From Wormholes and Baby Universes to Inflation and the Measure Problem

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based on review with P. Soler / T. Mikhail

and including some results from work with Mangat/Theisen/Witkowski, with Daus/March-Russell/Leonhardt, and with Friedrich/Salmhofer/Strauss/Walcher

Outline

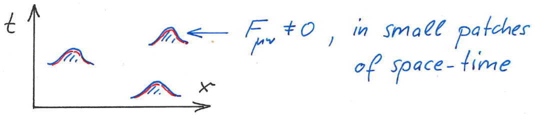
- Axions, Weak Gravity and
  Euclidean wormholes as the 'WGC-objects'
- 'Coleman's wormholes' and their problems
  (BH entropy, AdS/CFT, Global Symmetries)
- The need for topology change in eternal inflation

Axions

- One of our key players will be axion-like scalars:

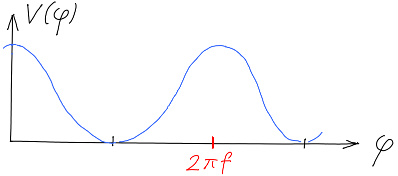
$$\mathcal{L} \supset -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{32\pi^2}\left(\frac{\varphi}{f}\right)\mathrm{tr}(F\tilde{F})\,.$$

- Their shift symmetry is generically broken by instantons:



$\Rightarrow \quad V_{eff} \sim \cos(\varphi/f)$,

$\varphi \equiv \varphi + 2\pi f$.



The weak gravity conjecture

Arkani-Hamed/Motl/Nicolis/Vafa '06

- Roughly speaking: 'Gravity is always the weakest force.'
- More concretely:

  For any U(1) gauge theory there exists a charged particle with

  $$\boxed{m < q} \qquad \text{(with } q = gn \text{ and } M_P = 1\text{)}.$$

  To apply it to axions + instantons, the WGC needs to be generalized ....

Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \sim \int (F_2)^2 + m \int_{1-dim.} d\ell + q \int_{1-dim.} A_1 \,.$$

- This generalizes to charged strings, domain walls etc. Crucially, the degree of the corresponding form-field (gauge-field) changes:

$$S \sim \int (F_{p+1})^2 + m \int_{p-dim.} dV + q \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

Generalizations to axions + instantons

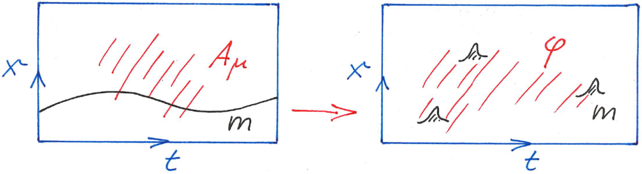
- One can also lower the dimension of the charged object, making it a point in space-time:

$$S \sim \int (d\varphi)^2 \; + m + q\,\varphi(x_{inst.})\,.$$

This should be compared with

$$\text{cf.} \quad S \sim \int (d\varphi)^2 + \int \text{tr}(F^2) + \int \frac{1}{f}\,\varphi\,\text{tr}(F\tilde{F})\,,$$

$$\text{where} \quad \int \text{tr}(F^2) \sim S_{inst.} \sim m\,.$$



WGC for instantons and axions

- First, recall the instanton-induced potential

$$V(\varphi) \;\sim\; e^{-S_{inst.}} \cos(\varphi/f)\,.$$

- Since, for instantons, $q \to 1/f$ and $m \to S_{inst.}$ we have

$$m < q \qquad \Rightarrow \qquad \boxed{S_{inst.} < 1/f}\,.$$

- Theoretical control (dilute instanton gas) requires $S_{inst.} > 1\,.$

- This implies $f < 1$ and hence
  large-field 'natural' inflation is in trouble.

Maybe more interesting: For $f < 1$ one gets a lower bound on the strength of instanton effects:

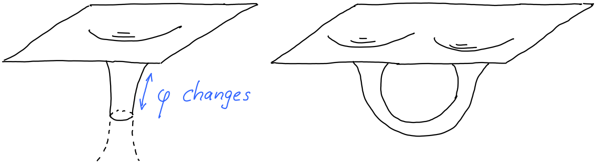
$$\boxed{\exp(-S_{inst.}) > exp(-1/f).}$$

Wormholes as the analogue of black holes in the axionic case

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ , instantonic solutions exist:

Giddings/Strominger '88

...



- The 'throat' is supported by the gradient energy of $\varphi = \varphi(r)$, with r the radial coordinate of the throat/instanton.

- The relevance for inflation arises through the induced instanton-potential for the originally shift-symmetric field φ.

Montero/Uranga/Valenzuela '15

Gravitational instantons (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim \mathcal{R} + f^2 |d\varphi|^2 \,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi \,.$$

- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim \mathcal{R} + \frac{1}{f^2} |dB_2|^2 \,.$$

- The 'throat' exists due the compensation of these two terms. Reinstating M_P, allowing n units of flux (of $H_3 = dB_2$) on the transverse S^3, and calling the typical radius R, we have

$$M_P^2 \, R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad M_P \, R^2 \sim \frac{n}{f} \,.$$

Gravitational instantons (continued)

- Returning to units with $M_P = 1$, their instanton action is

$$\boxed{S = \left(\pi\sqrt{6}/4\right)\; n/f} \qquad \text{(with } n \text{ the instanton number).}$$

- Parametrically, this agrees with the WGC instanton action.
  $\Rightarrow$ It may be taken to define the precise instanton WGC.

  AH/Mangat/Theisen/Witkowski

- The maximal WH-curvature scale is $\sqrt{f/n}$, which should not exceed the UV cutoff:

$$f/n < \Lambda^2$$

- This fixes the lowest n that we can trust and hence the minimal size of the instanton correction to the potential $V(\varphi)$:

$$\boxed{\delta V \sim e^{-S} \sim e^{-n/f} \sim e^{-1/\Lambda^2}}$$

Gravitational instantons (continued)

- For gravitational instantons not to prevent inflation, the relative correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda^2}}{H^2} \ll 1$$

- For a Planck-scale cutoff, $\Lambda \sim 1$, this is never possible
- However, the UV cutoff can in principle be as low as H
- Then, if also $H \ll 1$, everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H^2}}{H^2}$$

AH, Mangat, Rompineve, Witkowski '15

For more details see e.g. Heidenreich/Reece/Rudelius '15, AH/Mangat/Theisen/Witkowski '16, Hertog/Trigiante/Van Riet '17, ... Andriolo/Huang/Noumi/Ooguri/Shiu '20 ...

...However, beyond inflation, wormholes remain very interesting, both conceptually and phenomenologically

Gravitational instantons - Small-f axions

Coleman/Lee, Rey ∼'90 .... Alonso/Urbano '17 .... Alvey/Escudero '20, ... Andriolo/Shiu/Soler/Van Riet '22

- For example, for a QCD axion with (relatively) high f, the wormhole effect might be relevant:

$$V(\varphi) = \Lambda_{QCD}^4 \cos(\varphi) + r_c^{-4} e^{-S_w/2} \cos(\varphi + \delta) \,.$$

- It turns out that for $f \gtrsim 10^{16}$ GeV the solution to the strong CP problem is lost.

- Interesting positive observational consequences exist in the context of black-hole superradiance and ultralight dark matter.

Example: Fuzzy Dark Matter

Alonso/Urbano '17
AH/Mikhail/Soler '18

- Fuzzy Dark Matter is, by definition, so light that its de Broglie wavelength affects sub-galactic-scale structure: $m \lesssim 10^{-21}$ eV
- If wormholes are the universal, model-independent effect of shift symmetry breaking, then this fixes f by the relation

$$m^2 \sim \exp(-1/f)$$

- At the same time, the abundance of Fuzzy Dark Matter is given by

$$\Omega_{FDM} h^2 \approx 0.1 \left(\frac{f}{10^{17}\text{GeV}} \right)^2 \left(\frac{m}{10^{-22}\text{eV}} \right)^{\frac{1}{2}}$$

- Together, these two relations lead to a slight clash (between wormholes/WGC and Fuzzy Dark Matter pheno):

One finds $m \gtrsim 10^{-19}\,\text{eV}\,,$ ...slightly too high, but....

Cicoli/Guidetti/Righi/Westphal '21

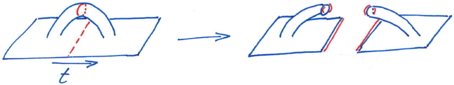
Gravitational instantons / wormholes - conceptual issues

- Motivated by the above, it is worthwhile revisiting some very fundamental conceptual issues of (euclidean) wormholes.

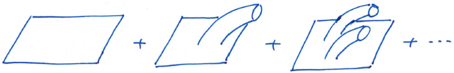
  Hawking '78..'88, Coleman '88, Preskill '89
  Giddings/Strominger/Lee/Klebanov/Susskind/Rubakov/Kaplunovsky/..
  Fischler/Susskind/...

  Recent review: AH, P. Soler, T. Mikhail '18

- First, once one allows for wormholes, one has to allow for baby universes.

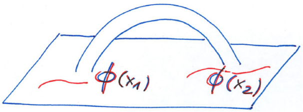


- Second, with baby universes comes a 'baby universe state' (α vacuum) encoding information on top of our 4d geometry.



Conceptual issues (continued)

- Crucially, α-parameters remove the disastrous-looking bilocal interaction.



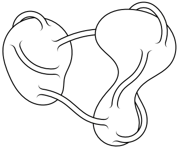
$$\exp\left(\int_{x_1}\int_{x_2}\Phi(x_1)\Phi(x_2)\right) \quad \to \quad \int_\alpha \exp\left(-\frac{1}{2}\alpha^2 + \alpha\int_x \Phi(x)\right)$$

- In our concrete (single-axion) case, an α parameter now governs the naively calculable $e^{-S}\cos(\varphi/f)$-term.

- But, what is worse, all coupling constants are 'renormalized' by α parameters and are hence not predictable in principle.

Conceptual issues (continued)

- Most naively, 4d measurements collapse some of the many α parameters to known constants.
- But in a global perspective, both different 4d geometries and α parameters have to be integrated over.
- This leads to the 'Fischler-Susskind-Kaplunovsky catastrophe'.



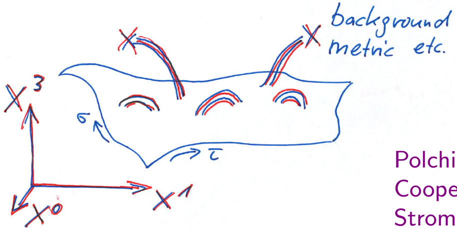
- Another key problem is a possible clash with locality on the CFT-side of AdS/CFT

Arkani-Hamed/Orgera/Polchinski '07, ..., 'SYK'

- Finally, just integrating over the α parameters is clearly not sufficient - one needs to consider their full quantum dynamics.

Conceptual issues (continued)

- Indeed, consider the case of 1+1 dimensions with a number of scalar fields (in addition to gravity).
- This is, of course, well known as string theory and the α parameters characterize the geometry the target space.



Polchinski, Banks/Lykken/O'Loughlin, Cooper/Susskind/Thorlacius, Strominger '89...'92

- The latter has a quantum dynamics of its own, the analogue of which in case of 3+1 dimensions is completely unknown.
- All this raises so many complicated issues, that one might want to dismiss wormholes altogether.

Conceptual issues (continued)

- But this is not easy, for example because we know that in string theory wormholes correspond to string loops and are a necessary part of the theory.

- Thus, forbidding for example topology change in general does not appear warranted.

- Is there a good reason to forbid topology change just in $d > 2$?

- Arguments have been given that the euclidean Giddings-Strominger solution has negative modes and should hence be dismissed.

  Rubakov/Shvedov '96, Maldacena/Maoz '04,
  see however Alonso/Urbano '17, ..., Loges/Shiu/Sudhir '22

- But the most recent results favor stability ...

Recent development: Wormholes and BH entropy

(very briefly)

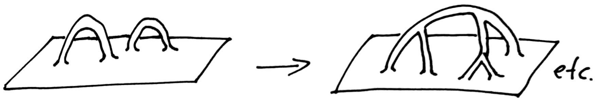
- Recently, a concrete proposal for calculating the entropy of an evaporating BH has emerged (method of 'Islands')

Penington, Almheiri/Engelhardt/Marolf/Maxfield, Almheiri/Mahajan/Maldacena/Zhao, ... '19/20

- The concrete mechanism by which entropy leaves the BH in this approach is related to euclidean WHs

- Motivated by this, a new 2d toy model developing Coleman's baby universe calculation has been suggested

Marolf/Maxfield '20



Recent development: Wormholes and BH entropy (continued)

- In particular, Marolf/Maxfield proposed to mod out the naive BU Hilbert space by a certain equivalence (related to 1 BU $\rightarrow$ 2 BU transitions, etc.)

- It has then be proposed that, in $d \geq 4$, this equivalence should be so strong that the BU Hilbert space is 1-dimensional

  McNamara/Vafa '20

- This would not remove the effect of BUs completely, but it would get rid of the arbitrariness of α parameters

  see also Betzios/Gaddam/Papadoulaki '22

- But can we do a proper calculation in $d \geq 4$? Is averaging over CFTs the solution? (cf. SYK)

  Chandra/Collier/Hartman/Maloney '22
  Schlenker/Witten '22

Recent development: Global Symmetry Conjecture

based on Daus/Leonhardt/March-Russell '20

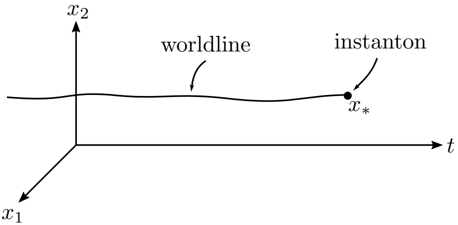
for an alternative view see Fichet/Saraswat '20

- Claim: For the important sub-class (gauge-derived, approximate global symmetries) the strength of violation may be derived from the WGC:
- **Gauge-derived global symmetry means:**

  Gauge an axion with a $U(1)$ vector field;

  The leftover in the IR are the light $U(1)$-charged states, but now only protected by a global symmetry.
- Instantons automatically destroy such globally-charged particles

  (cf. many stringy examples)



Recent development: Global Symmetry Conjecture (continued)

- Thus, by the WGC for axions the particle-number violation is suppressed by

$$\exp(-S_{inst.}) \;\sim\; \exp(-M_P/f)$$

- Moreover, according to the magnetic WGC for axions there is a UV-cutoff due to light strings:

AH/Soler '17

$$\Lambda \;\sim\; \sqrt{M_P f}$$

Hence, in total the global-symmetry violation is bounded below by

$$\exp(-S_{inst.}) \;\sim\; \exp(-M_P^2/\Lambda^2)$$

- Very intriguingly, this is the same as the wormhole-derived bound,

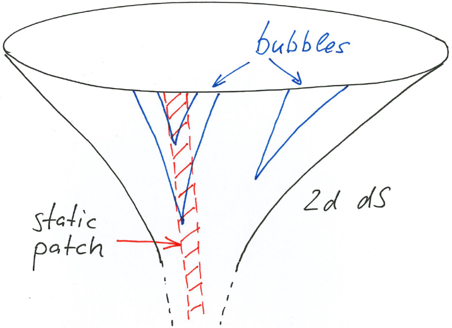
$$S_{WH} \;\sim\; M_P^2 \int \mathcal{R} \;\sim\; M_P^2/\Lambda^2 \,.$$

so wormholes fit really well into our limited QG-understanding.

A possible relation to eternal inflation and the measure problem

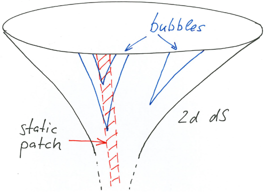
For details see our recent paper on the 'Local WDW Measure' (Friedrich/AH/Strauss/Salmhofer/Walcher)

- Cosmological central dogma:
  dS has Hilbert space of dimension $\exp(S) \sim \exp(M_P^2 R^2)$.
- Only causal patch is 'real' – the ever growing S^3 of global dS is unphysical



Eternal inflation / measure problem (continued)

- The WDW wave function of the universe describes a static patch tunnelling back and forth between different minima.



- Require a stationary solution in the presence of sinks ($\equiv$ terminal AdS/Mink. vacua)
- $\Rightarrow$ Must allow for sources: Linde/Vilenkin or Hartle-Hawking creation of universes our of nothing.
- Thus, topology change (in analogy to the wormhole/BU story described above) appears once again unavoidable.

Summary/Conclusions

- The WGC for axions demands certain minimal-action instantons and hence certain minimal potentials.
- Euclidean WHs may be the universal, semiclassical counterpart of WGC-instantons, making the bound precise:

$$S\, f = \left(\pi\sqrt{6}/4\right)\, M_P$$

- This number does actually matter: In inflation, for the strong CP problem, fuzzy DM, axionic dark energy, ... Rudelius '22
- But wormholes come at the price of α vacua (and possibly other disasters). Conceivably, they will deeply affect our view on AdS/CFT ....
- A key question is whether and how topology change finds its way into 'our' 4d quantum gravity.
- The WDW-view of the universe supports a positive answer....