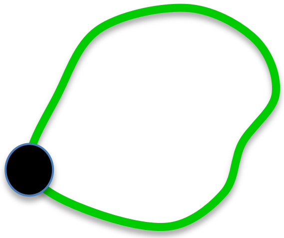
COSMIC STRINGS AND BLACK HOLES

A. Vilenkin

June 2022

Cosmic strings and black holes

String loops can be captured by black holes and can interact with them in interesting ways.



Work with:

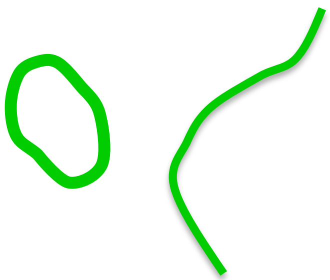
Yuri Levin, Andrei Gruzinov , Hengrui Xing, Heling Deng

Strings could be formed at a symmetry breaking phase transition in the early universe.

Nielsen & Olesen (1973)
Kibble (1976)

Predicted in a wide variety of particle physics models.

Can be either infinite or closed.

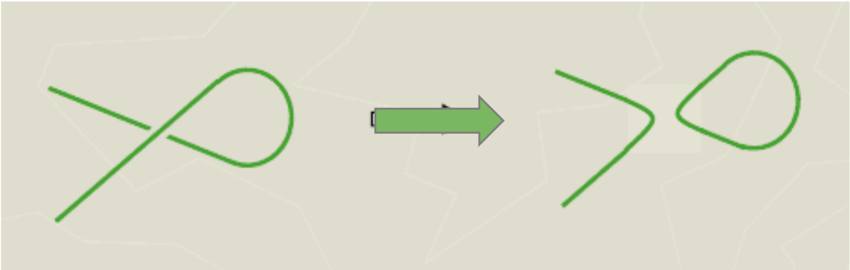


μ – mass per unit length

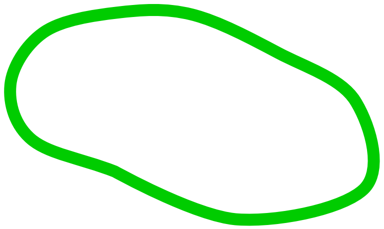
$$10^{-34} \lesssim G\mu \lesssim 10^{-10}$$

Tension = μ $\Rightarrow$ relativistic motion.

String reconnection



Loop dynamics



Worldsheet area

Nambu-Goto action: $S = -\mu \mathcal{A}$

Solution of NG eqs of motion:

$$\mathbf{x}(\sigma, t) = \frac{1}{2}\left[\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)\right] \qquad \mathbf{a}'^2 = \mathbf{b}'^2 = 1$$

$$0 < \sigma < L, \quad L = m/\mu$$

Invariant length

Loops oscillate with a period $T = L/2.$

Loop captured by a black hole

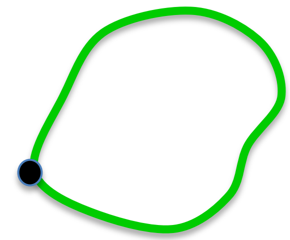
H. Xing, Y. Levin, A. Gruzinov, & A.V. (2020)

It is like a loop pinned at one point.

Boundary conditions: $\mathbf{x}(0,t) = \mathbf{x}(L,t) = 0.$

$$\mathbf{x}(\sigma,t) = \frac{1}{2}\left[\mathbf{a}(\sigma - t) - \mathbf{a}(-\sigma - t)\right]$$

The loop oscillates with a period *2L*.



$R = GM$

$M \gg \mu L$

$R \ll L$

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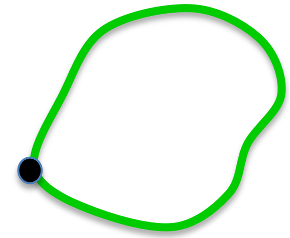

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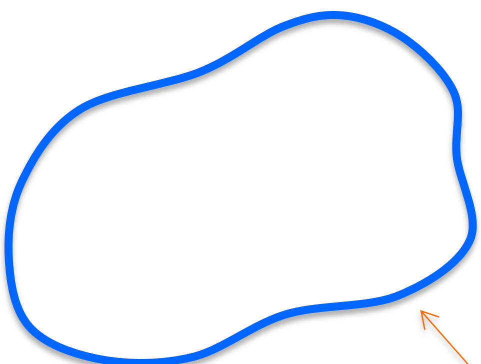
Fixed auxiliary curve

$\mathbf{a}(\sigma)$

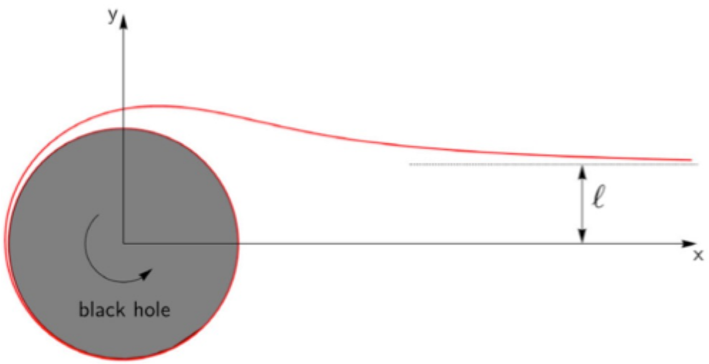


$\mathbf{a}'^2 = 1$

$0 < \sigma < 2L$

Not the shape of the physical loop!

Rotating black hole

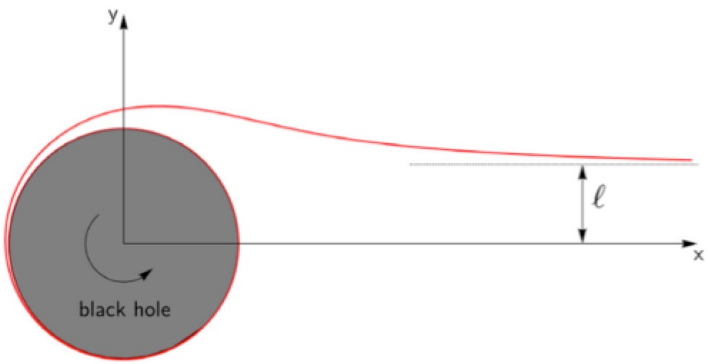


If the string is in the equatorial plane, $\ell = 4R^2\Omega$.

The torque is $Q = \mu\ell = 4\mu R^2\Omega\,.$

Frolov et al (!989)

Rotating black hole



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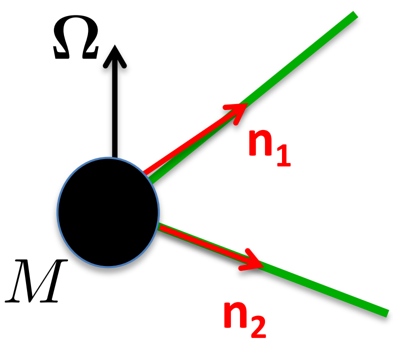
Equal and opposite torque acts on the string:

$$\mathbf{Q} = 4\mu R^2[\boldsymbol{\Omega} - (\mathbf{n}\cdot\boldsymbol{\Omega})\mathbf{n} - \mathbf{n}\times\dot{\mathbf{n}}]$$



$\omega = \mathbf{n}\times\dot{\mathbf{n}}$ – angular velocity of the string.

$L \gg R$ ⇨ quasistationary

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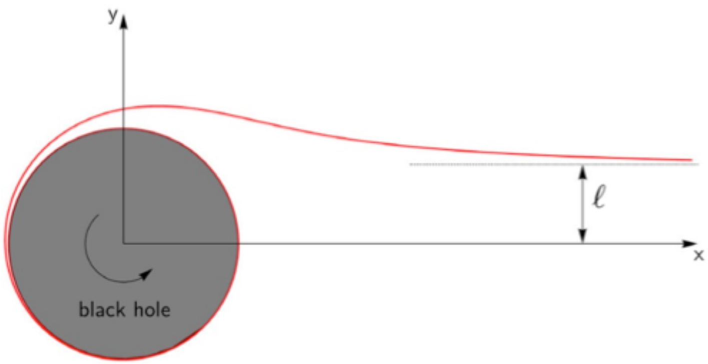

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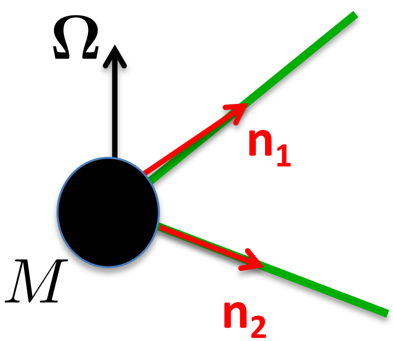

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Rate of energy change:

$$\dot{E} = \mathbf{Q}_1\cdot\omega_1 + \mathbf{Q}_2\cdot\omega_2$$

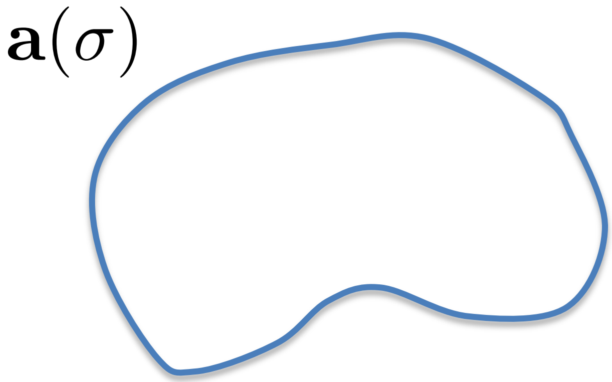
For $\mathbf{\Omega} = 0$: $\dot{E} = -4\mu R^2(\omega_1^2 + \omega_2^2)$

Horizon friction

The loop loses all its energy at $t \sim L^3/R^2 \gg L$.

Loop orbit evolves slowly compared to the oscillation period.

This can be described as *slow deformation of the auxiliary curve*.



Averaged over oscillation period:

$$\mathbf{v}(\sigma) = \frac{8R^2}{L}[\mathbf{a}'(\sigma) \times \mathbf{\Omega} + \mathbf{a}''(\sigma)]$$

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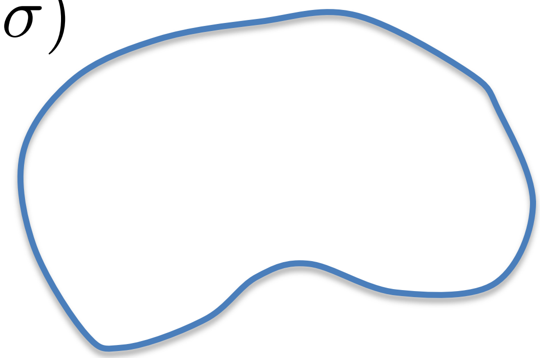
$\mathbf{a}(\sigma)$



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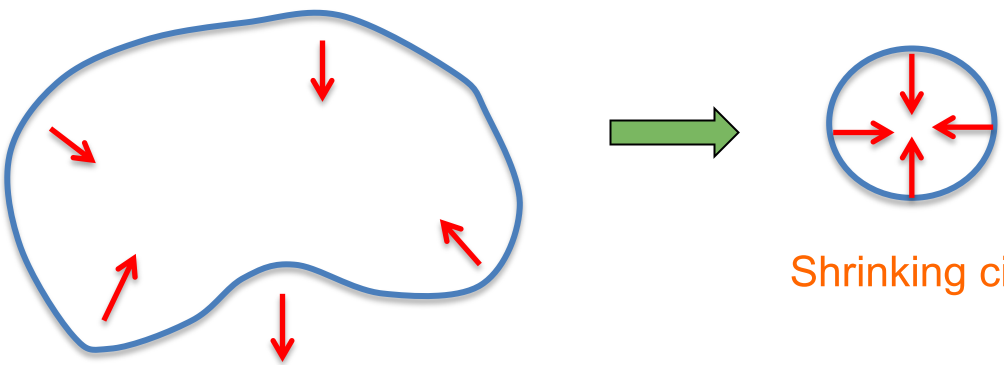
For $\mathbf{\Omega} = 0$: $\mathbf{v}(\sigma) \propto \mathbf{a}''(\sigma)$

Curve shortening flow

Gage (!984), Gage & Hamilton (1986), …



Shrinking circle

In the end the loop is swallowed by the BH.

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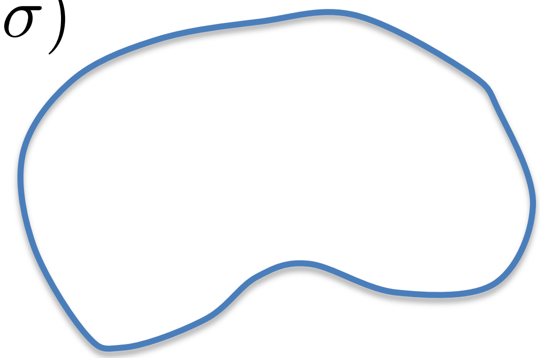

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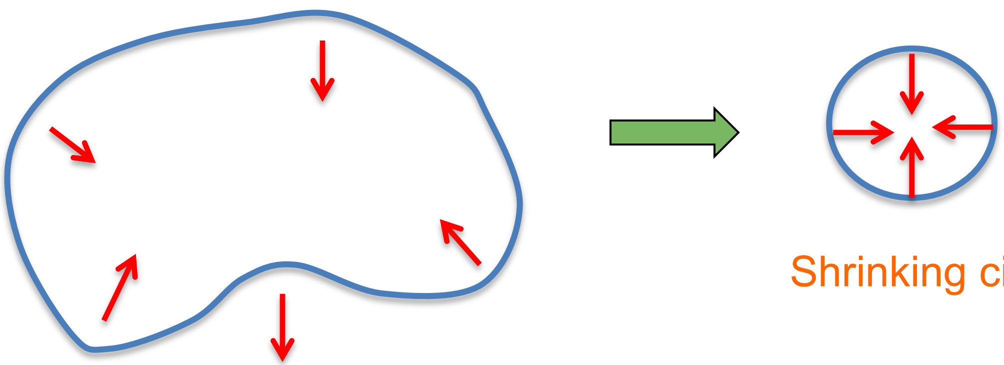

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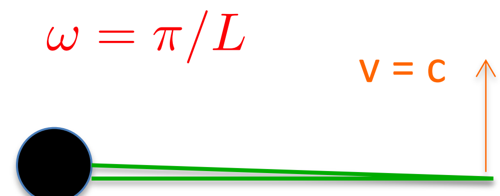


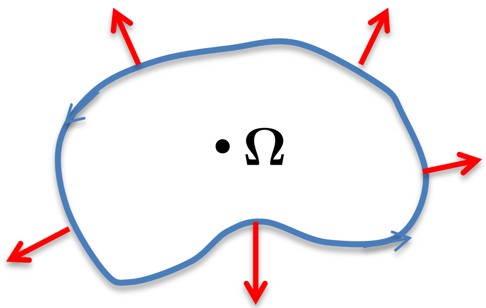
The physical loop is a rotating double line.

A strong emitter of gravitational waves.

In the end the loop is swallowed by the BH.

Now consider $\boldsymbol{\Omega} \neq 0$

$$\mathbf{v}(\sigma) = \frac{8R^2}{L}[\mathbf{a}'(\sigma) \times \boldsymbol{\Omega} + \mathbf{a}''(\sigma)]$$



The 1st term dominates if $\Omega L \gg 1$.

Auxiliary curve expands, approaching a circle.

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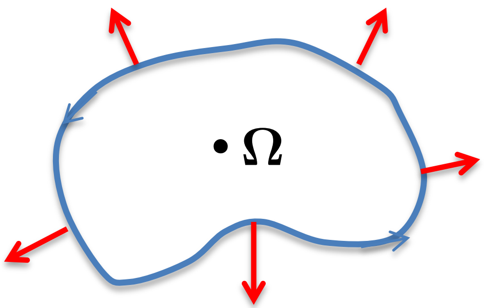

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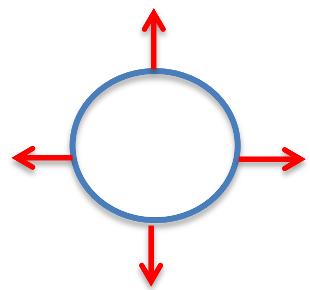
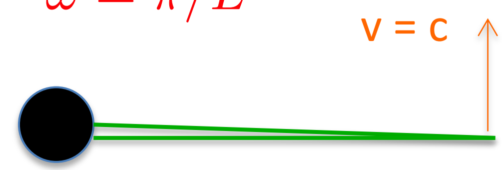
For a circular auxiliary curve:



$\omega = \pi / L$



$$L = \sqrt{L_0^2 + 16\pi R^2 \Omega t}$$

The loop grows by extracting rotational energy from the BH.

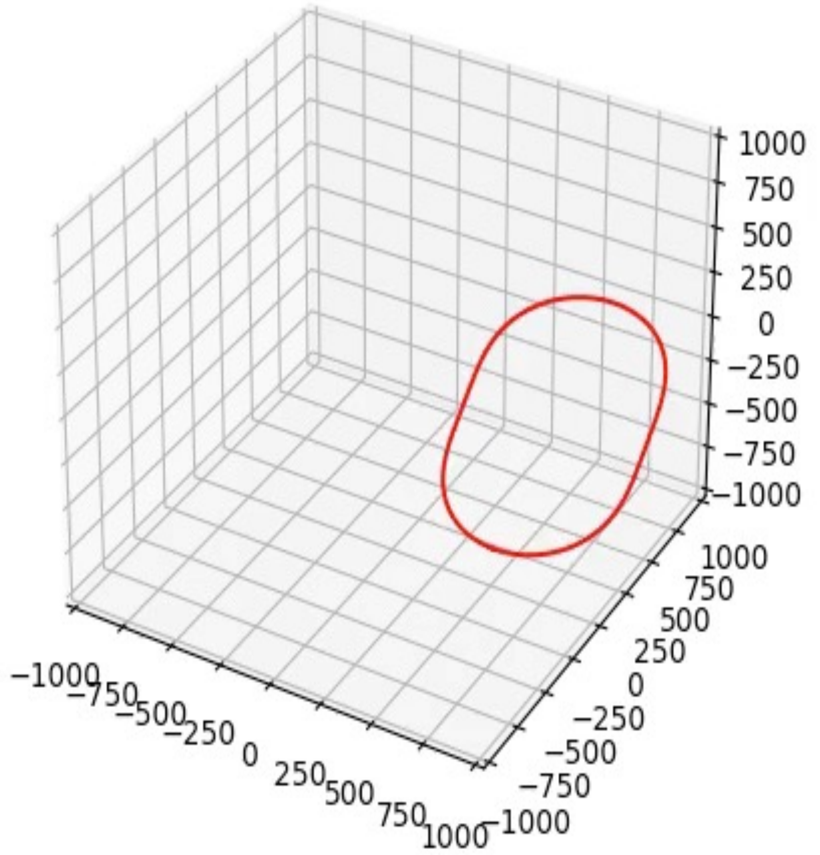
Complete spin down of a supermassive BH in 10^{10} yrs for $G\mu \gtrsim 10^{-15}$.

Numerical simulations

Heling Deng

Loop in Schwarzschild spacetime

$$L_i/R \sim 100$$



Numerical simulations

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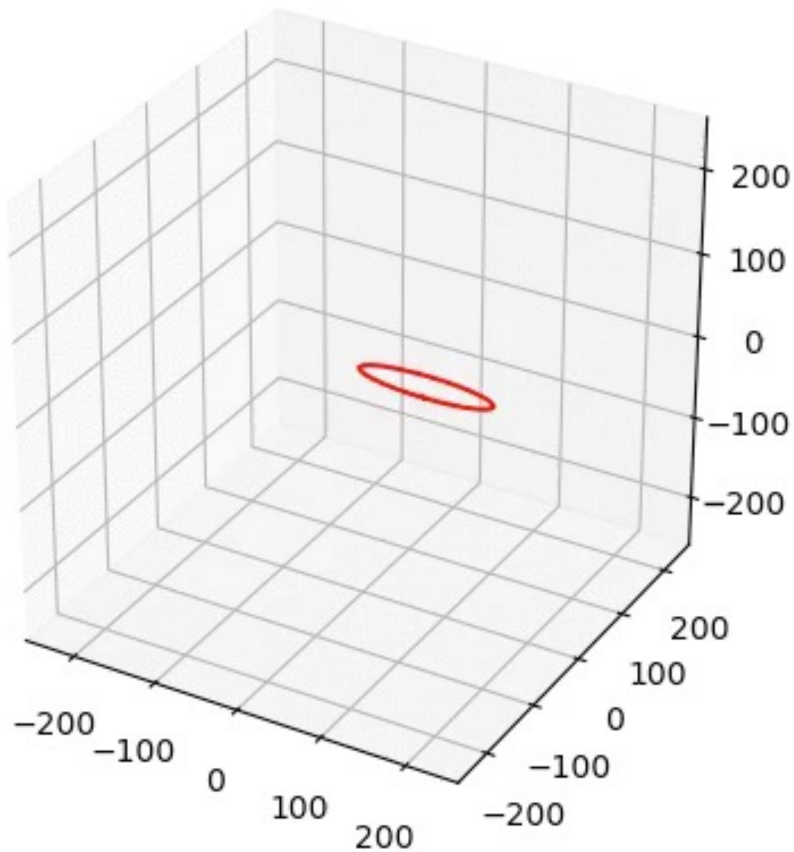
Loop in Kerr spacetime

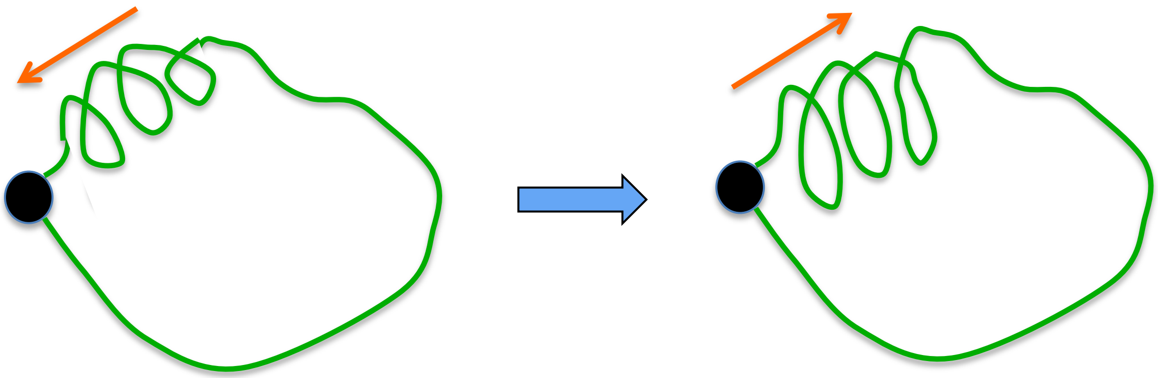
$$L_i/R \sim 50$$



Superradiance

Helical wave amplifies upon reflection.

Zel'dovich



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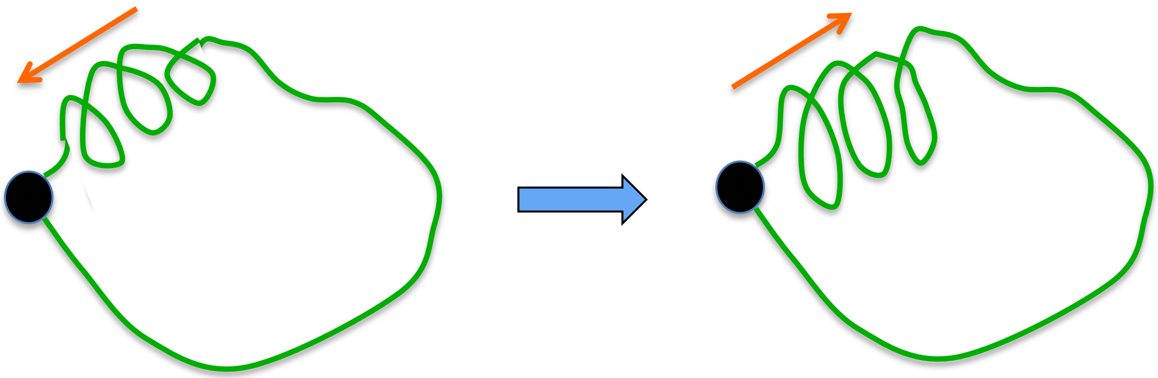
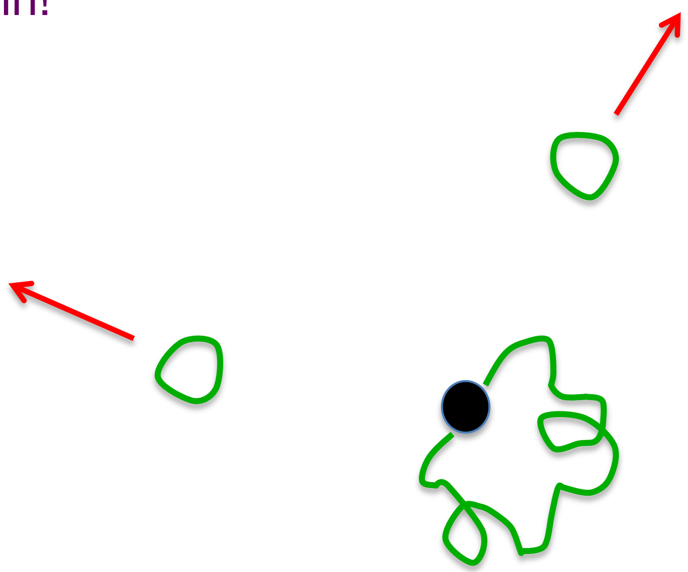


Then travels to the other side – and amplifies again!

Perturbations quickly become nonlinear.

May lead to continuous loop production.



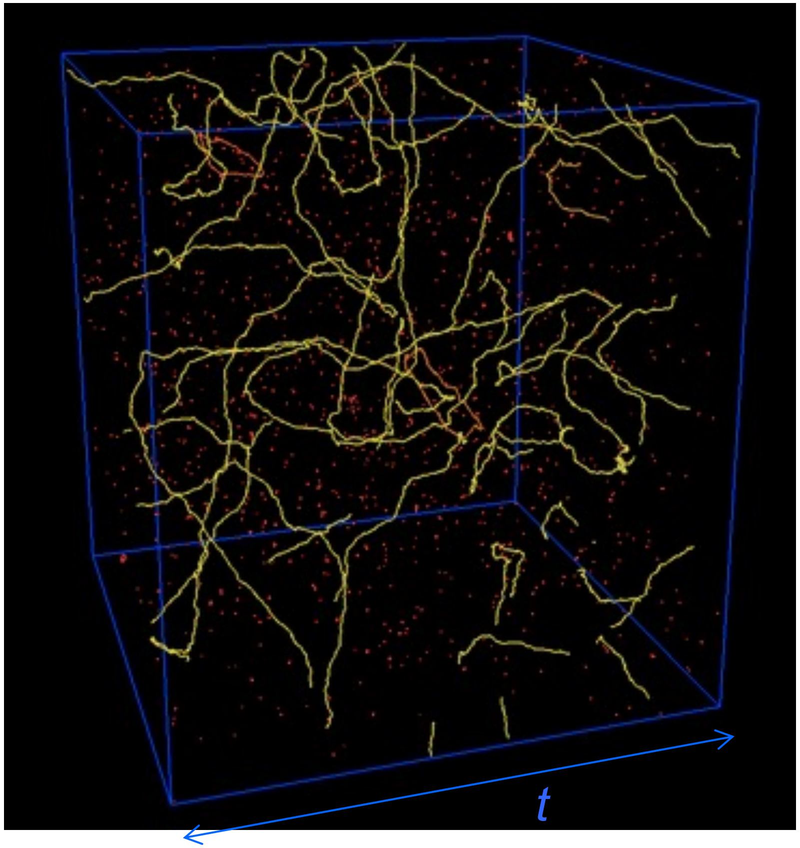
STRING EVOLUTION AND CAPTURE

Self-similar evolution

- Each horizon volume contains several long strings and a large number of loops with a wide distribution of sizes.

- Loops oscillate and decay by emitting gravitational waves.

- Loop density: $n \propto (G\mu)^{-3/2}$



Bennett & Bouchet (1990)

Allen & Shellard (1990)

Ringeval, Sakellariadou & Bouchet (2005)

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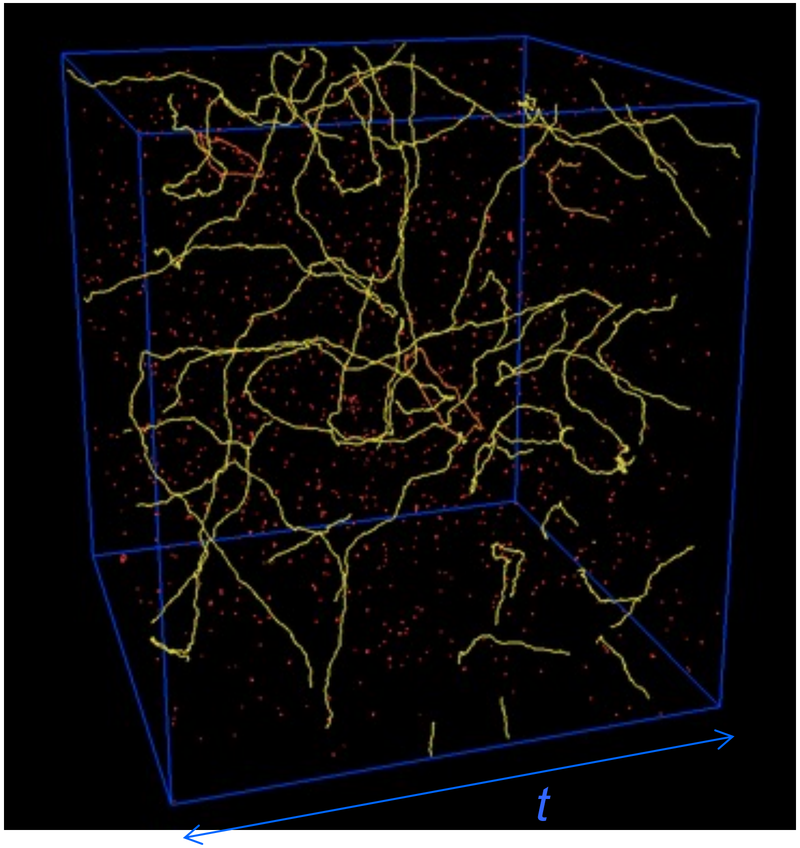

Loop capture

The probability of capture by a SMBH

is *P ~ 1* for $G\mu \lesssim 10^{-17}$.

H. Xing, Y. Levin,
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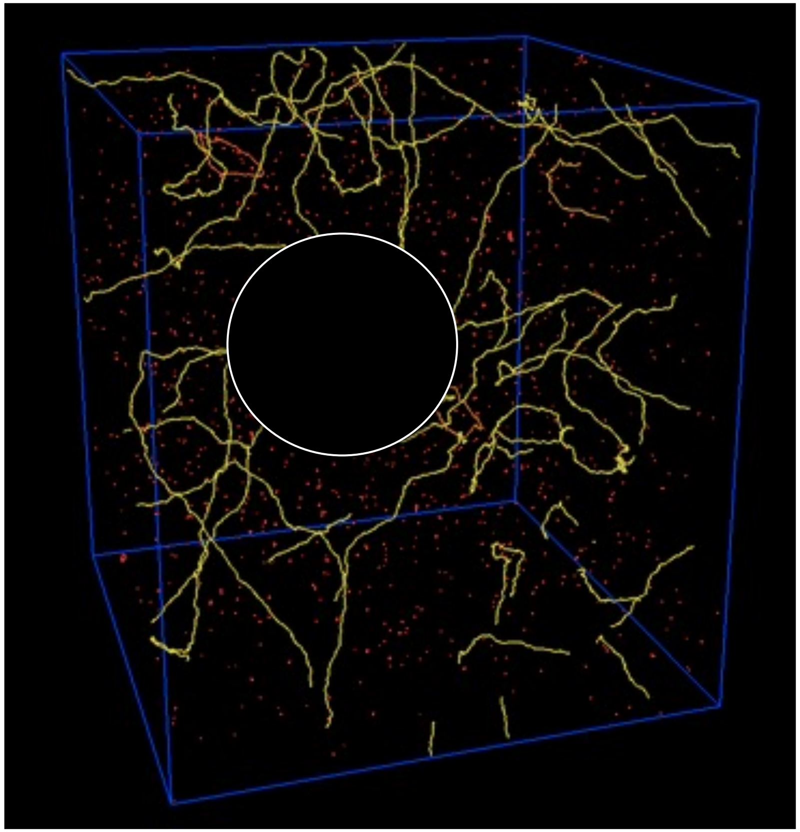
Strings and primordial BHs

A. Gruzinov, Y. Levin & A.V. (2020)

A. Lopez, K. Olum & A.V. (in progress)

- BHs have size ~ horizon at formation.
- A few strings are captured by each BH.

→ BH-string network.



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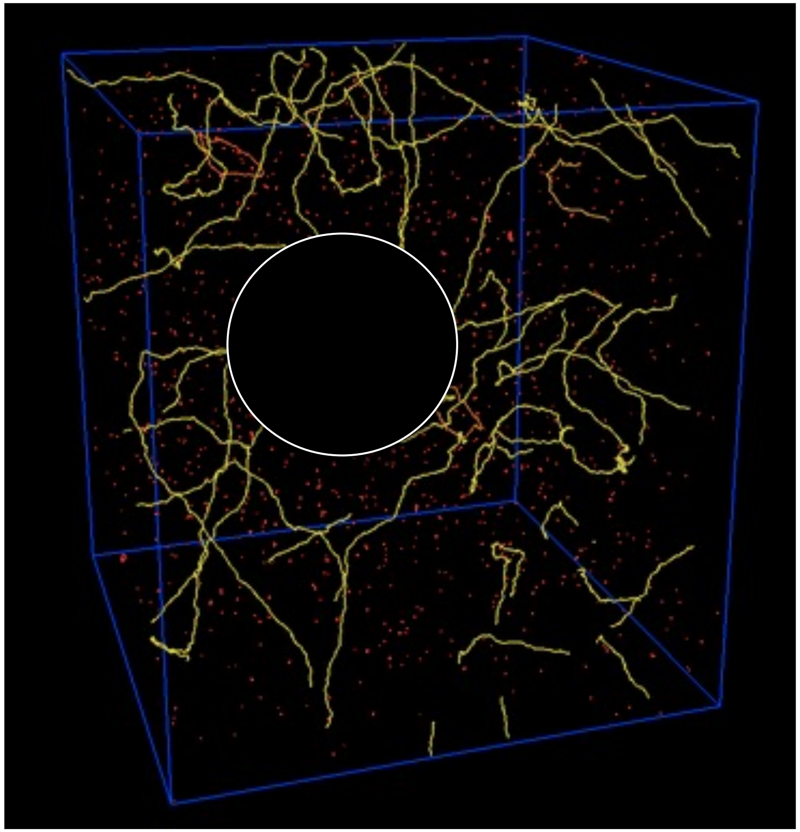

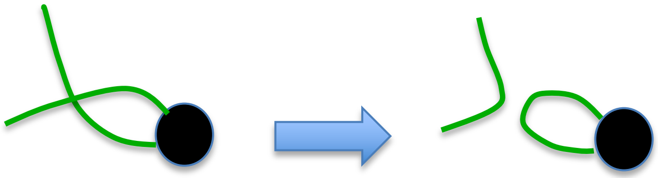

BHs can disconnect from the network, but only with loops attached.

Conclusions

String loops are likely to be captured by SMBH in galactic centers (for sufficiently small $G\mu$) and by primordial BHs (for BH mass and any $G\mu$).

A variety of physical effects:

- *BH spin down*
- *Superradiance*
- *GW emission*

Work in progress: reconnections, evolution of BH-string networks, etc.