

# Lattice simulations of inflation and reheating



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**ILLINOIS**  
Physics

GRAINGER COLLEGE OF ENGINEERING

# The equation of state after inflation

# What is $w$ after inflation?

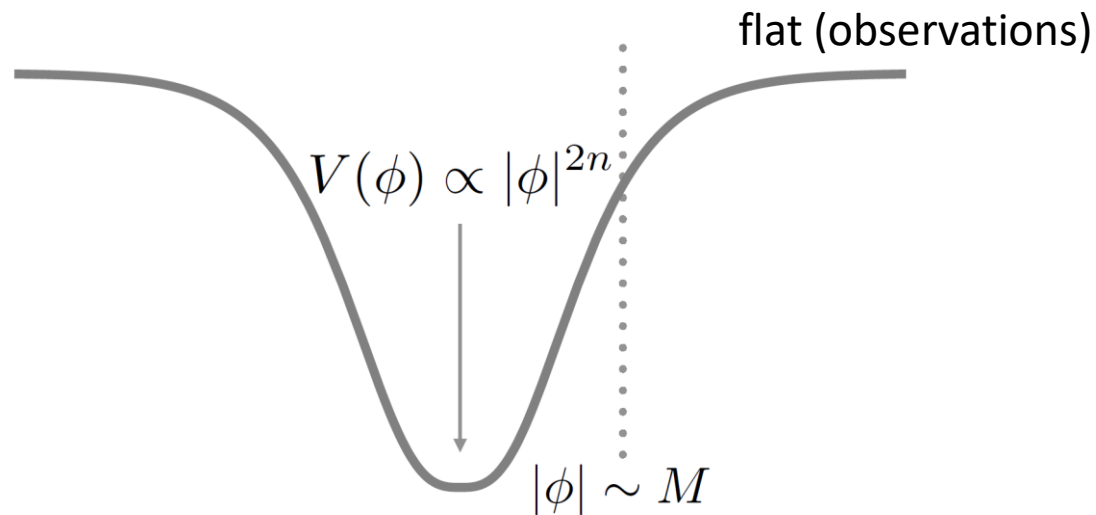
$$w = \frac{\text{pressure}}{\text{energy density}}$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} \mathcal{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

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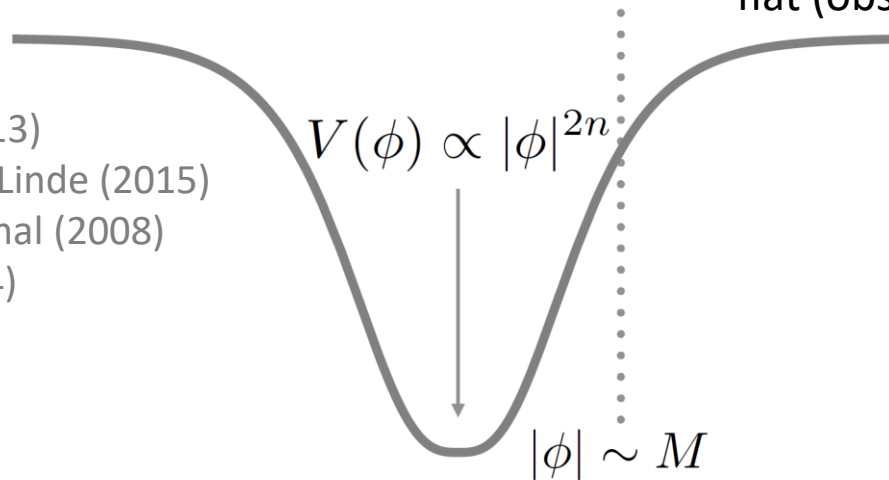
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flat (observations)

$$V(\phi) \propto |\phi|^{2n}$$

$$|\phi| \sim M$$

Kalosh and Linde (2013)  
Carrasco, Kalosh and Linde (2015)  
Silverstein and Westphal (2008)  
McAllister, et al. (2014)



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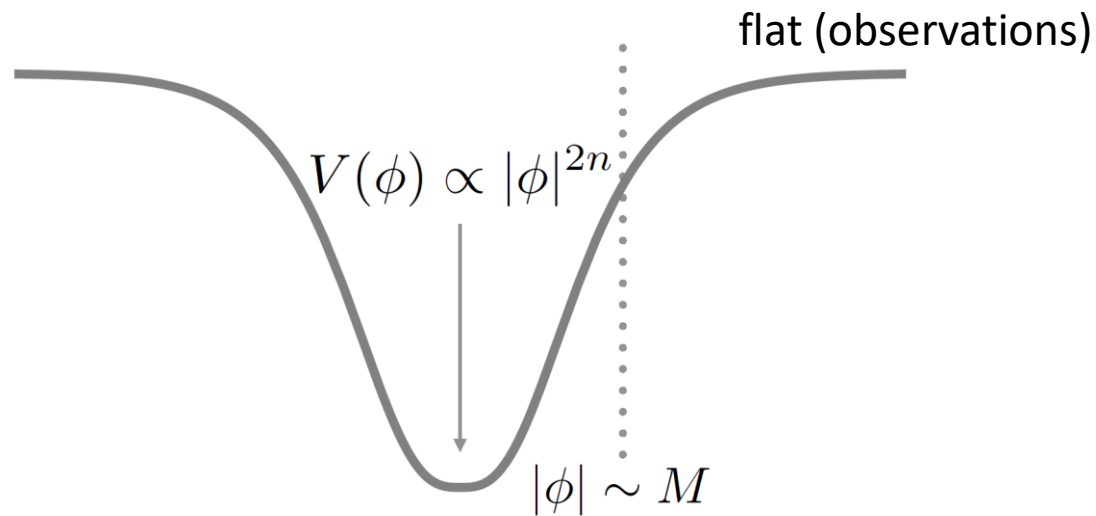
assumption: self-couplings dominate over others

# What is $w$ after inflation?

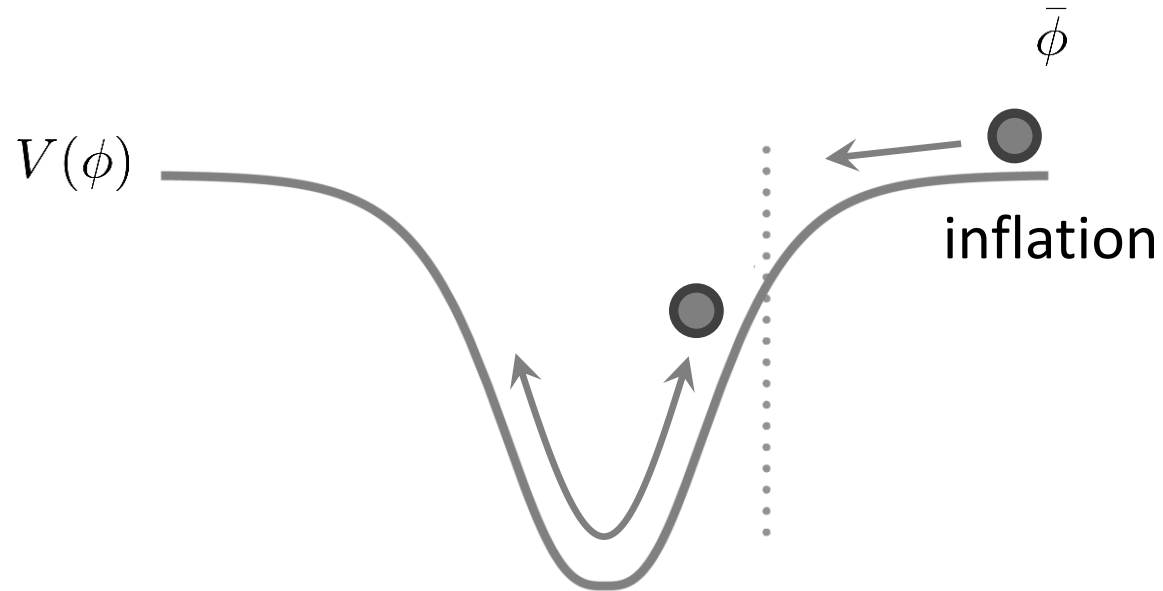
at sufficiently late times:

$$w = \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

(even without couplings to other fields!)



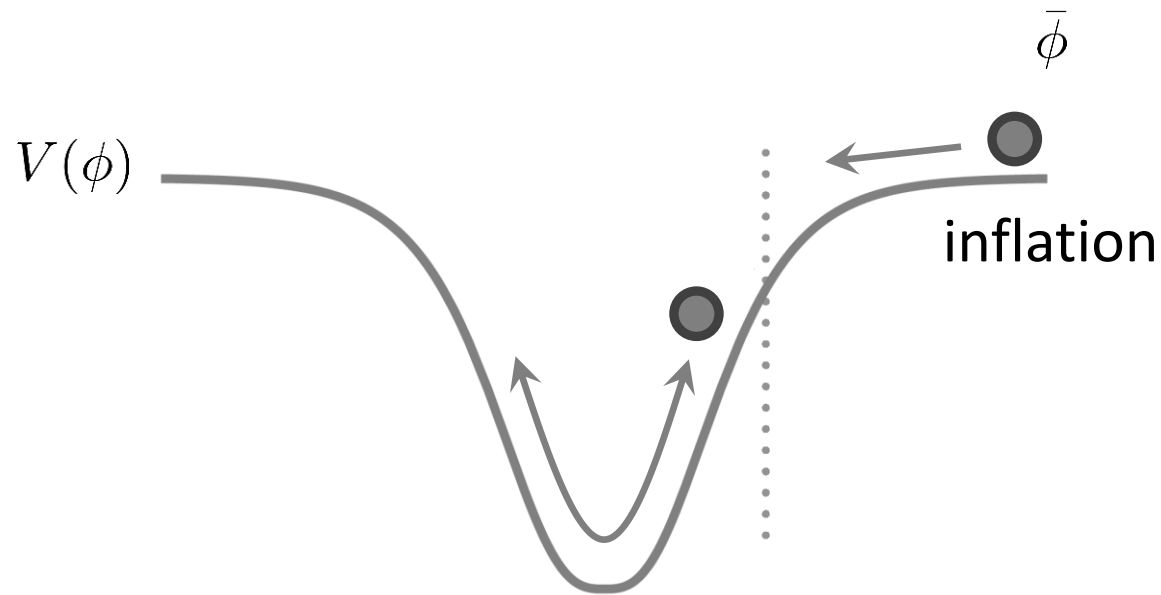
# Inflaton dynamics



inflation ends:  
oscillatory phase

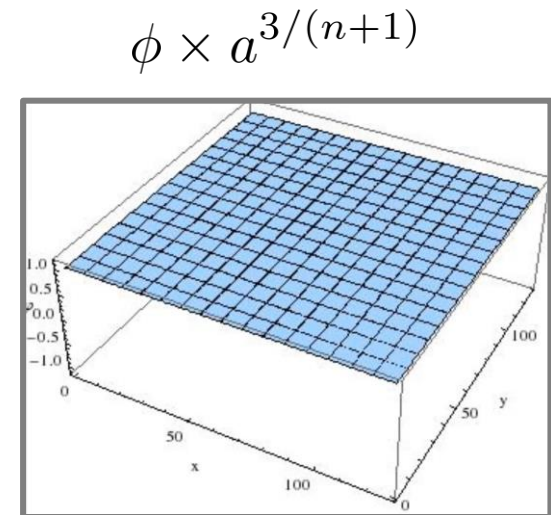


# Inflaton (homogeneous) dynamics

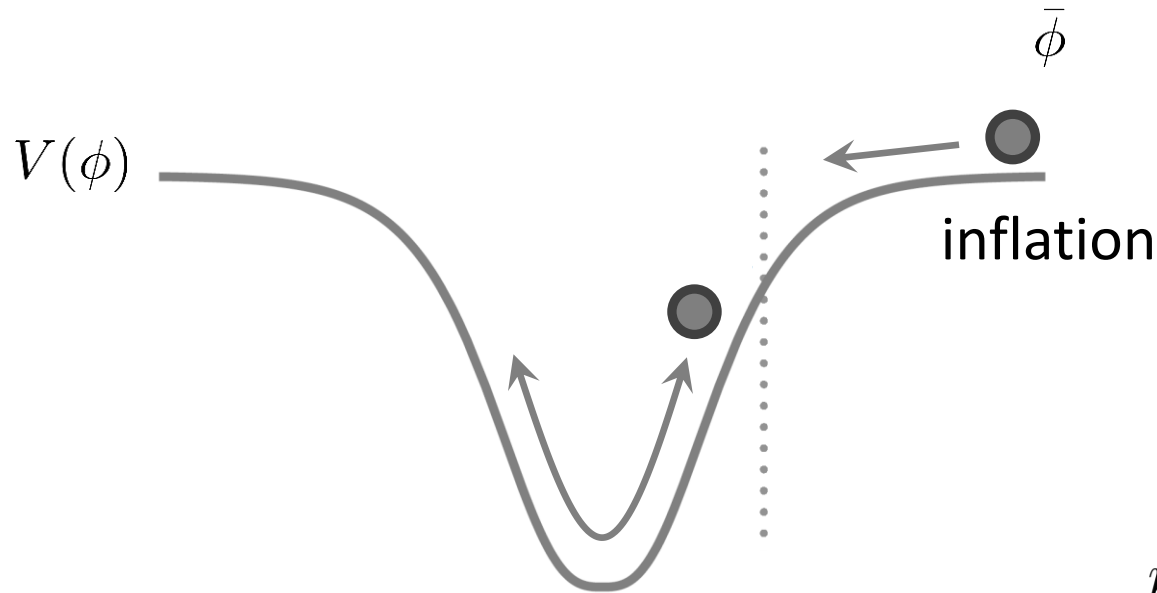


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$$\bar{\phi}(t) \propto a(t)^{-3/(n+1)}$$



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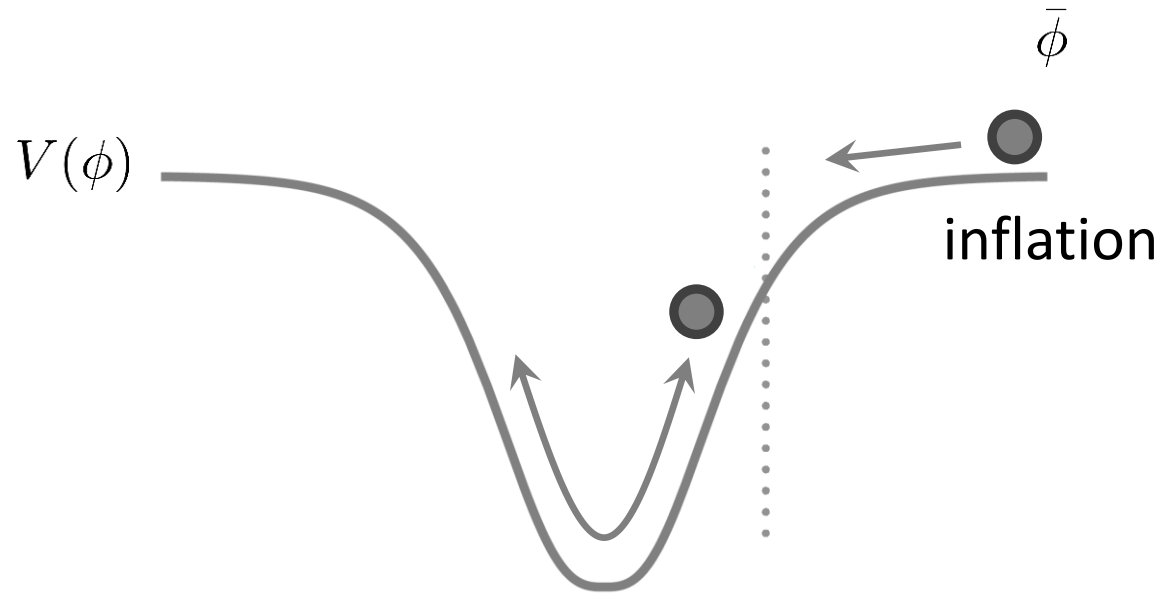


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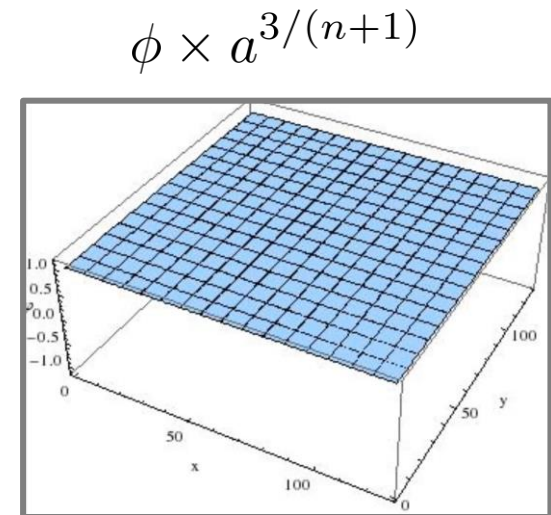
$$w \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - (\nabla\phi)^2/6 - V(\phi)}{\dot{\phi}^2/2 + (\nabla\phi)^2/2 + V(\phi)}$$

# Inflaton (homogeneous) dynamics



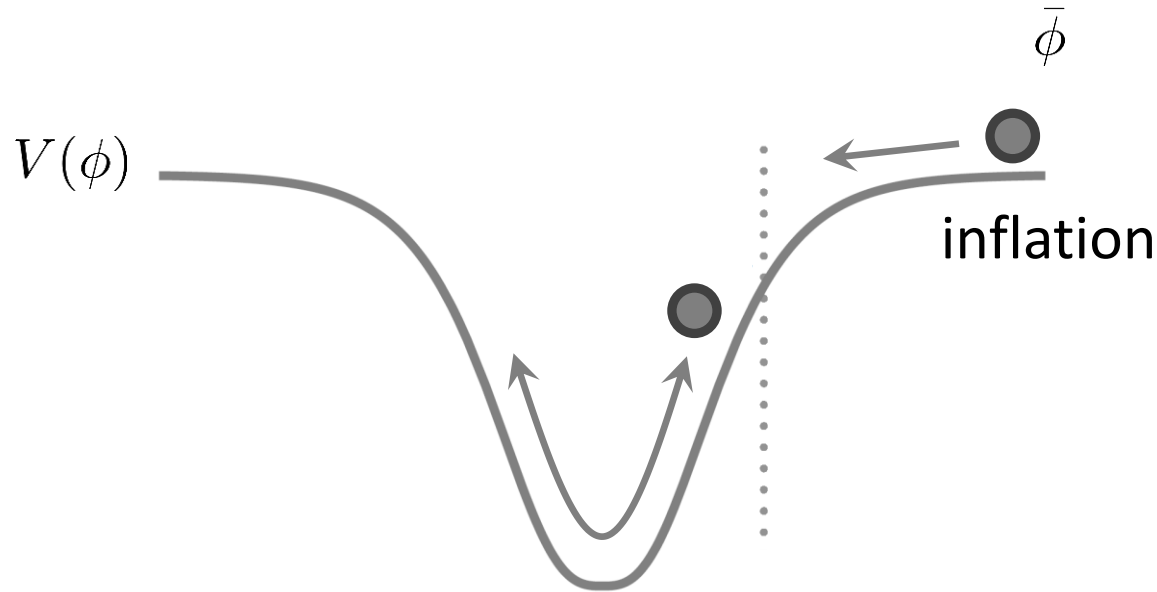
inflation ends:  
oscillatory phase

$$w_{\text{hom}} = \frac{n-1}{n+1}$$



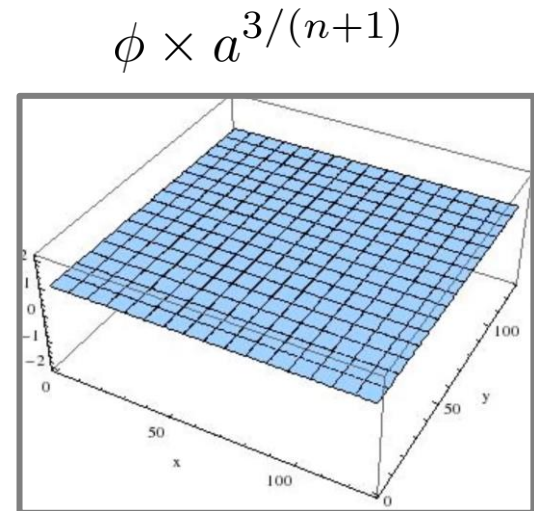
Turner (1983)

# Inflaton (actual) dynamics



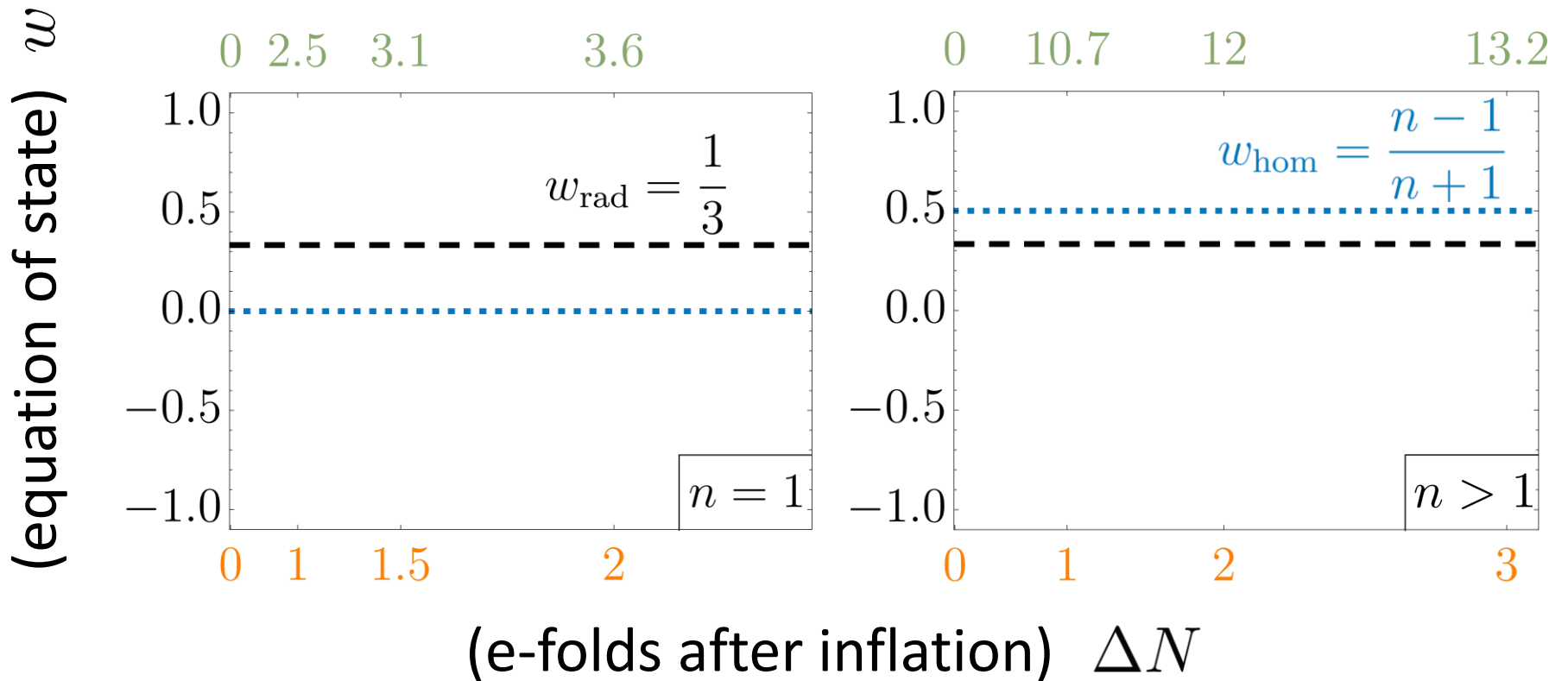
inflation ends:  
oscillatory phase

- parametric resonance of  $\delta\phi(t, \mathbf{x})$
- $\bar{\phi}$  fragments



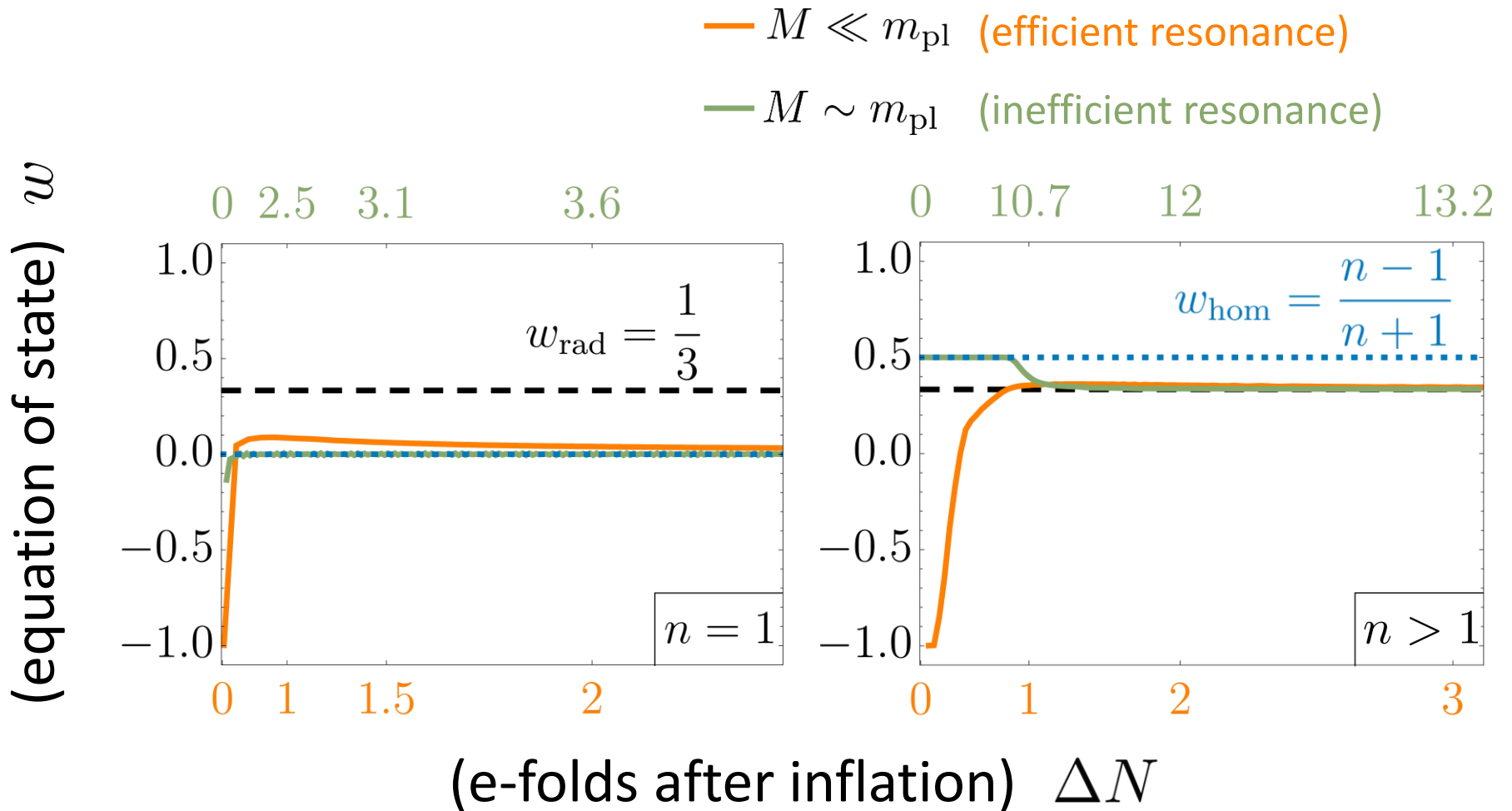
KL and M. Amin (2017)  
KL and M. Amin (2018)

# Equation of state



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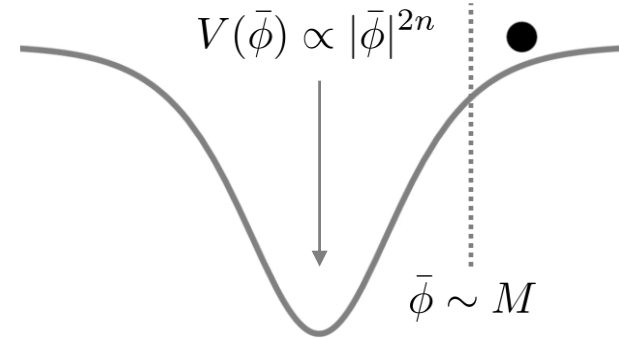
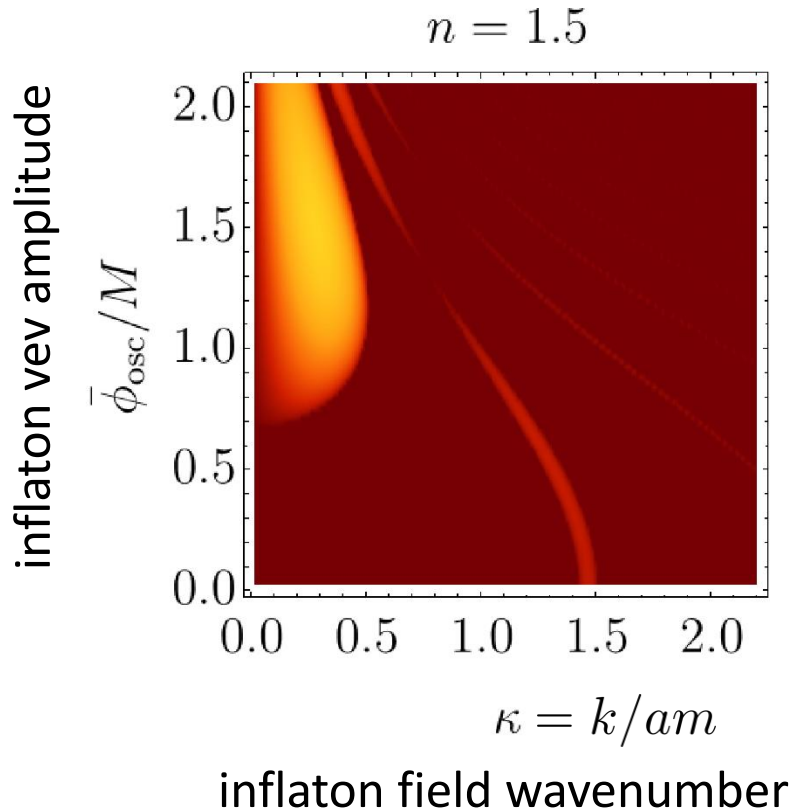
# Towards radiation domination

$$n > 1$$

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Non-perturbative decay (parametric self-resonance)



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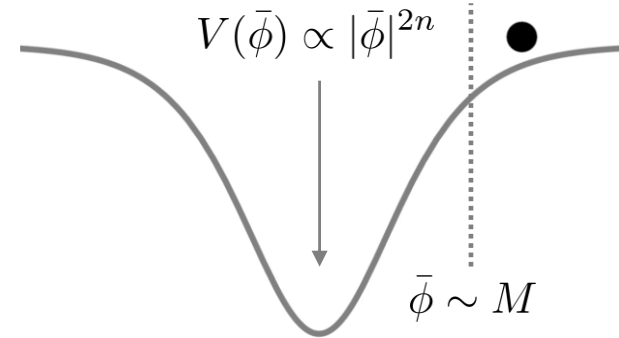
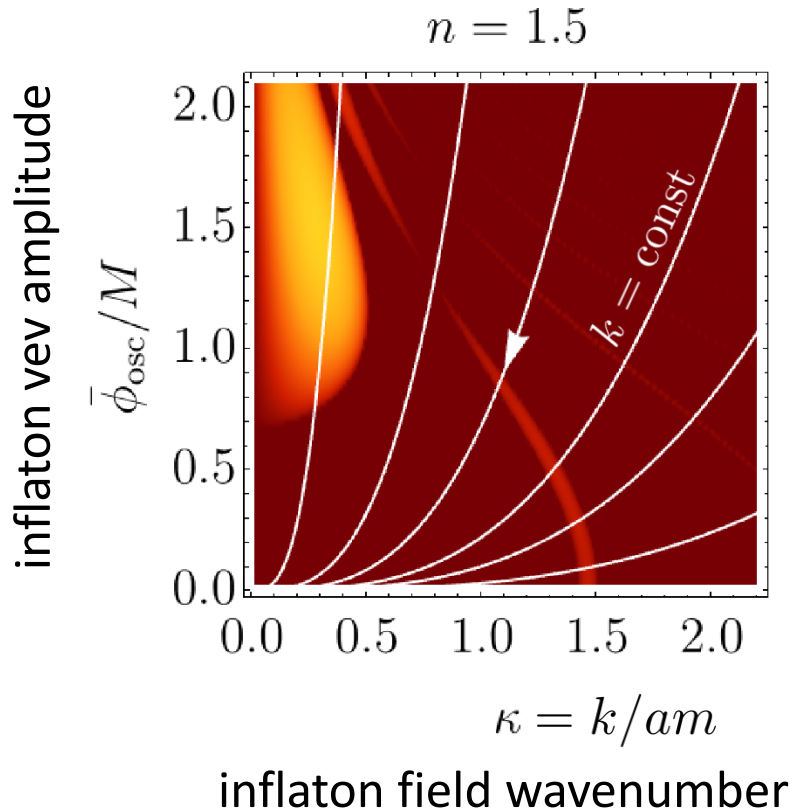
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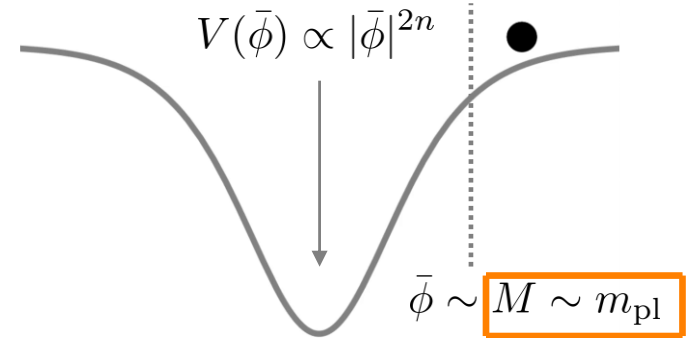
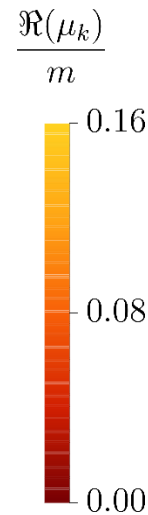
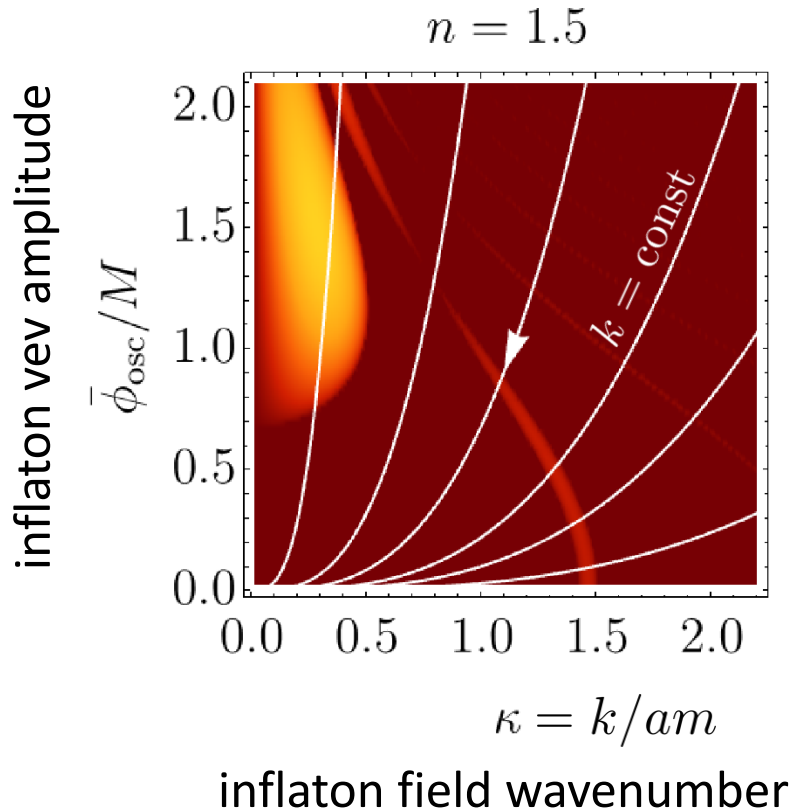
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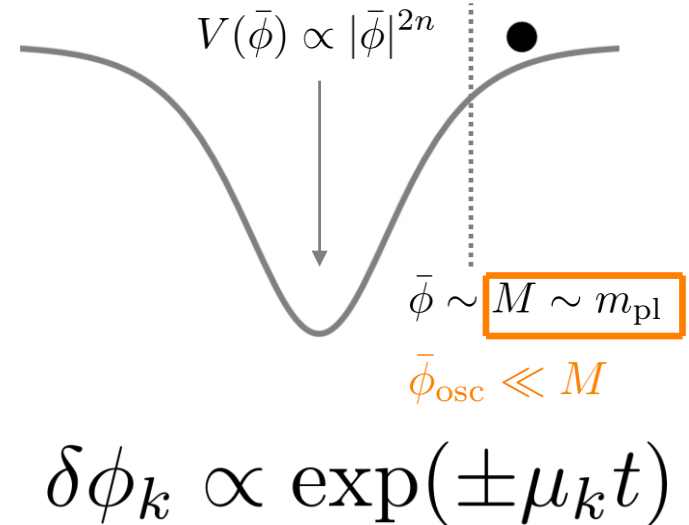
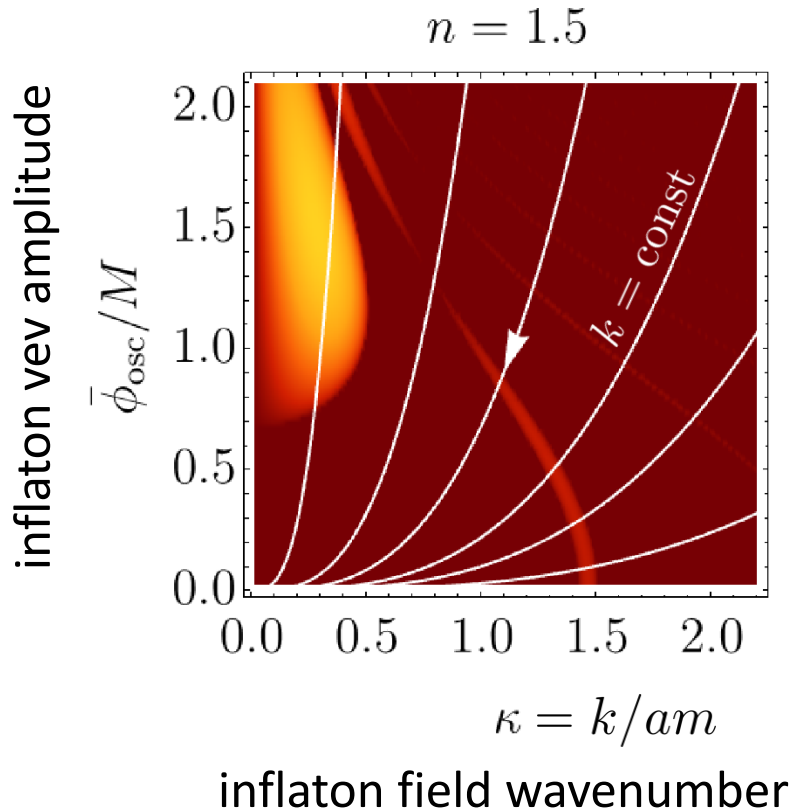
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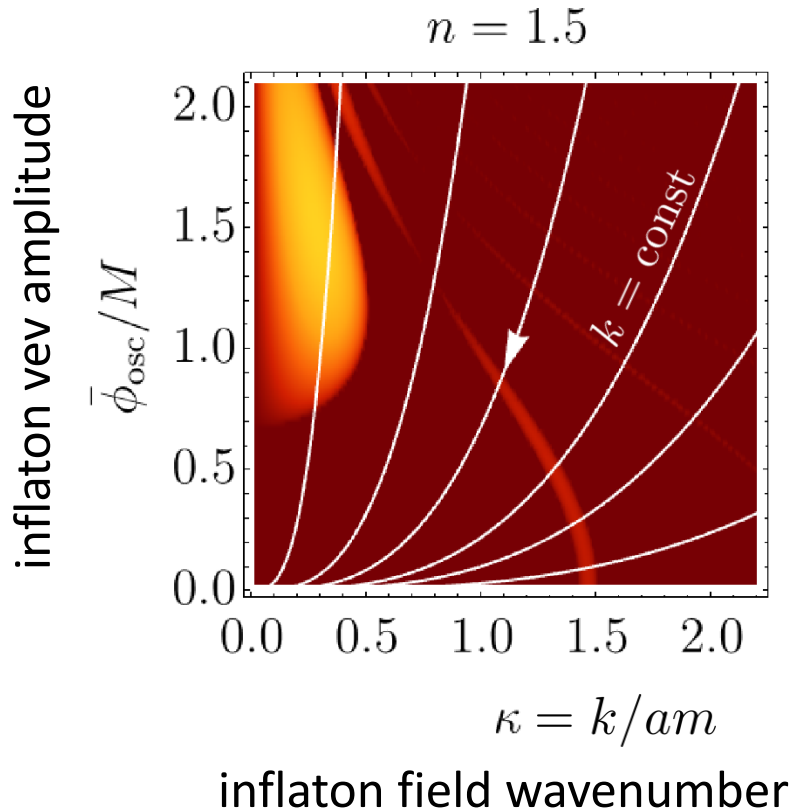


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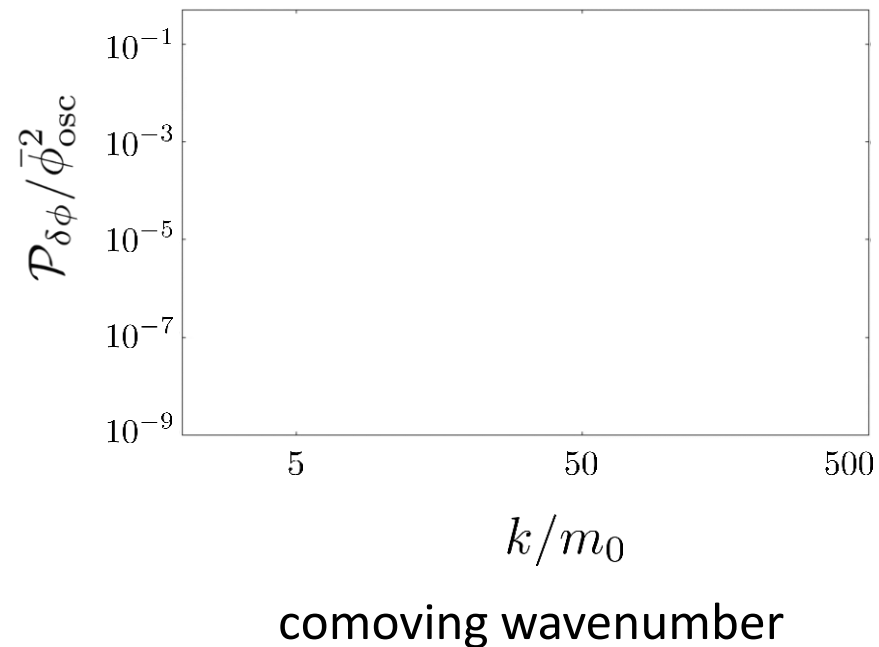
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Power spectrum:

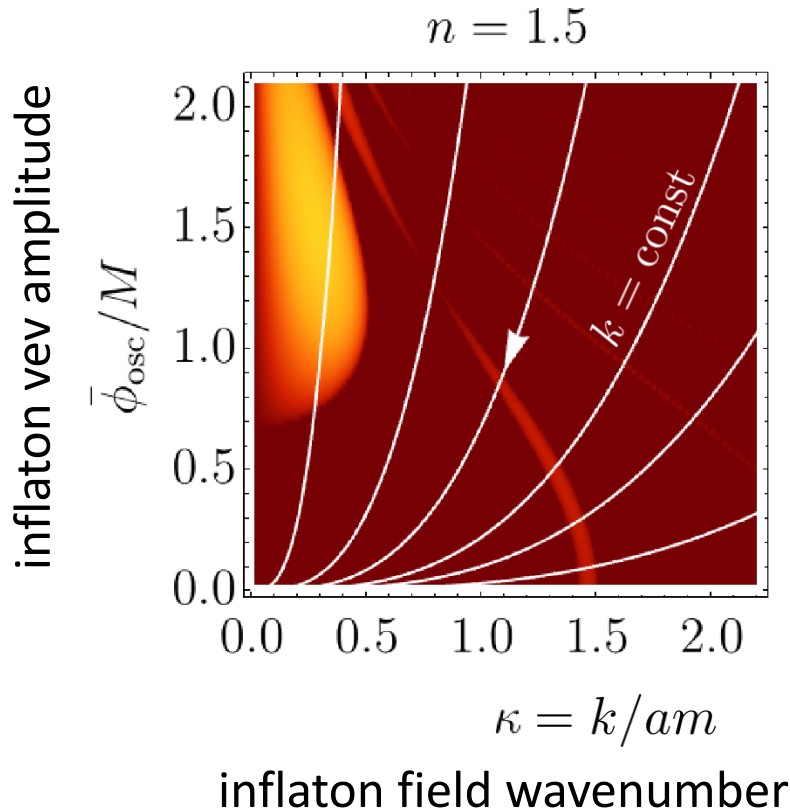
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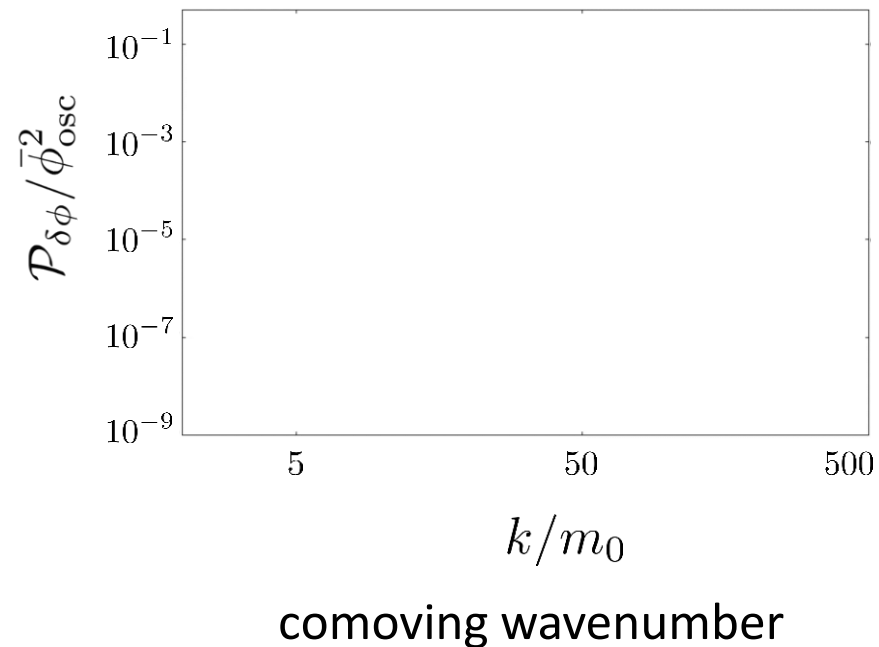
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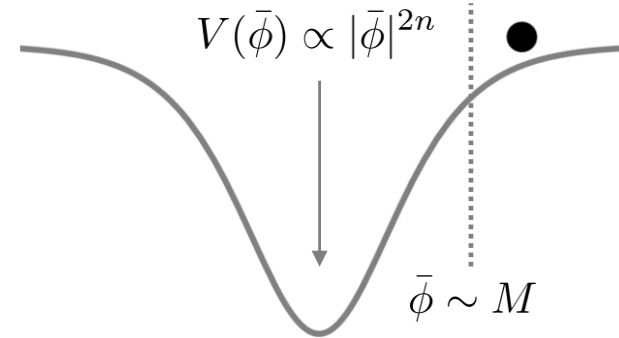
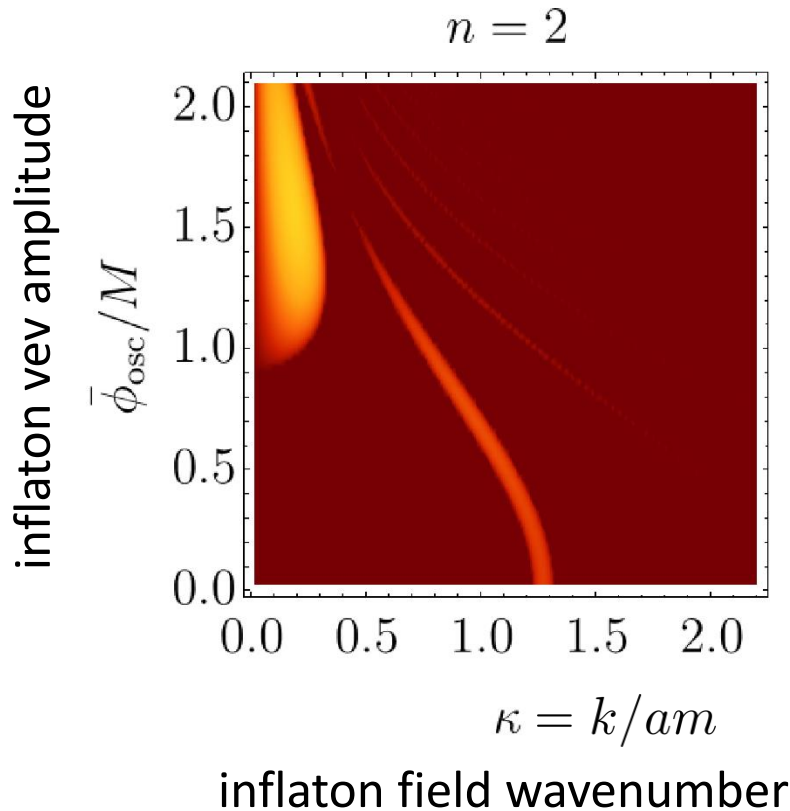
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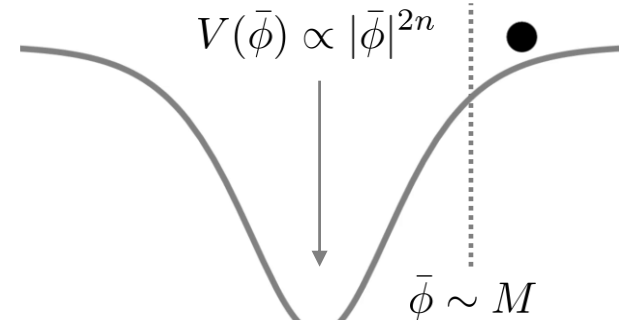
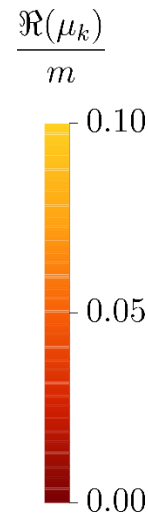
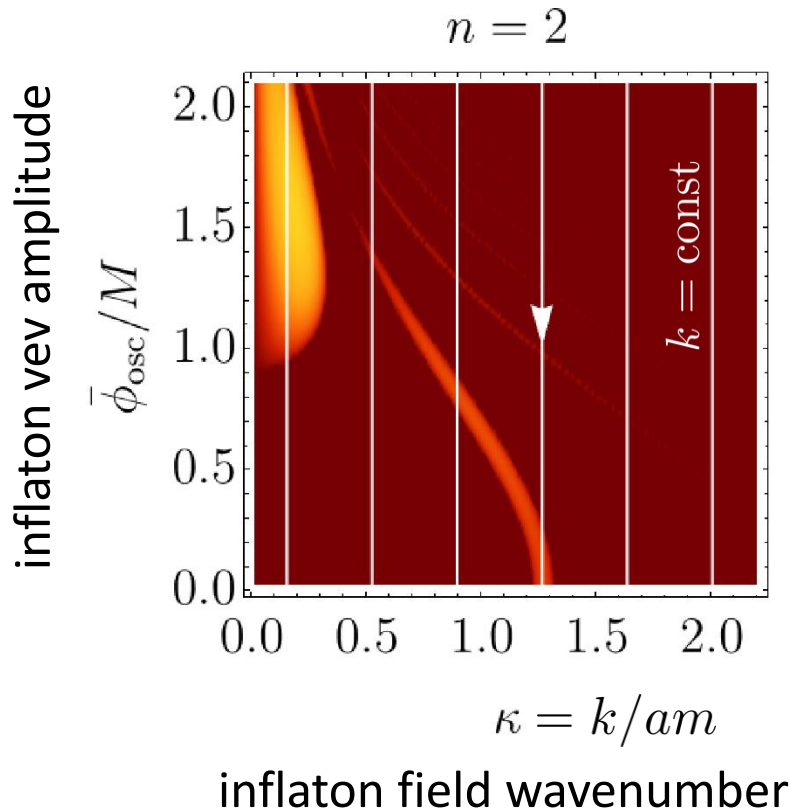
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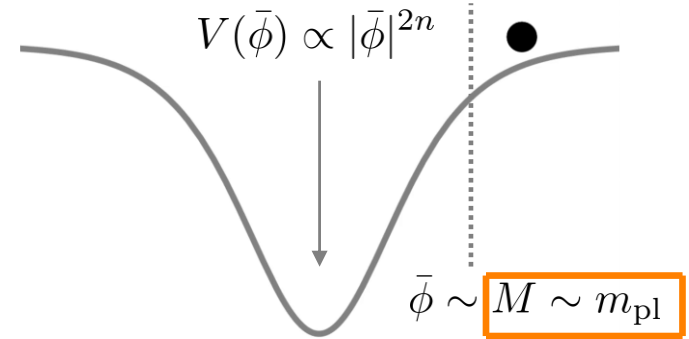
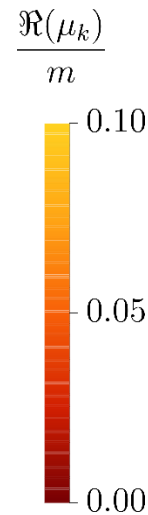
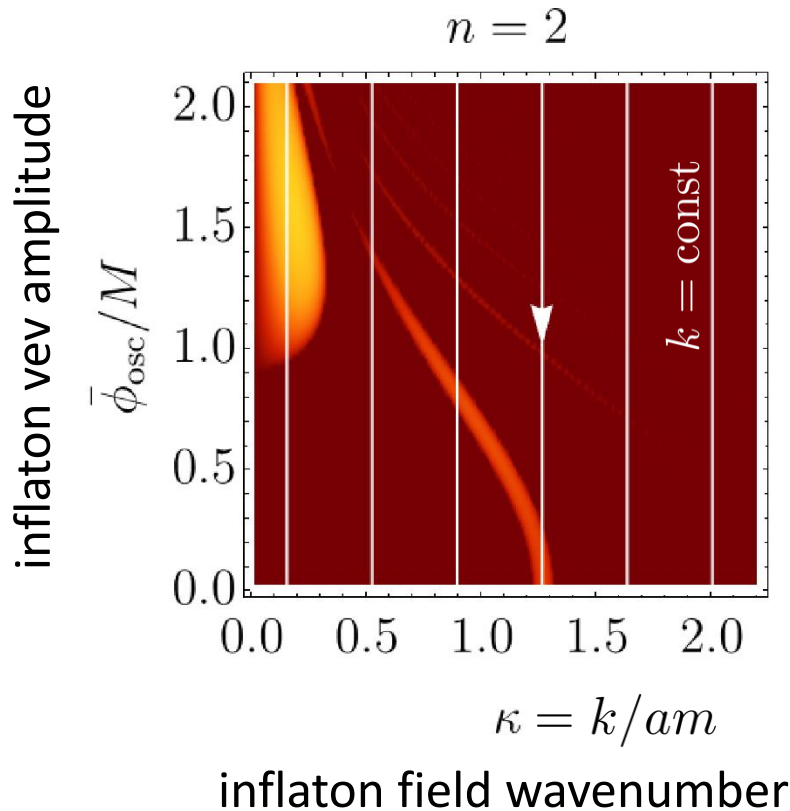
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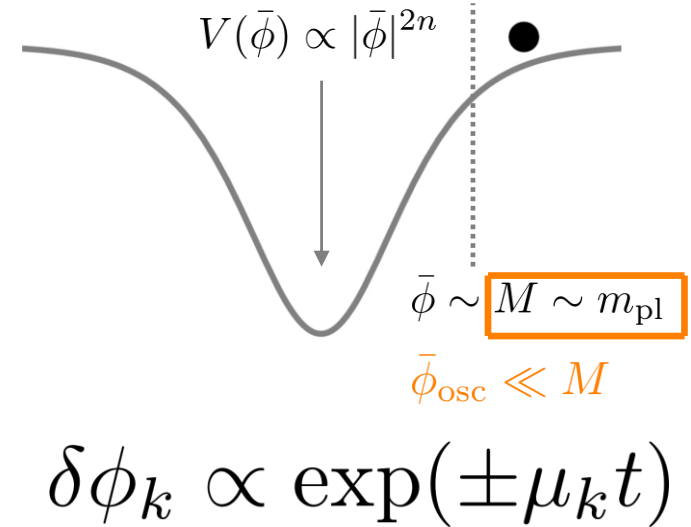
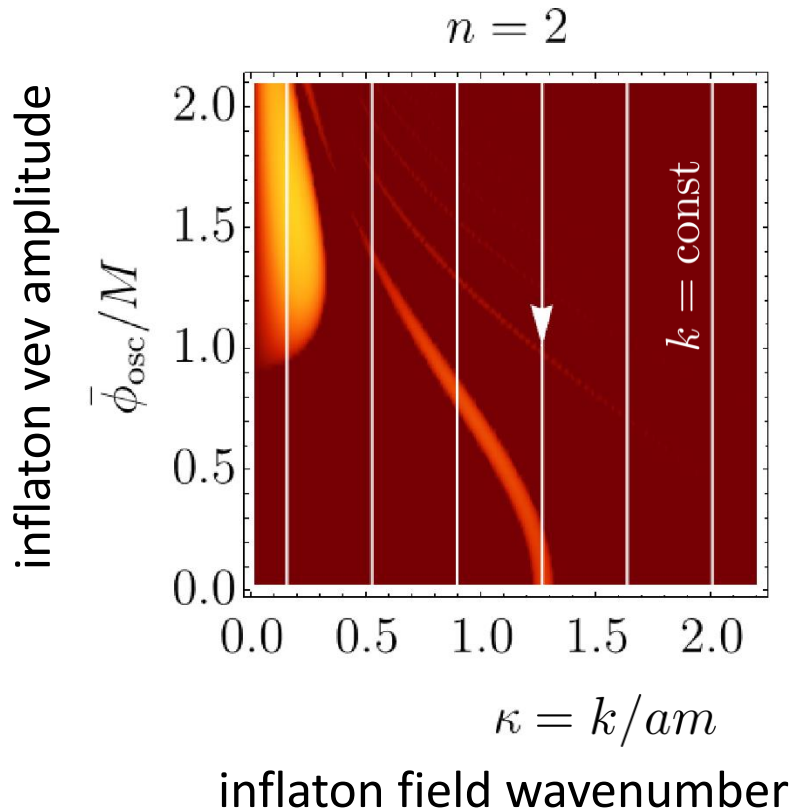
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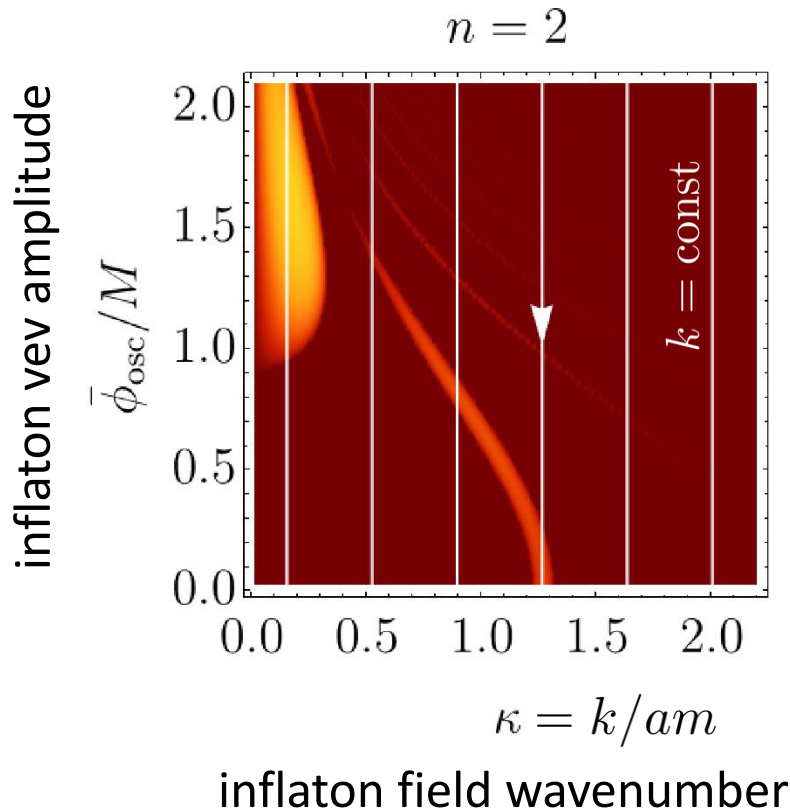


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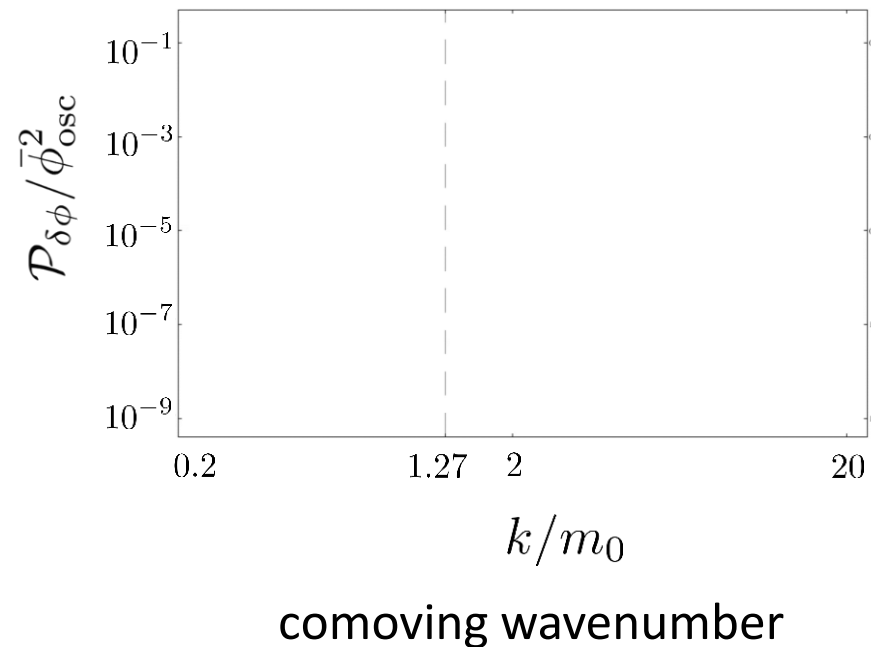
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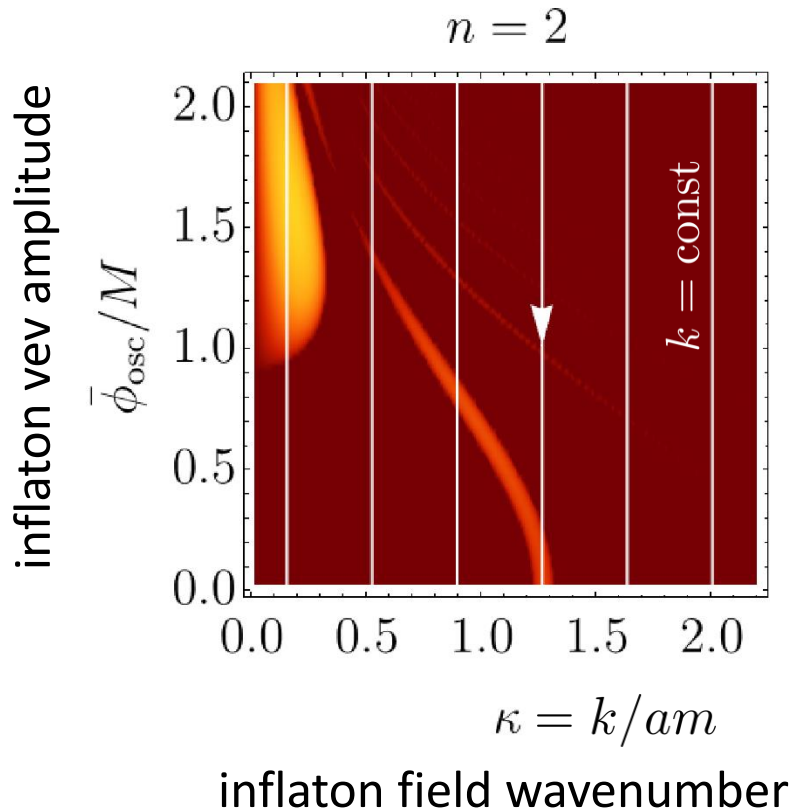
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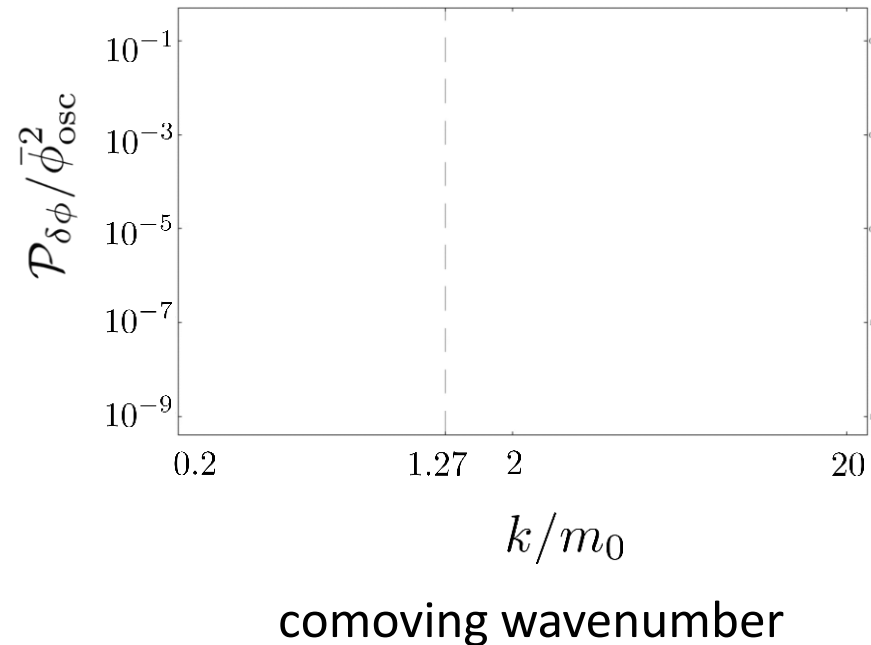
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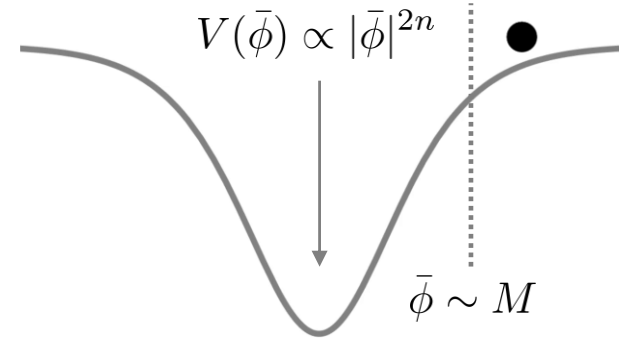
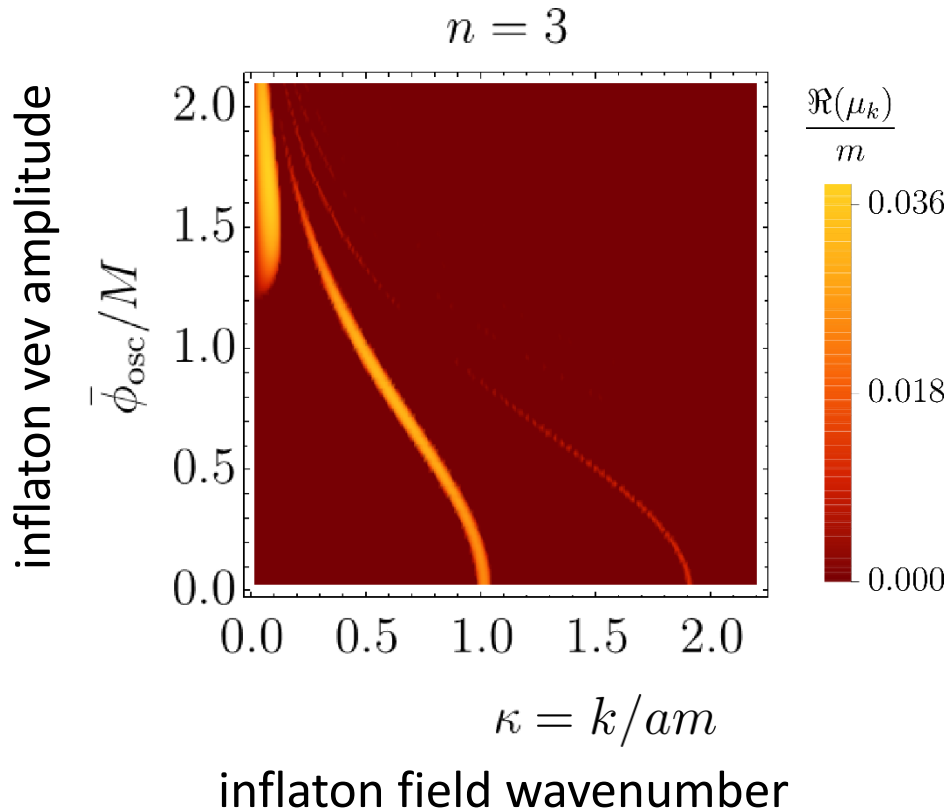
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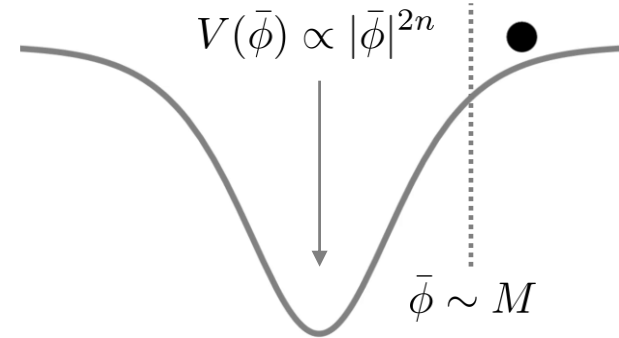
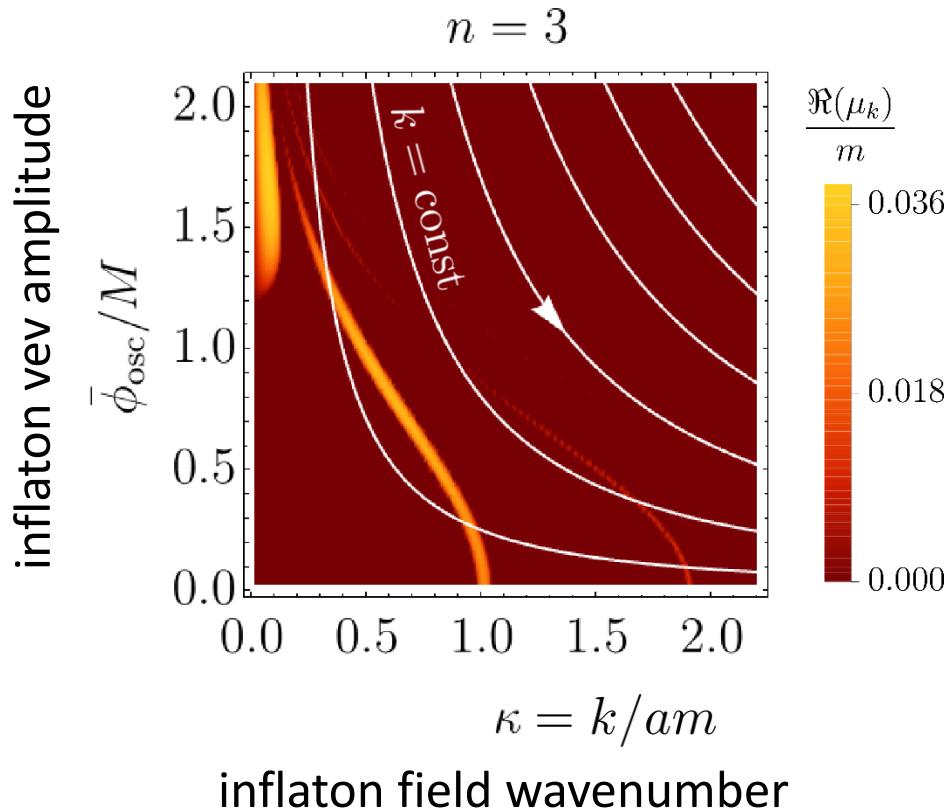
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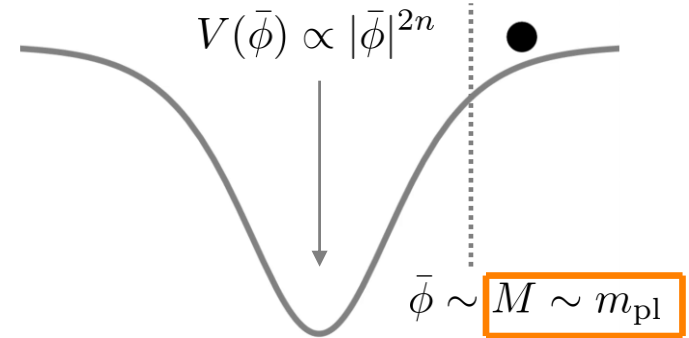
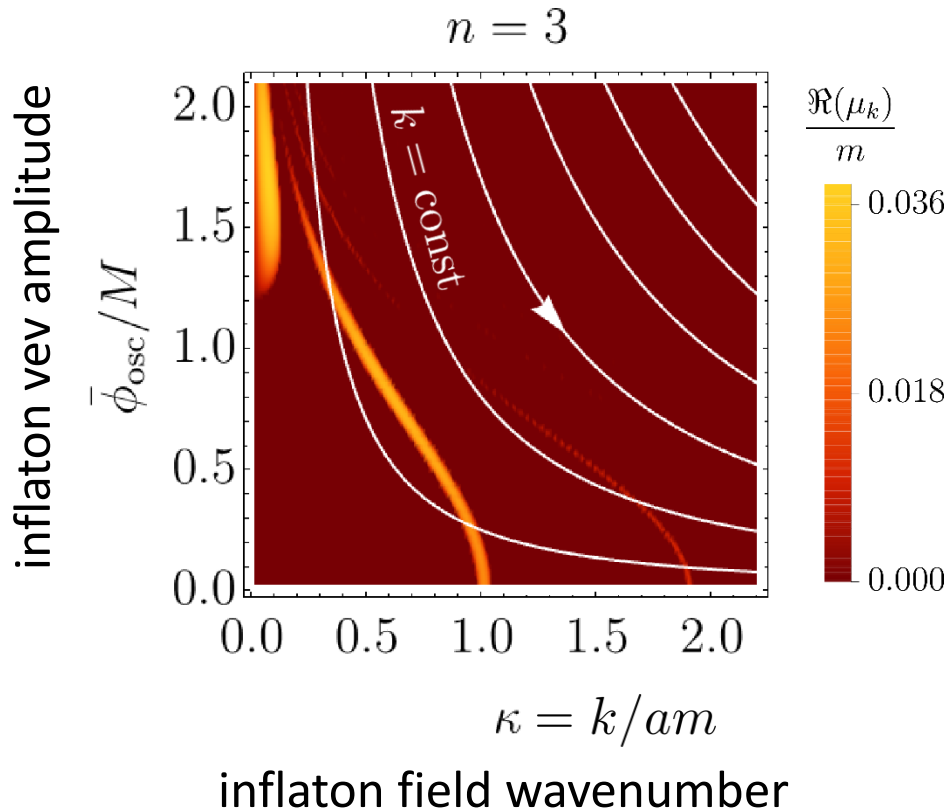
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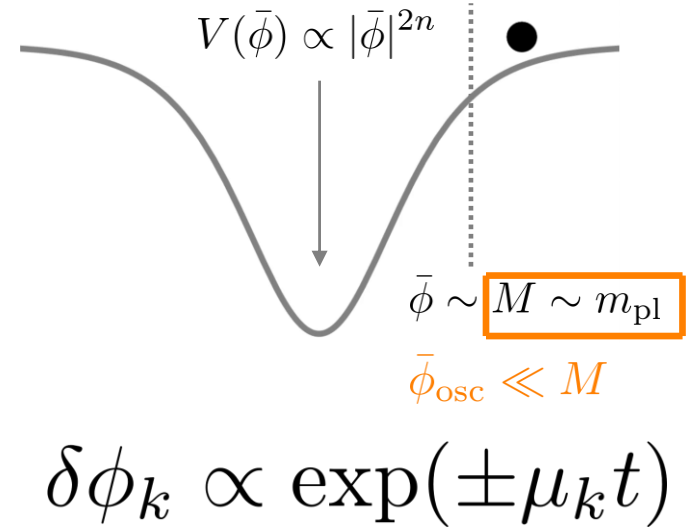
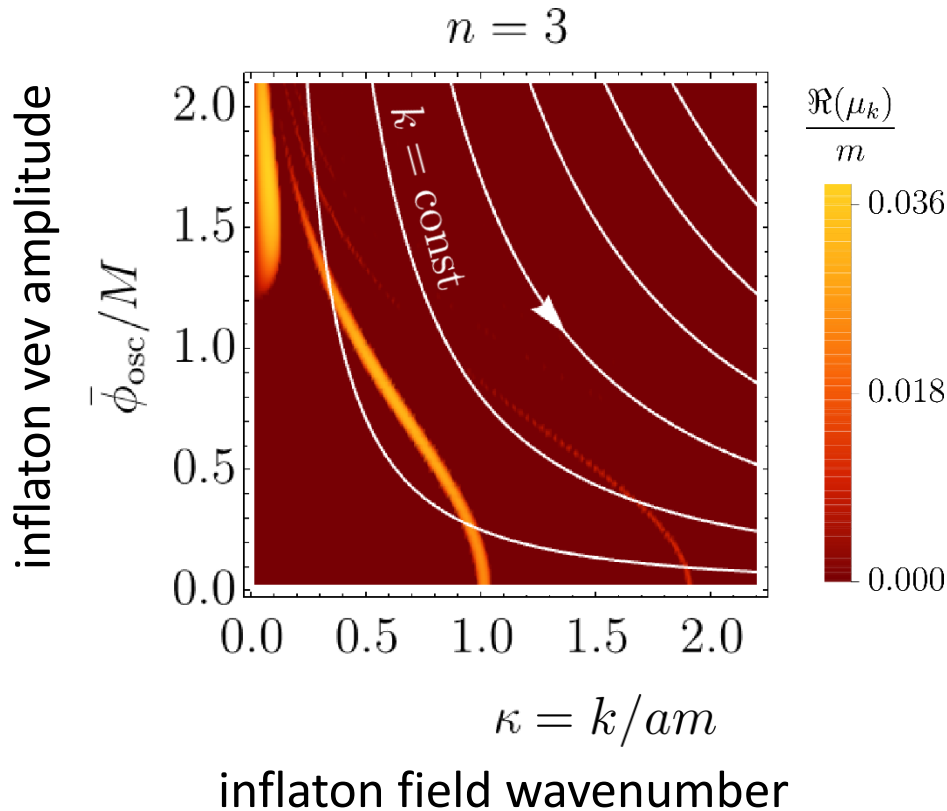
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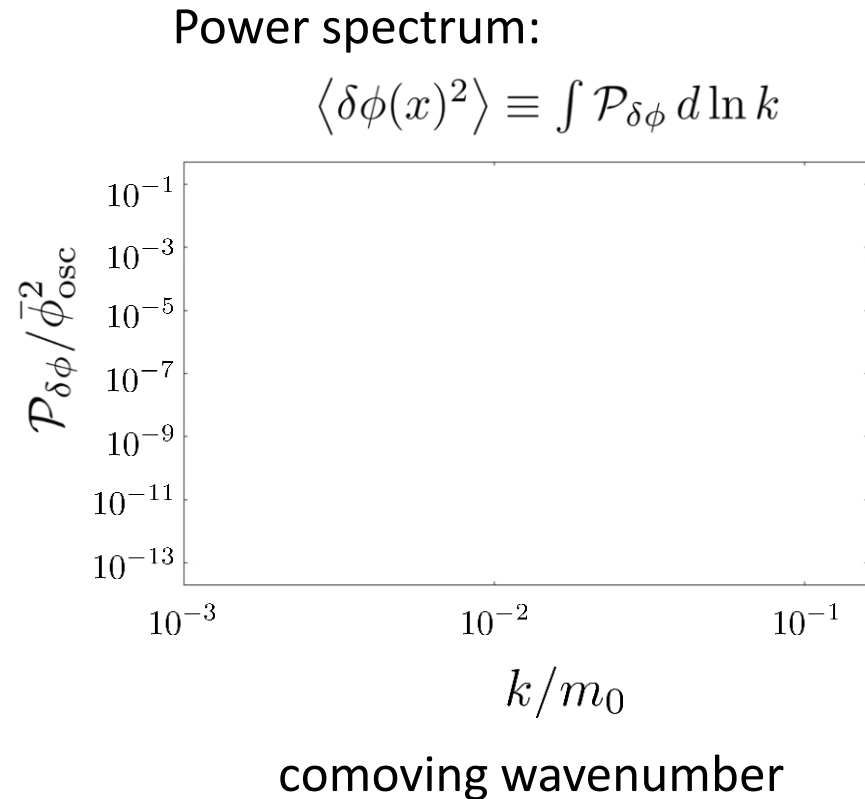
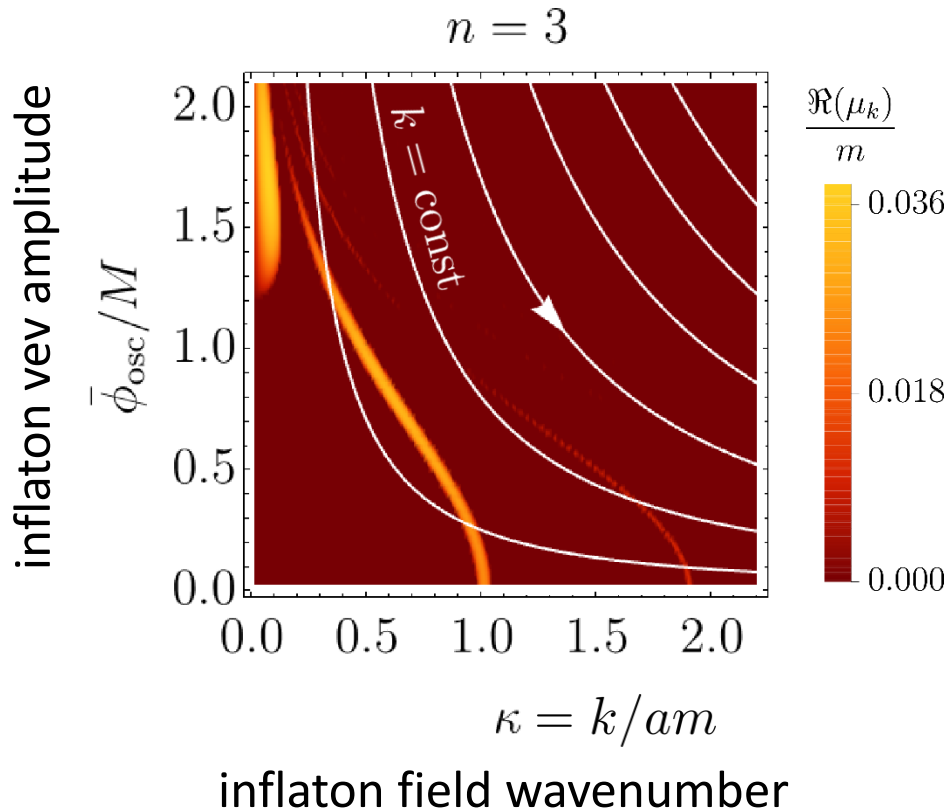


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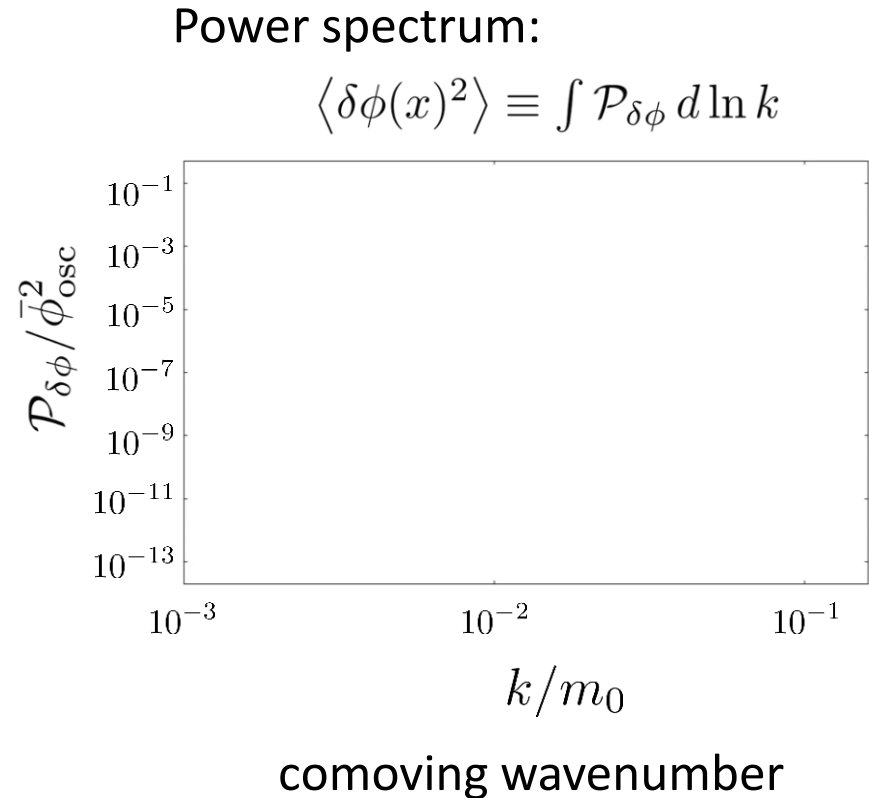
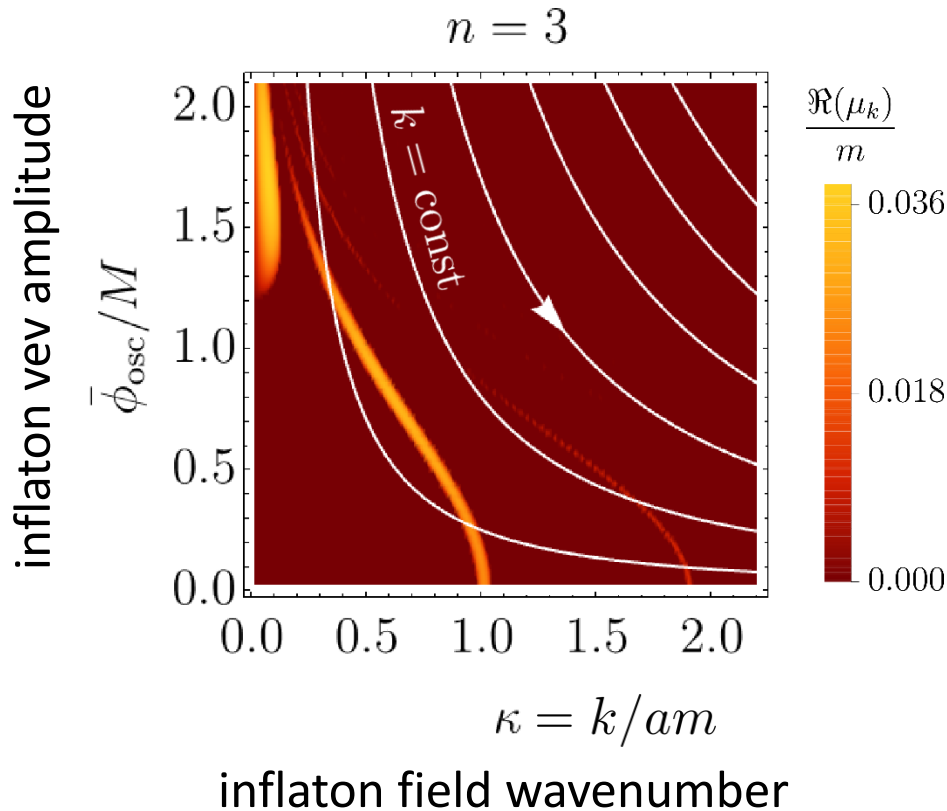




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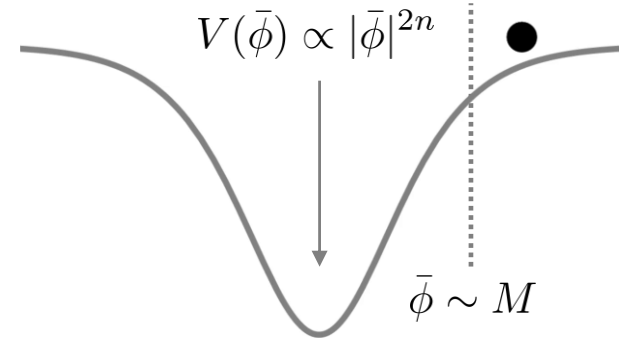
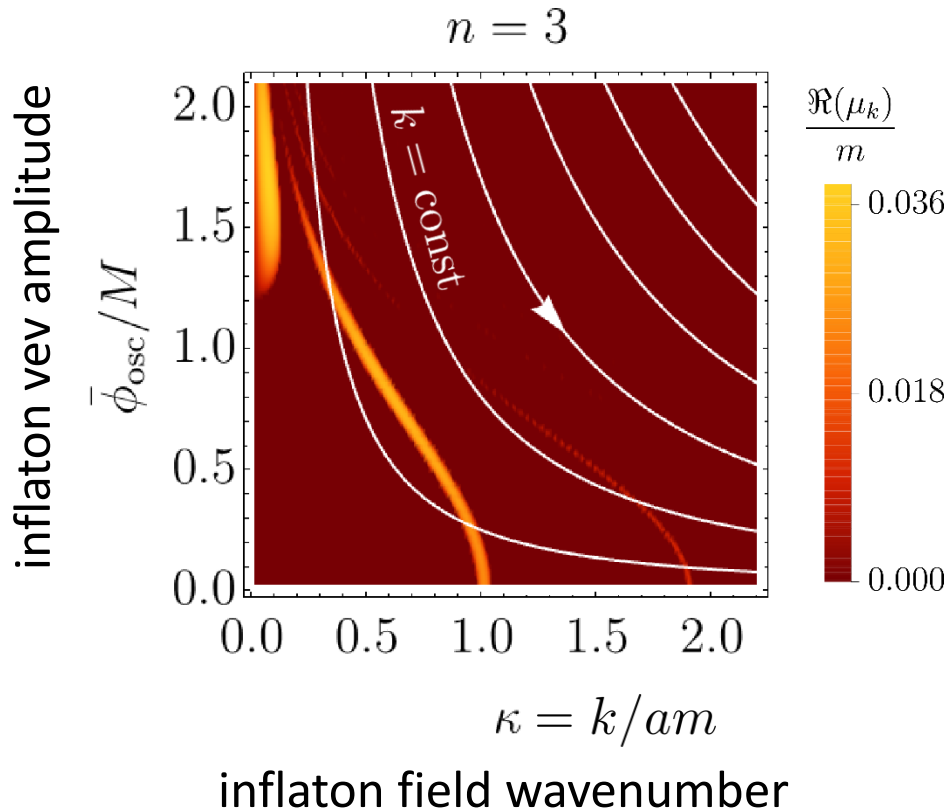
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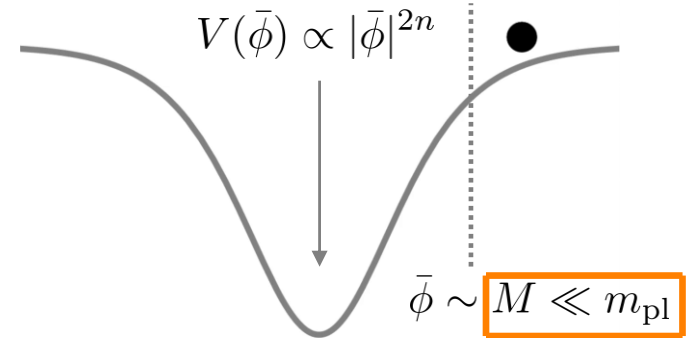
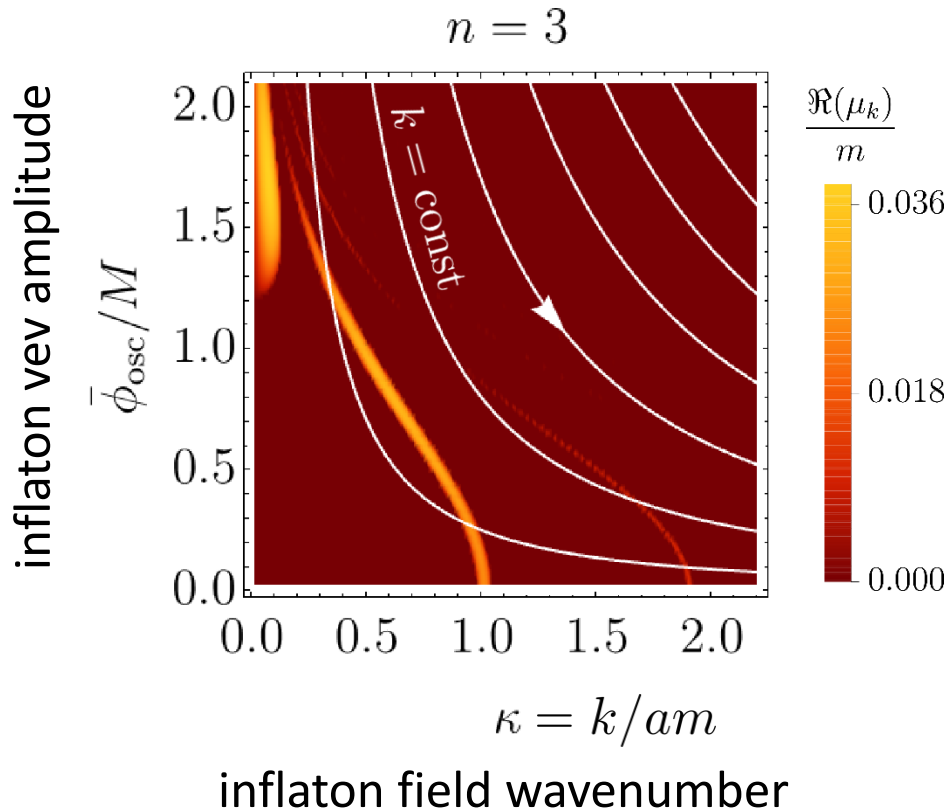
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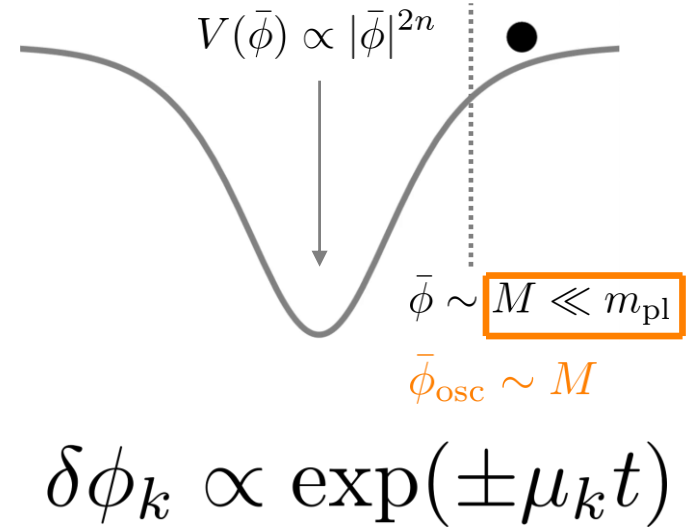
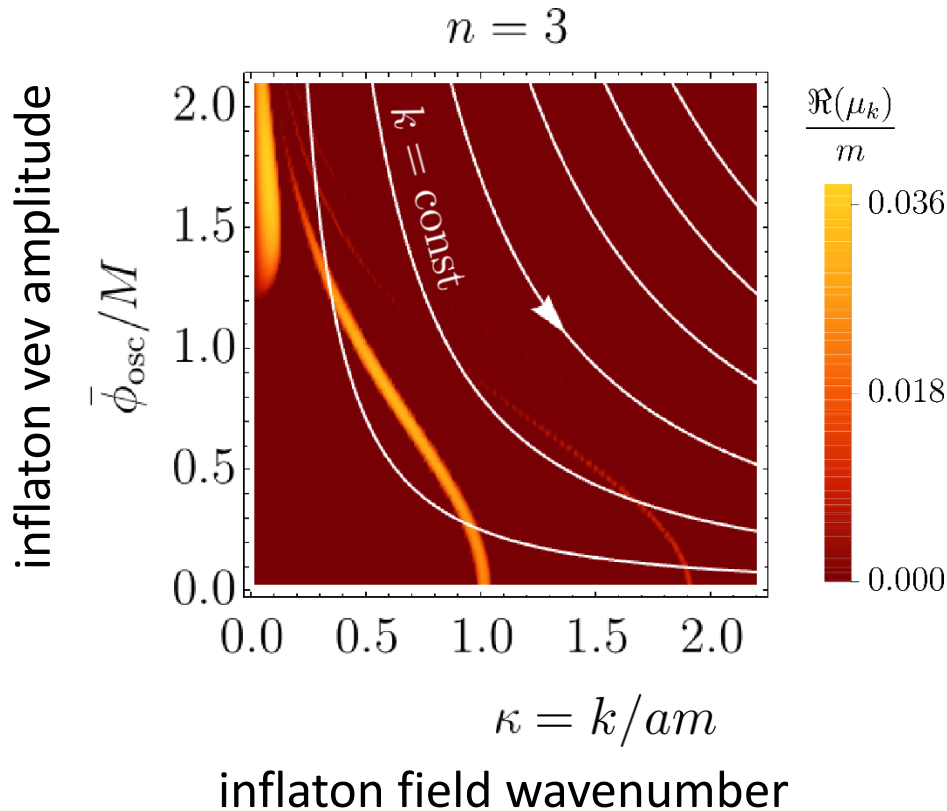
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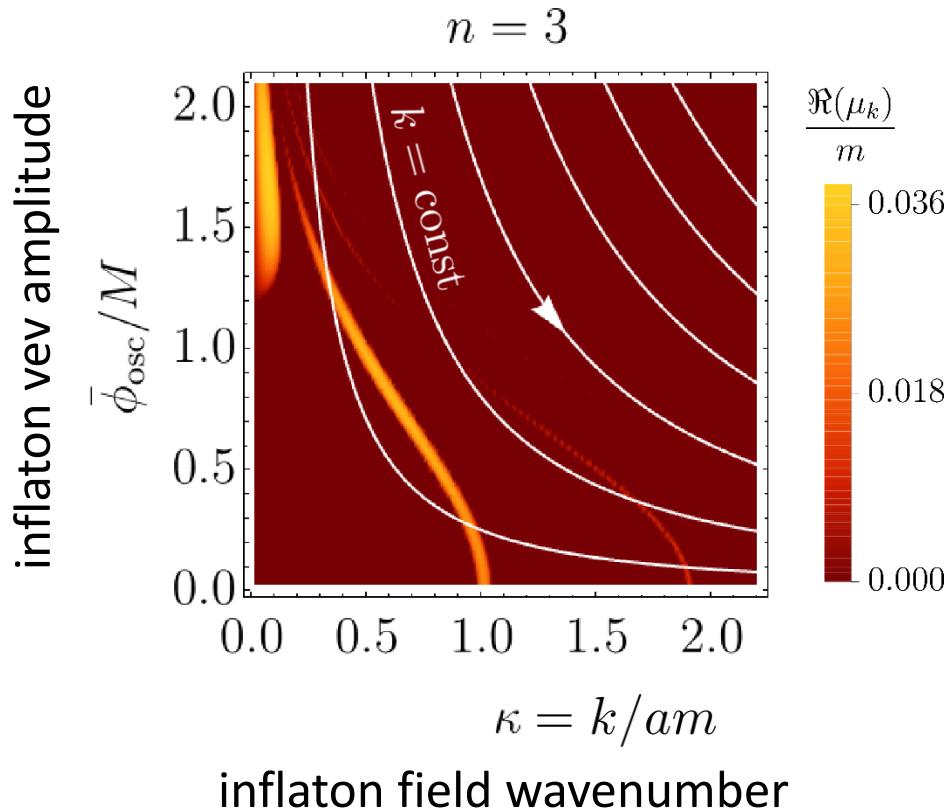


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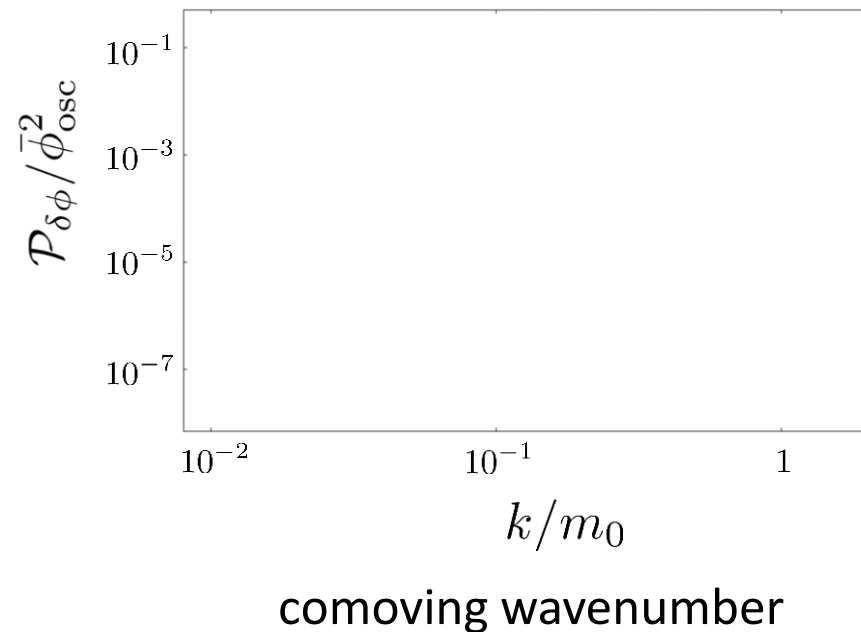
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Non-perturbative decay (parametric self-resonance)



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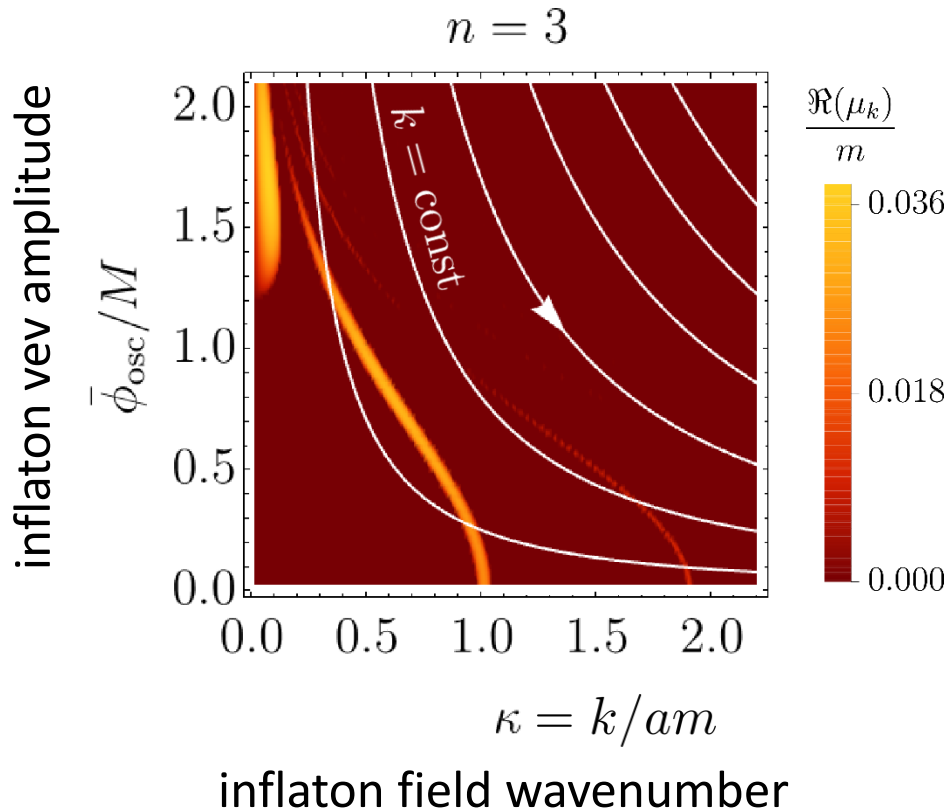
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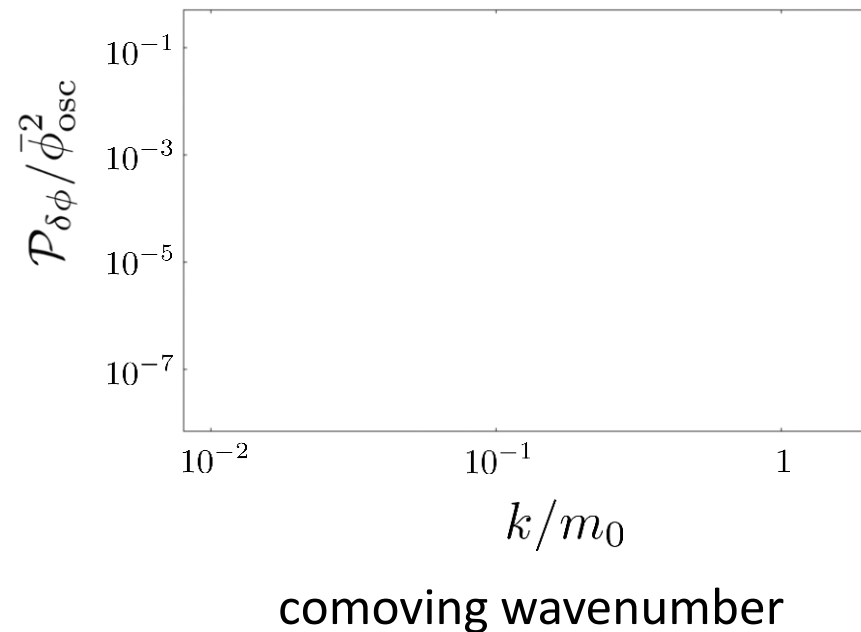
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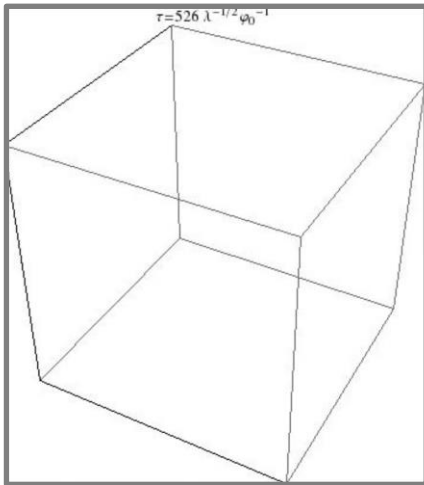


# Towards radiation domination

$$M \ll m_{\text{pl}}$$

$$n > 1$$

$$M \sim m_{\text{pl}}$$



- $\bar{\phi}$  fragments quickly

- slow production of  $\delta\phi(t, \mathbf{x})$
- $\bar{\phi}$  fragments gradually

$$\Delta N_{\text{fr}} \approx \frac{n+1}{3} \ln \left( 10^3 \frac{M}{m_{\text{pl}}} \right)$$

at sufficiently late times:

$$\phi \text{ virialized + turbulent} \rightarrow w = \frac{1}{3}$$

# Expansion history effects



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Spectral index:  $n_s$

Tensor-to-scalar ratio:  $r$

# Expansion history effects

Spectral index:  $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio:  $r = r(M, n, N_*)$

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Spectral index:  $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio:  $r = r(M, n, N_*)??$

$$50 \leq N_* \leq 60$$

# Expansion history effects

Spectral index:  $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio:  $r = r(M, n, N_*)$  ✓

$$\cancel{50 \leq N_* \leq 60}$$

# Expansion history effects

Spectral index:  $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio:  $r = r(M, n, N_*)$  ✓

$$N_* = 66.89 - \frac{1}{12} \ln g_{\text{th}} + \frac{1}{4} \ln \frac{V_*^4}{m_{\text{pl}}^4 \rho_{\text{end}}} - \ln \frac{k_*}{a_0 H_0} + \frac{3\bar{w}_{\text{int}} - 1}{4} \Delta N_{\text{rad}}$$

$$\Delta N_{\text{rad}} \equiv \int_{a_{\text{end}}}^{a_{\text{rad}}} d \ln a$$

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reheating

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$$\Delta N_{\text{fr}} = \begin{cases} 1 & \text{if } M \ll m_{\text{pl}} \\ \frac{n+1}{3} \ln \left( 10 \frac{\kappa}{\Delta \kappa} \frac{M}{m_{\text{pl}}} \right) & \text{if } M \sim m_{\text{pl}} \end{cases}$$

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Spectral index:  $n_s = n_s(M, n, N_*)$

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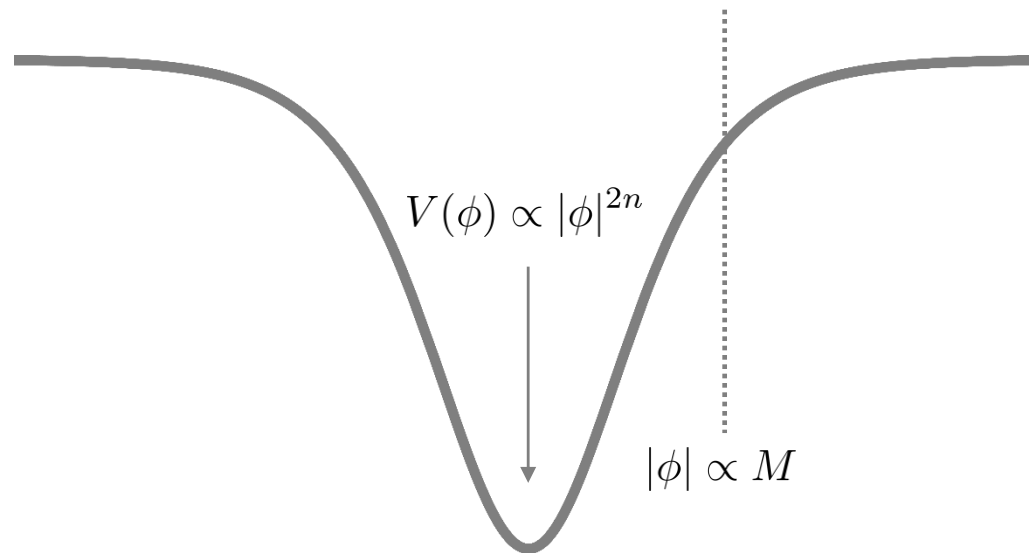
$$w_{\text{int}}(\Delta N) = \begin{cases} \frac{n-1}{n+1} & \text{if } 0 < \Delta N < \Delta N_{\text{rad}} \\ \frac{1}{3} & \text{if } \Delta N > \Delta N_{\text{rad}} \end{cases}$$

$$\Delta N_{\text{rad}} \equiv \int_{a_{\text{end}}}^{a_{\text{rad}}} d \ln a$$

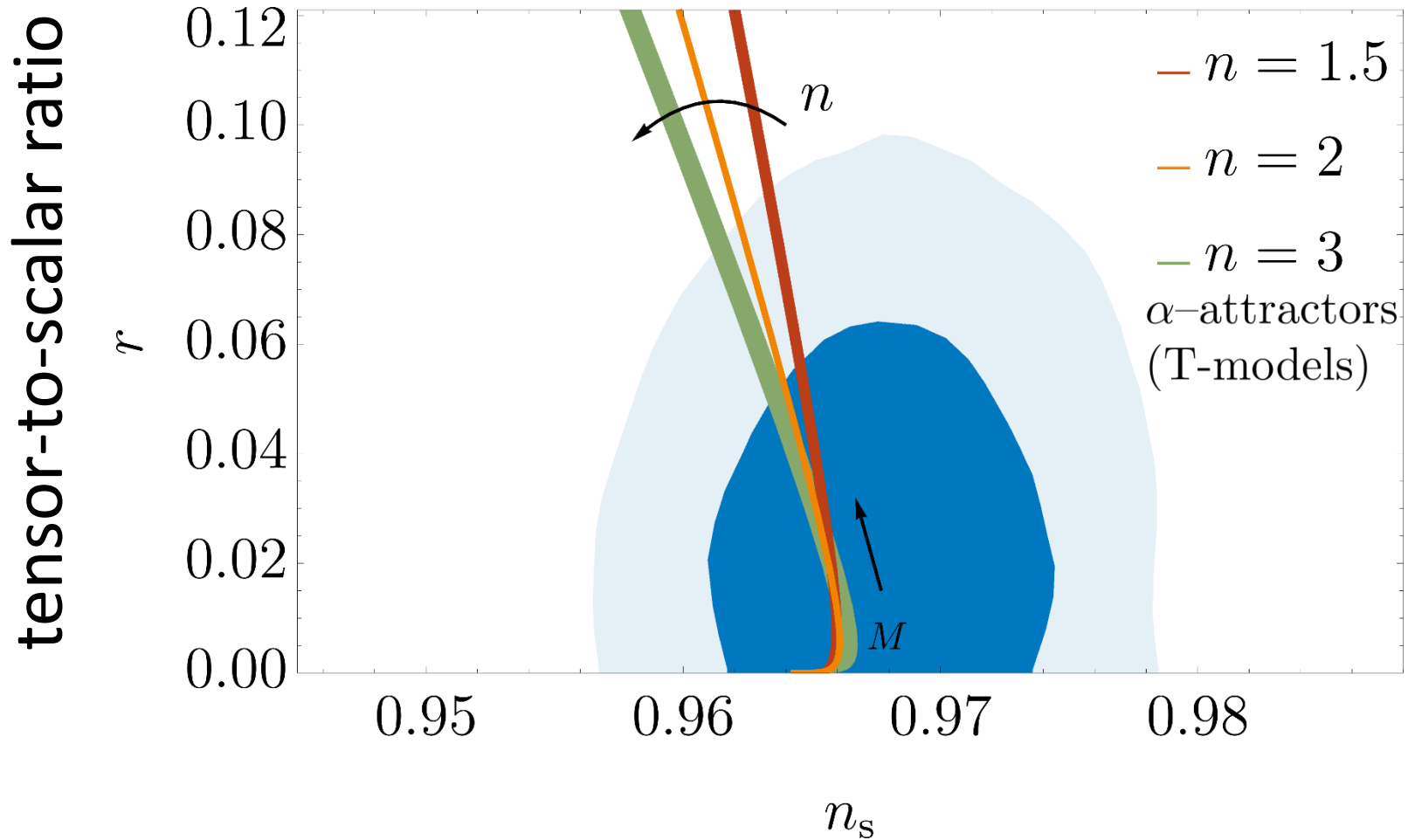
# Expansion history effects

$$V(\phi) \propto \tanh^{2n} \left( \frac{|\phi|}{M} \right)$$

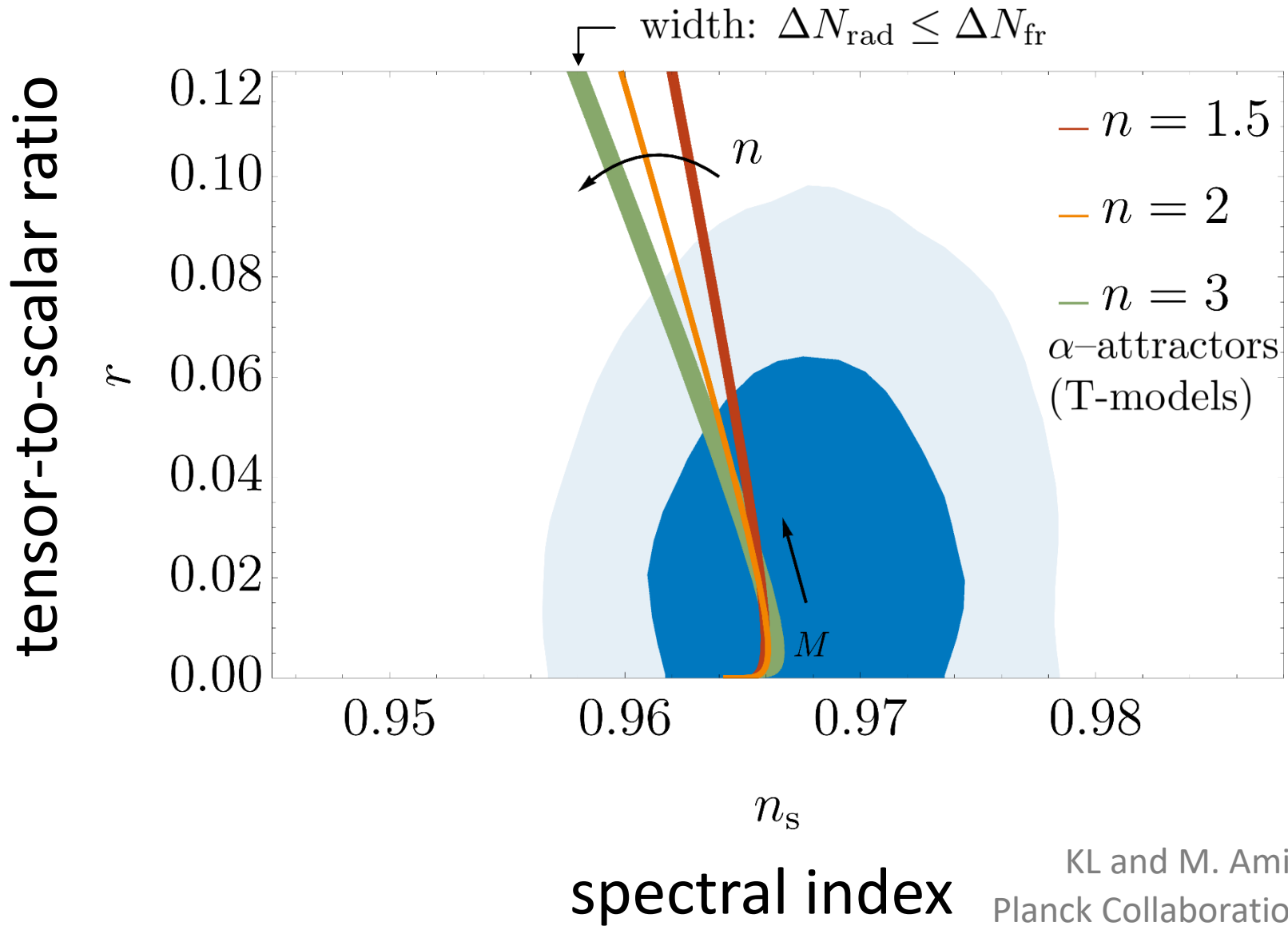
$\alpha$ -attractors  
(T-models)



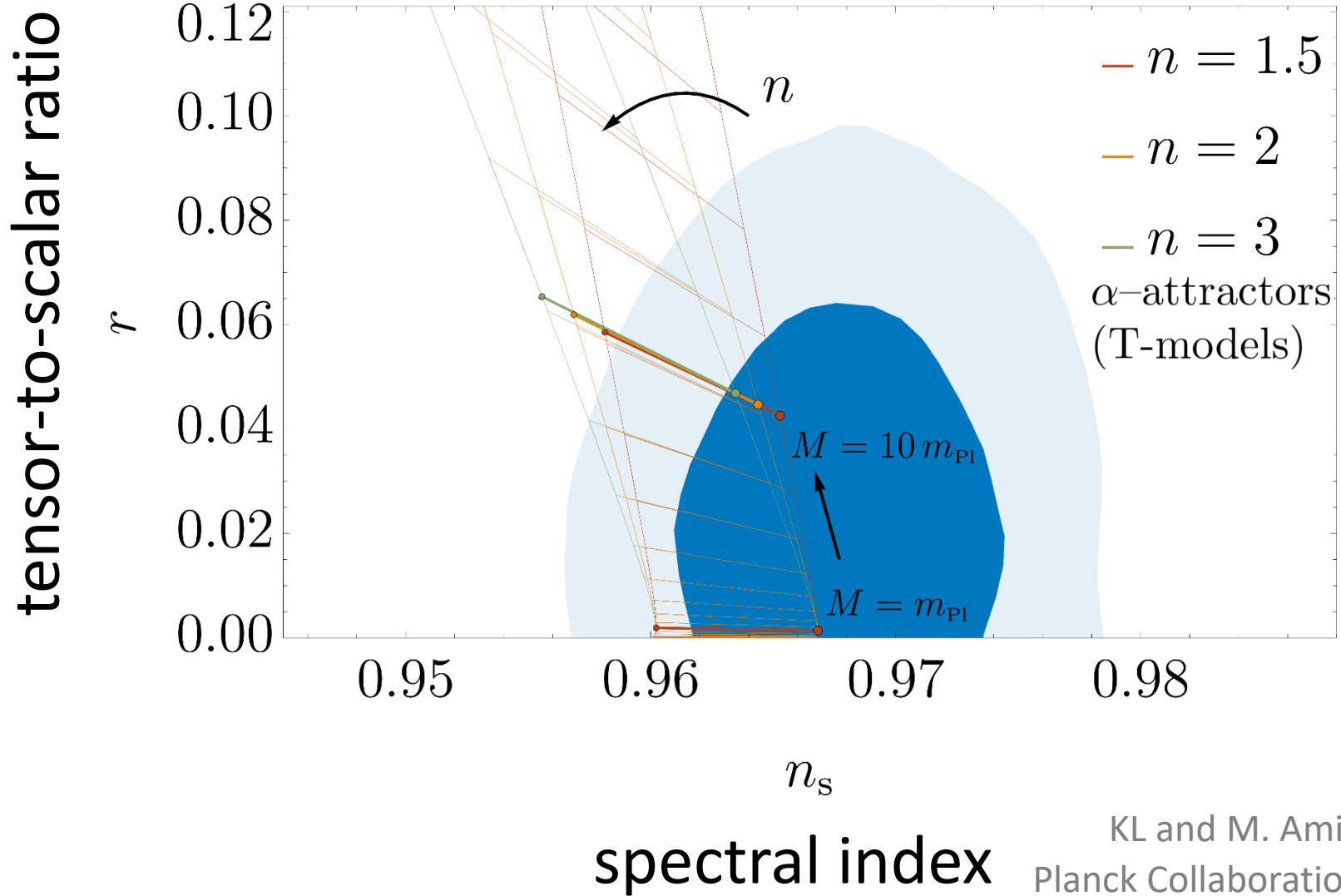
# Expansion history effects



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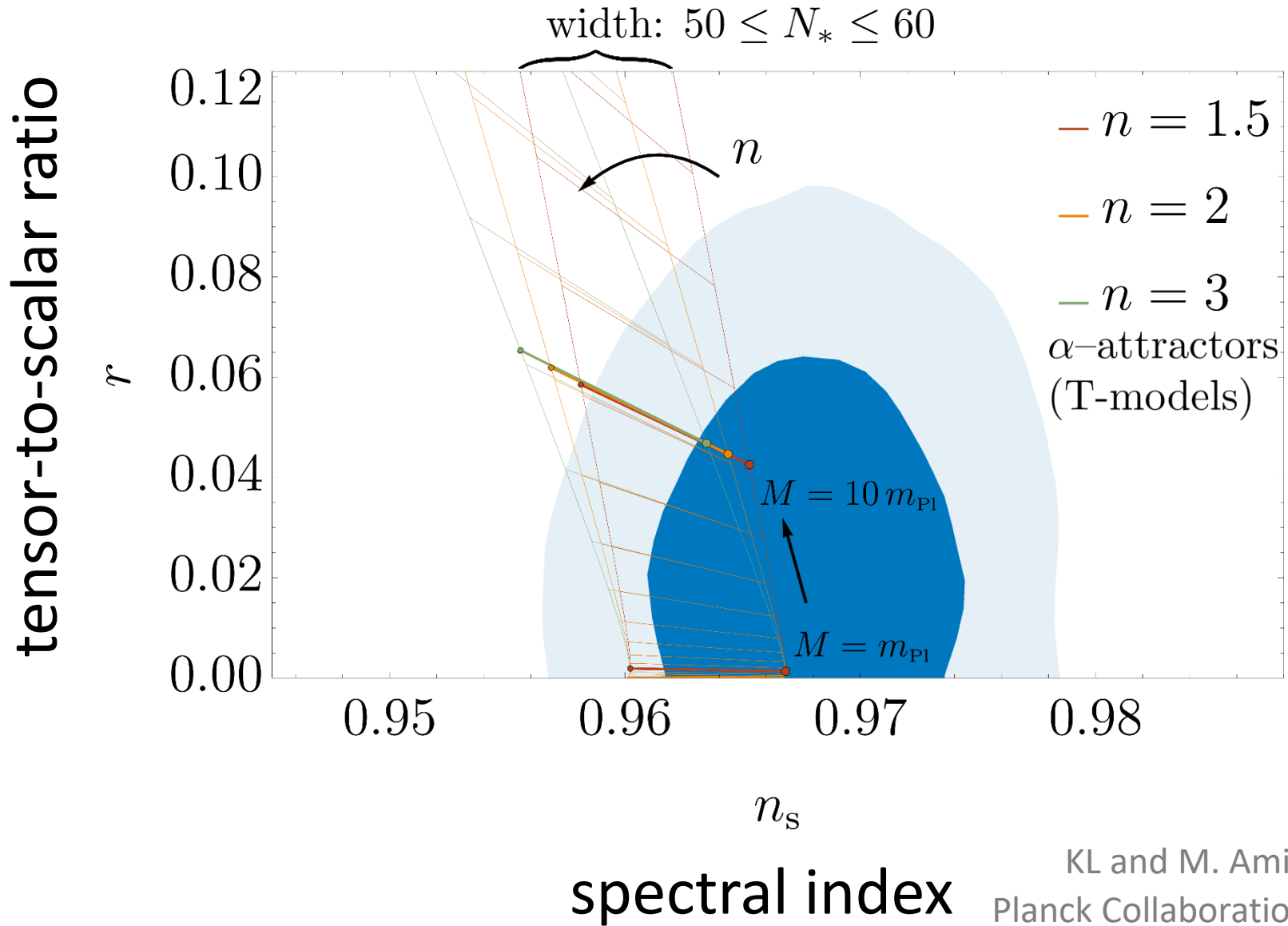


# Expansion history effects

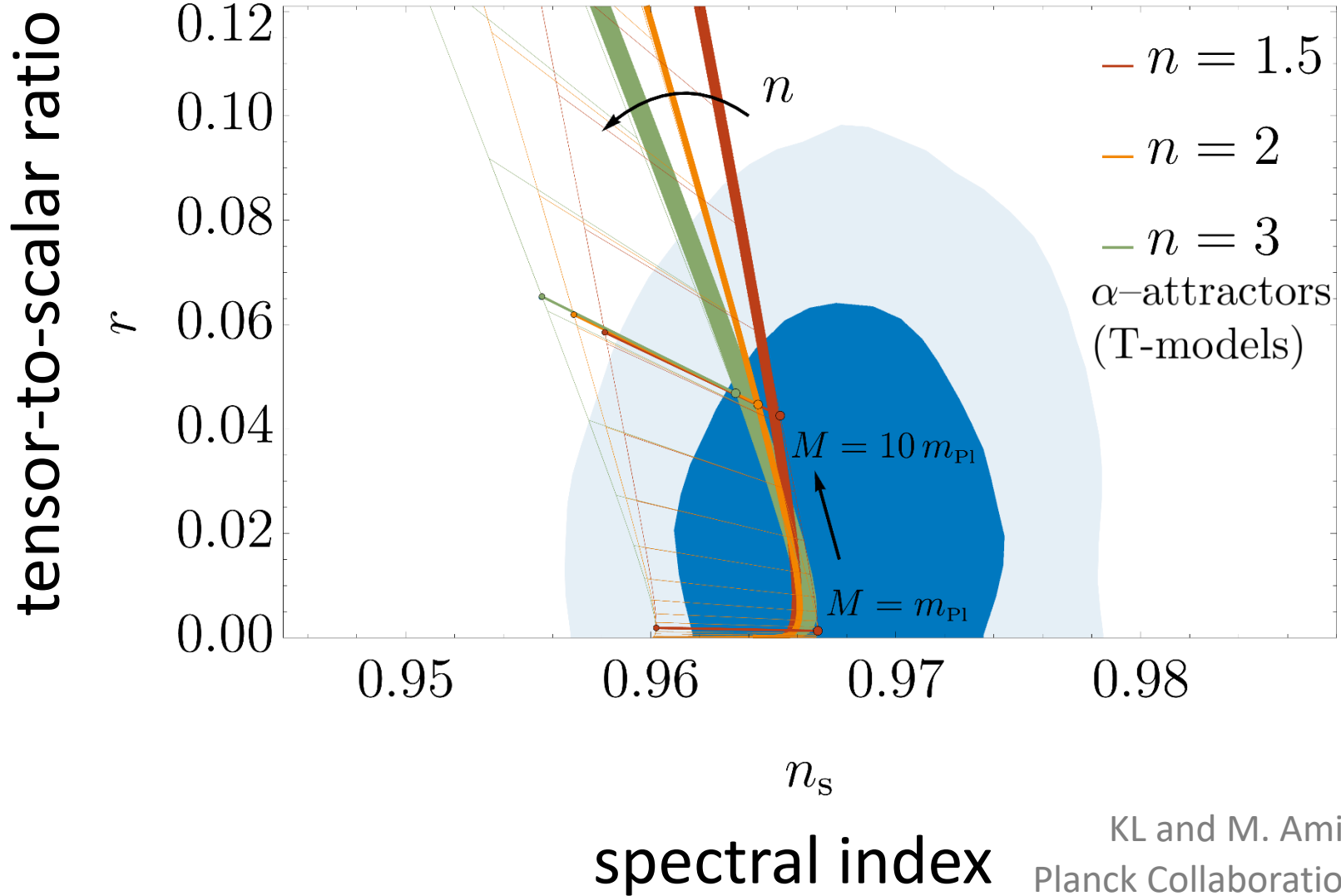




# Expansion history effects



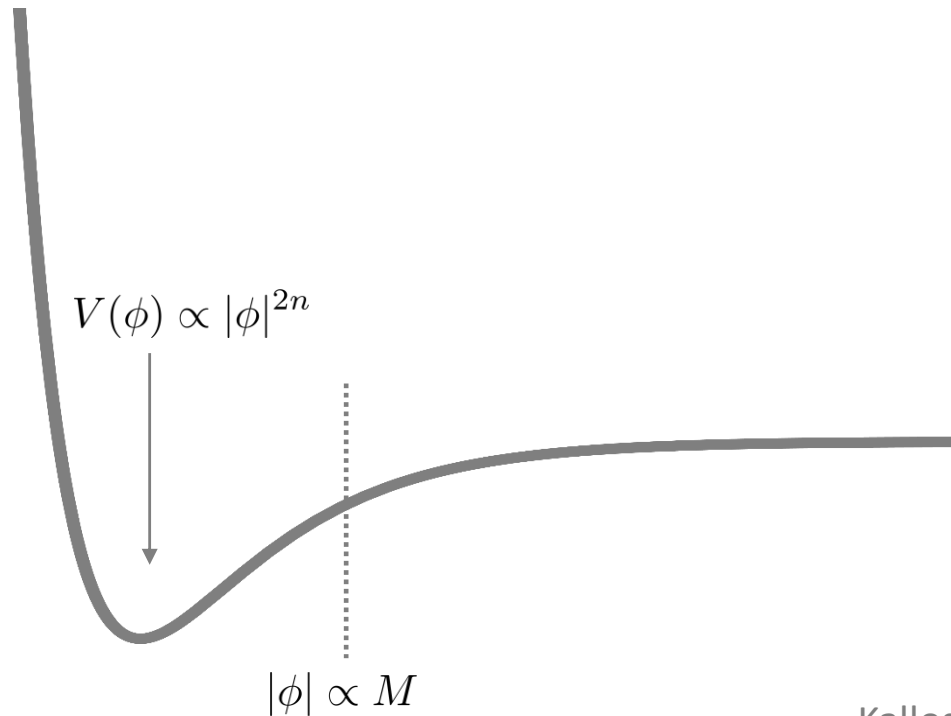
# Expansion history effects



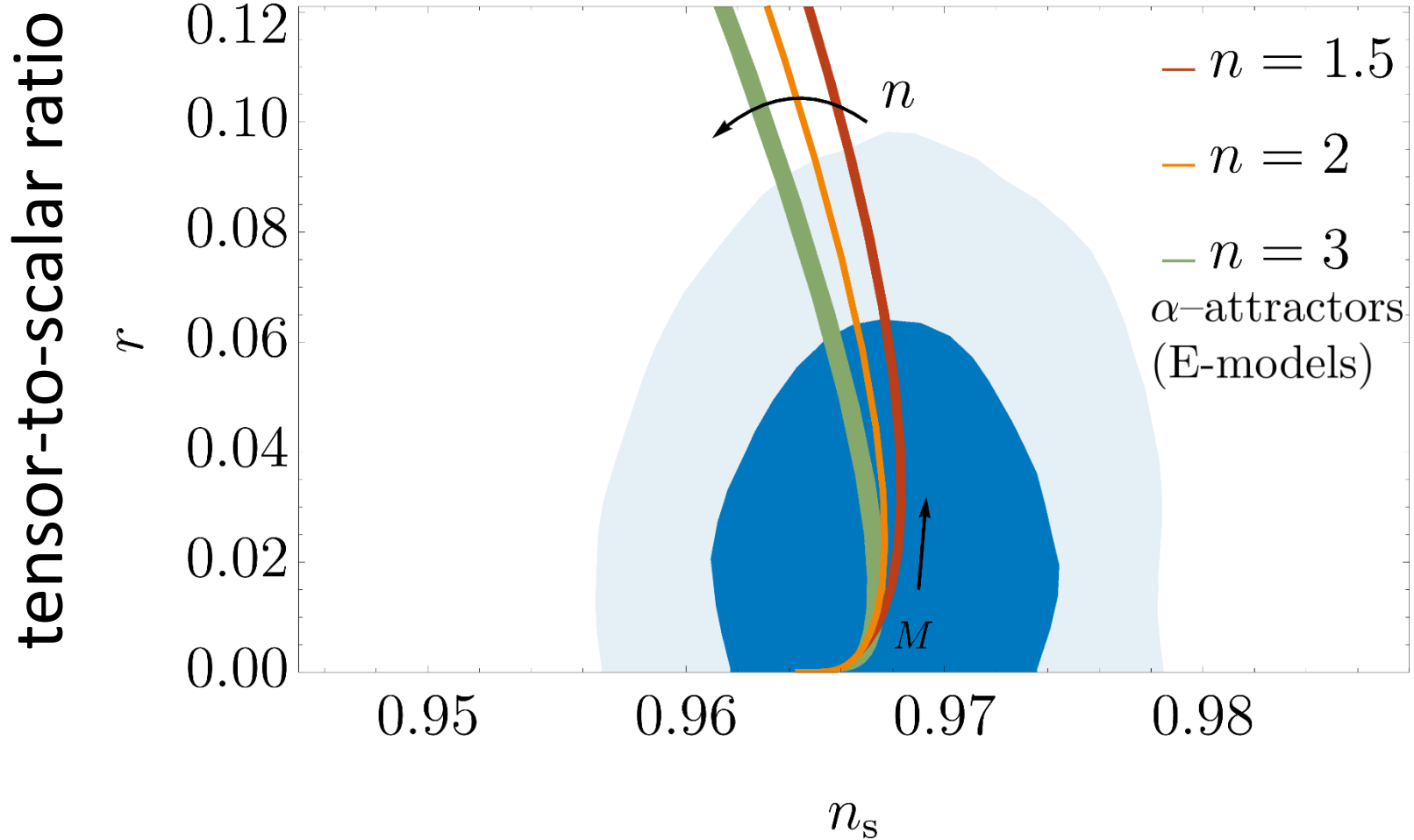
# Expansion history effects

$$V(\phi) \propto \left| 1 - e^{-\phi/M} \right|^{2n}$$

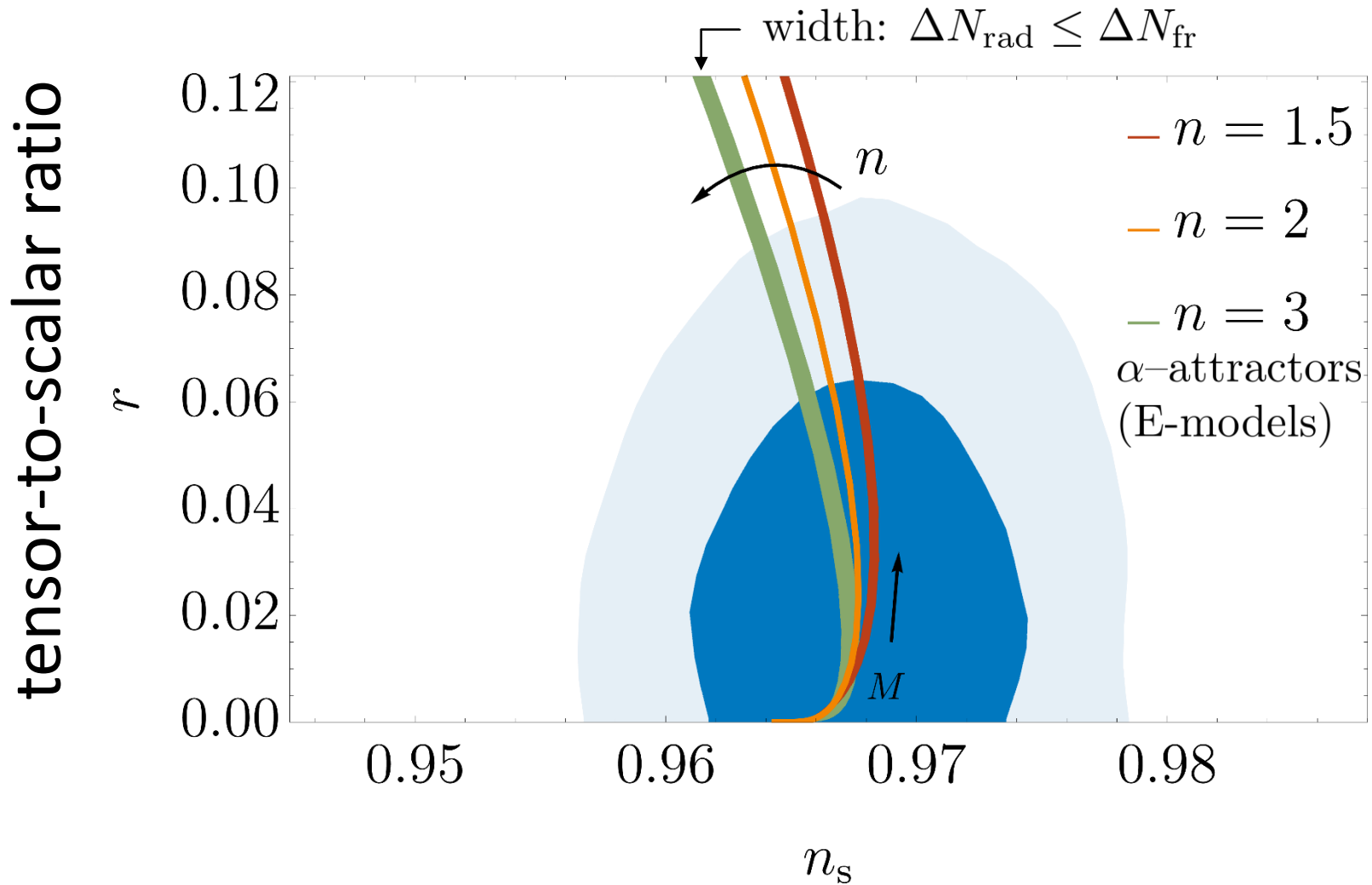
$\alpha$ -attractors  
(E-models)



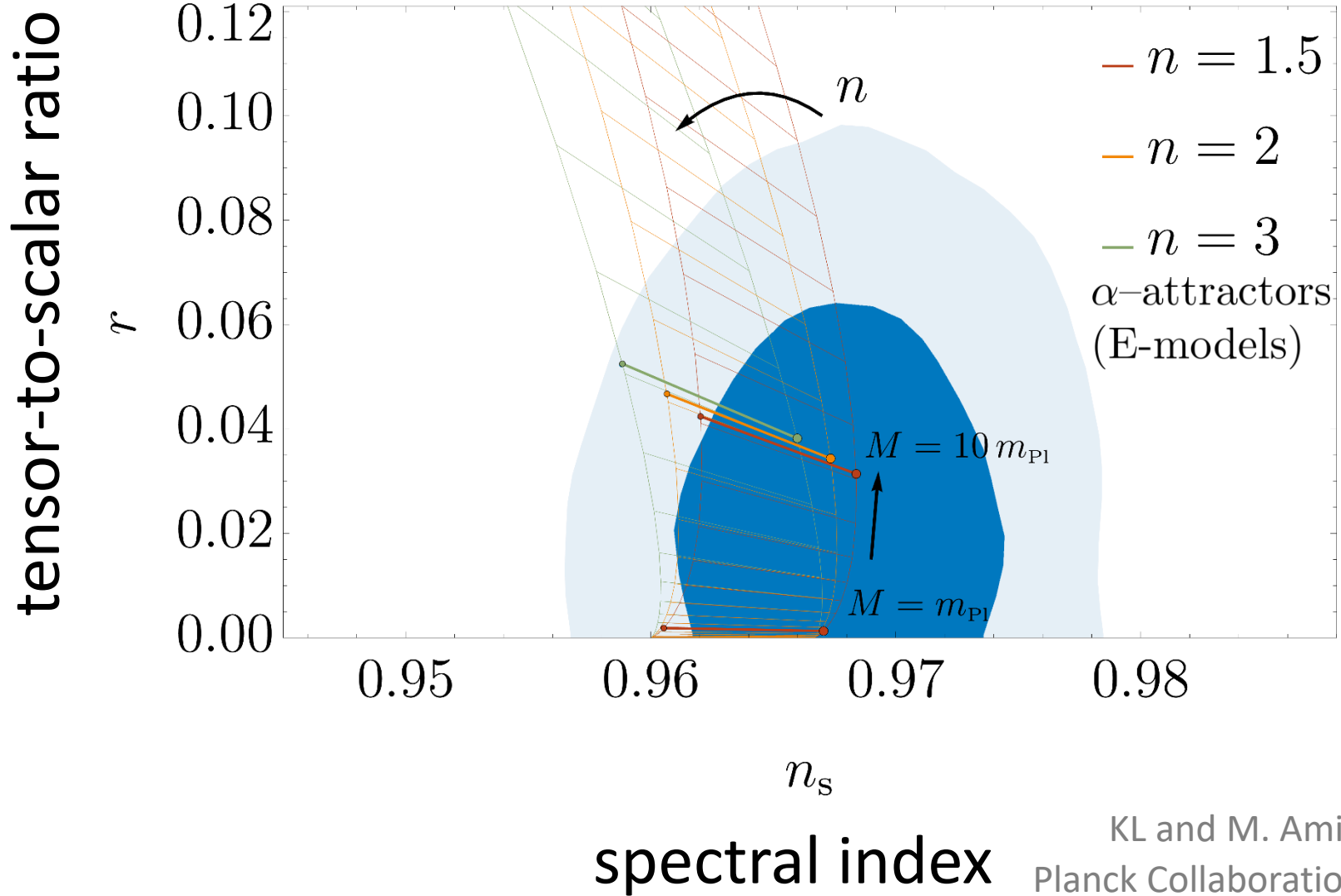
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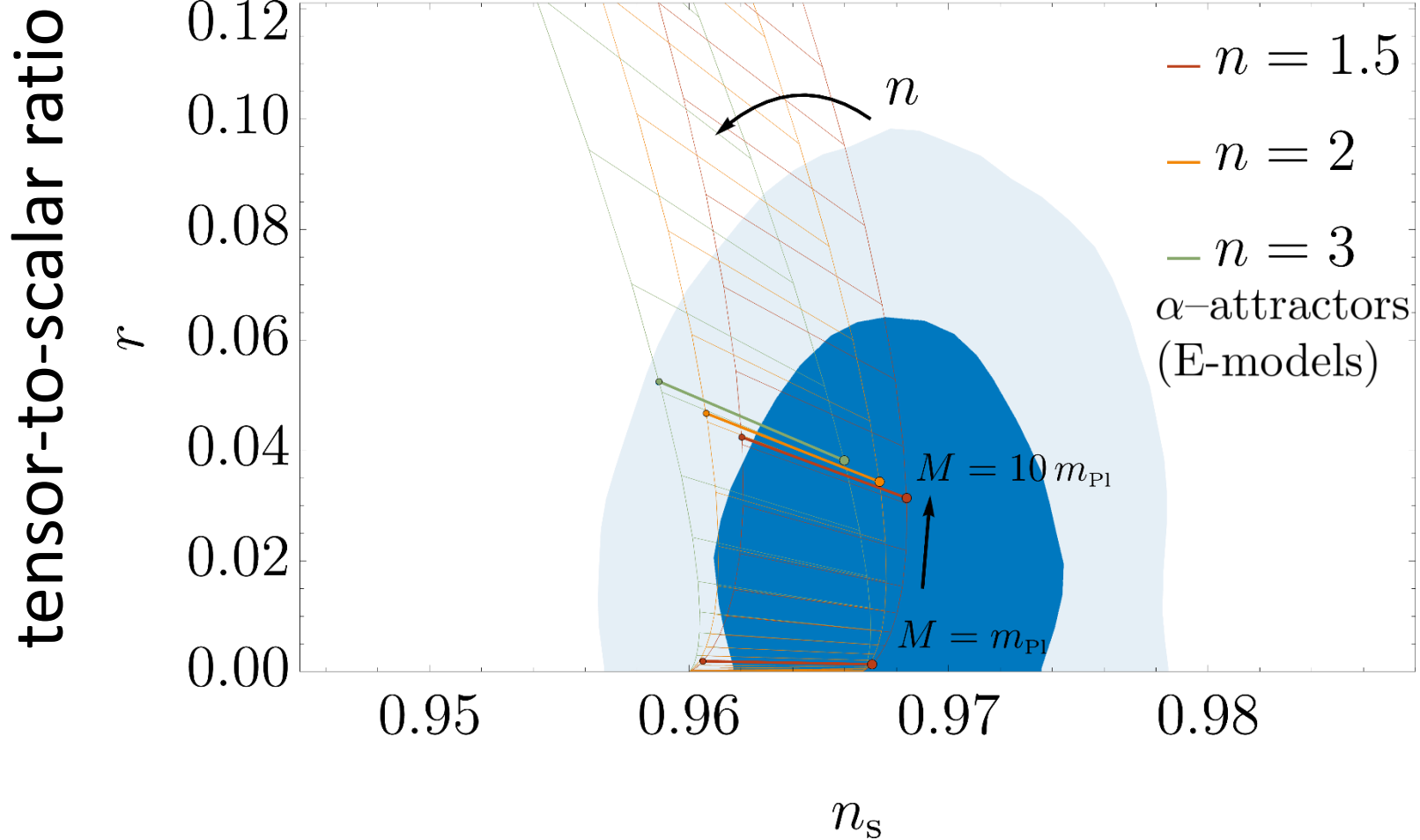


# Expansion history effects



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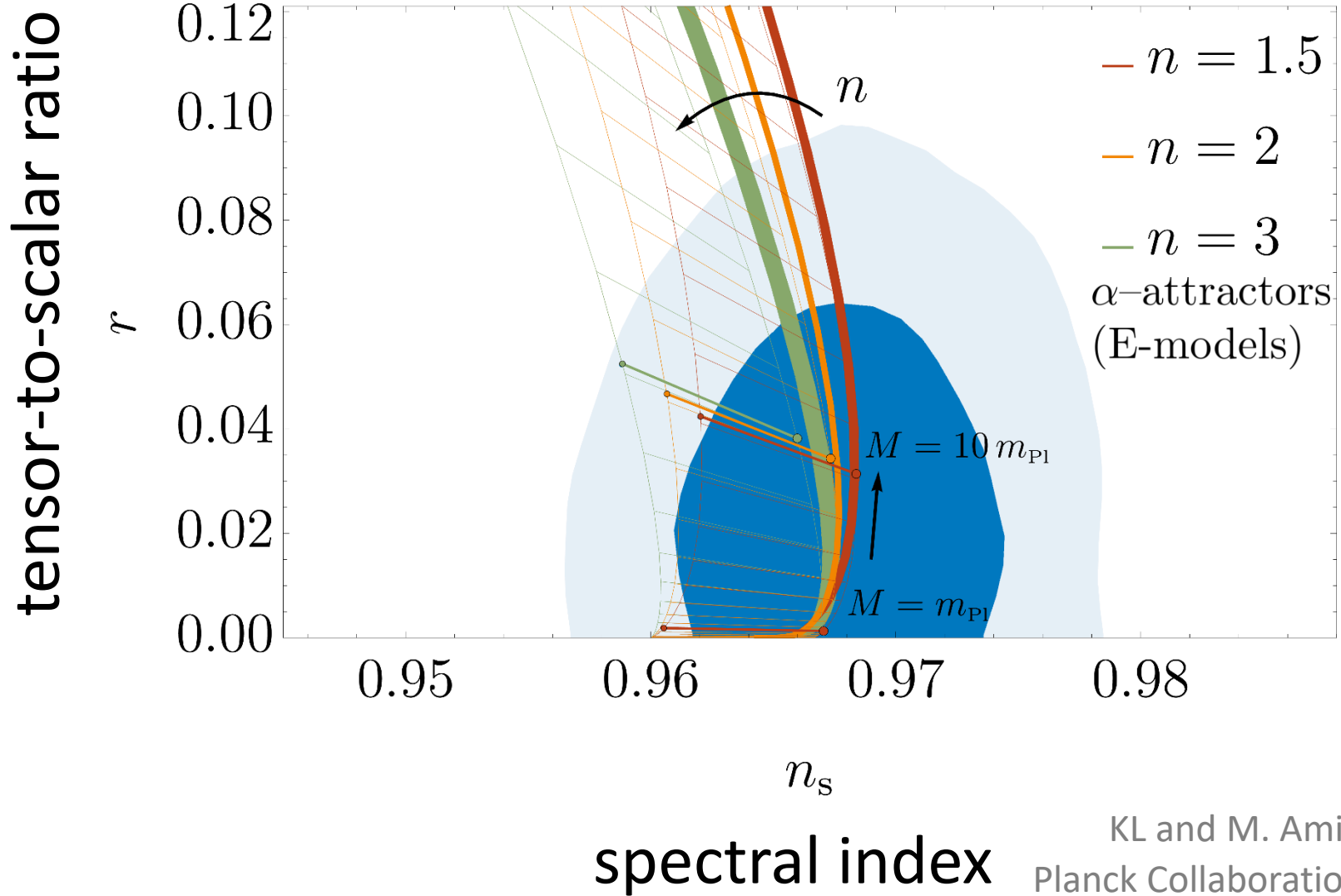
width:  $50 \leq N_* \leq 60$



spectral index

KL and M. Amin (2018)  
Planck Collaboration (2015)

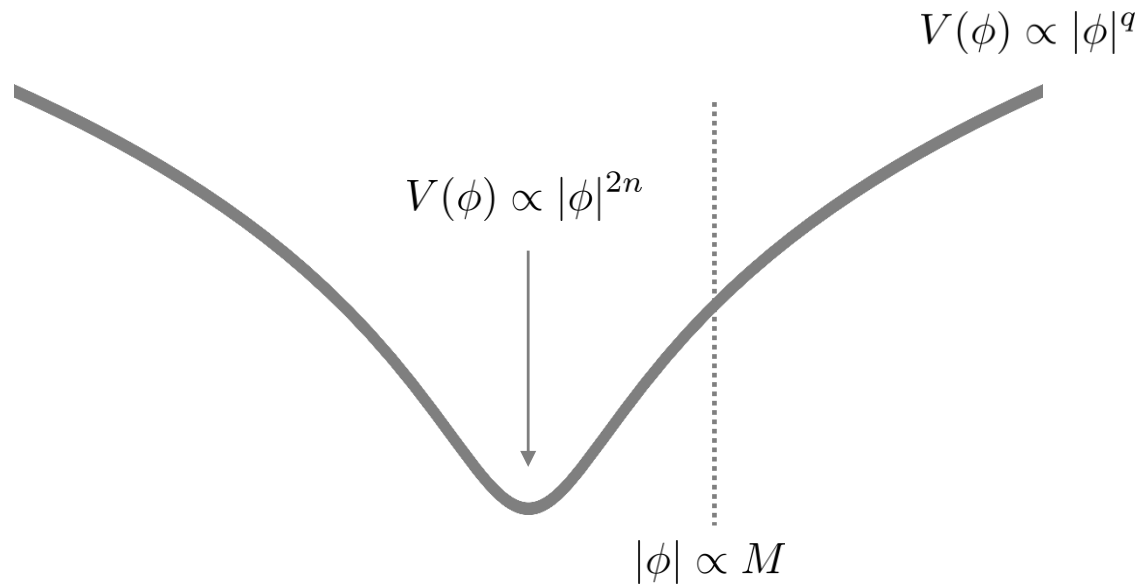
# Expansion history effects





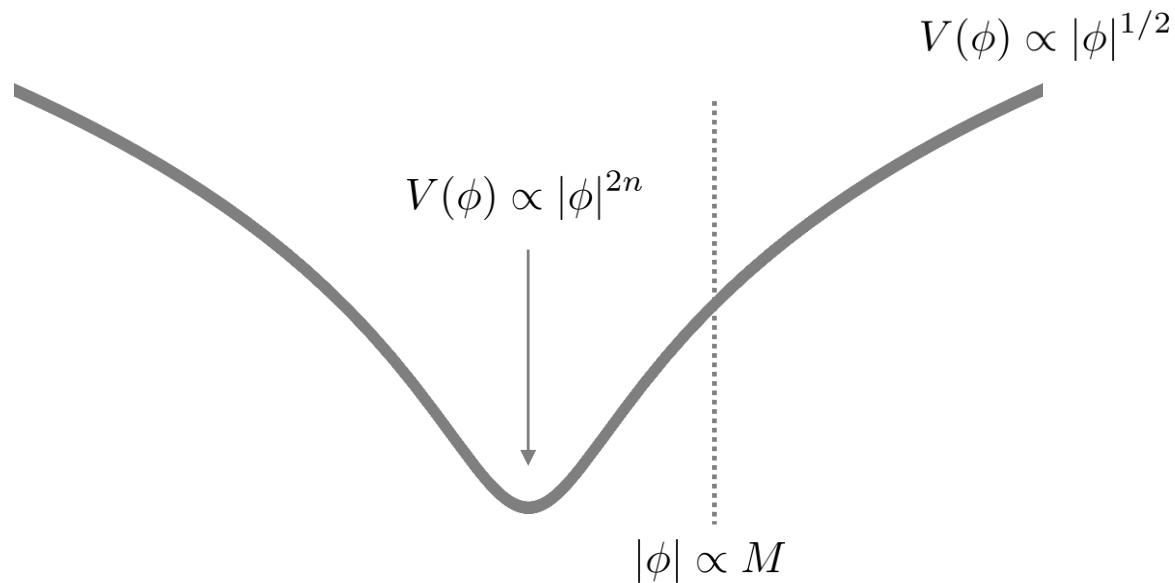
# Expansion history effects

$$V(\phi) \propto \left[ 1 + \left| \frac{\phi}{M} \right|^{2n} \right]^{\frac{q}{2n}} - 1 \quad \text{Monodromy}$$

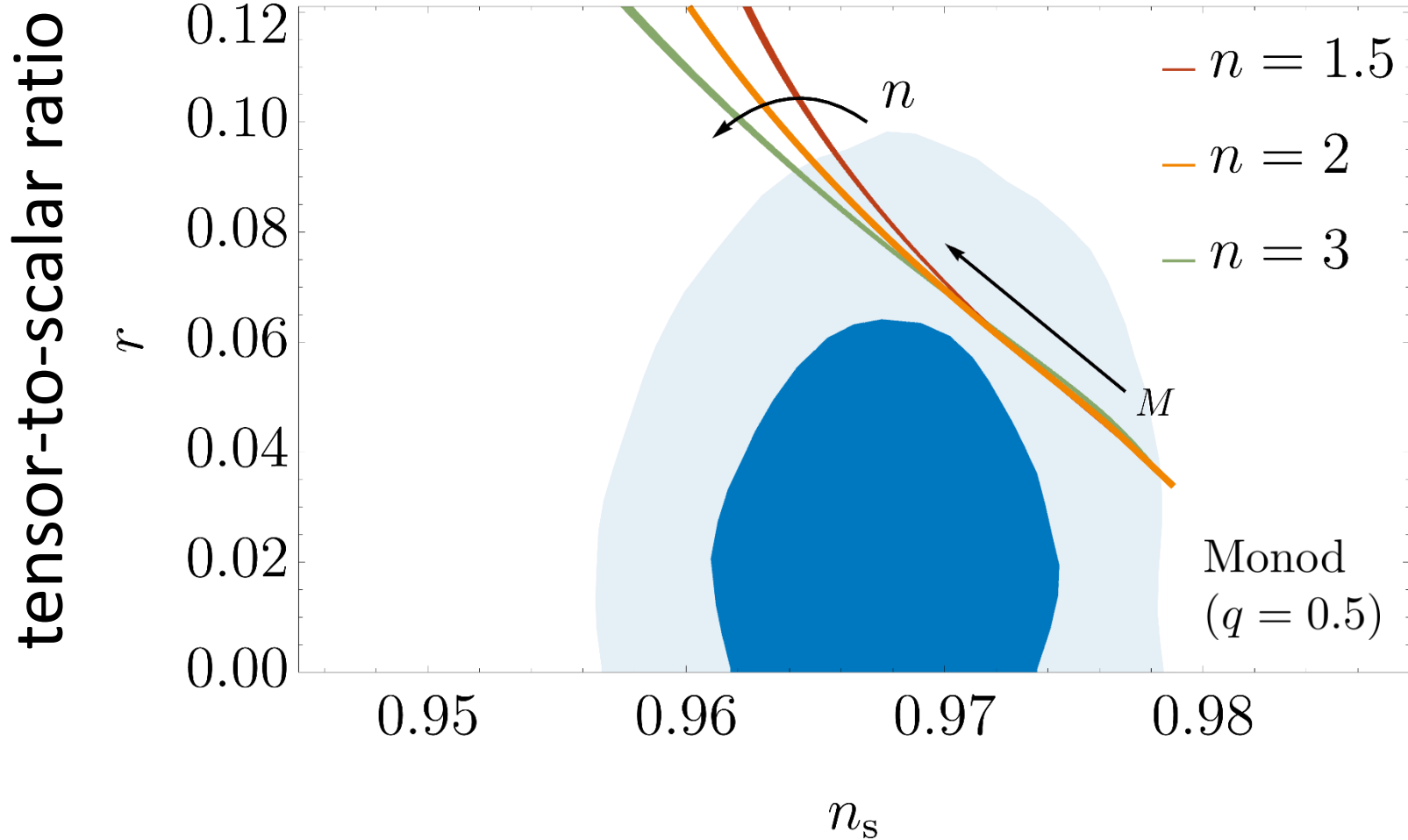


# Expansion history effects

$$V(\phi) \propto \left[ 1 + \left| \frac{\phi}{M} \right|^{2n} \right]^{\frac{1}{4n}} - 1 \quad \text{Monodromy} \\ q = 1/2$$



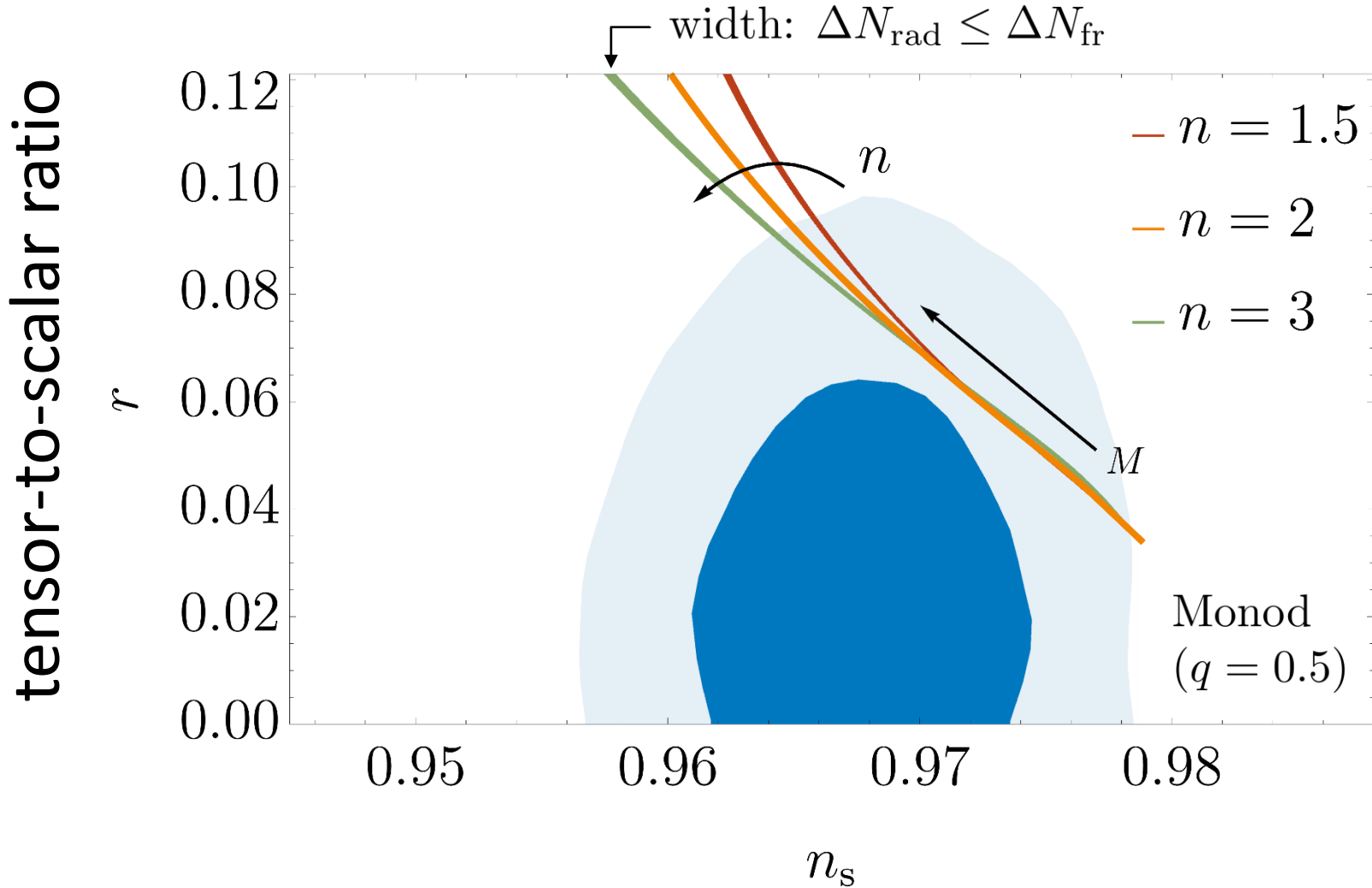
# Expansion history effects



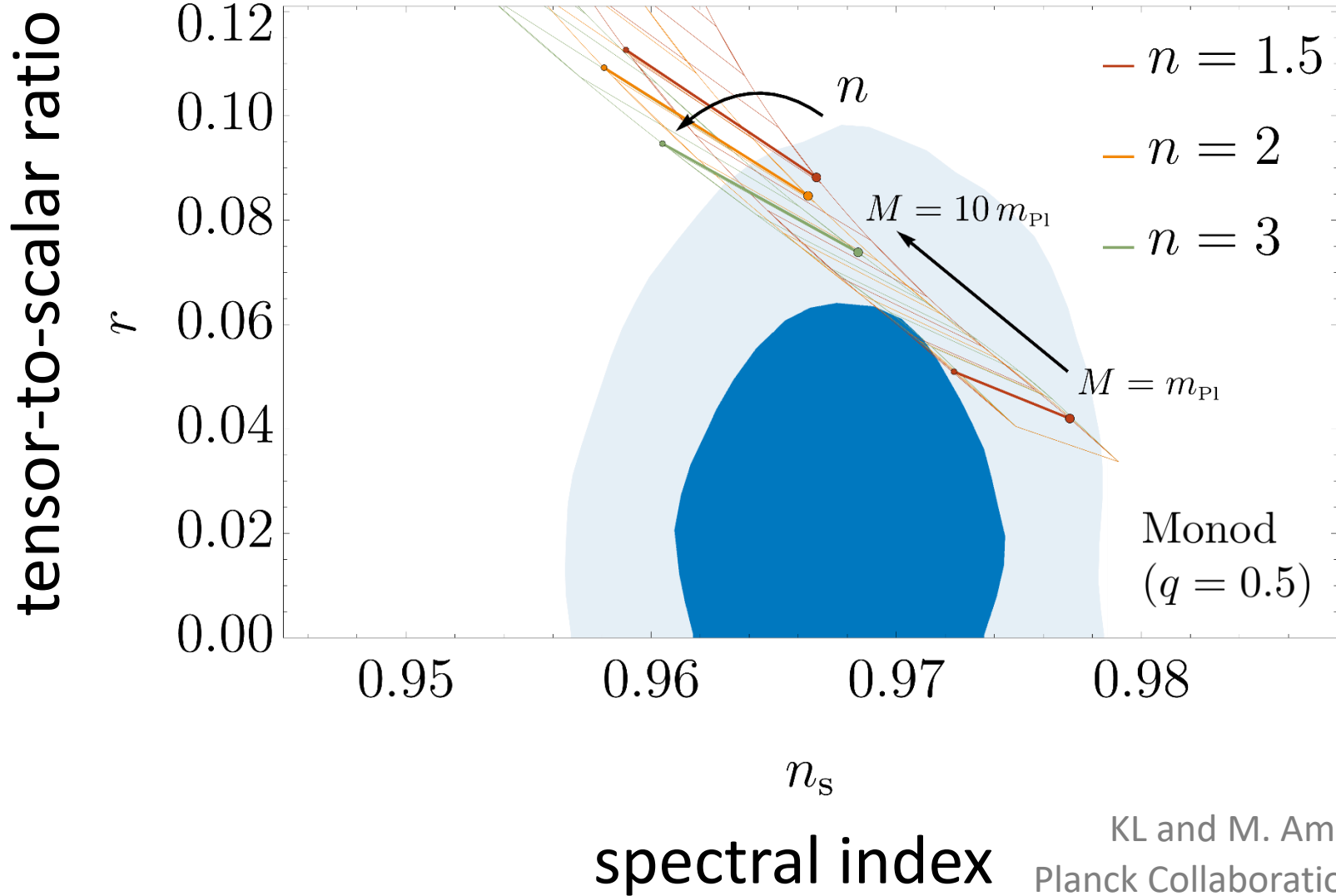
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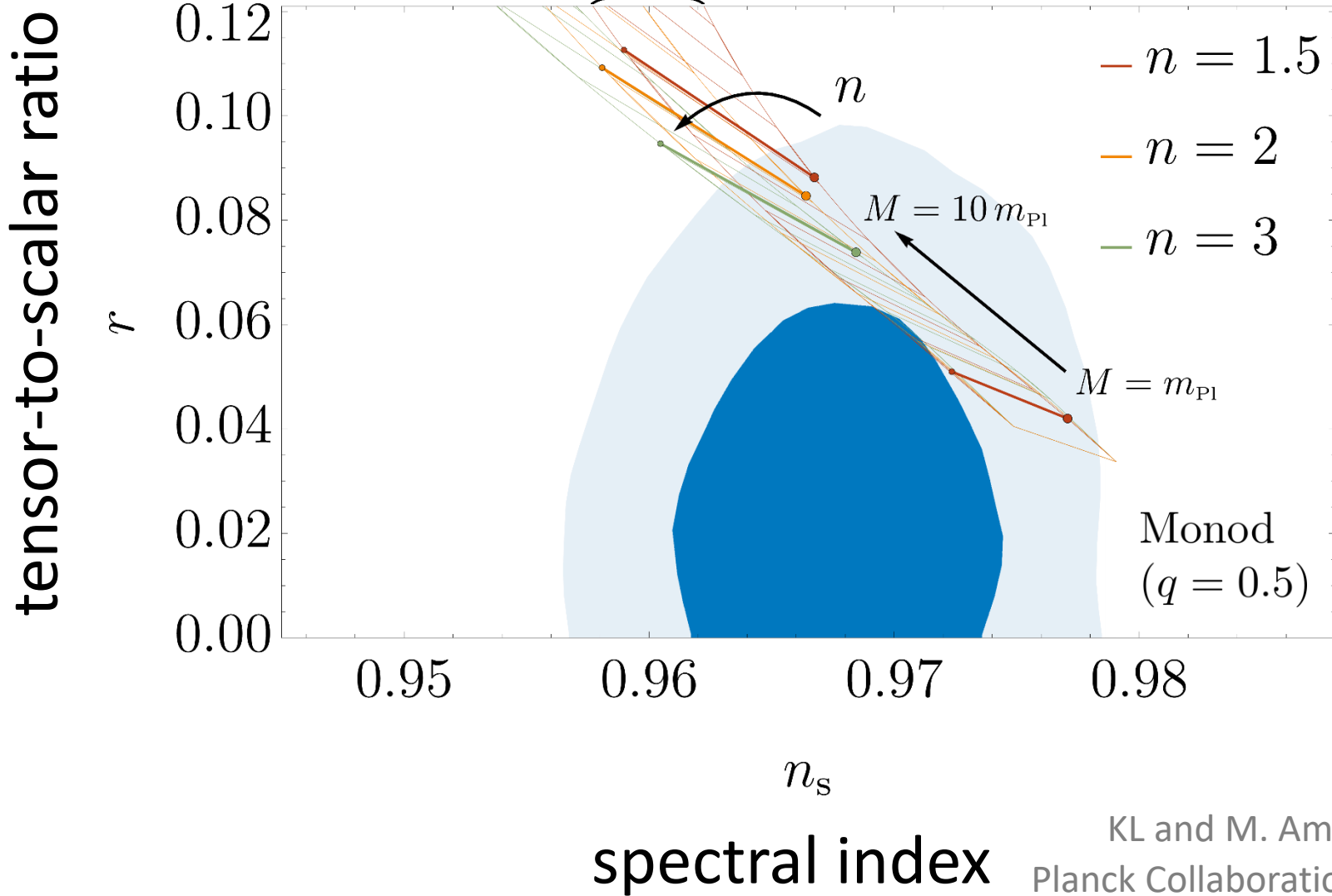


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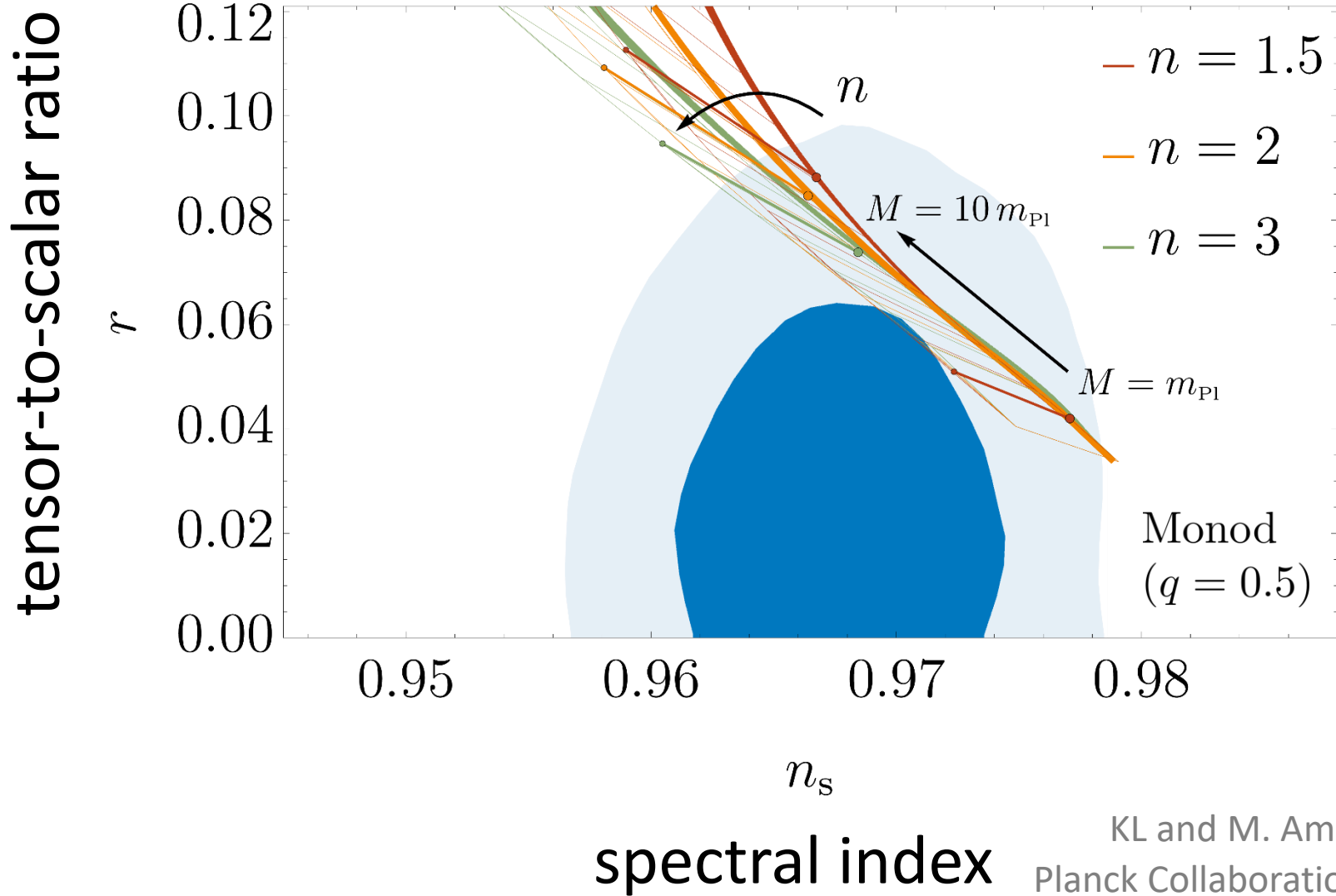


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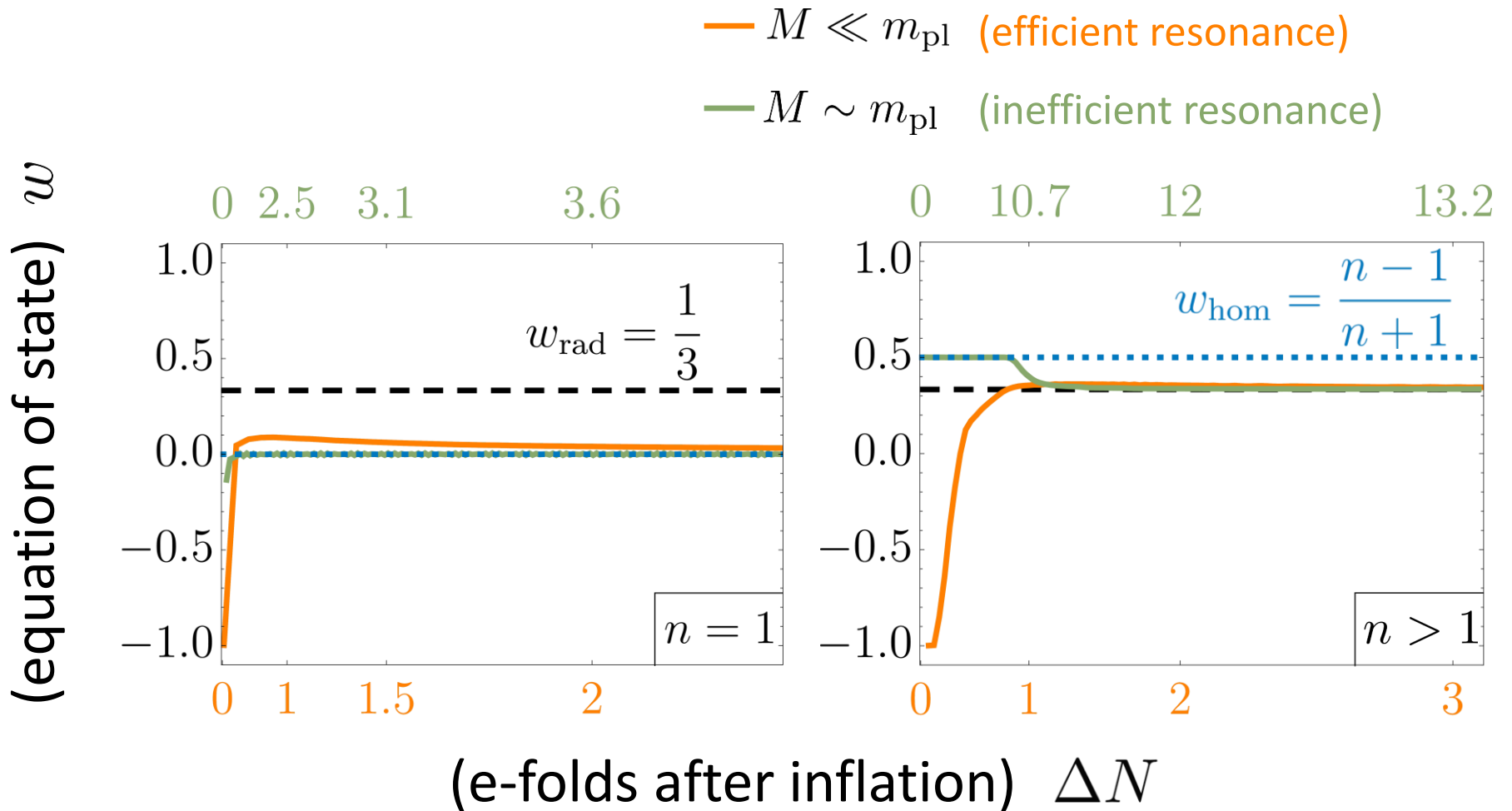
width:  $50 \leq N_* \leq 60$



# Expansion history effects



# Equation of state



$$\Delta N \equiv \int_{a_{\text{end}}}^a d \ln a$$



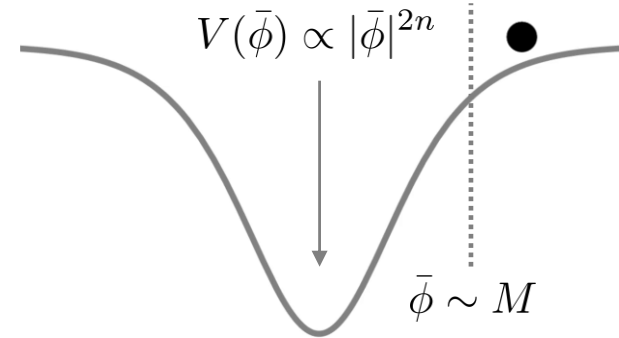
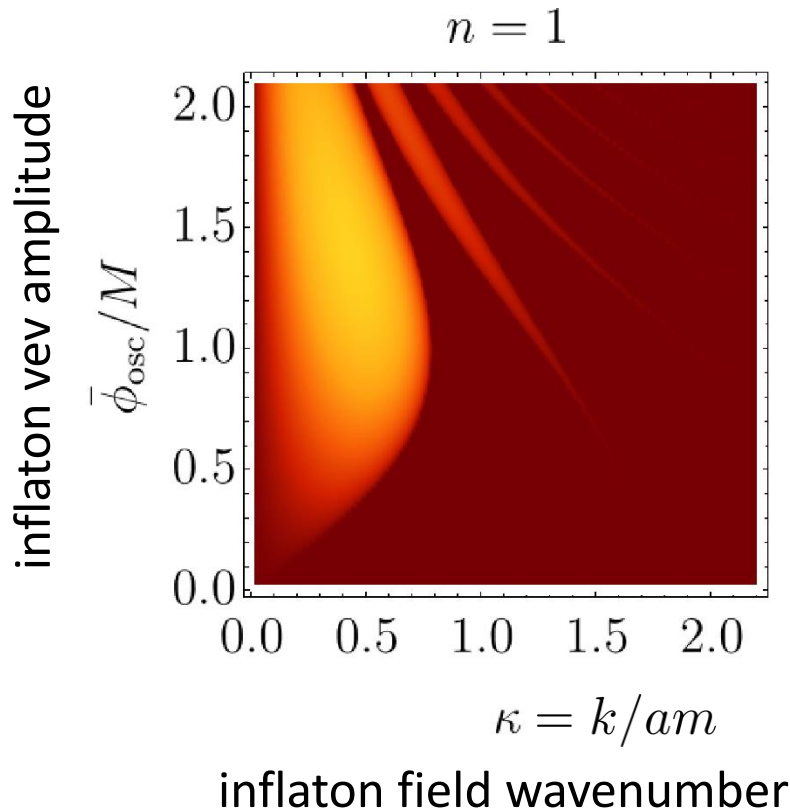
# Matter domination?

$$n = 1$$

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Non-perturbative decay (parametric self-resonance)



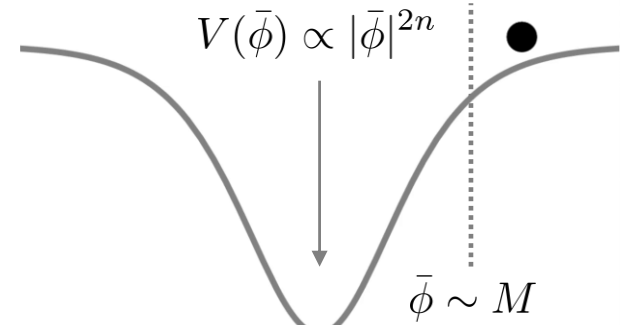
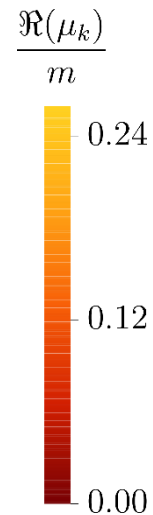
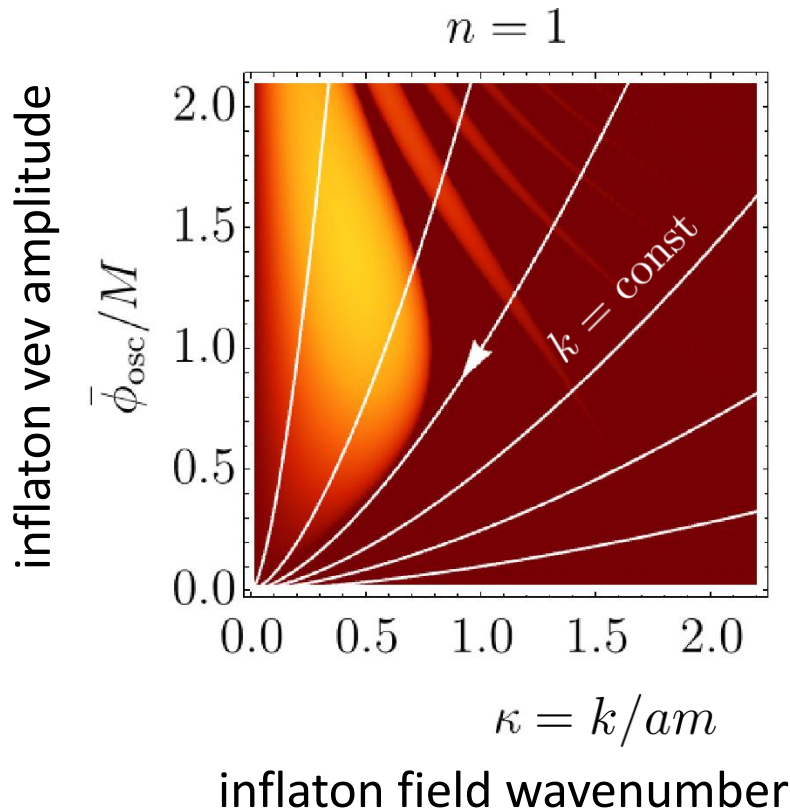
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

$$m^2 \equiv V'(\bar{\phi}_{\text{osc}})/\bar{\phi}_{\text{osc}}$$

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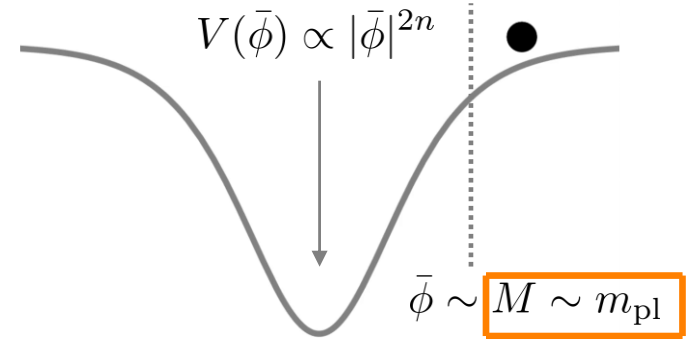
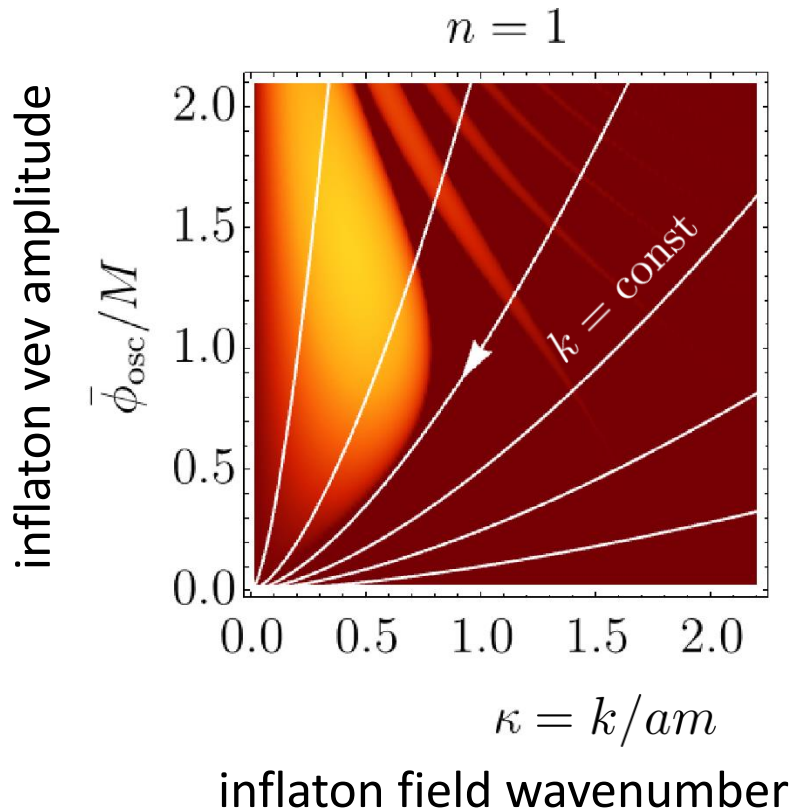
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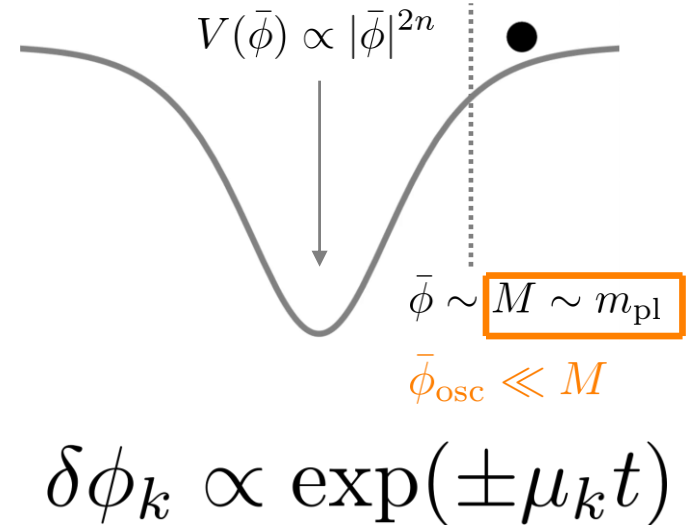
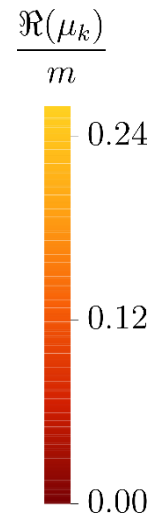
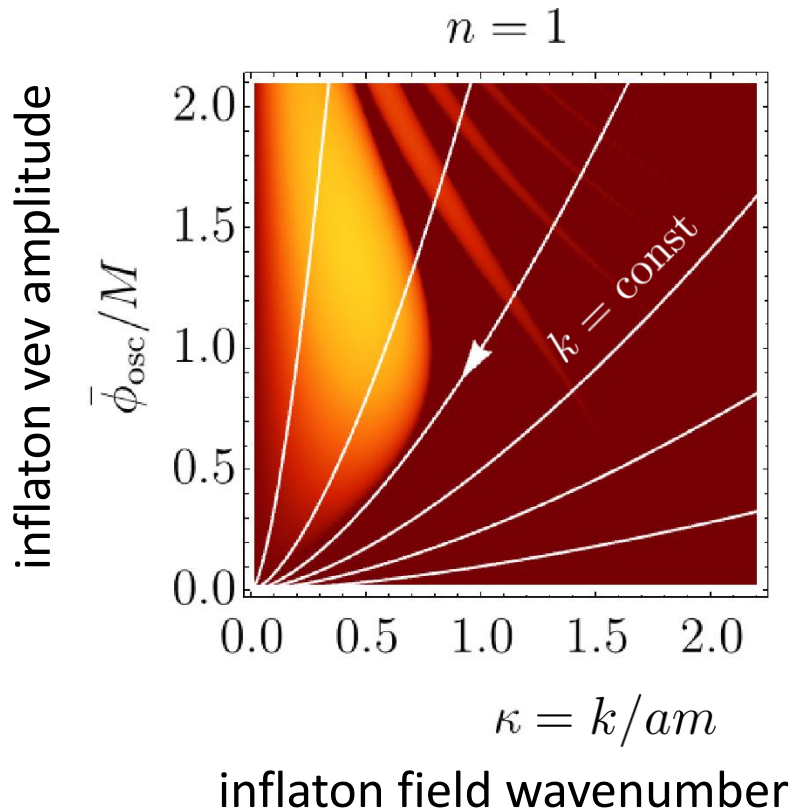
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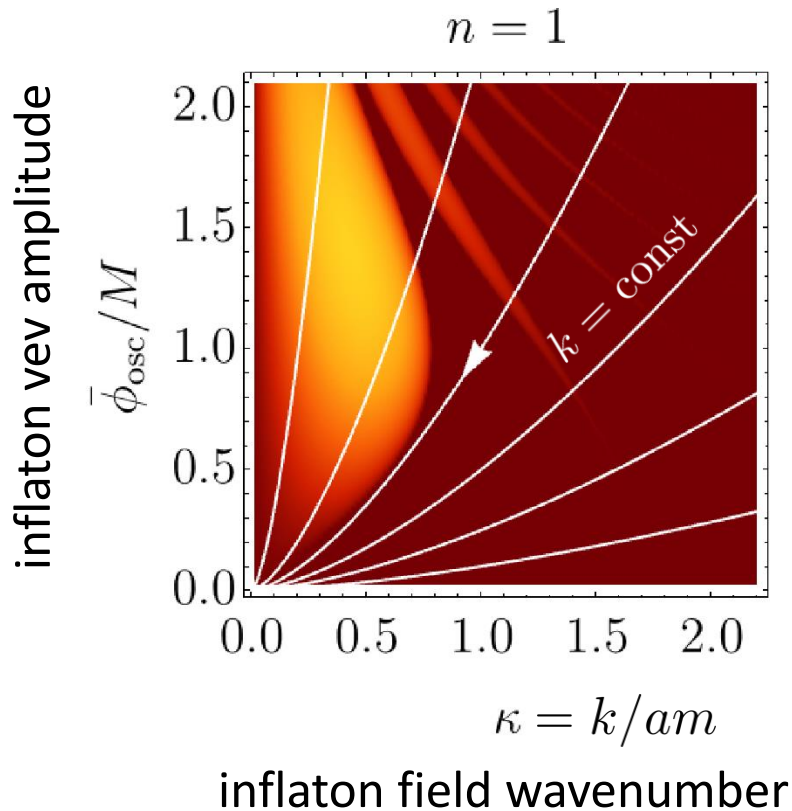


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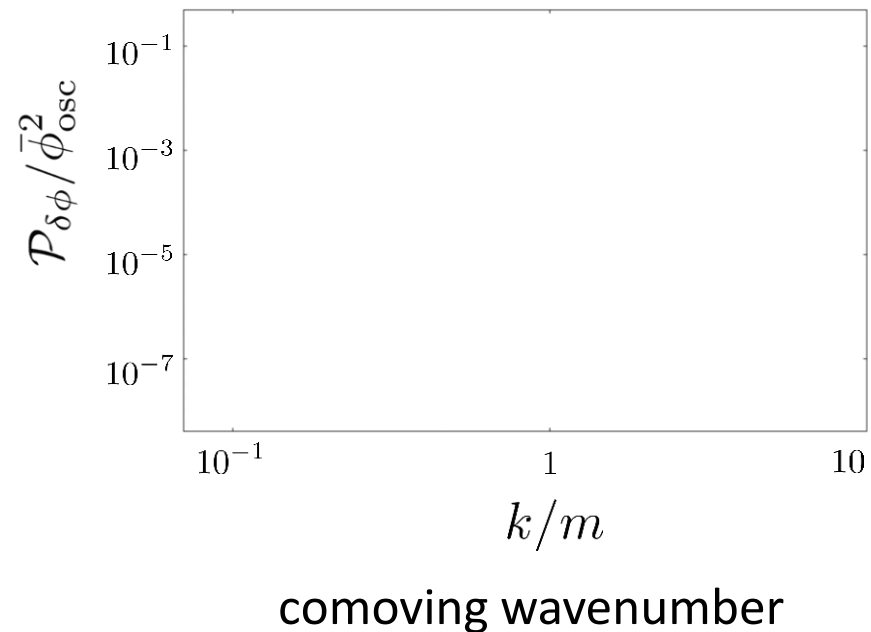
$$n = 1$$

Non-perturbative decay (parametric self-resonance)



Power spectrum:

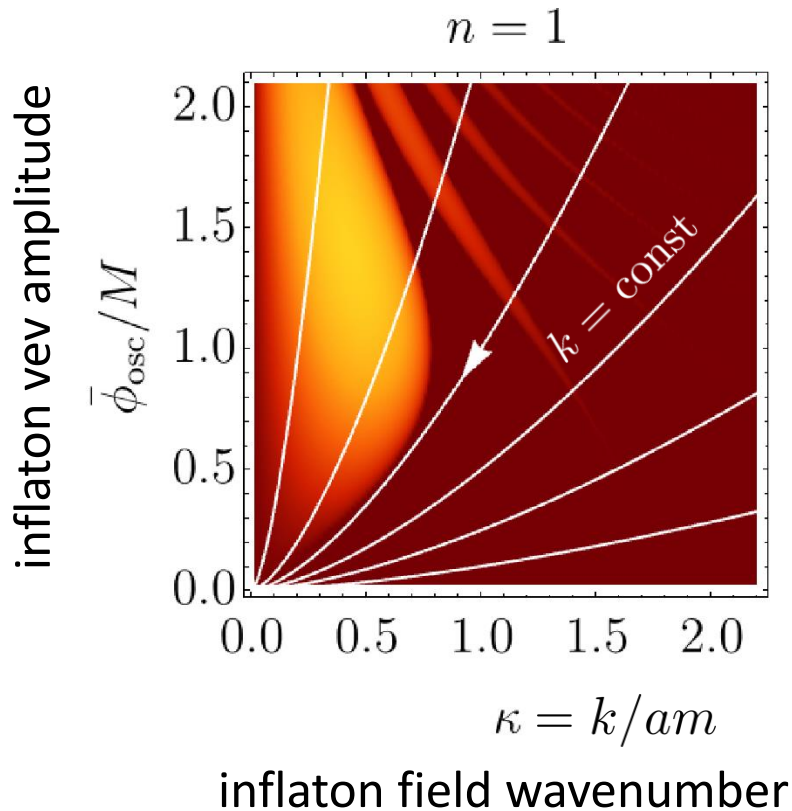
$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi} d \ln k$$



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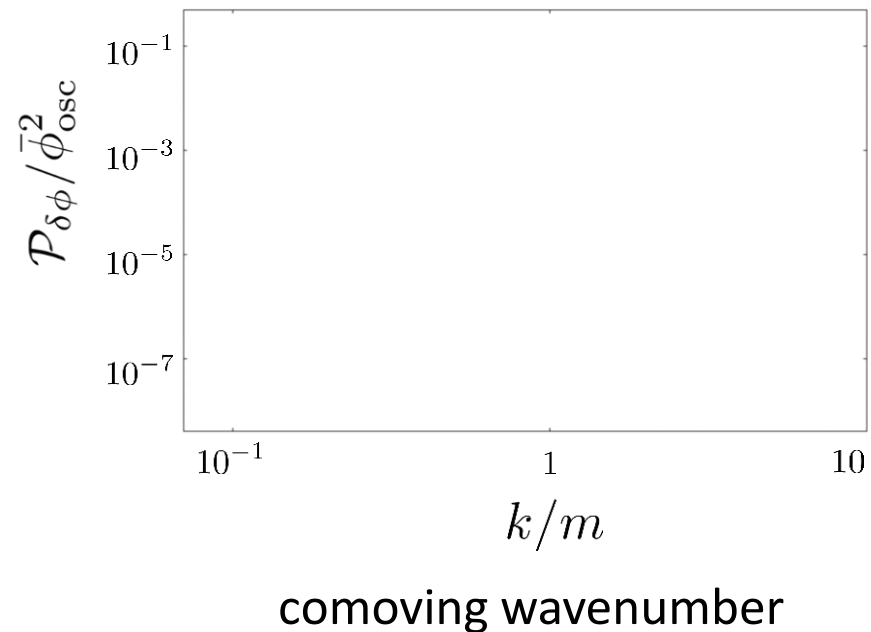
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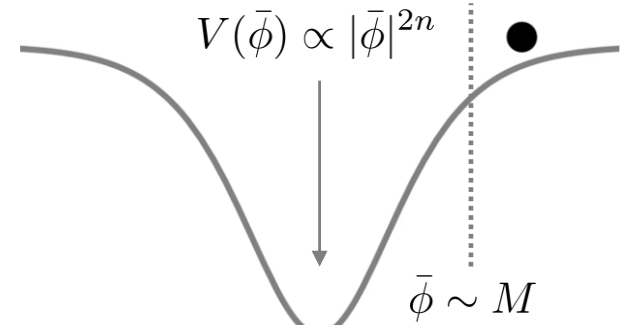
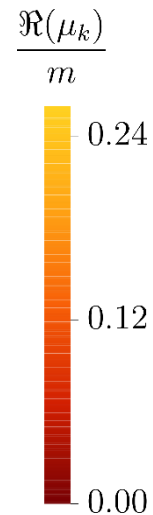
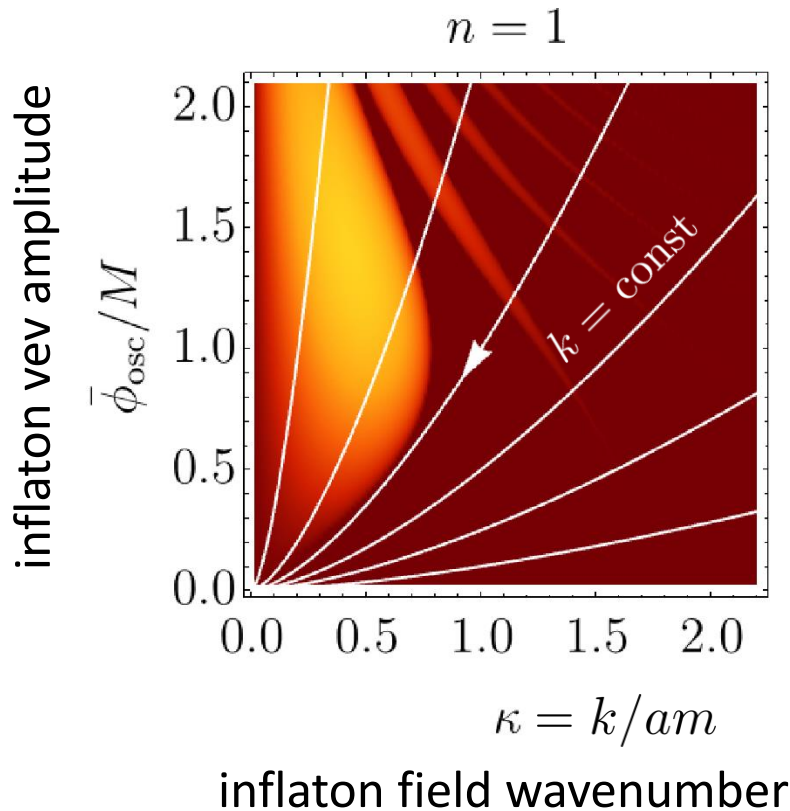
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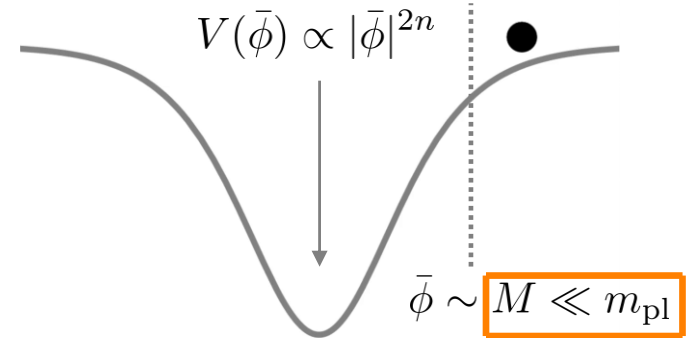
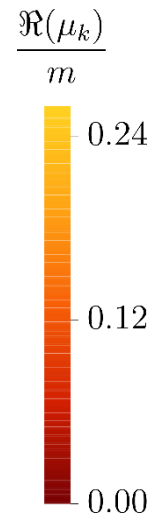
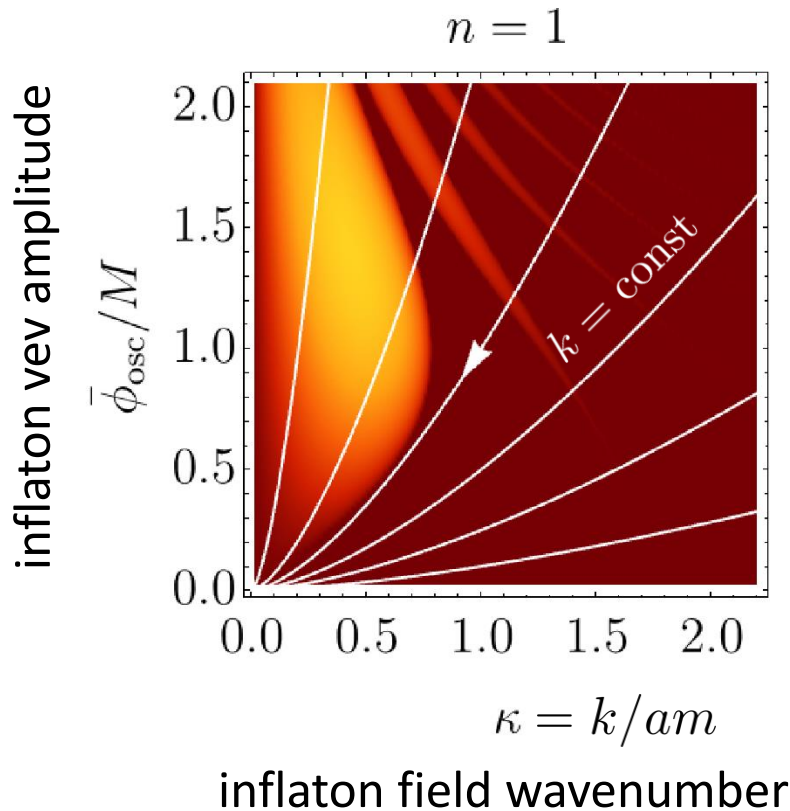
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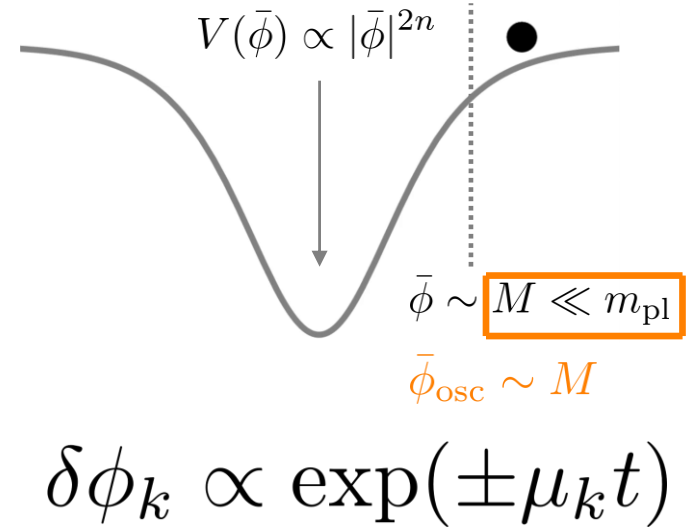
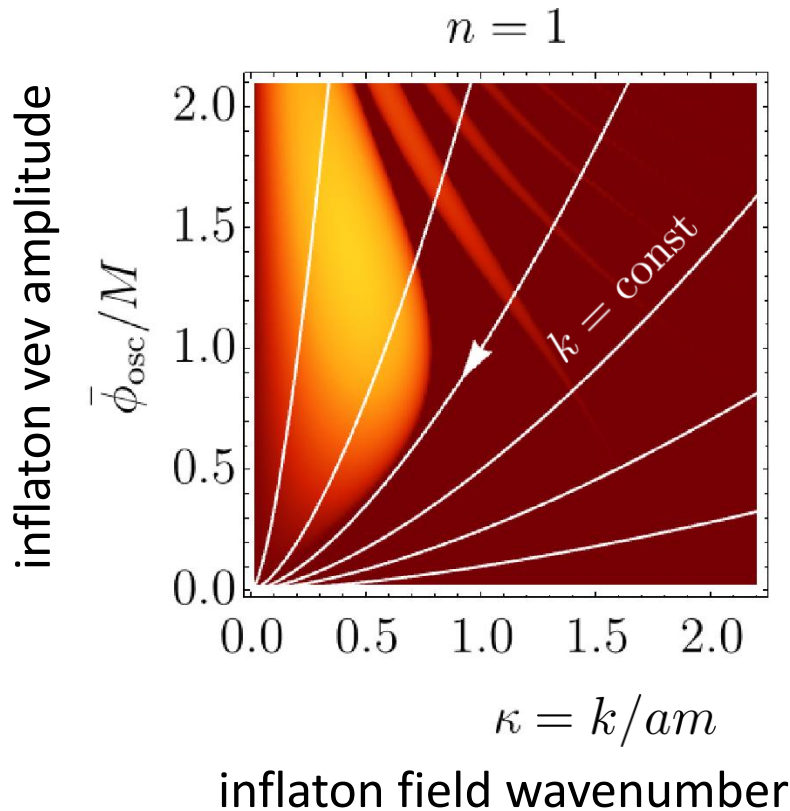
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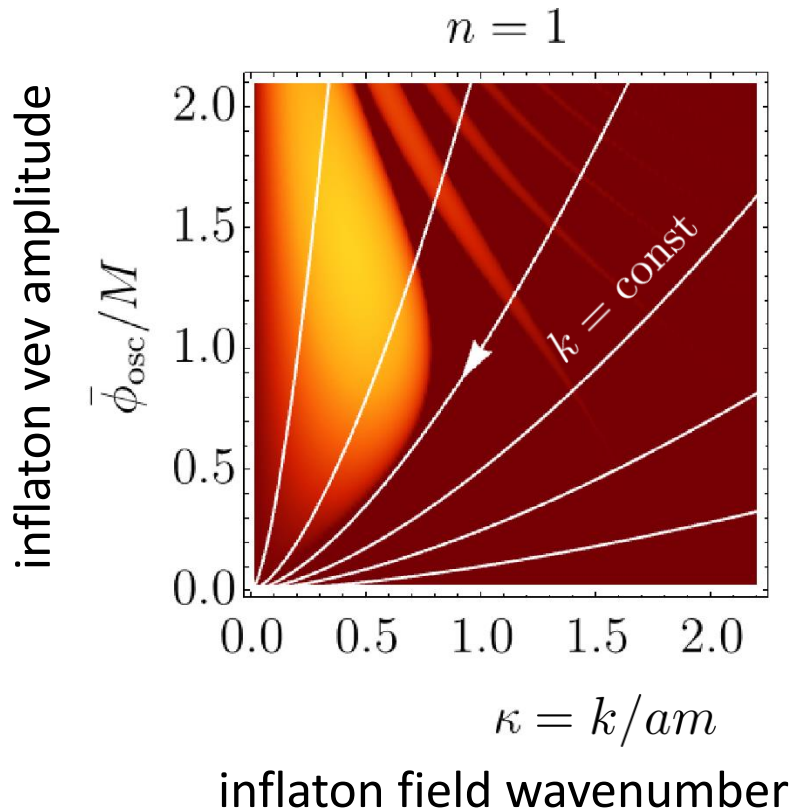


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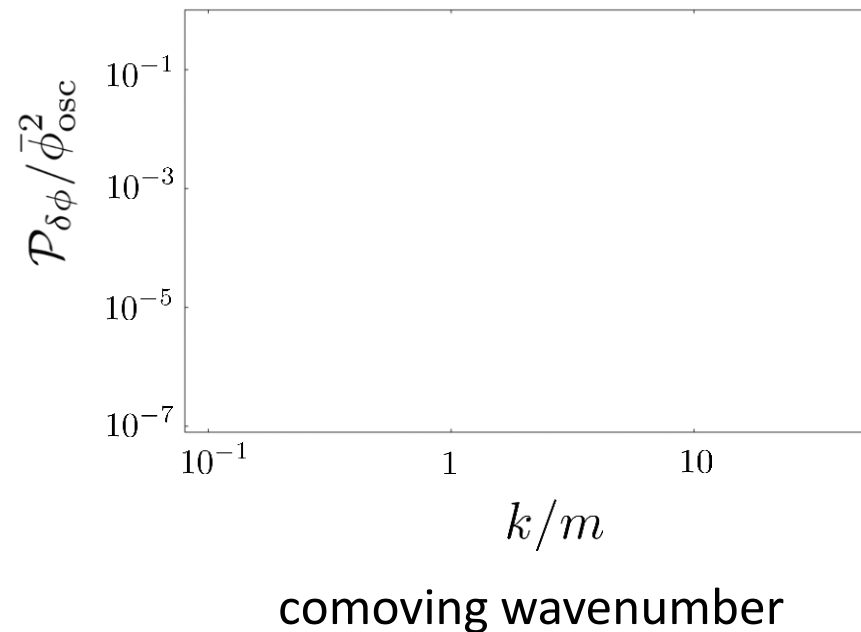
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Non-perturbative decay (parametric self-resonance)



Power spectrum:

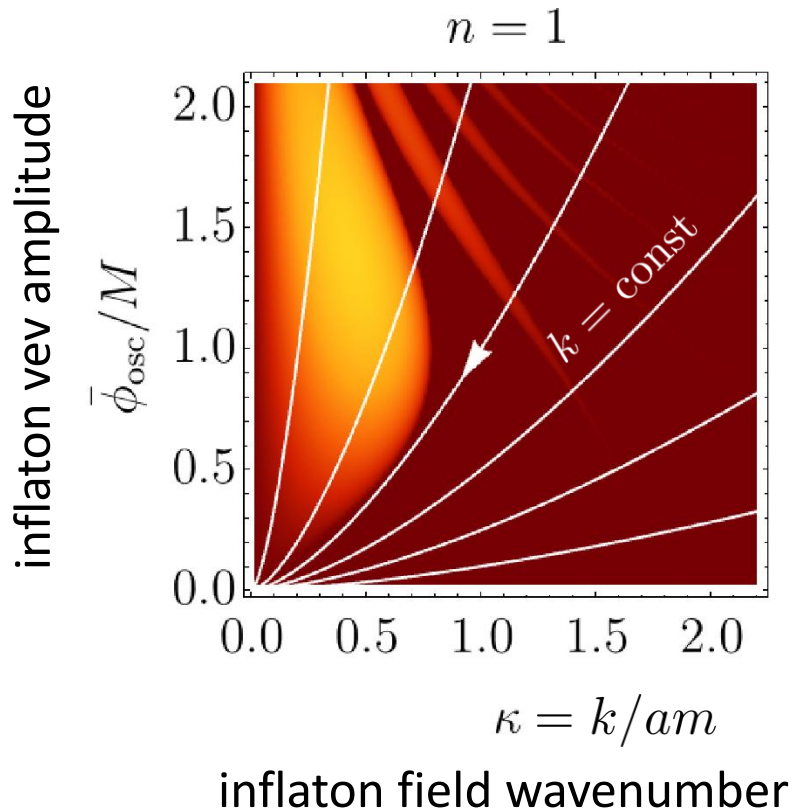
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# Matter domination?

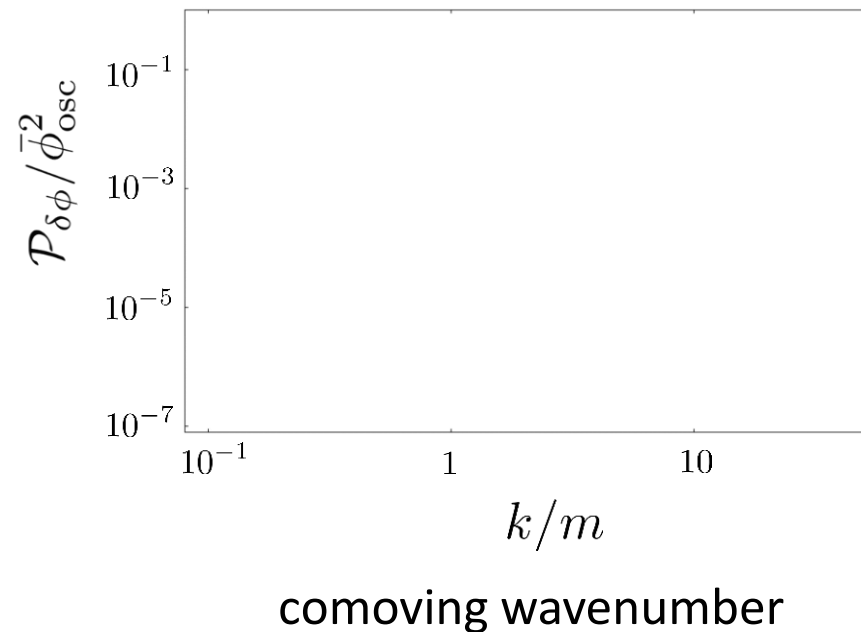
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Non-perturbative decay (parametric self-resonance)



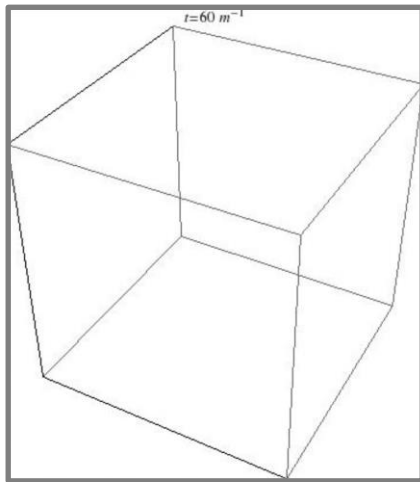
Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi} d \ln k$$



# Matter domination?

$$M \ll m_{\text{pl}}$$



$$n = 1$$

$$M \sim m_{\text{pl}}$$

- $\delta\phi(t, \mathbf{x})$  production shut off
- $\bar{\phi}_{\text{osc}}(t) = \text{pressureless dust}$

- $\bar{\phi}$  forms oscillons (stable)

KL and Amin (2017, 18, 19)

See also Amin, Easter, Finkel, Flauger, Hertzberg (2011)

matter-like eos:  $w = 0$

couplings to other fields?

See also Antusch, Figueroa, Marschall, Torrenti (2020)

Antusch, Marschall, Torrenti (2022)

Other connections ...

# Other connections ...

- stochastic GWs from fragmentation

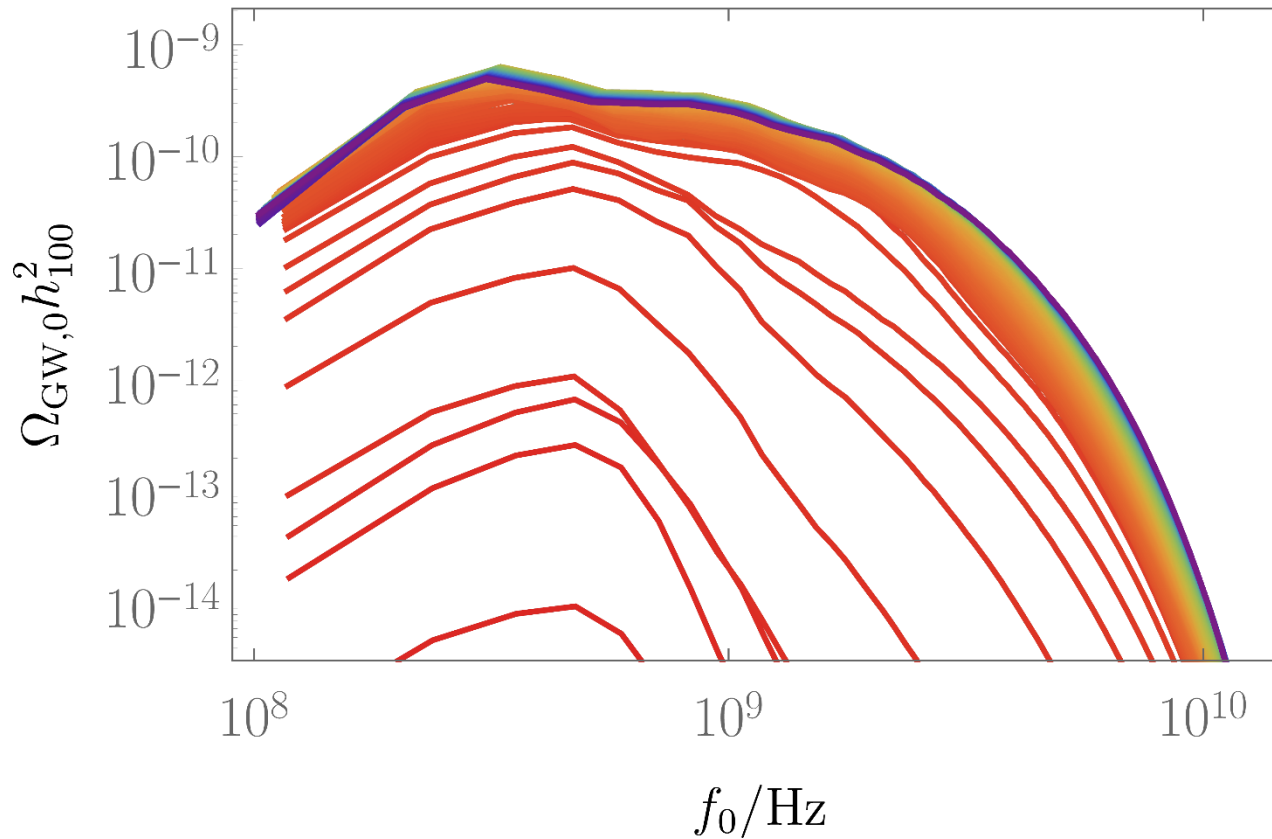
KL and M. Amin (2019)

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KL and M. Amin (2019)

Oscillons



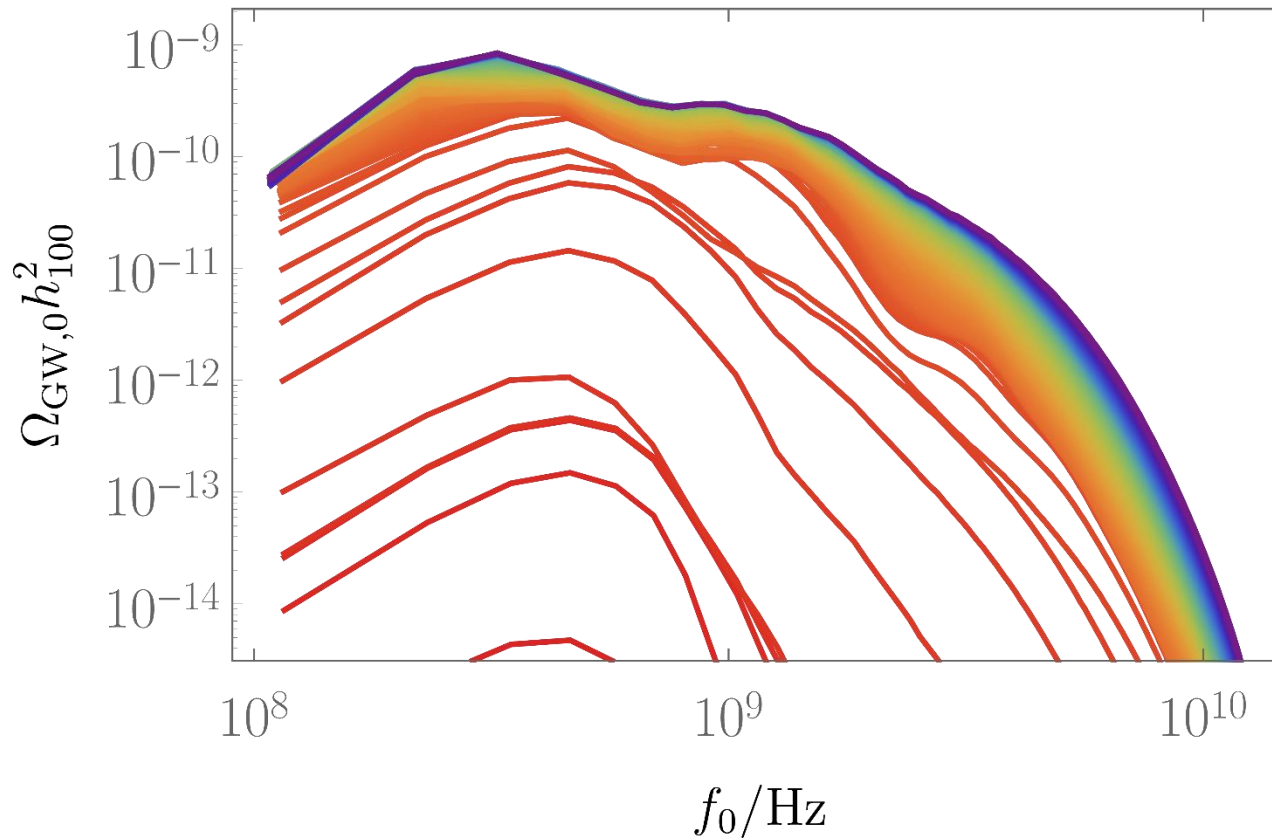


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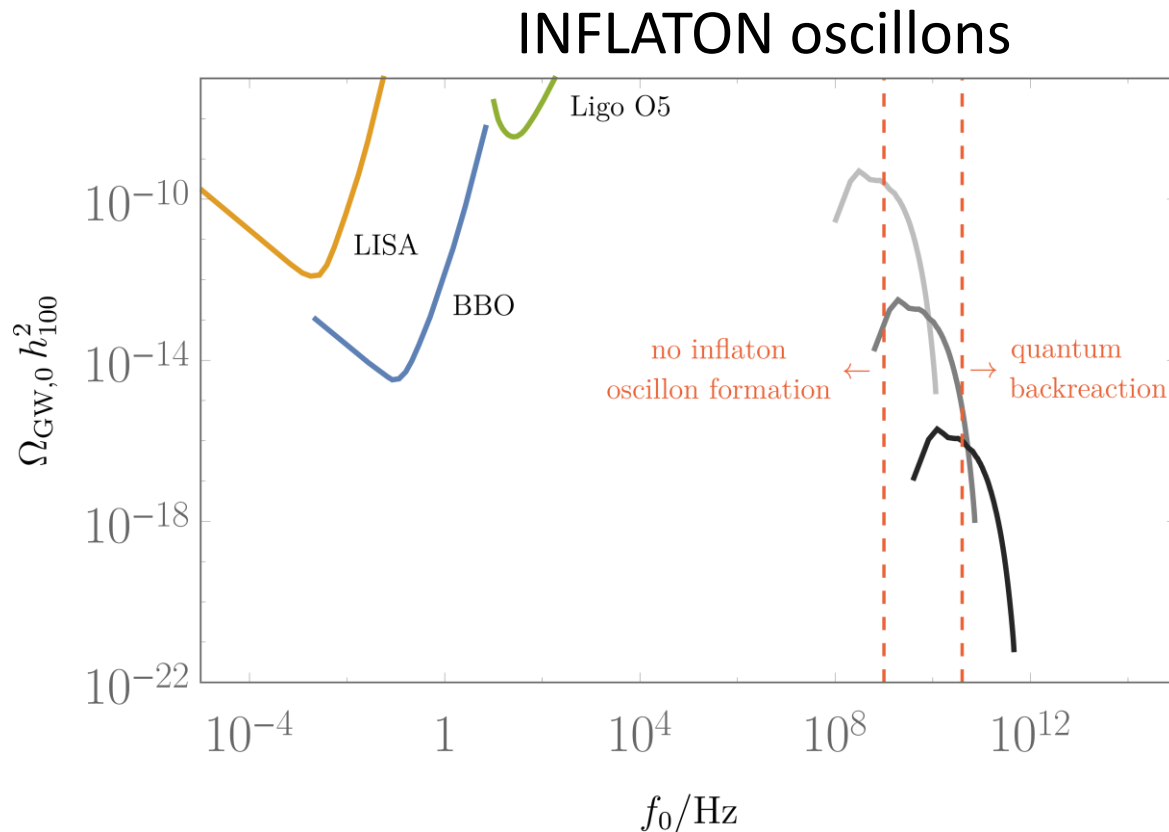
Transients



# Other connections ...

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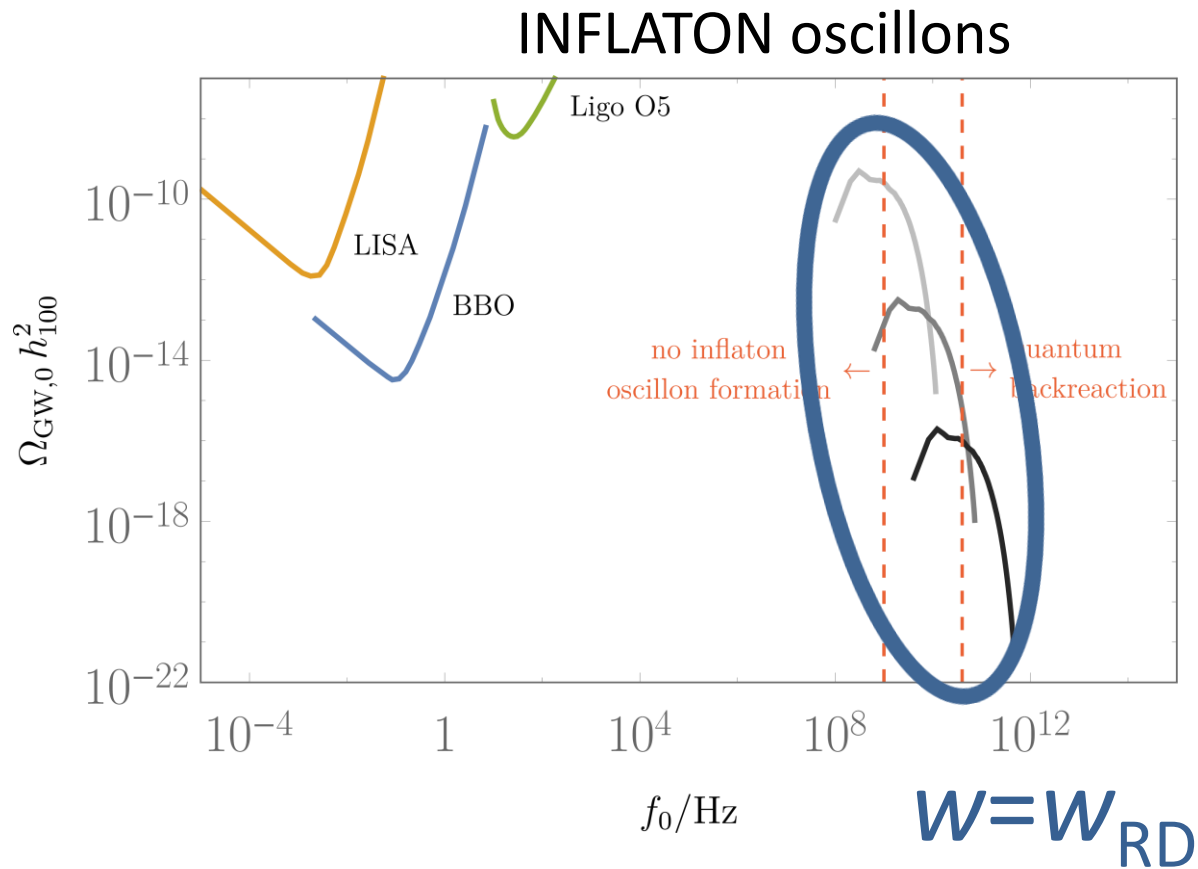


See also Adshead, Giblin, Pieroni, Weiner (2019a, 2019b)  
Figueroa and Tanin (2019)

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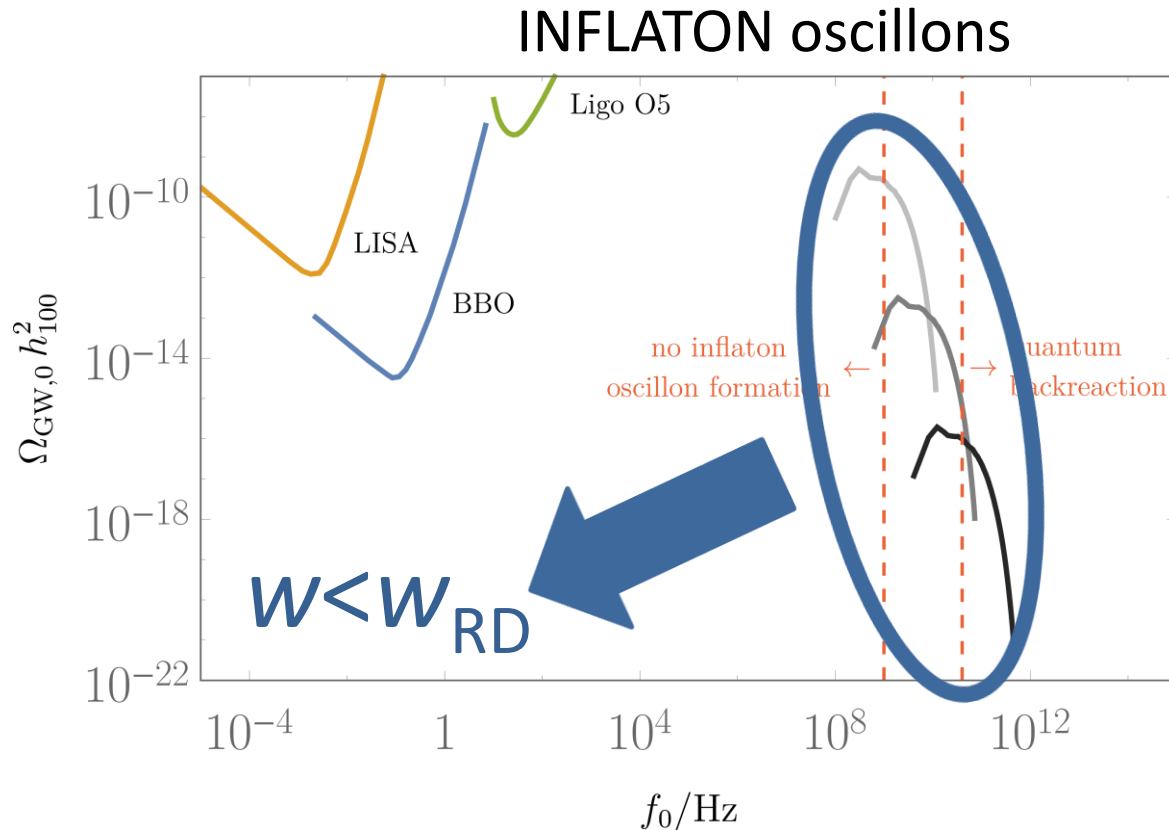


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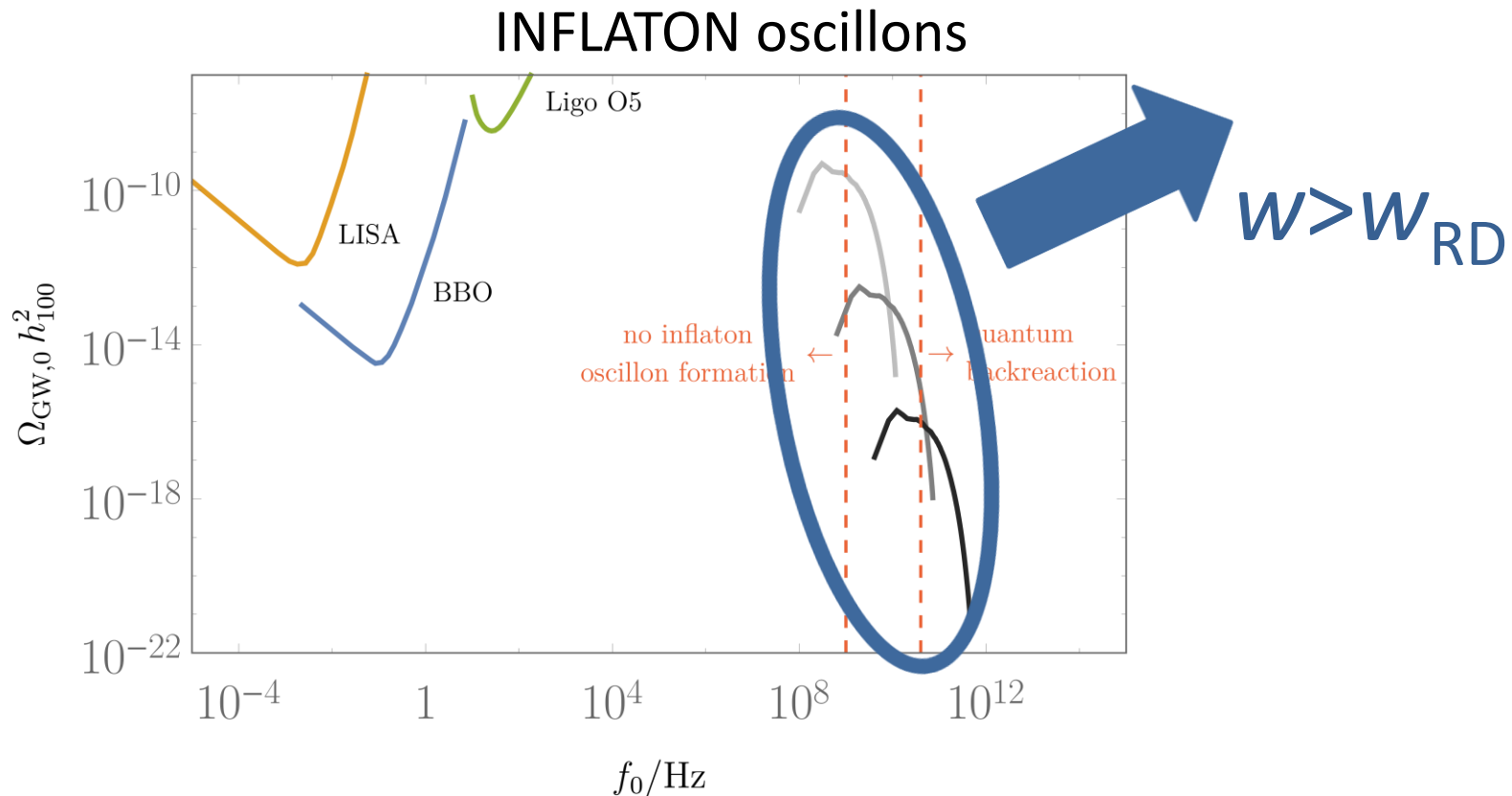


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M. Amin, KL and T. Smith (in progress)

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Weiner, Adshead, Giblin (2020)  
M. Amin, KL and T. Smith (in progress)
- dark matter ( $n=1$  oscillons  $\rightarrow$  Q-balls) Kusenko and Shaposhnikov (1997)

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Weiner, Adshead, Giblin (2020)  
M. Amin, KL and T. Smith (in progress)
- dark matter ( $n=1$  oscillons  $\rightarrow$  Q-balls) Kusenko and Shaposhnikov (1997)
- matter-antimatter asymmetry... KL and M. Amin (2014)



# Oscillons & baryogenesis

KL and M. Amin, PRD 90, 083528 (2014)

complex  
inflaton  $\phi$   
+  ~~$U(\mathbf{I})$~~



dynamics at the end of inflation

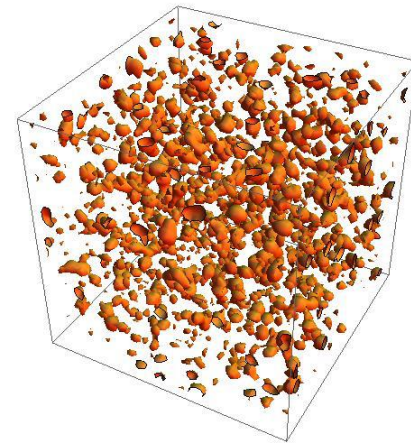
inflaton/anti-inflaton  
asymmetry



decay

matter-antimatter  
asymmetry

$$\eta \approx 6 \times 10^{-10}$$



very different dynamics from homogeneous case!

# Gauge fields, inflation & reheating

KL and M. Amin, JCAP 1606 032 (2016)

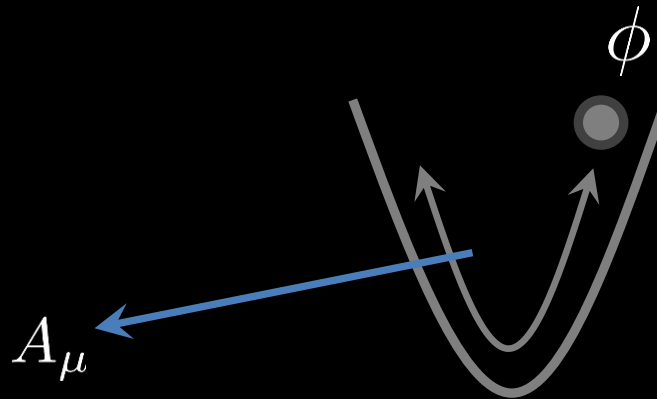
$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} \mathcal{R} - (D_\mu \phi)^\dagger D^\mu \phi - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

See also Figueroa, Garcia-Bellido, Torrenti, (2015, 16)

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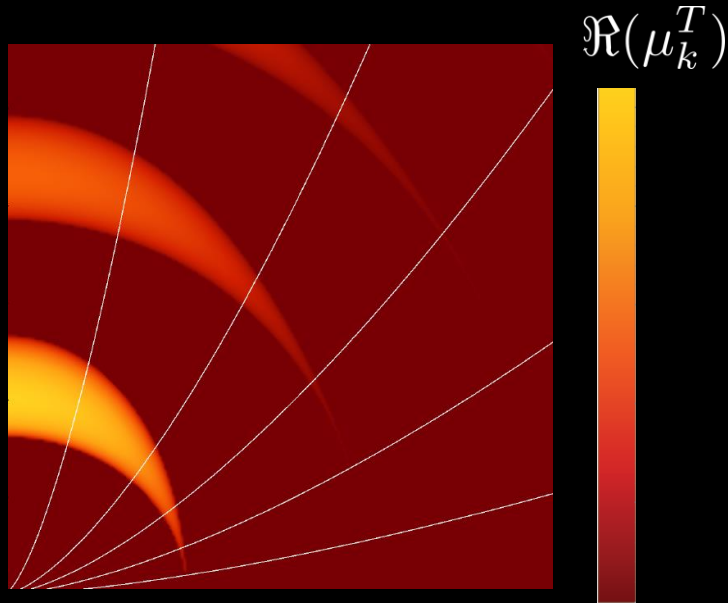
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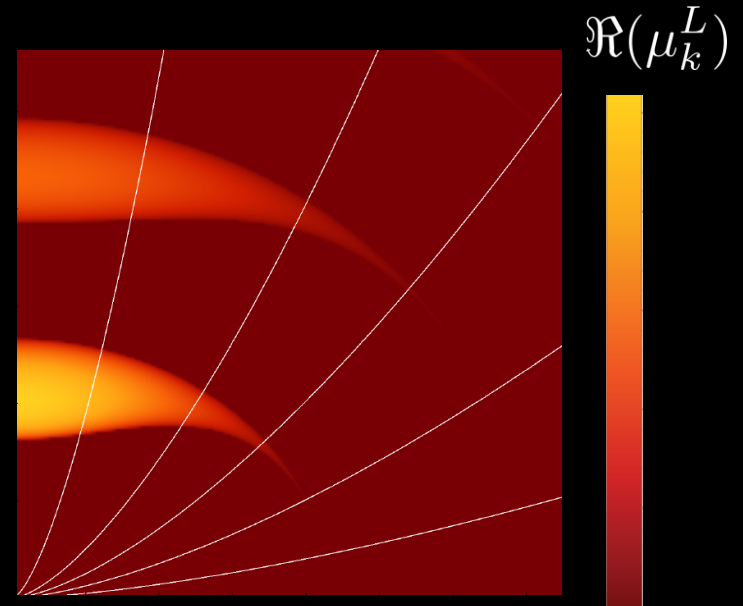
$$A_k^T \propto \exp(\pm \mu_k^T t)$$

$$A_k^L \propto \exp(\pm \mu_k^L t)$$

inflaton amplitude



wavenumber

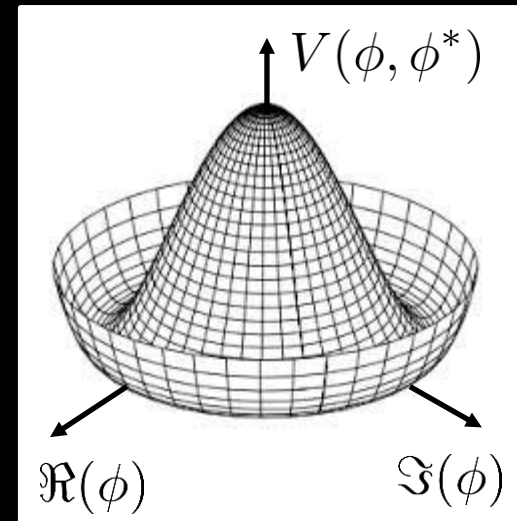
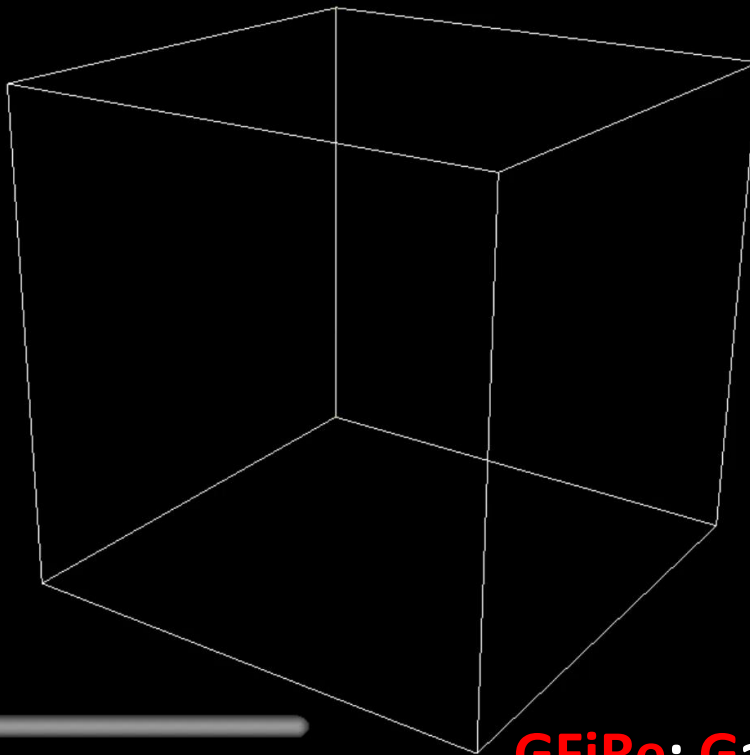


wavenumber

# Gauge fields, inflation & reheating

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**Non-Abelian code in collaboration with  
M. Amin, A. Caravano, KL, E. Komatsu...  
(in progress)**



Abelian code: KL, M. Amin (2020)

See also Figueroa, Florio,  
Torrenti, Valkenburg (2020, 2021)

**GFiRe: Gauge Field integrator for Reheating**  
**NEW LATTICE CODE!!! JCAP 058 04 (2020)**

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Lattice theory: Angelo Caravano, Komatsu, KL, Weller (2021a,b, 2022)

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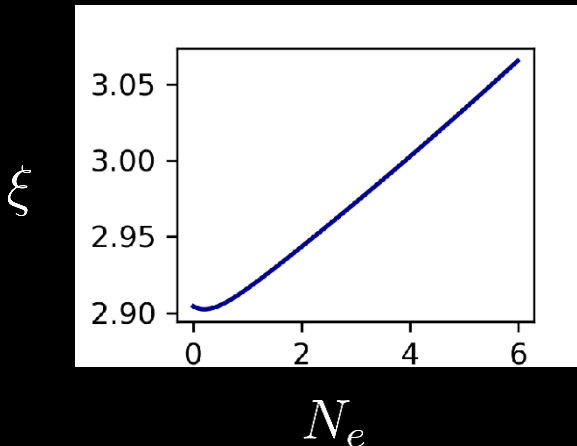
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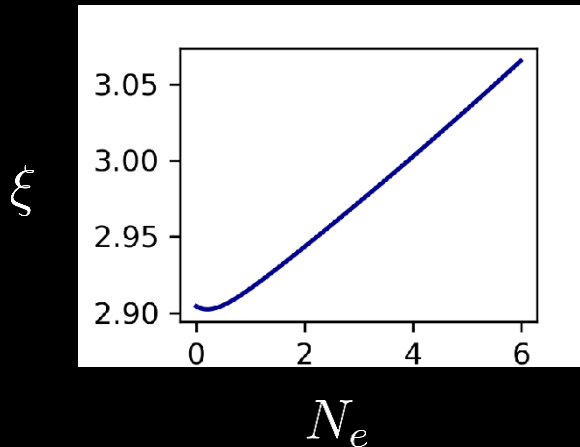
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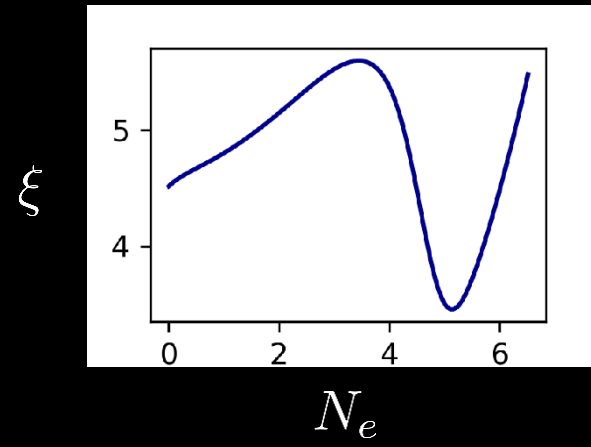
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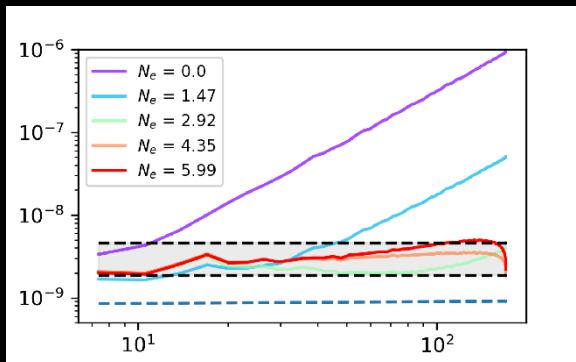
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$k/m$

$\mathcal{P}_\zeta$

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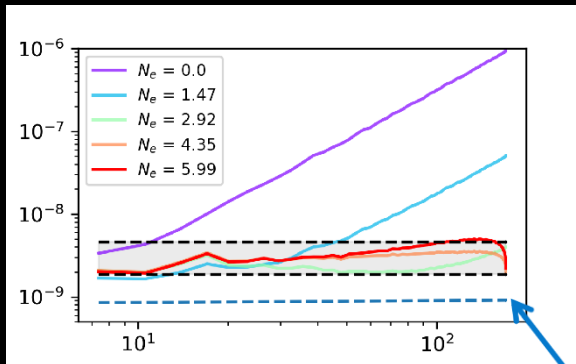
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$$k/m$$

$$\mathcal{P}_{\text{vac}} = H^4 / (2\pi \dot{\bar{\phi}})^2$$

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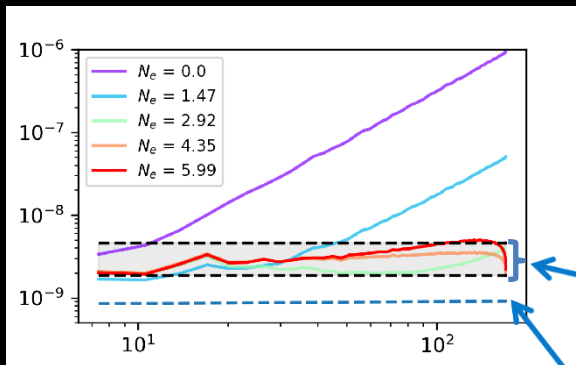
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$$\mathcal{P}_\zeta^a = \mathcal{P}_{\text{vac}} + \mathcal{P}_{\text{vac}}^2 f(\xi) e^{4\pi\xi}$$

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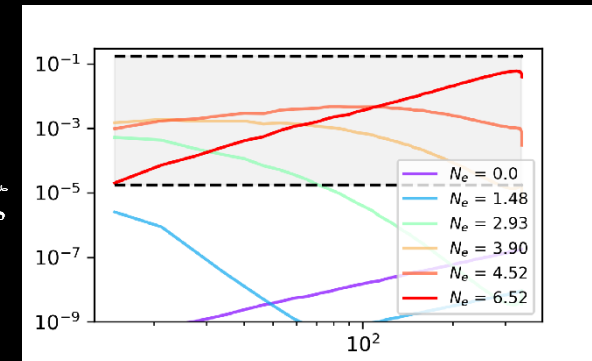
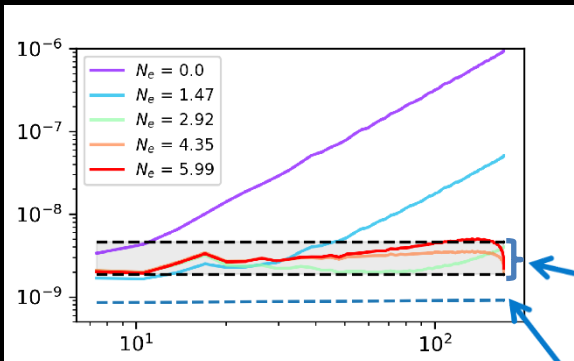
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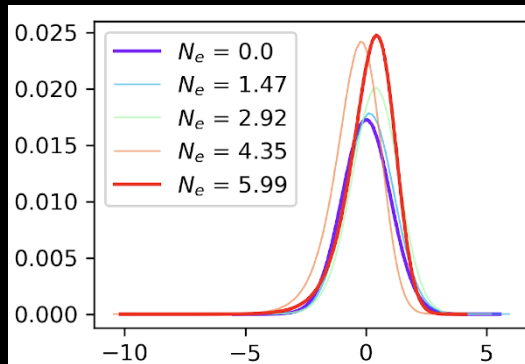
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$$\zeta/\sigma$$

$$\sigma^2 = \langle \zeta^2 \rangle$$

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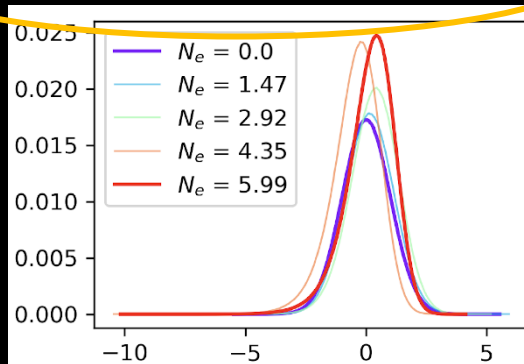
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$\xi \gtrsim 1$  **non-Gaussianity**

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$\zeta/\sigma$

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# Gauge fields, inflation & reheating



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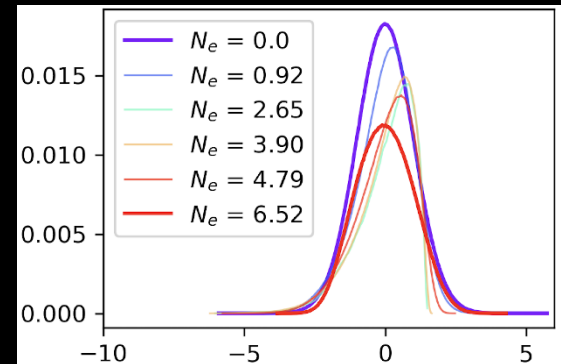
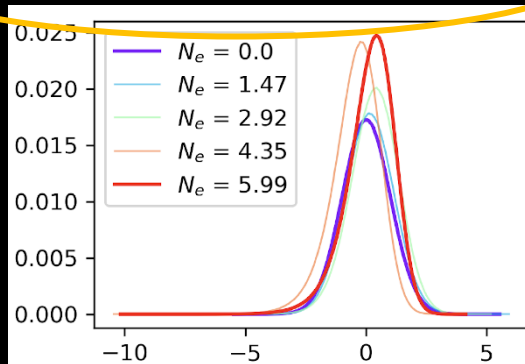
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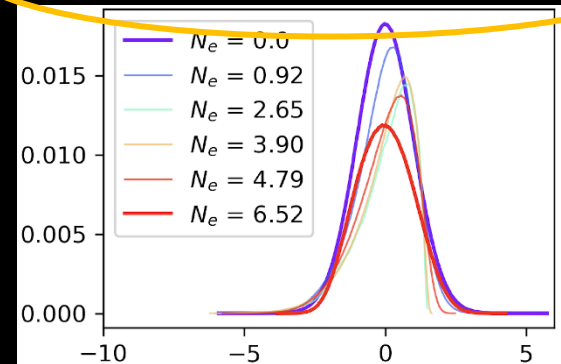
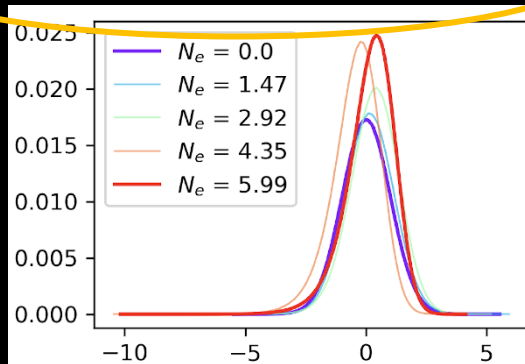
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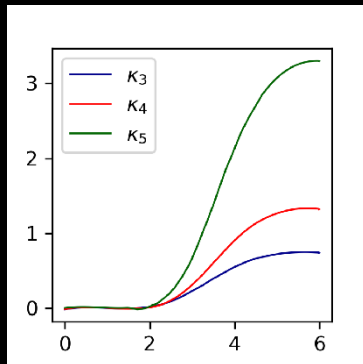
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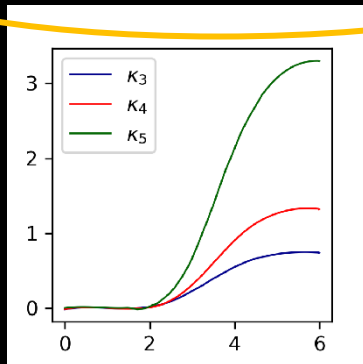
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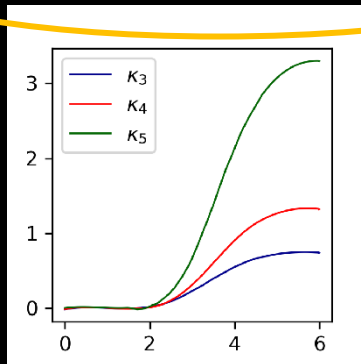
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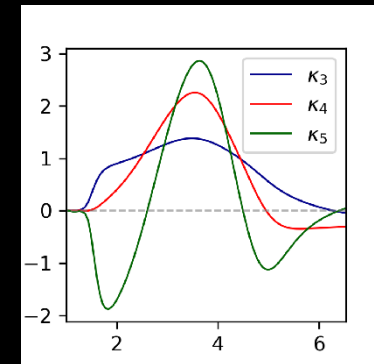
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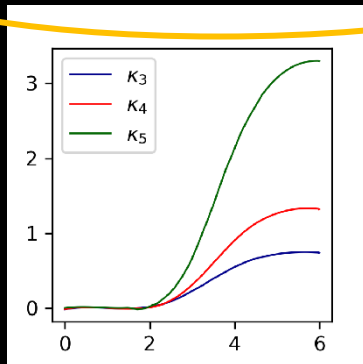
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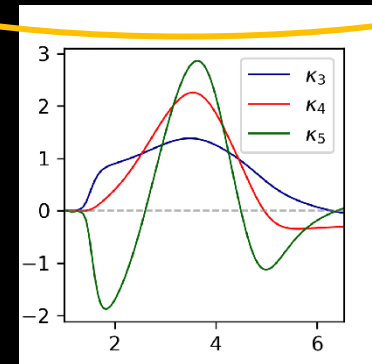
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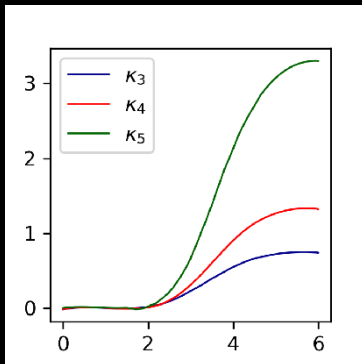
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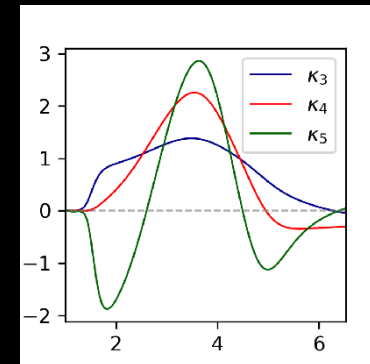


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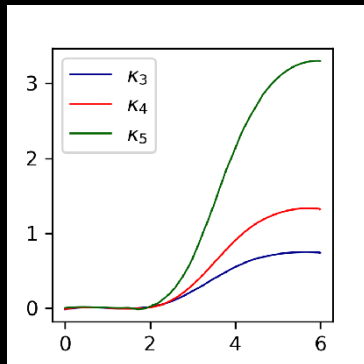
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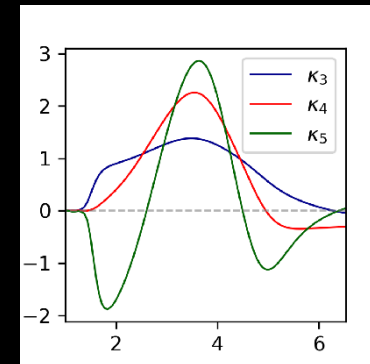
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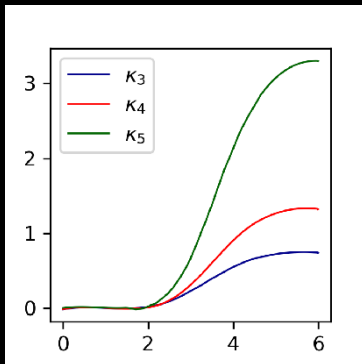
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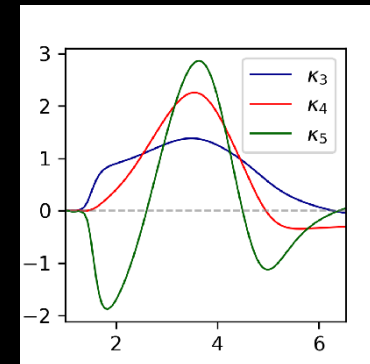
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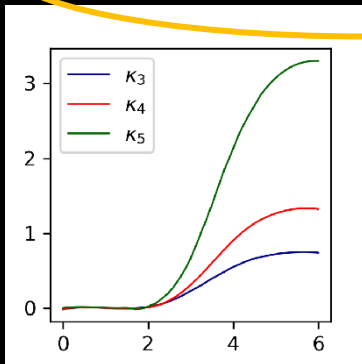
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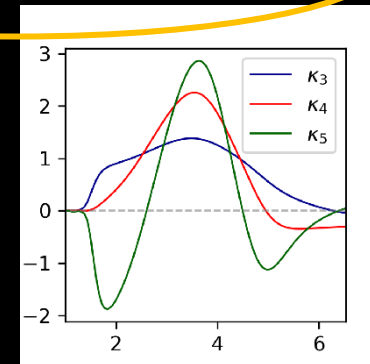


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**CLT**

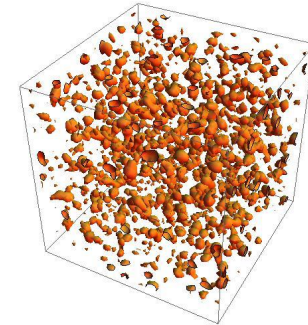


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# Conclusions

Reheating at the end of inflation:

- very rich dynamics



Future plans:

- more realistic models (e.g. gauge fields)
- observational signatures
  - expansion history, relics, GWs

