Starting the Universe from strong gravity regime

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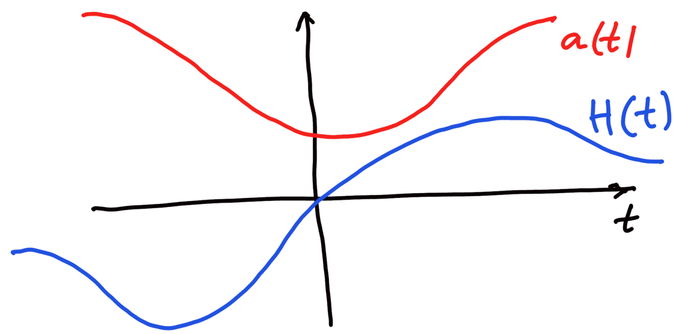
Introduction

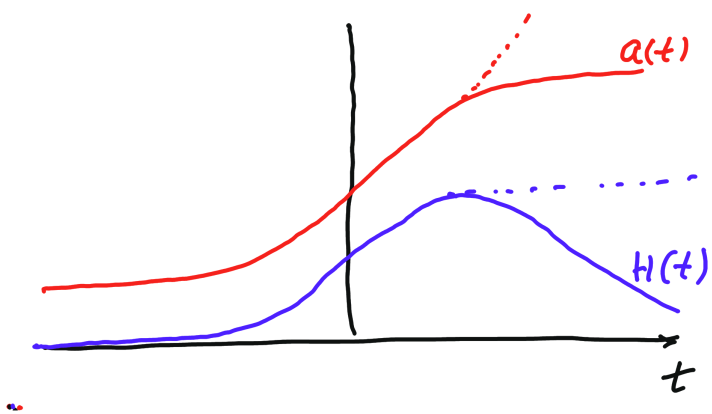
Homogeneous isotropic Universe without initial singularity

Bounce



Genesis

Creminelli, Nicolis, Trincherini' 2010



Motivation

- Curiuosity. Always good to have alternatives even to compelling scenarios like inflation.
- No initial singularity.
- Horizon, curvature problems "solved" by assumption about initial state.
- Very long prehistory without matter energy density $\Longrightarrow$ useful for relaxing the cosmological constant

V.R. '99;
Mukohyama, Randall '2003

DRAWBACK

Generation of (nearly) flat power power spectrum of scalar perturbations not so automatic as compared to in inflation

Obstacle in classical GR (if spatial curvature negligible): both bounce and Genesis need exotic matter which violates the Null Energy Condition,

$$T_{\mu\nu} n^{\mu} n^{\nu} > 0 \, , \quad n_{\mu} n^{\mu} = 0$$

i.e. has $p < -\rho$; where $\rho = T^0_0$, energy density; $p = T^1_1 = T^2_2 = T^3_s$, effective pressure.

Generalization to modified gravities (including scalar-tensor): the Null Convergence Condition, $R_{\mu\nu} n^{\mu} n^{\nu} > 0$

Penrose theorem for expanding Universe: if NCC holds and spatial curvature negligible, then there was a singularity in the past, $H = \infty$.

NCC can be violated in Horndeski theories – scalar-tensor gravities with fields π and $g_{\mu\nu}$.

Straightforward to construct healthy bounce or genesis stages.

No-Go

However, problem with “complete cosmologies”: $-\infty < t < +\infty$

No-go in Horndeski! Libanov, Mironov, V.R.’ 16; Kobayashi’ 16

General Horndeski theory

Require: both “Einstein” equations and π-field equation second order

Four arbitrary functions of π and $X = (\partial\pi)^2$:
$F \equiv G_2;\ K \equiv G_3;\ G_4;\ G_5$

Horndeski’ 1974; Deffayet, Esposito-Farese, Vikman’ 09

$$L = F(\pi, X) - K(\pi, X)\Box\pi$$
$$+ G_4(\pi, X)R + G_{4,X} \cdot \left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$
$$+ G_5 \cdot G^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \frac{1}{6}G_{5,X} \cdot \left[(\Box\pi)^3 - 3\Box\pi \cdot (\nabla_\mu\nabla_\nu\pi)^2 + 2(\nabla_\mu\nabla_\nu\pi)^3\right]$$

- Modified gravity (scalar-tensor).
- NB: always in Jordan frame.

No-go theorem for Genesis in Horndeski: gradient/ghost instability at some stage (which may be quite late)

Libanov, Mironov, V.R.' 16; Kobayashi' 16

Choose unitary gauge $\delta\pi = 0$.

$$ds^2 = N^2 dt^2 - a^2 e^{2\zeta} (\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj})(N^i dt + dx^i)(N^j dt + dx^j)$$

Dynamical variables: transverse traceless h_{ij} and ζ (lapse δN and shift N^i are not dynamical, as usual).

Upon solving for constraints, find quadratic Lagrangians for perturbations

$$L_S = \mathcal{G}_{\mathcal{S}} \dot{\zeta}^2 - a^{-2} \mathcal{F}_{\mathcal{S}} (\partial_i \zeta)^2 \,, \quad L_T = \mathcal{G}_{\mathcal{T}} \dot{h}_{ij}^2 - a^{-2} \mathcal{F}_{\mathcal{T}} (\partial_k h_{ij})^2$$

NB: $\mathcal{G}_{\mathcal{T}}$, $\mathcal{F}_{\mathcal{T}}$ = effective M_{Pl}^2.
Stable background $\Longleftrightarrow$ $\mathcal{G}_{\mathcal{T}}, \mathcal{F}_{\mathcal{T}}, \mathcal{G}_{\mathcal{S}}, \mathcal{F}_{\mathcal{S}} > 0$.

To simplify: $G_5 = 0$. Tensor sector:

$$\mathscr{G}_{\mathscr{T}} = 2G_4 - 4G_{4X}X,$$
$$\mathscr{F}_{\mathscr{T}} = 2G_4$$

Scalar sector:

$$\mathscr{G}_{\mathscr{S}} = \frac{\Sigma \mathscr{G}_{\mathscr{T}}^2}{\Theta^2} + 3\mathscr{G}_{\mathscr{T}},$$
$$\mathscr{F}_{\mathscr{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathscr{F}_{\mathscr{T}},$$
$$\xi = \frac{a\mathscr{G}_{\mathscr{T}}^2}{\Theta}.$$

Where

$$\Theta = -K_X X\dot{\pi} + 2G_4 H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi}$$
$$\Sigma = F_X X + 2F_{XX}X^2 + 12HK_X X\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2$$
$$-6H^2 G_4 + 42H^2 G_{4X}X + 96H^2 G_{4XX}X^2 + 24H^2 G_{4XXX}X^3 - 6HG_{4\pi}\dot{\pi}$$
$$-30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi}$$

Key relation (by explicit calculation in general Horndeski)

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

$$\xi = \frac{a(t)\mathscr{G}_{\mathscr{T}}{}^2(t)}{\Theta(t)}$$

where $\Theta(t) = -2HG_4 + \dot{\pi}XK_X + \ldots$, a complicated expression.

Main property: ξ never crosses zero ($\Theta = \infty$ is a singularity).

$$\xi(t_f) - \xi(t_i) = \int_{t_i}^{t_f} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

Impossible for $\mathscr{F}_{\mathscr{S}} > 0$, $\mathscr{F}_{\mathscr{T}} > 0$, and

$$\int_{-\infty}^{t_f} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty\,, \quad \int_{t_i}^{+\infty} dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty$$

Recall that $a(t) \to \infty/\mathrm{const}$ as $t \to -\infty$ and $a(t) \to \infty$ as $t \to +\infty$ for bounce/Genesis No-go

$$\xi(t) - \xi(0) = \int_0^t dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) \implies \xi(t) \to +\infty \text{ as } t \to +\infty$$

$$\xi(0) - \xi(t) = \int_t^0 dt\ a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) \implies \xi(t) \to -\infty \text{ as } t \to -\infty$$

Thus, $\xi(t)$ crosses zero, QED.

Strong gravity in the past

- One way: give up Horndeski for beyond Horndeski and DHOST
  No-go theorem no longer holds

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016

Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017

Beyond Horndeski and DHOST have their own superluminality problem

Mironov, VR, Volkova ' 2020

- Yet another way out, still in Horndeski.
  Example: bounce, $a(t) \to \infty$ as $t \to -\infty$>

$\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \to 0$ as $t \to -\infty$, so that

Kobayashi '2016; Ijjas, Steinhardt '2016

$$\int_{-\infty}^{t_f} dt \; a(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) < \infty$$

No-go theorem does not work.

But gravity tricky as $t \to -\infty$: effective Planck mass vanishes.

Strong coupling?

Simple example: bounce

Explicit Lagrangian later on, such that the field equations admit contracting solution as $t \to -\infty$:

$$a = (-t)^{\alpha} \quad \Longrightarrow \quad H = -\frac{\alpha}{(-t)} \,, \quad \alpha > 0$$

Initial stage of bounce.

Arrange functions in Horndeski Lagrangian in such a way that there is healthy bounce and then, say, kination (domination of massless conventional scalar field π), a là k-inflation.

Fairly straightforward

Ageeva, Petrov, VR ' 2021

Action for perturbations

$$S^{(2)} = \int d^4x \left(\mathscr{G}_{\mathscr{S}} \dot{\zeta}^2 - a^{-2} \mathscr{F}_{\mathscr{S}} (\partial_i \zeta)^2 + \mathscr{G}_{\mathscr{T}} \dot{h_{ij}}^2 - a^{-2} \mathscr{F}_{\mathscr{T}} (\partial_k h_{ij})^2 \right)$$

$$\mathscr{G}_{\mathscr{T}} = \mathscr{F}_{\mathscr{T}} \propto (-t)^{-2\mu} \to 0, \ \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \propto (-t)^{-2\mu} \to 0.$$

- Prerequisite for absence of instabilities

$$\int_{-\infty}^{t_f} dt \ a(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) < \infty \implies 2\mu - \alpha > 1$$

  Can be arranged by appropriate choice of the Lagrangian and $\mu > 1/2$.

- Strong coupling?
  - Cubic terms and higher order terms in perturbations ζ, h_{ij} are also suppressed at large $(-t)$.
    Strong coupling energy scale depends on t
  - Gravity is strong in the past, but does this ruin the whole picture?

Can one trust classical field theory treatment of cosmological evolution?

Energy scale of classical evolution $E_{class} = H$, $\dot{H}/H = (-t)^{-1} \to 0$

How does it compare with strong coupling scales E_{strong} inferred from interactions of ζ and h_{ij}?

Classical treatment of evolution legitimate for $E_{strong} >> E_{class}$ as $t \to -\infty$.

Example (part of the story): tensor sector up to cubic terms.
At given moment of time rescale spatial cordinates to set $a = 1$ (equivalently, work in terms of physical spatial momenta $\vec{p} = \vec{k}/a$).
Then (note that $\mathscr{G}_{\mathscr{T}} = \mathscr{F}_{\mathscr{T}}$)

$$S_{hh}^{(2,3)} = \int d^4x \left(\mathscr{F}_{\mathscr{T}} \dot{h}_{ij}^{\;2} - \mathscr{F}_{\mathscr{T}} (\partial_k h_{ij})^2 + \frac{\mathscr{F}_{\mathscr{T}}}{4} \left(h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl} \right) \partial_k \partial_l h_{ij} \right)$$

To figure out strong coupling energy scale, canonically normalize

$$h_{ij} = h_{ij}^{(c)} / \sqrt{\mathscr{F}_{\mathscr{T}}}$$

$$S^{(2,3)}_{hh} = \int d^4x \left(\dot{h}^{(c)\,2}_{ij} - (\partial_k h^{(c)}_{ij})^2 + \frac{1}{4\sqrt{\mathscr{F}_{\mathscr{T}}}} (h^{(c)}_{ik} h^{(c)}_{jl} - \frac{1}{2} h^{(c)}_{ij} h^{(c)}_{kl}) \partial_k \partial_l h^{(c)}_{ij} \right)$$

Dimension-5 operator "suppressed" by $1/\sqrt{\mathscr{F}_{\mathscr{T}}} \iff$
quantum strong coupling energy scale $E_{strong} = \sqrt{\mathscr{F}_{\mathscr{T}}} \propto (-t)^{-\mu}$

$E_{strong} \to 0$ as $t \to -\infty$, but $E_{strong} \gg E_{class} = (-t)^{-1}$ for $\mu < 1$.
Healthy early bounce stage within classcal field theory at weak coupling.

- This extends to scalar plus tensor sectors and all orders in perturbation theory.

Ageeva, Evseev, Melichev, V.R.' 18, 20;
Ageeva, V.R., Petrov' 20, 21

- "Calculate" action for δN, N^i, h_{ij}, ζ order by order in perturbation theory
- Solve constraint equations for δN, N^i, plug back $\Longrightarrow$ obtain unconstrained action for h_{ij}, ζ
- Canonically normalize $h^{(c)}_{ij} = t^{\mu} h_{ij}$, $\zeta^{(c)} = t^{\mu}\zeta$

Structure of interaction term in Lagrangian for perturbations

$$(\sqrt{-g}\mathscr{L})_{(pq)} = \sum_l \Lambda_l(t) \cdot (\partial)^{c_l} \cdot [\zeta^{(c)}]^p \cdot [h_{ij}^{(c)}]^q$$

Strong coupling scale “on dimensional grounds”

$$E_l(t) = [\Lambda_l(t)]^{-\frac{1}{c_l+p+q-4}}$$

Outcome:

Both cubic and higher order terms generate quantum strong coupling scale with

$$\frac{E_{strong}}{E_{class}} \propto |t|^{\alpha(1-\mu)} \,, \quad \alpha > 0$$

so that

$$E_{strong} \gg E_{class} \quad \text{for} \;\; \mu < 1 \,.$$

In a large region of parameter space, classical field theory treatment of cosmological evolution is legitimate, even though "effective Planck masses squared" $\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \to 0$ as $t \to -\infty$. Viable scenario.

Overall picture: Universe starts at very low quantum gravity scale

$$E_{strong} \propto \frac{1}{|t|} \cdot |t|^{\alpha(1-\mu)} \to 0 \quad \text{as} \;\; t \to -\infty \, ,$$

but expands so slowly that $E_{class} \ll E_{strong}$. Standard Model scales are above E_{strong}. Gravity is the strongest force.

Similar construction works for Genesis

Concrete Lagrangians

Equivalent ADM formulation of Horndeski theory

Gleyzes, Langlois, Piazza, Vernizzi '2014

- Make use of ADM form of metric (perturbations included to all orders)

$$ds^2 = N^2 dt^2 - \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

- Choose unitary gauge with $\pi = t$
- Field variables are all in metric

- Horndeski action reads ($G_5 = 0$, $G_4 = G_4(\pi)$ for brevity)

$$S = \int \sqrt{-g} d^4x \, [A_2(t,N) + A_3(t,N)\mathcal{K} + A_4(t)(\mathcal{K}^2 - \mathcal{K}_j^i \mathcal{K}_i^j - {}^{(3)}R)]$$

where $\mathcal{K} = \gamma^{ij}\mathcal{K}_{ij}$,

$$\mathcal{K}_{ij} \equiv \frac{1}{2N}\left(\dot{\gamma}_{ij} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i\right)$$

is extrinsic curvature of hypersurfaces $t = const$.

$F(\pi,X)$, $K(\pi,X)$, $G_4(\pi) \Longleftrightarrow A_2(t)$, $A_3(t,N)$, $A_4(t,N)$

Conversion formulas:

$$G_2 = A_2 - 2XF_\pi, \quad G_3 = -2XF_X - F, \quad G_4 = A_4,$$

where N and X are related by $N^{-1}d\pi/dt = \sqrt{2X}$ and

$$F_X = -\frac{A_3}{(2X)^{3/2}} - \frac{A_{4\pi}}{X}.$$

- Homogeneous field eqs. are simple

$$(NA_2)_N + 3NA_{3N}H + 6N^2(N^{-1}A_4)_N H^2 = 0\,,$$

$$A_2 - 6A_4H^2 - \frac{1}{N}\frac{d}{dt}(A_3 + 4A_4H) = 0$$

Concrete example of contraction before bounce ($\mu > 1/2$):

$$A_2 = g\cdot(-t)^{-2\mu-2}\cdot\left(-\frac{a_1}{N^2} + \frac{a_2}{N^4}\right)$$

$$A_3 = g\cdot(-t)^{-2\mu-1}\cdot\frac{a_3}{N^3}$$

$$A_4 = -g\cdot\frac{1}{2}(-t)^{-2\mu}\,,\ \text{ as }\ t\to-\infty$$

When converted to covariant Lagrangian formalism

$$G_2 = g\cdot a_1X\cdot e^{2\mu\pi} + g\cdot a_2X^2\cdot e^{(2\mu-2)\pi} + 4g\cdot\mu^2\cdot X\cdot \ln X\cdot e^{2\mu\pi},$$

$$G_3 = g\cdot a_3X\cdot e^{(2\mu-2)\pi} + 2g\cdot\mu e^{2\mu\pi} + g\cdot\mu\cdot\ln X\cdot e^{2\mu\pi},$$

$$G_4 = \frac{1}{2}g\cdot e^{2\mu\pi}.$$

Homogeneous solution

$$N = \text{const}\,, \qquad a = d(-t)^{\alpha}\,, \quad \alpha > 0$$

N and α obey algebraic equations.

Cosmic time in what follows: $Nt \to t$.

Cosmological perturbations

$$\mathscr{S}_{hh} = \int dt d^3x \, \frac{a^3}{8} \left[\mathscr{G}_T \dot{h}_{ij}^2 - \frac{\mathscr{F}_T}{a^2} h_{ij,k} h_{ij,k} \right] ,$$

$$\mathscr{S}_{\zeta\zeta} = \int dt d^3x \, a^3 \left[\mathscr{G}_S \dot{\zeta}^2 - \frac{\mathscr{F}_S}{a^2} \zeta_{,i} \zeta_{,i} \right] .$$

with

$$\mathscr{G}_T = \mathscr{F}_T = \frac{g}{(-t)^{2\mu}}$$

and

$$\mathscr{G}_S = g \frac{g_S}{2(-t)^{2\mu}} , \qquad \mathscr{F}_S = g \frac{f_S}{2(-t)^{2\mu}}$$

Sound speed of scalar perturbations

$$u_S^2 = \frac{\mathscr{F}_S}{\mathscr{G}_S} = \frac{f_S}{g_S} \neq 1 ,$$

while $u_T = 1$ for tensors.

Analogy to power-law inflation

Scale factor $a = |t|^{\alpha}$, $0 < \alpha < 1$.

Conformal time $\eta = \int \frac{dt}{a} = -|t|^{1-\alpha} \Longrightarrow a = |\eta|^{\frac{\alpha}{1-\alpha}}$.

$$\mathscr{S}_{hh} = \int d\eta d^3x \, \frac{a^2}{8} \left[\mathscr{G}_T \cdot \left(\frac{\partial h_{ij}}{\partial \eta} \right)^2 - \mathscr{F}_T h_{ij,k} h_{ij,k} \right]$$

with

$$\sqrt{a^2 \mathscr{G}_T} \equiv a_{eff} = \frac{|\eta|^{\frac{\alpha}{1-\alpha}}}{|\eta|^{\frac{\mu}{1-\alpha}}} = \frac{1}{|\eta|^{\frac{\mu-\alpha}{1-\alpha}}}$$

Effective physical time

$$t_{eff} = \int \, a_{eff} d\eta = \frac{1}{|\eta|^{\frac{\mu-1}{1-\alpha}}} = \frac{1}{|t|^{\mu-1}}$$

Power-law inflation (almost exponential for small $(\mu - 1)$)

$$a_{eff}(t_{eff}) = t_{eff}^{\frac{\mu-\alpha}{\mu-1}}$$

NB: $\mu < 1$ guarantees weak coupling all the way to $t \to -\infty$

$\Longrightarrow$ effective metric describes accelerated acceleration a là Big Rip.

Wrong scalar spectral tilt!

Sign of tension with data.

Consider in detail.

Power spectra

$$\mathscr{P}_\zeta \equiv \mathscr{A}_\zeta \left(\frac{k}{k_*}\right)^{n_s-1}, \quad \mathscr{P}_T \equiv \mathscr{A}_T \left(\frac{k}{k_*}\right)^{n_T},$$

where k_* is pivot scale, the spectral tilts are

$$n_s - 1 = n_T = -2 \cdot \left(\frac{\mu - 1}{1 - \alpha}\right)$$

and the amplitudes are given by

$$\mathscr{A}_\zeta = \frac{C}{g} \cdot \frac{1}{g_S u_S^{2\nu}} \left(\frac{k_*}{k_0}\right)^{2\frac{1-\mu}{1-\alpha}},$$

$$\mathscr{A}_T = \frac{8C}{g} \left(\frac{k_*}{k_0}\right)^{2\frac{1-\mu}{1-\alpha}},$$

where

$$\nu = \frac{1 + 2\mu - 3\alpha}{2(1-\alpha)} \approx \frac{3}{2}.$$

Problems

- Red tilt, $n_S - 1 = -0.035 \pm 0.05$ — experiment $\Longrightarrow$ $\mu > 1$ while absence of strong coupling requires $\mu < 1$ as $t \to -\infty$.

  Viewpoints:

  - Time-dependent μ; increases from $\mu < 1$ to $\mu > 1$.
  - Dive out from strong coupling regime (!)

    Negative spectral tilt may suggest that the evolution started from quantum gravity regime with low effective Planck mass

- Small tensor to scalar ratio

$$r = \frac{\mathscr{A}_T}{\mathscr{A}_\zeta} = 8 g_S u_S^{2\nu} \sim 8 u_S^3 < 0.036 \quad - \quad \text{experiment}$$

  Cure: tune $u_S \ll 1$. Similar to k-inflation.

Tension!

Perturbations in some detail.

- Effective horizon exit:

$$\frac{k}{a_{eff}(t_{eff,k})} = H_{eff}(t_{eff,k})$$

$$a_{eff} = t_{eff}^{\frac{\mu-\alpha}{\mu-1}}, \; H_{eff} = 1/t_{eff} \Longrightarrow t_{eff,k} = k^{\frac{\mu-1}{1-\alpha}}$$

- This is how perturbations are generated.

  But horizon exit must occur in weak coupling regime!

Tensor perturbations before horizon exit

$$h = \frac{1}{\sqrt{g}} \int d^3k \, \frac{1}{a_{eff}(t)\sqrt{2k}} \mathrm{e}^{i\mathbf{kx}} \left(\mathrm{e}^{\int dt \;\; \omega_k(t)} A^\dagger_{\mathbf{k}} + h.c. \right)$$

freezes out at horizon exit at

$$h = \frac{1}{\sqrt{g}} \int d^3k \, \frac{1}{a_{eff}(t_k)\sqrt{2k}} \mathrm{e}^{i\mathbf{kx}} \left(A^\dagger_{\mathbf{k}} + h.c. \right) = \frac{1}{\sqrt{g}} \int d^3k \, \frac{H_{eff}(t_k)}{k\sqrt{2k}} \mathrm{e}^{i\mathbf{kx}} \left(A^\dagger_{\mathbf{k}} + h.c. \right)$$

Power spectrum

$$\mathscr{P}_T(k) = \langle h^2(\mathbf{k}) \rangle \cdot k^3 = \frac{1}{g} H^2_{eff}(t_k) = \frac{1}{g} \frac{1}{t^2_{eff,k}} = \frac{1}{g} k^{-2\frac{\mu-1}{1-\alpha}} \; .$$

NB: Horizon exit physical time:

$$|t_k| = \frac{1}{t_{eff,k}^{\frac{1}{\mu-1}}} = \frac{1}{k^{\frac{1}{1-\alpha}}}$$

Strong coupling time:

- Strong coupling energy scale

$$E_{strong} = \sqrt{\mathscr{F}_T} = \frac{g^{1/2}}{t^{\mu}}$$

Require $E^2_{strong}(t_k) > 1/t_k^2 \Longrightarrow$

$$\frac{g}{t_k^{2(\mu-1)}} = \frac{g}{k^{2\frac{\mu-1}{1-\alpha}}} > 1$$

- Good so far:

$$\frac{g}{k^{2\frac{\mu-1}{1-\alpha}}} \approx \frac{1}{\mathscr{A}_T} \gg 1$$

- Problem in scalar sector.
  Strong coupling energy scale is suppressed by u_S^{11} (!!)

$$\frac{g u_S^{11}}{t_k^{2(\mu-1)}} = \frac{g u_S^{11}}{k^{2\frac{\mu-1}{1-\alpha}}} > 1$$

while $\mathscr{A}_\zeta \propto u_S^{-3}$. This gives

$$\frac{u_S^8}{\mathscr{A}_\zeta} > 1$$

On the other hand

$$r = \frac{\mathscr{A}_T}{\mathscr{A}_\zeta} \approx 8u_S^3$$

So

$$r > 8\mathscr{A}_\zeta^{3/8}$$

We know that $\mathscr{A}_\zeta = 2.1 \cdot 10^{-9} \Longrightarrow 8\mathscr{A}_\zeta^{3/8} = 0.005$
while $r < 0.036$. Suspiciously close!

- Precise calculation required!

  Starting from accurate calculation of strong coupling scale.

Work in progress

To summarize:

- Bounce and genesis within Horndeski theories with strong gravity in the past appear viable scenarios.

  No pathologies found so far.

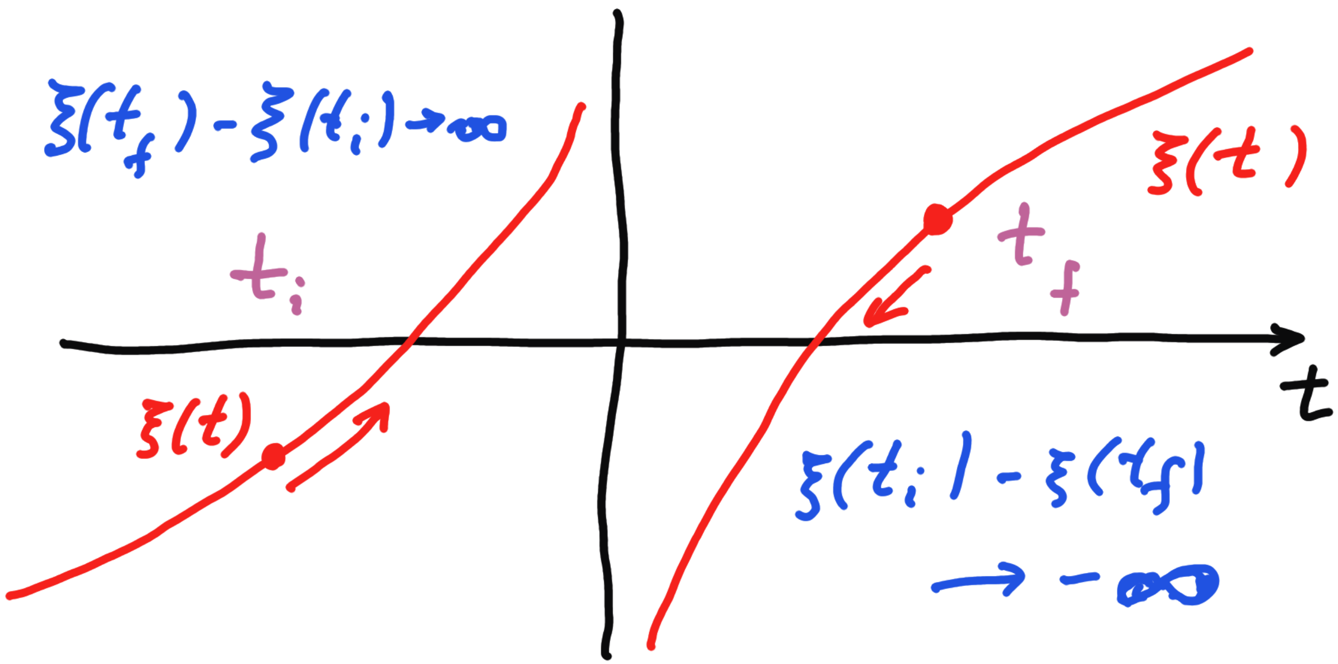
- Correct spectra of comological perturbations may be generated, possibly with certain fine-tuning to ensure $0 < (1 - n_S) \ll 1$ and $r \ll 1$.

- r cannot be arbitrarily small because of the strong coupling problem.

- Scalar and tensor spectral tilts equal for power-law initial contraction

- Strong coupling "nearby" $\Longrightarrow$ non-Gaussianity ???

Even if $\Theta = 0$ at some time $\Longleftrightarrow$ $\xi = \infty$, there is necessarily ξ-crossing:



Side remark: Θ-crossing $\Theta = 0$ at some t is not a problem by itself. $\mathscr{F}_{\mathscr{S}}, \mathscr{G}_{\mathscr{S}} = \infty$, but solutions for ζ remain finite. Also: no singularity in equations in Newtonian gauge

Ijjas' 17; Mironov, V.R., Volkova' 18

Complete cosmologies

Intelligent design: proof by example

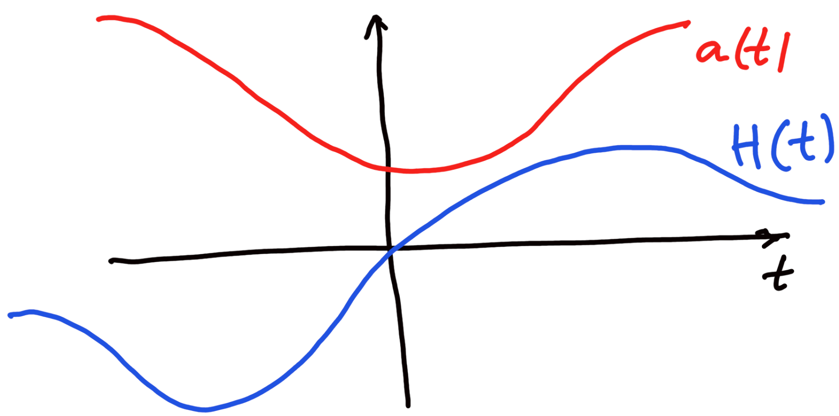
Dubbed "Inverse method" by Ijjas, Steinhardt' 2016

- Choose background $\pi(t) = t$, no loss of generality (field redefinition).

  Then $X = (\partial\pi)^2 = 1$.

  Field equations and stability conditions involve Lagrangian functions F, K, G_4 and their X-derivatives F_X, F_{XX}, etc, all at $\pi(t) = t$, $X = 1$.

  These are yet undetermined independent functions of time $f_0(t) = F(\pi(t), X = 1)$, $f_1(t) = F_X(\pi(t), X = 1)$, etc..
- Choose your favorite $H(t)$.



In particular, theory at late times becomes GR + conventional massless scalar field $\phi = (2/3)^{1/2} \log \pi$ ("kination"), i.e., at late times $\phi = \sqrt{\frac{2}{3}} \log\, t$, $H = \frac{1}{3t}$ and

$$L = -\frac{1}{2}R + \frac{1}{3}\left(\frac{\partial\pi}{\pi}\right)^2 \Longleftrightarrow G_4 = -\frac{1}{2}\,, \quad F(\pi, X) = \frac{1}{3}\frac{X}{\pi^2}\,, \quad K = F_4 = 0\,.$$

- Cook up Lagrangian functions in such a way that
  - Field equations are satisfied
  - Stability conditions are satisfied at all times
  - Classical field theory description of background is reliable at all times, including $t \to -\infty$

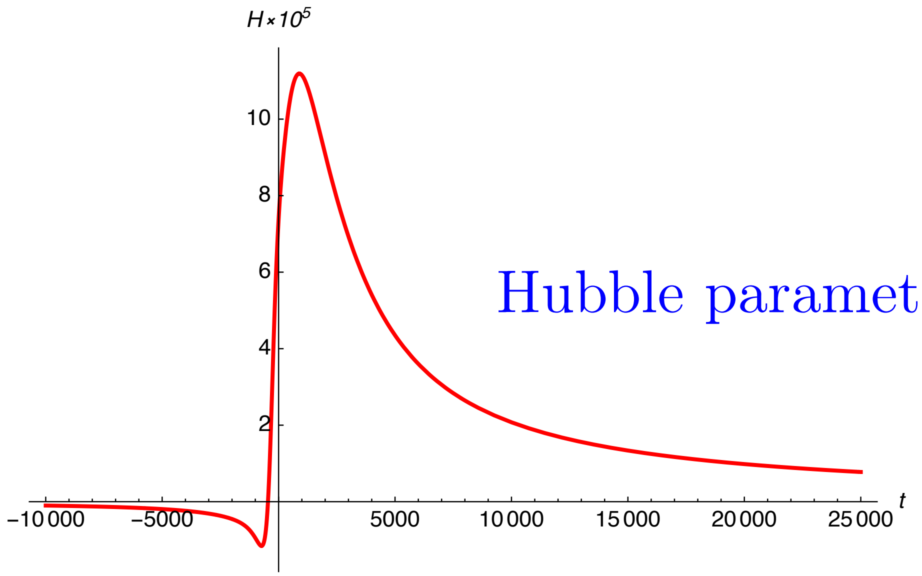
All this can be done for bounce (and also Genesis)

Ageeva, Petrov, V.R.' 2021

Moreover, one can design a model in such a way that

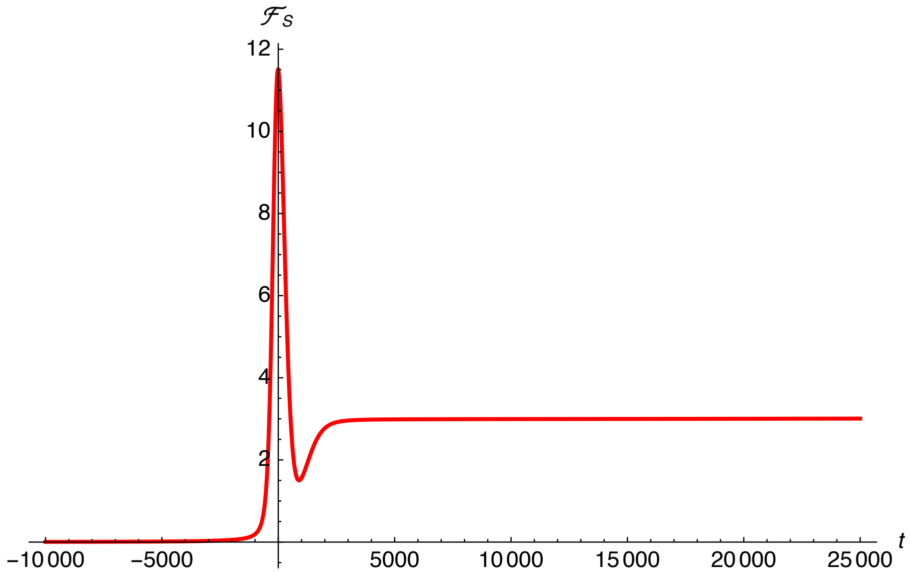
- Tensor and scalar perturbations are subluminal at all times (or luminal, if one wishes so)

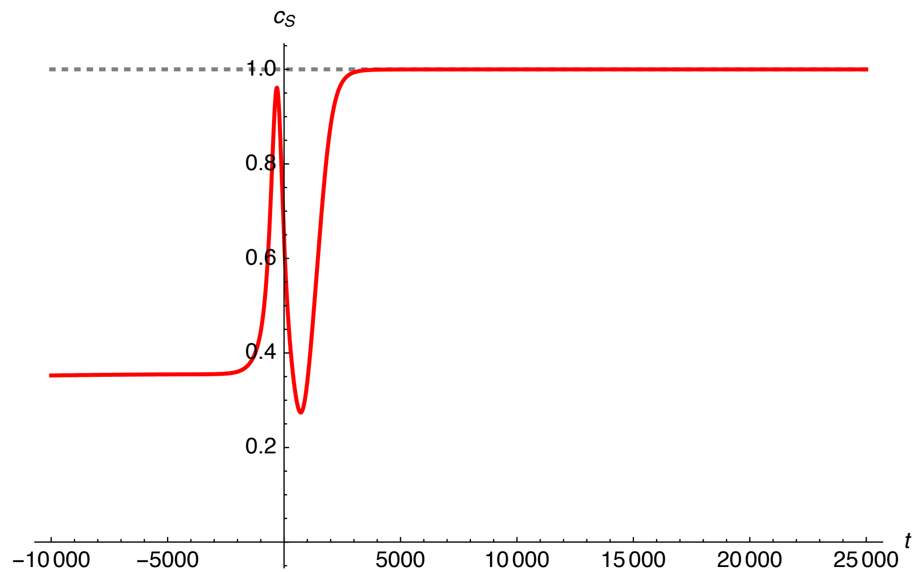
Bounce to kination



Hubble parameter H

t

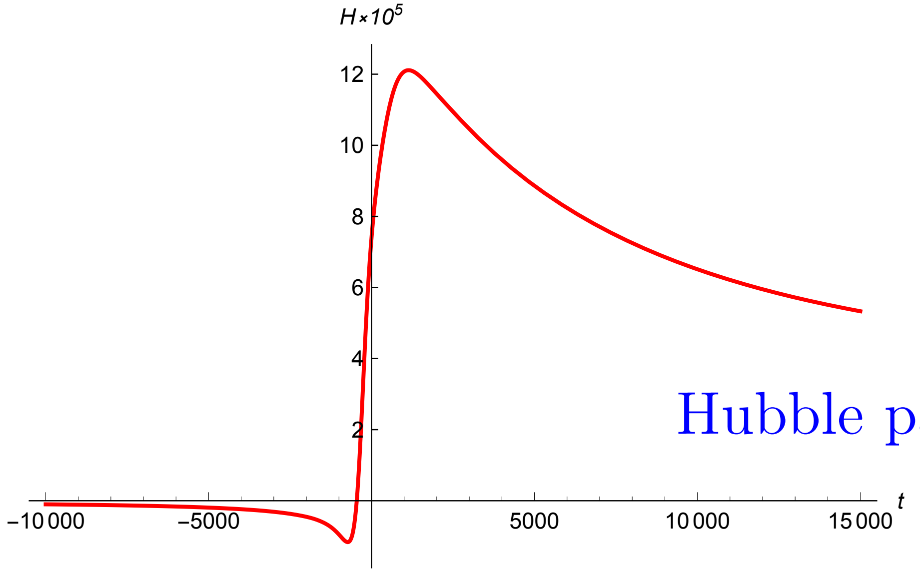




$\mathscr{F}_{\mathscr{S}}$

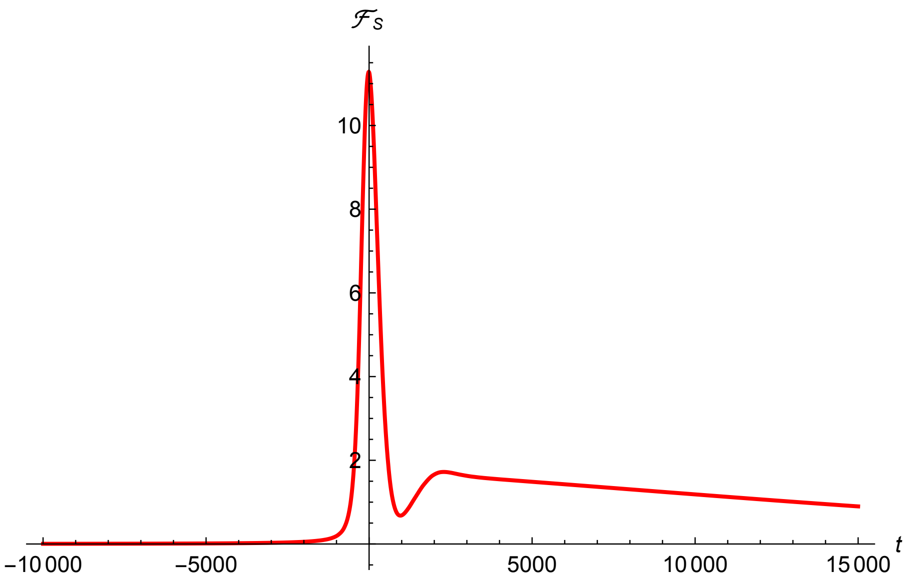
speed of scalar prturbations

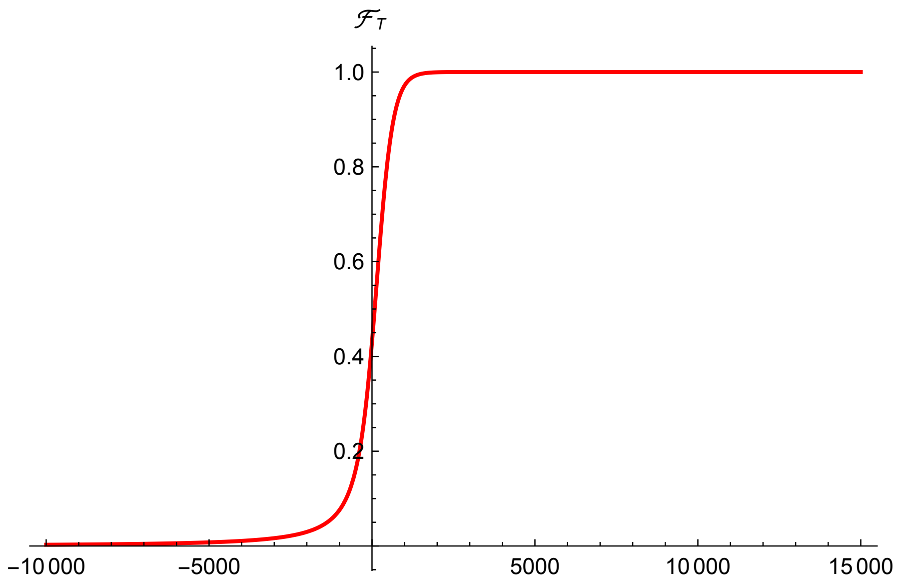
Bounce to inflation



Hubble parameter $\boldsymbol{H}$

t





$\mathscr{F}_{\mathscr{S}}$

$\mathscr{F}_{\mathscr{T}}$

Concrete example of bounce ($1 > \mu > 1/2$): as $t \to -\infty$

$$A_2 = (-t)^{-2\mu-2} \cdot \left(-\frac{a_1}{N^2} + \frac{a_2}{N^4}\right)$$

$$A_3 = (-t)^{-2\mu-1} \cdot \frac{a_3}{N^3}$$

$$A_4 = -\frac{1}{2}(-t)^{-2\mu}$$

When converted to covariant Lagrangian formalism it becomes the early bounce model with

$$F = c_1 X \cdot e^{2\mu\pi} + c_2 X^2 \cdot e^{(2\mu-2)\pi} + 4\mu^2 \cdot X \cdot \ln X \cdot e^{2\mu\pi},$$

$$K = c_3 X \cdot e^{(2\mu-2)\pi} + 2\mu e^{2\mu\pi} + \mu \cdot \ln X \cdot e^{2\mu\pi},$$

$$G_4 = \frac{1}{2} e^{2\mu\pi}.$$

Backup

NEC is not violated in conventional field theories

with Lagrangians involving first derivatives only.

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

Prototype example: scalar field theory with field π and

$$L = F(X, \pi)$$

with $X = (\partial\pi)^2 \equiv g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi \Longrightarrow$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X}\partial_\mu\pi\partial_\nu\pi - g_{\mu\nu}F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X}X - F \ , \qquad T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X}X = 2\frac{\partial F}{\partial X}\dot{\pi}^2$$

NEC-violation: $\partial F/\partial X_c < 0$. But perturbations $\pi = \pi_c(t) + \delta\pi(\vec{x},t)$

$$L_{\delta\pi} = F\Big(X = (\dot{\pi}_c + \delta\dot{\pi})^2 - (\vec{\nabla}\delta\pi)^2;\ \pi + \delta\pi\Big)$$
$$= \mathscr{F} \cdot (\delta\dot{\pi})^2 - \frac{\partial F}{\partial X_c} \cdot (\vec{\nabla}\delta\pi)^2 + \dots$$

Wrong sign of spatial gradient term.

- $\mathscr{F} < 0$: both terms have wrong sign. Hyperbolic equation of motion, but negative energies $\Longleftrightarrow$ ghosts: $E = -\sqrt{p^2 + m^2}$
  Catastrophic vacuum instability

- $\mathscr{F} > 0$: only gradient term has wrong sign. Elliptic equation of motion $\Longrightarrow$ gradient instability

$$E^2 = -(p^2 + m^2) \quad \Longrightarrow \quad \delta\pi \propto \mathrm{e}^{|E|t}$$

Also catastrophic

Straightforward to generalize to several conventional fields.

Reminder: Horndeski and $p < -\rho$

Twist: scalar-tensor theories with second derivatives in the Lagrangian.

Danger: higher order equations of motion $\Longrightarrow$ extra degrees of freedom = Ostrogradsky ghosts

Not necessarily!

- Emphasis of this talk: Horndeski   Horndeski' 1974
  aka Euler hierarchies, aka generalized Galileons, aka KGB, aka generalized Fab Four
  - Second derivatives in Lagrangian, second order field equations
  - Simplest case: Creminelli, Nicolis, Trincherini' 10, Deffayet, Pujolas, Sawicki, Vikman' 10, Kobayashi, Yamaguchi, Yokoyama' 10

$$L = -\frac{1}{16\pi G}R + F(\pi, X) - K(\pi, X)\Box\pi$$

where again $X = (\partial\pi)^2$.

  - Explicit examples of stable NEC-violation.

Simple example: scale-invariant model, $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$

$$L = F(Y)\cdot \mathrm{e}^{4\pi} + K(Y)\cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi\ , \qquad Y = \mathrm{e}^{-2\pi}\cdot(\partial\pi)^2$$

Homogeneous solution in Minkowski space (attractor)

$$\mathrm{e}^{\pi_c} = \frac{1}{\sqrt{Y_*}\,|t|}\ , \qquad t < 0$$

- $Y \equiv \mathrm{e}^{-2\pi_c}\cdot(\partial_\mu \pi_c)^2 = Y_* = \mathrm{const}$, a solution to

$$Z(Y_*) \equiv -F + 2Y_* F_Y - 2Y_* K + 2Y_*^2 K_Y = 0$$

$$F_Y = dF/dY\ .$$

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathscr{F}(\partial_t \delta\pi)^2 - \mathscr{G}(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

Absence of ghosts: $\mathscr{F} = \mathrm{e}^{2\pi_c} Z_Y \equiv \mathrm{e}^{2\pi_c} dZ/dY > 0$ at $Y = Y_*$, no problem.

- NEC-violation and absence of gradient/ghost instabilities:

$$\rho + p = \mathrm{e}^{4\pi_c}(F_Y - 2K + Y_* K_Y) \cdot 2Y_* \quad < 0$$

$$\mathscr{G} = \mathrm{e}^{2\pi_c}(F_Y - 2K + 4Y_* K_Y) \quad > 0$$

Easy to arrange.

NB: $\rho = 0,\ p < 0$ $\qquad p \to 0$ as $t \to -\infty$

Turning on gravity

Creminelli, Nicolis, Trincherini' 10

$$p = \mathrm{e}^{4\pi_c} (F - 2Y_* K) = -\frac{M^4}{Y_*^2 |t|^4}, \qquad \rho = 0$$

M: mass scale characteristic of π

- Use $\dot{H} = -4\pi G(p+\rho) \Longrightarrow$

$$H = \frac{4\pi}{3} \frac{M^4}{M_{Pl}^2 Y_*^2 |t|^3}$$

NB:

$$\rho \sim M_{Pl}^2 H^2 \propto \frac{1}{M_{Pl}^2 |t|^6}$$

Early stage of Genesis.

NB: Early times $\Longrightarrow$ weak gravity, $\rho \ll p$. Expansion, $H \neq 0$, is negligible for dynamics of π and $\delta\pi$.

- Option. Power law behavior with $a(t) \to 0$ as $t \to -\infty$, so that

$$\int_{-\infty}^{t} a(t)\, dt < \infty$$

Say

$$a = \frac{1}{|t|^{\alpha}}\,, \quad \alpha > 1$$

Hubble parameter and its derivatives vanish as $t \to -\infty$.

Modified Genesis, Libanov, Mironov, V.R.' 16

Naively: space in nearly Minkowskian as $t \to -\infty$.

Does not work: past geodesic incompleteness.

[Recall: we are in Jordan frame.]

Geodesic (in)completeness in FLRW

Solution to massive geodesic equation in spatially flat FLRW metric

$$u^0 \equiv \frac{dt}{ds} = \frac{\sqrt{m^2a^2 + k^2}}{ma}$$

where $k =$ conformal momentum. Proper time along geodesic

$$s = \int \frac{dt}{u^0} = \int dt \, \frac{ma}{\sqrt{m^2a^2 + k^2}}$$

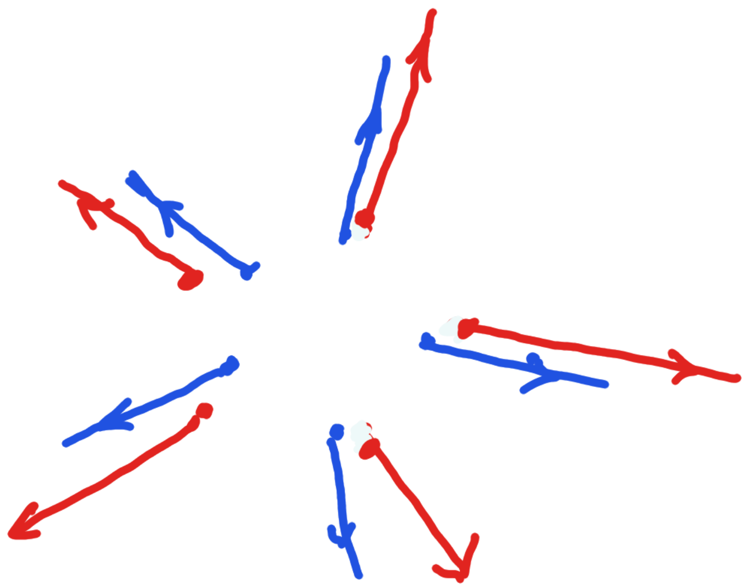
If $a(t) \to 0$ as $t \to -\infty$, space-time patch is past geodesically incomplete for

$$\int_{-\infty}^{t} a dt < \infty$$

Borde, Guth, Vilenkin '2003

Can space-time be geodesically completed?

Method of geodesic completion: new coordinates associated with congruence of radial geodesics (with one and the same $|\vec{k}| \equiv k$)



Time coorinate τ: proper time along geodesic (negative for geodesic directed to past)
Radial coordinate r: initial position

FLRW metric $ds^2 = dt^2 - a^2(t)[dR^2 + R^2 d\Omega^2]$ in new coordinates

$$ds^2 = d\tau^2 - (k^2 + a^2)dr^2 + a^2 R^2 d\Omega^2$$

Coordinate transformation (past directed geodesics, $t < 0$)

$$\tau + kr = -\int_t^0 \frac{a dt}{\sqrt{k^2 + a^2}}, \quad R = r + \int_t^0 \frac{k dt}{a\sqrt{k^2 + a^2}}$$

Necessary condition for geodesic completion:

- (aR) finite as $t \to -\infty \Longrightarrow a(t) \int_t^0 \frac{dt'}{a(t')} < \infty$ as $t \to -\infty$;
  OK for de Sitter, does not hold for power law $a = |t|^{-\alpha}$, $\alpha > 1$
- NB: Another condition: components of Riemann tensor finite in new coordinates $\Longrightarrow$

$$\frac{\dot{H}}{a^2} < \infty \quad \text{as} \quad t \to -\infty$$

Yoshida, Quentin '2018

OK for de Sitter, does not hold for power law $a = |t|^{-\alpha}$, $\alpha > 1$

Space-time with $a = |t|^{-\alpha}$, $\alpha > 1$ as $t \to -\infty$ is singular in the past

NB. Often said: geodesic incompleteness of tensor/scalar modes for

$$\int_{-\infty}^{t} dt\ a(t)\ \mathscr{G}_{\mathscr{T},\mathscr{S}} < \infty$$

But: geodesic incompleteness is not well defined notion for massless excitations. Upon field redefinition to canonically normalized field

$$L_T = \mathscr{G}_{\mathscr{T}} \dot{h}^2 - a^{-2} \mathscr{F}_{\mathscr{T}} (\partial_k h)^2 \Longrightarrow \dot{h}_{(c)}^2 - a^{-2} (\partial_k h_{(c)})^2 + \text{non-derivative terms}$$

i.e. $\mathscr{G}_{\mathscr{T}} \Longrightarrow 1$.

This observation does not apply to masive particles: proper time is measured in units of m^{-1}, where m becomes time-dependent after field redefinition.