The energy dependence of p_T-integrated quarkonium cross sections [1]

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Fixed-target experinemts at the LHC
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[1] Based on JHEP 05 (2022) 083 [hep-ph/2112.06789] and ongoing work
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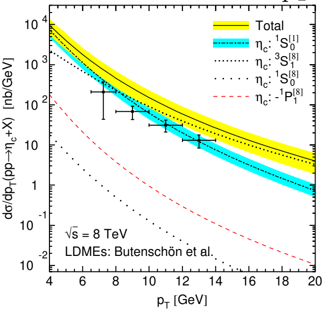
(Unexpected) ${}^1S_0^{(1)}$-dominance in η_c-production

The η_c hadroproduction was found to be dominated by the $c\bar{c}\left[{}^1S_0^{(1)}\right]$ state **for all values of** p_T:



[Butenschoen, He, Kniehl, 2015] This is a problem for NRQCD factorization, because roughly the same contribution of color-octet states **at** $p_T > M$ as for J/ψ was expected.

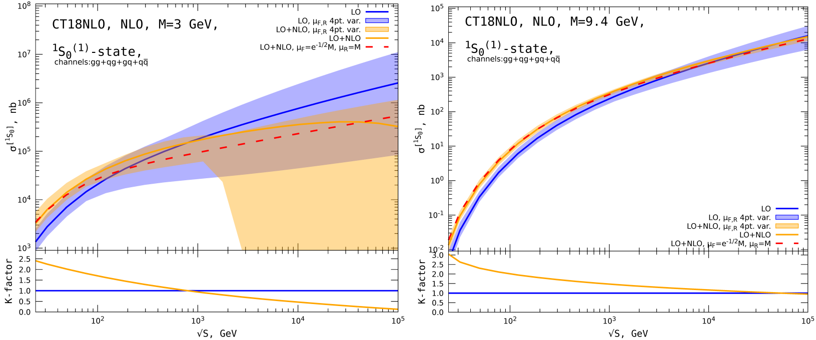
- Color-octet LDMEs for η_c are related (up to v^2 corrections) to LDMEs of J/ψ by *heavy-quark spin symmetry* (long wavelength gluons do not "see" heavy quark spin). Strong HQSS violation?
- But CS-dominance **at** $p_T \ll M$ is consistent with NRQCD! $\Rightarrow$
  - TMD factorisation should hold
  - p_T-integrated cross section should be computable
- For J/ψ and χ_c-production large CO contribution to p_T-integrated cross sections is expected

Perturbative instability of the η_c total cross section

For the p_T**-integrated** cross section of η_Q hadroproduction ($p + p \to \eta_c + X$), the LO partonic subprocess is simply:

$$g + g \to Q\bar{Q}\left[{}^1S_0^{(1)}\right].$$

The NLO correction can be computed in closed form [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'], and:



Outline

1. η_c inclusive hadroproduction, resummation of: $\alpha_s^n \ln^{n-1} \frac{\hat{s}}{M^2}$ and matching with NLO
   - Total cross section
   - Rapidity-differential cross section
2. J/ψ **inclusive** photoproduction

Part 1: η_c or η_b inclusive hadroproduction: total cross section

Why NLO calculation is unstable?

Collinear factorization for total CS for the state $m = ^{2S+1}L_J^{(0)}$:

$$\sigma^{[m]}(\sqrt{S}) = \int\limits_{z_{\min}}^{1} \frac{dz}{z} \mathcal{L}_{ij}(z, \mu_F) \hat{\sigma}_{ij}^{[m]}(z, \mu_F, \mu_R),$$

where $i, j = q, \bar{q}, g$, $\boxed{z = M^2/\hat{s}}$ and *partonic luminosity*:

$$\mathcal{L}_{ij}(z, \mu_F) = \int\limits_{-y_{\max}}^{+y_{\max}} dy\ \tilde{f}_i\left(\frac{M}{\sqrt{Sz}} e^{y}, \mu_F\right) \tilde{f}_j\left(\frac{M}{\sqrt{Sz}} e^{-y}, \mu_F\right),$$

with $\tilde{f}_j(x, \mu_F)$ – momentum density PDFs.
NLO coefficient function [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'] in the $z \to 0$ limit

$$\hat{\sigma}_{ij}^{[m]} = \sigma_{\rm LO}^{[m]} \left[A_0^{[m]} \delta(1-z) + C_{ij} \frac{\alpha_s(\mu_R)}{\pi} \left(A_0^{[m]} \ln \frac{M^2}{\mu_F^2} + A_1^{[m]} \right) + O(z\alpha_s, \alpha_s^2) \right],$$

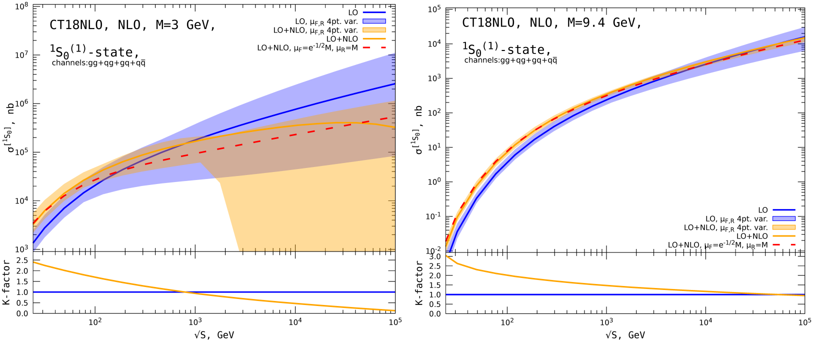
where $C_{gg} = 2C_A = 2N_c$, $C_{qg} = C_{gq} = C_F = (N_c^2 - 1)/(2N_c)$, $C_{q\bar{q}} = 0$ and $A_1^{[m]} < 0$.

Optimal μ_F choice?

It is natural to choose μ_F such a way, that the negative $A_1^{[m]}$ is cancelled [Lansberg, Ozcelik, 2020]:

$$\hat{\mu}_F = M \exp\left[\frac{A_1^{[m]}}{2A_0^{[m]}}\right],$$

is equivalent to resummation of **some of the** terms $\propto \alpha_s^n \ln^{n-1}\frac{1}{z}$ (more on this later). The result (red curve):



Is the systematic resummation of $\propto \alpha_s^n \ln^{n-1}\frac{1}{z}$ possible?

Resumed coefficient function

Small parameter: $\boxed{z = \frac{M^2}{\hat{s}}.}$

LLA ($\alpha_s^n \ln^{n-1} \frac{1}{z}$) in High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:

$$\hat{\sigma}_{ij}^{[m],\ \mathrm{HEF}}(z,\mu_F,\mu_R) = \int\limits_{-\infty}^{\infty} d\eta \int\limits_{0}^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\, \mathcal{C}_{gi}\left(\frac{M_T}{M}\sqrt{z}e^{\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right)$$

$$\times \mathcal{C}_{gj}\left(\frac{M_T}{M}\sqrt{z}e^{-\eta}, \mathbf{q}_{T2}^2, \mu_F, \mu_R\right) \int\limits_{0}^{2\pi} \frac{d\phi}{2} \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4} + O(z) + \mathrm{NLL},$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)}, {}^3P_J^{(1,8)}, {}^3S_1^{(8)}$.
The $H^{[m]}$ is a tree-level "squared matrix element" of the $2 \to 1$-type process:

$$R_+(\mathbf{q}_{T1}, q_1^+) + R_-(\mathbf{q}_{T2}, q_2^-) \to c\bar{c}[m].$$

LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(x, \mathbf{q}_T) = \delta(1-z)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int\limits_x^1 \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{x}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{p}_T^2) = \delta^{(2-2\epsilon)}(\mathbf{k}_T)\, \frac{(\mathbf{p}_T^2)^{-\epsilon}}{\epsilon} \frac{(4\pi)^{\epsilon}\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2}.$$

It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T\; e^{i\mathbf{x}_T\mathbf{q}_T} \int\limits_0^1 dx\; x^{N-1}\; \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \to \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^{k} \frac{1}{z} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Exact LL solution and DLA

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ has the form:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, 82']:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \dots}_{\text{LLA}}$$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Does this work?

The resummation has to reproduce the $A_1^{[m]}$ NLO coefficient when expanded up to NLO in α_s. And it does. We have performed expansion up to NNLO:

State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
$^1S_0$	1	$\mathbf{-1}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
$^3S_1$	0	1	0	$\frac{\pi^2}{6}$
$^3P_0$	1	$\mathbf{-\frac{43}{27}}$	$\frac{\pi^2}{6}+\frac{2}{3}$	$\frac{\pi^2}{6}+\frac{40}{27}$
$^3P_1$	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
$^3P_2$	1	$\mathbf{-\frac{53}{36}}$	$\frac{\pi^2}{6}+\frac{1}{2}$	$\frac{\pi^2}{6}+\frac{11}{9}$

for e.g.

$$\hat{\sigma}_{gg}^{[m],\,\mathrm{HEF}}(z\to 0)=\sigma_{\mathrm{LO}}^{[m]}\left\{A_0^{[m]}\delta(1-z)+\frac{\alpha_s}{\pi}2C_A\left[A_1^{[m]}+A_0^{[m]}\ln\frac{M^2}{\mu_F^2}\right]\right.$$
$$\left.+\left(\frac{\alpha_s}{\pi}\right)^2\ln\frac{1}{z}\cdot C_A^2\left[2A_2^{[m]}+B_2^{[m]}+4A_1^{[m]}\ln\frac{M^2}{\mu_F^2}+2A_0^{[m]}\ln^2\frac{M^2}{\mu_F^2}\right]+O(\alpha_s^3)\right\},$$

Matching with NLO of CF

The HEF works only at $z \ll 1$, misses power corrections $O(z)$, while NLO CF is exact in z, but only NLO in α_s. **We need to match them.**

- Simplest prescription: just subtract the overlap at $z \ll 1$:

$$\sigma^{[m]}_{\rm NLO+HEF} = \sigma^{[m]}_{\rm LO\ CF} + \int\limits_{z_{\rm min}}^{1} \frac{dz}{z} \left[\check{\sigma}^{[m],ij}_{\rm HEF}(z) + \hat{\sigma}^{[m],ij}_{\rm NLO\ CF}(z) - \hat{\sigma}^{[m],ij}_{\rm NLO\ CF}(0) \right] \mathcal{L}_{ij}(z),$$

- Or introduce **smooth weights**:

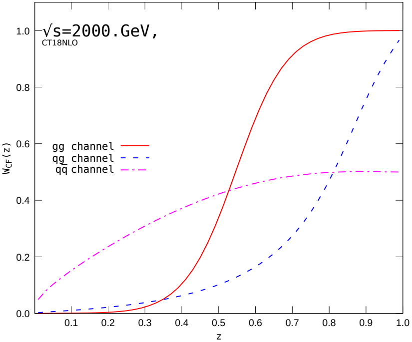
$$\sigma^{[m]}_{\rm NLO+HEF} = \sigma^{[m]}_{\rm LO\ CF} + \int\limits_{z_{\rm min}}^{1} dz \left\{ \left[\check{\sigma}^{[m],ij}_{\rm HEF}(z) \frac{\mathcal{L}_{ij}(z)}{z} \right] w^{ij}_{\rm HEF}(z) + \left[\hat{\sigma}^{[m],ij}_{\rm NLO\ CF}(z) \frac{\mathcal{L}_{ij}(z)}{z} \right] (1 - w^{ij}_{\rm HEF}(z)) \right\},$$

Inverse error weighting method

In the InEW method [Echevarria, *et.al.*, 2018] the weights are calculated from **estimates of the error** of each contribution:

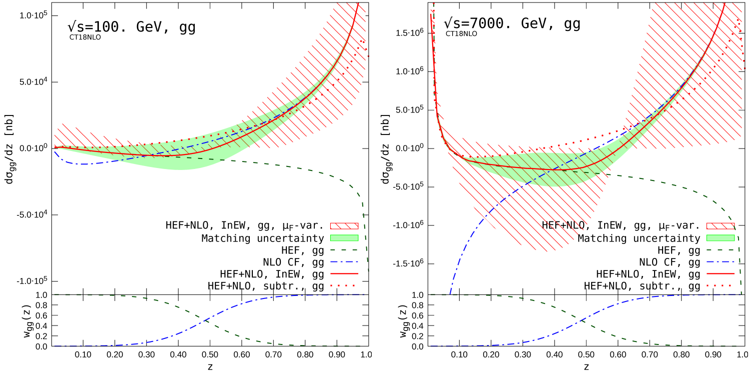
$$w^{ij}_{\rm HEF}(z) = \frac{[\Delta\sigma^{ij}_{\rm HEF}(z)]^{-2}}{[\Delta\sigma^{ij}_{\rm HEF}(z)]^{-2} + [\Delta\sigma^{ij}_{\rm CF}(z)]^{-2}},$$

- For $\Delta\sigma_{\rm CF}$ we take the NNLO $\alpha_s^2 \ln\frac{1}{z}$ term of $\hat{\sigma}(z)$ predicted by HEF,
- For $\Delta\sigma_{\rm HEF}$ we take the $\alpha_s O(z)$ part of the NLO CF result for $\hat{\sigma}(z)$.
- In both cases, stability against $O(\alpha_s^2)$ (constant in z, unknown) corrections is checked

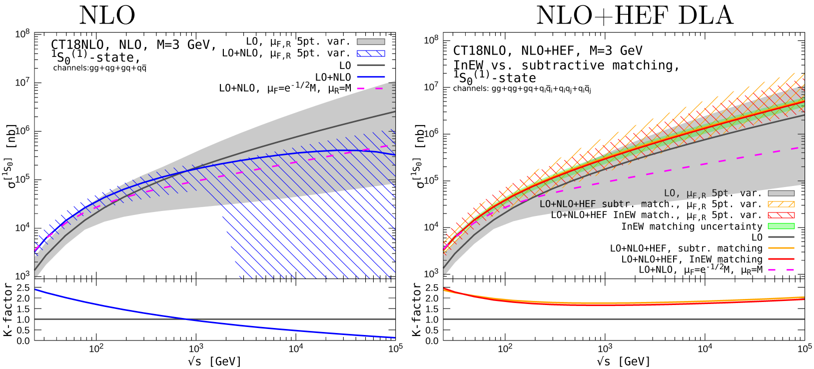


Matching plots

Plots of the integrand of the total cross section (gg channel) as function of $z = M^2/\hat{s}$:



Matched results for η_c

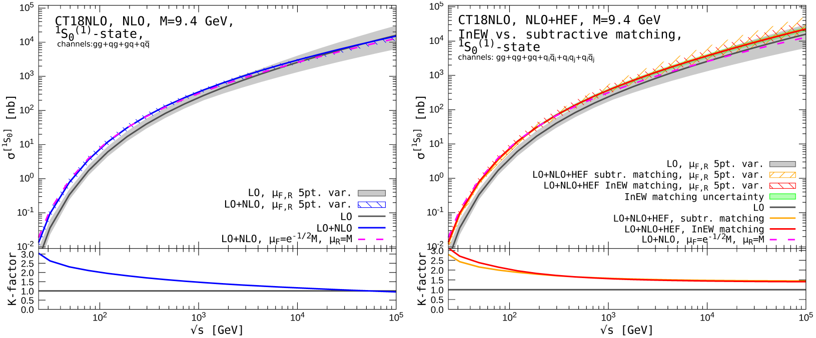


- Resummation and matching solves the high-energy instability
- The matched result is significantly different from $\hat{\mu}_F$-prescription one (dashed line)
- The description is valid in wide range of energies: **data from both fixed-target ($\sqrt{s} \leq 100$ GeV) and collider-mode ($\sqrt{s} \geq 1$ TeV) pp-collisions are required to test it!**
- However the **y-integrated cross section** is hard to measure at high energies. Predictions for $d\sigma/dy$ are needed.

Matched results for η_b

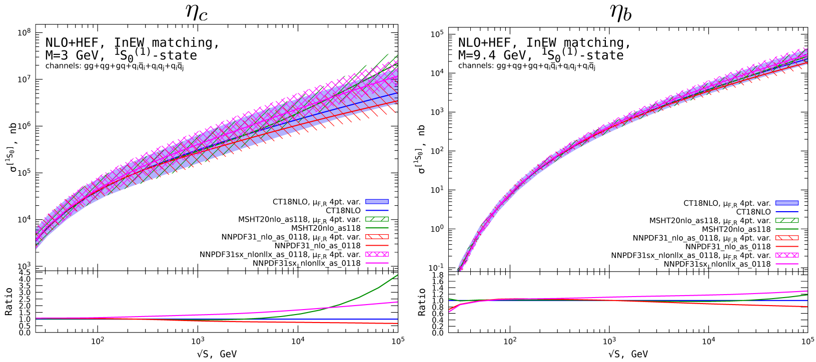
NLO

NLO+HEF DLA



The K-factor of the matched calculation is constant in energy, unlike in the NLO case. $\Rightarrow \sigma(\sqrt{s})$ is driven by PDFs, not by hard-scattering.

The PDF dependence



There is a spread up to a factor $1.5 - 2$ for different PDFs at $\sqrt{s} \sim 10$ TeV, but the scale-uncertainty covers it $\Rightarrow$ Beyond-DLA calculation is needed to reduce the uncertainty.

Part 2: η_c or η_b inclusive hadroproduction: $d\sigma/dy$

NLO Collinear factorisation for $d\sigma/dy$

Contribution of the gg-channel [Lansberg, Ozcelik, 2020], terms important at $z \to 0$ (for $y > 0$):

$$\frac{d\sigma_{\rm NLO}}{dy} = \sigma_0 \frac{\alpha_s C_A}{\pi} \left\{ \int\limits_{z_0}^{z_1} \frac{dz}{z} \int\limits_{w_{\min}(z,y)}^{w_{\max}(z,y)} \frac{dw}{1-w^2} 2\mathcal{L}(z,w,y) a_1(z,w) \right.$$

$$+ \int\limits_{z_1}^{z_2} \frac{dz}{z} \left[\int\limits_{w_{\min}(z,y)}^{1} dw \frac{\mathcal{L}(z,w,y)a_1(z,w) - \mathcal{L}(z,1,y)}{1-w} + \mathcal{L}(z,1,y) \ln \frac{1-w_{\min}(z,y)}{2} \right]$$

$$\left. + \int\limits_{z_2}^{1} \frac{dz}{z} \int\limits_{-1}^{1} dw \left[\frac{\mathcal{L}(z,w,y)a_1(z,w) - \mathcal{L}(z,1,y)}{1-w} + \frac{\mathcal{L}(z,w,y)a_1(z,w) - \mathcal{L}(z,-1,y)}{1+w} \right] + \dots \right\}$$

where

$$\mathcal{L}(z,w,y) = \tilde{f}_g\left(\sqrt{\frac{z_0}{z}\frac{(1+z)-w(1-z)}{(1+z)+w(1-z)}}e^{y}, \mu_F\right) \tilde{f}_g\left(\sqrt{\frac{z_0}{z}\frac{(1+z)+w(1-z)}{(1+z)-w(1-z)}}e^{-y}, \mu_F\right),$$

$$\boxed{a_1^{(\rm NLO)}(z,w) = \frac{z^2(3+w^2)^2(w^4+6w^2+9)}{16[(1+z)^2 - w^2(1-z)^2]^2} + O(z),} \quad z_0 = \frac{M^2}{S},\ z_1 = \frac{M}{\sqrt{S}}e^{-y},$$

$$z_2 = \frac{M}{\sqrt{S}}e^{y},\ w_{\min}(z,y) = -\frac{1+z}{1-z}\frac{ze^{-2y}-z_0}{ze^{-2y}+z_0},\ w_{\max}(z,y) = \frac{1+z}{1-z}\frac{z-z_0e^{-2y}}{z+z_0e^{-2y}}.$$

HEF resummation formula for $d\sigma/dy$

$$\frac{d\sigma^{[m]}}{dy} = \int_{z_0}^{1} \frac{dz}{z} \int_{\bar{w}_{\min}(z,y)}^{\bar{w}_{\max}(z,y)} \frac{d\bar{w}}{1-\bar{w}^2} \mathcal{L}(z,\bar{w},y)\mathcal{H}^{[m]}(z,\bar{w}),$$

where $\mathcal{L}(z,\bar{w},y) = \tilde{f}_g\left(\sqrt{\frac{z_0}{z}\frac{1-\bar{w}}{1+\bar{w}}}e^{y},\mu_F\right)\tilde{f}_g\left(\sqrt{\frac{z_0}{z}\frac{1+\bar{w}}{1-\bar{w}}}e^{-y},\mu_F\right)$,
$\bar{w}_{\min}(z,y) = -\frac{ze^{-2y}-z_0}{ze^{-2y}+z_0}$, $\bar{w}_{\max}(z,y) = \frac{z-z_0e^{-2y}}{z+z_0e^{-2y}}$, and:

$$\begin{aligned} \mathcal{H}^{[m]}(z,\bar{w}) &= \int_0^{2\pi} \frac{d\phi}{2} \int_0^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\ \mathcal{C}_{gg}\left(\sqrt{\frac{zM_T^2}{M^2}\frac{1-\bar{w}}{1+\bar{w}}},\mathbf{q}_{T1}^2,\mu_F,\mu_R\right) \\ &\times\ \mathcal{C}_{gg}\left(\sqrt{\frac{zM_T^2}{M^2}\frac{1+\bar{w}}{1-\bar{w}}},\mathbf{q}_{T2}^2,\mu_F,\mu_R\right)\frac{H^{[m]}(\mathbf{q}_{T1},\mathbf{q}_{T2})}{M_T^4}, \end{aligned}$$

where $M_T^2 = M^2 + (\mathbf{q}_{T1}+\mathbf{q}_{T2})^2$.

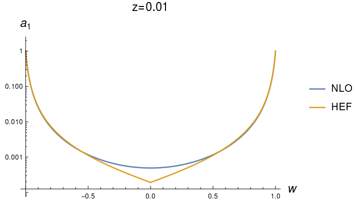
Expanding the resummation up to NLO

Cross-check $\int\limits_{-\infty}^{+\infty} dy \frac{d\sigma}{dy} = \sigma \Leftrightarrow$

$$\int\limits_{-1}^{1} \frac{d\bar{w}}{1-w^2} \mathcal{H}_{\rm NLO}^{[^1S_0]}(z,\bar{w}) = \sigma_0^{[^1S_0]} \frac{2C_A\alpha_s}{\pi} \left[-1 + \ln\frac{M^2}{\mu_F^2} + O(z) \right].$$

The $d\sigma^{\rm (HEF)}/dy$, expanded uo to NLO in α_s, can be put into the same form as NLO partonic cross section on slide 17, but with slightly different function a_1:

$$a_1^{\rm (HEF)}(z,w) = \frac{2z^2 \left[(1+w)^3\theta(w) + (1-w)^3\theta(-w) \right]}{[(1+z)^2 - w^2(1-z)^2]^2}$$



The difference between $a_1^{\rm (NLO)}$ and $a_1^{\rm (HEF)}$ leads to $O(z)$-effects.
Should the matching be done in $2D$ (z and w)?

Part 3: J/ψ **inclusive** photoproduction

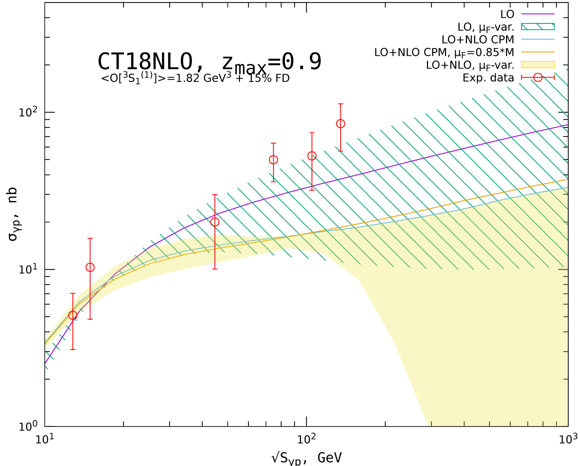
$\gamma(q) + p(P) \to J/\psi(p) + X$ @ NLO

The LO CS partonic process is

$$\gamma + g \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g.$$

The CS contribution is $> 50\%$ of p_T-diff. cross section even at NLO.

For p_T**-integrated** CS one has ($z = \frac{p_{J/\psi}P}{q_\gamma P}$):



J/ψ photoproduction at FT LHC?

Table from [Lansberg *et al.*, 2018]

System	$\sqrt{s_{NN}}$ (GeV)	$\mathcal{L}_{AB}{}^5$ ($\text{pb}^{-1}\text{yr}^{-1}$)	$E_{\gamma\ \max}^{\text{B rest}}$ (GeV)	$\sqrt{s_{\gamma_N}^{\max}}$ (GeV)	$E_{\gamma\ \max}^{\text{cms}}$ (GeV)
AFTER@LHC					
$p\text{H}^{\uparrow}$	115	1.0×10^4	1050	44	8.6
$p\text{D}^{\uparrow}$	115	1.1×10^4	520	30	4.2
$p^3\text{He}^{\uparrow}$	115	3.7×10^4	520	30	4.2
$\text{PbH}^{\uparrow}$	72	0.12	74	12	0.97
$\text{PbD}^{\uparrow}$	72	8.8×10^{-2}	62	11	0.82
$\text{Pb}^3\text{He}^{\uparrow}$	72	8.3×10^{-2}	62	11	0.82
RHIC (STAR)					
$p^{\uparrow}p^{\uparrow}$ (2017)	510	400	3190	77	15
$\text{Au}^{\uparrow}p^{\uparrow}$ (2023)	200	1.75	570	33	2.7

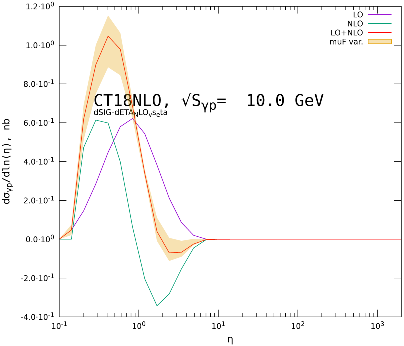
But could the cuts on $z = \frac{p_{J/\psi}P}{q_\gamma P}$ be made?

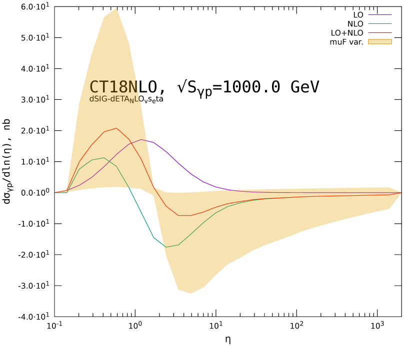
Why?

$$\frac{d\sigma_{\gamma p}}{dz} = \frac{M^2}{S_{\gamma p}} \int_0^{S_{\gamma p}/M^2-1} d\eta \; f_i\left(\frac{M^2}{S_{\gamma p}}(\eta+1), \mu_F\right) \frac{d\hat{\sigma}_{i\gamma}(\eta, z)}{dz},$$

where $\boxed{z = \frac{Pp}{Pq} = \frac{p^-}{q^-}, \; \eta = \frac{\hat{s}}{M^2} - 1}$ with $\hat{s} = S_{\gamma p} x$.

Plots of the integrand:



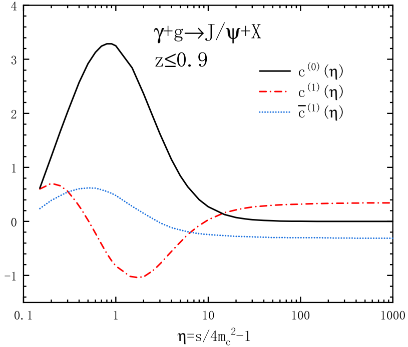


Asymptotics $\hat{\sigma}_{\rm NLO}(\eta \to \infty)$

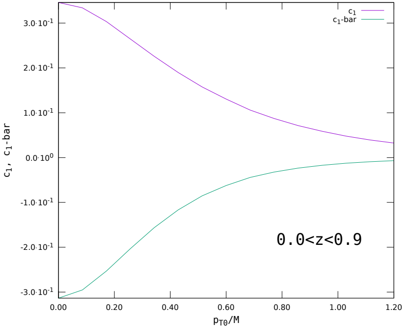
[Kraemer, 1995]:

$$\hat{\sigma}_{i\gamma} \propto c^{(0)}_{i\gamma}(\eta) + 4\pi\alpha_s \left[c^{(1)}_{i\gamma}(\eta) + \bar{c}^{(1)}_{i\gamma}(\eta) \ln \frac{\mu_F^2}{m_c^2} + \frac{\beta_0}{8\pi^2} c^{(0)}_{i\gamma}(\eta) \right]$$

Numerical NLO result (**FDC** code, Yu Feng)



$c^{(1)}$ and $\bar{c}^{(1)}$ at $\eta \to \infty$ from HEF as function of $p_{T\,\min}^{J/\psi}$



$\gamma(q) + p(P) \to J/\psi(p) + X$ in HEF

HEF resummed partonic cross section:

$$\frac{d\hat{\sigma}_{i\gamma}^{\rm HEF}(\eta,z)}{dz} = \frac{1}{2zM^2} \int\limits_{1/z}^{1+\eta} \frac{dy}{y} \int\limits_{0}^{\infty} d\mathbf{q}_T^2\, \mathcal{C}_{gi}\left(\frac{y}{\eta+1}, \mathbf{q}_T^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_T^2, z),$$

actually resums $\ln\frac{1}{z_+} = \ln\frac{\eta+1}{y}$. *Is resummation of only* $\ln(1+\eta)$ *possible? Yes.*

The $\mathcal{H}$ is the integral of the HEF coefficient function (H):

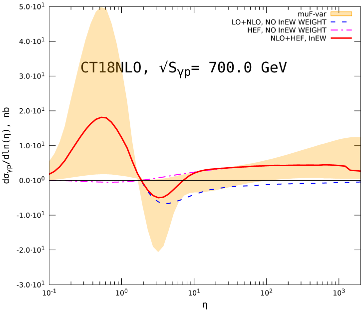
$$R_+(\mathbf{q}_{T1}, q_1^+) + \gamma(q) \to c\bar{c}\left[{}^3S_1^{(1)}\right](p) + g(k),$$

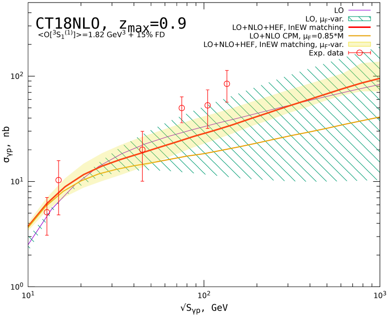
over the PS of the gluon ($y = q_1^+ q^-/M^2$):

$$\mathcal{H}(y, \mathbf{q}_{T1}^2, z) = \int\limits_0^\infty \frac{dk^+ dk^-}{2(2\pi)^2} \int d^2\mathbf{p}_T\ H(\hat{s}, \hat{t}, \hat{u}, (\mathbf{q}_{T1}\cdot\mathbf{p}_T), \mathbf{q}_{T1}^2)$$

$$\times\delta(q^-(1-z) - k^-)\delta(q_1^+ - \frac{M_T^2}{q_- z} - k^+)\delta(k^+k^- - (\mathbf{q}_{T1} - \mathbf{p}_T^2)^2),$$

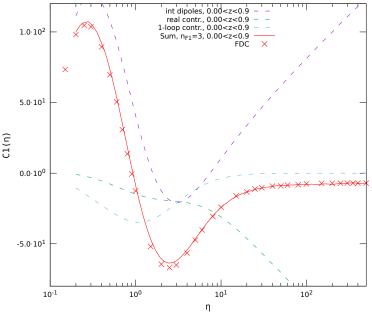
InEW matching results



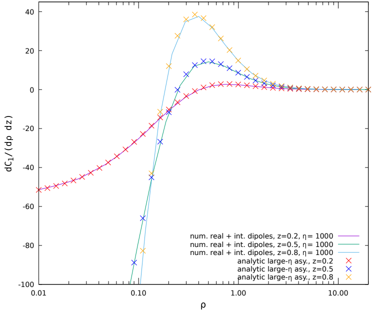


Cross-check with our “in-house” NLO code

We have developed the numerically-efficient NLO code for calculation of *scaling-functions* c_1 and $\bar{c}_1$ which is based on the **Catani-Seymour dipole-subtraction formalism** (as opposed to **phase-space slicing** used by **FDC**). The validation of the code is in progress:



Comparison of $c_1(\eta)$ with the FDC result (crosses). Small systematic difference is still present at $\eta < 1$.



Comparison of $dc_1/dzd\rho$ at $\eta = 1000$ against analytic $\eta \to \infty$ asymptotics from HEF (crosses). Strong cross-check of analytics and the NLO code.

Analytic results for $\eta \to \infty$ asymptotics of $d\hat{\sigma}/dp_T$

Can be derived via expansion of HEF formula up to NLO and applying **IBP-reduction** to $\mathbf{q}_T$-integrals.

$$\frac{dc_1(z,\rho,\eta\to\infty)}{dzd\rho} = c_1^{(\mathrm{R})}(z,\rho)$$

$$+c_1^{(1)}(z,\rho)\ln\left[\frac{z^2(1-z)^2}{(\rho+(1-z)^2)^2}\right] + c_1^{(2)}(z,\rho)\ln\left[\frac{(\rho+1-z)^2}{(1-z)(\rho+2-z)}\right]$$

$$+\frac{c_1^{(3)}(z,\rho)}{\sqrt{(1+\rho)((2-3z)^2+(2-z)^2\rho)}}$$

$$\times\ln\left[\frac{\rho(2-z)-(3-2z)z+2-\sqrt{(\rho+1)\left(\rho(z-2)^2+(2-3z)^2\right)}}{\rho(2-z)-(3-2z)z+2+\sqrt{(\rho+1)\left(\rho(z-2)^2+(2-3z)^2\right)}}\right],$$

where $\rho = \mathbf{p}_T^2/M^2$ and

$$c_1^{(1)}(z,\rho) = \frac{-z^3}{(\rho+1)^2\left(\rho+(z-1)^2\right)^2(\rho-2z+1)^4}$$

$$\times\left\{5(\rho+1)^4+4(2\rho+1)z^6-(\rho+1)(23\rho+31)z^5+(\rho+1)(\rho(12\rho+77)+89)z^4\right.$$

$$-2(\rho+1)(\rho+3)(\rho(\rho+18)+21)z^3+2(\rho+1)^2(\rho(3\rho+32)+47)z^2$$

$$\left.-(\rho+1)^3(11\rho+35)z\right\},$$

and so on...

Conclusions and outlook

- The high-energy instability of the NLO cross section is related with lack of the $\alpha_s^n \ln^{n-1} \hat{s}/M^2$ corrections in $\hat{\sigma}$ at $\hat{s} \gg M^2$.
- The HEF at DLA is the formalism to solve this problem if the standard fixed-order PDFs are to be used.
- Matching between NLO CF ($\hat{s} \sim M^2$) and HEF($\hat{s} \gg M^2$) has been performed for η_c inclusive hadroproduction and J/ψ inclusive photoproduction **total cross sections**.
- The matched calculation for $d\sigma/dy$ is in progress.
- Future plans (6 months):
  - χ_{cJ} production in DLA+NLO CF (relevant for FT LHC phenomenology),
  - p_T-distribution of J/ψ **in photoproduction** in DLA+NLO CF,
- More distant future:
  - Next-to-DLA corrections

Thank you for your attention!

Expanding the resummation up to NLO, part 1

Substituting the Mellin-space expressions for $\mathcal{C}_{gg}$ we obtain (recall that $\gamma_N = \frac{\alpha_s C_A}{\pi N}$):

$$\mathcal{H}^{[m]}(z,\bar{w}) = \int \frac{dN_1 dN_2}{(2\pi i)^2} z^{-\frac{N_1+N_2}{2}} \left(\frac{1-\bar{w}}{1+\bar{w}}\right)^{-\frac{N_1-N_2}{2}} F^{[m]}(N_1,N_2),$$

$$F^{[m]}(N_1,N_2) = \gamma_{N_1}\gamma_{N_2} \int_0^{2\pi} \frac{d\phi}{2} \int_0^{\infty} \frac{d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2}{\mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2} \left(\frac{\mathbf{q}_{T1}^2}{\mu_F^2}\right)^{\gamma_{N_1}} \left(\frac{\mathbf{q}_{T2}^2}{\mu_F^2}\right)^{\gamma_{N_2}}$$
$$\times \frac{H^{[m]}(\mathbf{q}_{T1},\mathbf{q}_{T2})}{M_T^4} \left(\frac{M^2}{M_T^2}\right)^{\frac{N_1+N_2}{2}} = \mathrm{LO} + \frac{\alpha_s C_A}{\pi}\left[\frac{1}{N_1}+\frac{1}{N_2}\right]$$
$$\times \int_0^{2\pi} \frac{d\phi}{2} \int_0^{\infty} \frac{d\mathbf{q}_{T1}^2}{\mathbf{q}_{T1}^2} \left[\frac{H^{[m]}(\mathbf{q}_{T1},0)}{(M^2+\mathbf{q}_{T1}^2)^2}\left(\frac{M^2}{M^2+\mathbf{q}_{T1}^2}\right)^{\frac{N_1+N_2}{2}} - \frac{H^{[m]}(0,0)}{M^4}\theta(\mu_F^2-\mathbf{q}_{T1}^2)\right]$$
$$+O(\alpha_s^2),$$

for $m = {}^1S_0$: $H^{[{}^1S_0]}(\mathbf{q}_{T1},0) = \frac{M^4 \sigma_0^{[{}^1S_0]}}{\pi} 2\sin^2\phi$.

Expanding the resummation up to NLO, part 2

$$F_{\rm NLO}^{[^1S_0]}(N_1,N_2)=\sigma_0^{[^1S_0]}\frac{\alpha_s C_A}{\pi}\frac{N_1+N_2}{N_1N_2}\int\limits_0^\infty\frac{d\mathbf{q}_{T1}^2}{\mathbf{q}_{T1}^2}\left[\left(\frac{M^2}{M^2+\mathbf{q}_{T1}^2}\right)^{2+\frac{N_1+N_2}{2}}-\theta(\mu_F^2-\mathbf{q}_{T1}^2)\right]$$

$$=\sigma_0^{[^1S_0]}\frac{\alpha_s C_A}{\pi}\left[\frac{1}{N_1}+\frac{1}{N_2}\right]\left[\ln\frac{M^2}{\mu_F^2}-\left(\gamma_E+\psi\left(2+\frac{N_1+N_2}{2}\right)\right)\right].$$

Using the fact that $-\gamma_E-\psi(N)=\int\limits_0^1 dx\frac{x^{N-1}}{(1-x)_+}$ we can express:

$$\int\limits_{\bar{w}_{\rm min}}^{\bar{w}_{\rm max}}\frac{d\bar{w}}{1-\bar{w}^2}\mathcal{H}_{\rm NLO}^{[^1S_0]}(z,\bar{w})\mathcal{L}(z,\bar{w},y)=\sigma_0^{[^1S_0]}\frac{\alpha_s C_A}{\pi}\left\{\int\limits_{\bar{w}_{\rm min}}^{\bar{w}_{\rm max}}d\bar{w}\Delta(z,\bar{w})\mathcal{L}(z,\bar{w},y)\ln\frac{M^2}{\mu_F^2}\right.$$

$$\left.+\mathcal{F}\left[\mathcal{L}(z,\bar{w},y)\theta(\bar{w}-\bar{w}_{\rm min})\theta(\bar{w}_{\rm max}-\bar{w})\right]\right\},$$

where $\Delta(z,w)=\delta\left(w-\frac{1-z}{1+z}\right)+\delta\left(w+\frac{1-z}{1+z}\right)$ and the functional

$$\mathcal{F}[f]=2z^2\int\limits_{-1}^{1}\frac{d\bar{w}}{1-\bar{w}^2}\left\{\frac{(1-\bar{w})\theta\left(\frac{1-z}{1+z}+\bar{w}\right)}{(1+\bar{w})-z(1-\bar{w})}\left[\frac{1-\bar{w}}{1+\bar{w}}\theta(-\bar{w})f(\bar{w})-\frac{1}{z}f\left(-\frac{1-z}{1+z}\right)\right]\right.$$

$$\left.+\frac{(1+\bar{w})\theta\left(\frac{1-z}{1+z}-\bar{w}\right)}{(1-\bar{w})-z(1+\bar{w})}\left[\frac{1+\bar{w}}{1-\bar{w}}\theta(\bar{w})f(\bar{w})-\frac{1}{z}f\left(\frac{1-z}{1+z}\right)\right]\right\}.$$