{\mu}$$

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- Teleparallelism 1.
 - Teleparallel Geometry
 - Symmetry
 - Teleparallale Gravity
- 2. Black Holes in $f(T,B,\phi)$ teleparallel gravity
 - Born-Infeld f(T)-gravity
 - Teleparallel perturbations of GR
 - Scalar-Torsion gravity
- Conclusion and Outlook 3.





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Weitze
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The theory

C. Pfeifer, ZARM, Workshop on Astro-particles and Gravity at Cairo University 2022

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$$f = \hat{\lambda} \left(\sqrt{1 + \frac{T}{\hat{\lambda}}} - 1 \right)$$

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Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$







Geometric fields

 $\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$

The theory

$$f = \hat{\lambda} \left(\sqrt{1 + \frac{T}{\hat{\lambda}}} - 1 \right)$$

• Spherically symmetric tetrads solving the antisymmetric field equations

$$\begin{aligned} & \text{Real} \\ \theta_{1\mu}^{a} = \begin{pmatrix} \mathscr{A}(r) & 0 & 0 & 0 \\ 0 & \mathscr{B}(r)\sin\vartheta\cos\varphi & \xi r\cos\vartheta\cos\varphi & -r\xi\sin\vartheta\sin\varphi \\ 0 & \mathscr{B}(r)\sin\vartheta\sin\varphi & \xi r\cos\vartheta\sin\varphi & \xi r\sin\vartheta\cos\varphi \\ 0 & \mathscr{B}(r)\cos\vartheta & -r\xi\sin\vartheta & 0 \end{pmatrix} & \xi = \pm i \\ \\ & \text{Complex} \\ \theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} & \chi = \pm i \\ \end{aligned}$$
With metric
$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

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Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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• Spherically symmetric tetrads solving the antisymmetric field equations
$$\theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\varphi\cos\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1$$

With metric
$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

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Weitzenböck gauge $(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$

f(T,B, ϕ) gravity

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$

e solution







Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

The theory

$$f = \hat{\lambda} \left(\sqrt{1 + \frac{T}{\hat{\lambda}}} - 1 \right)$$

• Spherically symmetric tetrads solving the antisymmetric field equations

Non-perturbative solution

$$\mathscr{A}(r)^{2} = 1 - \frac{2M}{r} - \frac{2M}{r\lambda} \mathscr{T}, \, \mathscr{B}(r)^{2} = \frac{r^{4}\lambda^{4}}{16M^{4}\mathscr{A}(r)^{2}} \left(1 + \frac{\lambda^{2}r^{2}}{4M^{2}}\right)^{-2}$$
$$\mathscr{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \qquad \lambda = M\sqrt{\lambda}$$

$$\begin{aligned} & \text{Complex} \\ \theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1 \end{aligned}$$
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$$& ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2} \end{aligned}$$

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$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$







Geometric fields

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• Spherically symmetric tetrads solving the antisymmetric field equations

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$$\mathscr{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \qquad \lambda = M\sqrt{\lambda}$$

• Schwarzschild limit for $\lambda \to \infty$

$$\begin{aligned} & \text{Complex} \\ \theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm \end{aligned}$$
With metric
$$& ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2} \end{aligned}$$

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Weitzenböck gauge $(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$

f(T,B, ϕ) gravity $d^4x \,|\, \theta \,| f(T, B, \phi)$ $S[\theta] =$

e solution







Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

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• Spherically symmetric tetrads solving the antisymmetric field equations

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$$\mathscr{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \qquad \lambda = M\sqrt{\lambda}$$

- •

$$\gamma = -1,$$

$$\begin{aligned} & \theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1 \end{aligned}$$
With metric

$$ds^2 = -\mathscr{A}^2 dt^2 + \mathscr{B}^2 dr^2 + r^2 d\Omega^2$$

Weitzenböck gauge $(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$

e solution

Schwarzschild limit for $\lambda \to \infty$

• weak field expansion PPN parameters

$$\beta - 1 = \frac{8}{(2\lambda - \pi)^2}$$
$$\Rightarrow \lambda \gtrsim 140$$

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$







Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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With metric
$$ds^2 = -\mathscr{A}^2 dt^2 + \mathscr{B}^2 dr^2 + r^2 d\Omega^2$$

Non-perturbative solution

$$\mathcal{A}(r)^{2} = 1 - \frac{2M}{r} - \frac{2M}{r\lambda} \mathcal{T}, \ \mathcal{B}(r)^{2} = \frac{r^{4}\lambda^{4}}{16M^{4}d(r)^{2}} \left(1 + \frac{\lambda^{2}r^{2}}{4M^{2}}\right)^{-2}$$

$$\mathcal{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \quad \lambda = M\sqrt{\lambda}$$
• Schwarzschild limit for $\lambda \to \infty$
• weak field expansion PPN parameters
$$\gamma = -1, \quad \beta - 1 = \frac{8}{(2\lambda - \pi)^{2}}$$

$$\Rightarrow \lambda \gtrsim 140$$
• marginally stable and marginally bound orbits
$$q = -1, \quad \beta - 1 = \frac{8}{(2\lambda - \pi)^{2}}$$

$$\gamma = -1,$$

f(T,B, ϕ) gravity

 $S[\theta] =$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$









Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

The theory

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• Spherically symmetric tetrads solving the antisymmetric field equations

$$\mathcal{O}_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0\\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi\\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi\\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix}$$

With metric

$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

Non-perturbative solution

$$\mathscr{A}(r)^{2} = 1 - \frac{2M}{r} - \frac{2M}{r\lambda} \mathscr{T}, \, \mathscr{B}(r)^{2} = \frac{r^{4}\lambda^{4}}{16M^{4}\mathscr{A}(r)^{2}} \left(1 + \frac{\lambda^{2}r^{2}}{4M^{2}}\right)^{-2}$$
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- Schwarzschild limit for $\lambda \to \infty$

$$\gamma = -1,$$

- orbits
- •

 $\chi = \pm 1$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

• weak field expansion PPN parameters

$$\beta - 1 = \frac{8}{(2\lambda - \pi)^2}$$

 $\Rightarrow \lambda \gtrsim 140$ marginally stable and marginally bound

Equipotential surfaces of Thick Disc models

 $\lambda = 140$

**f(T,B,
$$\phi$$
) gravity**

$$S[\theta] = \int d^4x |\theta| f(T, B, \phi)$$









Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

The theory

$$f = \hat{\lambda} \left(\sqrt{1 + \frac{T}{\hat{\lambda}}} - 1 \right)$$

• Spherically symmetric tetrads solving the antisymmetric field equations

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$$\mathscr{A}(r)^{2} = 1 - \frac{2M}{r} - \frac{2M}{r\lambda} \mathscr{T}, \, \mathscr{B}(r)^{2} = \frac{r^{4}\lambda^{4}}{16M^{4}\mathscr{A}(r)^{2}} \left(1 + \frac{\lambda^{2}r^{2}}{4M^{2}}\right)^{-2}$$
$$\mathscr{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \qquad \lambda = M\sqrt{\hat{\lambda}}$$

- Schwarzschild limit for $\lambda \to \infty$ •

$$\gamma = -1,$$

ulletorbits

$$\lambda =$$

 $\theta^a_{2\mu} =$

Complex

 $i\mathscr{A}(r)\sin\vartheta\cos\varphi$

 $i\mathscr{A}(r)\sin\vartheta\sin\varphi$

 $i\mathscr{A}(r)\cos\vartheta$

 $i\mathscr{B}(r)$

$$ds^2 = -\mathscr{A}^2 dt^2 + \mathscr{B}^2 dr^2 + r^2 d\Omega^2$$

 $\chi r \cos \varphi$

0

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 $-\chi r \sin \varphi - r\chi \sin \vartheta \cos \vartheta \cos \varphi$

 $-r\chi\sin\vartheta\cos\vartheta\sin\varphi$

 $\chi r \sin^2 \vartheta$

Weitzenböck gauge

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> Irfaces of Thick Disc models $140 \text{ vs } \lambda = 10$

**f(T,B,
$$\phi$$
) gravity**
 $S[\theta] = \int d^4x |\theta| f(T, B)$



Teleparallelism - Black Holes in $f(T,B,\phi)$ gravity - Conclusion 68





Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

The theory

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$$\begin{aligned} & \theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0\\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi\\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi\\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \end{aligned}$$

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Non-perturbative solution

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$$\mathscr{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \qquad \lambda = M\sqrt{\lambda}$$

- Schwarzschild limit for $\lambda \to \infty$

$$\gamma = -1,$$

- orbits
- •

 $\chi = \pm 1$

$$\lambda =$$

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Weitzenböck gauge

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Equipotential surfaces of Thick Disc models $140 \text{ vs } \lambda \to \infty$

**f(T,B,
$$\phi$$
) gravity**
 $S[\theta] = \int d^4x |\theta| f(T, B, \phi)$









Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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• Spherically symmetric tetrads solving the antisymmetric field equations

$$\begin{aligned} & \mathcal{C} \text{omplex} \\ \theta^{a}_{2\mu} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\vartheta \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\vartheta \end{aligned}$$

$$\begin{array}{c} -\chi r \sin \varphi & -r\chi \sin \vartheta \cos \vartheta \cos \varphi \\ \chi r \cos \varphi & -r\chi \sin \vartheta \cos \vartheta \sin \varphi \\ 0 & \chi r \sin^2 \vartheta \end{array} \right) \qquad \chi = \pm 1$$

With metric
$$ds^2 = -\mathscr{A}^2 dt^2 + \mathscr{B}^2 dr^2 + r^2 d\Omega^2$$

0

 $i\mathscr{A}(r)\cos\vartheta$

Non-perturbative solution

$$\mathscr{A}(r)^{2} = 1 - \frac{2M}{r} - \frac{2M}{r\lambda} \mathscr{T}, \, \mathscr{B}(r)^{2} = \frac{r^{4}\lambda^{4}}{16M^{4}\mathscr{A}(r)^{2}} \left(1 + \frac{\lambda^{2}r^{2}}{4M^{2}}\right)^{-2}$$
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- Schwarzschild limit for $\lambda \to \infty$ •

$$\gamma = -1,$$

- orbits

$$\lambda =$$

torus with cusp, tori, bound structures, inner surfaces

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Weitzenböck gauge

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Equipotential surfaces of Thick Disc models 140 vs $\lambda \to \infty$

$$f(\mathbf{T},\mathbf{B},\boldsymbol{\phi}) \text{ gravity}$$

$$S[\theta] = \int d^4x |\theta| f(T, B, \phi)$$









Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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- Schwarzschild limit for
$$\lambda \to \infty$$

$$\gamma = -1,$$

orbits

$$\lambda =$$

With metric

 $\theta^a_{2\,\mu} =$

Complex

 $i\mathscr{A}(r)\sin\vartheta\cos\varphi$

 $i\mathscr{A}(r)\sin\vartheta\sin\varphi$

 $i\mathscr{A}(r)\cos\vartheta$

 $i\mathscr{B}(r)$

$$ds^2 = -\mathscr{A}^2 dt^2 + \mathscr{B}^2 dr^2 + r^2 d\Omega^2$$

0

 $\chi r \cos \varphi$

0

In spherical symmetry: no strong effect of λ on accretion discs.

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0

 $-r\chi\sin\vartheta\cos\vartheta\sin\varphi$

 $\chi r \sin^2 \vartheta$

 $\chi = \pm 1$

 $-\chi r \sin \varphi - r\chi \sin \vartheta \cos \vartheta \cos \varphi$

Weitzenböck gauge

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> Irfaces of Thick Disc models 140 vs $\lambda \to \infty$

f(T,B, ϕ) gravity $d^4x \,|\, \theta \,| f(T, B, \phi)$ $S[\theta] \Rightarrow$









Geometric fields

 $\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$

Weitze
$$(\theta^a_{\mu}, \phi^a)$$

The theory

enböck gauge

$$\delta^a_b) \Rightarrow \omega^a{}_{b\mu} = 0, \ \Gamma^\mu{}_{\nu\rho} = e_a{}^\mu \partial_\rho \theta^a{}_{\nu},$$

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$






Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

The theory

$$f(T,B) = T + \frac{\epsilon}{2}(\alpha T^2 + \beta B^2 + \gamma BT)$$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

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Geometric fields

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The theory

$$f(T,B) = T + \frac{\epsilon}{2} (\alpha T^2 + \beta B^2 + \gamma BT)$$

 Spherically symmetric tetrads solving the antisymmetric field equations

$$\begin{aligned} \text{Real} \\ \theta_{1\mu}^{a} &= \begin{pmatrix} \mathscr{A}(r) & 0 & 0 & 0 \\ 0 & \mathscr{B}(r)\sin\vartheta\cos\varphi & \xi r\cos\vartheta\cos\varphi & -r\xi\sin\vartheta\sin\varphi \\ 0 & \mathscr{B}(r)\sin\vartheta\sin\varphi & \xi r\cos\vartheta\sin\varphi & \xi r\sin\vartheta\cos\varphi \\ 0 & \mathscr{B}(r)\cos\vartheta & -r\xi\sin\vartheta & 0 \end{pmatrix} \qquad \xi = \pm 1 \\ \end{aligned}$$

$$\begin{aligned} \mathcal{E}_{2\mu}^{a} &= \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1 \end{aligned}$$

$$\begin{aligned} \text{With metric} \\ ds^{2} &= -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2} \end{aligned}$$

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Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$







Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

The theory

$$f(T,B) = T + \frac{\epsilon}{2}(\alpha T^2 + \beta B^2 + \gamma BT)$$

• Spherically symmetric tetrads solving the antisymmetric field equations

Real

$$\theta_{1\mu}^{a} = \begin{pmatrix} \mathscr{A}(r) & 0 & 0 & 0 \\ 0 & \mathscr{B}(r)\sin\vartheta\cos\varphi & \xi r\cos\vartheta\cos\varphi & -r\xi\sin\vartheta\sin\varphi \\ 0 & \mathscr{B}(r)\sin\vartheta\sin\varphi & \xi r\cos\vartheta\sin\varphi & \xi r\sin\vartheta\cos\varphi \\ 0 & \mathscr{B}(r)\cos\vartheta & -r\xi\sin\vartheta & 0 \end{pmatrix} \qquad \xi = \pm \begin{pmatrix} \xi = \pm r \\ 0 & \mathfrak{B}(r)\cos\vartheta & -r\xi\sin\vartheta & 0 \end{pmatrix} \qquad \xi = \pm \begin{pmatrix} 0 & \mathfrak{B}(r) & 0 & 0 \\ \mathfrak{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ \mathfrak{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ \mathfrak{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm \begin{pmatrix} \Psi = \frac{1}{2} & \Psi =$$

Perturbative solutions

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Weitzenböck gauge $(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$







Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

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\theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm \\ With metric
ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

Perturbative solutions

$$\mathscr{A}^{2}(r) = 1 - \frac{2M}{r} + \epsilon a(r), \, \mathscr{B}^{2}(r) = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon b(r)$$

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Weitzenböck gauge

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu}, \qquad f(T,B,\phi) \text{ gravit}$$

$$S[$$

gravity

$$S[\theta] = \int d^4x |\theta| f(T, B, d)$$







Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

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With metric

$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

Perturbative solutions

$$\mathcal{A}^{2}(r) = 1 - \frac{2M}{r} + \epsilon a(r), \, \mathcal{B}^{2}(r) = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon b(r)$$

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Weitzenböck gauge $(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$

f(T,B, ϕ) gravity $d^4x \,|\, \theta \,| f(T, B, \phi)$ $S[\theta] =$

complex and real tetrad







Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

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$$\begin{aligned} \text{With metric} \\ ds^{2} &= -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2} \end{aligned}$$

Perturbative solutions

$$\mathscr{A}^{2}(r) = 1 - \frac{2M}{r} + \epsilon a(r), \, \mathscr{B}^{2}(r) = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon b(r)$$

- ullet
- complex: independent of χ lacksquare
- real: dependent on ξ

Weitzenböck gauge $(\theta^{a}_{\mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$

f(T,B, ϕ) gravity

$$S[\theta] = \int d^4x \, |\, \theta| \, f(T, B, q)$$

exists for the complex and real tetrad







Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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$$\theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\theta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\theta\cos\varphi\sin\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1$$
With metric

$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

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Deflection of light

f(T,B, ϕ) gravity

 $S[\theta] =$

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu} \partial_{\rho} \theta^{a}_{\ \nu},$$

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Deflection of light

f(T,B, ϕ) gravity

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

exists for the complex and real tetrad



Μ

$$\Delta \varphi_{\text{real}} = \frac{4M}{r_0} + \epsilon \left(\frac{(\xi - 1)(M(4(44 - 9\pi)\alpha + 8(29 - 6\pi)\beta + 6(34 - 7\pi)\gamma) + \pi r_0(6\alpha + 8\beta))}{2\xi r_0^3} + \frac{2\xi r_0^3}{r_0^2} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 18$$

Teleparallelism - Black Holes in f(T,B, ϕ) gravity - Conclusion 80









Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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$$\theta_{1\mu}^{a} = \begin{pmatrix} \mathscr{A}(r) & 0 & 0 & 0 \\ 0 & \mathscr{B}(r)\sin\vartheta\cos\varphi & \xi r\cos\vartheta\cos\varphi & -r\xi\sin\vartheta\sin\varphi \\ 0 & \mathscr{B}(r)\sin\vartheta\sin\varphi & \xi r\cos\vartheta\sin\varphi & \xi r\sin\vartheta\cos\varphi \\ 0 & \mathscr{B}(r)\cos\vartheta & -r\xi\sin\vartheta & 0 \end{pmatrix} \qquad \xi = \pm 1$$
Complex

$$\theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1$$
With metric

$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

Perturbative solutions

$$\mathscr{A}^{2}(r) = 1 - \frac{2M}{r} + \epsilon a(r), \, \mathscr{B}^{2}(r) = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon b(r)$$

- ullet
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Deflection of light

f(T,B, ϕ) gravity

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

exists for the complex and real tetrad



 $S[\theta] =$

Μ

$$\Delta \varphi_{\text{real}} = \frac{4M}{r_0} + \epsilon \left(\frac{(\xi - 1)(M(4(44 - 9\pi)\alpha + 8(29 - 6\pi)\beta + 6(34 - 7\pi)\gamma) + \pi r_0(6\alpha + 8\beta))}{2\xi r_0^3} - \frac{4M}{r_0} + \epsilon \left(\frac{\pi (3\alpha + 5\beta + 4\gamma)}{r_0^2} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 27\pi)\alpha + 2(85 - 18\beta))}{3r_0^3} + \frac{2M((128 - 18\beta))}$$

Teleparallelism - Black Holes in f(T,B, ϕ) gravity - Conclusion 81









Geometric fields

$$\theta^{a}{}_{\mu}(x), \Lambda^{a}{}_{b}(x) \Rightarrow \omega^{a}{}_{b\mu} \text{ and } \Gamma^{\rho}{}_{\mu\nu}, \text{ Torsion } T^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\sigma}{}_{\mu\nu}$$

$$f(T,B) = T + \frac{\epsilon}{2}(\alpha T^2 + \beta B^2 + \gamma BT)$$

• Spherically symmetric tetrads solving the antisymmetric field equations

$$\mathsf{Real}$$

$$\theta_{1\mu}^{a} = \begin{pmatrix} \mathscr{A}(r) & 0 & 0 & 0 \\ 0 & \mathscr{B}(r)\sin\vartheta\cos\varphi & \xi r\cos\vartheta\cos\varphi & -r\xi\sin\vartheta\sin\varphi \\ 0 & \mathscr{B}(r)\sin\vartheta\sin\varphi & \xi r\cos\vartheta\sin\varphi & \xi r\sin\vartheta\cos\varphi \\ 0 & \mathscr{B}(r)\cos\vartheta & -r\xi\sin\vartheta & 0 \end{pmatrix} \qquad \xi = \pm 1$$

$$\mathsf{Complex}$$

$$\theta_{2\mu}^{a} = \begin{pmatrix} 0 & i\mathscr{B}(r) & 0 & 0 \\ i\mathscr{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathscr{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^{2}\vartheta \end{pmatrix} \qquad \chi = \pm 1$$
With metric
$$ds^{2} = -\mathscr{A}^{2}dt^{2} + \mathscr{B}^{2}dr^{2} + r^{2}d\Omega^{2}$$

Perturbative solutions

$$\mathscr{A}^{2}(r) = 1 - \frac{2M}{r} + \epsilon a(r), \, \mathscr{B}^{2}(r) = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon b(r)$$

- ullet
- complex: independent of χ lacksquare
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sensitive

- to complex vs real tetrad
- choice of ξ

f(T,B, ϕ) gravity

Weitzenböck gauge

$$(\theta^{a}_{\ \mu}, \delta^{a}_{b}) \Rightarrow \omega^{a}_{\ b\mu} = 0, \ \Gamma^{\mu}_{\ \nu\rho} = e_{a}^{\ \mu}\partial_{\rho}\theta^{a}_{\ \nu},$$

exists for the complex and real tetrad

Deflection of light, order of correction

$$\Delta \varphi_{\text{real}} = \frac{4M}{r_0} + \epsilon \left(\frac{(\xi - 1)(M(4(44 - 9\pi)\alpha + 8(29 - 6\pi)\beta + 6(34 - 7\pi)\gamma) + \pi r_0(6\alpha + 2\xi r_0^3))}{2\xi r_0^3} \right)$$
$$\Delta \varphi_{\text{emplx}} = \frac{4M}{r_0} + \epsilon \left(\frac{\pi (3\alpha + 5\beta + 4\gamma)}{r_0^2} + \frac{2M((128 - 27\pi)\alpha + (212 - 45\pi)\beta + 2(85\pi)\beta)}{3r_0^3} \right)$$

 $S[\theta] \Rightarrow$









Geometric fields

$$\theta^{a}_{\mu}(x), \Lambda^{a}_{b}(x) \Rightarrow \omega^{a}_{b\mu} \text{ and } \Gamma^{\rho}_{\mu\nu}, \text{ Torsion } T^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\mu\nu}$$

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• real: dependent on ξ

sensitive

- to complex vs real tetrad
- choice of ξ

f(T,B, ϕ) gravity

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Deflection of light, order of correction

Similar for Shapiro delay & perihelion shift. \Rightarrow Constraints on parameters









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Born-Infeld gravity - Perturbations of GR - Scalar-Torsion gravity

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[Bahamonde, Faraji, Hackmann, CP 2022; Bahamonde, Golovnev, Guzman, Said, CP 2022; Bahamonde, Ducobu, CP 2022]

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$$=0, \quad A=0$$

No-hair theorem







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- Teleparallelism 1.
 - Teleparallel Geometry
 - Symmetry
 - Teleparallale Gravity
- 2. Black Holes in $f(T,B,\phi)$ teleparallel gravity
 - Born-Infeld f(T)-gravity
 - Teleparallel perturbations of GR
 - Scalar-Torsion gravity
- Conclusion and Outlook 3.





Teleparallelism - Black Holes in f(T,B, ϕ) gravity - Conclusion 100





Geometric fields



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Lorentz transformations as gauge transformations

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