Bootstrapping large graviton non-Gaussianity

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The Cosmological Bootstrap

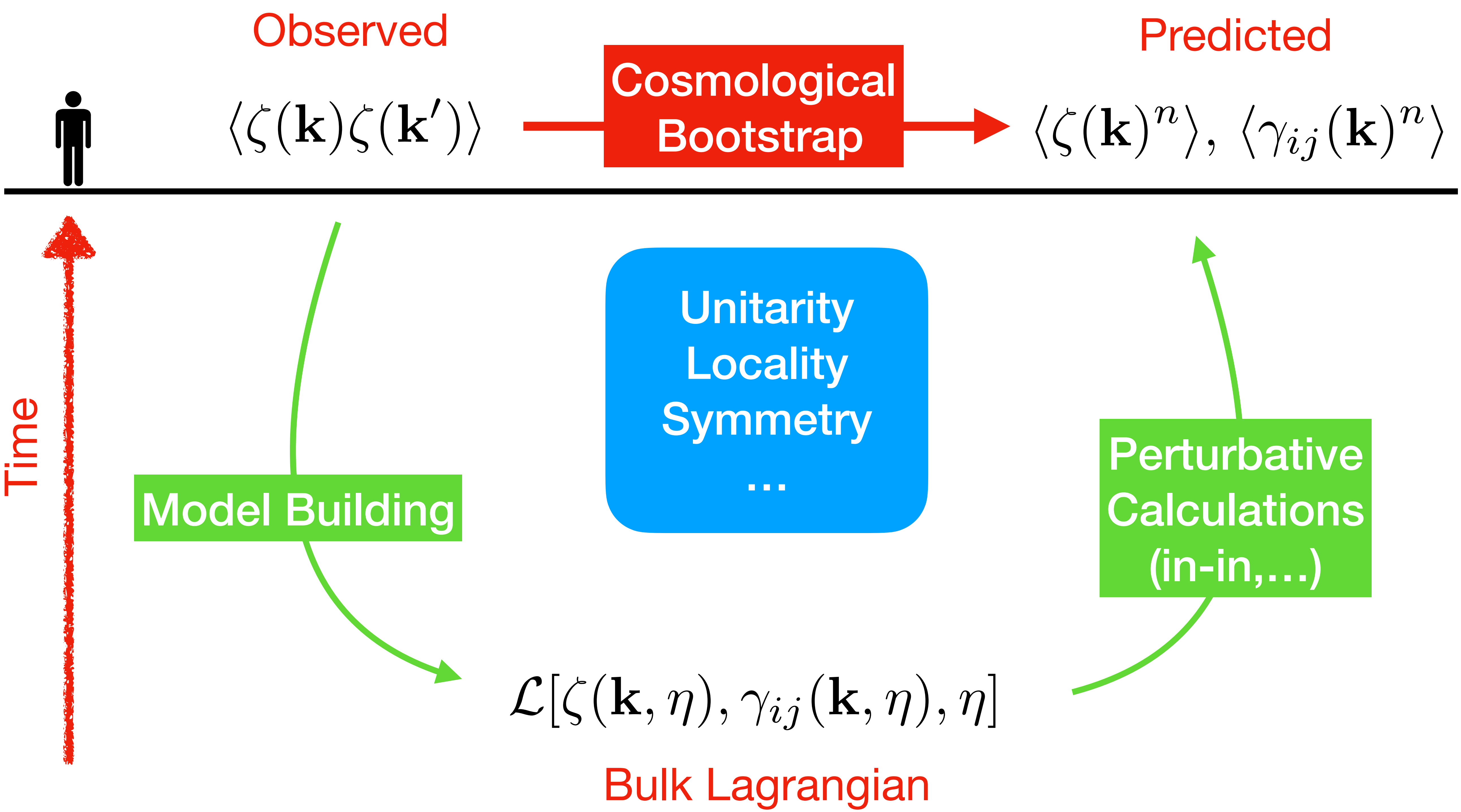


Table of contents

Primordial Cosmo = QFT in curved spacetime / Quantum Gravity

- On large scales (>> Mpc) cosmological surveys measure QFT correlators of metric fluctuation

$$\langle \prod^n \delta(k_a) \rangle \sim \int_k \left[\prod^n \Delta^{(Y)}(k_a) \right] \langle \prod^n \zeta(k_a) \rangle$$

CMB temperature
density of galaxies
dark matter, …

QFT / QG
in de Sitter

- The goal of primordial cosmology is to understand QFT and QG in (approximately, asymptotically) de Sitter

Aspirations

- We hope to learn about:
  - *New degrees of freedom and their interactions*: Inflation requires at least one degree of freedom and three energy scales beyond the standard model.
  - The laws of gravity at short distances/high energies: probe GR and beyond at high energies
  - QFT in de Sitter: which theories are consistent?
  - Quantum gravity in dS? Holography, string theory?

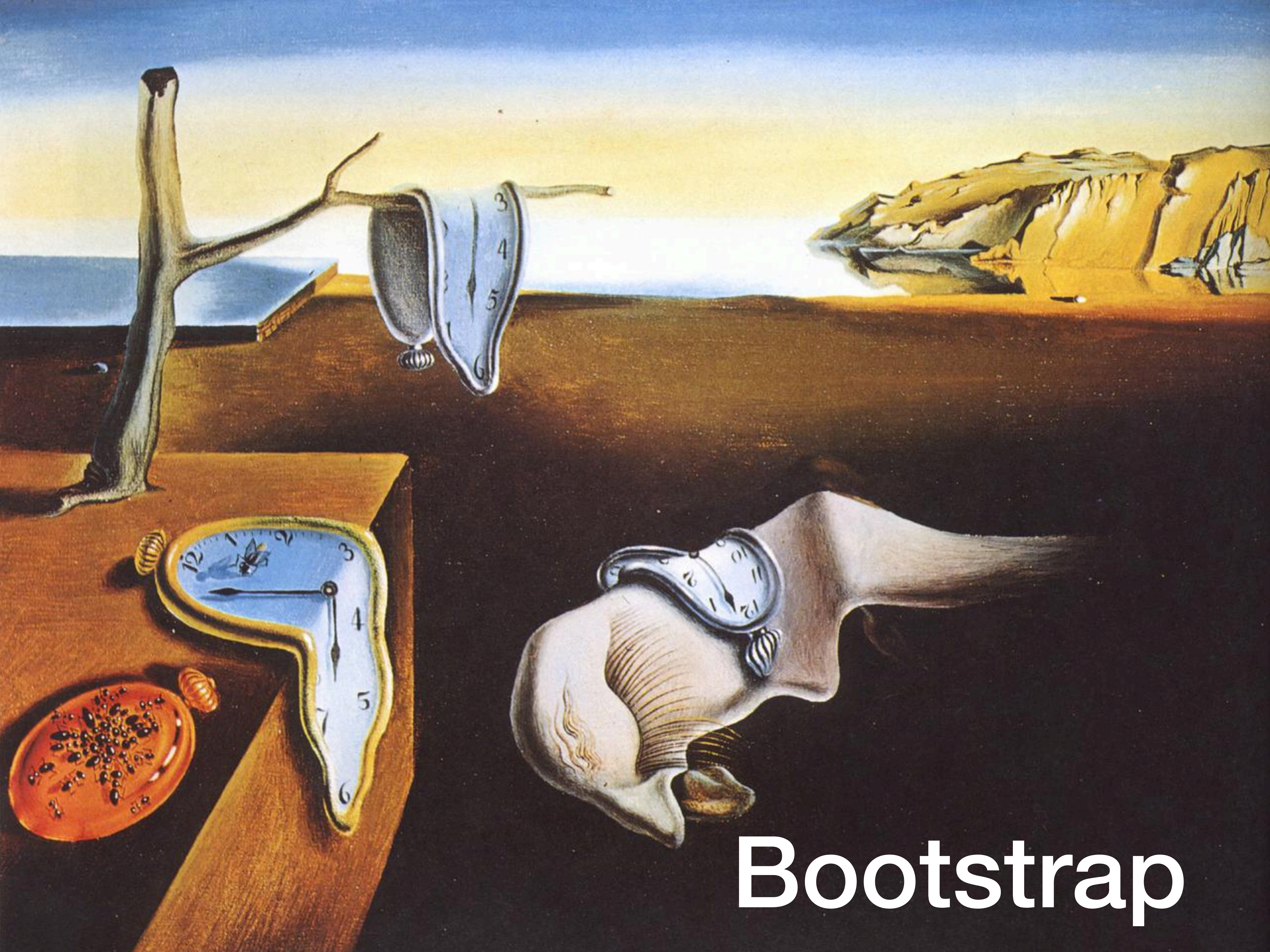
Problems

Here's some obstacles we are facing (in order of difficulty)

- *Too many models* for too little data on primordial universe. Concrete constructions model time evolution, which is not observable.
- Too little *theoretical guidance*: what are necessary conditions on EFT's to admit standard (e.g. unitary, causal,…) UV-completions?
- What's our estimate for non-perturbative effects from QFT and quantum gravity? Any guidance from bottom-up de Sitter holography?

To make progress, I will describe a new approach to compute observables in de Sitter and inflation, the *boostless cosmological bootstrap*. This gives us very general consequences of locality and unitarity within perturbative QFT on a curved background.

A poster-child of the approach is the calculation of all graviton non-Gaussianities



Bootstrap

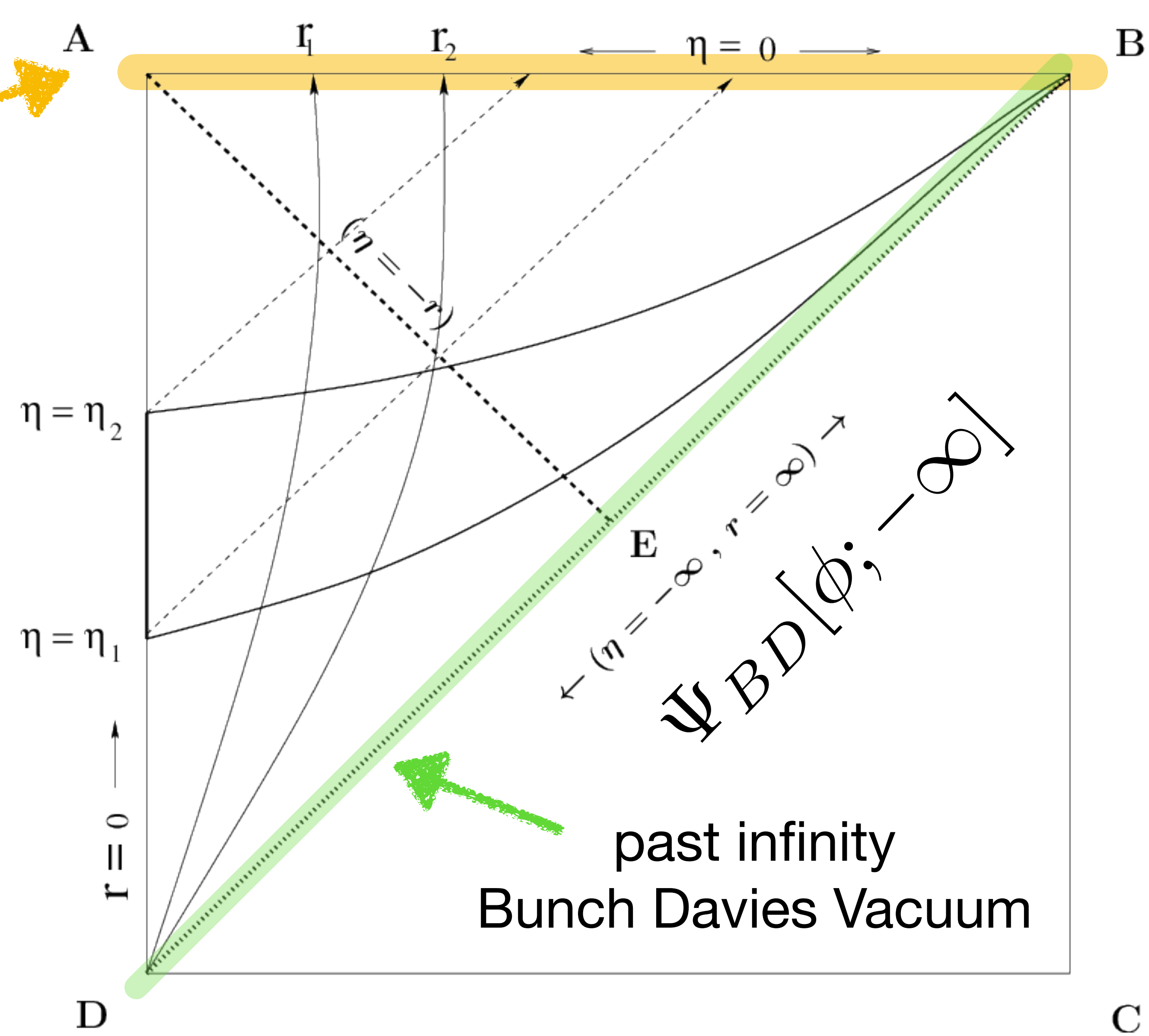
Penrose diagram

- We work in the Poincare' patch (half of dS)

$$ds^2 = -dt^2 + a^2 dx^2 = a^2(-d\eta^2 + dx^2)$$

$$\Psi[\phi; \eta \to 0]$$

The future (conformal) boundary can be thought as the reheating surface after inflation and determines the statistics of *LSS and CMB observations*



The wavefunction

- The *wavefunction of the universe,* is a functional of the all fields in the theory (including the metric) at some time:

$$\Psi[\phi] = \exp\left[-\sum_{n=2}^{\infty}\int \psi_n\, \phi(k_1)\ldots\phi(k_n)\right]$$

- All probabilities can be computed as in QM

$$\langle \mathcal{O} \rangle = \int d\phi \Psi^* \mathcal{O} \Psi$$

- The Ψ_n are closely related to *cosmological correlators*, which determine the statistics of the Cosmic Microwave Background and Large Scale Structures

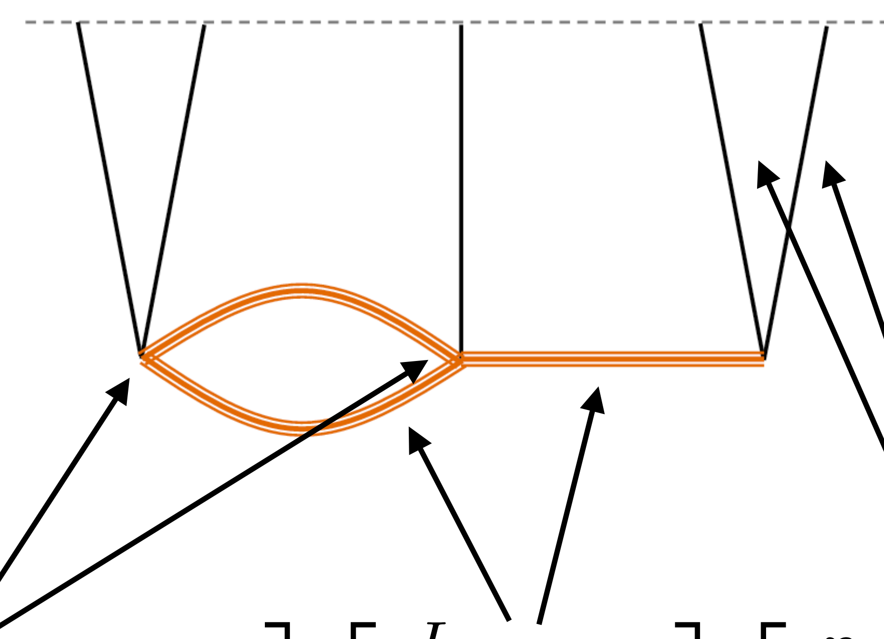
$$\langle \phi\phi \rangle = \left[2\mathrm{Re}\psi_2(k)\right]^{-1}, \qquad \langle \phi\phi\phi \rangle = -2\mathrm{Re}\psi_3 \left[\prod^{3} \mathrm{Re}\psi_2\right]^{-1}$$

Feynman Diagrams

- Given a model, the wavefunction can be computed perturbatively from a path integral

$$\Psi[\phi(\mathbf{x});\eta_0] = \int_{\text{vacuum}}^{\phi(\mathbf{x},\eta_0)} [\mathcal{D}\phi]\, e^{iS[\phi]}$$

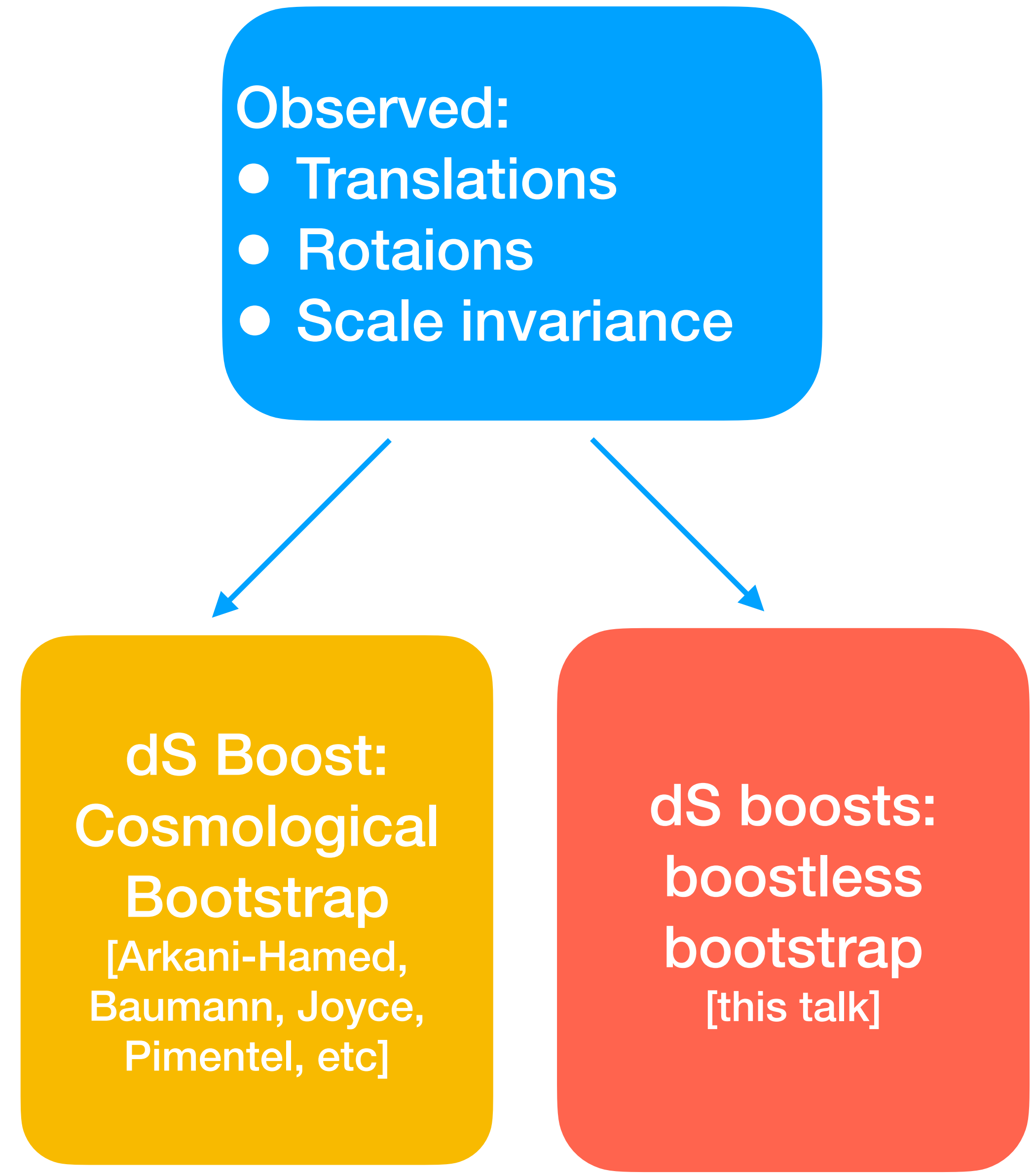
with the following Feynman rules (equivalent to in-in):



$$\psi_n = \left[\prod_A^V \int d\eta_A\, F_A\right] \left[\prod_m^I G(p)\right] \left[\prod_a^n K(k_a)\right]$$

Symmetries

- Cosmological perturbations are observed to be statistically *homogeneous* and *isotropic*
- Primordial perturbations are also observed to be approximately scale invariant
- Anything else?
  - With *de Sitter boost* we can derive general results and connect with Conformal Field Theory and holography. See beautiful progress by Maldacena, Pimentel, Arkani-Hamed, Baumann, Joyce, etc…
  - If we are instead more interested in phenomenology, we cannot assume Boost invariance.



All *dS-invariant* graviton non-Gaussianities

- In 2011 Maldacena & Pimentel showed non-perturbatively that there are only *three* possible dS invariant graviton cubic wavefunction coefficients ψ3.

$$\langle \gamma^+_{k_1} \gamma^+_{k_2} \gamma^+_{k_3} \rangle_R = (2\pi)^3 \delta^3 \left(\sum_i k_i \right) \left(\frac{H}{M_{Pl}} \right)^4 \frac{2}{(k_1 k_2 k_3)^5} \left[(k_1 + k_2 + k_3)^3 - \right.$$

$$\left. -(k_1 k_2 + k_1 k_3 + k_2 k_3)(k_1 + k_2 + k_3) - k_1 k_2 k_3 \right] \left[\langle \bar{1}, \bar{2} \rangle \langle \bar{2}, \bar{3} \rangle \langle \bar{3}, \bar{1} \rangle \right]^2$$

$$\langle \gamma^+_{k_1} \gamma^+_{k_2} \gamma^+_{k_3} \rangle_{W^3} = \mathcal{M} \frac{(-2^8 \times 3^2 \times 5)}{(k_1 + k_2 + k_3)^6 (k_1 k_2 k_3)^2} \left[\langle \bar{1}, \bar{2} \rangle \langle \bar{2}, \bar{3} \rangle \langle \bar{3}, \bar{1} \rangle \right]^2$$

- They lead to only *two* graviton bispectra [Soda Kodama Nozawa '11]

- We extended this result to *arbitrary violations of de Sitter boosts*, which appear in all phenomenologically models because of the coupling to the inflationary background that foliates de Sitter.

Boostless theories

- All cosmological models break Lorentz/de Sitter boosts. *The breaking of boosts can be large and is NOT slow-roll suppressed*, in contrast to the small breaking of dilation

assumed observed symmetries

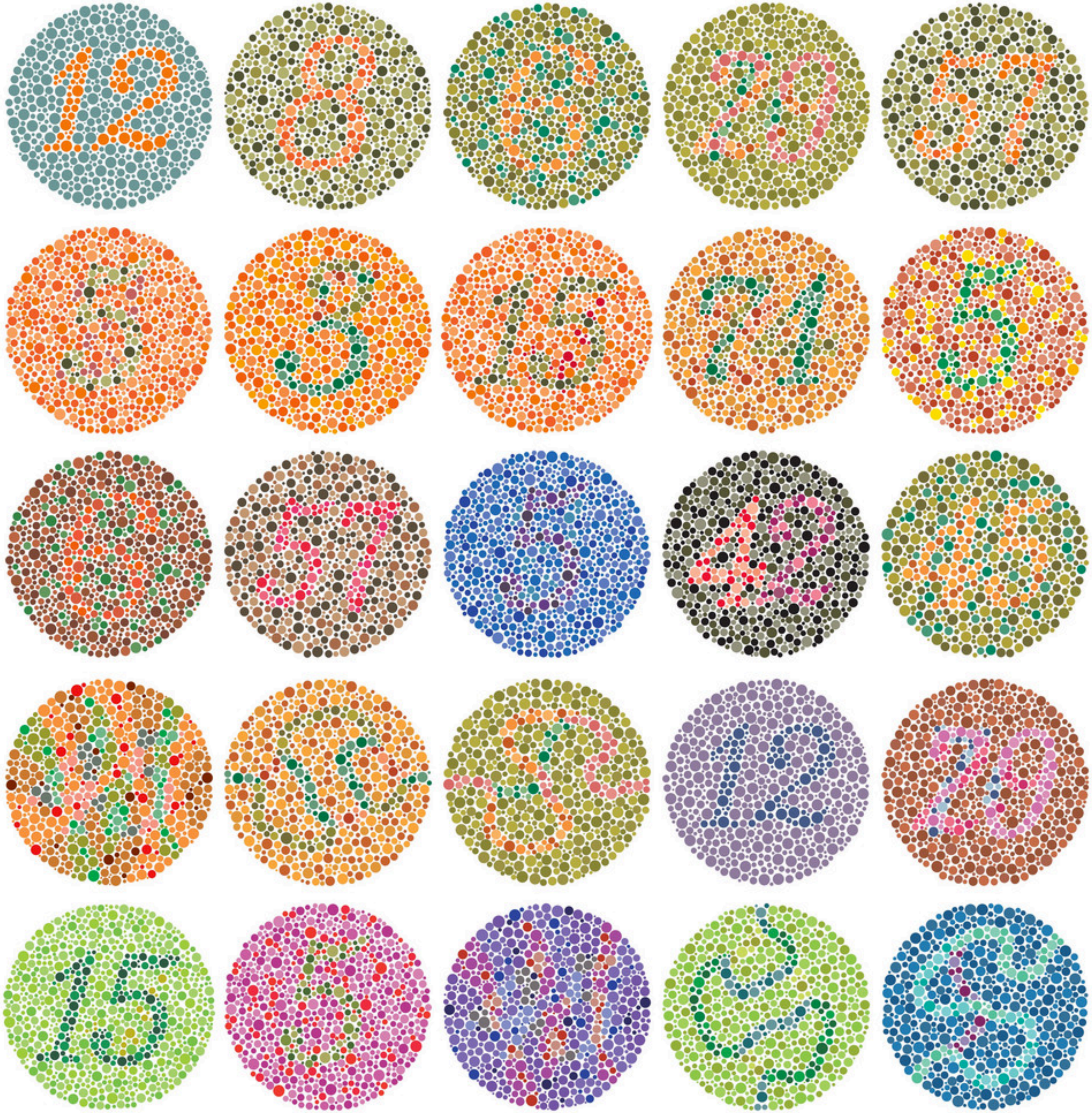
$$\sum_{a=1}^{n} \vec{k}_a \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ translations

$$\sum_{a=1}^{n} k_a^{[i} \partial_{k_a^{j]}} \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ rotations

$$\sum_{a=1}^{n} (3 - \Delta + k_a \partial_{k_a}) \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ dilations

$$\sum_{a=1}^{n} \left[2\vec{k} \cdot \vec{\partial} \partial_i - k_i \partial^2 + 2(3-\Delta)\partial_i \right] \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ dS boosts

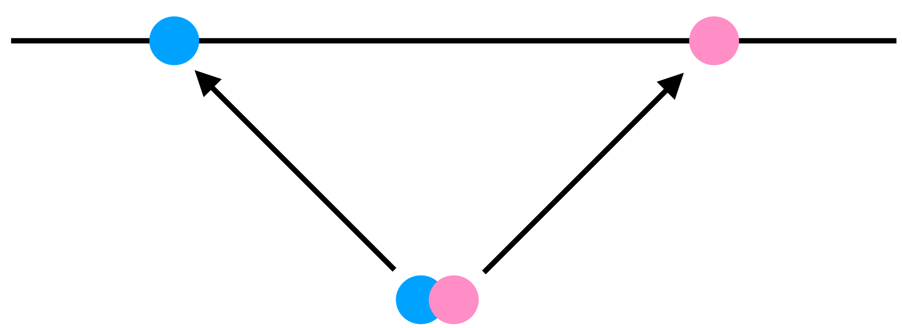
additional



Locality

Locality

- Locality: what happens here cannot affect what happens far away. Operators commute for space-like separation and correlators factorise at large distances (*cluster decomposition*).



- There is no cluster decomposition in dS

- A common sufficient condition is *Manifest Locality*: Lagrangian interactions are products of operators *at the same spacetime point*. No inverse laplacians are allowed.

- Remarkably, the wavefunction of *massless scalars and gravitons* must satisfy the very simple *Manifestly Local Test (MLT)* [Jazayeri, EP & Stefanyszyn '21]

$$\frac{\partial}{\partial k_c}\psi_n(k_1,...,k_n;\{p\};\{\mathbf{k}\})\Big|_{k_c=0}=0\,,\qquad \forall c=1,\ldots,n\,,$$

Manifest locality

- This is true as long as there are *only positive powers of k* in the interactions, i.e. the theory is *manifestly local.*

- All large non-Gaussianities in single field inflation come from manifestly local interactions in the EFT of inflation, e.g.

$$\mathcal{L} \supset \dot{\phi}^3 + (\partial\phi)^2\dot{\phi} + \dot{\phi}^4 + \dots$$

- Gravity has not manifestly local interactions for scalars after we integrate out the lapse and shift

$$\mathcal{L}_{GR} \supset \dot{\zeta}^2\nabla^{-2}\dot{\zeta} + \dots$$

  but this cannot happen for gravitons because they appear in the constraints only to quadratic order. *So the MLT applies to gravitons to all orders for all theories, including GR*

The Bootstrap Rules

Bootstrap Rules

- Instead of computing bispectra from a model we use a set of Bootstrap Rules based on fundamental principles [EP '20]

- As an example, let's bootstrap the bispectrum (3-point function) of a *scalar*

- It can only have kT poles by locality!

Scale invariance

Bose symmetry

$$\psi_3 = \frac{\text{Poly}_{3+p}(\overbrace{k_T, e_2, e_3})}{k_T^p}$$

$$k_T \equiv k_1 + k_2 + k_3$$
$$e_2 \equiv k_1 k_2 + k_2 k_3 + k_1 k_3$$
$$e_3 \equiv k_1 k_2 k_3$$

tree level in dS

Bunch Davies vacuum

The calculation

- The Bootstrap Rules reduced the problem to determining the numerical constants C_{mn} via the Manifestly Local Test

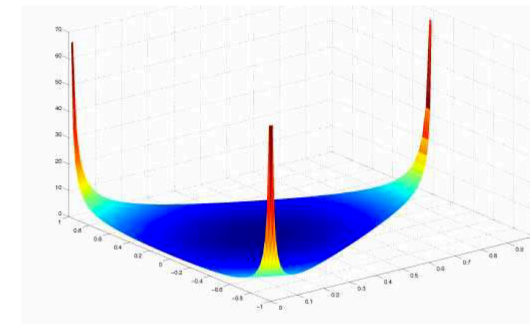
$$\psi_3^{(p)}(k_1,k_2,k_3) = \frac{1}{k_T^p} \sum_{n=0}^{\lfloor \frac{p+3}{3} \rfloor} \sum_{m=0}^{\lfloor \frac{p+3-3n}{2} \rfloor} C_{mn} k_T^{3+p-2m-3n} e_2^m e_3^n,$$

$$\partial_{k_1} \psi_3 \Big|_{k_1=0} = 0$$

- This yields all manifestly local bispectra for a scalar to *any order in derivatives* in the EFT of inflation

- This gives order by order the shapes of non-Gaussianity that are constraint e.g. by the Cosmic Microwave Background, e.g. the Planck mission

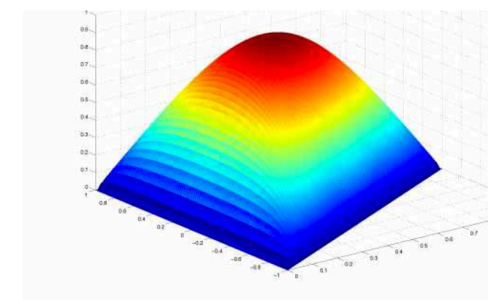
Shapes of non-Gaussianity



$$\psi_3^{(0)} = A_0 \left[4e_3 - e_2 k_T + (3e_3 - 3e_2 k_T + k_T^3) \log(-k_T \eta/\mu)\right]$$

$$\psi_3^{(1)} = 0$$

$$\psi_3^{(2)} = A_2 \left[-k_T^3 + 3k_T e_2 - 11e_3 + \frac{4e_2^2}{k_T} + \frac{4e_2 e_3}{k_T^2}\right]$$



$$\psi_3^{(3)} = A_3 \frac{1}{k_T^3}(k_T^6 - 3k_T^4 e_2 + 11k_T^3 e_3 - 4k_T^2 e_2^2 - 4k_T e_2 e_3 + 12e_3^2) + A_3' \frac{e_3^2}{k_T^3}$$

- $\Psi_3^{(0)}$ contains the famous *local non-Gaussianity,* while $\Psi_3^{(1,2)}$ the so-called *equilateral and orthogonal non-Gaussianities*, the main targets of non-Gaussian searches in the CMB and galaxy surveys!

- In the standard approach the numerical coefficient come from time integrations, here they're fixed algebraically


Unitarity

Unitary time evolution

- In Quantum Mechanics we compute probabilities, which must be between 0 and 1 to make sense

- This requires the positive norm of states in the Hilbert space *and* Unitary time evolution, $UU^\dagger=1$. Colloquially this is the *conservation of probabilities*

- The consequences of unitarity for particle physics amplitudes were discover over *60 years ago*: the Optical theorem and Cutkosky Cutting Rules.

- In cosmology we don’t see the time evolution, so how can we see it’s unitary?!

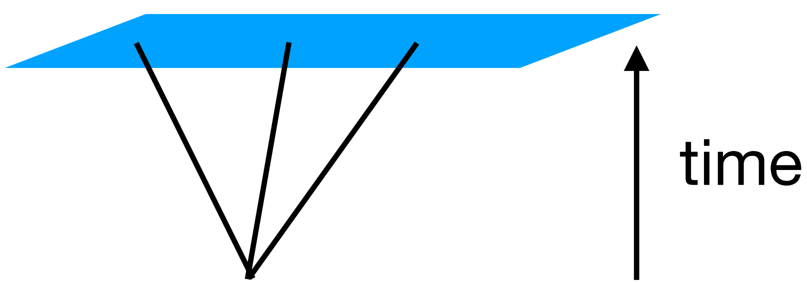
The Cosmological Optical Theorem (COT)

[Goodhew, Jazayeri, EP ’20]

- From unitarity, $UU^\dagger=1$, we found infinitely many relations.

- The simplest applies to contact n-point functions

$$\psi_n(\{k\},\{\mathbf{k}\}) + \psi_n^*(-\{k\},-\{\mathbf{k}\}) = 0$$



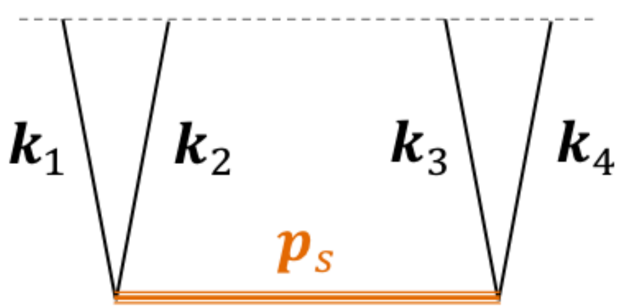
- It follows from unitarity time evolution, but the equation does not involve time! Time “emerges” at boundary as in holography…

- This is a Cosmological Optical Theorem (COT) and can be interpreted as fixing a “discontinuity”

$$\mathrm{Disc}\psi_n(\{k\},\{\mathbf{k}\}) \equiv \psi_n(\{k\},\{\mathbf{k}\}) + \psi_n^*(-\{k\},-\{\mathbf{k}\})$$

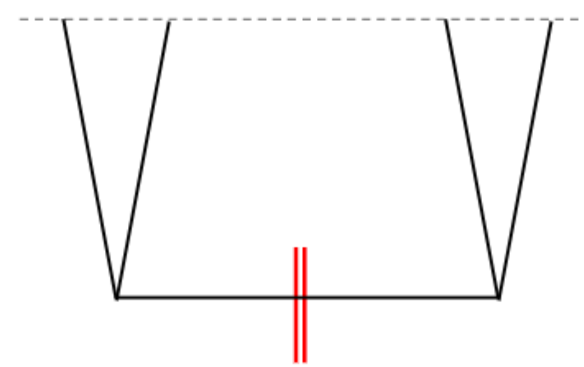
Exchange diagrams

- The next simplest case is a 4-particle exchange diagram (trispectrum). The Cosmo Optical Theorem (COT) is



$$i\,\mathrm{Disc}_{\boldsymbol{p}_s}\left[i\psi^{(s)}_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4}\right]$$

$=$



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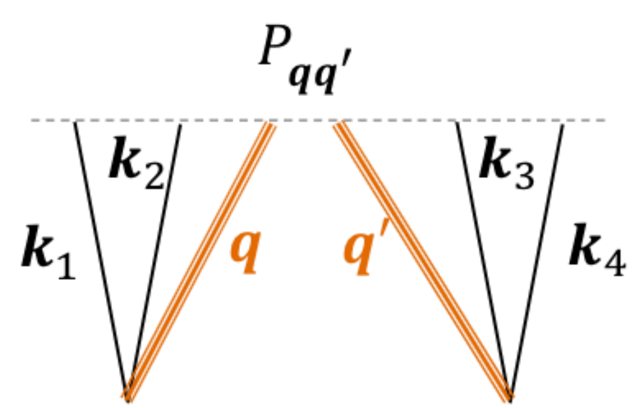
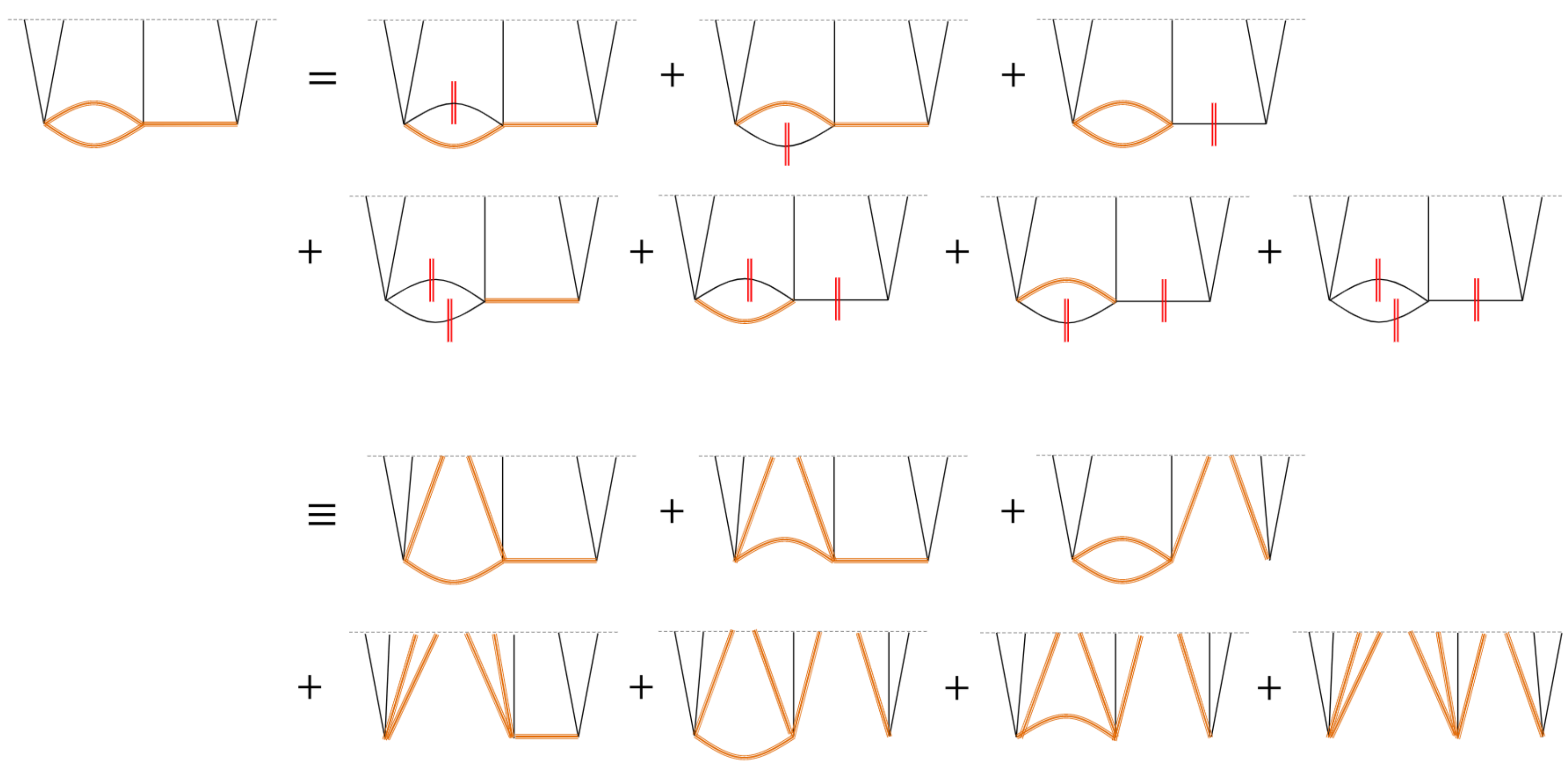


$$i\,\mathrm{Disc}_{\boldsymbol{q}}\left[i\psi_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{q}}\right]\; P_{\boldsymbol{q}\boldsymbol{q}'}\; i\,\mathrm{Disc}_{\boldsymbol{q}'}\left[i\psi_{\boldsymbol{q}'\boldsymbol{k}_3\boldsymbol{k}_4}\right]$$

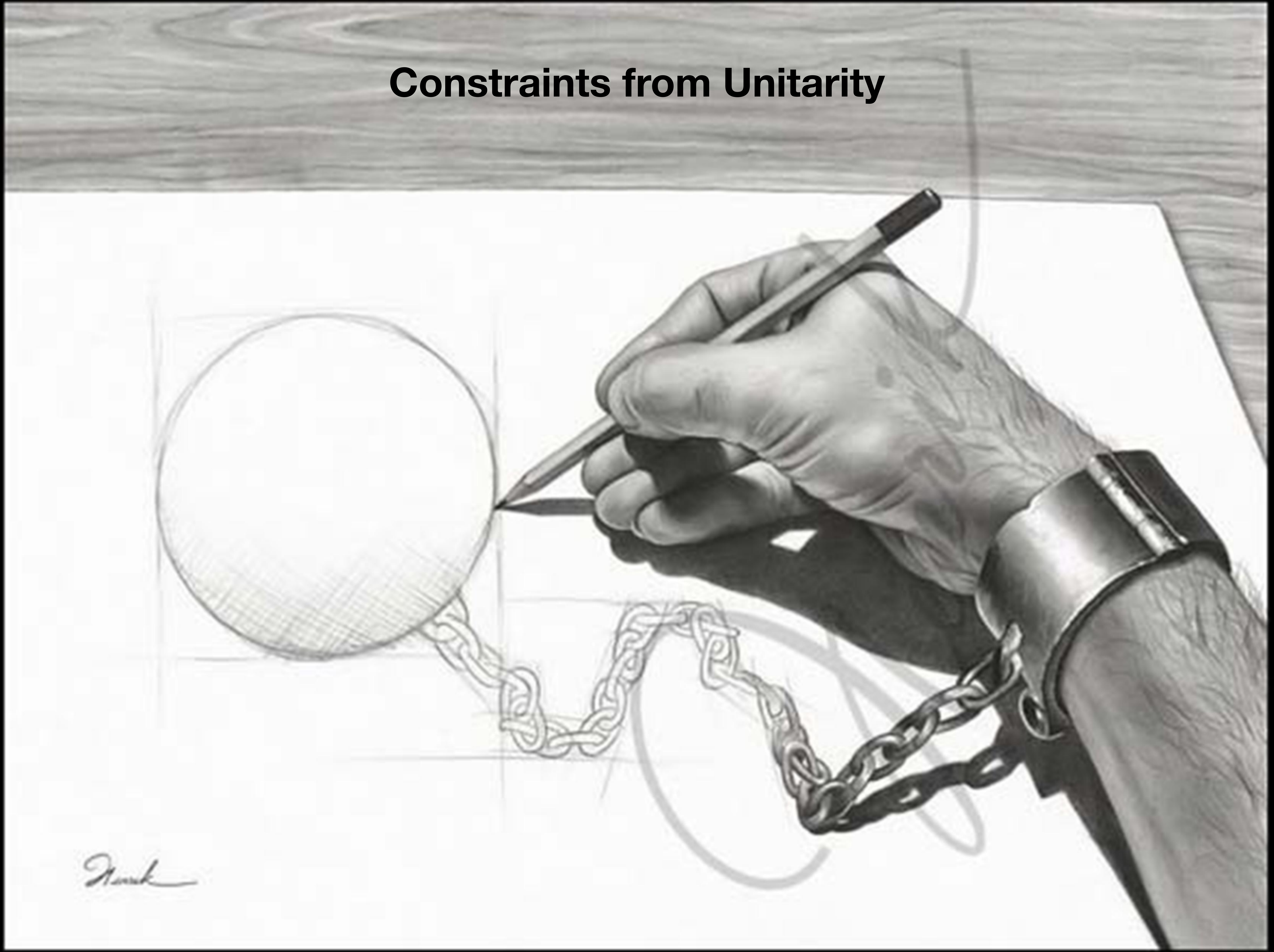
General diagrams

- These relations are valid to *all order in perturbation theory to any number of loops for fields of any mass and spin and arbitrary interactions (around any FLRW admitting a Bunch Davies initial condition)* [Goodhew, Jazayeri & EP ’21; Melville & EP ’21]



- These are Cosmological Cutting Rules. With a 60 year delay over particle physics, we finally understand unitarity in cosmology.

Constraints from Unitarity



Contact interactions

- Contact interactions contribute to correlators as

$$\langle\varphi(\mathbf{k}_1)\dots\varphi(\mathbf{k}_n)\rangle = \frac{\int \mathcal{D}\varphi\ \Psi\Psi^*\ \varphi(\mathbf{k}_1)\dots\varphi(\mathbf{k}_n)}{\int \mathcal{D}\varphi\ \Psi\Psi^*}\,,$$

$$B_n^{\mathrm{contact}}(\{k\};\{\mathbf{k}\}) = -\frac{\psi_n'(\{k\};\{\mathbf{k}\}) + \psi_n'^*(\{k\};-\{\mathbf{k}\})}{\prod_{a=1}^n 2\ \mathrm{Re}\ \psi_2'(k_a)}\,,$$

  which gives the Real or Imaginary part for parity even or odd interactions.

- Unitarity in the form of the Cosmo Optical Theorem (COT)

$$\psi_n(\{k\};\{\mathbf{k}\}) + \psi_n^*(\{-k\};-\{\mathbf{k}\}) = 0$$

- Combining the two we find that: A*ny contribution to Ψ that is invariant under {k}→{-k} does not contribute to correlators.*

What can contribute?

- The most generic tree-level cubic wavefunction must satisfy locality (MLT), unitarity (COT) and scaling (k^3)

- For example for *parity-even* interactions (**α** is even) we find *infinitely many terms* with *kT-poles of order p>0*

MLT

$$\alpha = 0: \quad \psi_n = \frac{\text{Poly}_{3+p}}{k_T^p} + \text{Poly}_3\left(\ln(k_T\eta) + i\frac{\pi}{2}\right) + \frac{\text{Poly}_2}{\eta} + \frac{\text{Poly}_1}{\eta^2} + \frac{1}{\eta^3}$$

$$\alpha = 2 \quad \psi_n = \mathbf{k}^2\left[\frac{\text{Poly}_{1+p}}{k_T^p} + \text{Poly}_1\left(\ln(k_T\eta) + i\frac{\pi}{2}\right) + \frac{1}{\eta}\right]$$

MLT

Not in B_3 because of unitarity (COT)

Btw, this shows there are no logs in GR in 3+1 dim. This implies all correlators and wavefunction coefficients in GR on dS at tree level are rational functions (no polylogs)

What can contribute?

- The most generic tree-level cubic wavefunction must satisfy locality (MLT), unitarity (COT) and scaling (k^3)

- For example for *parity-odd* interactions (**α** is odd) we find only a handful of terms *without any kT poles*!

$$\alpha = 1: \quad \psi_n = \mathbf{k}\left[\frac{\text{Poly}_{2+p}}{k_T^p} + \text{Poly}_2\left(\ln(k_T\eta) + i\frac{\pi}{2}\right) + \frac{\text{Poly}_1}{\eta} + \frac{1}{\eta^2}\right]$$

MLT

$$\alpha = 3: \quad \psi_n = \mathbf{k}^3\left[\frac{\text{Poly}_p}{k_T^p} + \left(\ln(k_T\eta) + i\frac{\pi}{2}\right)\right]$$

Not in B_3 because of unitarity (COT)

Summary

- For massless scalars and gravitons, locality, unitarity and scale invariance imply that:

  - *parity-odd contact correlators, such as the tree-level bispectrum, cannot have kT-poles and come in a small number, even though there are infinitely many interactions in the Lagrangian!*

  - Tree-level GR correlators are rational functions (in even spacetime dimensions). No logs or polylogs.

All Graviton non-Gaussianities

All graviton bispectra

- All tree-level, scale-invariant graviton bispectra on de Sitter must take the form

$$\psi_3^{+++} = \sum \left[e^+(\mathbf{k}_1) e^+(\mathbf{k}_2) e^+(\mathbf{k}_3) \mathbf{k}_1^{\alpha_1} \mathbf{k}_2^{\alpha_2} \mathbf{k}_3^{\alpha_3} \right] \psi_3^{\mathrm{trim}}(k_1, k_2, k_3) \,,$$

- In terms of spinors this is (all other polarizations are fixed by this)

$$\psi_3^{+++} = \frac{[12]^2[23]^2[31]^2}{e_3^2} \sum_{\mathrm{perm's}} h_\alpha(k_1, k_2, k_3) \psi_3^{\mathrm{trim}}(k_1, k_2, k_3) \,,$$

$$h_0 = 1 \,, \quad h_1 = ik_1 \,, \quad h_2 = k_2 k_3 \,, \quad h_3 = iI_1 I_2 I_3 \,, \quad h_4 = I_1^2 I_2 I_3 \,,$$

$$h_{5a,b} = iI_1^3 I_2 I_3, iI_1 I_2^2 I_3^2 \,, \quad h_6 = I_1^2 I_2^2 I_3^2 \,, \quad h_7 = iI_1^3 I_2^2 I_3^2 \,.$$

- The trimmed wavefunction is the most general solution to the Manifestly Local Test (MLT)

Parity-odd graviton bispectra

- *Parity-odd* graviton non-Gaussianity is a poster child of the *boostless* cosmo bootstrap. The are infinitely many interactions

$$L \supset \dots + \epsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\lambda\delta} R^{\lambda\delta}{}_{\gamma\theta} R^{\gamma\theta}{}_{\mu\nu} R^n + \dots$$

- Yet, *to all orders in derivatives* there are only *three* possible bispectra (notice no kT poles)!

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{k_T \left(k_T^2 - 2e_2\right)}{e_3^3} \,,$$

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{(-3e_3 + k_T e_2)}{e_3^3} \,,$$

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{(-k_1 + k_2 + k_3)(k_1 - k_2 + k_3)(k_1 + k_2 - k_3)}{e_3^3} \,.$$

Parity-odd mixed bispectra

- Similarly, parity-odd Scalar-Scalar-Tensor and Scalar-Tensor-Tensor bispectra to all orders in derivates can only be:

$$B_3^{00+} = \frac{[13]^2[23]^2}{k_3^2[12]^2}\frac{(k_1+k_2-k_3)^2 k_3}{e_3^3}\,,$$

$$B_3^{0++} = \frac{[23]^4}{k_2^2 k_3^2 e_3^3}[q_{1,1}(k_2+k_3)k_1^2 + q_{1,2,a}(k_2^3+k_3^3) + q_{1,2,b}(k_2 k_3^2 + k_3 k_2^2)]\,,$$

$$B_3^{0+-} = \frac{[12]^4}{[31]^4}\frac{I_2^4}{k_2^2 k_3^2 e_3^3}\left[q_{1,1}(k_2-k_3)k_1^2 + q_{1,2,a}(k_2^3-k_3^3) + q_{1,2,b}(k_2 k_3^2 - k_3 k_2^2)\right]\,,$$

Parity-even bispectra

- There are infinitely many parity-even graviton bispectra, corresponding to interactions with a larger and larger number of derivatives

- In a technically natural theory, the larger the number of derivatives, i.e. the order of the polynomial, the smaller the contribution should be.

$$B_3^{+++} = \frac{\mathrm{SH}_{+++}}{e_3^3}\Bigg[g_{0,0}\left(4e_3 - e_2 k_T + (k_T^3 - 3k_T e_2 + 3e_3)\log(-k_T\eta_0/\mu)\right) + g_{0,2}\frac{e_2 e_3 + e_2^2 k_T - 2e_3 k_T^2}{k_T^2} + g_{0,3}\frac{e_3^2}{k_T^3} + \dots \Bigg],$$

Model Building

- All of above correlators can arise in a variety of concrete models.

- For example, all parity-odd graviton and mixed bispectra can appear in *Solid inflation* with arbitrary coefficients and no corrections to the quadratic theory (power spectrum)

- *EFT of Inflation: only one linear combination* of parity-odd graviton bispectra can arise from the 3d Chern-Simons term

$$\int a(\eta)\epsilon_{ijk}\left[\frac{{}^{(3)}\Gamma^{l}_{im}\partial_j{}^{(3)}\Gamma^{m}_{kl}}{2}+\frac{{}^{(3)}\Gamma^{l}_{im}{}^{(3)}\Gamma^{m}_{jn}{}^{(3)}\Gamma^{n}_{kl}}{3}\right],$$

  but must come with a parity-odd correction to the power spectrum.

- The parity-odd <scalar^2 tensor> cannot appear in the EFT of I [cfr Bartolo Orlando ’17]

Phenomenology

Perturbativity and the bounds on the tensor-to-scalar ratio imply various phenomenological constraints:

- Since there are only a few parity-odd shape, they should be a *primary targets* of observations (mostly CMB B-modes)
- Graviton bispectra have a smaller signal-to-noise ratio (S/N) than the graviton power spectrum, so they can be seen only after a detection of primordial tensor modes
- <scalar^2 tensor> has always a larger S/N than <tensor^3> or <scalar tensor^2> by a factor of ϵ^{-1}, so it should be the *first target,* unless one probes only the tensor sector

Horizons

- There are still basic and very general facts about quantum field theory on cosmological spacetimes that are awaiting to be discovered: *it's a wide open field of research*!

- Questions for the future include:

  - Can we derive "positivity bounds" for cosmology that encode the constraints of a consistent UV completion?

  - Are there measurable non-perturbative quantum gravity effects in cosmological correlators as e.g. in Black Hole physics?

  - Numerically bootstrap fully non-perturbative correlators in dS?

- Because of the ever growing body of cosmological dataset, advancements on the theory side are likely to have important repercussion on the phenomenology and ultimately make a long standing contribution to our understanding of the very early universe.