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Cosmological Implications of Higgs Vacuum Instability

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Physics of the Early Universe
16 June 2022

Based on:

Review Article:

- Markkanen, AR & Stopyra, Front.Astron.Space Sci. 5 (2018) 40

Original works:

- Herranen, Markkanen, Nurmi & AR, PRL113 (2014) 211102
- Herranen, Markkanen, Nurmi & AR, PRL115 (2015) 241301
- AR & Stopyra, PRD95 (2017) 025008
- AR & Stopyra, PRD97 (2018) 025012
- Figueroa, AR & Torrenti, PRD98 (2018) 023532
- Markkanen, Nurmi, AR & Stopyra, JHEP 1806 (2018) 040
- Mantziris, Markkanen & AR, JCAP03 (2021) 077

Standard Model of Particle Physics

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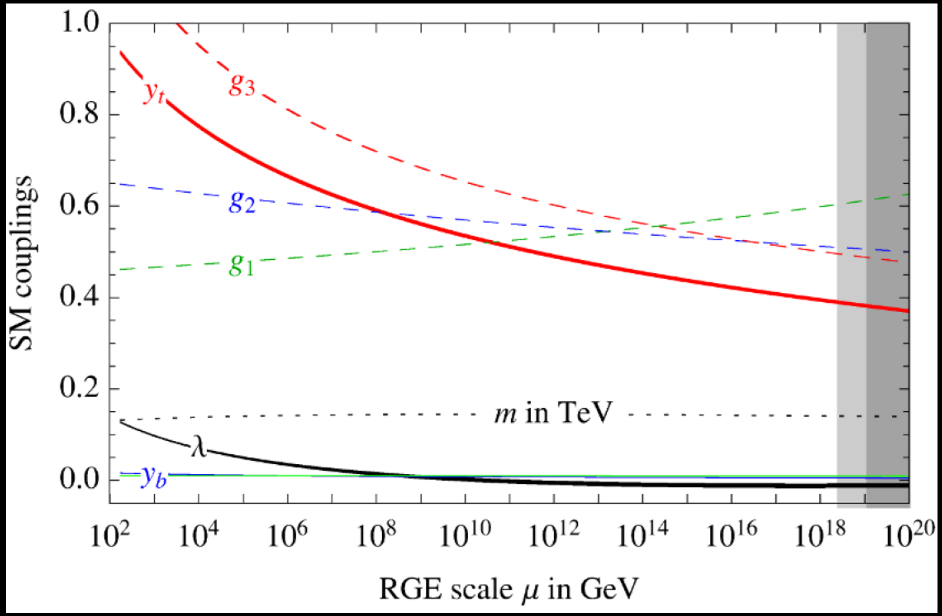
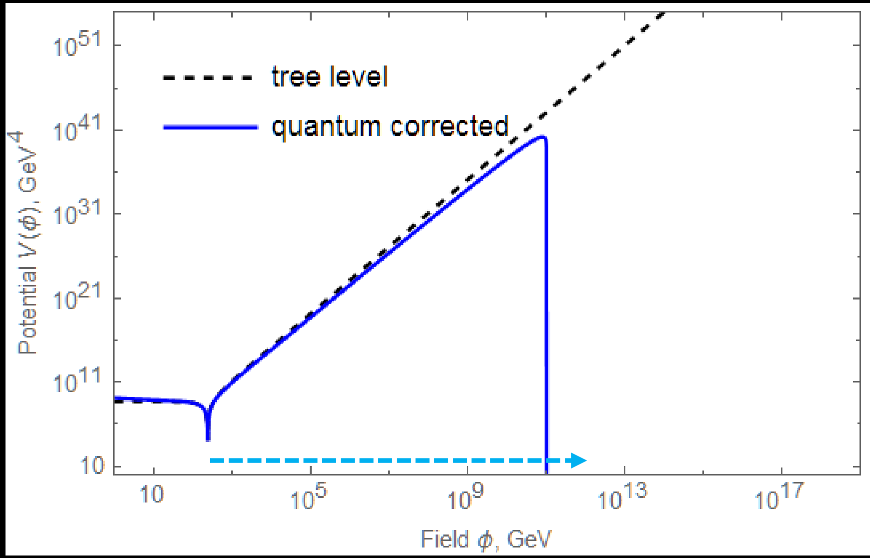
- All renormalisable terms allowed by symmetries in Minkowski space
- 19 parameters – all have been measured
- Can be extrapolated all the way to Planck scale
- For central experimental values $M_{\mathrm{H}} = 125.18\ \mathrm{GeV}$ and $M_{\mathrm{t}} = 173.1\ \mathrm{GeV}$
  - λ becomes negative at $\mu_\Lambda \approx 9.9 \times 10^9\ \mathrm{GeV}$
  - Minimum value $\lambda_{\mathrm{min}} \approx -0.015$ at $\mu_{\mathrm{min}} \approx 2.8 \times 10^{17}\ \mathrm{GeV}$



(Buttazzo et al 2013)

Vacuum Instability

- Renormalisation group improved Higgs effective potential
$$V(\phi) \approx \lambda(g\phi)\phi^4$$
- Becomes negative at $\phi > \phi_c \approx 10^{10}\mathrm{GeV}$
- True vacuum at Planck scale?
- Current vacuum metastable against quantum tunnelling
- Barrier at $\phi_{\mathrm{bar}} \approx 4.6 \times 10^{10}\ \mathrm{GeV}$, height $V(\phi_{\mathrm{bar}}) \approx \left(4.3 \times 10^9\ \mathrm{GeV}\right)^4$ (Based on a 3-loop calculation by Bednyakov et al. 2015)



Tunneling Rate

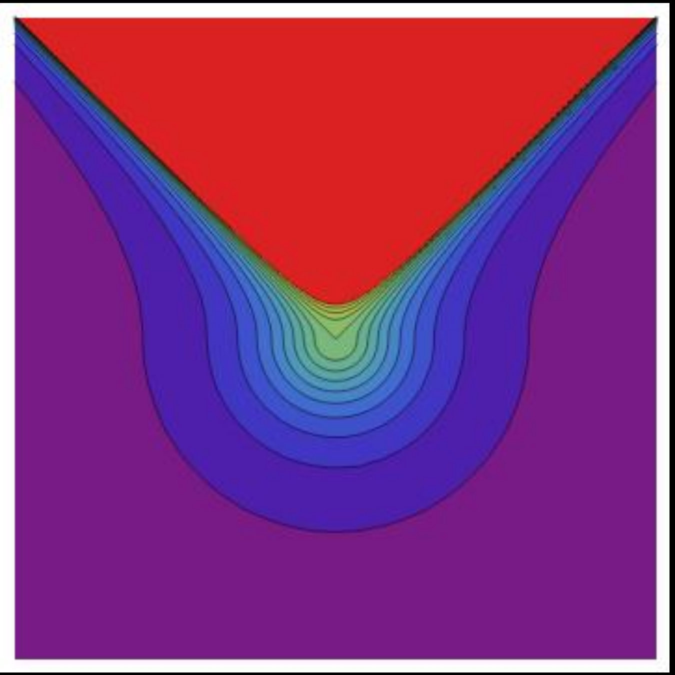
- Bubble nucleation rate:
  - $\Gamma \sim e^{-B}$, where
  - $B =$ "bounce" action (Coleman 1977)
  - Solution of Euclidean equation of motion
- Constant $\lambda < 0$: (Fubini 1976)

$$\phi(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2+R^2}$$

- Action $B = \frac{8\pi^2}{3|\lambda|}$
- When λ runs, $B \approx \frac{8\pi^2}{3|\lambda_{\min}|} \approx 1800$

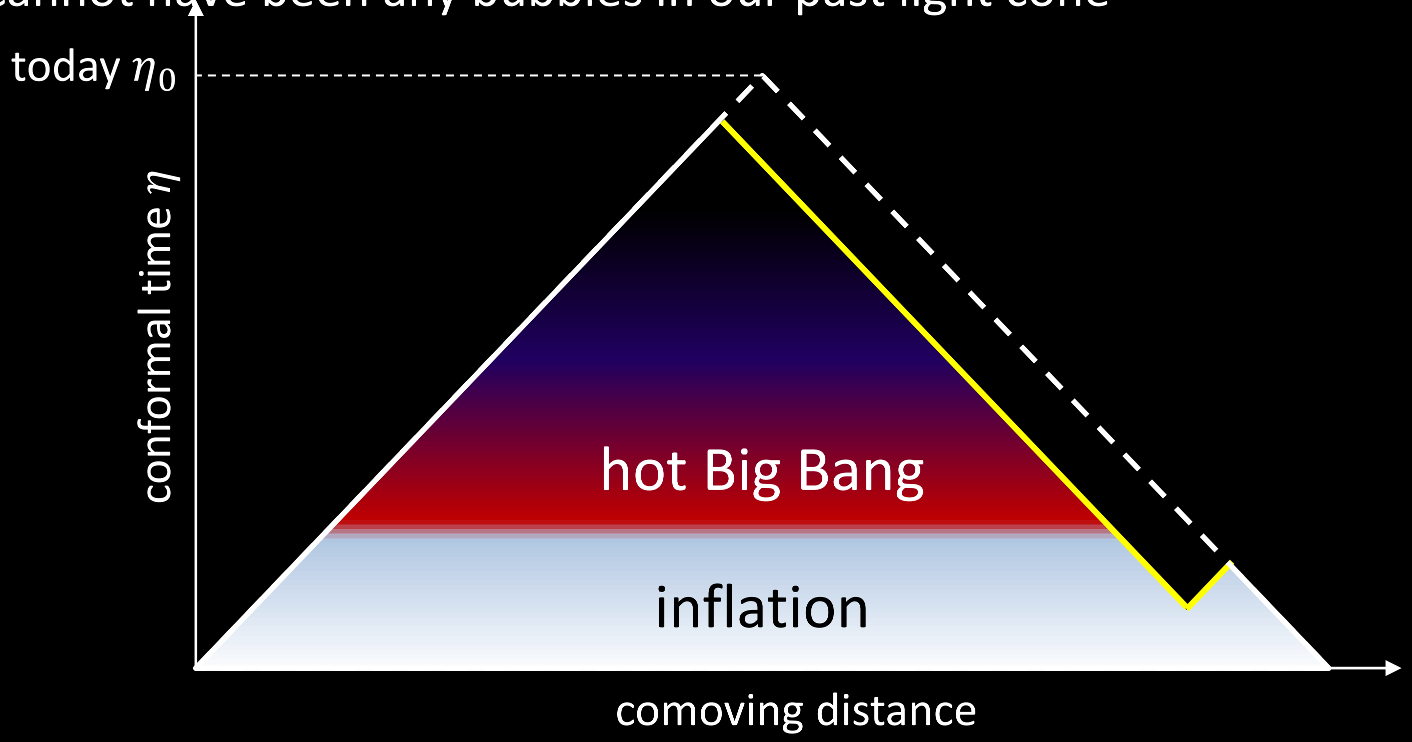
  (depending on Higgs and top masses)

  $\Rightarrow$ extremely slow rate $\Gamma \sim \mu_{\min}^4 e^{-B}$ - but is it slow enough?



Past Light Cone

- Assume: Bubbles grow at the speed of light and destroy everything they hit (see, however, De Luca, Kehagias & Riotto arXiv:2205.10240)
  $\Rightarrow$ There cannot have been any bubbles in our past light cone



Past Light Cone

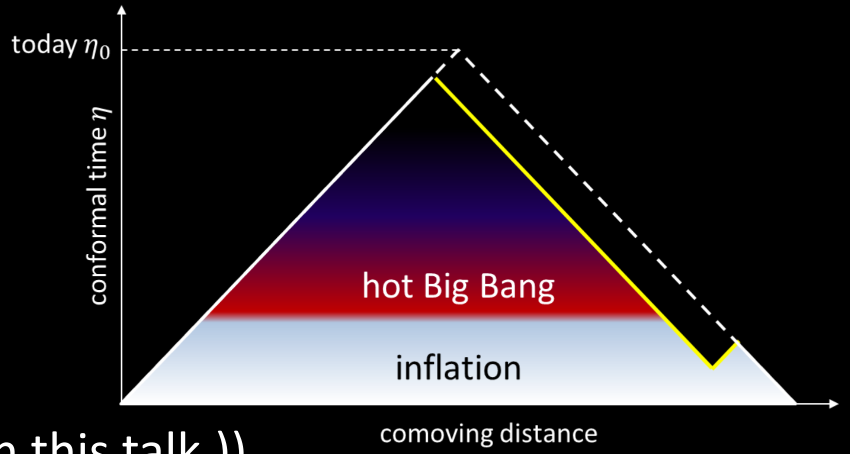
- Probability of no bubble in the past light cone:

$$P(\mathcal{N}=0)=\mathrm{e}^{-\langle\mathcal{N}\rangle},$$

where $\langle\mathcal{N}\rangle$ is the expected number of bubbles ($d\eta = dt/a$),

$$\langle\mathcal{N}\rangle=\frac{4\pi}{3}\int^{\eta_0} d\eta\, a(\eta)^4(\eta_0-\eta)^3\Gamma(\eta)$$

- Therefore, we must have $\langle\mathcal{N}\rangle \lesssim 1$
- Integrate over the whole history of the Universe: inflation, reheating, hot Big Bang, and late Universe
- (For anthropists: $\frac{d\langle\mathcal{N}\rangle}{dt}\Delta t \lesssim 1$)
- ((For quantum immortalists: You may go and make a coffee. There is nothing for you in this talk.))



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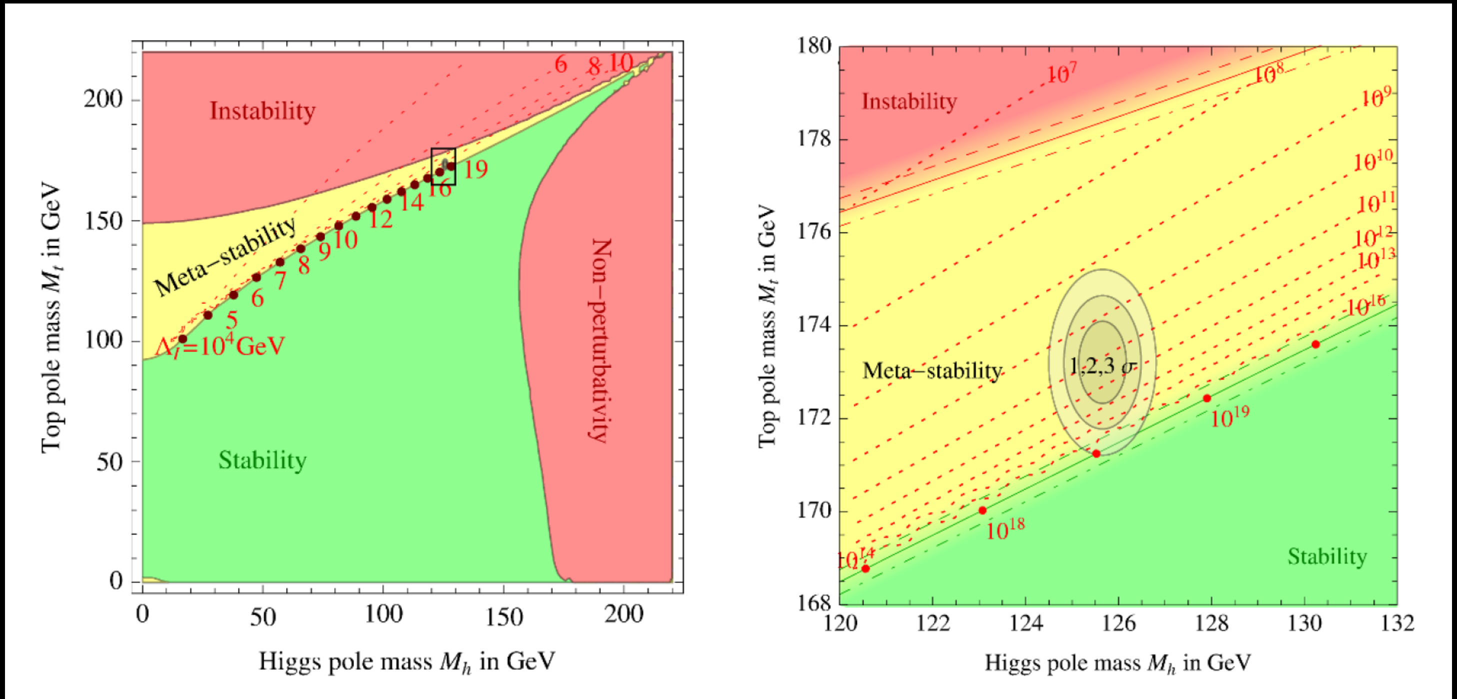
Late Universe Stability Bounds

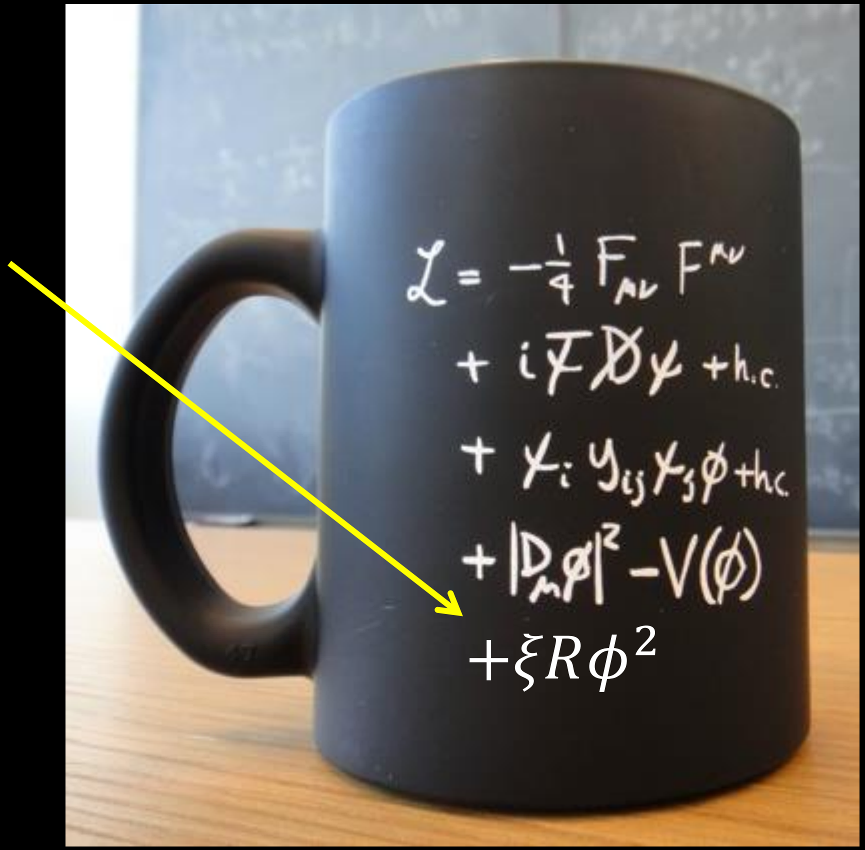


(Buttazzo et al. 2013)

- Number of bubbles in past lightcone: $\langle\mathcal{N}\rangle \approx 0.125\Gamma/H_0^4$
- If $\langle\mathcal{N}\rangle \ll 1$, no contradiction $\Rightarrow$ Metastability

Higgs-Curvature Coupling

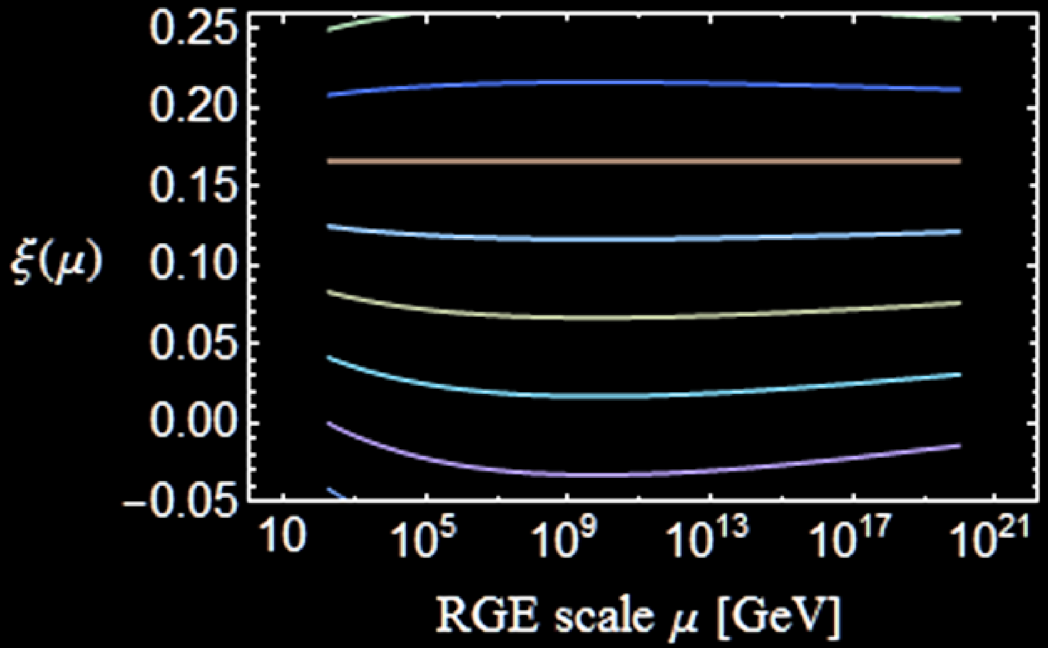
- Curved spacetime:
  $$\mathcal{L} = \mathcal{L}_{\rm SM} + \xi R \phi^\dagger \phi$$
  (Chernikov&Tagirov 1968)
- Symmetries allow one more renormalisable term: Higgs-curvature coupling ξ
- Required for renormalisability, runs with energy – Cannot be set to zero!
- Last unknown parameter in the Standard Model



Running ξ

$$\mu\frac{d\xi}{d\mu}=\left(\xi-\frac{1}{6}\right)\frac{12\lambda+6y_t^2-\frac{3}{2}g'^2-\frac{9}{2}g^2}{16\pi^2}$$

- Becomes negative if $\xi_{\rm EW}=0$
- Conformal value $\xi=1/6$
  RG invariant at 1 loop but not beyond



Measuring ξ

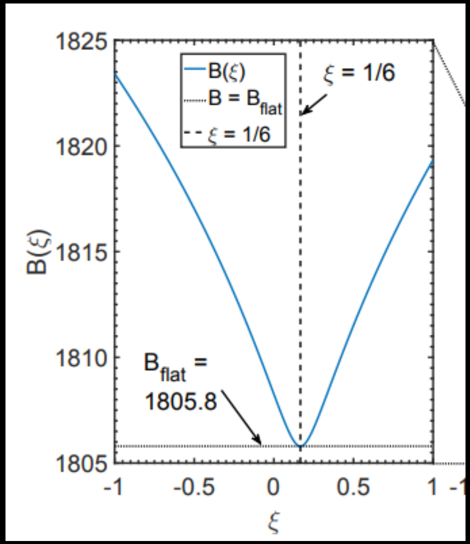
- Curved spacetime:

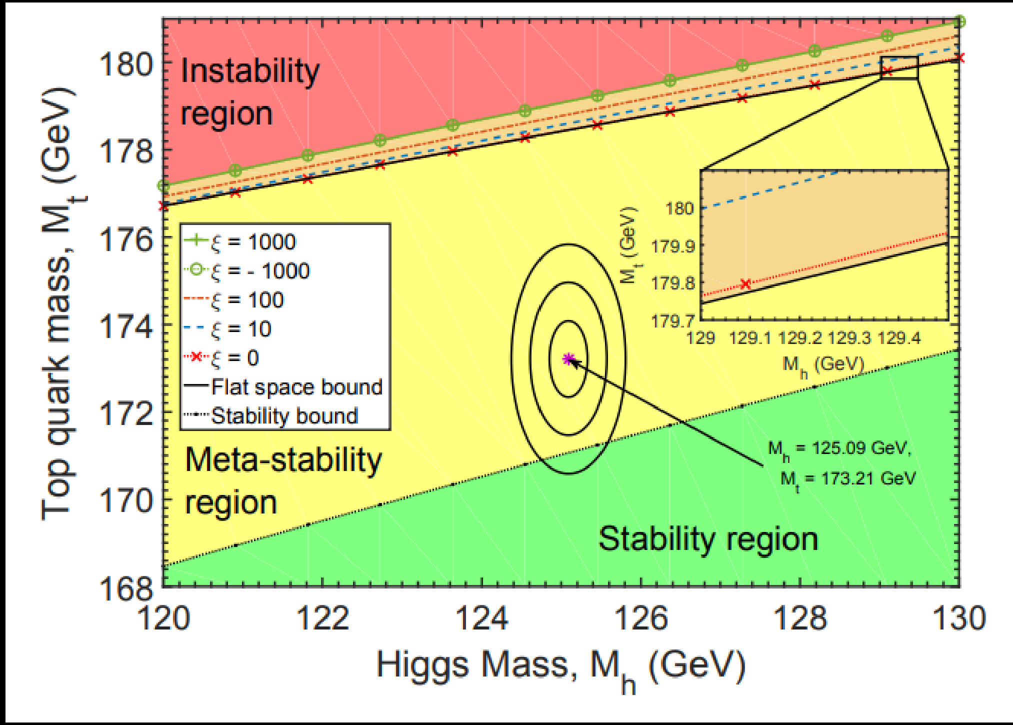
$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

- Ricci scalar R very small today
  $\Rightarrow$ Difficult to measure ξ
- Colliders: Suppresses Higgs couplings (Atkins&Calmet 2012)
  - LHC Bound $|\xi| \lesssim 2.6 \times 10^{15}$
  - Future (?) ILC: $|\xi| \lesssim 4 \times 10^{14}$
- In contrast, R was high in the early Universe

Late Universe Stability Bounds

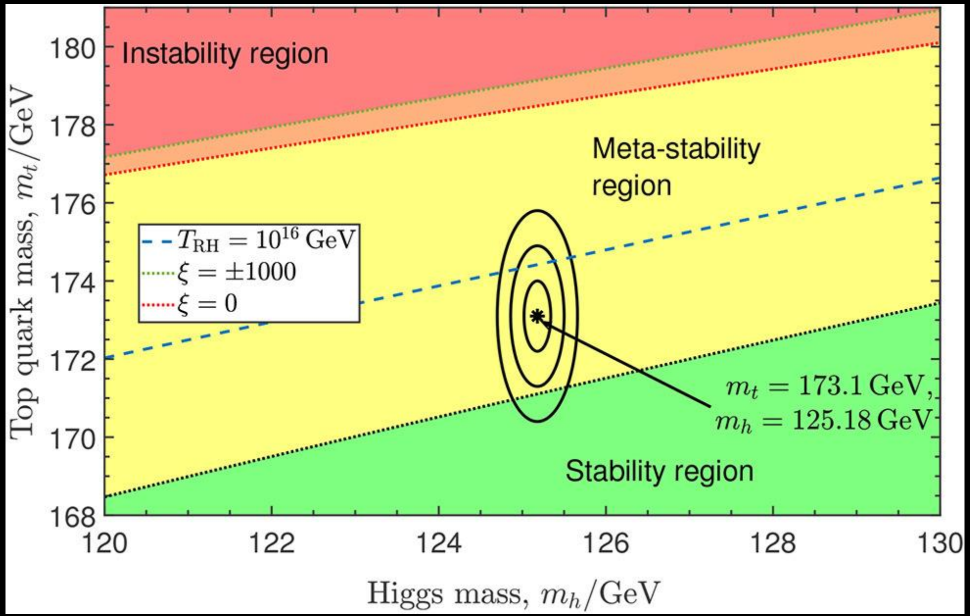
- Find the gravitational instanton by solving field + Einstein equations numerically (AR&Stopyra 2016)





Hot Big Bang

- High temperature: Higher bubble nucleation rate (Espinosa et al 2008)
- If reheat temperature $T_{\rm RH}$ is high enough, this dominates over late-time contribution
- Top mass bound (Delle Rose et al 2016):



(Markkanen, AR, Stopyra, 2018)

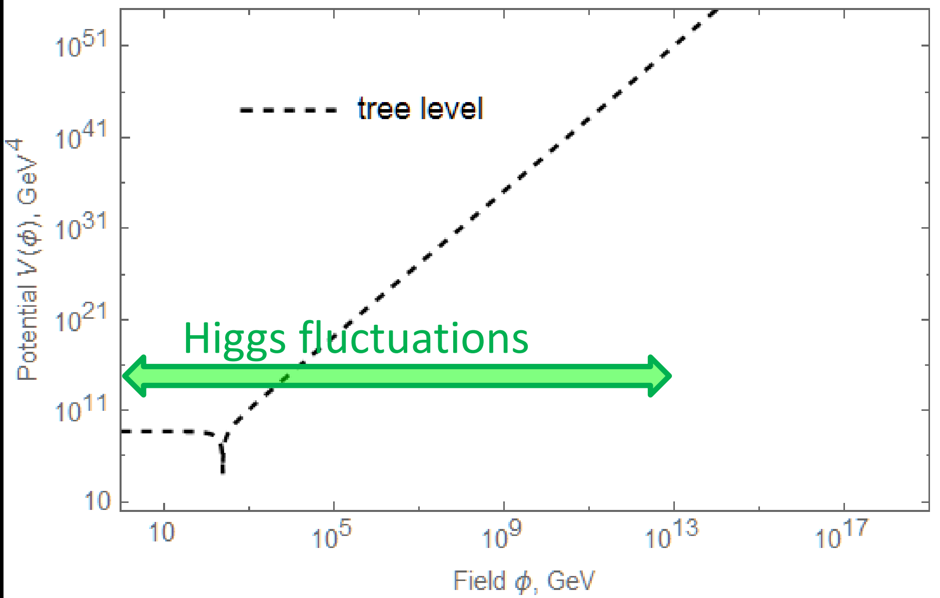
$$\frac{M_t}{\rm GeV} < 0.283\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.4612\frac{M_h}{\rm GeV}$$
$$+1.907\log_{10}\frac{T_{\rm RH}}{\rm GeV} + \frac{1.2\times 10^3}{0.323\log_{10}\frac{T_{\rm RH}}{\rm GeV} + 8.738}.$$

Higgs Fluctuations from Inflation

- Inflation: $H \lesssim 9 \times 10^{13}$ GeV (Planck+BICEP2 2015)
- Assume light Higgs, no direct coupling to inflaton
- Equilibrium field distribution (Starobinsky&Yokoyama 1994)

$$P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$$

- Tree-level potential
  $V(\phi) = \lambda(\phi^2 - v^2)^2$
- Nearly scale-invariant fluctuations with amplitude $\phi \sim \lambda^{-1/4} H$



Higgs Fluctuations from Inflation

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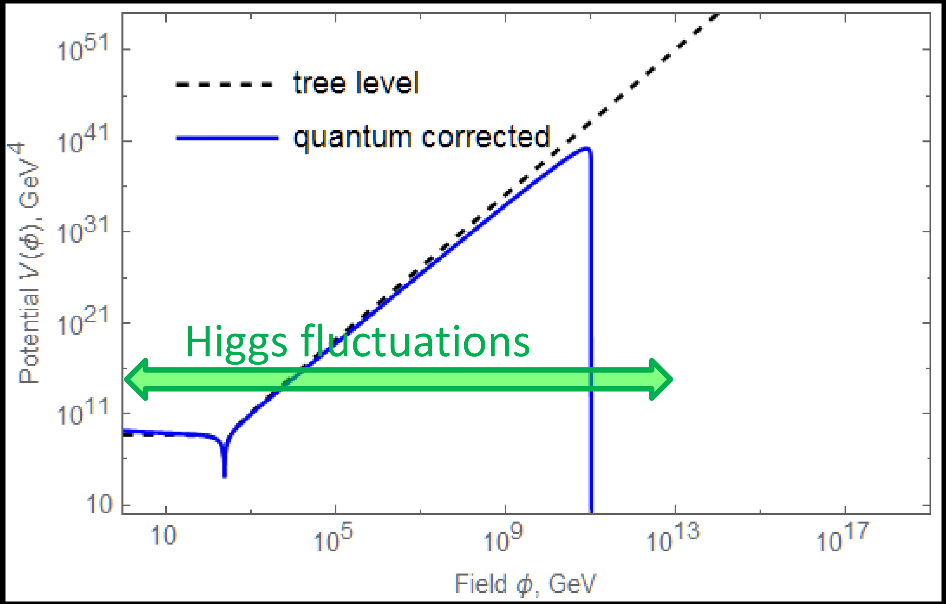
- Equilibrium $P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$
- Running λ:
  Fluctuations take the Higgs over the barrier if $H \gtrsim \phi_{\mathrm{bar}} \approx 10^{10}\mathrm{GeV}$
  (Espinosa et al. 2008; Lebedev&Westphal 2013; Kobakhidze&Spencer-Smith 2013; Fairbairn&Hogan 2014; Hook et al. 2014)
- Does this imply an upper limit on the scale of inflation $H \lesssim 10^{10}\mathrm{GeV}$?



Spacetime Curvature

- Effective Higgs mass term $m_{\mathrm{eff}}^2(t) = m_{\mathrm{H}}^2 + \xi R(t)$
- Ricci scalar in FRW spacetime:

$$R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = 3(1-3w)H^2$$

  - Radiation dominated $w = 1/3$ $R = 0$
  - Matter dominated $w = 0$ $R = 3H^2$
  - Inflation / de Sitter $w = -1$ $R = 12H^2$

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Higgs During Inflation

- Inflation: Constant $R = 12H^2$
- Effective mass term

$$m_{\mathrm{eff}}^2 = m_{\mathrm{H}}^2 + \xi R = m_{\mathrm{H}}^2 + 12\xi H^2$$

- Tree level: (Espinosa et al 2008)
  - $\xi > 0$: Increases barrier height
    Makes the low-energy vacuum more stable
  - $\xi < 0$: Decreases barrier height
    Makes the low energy vacuum less stable
- H contributes to loop corrections:
  For $H \gg \phi$, the RGI scale is $\mu \approx H$

$$V(\phi) \approx \lambda(H)\phi^4$$

⇨ No barrier if $H \gtrsim 10^{10}$ GeV (HMNR 2014)

Effective Potential in Curved Spacetime

- One-loop computation in de Sitter:

$$V_{\rm SM}^{\rm eff}(\varphi(\mu)) = -\frac{1}{2}m^2(\mu)\varphi^2(\mu) + \frac{\xi(\mu)}{2}R\varphi^2(\mu) + \frac{\lambda(\mu)}{4}\varphi^4(\mu) + V_\Lambda(\mu) - 12\kappa(\mu)H^2 + \alpha(\mu)H^4$$
$$+\frac{1}{64\pi^2}\sum_{i=1}^{31}\left\{n_i\mathcal{M}_i^4(\mu)\left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right) - d_i\right] + n_i' H^4 \log\left(\frac{|\mathcal{M}_i^2(\mu)|}{\mu^2}\right)\right\}. \quad (5.3)$$

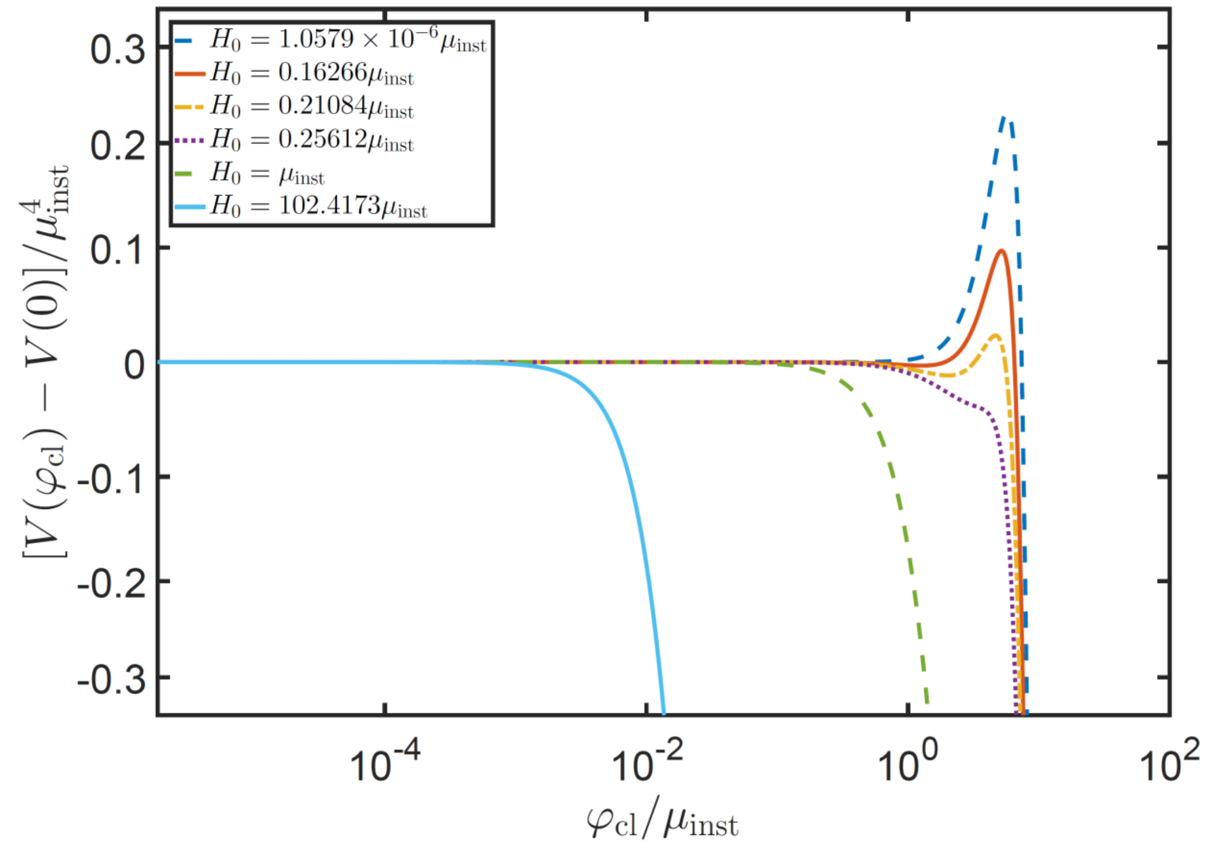
Ψ	i	n_i	d_i	n_i'	$\mathcal{M}_i^2$
$W^\pm$	1	2	3/2	−34/15	$m_W^2 + H^2$
	2	6	5/6	−34/5	$m_W^2 + H^2$
	3	−2	3/2	4/15	$m_W^2 - 2H^2$
Z^0	4	1	3/2	−17/15	$m_Z^2 + H^2$
	5	3	5/6	−17/5	$m_Z^2 + H^2$
	6	−1	3/2	2/15	$m_Z^2 - 2H^2$
q	7 − 12	−12	3/2	38/5	$m_q^2 + H^2$
l	13 − 15	−4	3/2	38/15	$m_l^2 + H^2$
h	16	1	3/2	−2/15	$m_h^2 + 12(\xi - 1/6)H^2$
χ_W	17	2	3/2	−4/15	$m_\chi^2 + \zeta_W m_W^2 + 12(\xi - 1/6)H^2$
χ_Z	18	1	3/2	−2/15	$m_\chi^2 + \zeta_Z m_Z^2 + 12(\xi - 1/6)H^2$
c_W	19	−2	3/2	4/15	$\zeta_W m_W^2 - 2H^2$
c_Z	20	−1	3/2	2/15	$\zeta_Z m_Z^2 - 2H^2$

Ψ	i	n_i	d_i	n_i'	$\mathcal{M}_i^2$
γ	21	1	3/2	−17/15	H^2
	22	3	5/6	−17/5	H^2
	23	−1	3/2	2/15	$-2H^2$
g	24	8	3/2	−136/15	H^2
	25	24	5/6	−136/5	H^2
	26	−8	3/2	16/15	$-2H^2$
ν	27 − 29	−2	3/2	19/15	H^2
c_γ	30	−1	3/2	2/15	$-2H^2$
c_g	31	−8	3/2	16/15	$-2H^2$

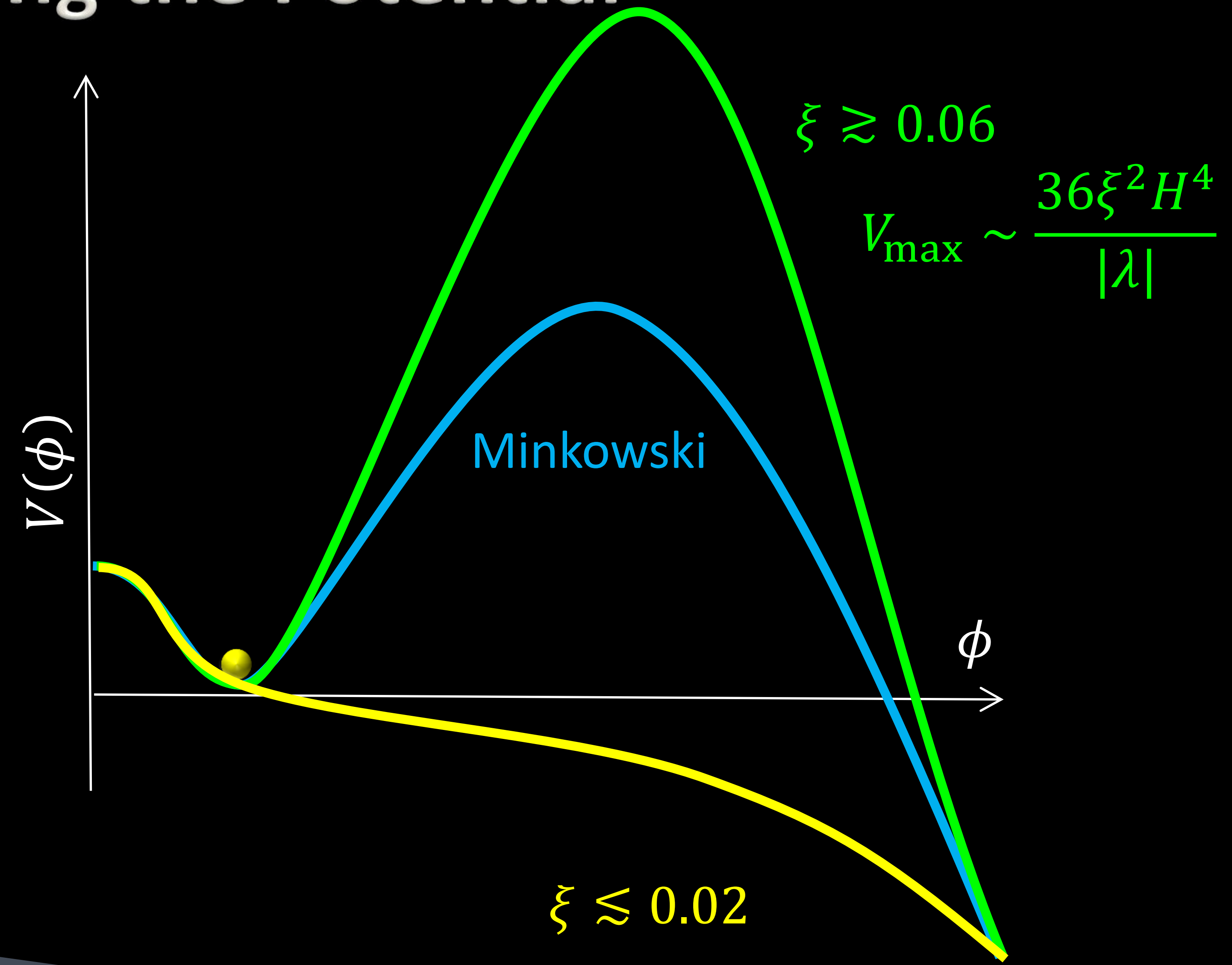
(MNRS 2018)

Potential in Curved Spacetime

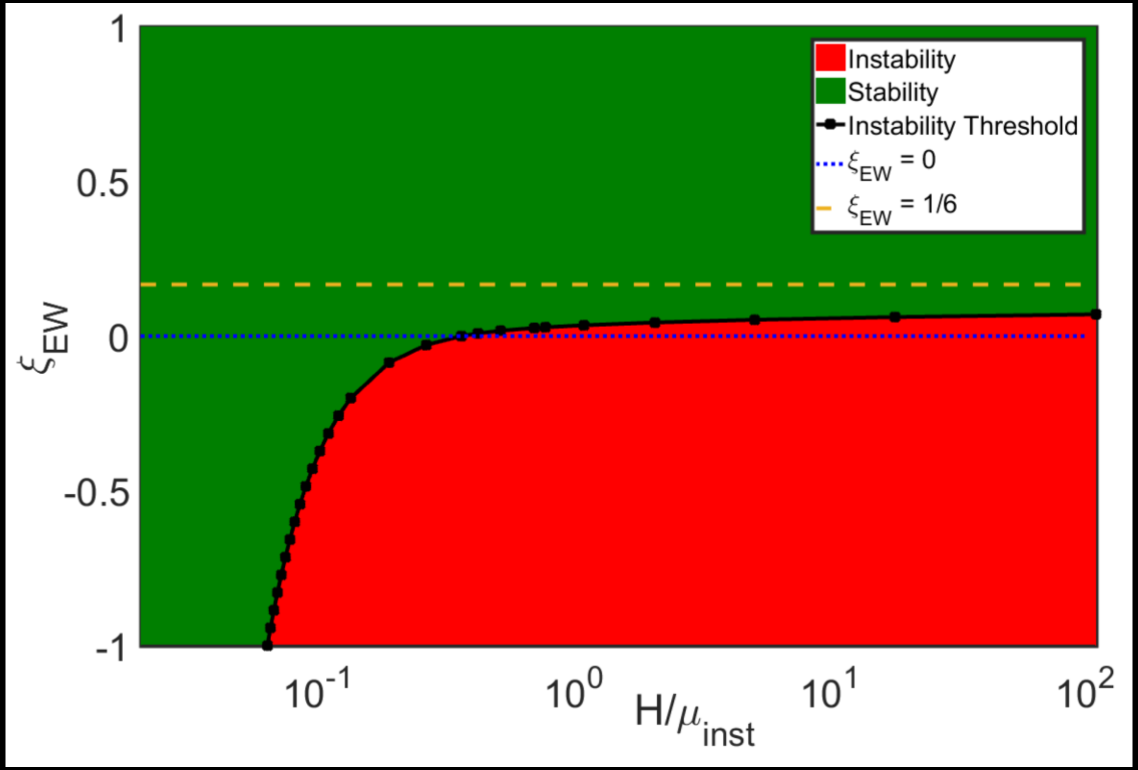
- One-loop computation for $\xi = 0$ (in units of $\mu_{\rm inst} \approx 6.6 \times 10^9$ GeV)
- When spacetime curvature is high, the barrier disappears (MNRS 2018)



(De-)Stabilising the Potential



(De)Stabilising the Potential

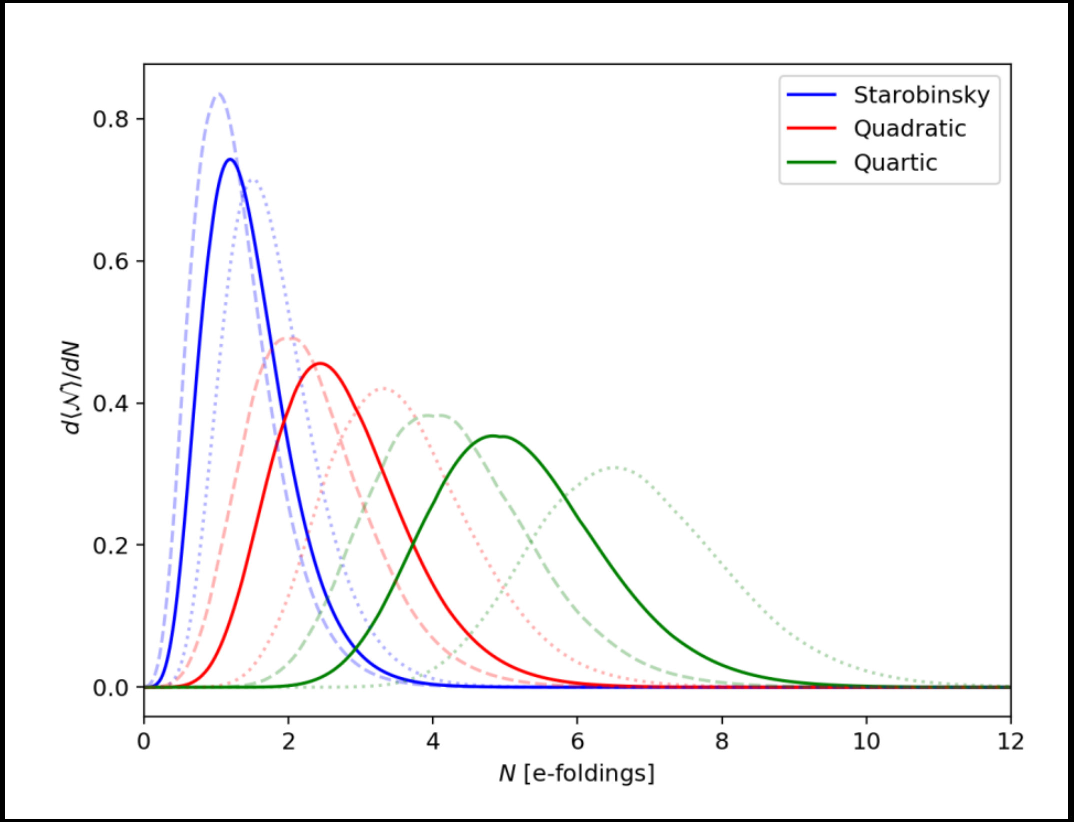


(MNRS 2018)

- If $H \gtrsim \mu_{\rm inst} = 6.6 \times 10^9 \rm GeV$ and there is no new physics, vacuum stability during inflation requires $\xi \gtrsim 0$

Time-Dependent Hubble Rate

- In real inflationary models, H depends on time: Affects decay rate Γ and volume of past light cone
- (Mantziris, Markkanen & AR, 2021): Consider three single-field inflation models
- Bubbles most likely produced during the last few e-foldings



Time-Dependent Hubble Rate

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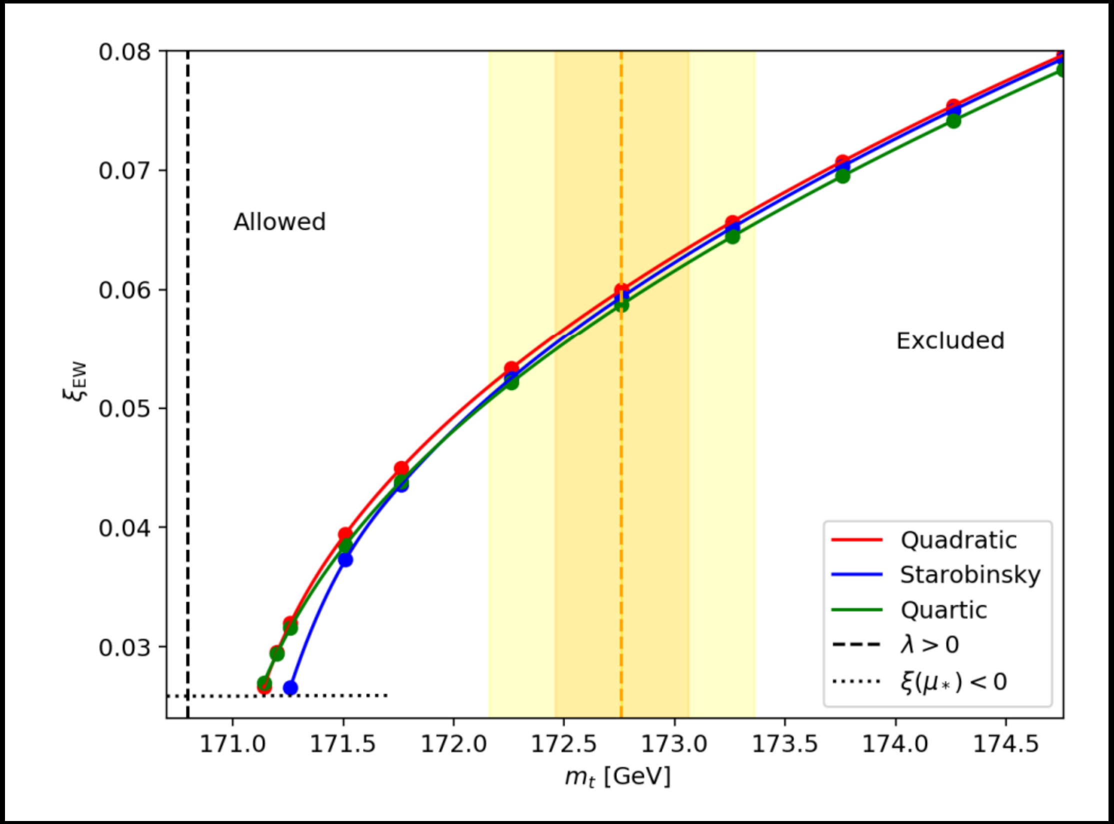
- (Mantziris, Markkanen & AR, 2021): Consider three single-field inflation models
- Bubbles most likely produced during the last few e-foldings
- Stability requires

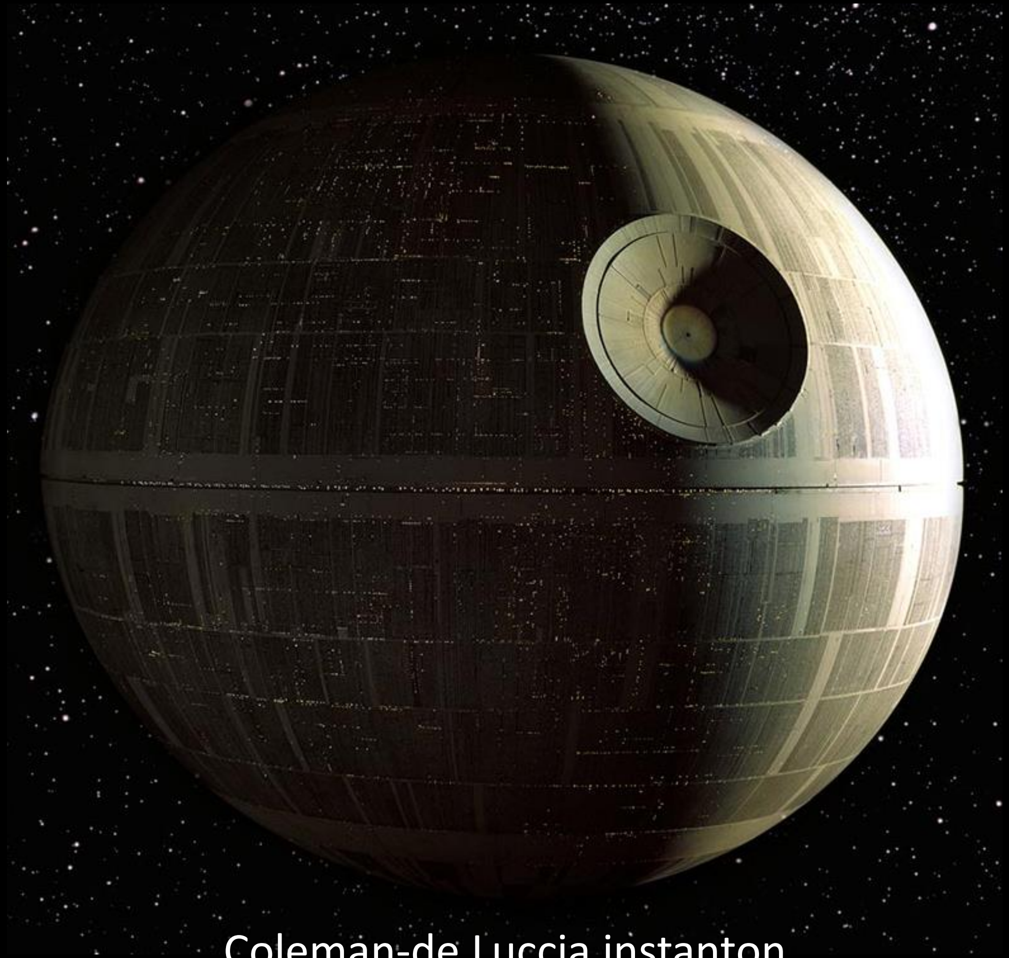
$$\xi \gtrsim 0.06$$

in all three model



Quantum Tunneling

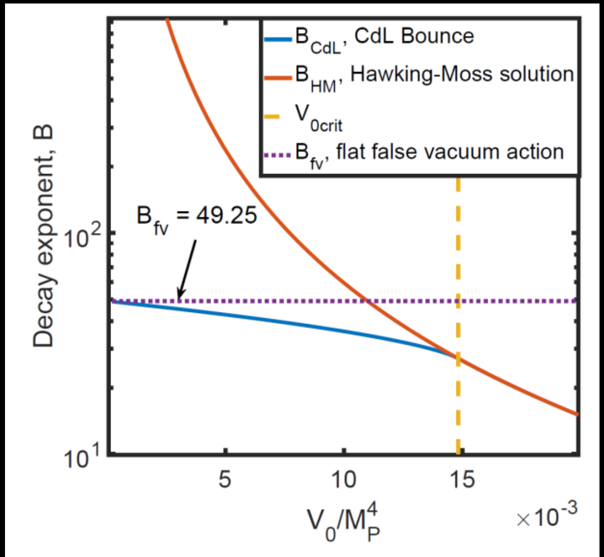
Hawking-Moss instanton



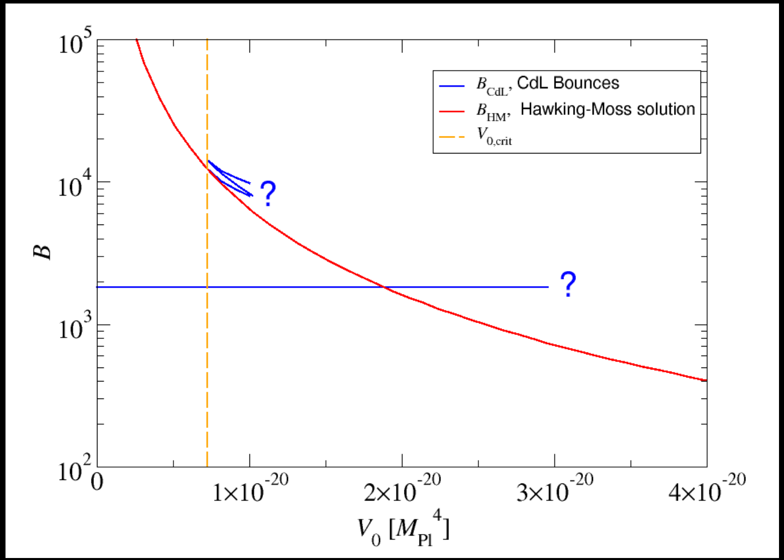
Coleman-de Luccia instanton

Quantum Tunneling

Toy model

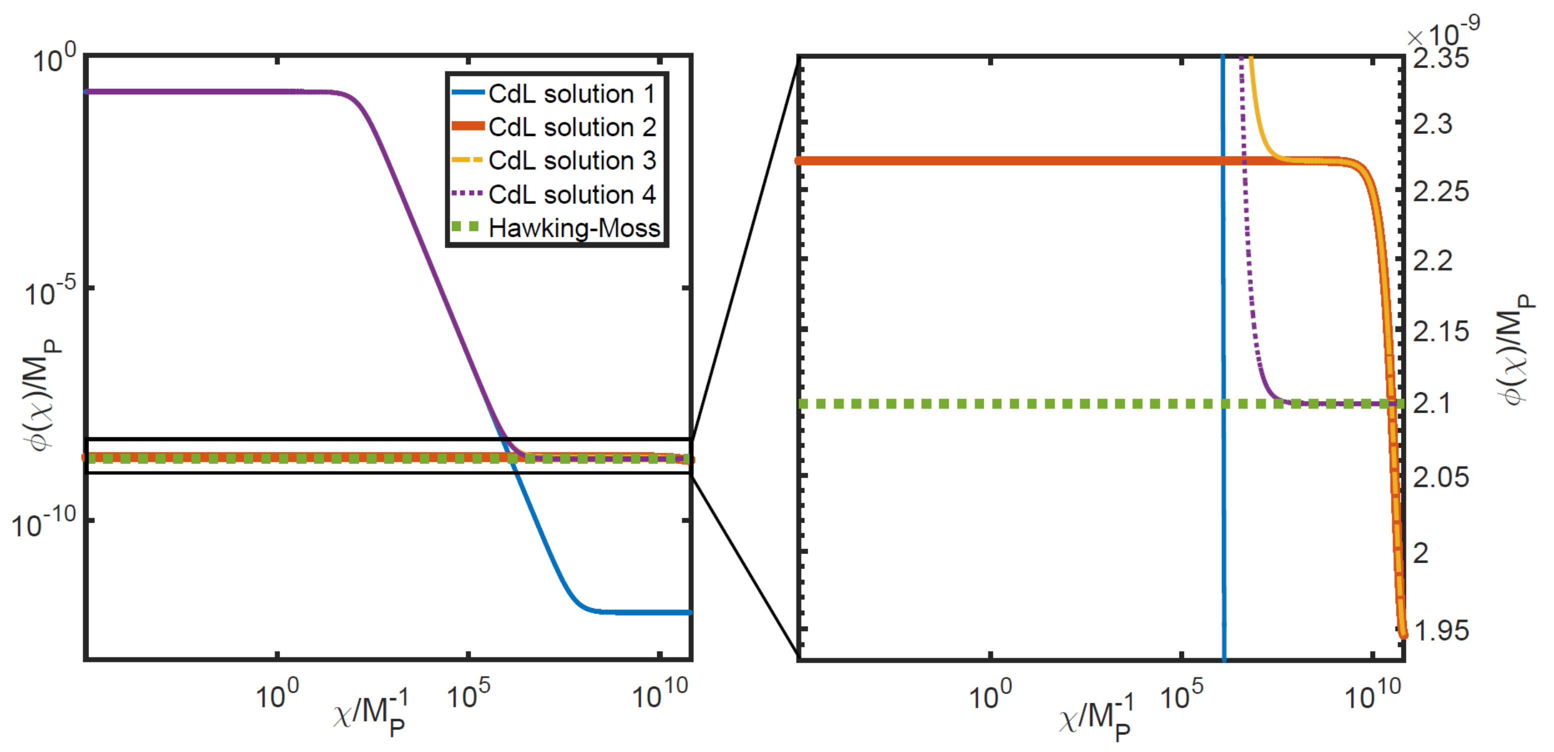


Standard Model



- Multiple coexisting solutions (AR&Stopyra, PRD 2018)
- Quantum (Coleman-de Luccia) tunnelling rate $\Gamma \sim e^{-B}$ nearly constant until Hawking-Moss starts to dominate $\Rightarrow$ Always the relevant process for the constraint

Multiple Solutions



(AR&Stopyra, PRD 2018)

End of Inflation

- Reheating: Inflation ($R = 12H^2$) → radiation ($R = 0$)

$$R(t) = \frac{2m^2\chi^2 - \dot{\chi}^2}{M_{\rm Pl}^2}$$

- Effective Higgs mass $m_{\rm eff}^2 = m_{\rm H}^2 + \xi R$ oscillates:
  - Parametric resonance ("Geometric preheating")
    (Bassett&Liberati 1998, Tsujikawa et al. 1999)
- R goes negative when $\chi \sim 0$
  - If $\xi > 0$, Higgs becomes tachyonic (HMNR 2015)
  - Exponential amplification

$$\langle\phi^2\rangle_H \sim \frac{2}{3\sqrt{3}\xi}\left(\frac{H}{2\pi}\right)^2 e^{\frac{2\sqrt{\xi}\chi_{\rm ini}}{M_{\rm Pl}}} \sim \frac{H^2}{\xi} e^{2\sqrt{\xi}}$$

Vacuum Decay at the End of Inflation

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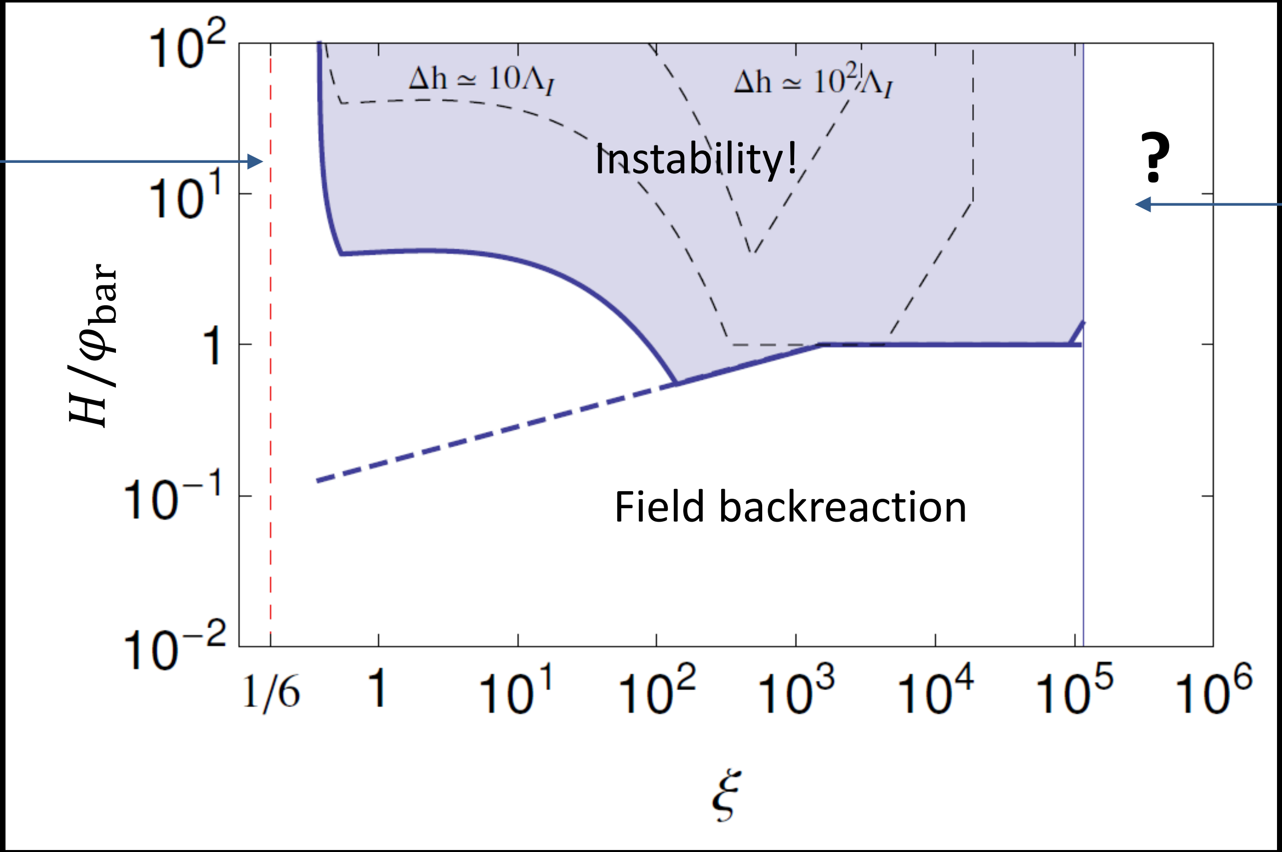
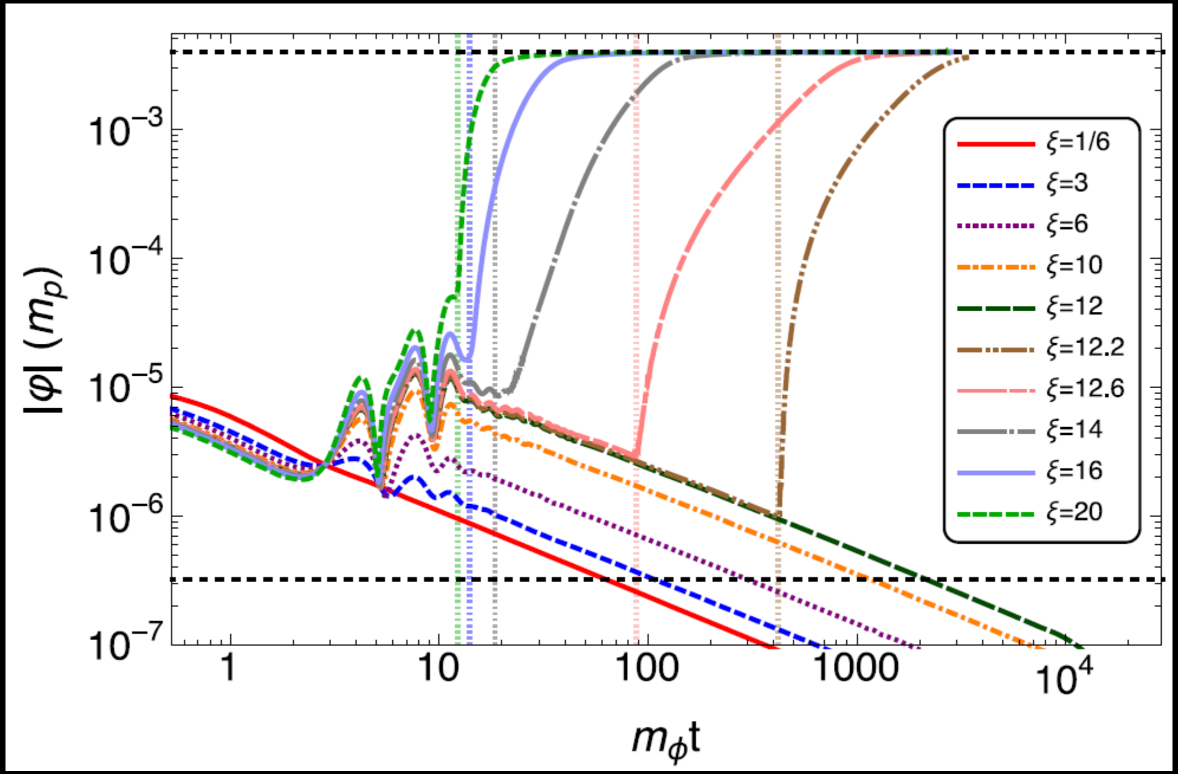
Not enough growth



Becomes nonlinear

(HMNR 2015)

Lattice Simulations



true vacuum

Figueroa, AR & Torrenti, 2018

- $V(\chi) = \frac{1}{2}m^2\chi^2, M_{\text{top}} = 172.12\ \text{GeV}$

Instability Time

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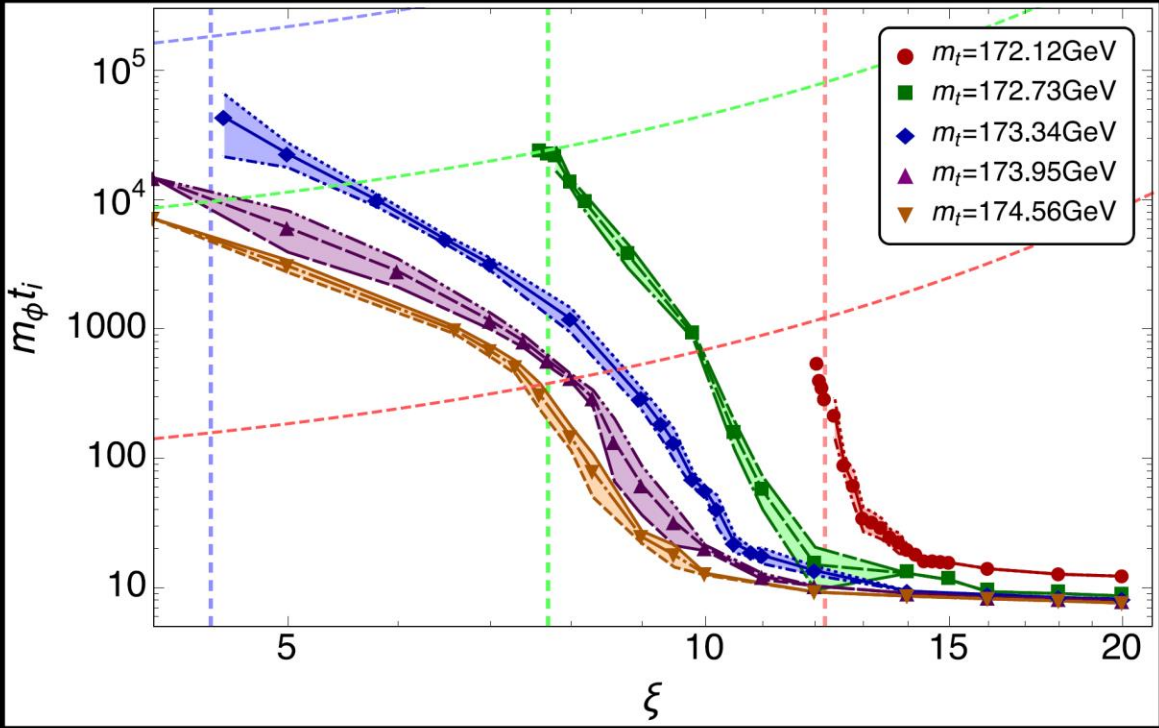
- Stability depends on top mass and speed of reheating
- $M_{\mathrm{top}} = 173.34\ \mathrm{GeV}$:
  vacuum survival until $t = 100/m$
  requires $\xi \lesssim 9$
- Li et al (arXiv:2206.05926):
  Much stronger bound $\xi \lesssim 2$
  in Starobinsky inflation



Figueroa, AR & Torrenti, 2018

Constraints on ξ

- Minimal scenario:
  Standard Model + $m^2\chi^2$ chaotic inflation,
  no direct coupling to inflaton

$$0.06 \lesssim \xi \lesssim 9$$

- 15 orders of magnitude stronger than the LHC bound

$$|\xi| \lesssim 2.6 \times 10^{15}$$

- Caveats:
  - Assumes no direct coupling to inflaton (see, e.g., Ema et al. 2016, 2017)
    – Would still need $|\xi| \lesssim O(1)$
  - Assumes no new physics
    – Could stabilise potential altogether, or destabilise further
  - Assumes high scale inflation $H \gtrsim 10^9$ GeV