A novel technique for the description of gauge-field production during axion inflation

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Overview

Magnetic fields in Universe

Magnetic fields exist in all astrophysical objects on all observable scales of the visible Universe:

- **Neutron stars**: $10^{12} - 10^{15}$ G
- **Stars**: $1 - 10^3$ G
- **Planets**: ~ 1 G
- **Galaxies**: $\sim 10^{-5} - 10^{-6}$ G
- **Galaxy clusters**: $\sim 10^{-6} - 10^{-7}$ G

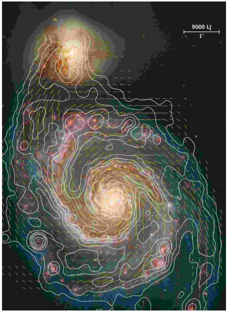


Figure: Optical and radio image of the Whirlpool galaxy M51 with MF configuration. Credit MPIfR Bonn

Since 2010, there is evidence of MF detection also on a cosmological scale — **in the cosmic voids**: $10^{-16}\,\mathrm{G} \lesssim B_0 \lesssim 10^{-10}$ G

Existing constrains on MF

- Constraint from **below** is from the analysis of γ-radiation of blazars: $B \geq 10^{-16}$ G.
  [Tavecchio *et al.*, MNRAS **406**; Ando & Kusenko, Astrophys. J. Lett. **722**; Neronov & Vovk, Science **328**]
- Constrains from **above** follow from the analysis of the anisotropy spectrum of CMB and UHECR deviation: $B \leq 10^{-10}$ G.
  [Neronov et al., arXiv:2112.08202]

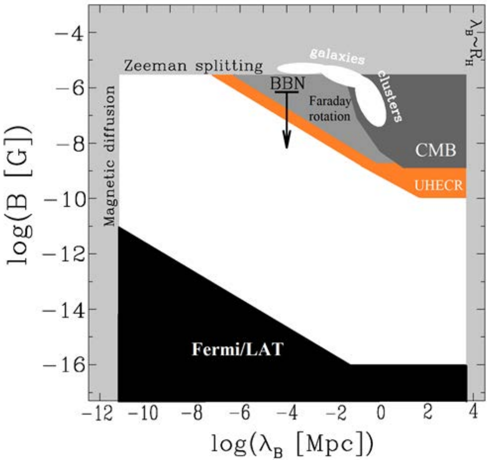
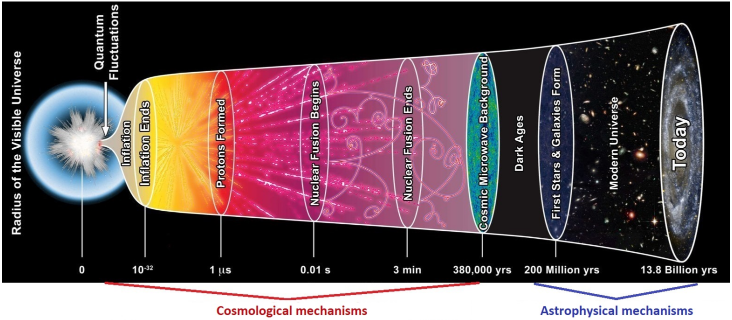


Figure: Summary of onstrains on B and λ_B [Neronov&Vovk, Science **328**, 73 (2010)] and [Neronov et al., arXiv:2112.08202]

When and how could MFs arise in the Universe?

There are two different hypotheses for the generation of seed MF:

- **Astrophysical** — generation during the structures formation: Biermann battery, adiabatical contraction, dynamos, ...
- **Cosmological** — generation in a very early Universe: phase transitions, reheating, inflation, ...



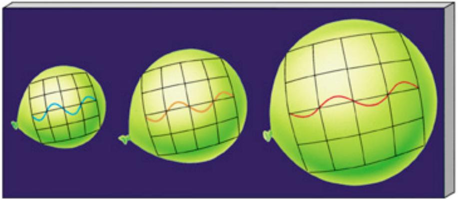
How to generate the gauge field?

- FLRW metric is **conformally flat**: $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$.
- The action for free gauge field is **conformally invariant**:

$$S_{GF} = -\frac{1}{4}\int d^4x\,\sqrt{-g}\,g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} = \left|\begin{array}{c}\sqrt{-g} = a^4,\\ g^{\mu\nu} = a^{-2}\eta^{\mu\nu}\end{array}\right| =$$

$$= -\frac{1}{4}\int d^4x\,\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$ – the same as in Minkovski spase.

- Solutions of the equations of motion are plain waves, the frequency of which undergoes only a redshift.
- **Generation of the gauge field in the early Universe requires a violation of conformal invariance of the action!!!**



Axion inflation

- For successful slow-roll inflation we need the inflaton potential to be **sufficiently flat**. However, the **radiative corrections** may **break** this **flatness** and spoil inflation.
- This usually happens unless the flatness of the potential is **protected by a shift symmetry** $\phi \to \phi + \text{const}$.
  E.g., natural inflation model [Freese et al., PRL **65** (1990)]
- Interaction terms with matter fields should also be shift-symmetric. The simplest choice for the gauge field is [Garretson et al., PRD **46** (1992)]

$$S_{GF} = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\beta}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right].$$

- Such scalar field ϕ is often called **axion** (or axion-like field).

Parity violation

Axion-like coupling leads to the effective current in the Maxwell equation:

$$\boldsymbol{j} = \alpha \boldsymbol{B}, \qquad \alpha \propto \dot{\phi}.$$

Similar term also appears

- due to the *chiral magnetic effect* in a chirally-asymmetric plasma of fermions [Vilenkin, PRD **22** (1980)]

$$\alpha \propto \mu_5 = \mu_R - \mu_L.$$

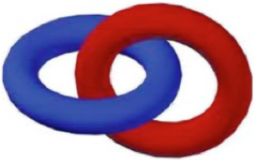
- in the *mean-field dynamo* theory [Steenbeck et al., Z. Naturforsch. **21** (1966)]

$$\alpha \propto \langle \boldsymbol{v} \cdot [\boldsymbol{\nabla} \times \boldsymbol{v}] \rangle.$$

Note that in all three cases α is a **pseudoscalar**. If it is nonzero, the **parity** symmetry is **violated**.

Phenomenology of the axion inflation

- The current $\boldsymbol{j} = \alpha \boldsymbol{B}$ leads to the **tachyonic instability** for only one circular polarization of gauge fields
  $\Rightarrow$ **helical gauge fields are generated** [Anber&Sorbo, JCAP **10** (2006)].



Magnetic helicity

$$\mathcal{H} = \int \boldsymbol{A} \cdot \boldsymbol{B}\, d^3x \neq 0.$$

- Helical **gravitational waves** and **non-Gaussianities** in scalar power spectrum are generated [Barnaby et al., PRD **85** (2012)].
- Axion coupling leads to **effective reheating** of the Universe [Adshead et al., JCAP **12** (2015)].
- Decaying helical GF after inflation may lead to the **baryon asymmetry of the Universe** due to chiral anomaly [Giovannini&Shaposhnikov, PRD **57** (1998); Kamada&Long, PRD **94** (2016)].

Schwinger effect during inflation

Strong electric component is generated as well leading to the Schwinger pair production [Schwinger, PR **82** (1951)].

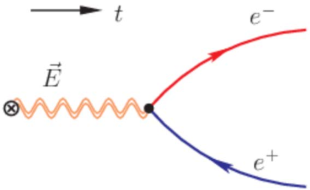
Studied analytically [Domcke, JHEP **02** (2020)]

- De Sitter space
- Constant and collinear electric and magnetic fields
- Strong-field regime

$$E_{\rm th} \triangleq \frac{2m_e c^3}{e\hbar} \simeq 10^{18}\ \mathrm{V/m}.$$

$$|e\boldsymbol{E}| \gg H^2$$

$$t_E = \frac{1}{\sqrt{|eE|}} \ll H^{-1} = t_H$$



$$\boldsymbol{j} = \frac{(e|Q|)^3}{6\pi^2}\frac{|B|\boldsymbol{E}}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eQE|}\right) = \sigma(|E|,|B|)\ \mathbf{E}.$$

Gauge-field generation during axion inflation

$$S = \int d^4x\sqrt{-g}\Big[\underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)}_{\text{pseudoscalar inflaton}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free gauge field}} - \underbrace{\frac{\beta}{4M_p}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{axion coupling GF with inflaton}} + \underbrace{\mathcal{L}_{ch}(A_\nu,\chi)}_{\text{charged field (Schwinger eff.)}}\Big]$$

Equations of motion:

- Friedmann eq.: $H^2 = \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2\rangle + \rho_\chi\right]$
- Klein-Gordon eq.: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{\beta}{M_p}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$
- Maxwell equations: $\dot{\boldsymbol{E}} + 2H\boldsymbol{E} - \frac{1}{a}\mathrm{rot}\,\boldsymbol{B} + \frac{\beta}{M_p}\,\dot{\phi}\,\boldsymbol{B} + \boldsymbol{J}_\chi = 0,$
  $\dot{\boldsymbol{B}} + 2H\boldsymbol{B} + \frac{1}{a}\mathrm{rot}\,\boldsymbol{E} = 0,\ \mathrm{div}\,\boldsymbol{E} = 0,\ \mathrm{div}\,\boldsymbol{B} = 0.$
- Eq. for charged particles: $\dot{\rho}_\chi + 4H\rho_\chi = \boldsymbol{J}_\chi\cdot\boldsymbol{E}.$

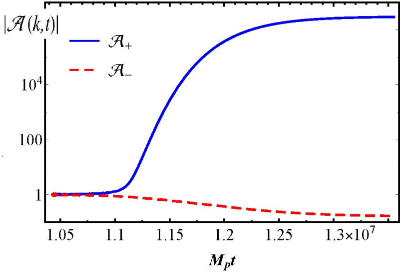
Standard mode-by-mode approach

If there is (1) no backreaction, (2) no Schwinger effect, Fourier modes of the gauge field evolve independently (on the known inflationary background)

Equation for the mode function with circular polarization $\lambda = \pm$

$$\mathcal{A}''_{\pm}(\eta, k) + \left[k^2 \pm \frac{2k\xi}{\eta}\right]\mathcal{A}_{\pm}(\eta, k) = 0,$$

$$\xi = \frac{\beta\dot{\phi}}{2M_p H}.$$



Only one of the two polarizations is amplified. Therefore, the generated MF will be **helical**!

$$\mathcal{H} \sim \int k^3(|\mathcal{A}_+|^2 - |\mathcal{A}_-|^2)dk \neq 0.$$

$$\rho_B = \sum_{\lambda=\pm1} \int_0^{k_h} \frac{dk}{k} \frac{k^5}{4\pi^2 a^4} |\mathcal{A}_\lambda(t,k)|^2, \quad \rho_E = \sum_{\lambda=\pm1} \int_0^{k_h} \frac{dk}{k} \frac{k^3}{4\pi^2 a^2} |\dot{\mathcal{A}}_\lambda(t,k)|^2$$

Gradient expansion formalism

We introduce an infinite set of quantities:

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{E}\rangle, \qquad \mathcal{G}^{(n)} = -\frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle,$$

$$\mathcal{B}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{B}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle.$$

They satisfy the following chain of equations:

$$\dot{\mathcal{E}}^{(n)} + [(n+4)H + 2\sigma]\,\mathcal{E}^{(n)} - 2(\beta/M_p)\dot{\phi}\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{E}}^{(n)}]_b,$$

$$\dot{\mathcal{G}}^{(n)} + [(n+4)H + \sigma]\,\mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - (\beta/M_p)\dot{\phi}\,\mathcal{B}^{(n)} = [\dot{\mathcal{G}}^{(n)}]_b,$$

$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{B}}^{(n)}]_b.$$

Thus, we trade an **infinite number of Fourier-modes** for an **infinite set** of scalar **functions** in the coordinate space – **what's the gain?**

The chain can be truncated!

Boundary terms

Any function $X^{(n)}$ has the following spectral decomposition:

$$X = \int_0^{k_\mathrm{h}(t)} \frac{dk}{k} \frac{dX}{d\ln k}.$$

There are two sources of time dependence:

- The spectral density depends of $\mathcal{A}_\lambda(k, t)$ and its derivatives.
- The upper integration limit $k_\mathrm{h}(t)$ is time dependent!
  E.g., w/o Schwinger effect, $k_\mathrm{h}(t) = 2a(t)H(t)|\xi(t)|$.

Boundary terms describe the latter time dependence, i.e., they take into account the fact that the **number of physically relevant modes grows in time** during inflation.

$$(\dot{X})_b = \frac{dX}{d\ln\, k}\bigg|_{k=k_\mathrm{h}} \cdot \frac{d\ln k_\mathrm{h}}{dt}.$$

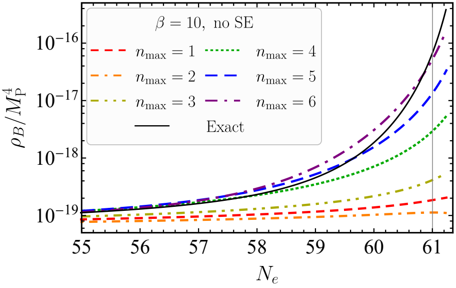
They are expressed in terms of Whittaker functions.

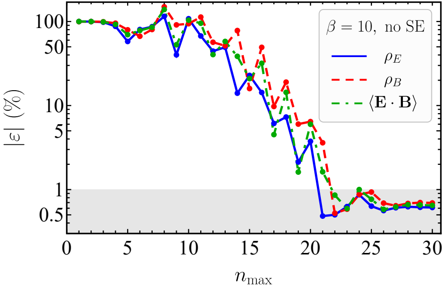
Truncation and convergence

For large enough n, the integral is dominated by the upper integration limit:

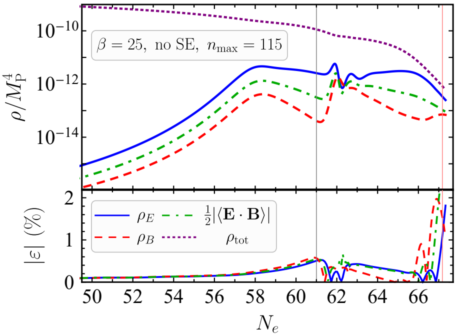
$$\mathcal{X}^{(n)} = \sum_{\lambda=\pm} \lambda^n \int_0^{k_\mathrm{h}} \left(\frac{k}{a}\right)^{n+4} \mathcal{X}_\lambda(t,k)\, dk, \qquad \Rightarrow \qquad \boxed{\mathcal{X}^{(n+2)} \overset{n\gg 1}{\approx} \left(\frac{k_\mathrm{h}}{a}\right)^2 \mathcal{X}^{(n)}}$$

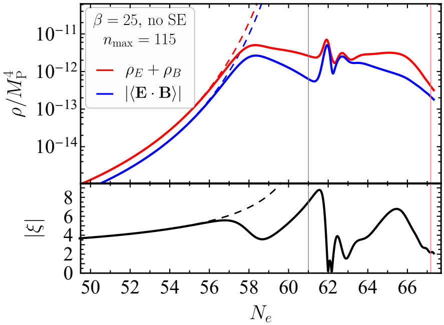
This allows to truncate the chain at some finite order.





Backreaction regime





Qualitative features of BR regime:

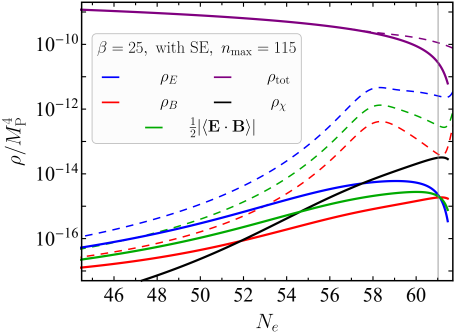
- Inflation is extended
- Rolling of the inflaton is slowed down
- Oscillatory behavior of the generated fields

Reproduced the results of iterative mode-by-mode approach of Ref. [Domcke et al., JCAP **09** (2020)].

Impact of the Schwinger effect

Finite conductivity due to Schwinger effect damps the Fourier modes inside the horizon:

$$\mathcal{A}_\lambda(t,k) = \sqrt{\Delta(t)} f_\lambda(t,k), \qquad \Delta(t) \equiv \exp\left(- \int_{-\infty}^{t} \sigma(t')dt' \right).$$

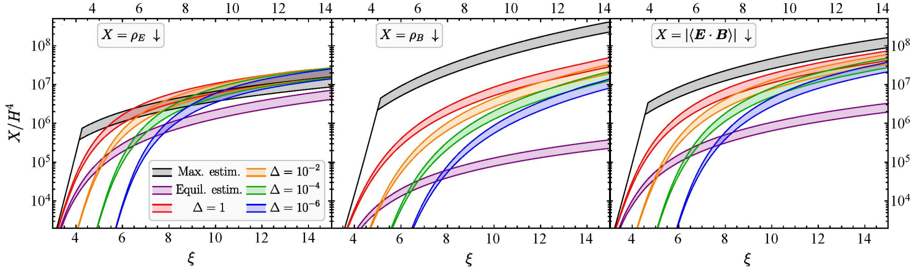


Qualitative features:

- Gauge fields are strongly damped
- Parameter $\Delta(t)$ introduces "memory effects", i.e., the system is nonlocal in time
- Energy density of charged particles may be significant – reheating starts already during inflation!

Model-independent predictions

Any axion inflation model can be characterized by two parameters: the Hubble parameter H and gauge-field production parameter ξ. Both of them change ver slowly during inflation.
The Schwinger effect introduces "memory effects" by the parameter Δ.



Black and purple lines represent the upper and lower estimates for the generated fields derived in the literature [Domcke et al., JCAP **11** (2018); Domcke et al., JCAP **10** (2019)].

Conclusion

1. Axion inflation model has many phenomenological applications: from **magnetogenesis** to **baryogenesis** and **stochastic GW production**. In order to consider these phenomena, it is very important to have a reliable solution for the inflaton and gauge fields.
2. Standard mode-by-mode approach is hard to apply when the backreaction and the Schwinger effect are present. At least they require the **iterative procedure** which is time consuming.
3. Gradient expansion formalism allows to deal with these highly nonlinear phenomena as it **captures all physically relevant modes at once**. It allows to get the solution **in a single numerical run**.
4. In the case with the Schwinger effect, there is still some ambiguity in the way of representing the Schwinger current. Also the current should be generalized to the case of noncollinear electric and magnetic fields. This is work in progress.

Thank you very much for your attention!

Peace to all of us!