Black Holes in teleparallel gravity

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1. Teleparallelism
    - Teleparallel Geometry
    - Symmetry
    - Teleparallale Gravity

2. Black Holes in f(T,B,ϕ) teleparallel gravity
    - Born-Infeld f(T)-gravity
    - Teleparallel perturbations of GR
    - Scalar-Torsion gravity

3. Conclusion and Outlook

Teleparallel Geometry - Symmetry - Teleparallel Gravity

Geometric fields

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Tetrad components $\theta^a{}_\mu(x)$

An independent flat and metric compatible connection $\Gamma^\sigma{}_{\mu\nu}(x)$

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- technically: basis 1-forms $\theta^a = \theta^a{}_\mu \mathrm{dx}^\mu$
- practically: 16 field components $\theta^a{}_\mu$ with inverse $e_a{}^\mu$ $\quad \theta^a{}_\mu e_a{}^\nu = \delta^\nu_\mu,\ \theta^a{}_\nu e_b{}^\nu = \delta^a_b$
- the metric is a derived object $g_{\mu\nu} = \eta_{ab}\theta^a{}_\mu\theta^b{}_\nu,\ \eta_{ab} = \mathrm{diag}(-,+,+,+)$

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Curavature (rotation) | Non-Metricity (changing lengths) | Torsion (non closing)

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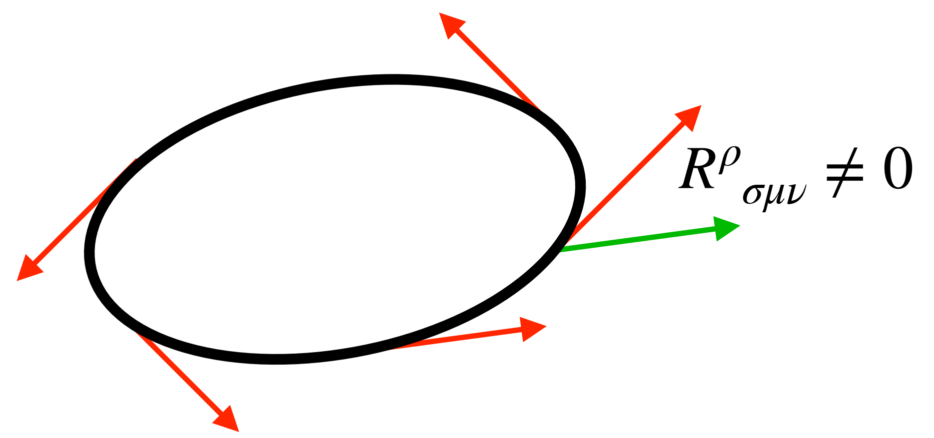
$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho{}_{\sigma\nu} - \partial_\nu\Gamma^\rho{}_{\sigma\mu} + \Gamma^\rho{}_{\lambda\mu}\Gamma^\lambda{}_{\sigma\nu} - \Gamma^\rho{}_{\lambda\nu}\Gamma^\lambda{}_{\sigma\mu}$$



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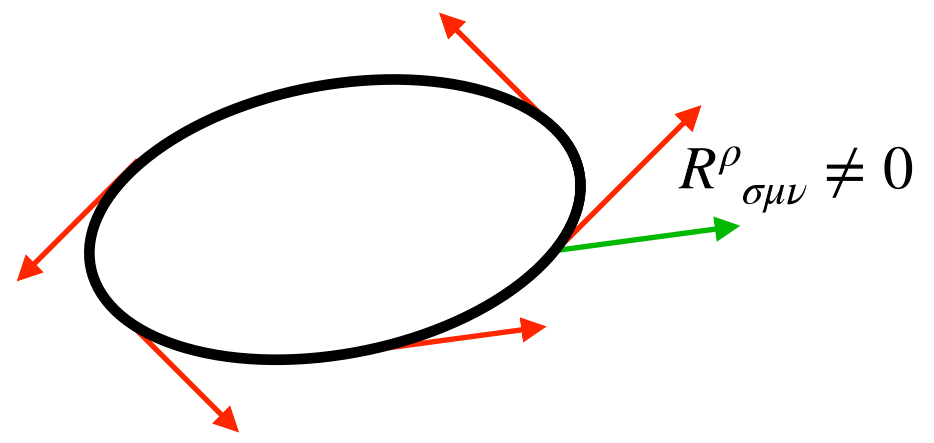

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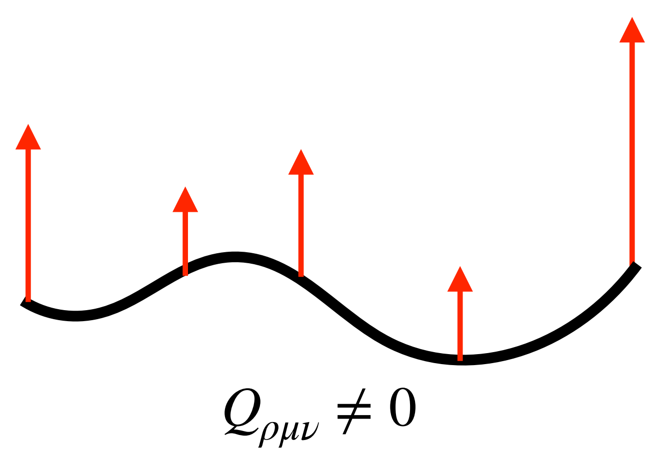
$$Q_{\rho\mu\nu} = \nabla_\rho g_{\mu\nu}$$



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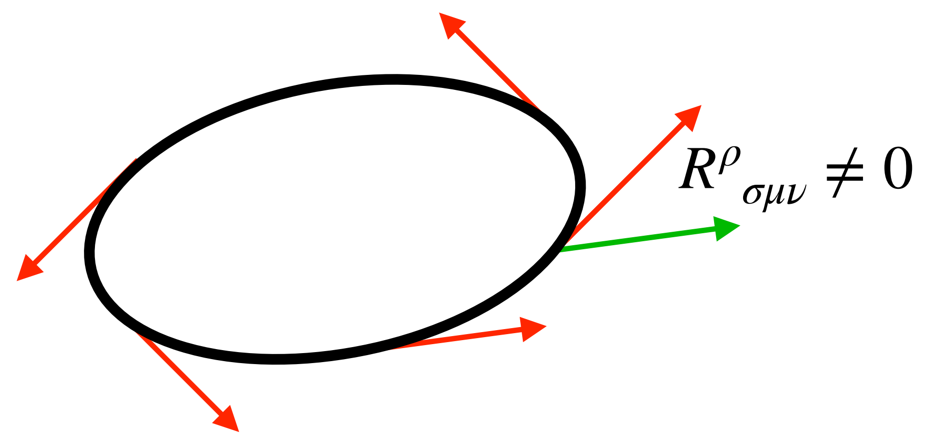

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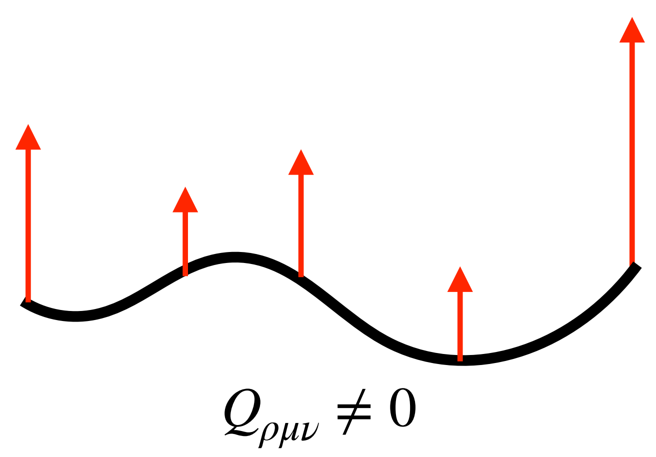

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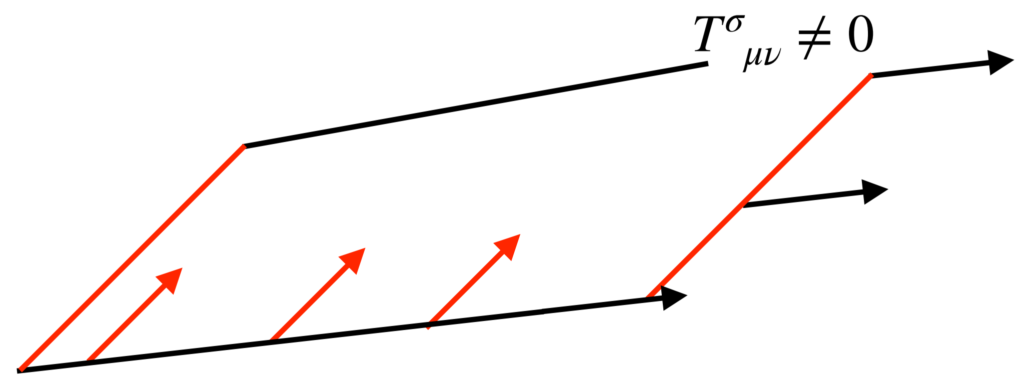

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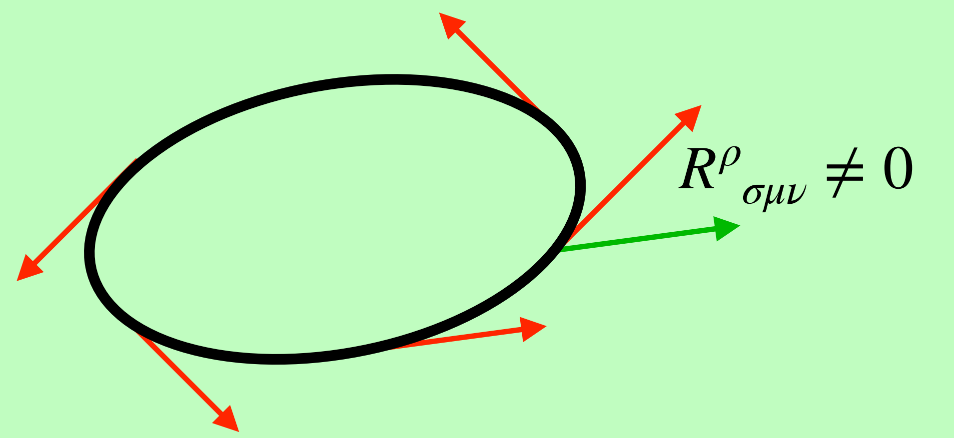

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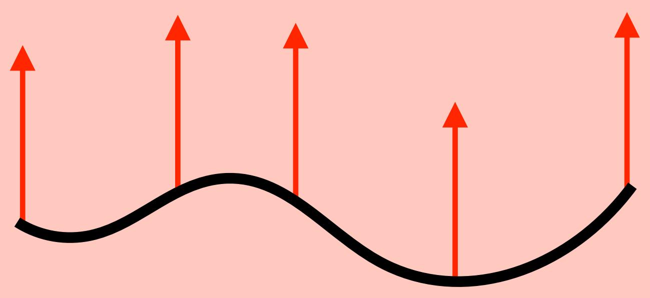

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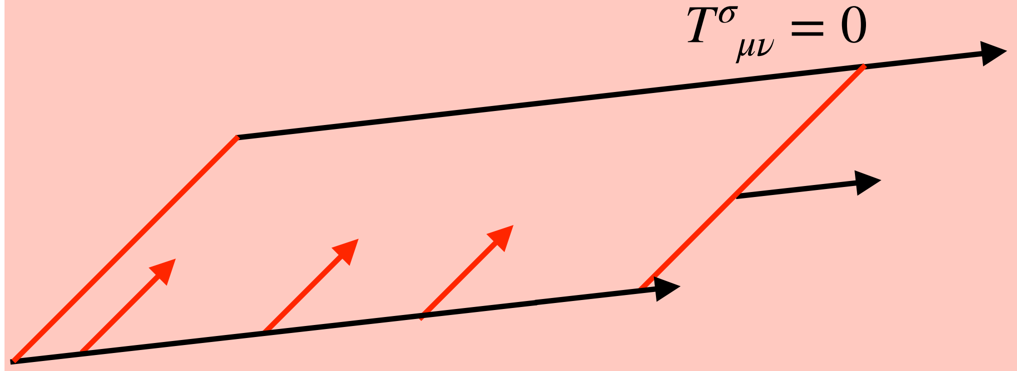

- Levi-Civita $R^\rho{}_{\sigma\mu\nu} \neq 0,\ Q_{\rho\mu\nu} = 0,\ T^\rho{}_{\mu\nu} = 0 \Rightarrow \Gamma^\sigma{}_{\mu\nu} = \frac{1}{2}g^{\rho\lambda}\left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu}\right)$

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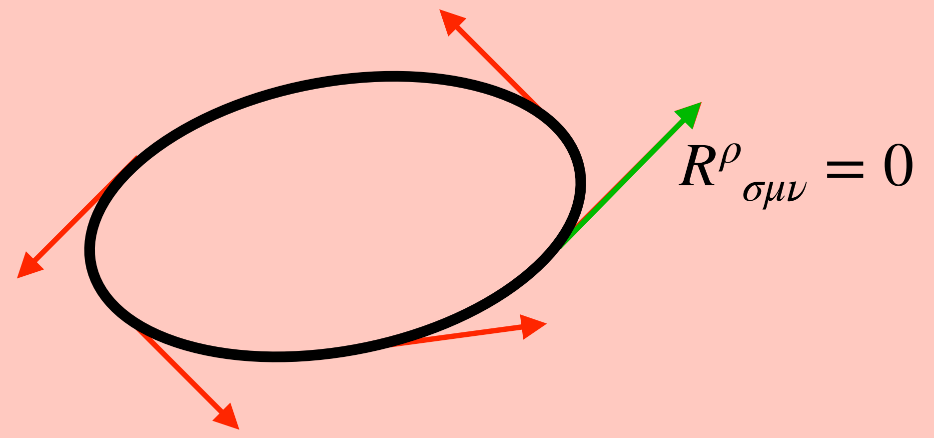

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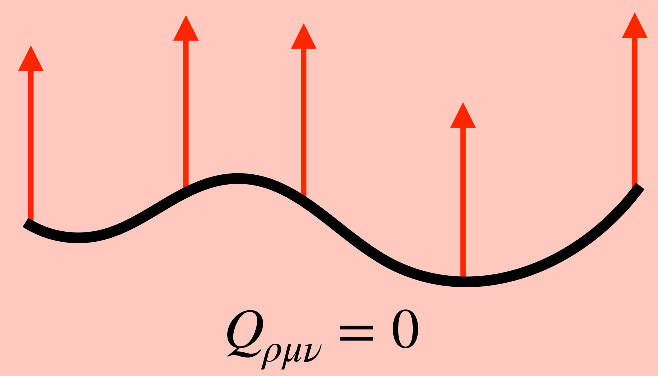

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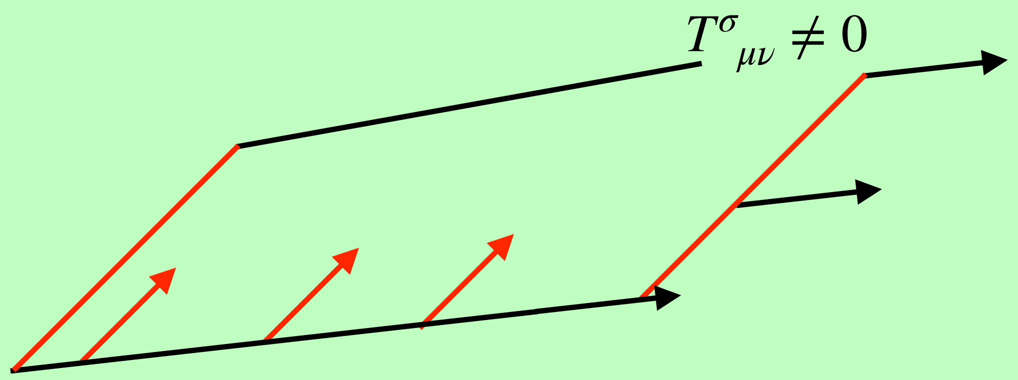

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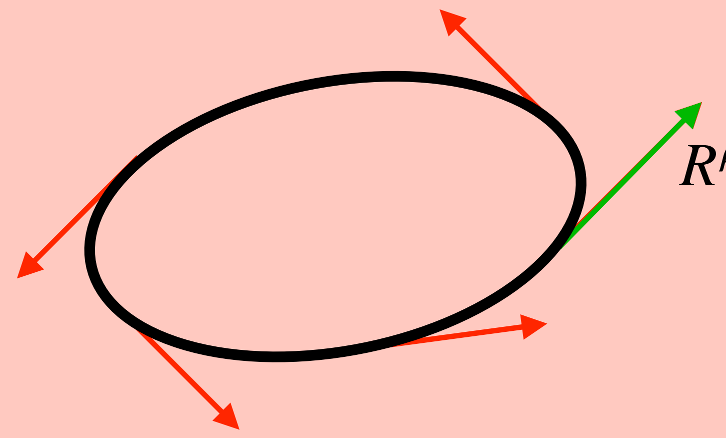

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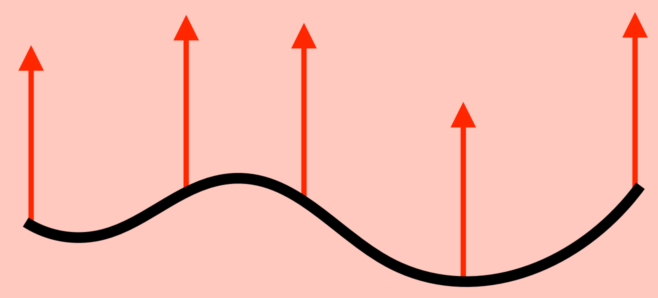

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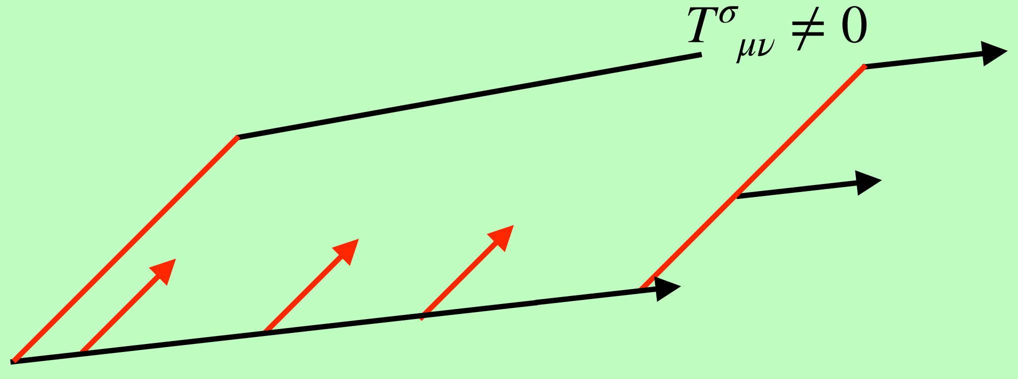

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[Ferraro 2007; Pereira 2013; Krššák 2016; MH, CP 2018]

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Tetrad components $\theta^a{}_\mu(x)$, Lorentz transformations $\Lambda^a{}_b(x)$ generating $\omega^a{}_{b\mu}$ and $\Gamma^\rho{}_{\mu\nu}$, Torsion $T^\sigma{}_{\mu\nu} = \Gamma^\sigma{}_{\nu\mu} - \Gamma^\sigma{}_{\mu\nu}$

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$$g_{\mu\nu}[\hat{\theta}] = g_{\mu\nu}[\theta], \quad \Gamma^\mu{}_{\nu\rho}[\hat{\theta}, \hat{\Lambda}] = \Gamma^\mu{}_{\nu\rho}[\theta, \tilde{\Lambda}], \quad T^\rho{}_{\mu\nu}[\hat{\theta}, \hat{\Lambda}] = T^\rho{}_{\mu\nu}[\theta, \tilde{\Lambda}]$$

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- Choose $\Lambda^a{}_b = (\hat\Lambda^{-1})^a{}_b \Rightarrow \tilde\Lambda^a{}_b = \delta^a_b$

$\Rightarrow$ Affine connection: $\Gamma^\mu{}_{\nu\rho}[\theta^a{}_\mu, \delta^a_b] = e_a{}^\mu \partial_\rho \theta^a{}_\nu$, Torsion: $T^\rho{}_{\mu\nu}[\theta^a{}_\mu, \delta^a_b] = 2 e_c{}^\rho \partial_{[\mu}\theta^c{}_{\nu]}$.

[Ferraro 2007; Pereira 2013; Krššák 2016; MH, CP 2018]

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Without loss of generality one can always work with the pair $(\theta^a{}_\mu, \delta^a_b)$, called **Weitzenböck** gauge.

[Ferraro 2007; Pereira 2013; Krššák 2016; MH, CP 2018] [Hohmann, Jarv, Krssak, CP 2019]

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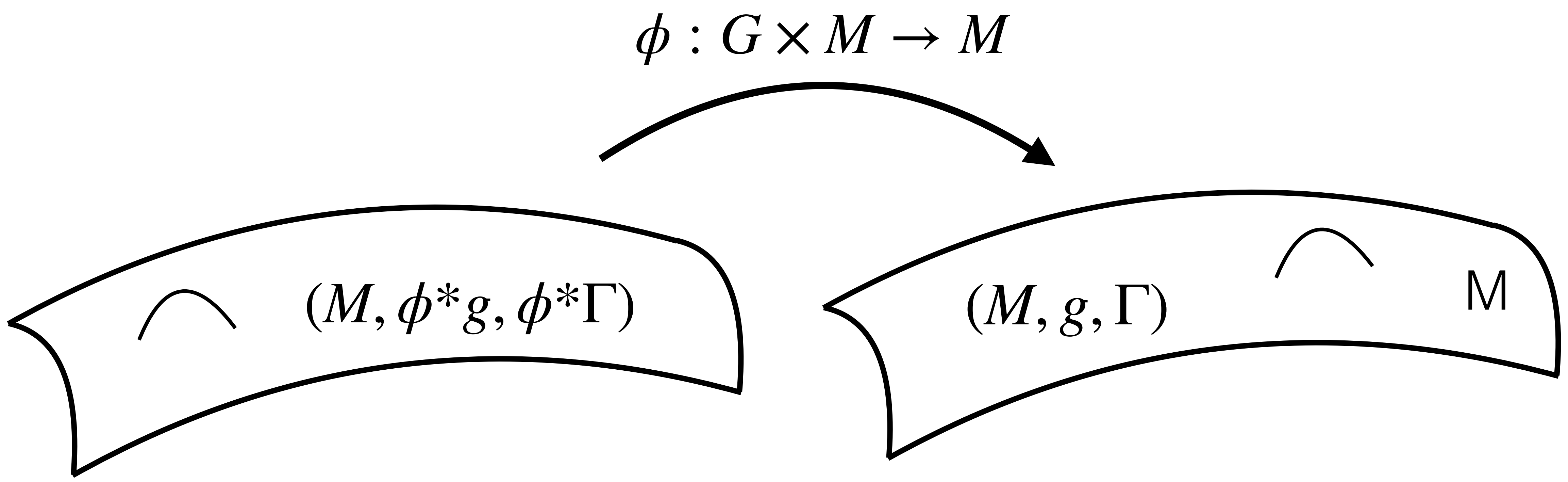

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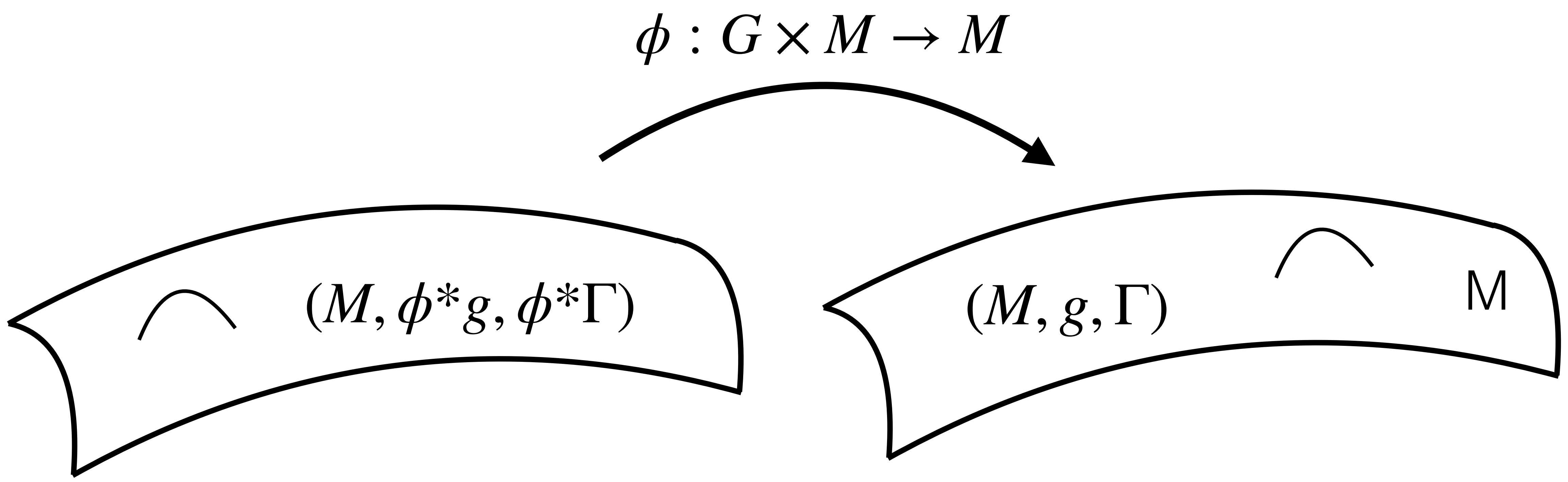

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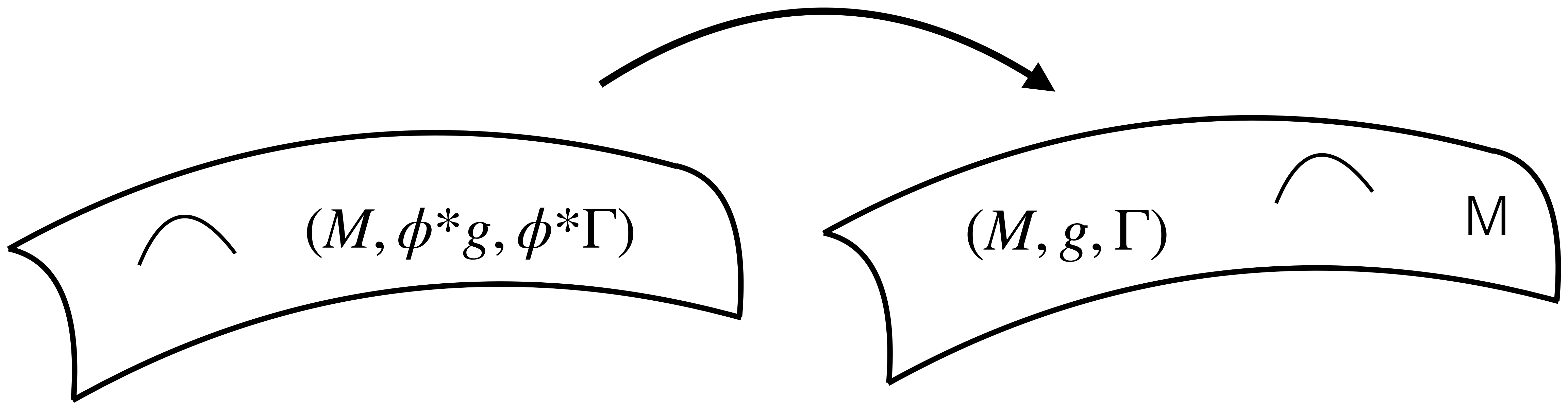

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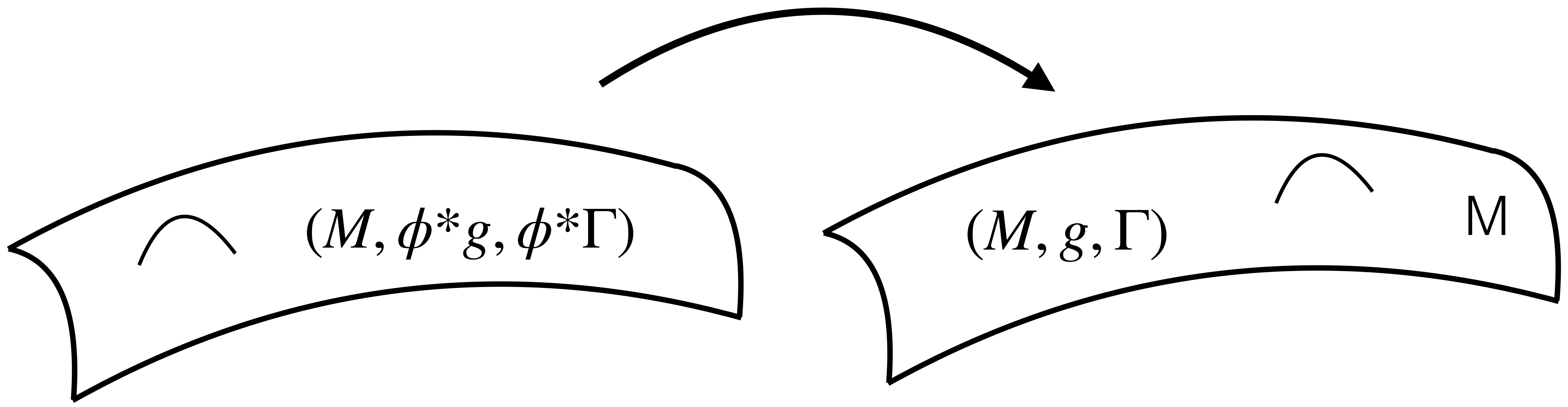

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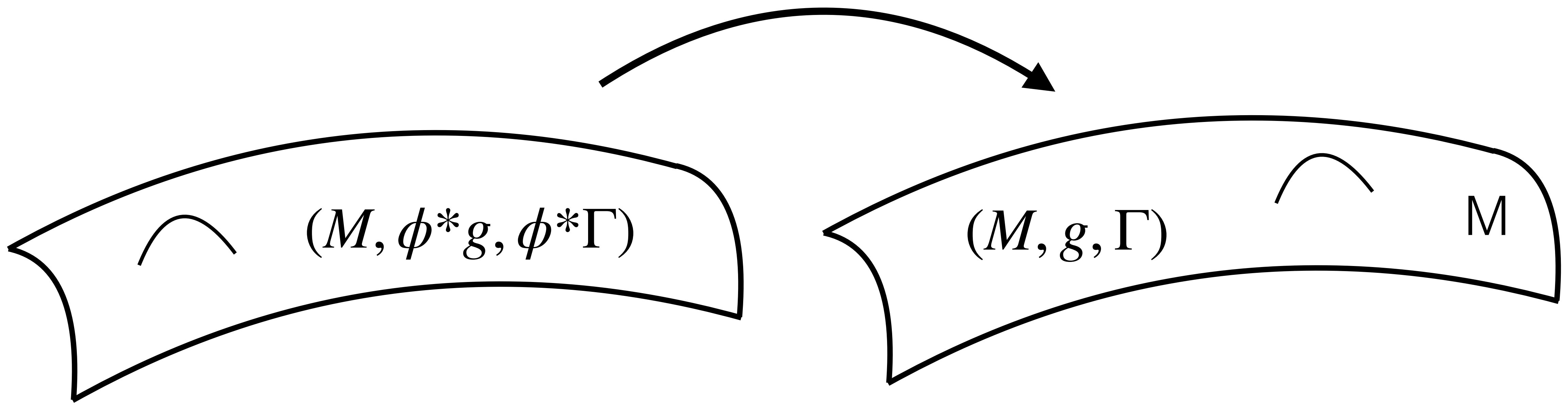

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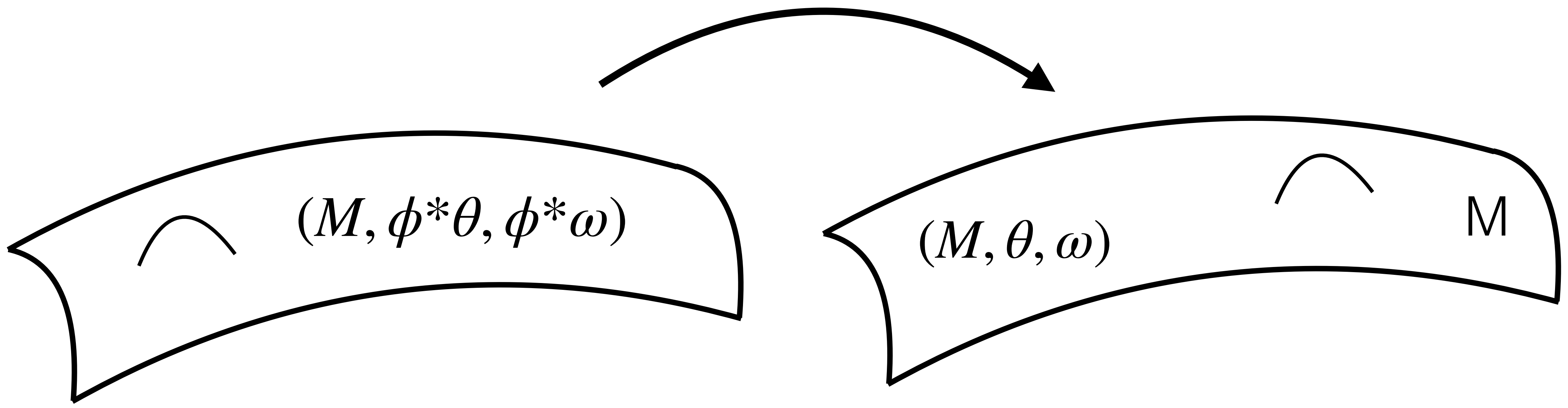

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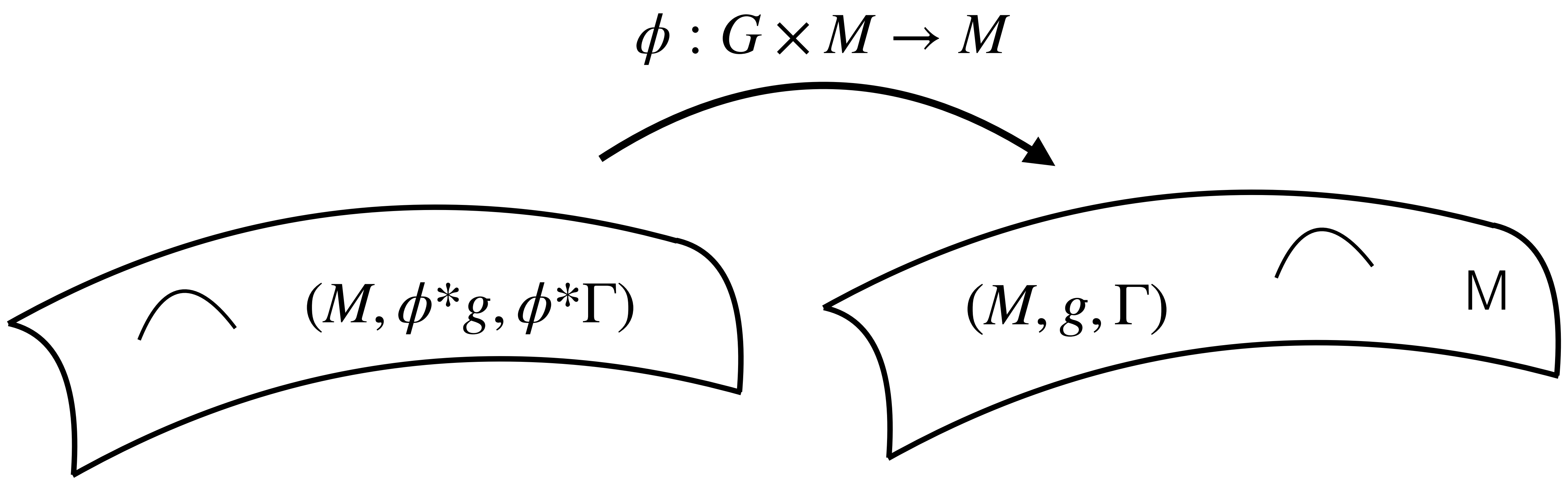

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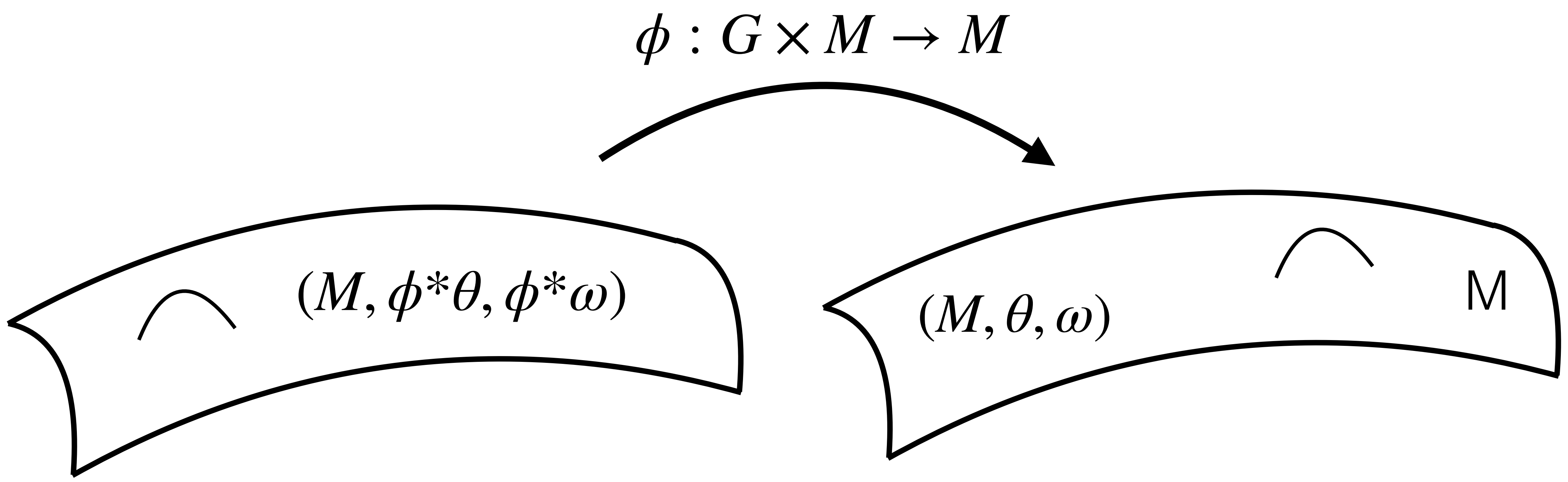

- $g = \phi^*g,\ \Gamma = \phi^*\Gamma$ indendent of Lorentz frame
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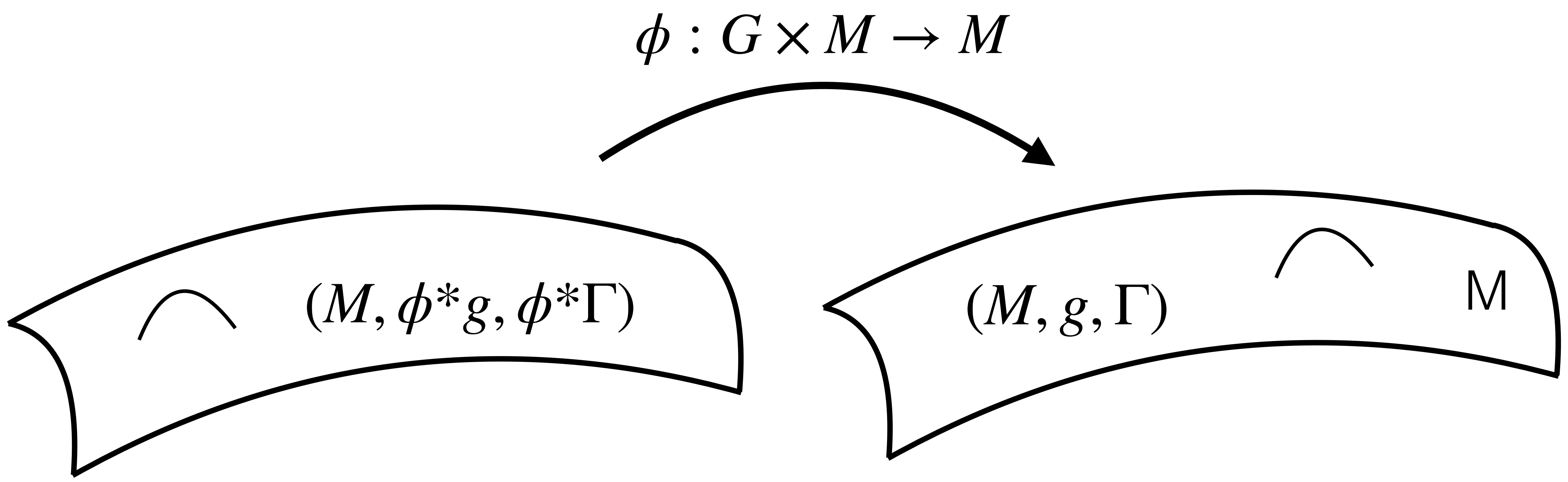

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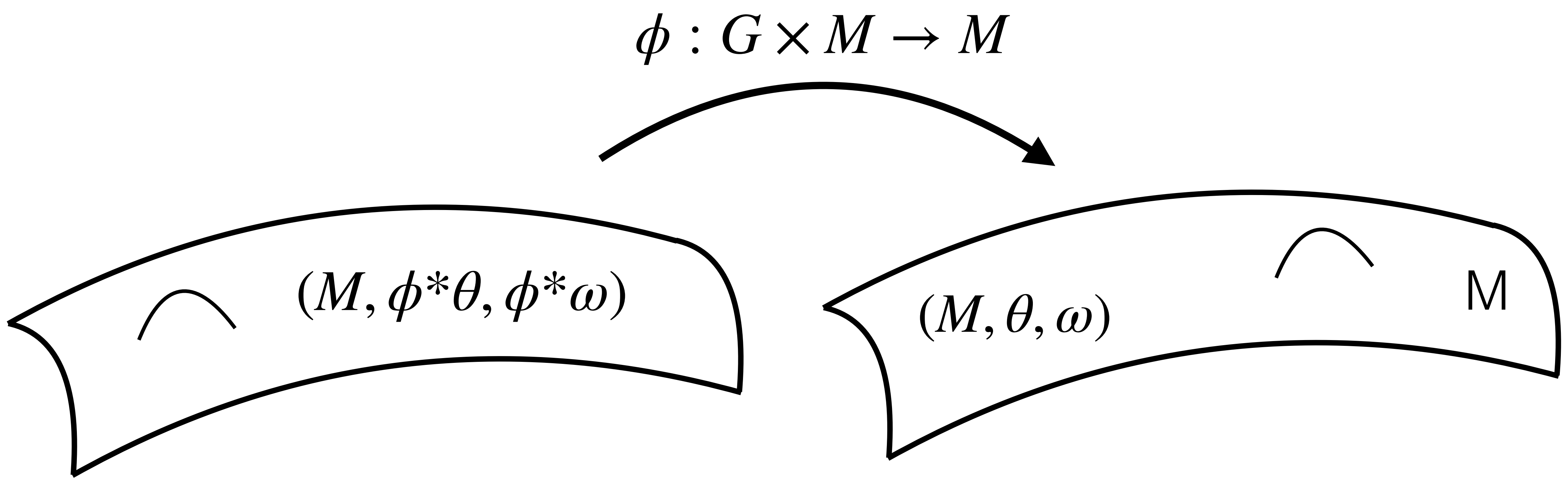

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The general spherically symmetric tetrad, $C_i = C_i(t, r)$

$$\theta^a{}_\mu = \begin{pmatrix} C_1 & C_2 & 0 & 0 \\ C_3 \sin\vartheta\cos\varphi & C_4 \sin\vartheta\cos\varphi & C_5\cos\vartheta\cos\varphi - C_6\sin\varphi & -\sin\vartheta(C_5\sin\varphi + C_6\cos\vartheta\cos\varphi) \\ C_3\sin\vartheta\sin\varphi & C_4\sin\vartheta\sin\varphi & C_5\cos\vartheta\sin\varphi + C_6\cos\varphi & \sin\vartheta(C_5\cos\varphi - C_6\cos\vartheta\sin\varphi) \\ C_3\cos\vartheta & C_4\cos\vartheta & -C_5\sin\vartheta & C_6\sin^2\vartheta \end{pmatrix}$$

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The standard spherically symmetric metric

$$g = (C_1^2 - C_3^2)dt^2 - (C_4^2 - C_2^2)dr^2 - (C_5^2 + C_6^2)(d\vartheta^2 + sin^2\vartheta d\varphi^2) - (C_3C_4 - C_1C_2)dtdr$$

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$$\mathfrak{v}_\mu = T^\rho{}_{\rho\mu}, \quad \mathfrak{a}_\mu = \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma}, \quad \mathfrak{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3}(g_{\rho(\mu}\mathfrak{v}_{\nu)} - g_{\nu\mu}\mathfrak{v}_\rho)$$

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$$\mathfrak{v}_\mu = T^\rho{}_{\rho\mu}, \quad \mathfrak{a}_\mu = \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma}, \quad \mathfrak{t}_{\mu\nu\rho} = T_{(\mu\nu)\rho} + \frac{1}{3}(g_{\rho(\mu}\mathfrak{v}_{\nu)} - g_{\nu\mu}\mathfrak{v}_\rho)$$

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Most prominent theories in the literature

- Teleparallel equivalent of general relativity TEGR

$$S[\theta] = \int d^4x\,|\theta|\,T = \int d^4x\,|\theta|\,(-\mathring{R} + B)$$

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- f(T,B,ϕ,X) gravity

$$S[\theta] = \int d^4x\,|\theta|\,f(T, B, \phi, X)$$

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f(T,B,Φ,X)-Gravity and antisymmetric field eqs in spherical symmetry

$$S[\theta] = \int d^4x\,|\theta|\,f(T,B,\Phi,X)\,,\ X = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi, T = TEGR$$

$$\theta^a{}_\mu = \begin{pmatrix} C_1 & C_2 & 0 & 0 \\ C_3\sin\vartheta\cos\varphi & C_4\sin\vartheta\cos\varphi & C_5\cos\vartheta\cos\varphi - C_6\sin\varphi & -\sin\vartheta(C_5\sin\varphi + C_6\cos\vartheta\cos\varphi) \\ C_3\sin\vartheta\sin\varphi & C_4\sin\vartheta\sin\varphi & C_5\cos\vartheta\sin\varphi + C_6\cos\varphi & \sin\vartheta(C_5\cos\varphi - C_6\cos\vartheta\sin\varphi) \\ C_3\cos\vartheta & C_4\cos\vartheta & -C_5\sin\vartheta & C_6\sin^2\vartheta \end{pmatrix}$$

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$$\theta^a{}_\mu = \begin{pmatrix} C_1 & C_2 & 0 & 0 \\ C_3\sin\vartheta\cos\varphi & C_4\sin\vartheta\cos\varphi & C_5\cos\vartheta\cos\varphi - C_6\sin\varphi & -\sin\vartheta(C_5\sin\varphi + C_6\cos\vartheta\cos\varphi) \\ C_3\sin\vartheta\sin\varphi & C_4\sin\vartheta\sin\varphi & C_5\cos\vartheta\sin\varphi + C_6\cos\varphi & \sin\vartheta(C_5\cos\varphi - C_6\cos\vartheta\sin\varphi) \\ C_3\cos\vartheta & C_4\cos\vartheta & -C_5\sin\vartheta & C_6\sin^2\vartheta \end{pmatrix}$$

$$E_{[tr]} = 0 \Leftrightarrow C_3C_5(f'_T + f'_B) = 0$$

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1. Teleparallelism
    - Teleparallel Geometry
    - Symmetry
    - Teleparallale Gravity

2. Black Holes in f(T,B,ϕ) teleparallel gravity
    - Born-Infeld f(T)-gravity
    - Teleparallel perturbations of GR
    - Scalar-Torsion gravity

3. Conclusion and Outlook

[Bahamonde, Faraji, Hackmann, CP 2022; Bahamonde, Golovnev, Guzman, Said, CP 2022; Bahamonde, Ducobu, CP 2022]

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$$\mathscr{A}(r)^2 = 1 - \frac{2M}{r} - \frac{2M}{r\lambda}\mathscr{T},\ \mathscr{B}(r)^2 = \frac{r^4\lambda^4}{16M^4\mathscr{A}(r)^2}\left(1+\frac{\lambda^2r^2}{4M^2}\right)^{-2}$$

$$\mathscr{T} = \tan^{-1}\left(\frac{\lambda r}{2M}\right), \qquad \lambda = M\sqrt{\hat{\lambda}}$$

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[Bahamonde, Faraji, Hackmann, CP 2022; Bahamonde, Golovnev, Guzman, Said, CP 2022; Bahamonde, Ducobu, CP 2022]

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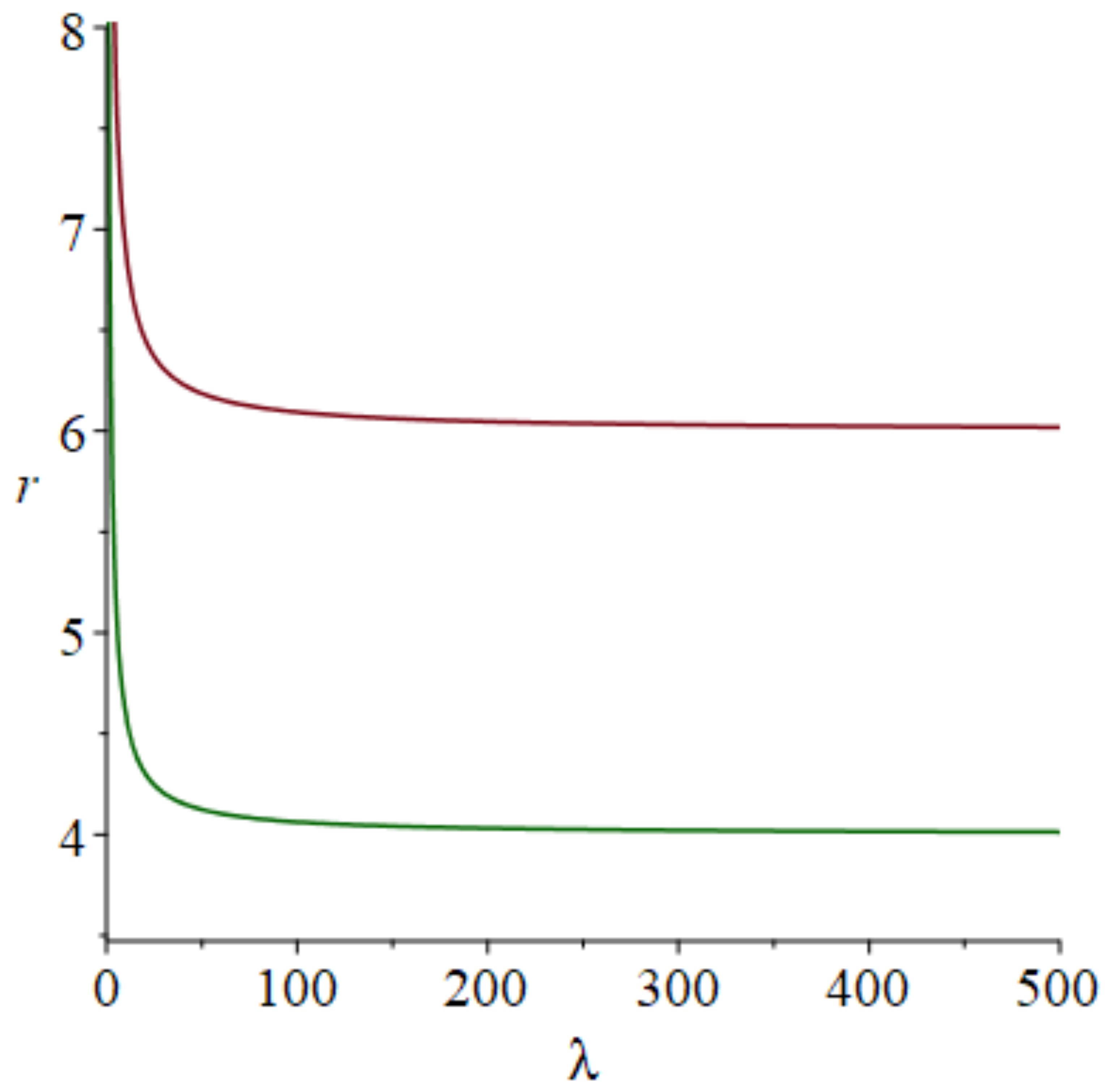

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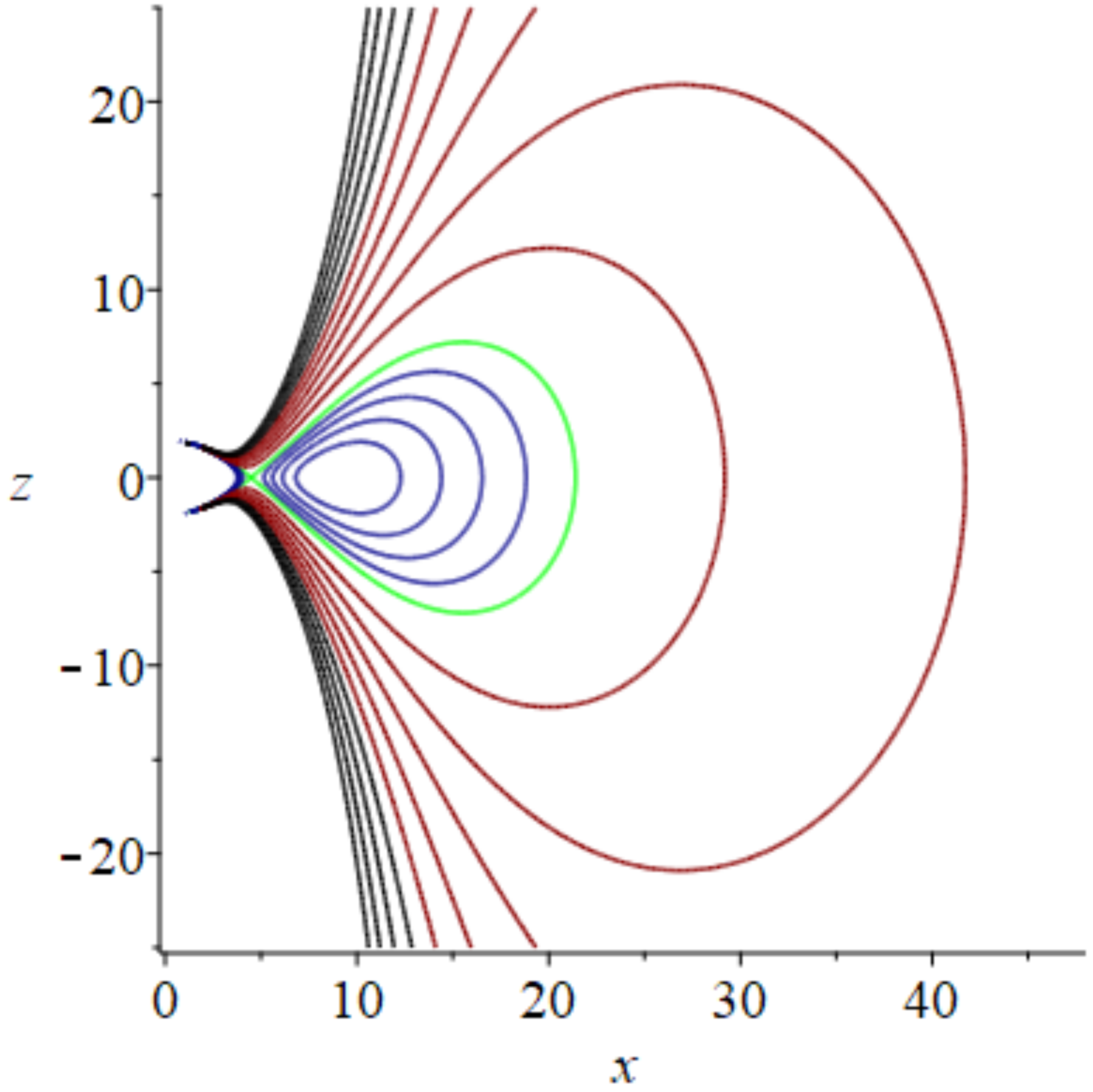
$$\lambda = 140$$



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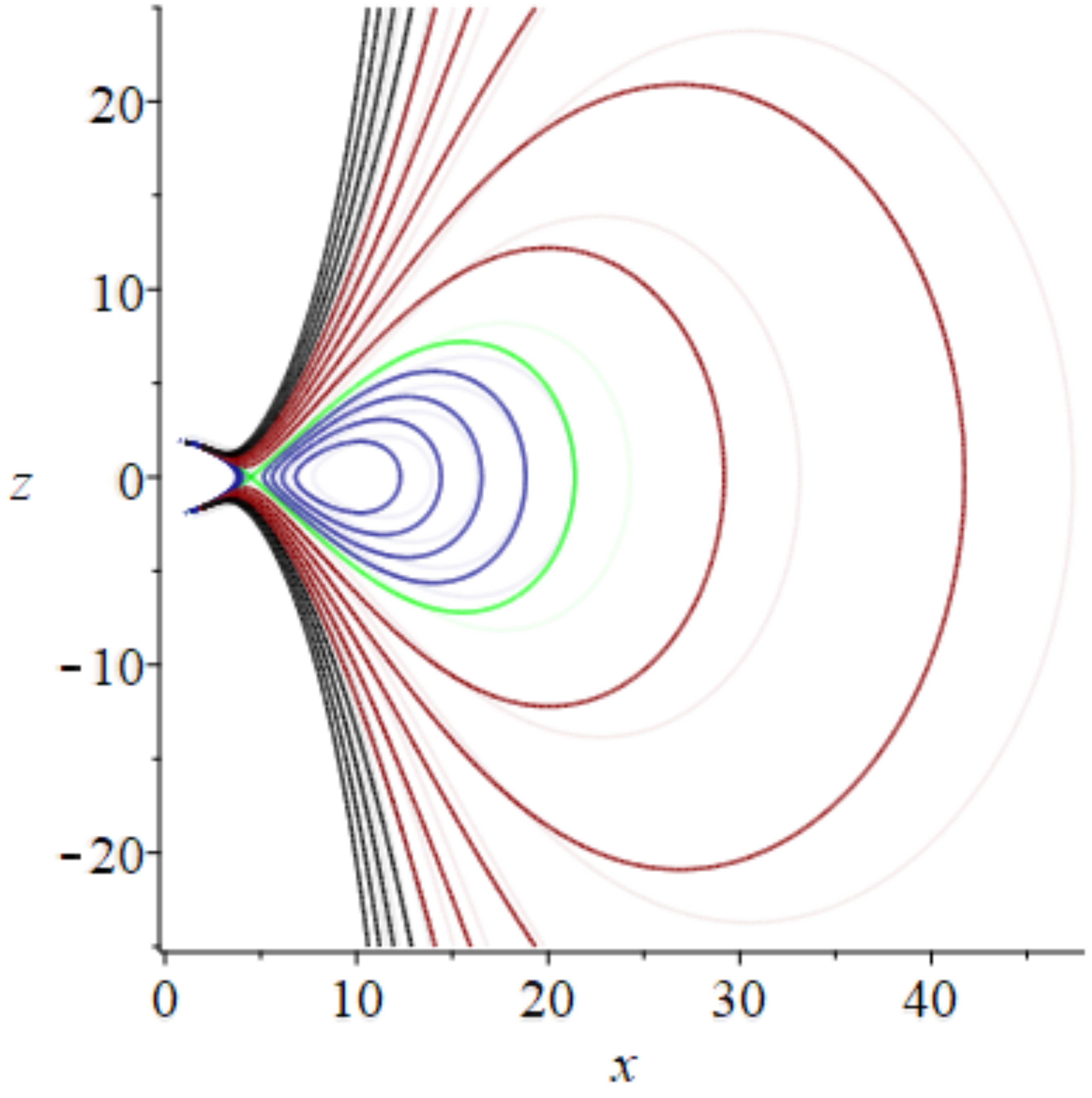
$$\lambda = 140 \text{ vs } \lambda = 10$$



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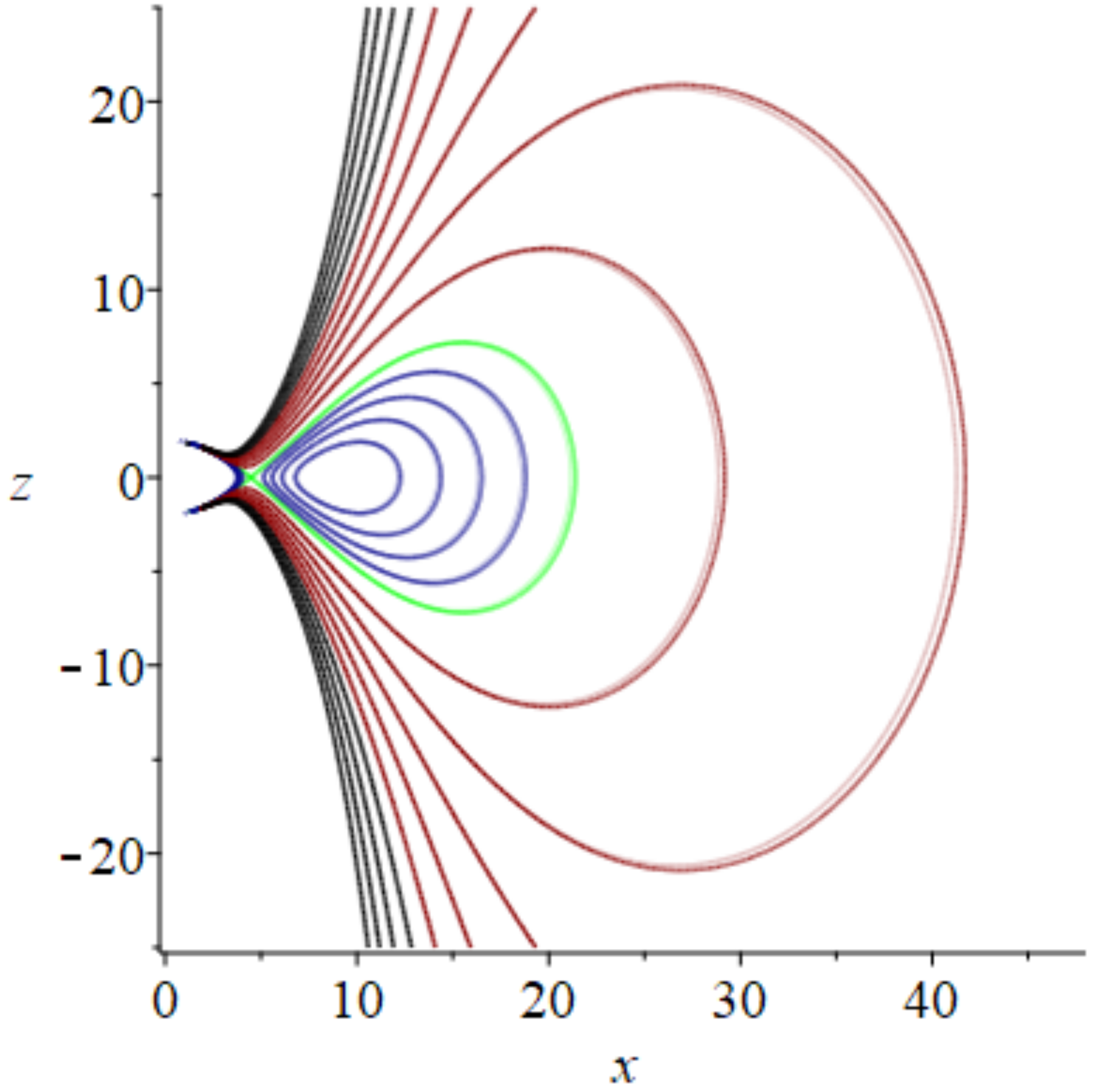

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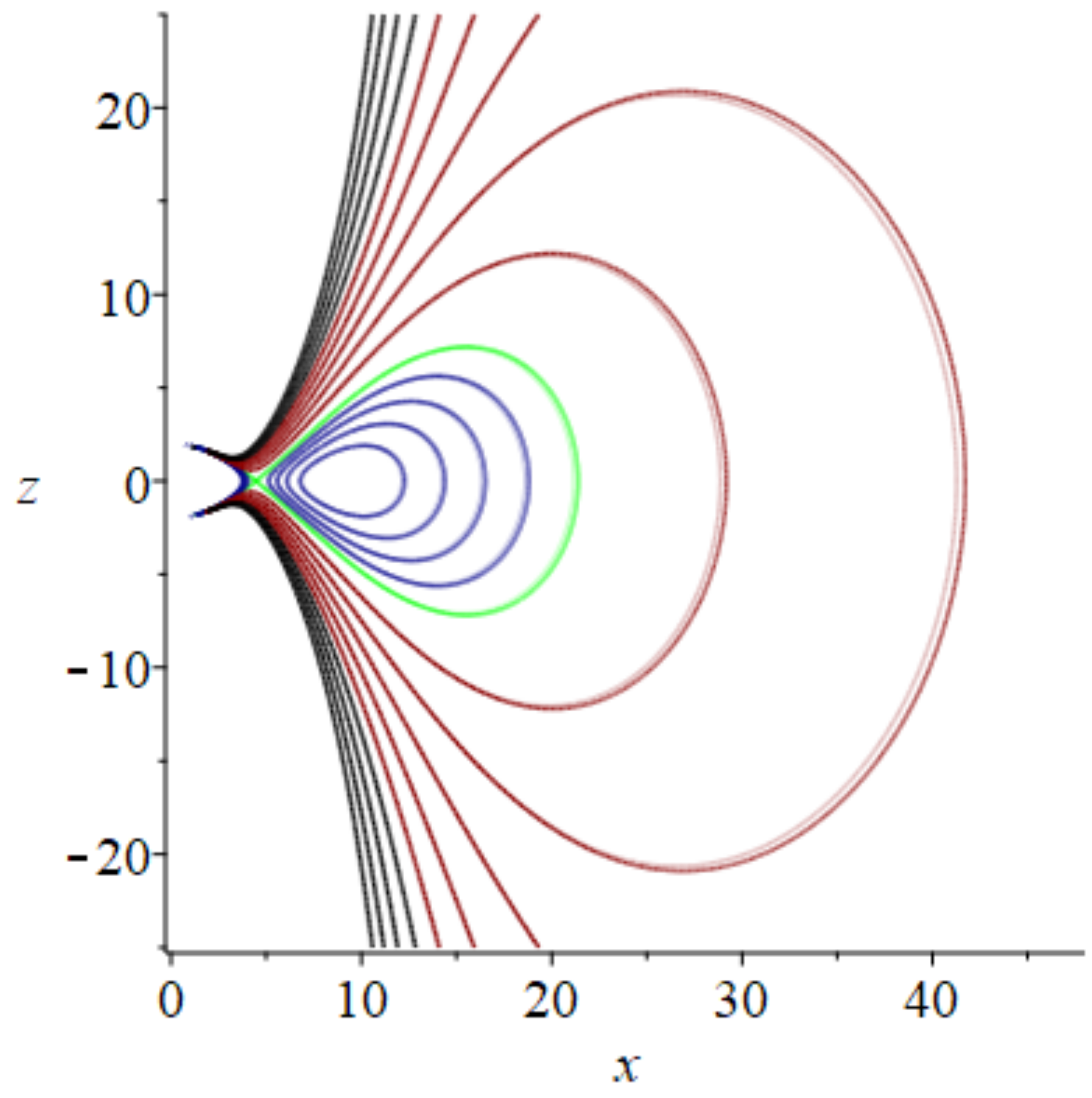
torus with cusp, tori,
bound structures, inner surfaces



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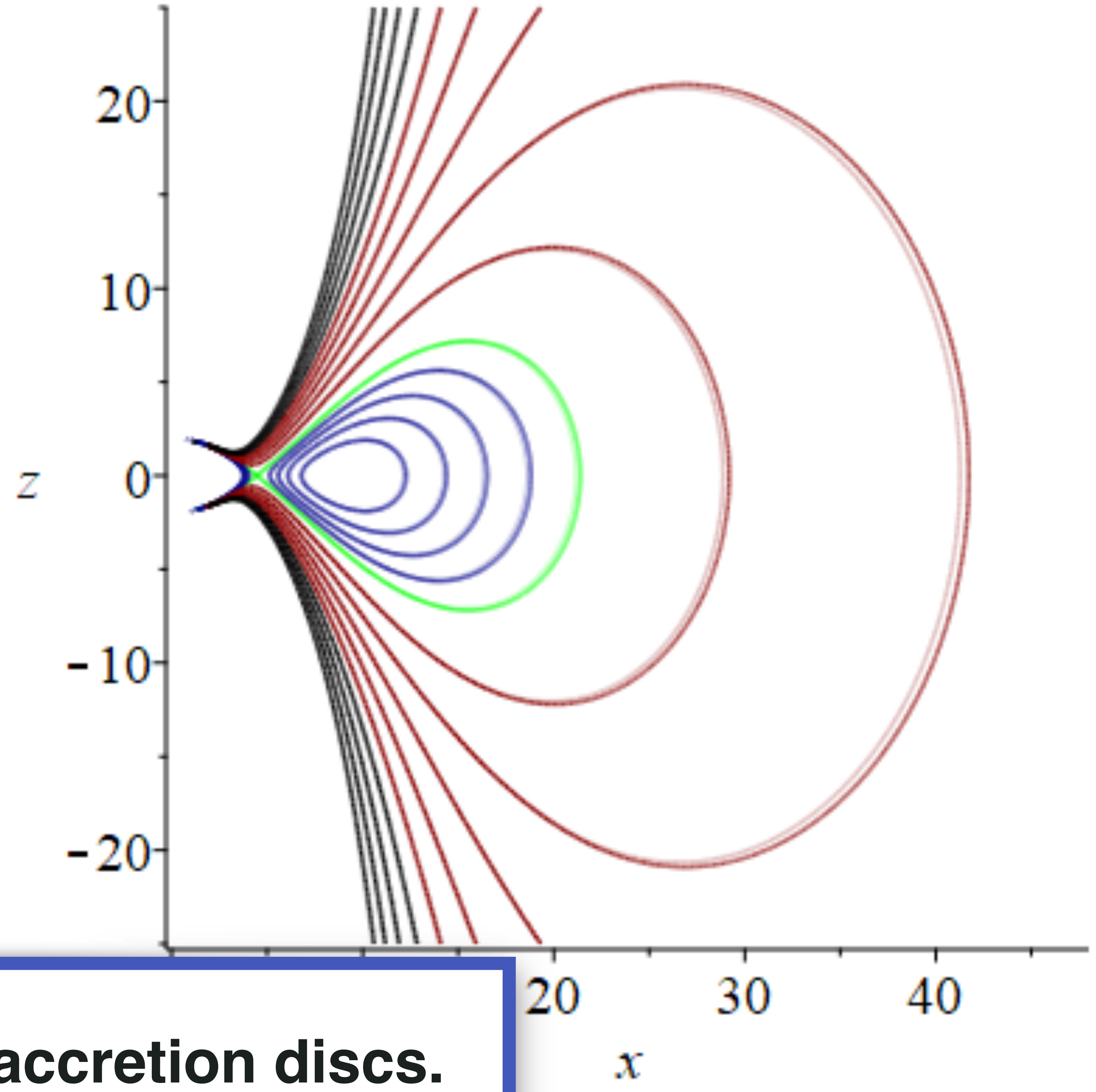

In spherical symmetry: no strong effect of λ on accretion discs.

[Bahamonde, Faraji, Hackmann, CP 2022; Bahamonde, Golovnev, Guzman, Said, CP 2022; Bahamonde, Ducobu, CP 2022]

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Perturbative solutions

[Bahamonde, Faraji, Hackmann, CP 2022; Bahamonde, Golovnev, Guzman, Said, CP 2022; Bahamonde, Ducobu, CP 2022]

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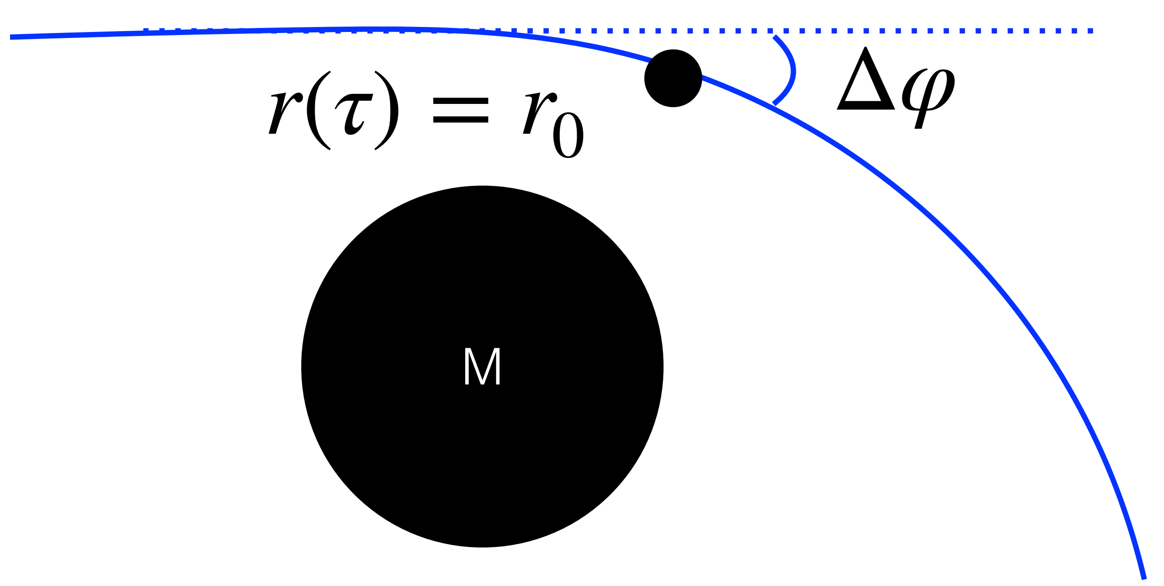

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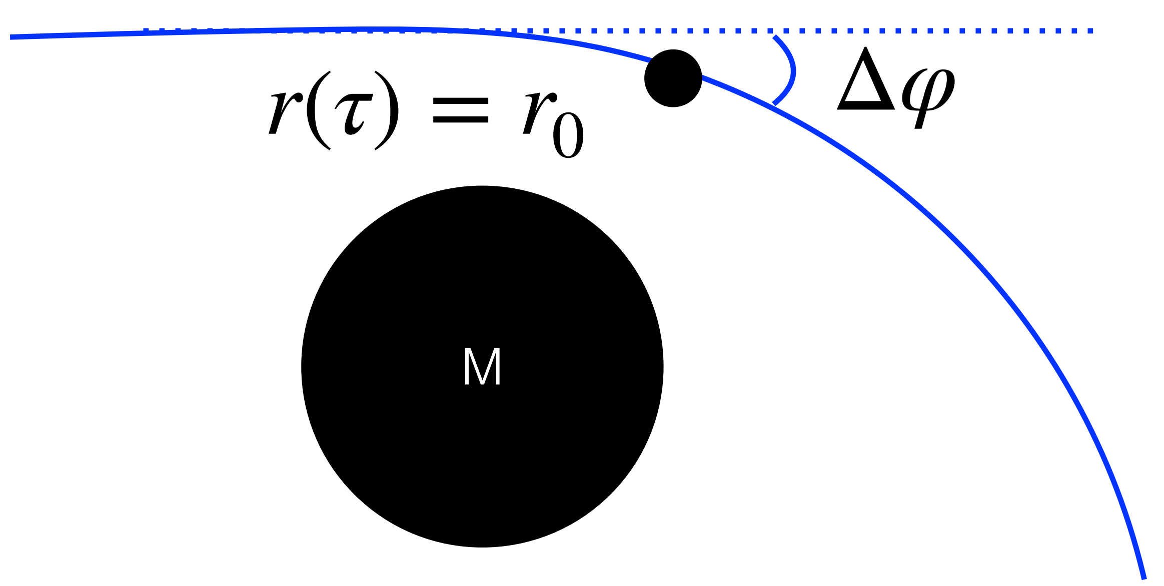

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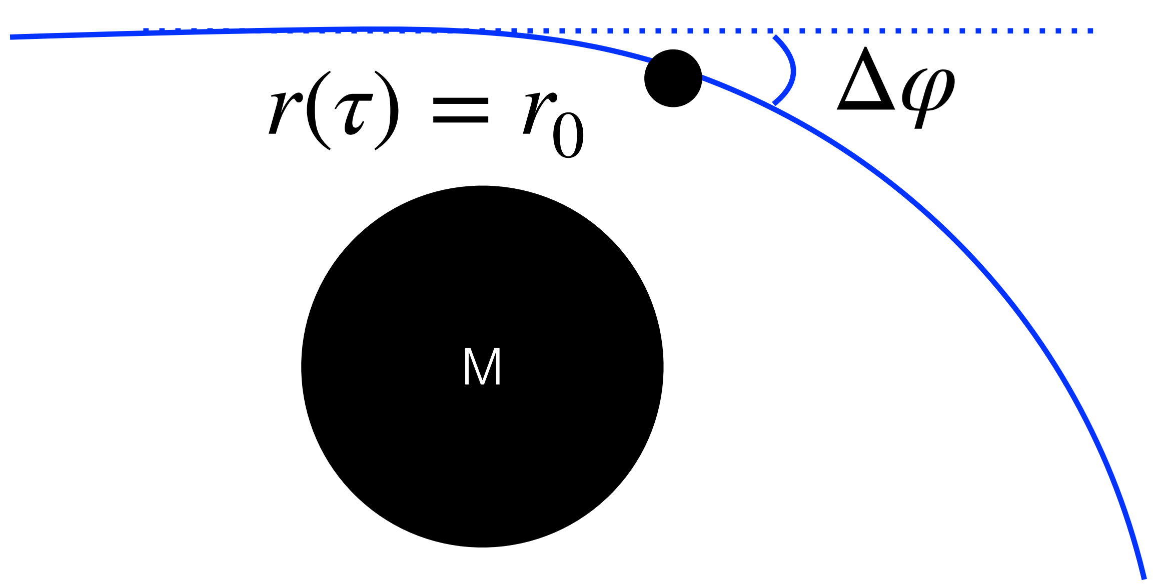

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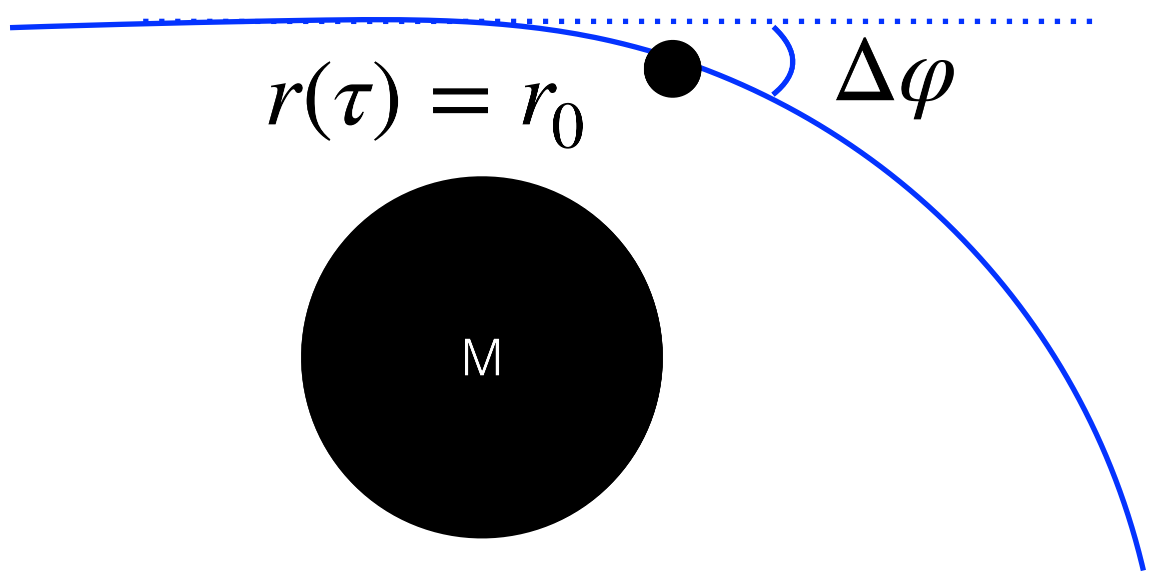

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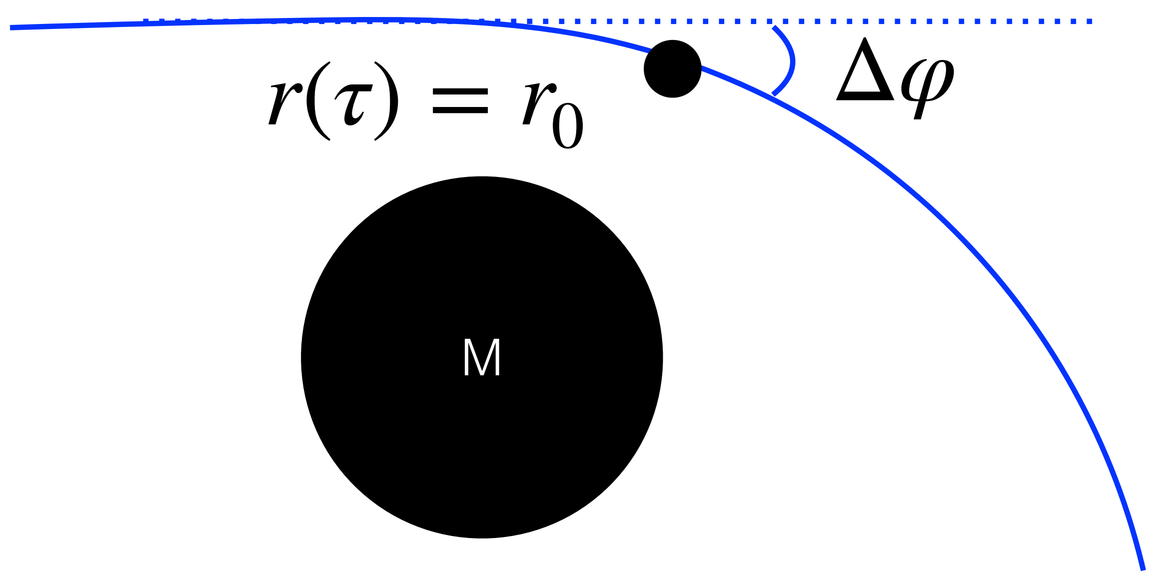

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No spherically symmetric asymptotically flat scalarized solutions exist if

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Solutions (numerous exist)

Example 1: (real tetrad)

- $C = 0, \quad V = 0, \quad A = -\frac{1}{8}\beta\psi^2$

yields the BBMB solution

$$ds^2 = \left(1 - \frac{K}{r}\right)^2 dt^2 - \left(1 - \frac{K}{r}\right)^{-2} dr^2 - r^2 d\Omega^2$$

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No spherically symmetric asymptotically flat scalarized solutions exist if

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4. $A = \alpha$, $C = \frac{\gamma}{m+1}\phi^{m+1}$ and
either $\frac{1}{\beta}\left(\psi V' - (m+1)\gamma\psi^m T^r\psi'\right) \leq 0$
or $\frac{(m+1)}{m-1}\frac{1}{\beta}\left(\alpha\mathring{R} + \kappa^2(\psi V - 4V)\right) \leq 0$

1. Teleparallelism
   - Teleparallel Geometry
   - Symmetry
   - Teleparallale Gravity

2. Black Holes in f(T,B,ϕ) teleparallel gravity
   - Born-Infeld f(T)-gravity
   - Teleparallel perturbations of GR
   - Scalar-Torsion gravity

3. Conclusion and Outlook

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Geometric fields

Tetrad components $\theta^a{}_\mu(x)$, Lorentz transformations $\Lambda^a{}_b(x)$ generating $\omega^a{}_{b\mu}$ and $\Gamma^\rho{}_{\mu\nu}$, Torsion $T^\sigma{}_{\mu\nu} = \Gamma^\sigma{}_{\nu\mu} - \Gamma^\sigma{}_{\mu\nu}$

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Thank you for your attention

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