The problem of instability of gravitational baryogenesis and its possible resolution

Elena Arbuzova

Dubna State University & Novosibirsk State University

based on joint works with A.D. Dolgov, K. Dutta, R. Rangarajan

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Baryogenesis

Matter dominance in the Universe:

- The amount of antimatter is very small and it can be explained as the result of high energy collisions in space.
- The existence of large regions of antimatter in our neighbourhood would produce high energy radiation as a consequence of matter-antimatter annihilation, which is not observed.
- Any initial asymmetry at inflation could not solve the problem of observed excess of matter over antimatter, because the energy density associated with baryonic number would not allow for sufficiently long inflation.

On the other hand, matter and antimatter seem to have similar properties and therefore we could expect a matter-antimatter symmetric universe.

A satisfactory model of our Universe should be able to explain the origin of the matter-antimatter asymmetry.

The term **baryogenesis** is used to indicate the generation of the asymmetry between baryons and antibaryons.

Sakharov Principles (1967)

1. Non-conservation of baryonic number
2. Breaking of symmetry between particles and antiparticles
3. Deviation from thermal equilibrium

List of baryogenesis scenarios:

- Heavy particle decays (Sakharov 1967)
- Baryogenesis by primordial black hole evaporation (Hawking 1974, Zeldovich 1976, Dolgov 1980)
- Electroweak baryogenesis (Kuzmin et al. 1985)
- SUSY condensate baryogenesis (Affleck, Dine 1985)
- Baryo-through-leptogenesis (Fukugita, Yanagita 1986)
- Spontaneous baryogenesis (Cohen, Kaplan 1987)
- **Gravitational baryogenesis** (Davoudiasl et al. 2004)
- Baryogenesis due to CPT violation (Dolgov 2010)
- Baryogenesis in extended theories of gravity (...)

SBG and GBG do not demand an explicit C and CP violation and can proceed in Th.Eq.

Spontaneous Baryogenesis (SBG)

Cosmological baryon asymmetry can be created by SBG in thermal equilibrium:

- A. Cohen, D. Kaplan, Phys. Lett. B 199, 251 (1987); Nucl.Phys. B308 (1988) 913. A.Cohen, D.Kaplan, A. Nelson, Phys.Lett. B263 (1991) 86-92

Reviews:

- A.D.Dolgov, Phys. Repts 222 (1992) No. 6; V.A. Rubakov, M.E. Shaposhnikov, Usp. Fiz. Nauk, 166 (1996) 493; A.D. Dolgov, Surveys in High Energy Physics, 13 (1998) 83.

The term "spontaneous" is related to spontaneous breaking of underlying symmetry of the theory.

- Unbroken phase: the theory is invariant with respect to the global $U(1)$-symmetry, which ensures conservation of total baryonic number.
- Spontaneous symmetry breaking: the Lagrangian density acquires the term

$$\mathcal{L}_{SB} = (\partial_\mu \theta) J^\mu_B$$

$\boldsymbol{\theta}$ is the (pseudo) Goldstone field and $\boldsymbol{J^\mu_B}$ is the baryonic current of matter fields.

SBG: $\mathcal{L}_{SB} = (\partial_\mu \theta) J_B^\mu$

Spatially homogeneous field $\theta = \theta(t)$:

$$\mathcal{L}_{SB} = \dot{\theta}\, n_B\,, \qquad n_B \equiv J_B^0$$

- n_B is the baryonic number density, so it is tempting to identify $\dot{\boldsymbol{\theta}}$ with the chemical potential, $\boldsymbol{\mu}_B$, of the corresponding system.

The identification of $\dot{\theta}$ with chemical potential is questionable. It depends upon the representation chosen for the fermionic fields and is heavily based on the assumption $\dot{\theta} \approx const$:

- A.D. Dolgov, K. Freese, Phys.Rev. D **51** (1995) 2693-2702; A.D. Dolgov, K. Freese, R. Rangarajan, M. Srednicki, Phys.Rev. D **56** (1997) 6155
- E.V. Arbuzova, A.D. Dolgov, V.A. Novikov, Phys. Rev. D **94** (2016) 123501 – the assumption $\dot{\theta} \approx const$ is relaxed.

Still the scenario is operative and presents a beautiful possibility to create an excess of particles over antiparticles in the Universe.

Gravitational Baryogenesis (GBG)

Stimulated by SBG the idea of gravitational baryogenesis was put forward:

- H. Davoudiasl, R. Kitano, G. D. Kribs, H. Murayama, P. J. Steinhardt, Phys. Rev. Lett. **93** (2004) 201301, hep-ph/0403019.

The scenario of SBG was modified by the introduction of the coupling of the baryonic current to the derivative of the curvature scalar $\boldsymbol{R}$:

$$\mathcal{L}_{GBG} = \frac{1}{M^2}(\partial_\mu R) J_B^\mu$$

where M is a constant parameter with the dimension of mass.

The addition of the curvature dependent term to the Hilbert-Einstein Lagrangian of GR leads to higher order gravitational equations of motion which are strongly unstable with respect to small perturbations.

Gravitational Baryogenesis with Scalar Baryons

- EA, A.D. Dolgov, *"Intrinsic problems of the gravitational baryogenesis"*, Phys.Lett. B769 (2017) 171-175, arXiv:1612.06206.

GBG with scalars

The action of the scalar model has the form:

$$A = \int d^4x \sqrt{-g} \left[\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} (\partial_\mu R) J^\mu - g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi^* + U(\phi, \phi^*) \right] - A_m$$

- where $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass, A_m is the matter action.

The baryonic number is carried by scalar field ϕ with potential $U(\phi, \phi^*)$.

If the potential $U(\phi)$ is not invariant w.r.t. the $U(1)$-rotation $\phi \to e^{i\beta}\phi \Longrightarrow$ the baryonic current defined in the usual way is not conserved.

$$J_\mu = iq(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*)$$

- Here q is the baryonic number of ϕ

The corresponding equation of motion for the curvature scalar:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2} \left[(R + 3D^2) D_\alpha J^\alpha + J^\alpha \, D_\alpha R \right] - D_\alpha \phi \, D^\alpha \phi^* + 2U(\phi) = -\frac{1}{2} \, T^\mu_\mu$$

- D_μ is the covariant derivative, $T_{\mu\nu}$ is the energy-momentum tensor of matter

GBG with scalars: non-invariant potential $U(\phi)$

The equation of motion for field ϕ is:

$$D^2\phi + \frac{\partial U}{\partial \phi^*} = -\frac{iq}{M^2}\left(2D_\mu R\, D^\mu\phi + \phi D^2 R\right)$$

According to definition, the current divergence is:

$$D_\mu J^\mu = \frac{2q^2}{M^2}\left[D_\mu R\left(\phi^* D^\mu\phi + \phi D^\mu\phi^*\right) + |\phi|^2 D^2 R\right] + iq\left(\phi\frac{\partial U}{\partial\phi} - \phi^*\frac{\partial U}{\partial\phi^*}\right)$$

- If $U = U(|\phi|)$, the last term disappears, but the current is non-conserved.
- this non-conservation does not lead to any cosmological baryon asymmetry
- it can produce or annihilate an equal number of baryons and antibaryons, since the current non-conservation is bilinear in terms of $\phi^*\phi$.

To create cosmological baryon asymmetry we need new types of interactions, e.g.

$$U_4 = \lambda_4\phi^4 + \lambda_4^*\phi^{*4}$$

This potential is non invariant w.r.t. the phase rotation of ϕ and can induce the B-non-conserving process of transition $\mathbf{2\phi \to 2\bar{\phi}}$.

EoM in FLRW background: $ds^2 = dt^2 - a^2(t)d\mathbf{r}^2$

In the homogeneous case the equation for the curvature scalar:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2}\left[(R + 3\partial_t^2 + 9H\partial_t)D_\alpha J^\alpha + \dot{R}\, J^0\right] = -\frac{T^{(tot)}}{2}$$

- J^0 is the baryonic number density of the ϕ-field, $H = \dot{a}/a$ (Hubble par.)
- $T^{(tot)}$ is the trace of the energy-momentum tensor of matter including contribution from the ϕ-field.

In the homogeneous and isotropic cosmological plasma

$$T^{(tot)} = \varrho - 3P\,,$$

where ϱ and P are the energy density and the pressure of the plasma.

Relativistic plasma:

- $\varrho = \pi^2 g_* T^4/30$ with T and g_* being respectively the plasma temperature and the number of particle species in the plasma.
- The Hubble parameter: $H^2 = 8\pi\varrho/(3m_{Pl}^2) \sim T^4/m_{Pl}^2$

Equation for $\boldsymbol{R}$: the fourth order

The covariant divergence of the current in the homogeneous case:

$$D_\alpha J^\alpha = \frac{2q^2}{M^2}\left[\dot R\left(\phi^*\dot\phi + \phi\dot\phi^*\right) + \left(\ddot R + 3H\dot R\right)\phi^*\phi\right] + iq\left(\phi\frac{\partial U}{\partial \phi} - \phi^*\frac{\partial U}{\partial \phi^*}\right)$$

The expectation values of the products of the quantum operators ϕ, ϕ^*, and their derivatives after the thermal averaging:

$$\langle \phi^*\phi\rangle = \frac{T^2}{12}, \quad \langle \phi^*\dot\phi + \dot\phi^*\phi\rangle = 0$$

Equation of motion for the classical field $\boldsymbol{R}$ in the cosmological plasma:

$$\frac{m_{Pl}^2}{16\pi}R + \frac{q^2}{6M^4}\left(R + 3\partial_t^2 + 9H\partial_t\right)\left[\left(\ddot{\boldsymbol{R}} + 3H\dot R\right)T^2\right] + \frac{1}{M^2}\dot R\,\langle J^0\rangle = -\frac{T^{(tot)}}{2}$$

- $\langle J^0\rangle$ is the thermal average value of the baryonic number density of ϕ.
- This term can be neglected, since it is small initially and subdominant later.

NB: We obtained the 4th order differential equation for $\boldsymbol{R}$.

Exponential Solutions

Keeping only the linear in R terms and neglecting higher powers of R, such as R^2 or HR, we obtain the linear fourth order equation:

$$\frac{d^4 R}{dt^4} + \mu^4 R = -\frac{1}{2}\, T^{(tot)}\,, \quad \mu^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2}$$

The homogeneous part of this equation has exponential solutions:

$$R \sim e^{\lambda t}\,, \quad \lambda = |\mu| e^{i\pi/4 + i\pi n/2}\,, \quad n = 0, 1, 2, 3$$

- There are two solutions with positive real parts of $\boldsymbol{\lambda}$.

Curvature scalar is exponentially unstable w.r.t. small perturbations, so $\boldsymbol{R}$ should rise exponentially fast with time and quickly oscillate around this rising function.

The characteristic rate of the perturbation explosion is much larger than the rate of the universe expansion, if:

$$(Re\,\lambda)^4 > H^4 = \left(\frac{8\pi\varrho}{3m_{Pl}^2}\right)^2 = \frac{16\pi^6 g_*^2}{2025}\,\frac{T^8}{m_{Pl}^4}$$

- $\varrho = \pi^2 g_* T^4/30$ is energy density of the primeval plasma at temperature T
- $g_* \sim 10 - 100$ is the number of relativistic degrees of freedom in the plasma.

This condition is fulfilled if

$$\frac{2025}{2^9\pi^7 q^2 g_*^2}\,\frac{m_{Pl}^6 M^4}{T^{10}} > 1 \quad or \quad T \lesssim m_{Pl}^{3/5} M^{2/5}$$

Let us stress that at these temperatures the instability is quickly developed and the standard cosmology would be destroyed.

To preserve the successful BBN results $\Longrightarrow$ impose the condition that the development of the instability was longer than the Hubble time at the BBN epoch at $T \sim 1$ MeV $\Longrightarrow$ M should be extremely small: $M < 10^{-32}$ MeV $\Longrightarrow$

A tiny M leads to a huge strength of coupling

$$\mathcal{L}_{GBG} = \frac{1}{M^2}(\partial_\mu R) J_B^\mu$$

It surely would lead to pronounced effects in stellar physics.

Gravitational Baryogenesis with Fermions

- EA, A.D. Dolgov, *"Instability of gravitational baryogenesis with fermions"*, JCAP 1706 (2017) no.06, 001, arXiv:1702.07477.

GBG with fermions

We start from the action in the form:

$$A = \int d^4x \sqrt{-g} \left[\frac{m_{Pl}^2}{16\pi} R - \mathcal{L}[Q, L] \right] + A_{matt}$$

with

$$\begin{aligned} \mathcal{L}[Q, L] &= \frac{i}{2}(\bar{Q}\gamma^\mu \nabla_\mu Q - \nabla_\mu \bar{Q}\,\gamma^\mu Q) - m_Q \bar{Q}\, Q \\ &+ \frac{i}{2}(\bar{L}\gamma^\mu \nabla_\mu L - \nabla_\mu \bar{L}\gamma^\mu L) - m_L \bar{L}\, L \\ &+ \frac{g}{m_X^2} \left[(\bar{Q}\, Q^c)(\bar{Q} L) + (\bar{Q}^c Q)(\bar{L} Q) \right] + \frac{d}{M^2} (\partial_\mu R) J^\mu + \mathcal{L}_{matt} \end{aligned}$$

- Q is the quark (or quark-like) field with non-zero baryonic number, L is another fermionic field (lepton)
- ∇_μ is the covariant derivative of Dirac fermion in tetrad formalism
- $J^\mu = \bar{Q}\gamma^\mu Q$ is the quark current with γ^μ being the curved space gamma-matrices; $d = \pm 1$ is dimensionless coupling constant
- $\mathcal{L}_{matt}$ describes all other forms of matter.

The four-fermion interaction between quarks and leptons

$$\frac{g}{m_X^2}\left[(\bar{Q}\,Q^c)(\bar{Q}L)+(\bar{Q}^c Q)(\bar{L}Q)\right]$$

is introduced to ensure the necessary non-conservation of the baryonic number

- g is a dimensionless coupling constant.
- m_X is a constant parameter with dimension of mass, which may be of the order of $10^{14} - 10^{15}$ GeV in grand unified theories.

The gravitational EoM can be written as:

$$\frac{m_{Pl}^2}{8\pi}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right) = -g_{\mu\nu}\mathcal{L}_m +$$
$$\frac{i}{4}\left[(\bar{Q}(\gamma_\mu\nabla_\nu+\gamma_\nu\nabla_\mu)Q-(\nabla_\nu\bar{Q}\,\gamma_\mu+\nabla_\mu\bar{Q}\,\gamma_\nu)Q\right] +$$
$$\frac{i}{4}\left[(\bar{L}(\gamma_\mu\nabla_\nu+\gamma_\nu\nabla_\mu)L-(\nabla_\nu\bar{L}\,\gamma_\mu+\nabla_\mu\bar{L}\,\gamma_\nu)L\right] -$$
$$\frac{2d}{M^2}\left[R_{\mu\nu}+g_{\mu\nu}D^2-D_\mu D_\nu\right]D_\alpha J^\alpha + \frac{d}{2M^2}(J_\mu\partial_\nu R+J_\nu\partial_\mu R)$$

where D_μ is the usual tensor covariant derivative in background metric.

Gravitational EoM for Trace

Taking trace with respect to μ and ν we obtain:

$$-\frac{m_{Pl}^2}{8\pi} R = m_Q \bar{Q} Q + m_L \bar{L} L + \frac{2g}{m_X^2} \left[(\bar{Q}\, Q^c)(\bar{Q} L) + (\bar{Q}^c Q)(\bar{L} Q) \right] - \frac{2d}{M^2} (R + 3D^2) D_\alpha J^\alpha + T_{other}$$

- T_{other} is the trace of the energy momentum tensor of all other fields.
- At relativistic stage we can take $T_{other} = 0$.

To calculate the current divergence, we use kinetic equation, which leads to an explicit dependence of $D_\alpha J^\alpha$ on $\dot{R}$, if the current is not conserved.

As a result we obtain high (fourth) order equation for $\boldsymbol{R}$.

We study solutions in cosmology in homogeneous and isotropic FRW background:

$$ds^2 = dt^2 - a^2(t) d\mathbf{r}^2 , \quad D_\alpha J^\alpha = (\partial_t + 3H) J^t \equiv (\partial_t + 3H) n_B$$

Equilibrium w.r.t. Elastic Scattering and Annihilation

Equilibrium distribution functions of quarks and leptons:

$$f = \frac{1}{e^{(E/T-\xi)}+1} \approx e^{-E/T+\xi}$$

- $\xi = \mu/T$ is dimensionless chemical potential, different for quarks and leptons

The assumption of kinetic equilibrium is well justified since it is usually enforced by very efficient elastic scattering.

The baryonic number density is given by the expression:

$$\begin{aligned} n_B &= \int \frac{d^3q}{2E_q\,(2\pi)^3}(f_q - f_{\bar{q}}) \\ &= \frac{g_S B_Q}{6}\left(\mu T^2 + \frac{\mu^3}{\pi^2}\right) = \frac{g_S B_Q T^3}{6}\left(\xi + \frac{\xi^3}{\pi^2}\right) \end{aligned}$$

- T is the cosmological plasma temperature
- g_S and B_Q are respectively the number of the spin states and the baryonic number of quarks.

Kinetic Equation: the reaction $q_1 + q_2 \leftrightarrow \bar{q}_3 + l_4$

The kinetic equation for the variation of the baryonic number density $n_B \equiv J^t$

$$(\partial_t + 3H) n_B = I_B^{coll}$$

In the case of small dimensionless chemical potentials of quarks, $\boldsymbol{\xi_q}$, and leptons, $\boldsymbol{\xi_l}$:

$$I_B^{coll} \approx \frac{C_l g^2 T^8}{m_X^4} \left[\frac{3d\dot{R}(t)}{M^2\, T} + 3\xi_q - \xi_l \right]$$

- C_l is a positive dimensionless constant

The conservation of the sum of baryonic and leptonic numbers gives: $\xi_l = -\xi_q/3$

NB. Here we consider only the simple situation with quasi-stationary background:

$$R(t) \approx \dot{R}(t)\, t, \quad \dot{R} \approx const$$

The case of an essential variation of $\dot{R}(t)$ is analogous to fast variation of $\dot{\theta}(t)$ studied in

- *EA, A.D. Dolgov, V.A. Novikov, Phys. Rev. D* **94** *(2016) 123501*
  "General properties and kinetics of spontaneous baryogenesis"

It is much more complicated technically and is postponed for the future work.

Stationary Point Approximation

For small chemical potential the baryonic number density is equal to

$$n_B \approx \frac{g_s B_Q}{6} \xi_q T^3$$

If the temperature adiabatically decreases in the course of the cosmological expansion, according to $\dot{T} = -HT$, the kinetic equation

$$(\partial_t + 3H) n_B = I_B^{coll}$$

turns into

$$\dot{\xi}_q = \Gamma \left[\frac{9 d\dot{R}(t)}{10 M^2 T} + \xi_q \right], \quad \Gamma \sim g^2 T^5 / m_X^4$$

- where Γ is the rate of B-nonconserving reactions.

If Γ is large, this equation can be solved in stationary point approximation as

$$\xi_q = \xi_q^{eq} - \dot{\xi}_q^{eq} / \Gamma \,, \quad where \quad \xi_q^{eq} = -\frac{9}{10} \frac{d\dot{R}}{M^2 T} \,.$$

If we substitute $\boldsymbol{\xi_q^{eq}}$ into EoM for trace we arrive to the 4th order equation for $\boldsymbol{R}$.

Curvature Instability

The contribution of thermal matter into EoM for trace can be neglected

$$\frac{m_{Pl}^2}{8\pi} R = \frac{2d}{M^2} (R + 3D^2)(\partial_t + 3H) n_B$$

From the kinetic equation

$$n_B \sim \xi_q^{eq} = -\frac{9}{10} \frac{d\dot{R}}{M^2 T}$$

Neglecting the H-factor in comparison with time derivatives of R, we arrive to:

$$\frac{d^4 R}{dt^4} + \lambda^4 R = 0 \qquad (*)$$

where

$$\lambda^4 = C_\lambda m_{Pl}^2 M^4 / T^2 \quad with \quad C_\lambda = 5/(36\pi d^2 g_s B_Q).$$

Equation $(*)$ has extremely unstable solution with instability time by far shorter than the cosmological time. This instability would lead to an explosive rise of ***R***

Stabilization of gravitational baryogenesis

- EA, A.D. Dolgov, Koushik Dutta, and Raghavan Rangarajan, work in progress

Possible Stabilization

Gravitational baryogenesis:

$$\mathcal{S}_{GBG} = \frac{1}{M^2} \int d^4x \sqrt{-g}\, (\partial_\mu R) J^\mu_B$$

- Can successfully explain the magnitude of the cosmological baryon asymmetry of the universe.
- The back reaction of the created non-zero baryonic density leads to strong instability of the cosmological evolution.

Possible stabilization mechanism $\Longrightarrow$ R^2-modified gravity:

$$S_{Grav} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \, \sqrt{-g} \left(R - \frac{R^2}{6M_R^2} \right)$$

R^2-term:

- In the early universe generates inflation and density perturbations.
- leads to excitation of the scalar degree of freedom: scalaron with the mass M_R.
- Amplitude of the observed density perturbations demands: $M_R = 3 \cdot 10^{13}$GeV.

Stabilization: Bosonic Case

Baryonic number is carried by a complex scalar field ϕ:

$$S_{tot}[\phi] = -\int d^4x\sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}\left(R - \frac{R^2}{6M_R^2}\right) + \frac{1}{M^2}(\partial_\mu R)J^\mu_{(\phi)}\right] - \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi^* + U(\phi,\phi^*)\right] + S_{matt}$$

The equation for the curvature evolution:

$$\frac{m_{Pl}^2}{16\pi}\left(R + \frac{1}{M_R^2}D^2R\right) + \frac{1}{M^2}\left[(R+3D^2)D_\alpha J^\alpha_{(\phi)} + J^\alpha_{(\phi)}\,D_\alpha R\right] - D_\alpha\phi\,D^\alpha\phi^* + 2U(\phi) = -\frac{1}{2}\,T^{(matt)}$$

- For relativistic matter $T^{(matt)} = 0$.

In spatially flat FLRW-metric:

$$\frac{m_{Pl}^2}{16\pi}\left[R + \frac{1}{M_R^2}(\partial_t^2 + 3H\partial_t)R\right] + \frac{1}{M^2}\left[(R + 3\partial_t^2 + 9H\partial_t)D_\alpha J^\alpha_{(\phi)} + \dot{R}\,J^0_{(\phi)}\right] + 2U(\phi) - (D_\alpha\phi)(D^\alpha\phi^*) = 0$$

Stabilization: Bosonic Case

The divergence of the current is given by the expression:

$$D_\alpha J^\alpha_{(\phi)} = \frac{2q^2}{M^2}\left[\dot{R}\left(\phi^*\dot{\phi} + \phi\dot{\phi}^*\right) + \left(\ddot{R} + 3H\dot{R}\right)\phi^*\phi\right] + iq\left(\phi\frac{\partial U}{\partial\phi} - \phi^*\frac{\partial U}{\partial\phi^*}\right)$$

Performing thermal averaging of the bilinear products of field ϕ:

$$\langle\phi^*\phi\rangle = \frac{T^2}{12}\,, \quad \langle\phi^*\dot{\phi} + \dot{\phi}^*\phi\rangle = 0$$

we obtain the 4th order differential equation:

$$\frac{m_{Pl}^2}{16\pi}\left(R + \frac{1}{M_R^2}D^2R\right) + \frac{q^2}{6M^4}\left(R + 3\partial_t^2 + 9H\partial_t\right)\left[\left(\ddot{R} + 3H\dot{R}\right)T^2\right] + \frac{1}{M^2}\dot{R}\,\langle J^0_{(\phi)}\rangle = -\frac{T^\mu_\mu(\phi)}{2}$$

Keeping only the dominant terms we simplify the above equation to:

$$\frac{d^4R}{dt^4} + \frac{\kappa^4}{M_R^2}\frac{d^2R}{dt^2} + \kappa^4 R = -\frac{1}{2}\,T^\mu_\mu(\phi), \qquad \kappa^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2}$$

Stability condition: Bosonic Case

The characteristic equation for the solution $R \sim \exp(\lambda t)$:

$$(*) \quad \lambda^4 + \frac{\kappa^4}{M_R^2}\lambda^2 + \kappa^4 = 0 \quad \Longrightarrow \quad \lambda^2 = -\frac{\kappa^4}{2M_R^2} \pm \kappa^2 \sqrt{\frac{\kappa^4}{4M_R^4} - 1}$$

No instability, if $\lambda^2 < 0$ and Eq. $(*)$ has only oscillating solutions.

Stability condition:

$$\kappa^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2} > 4M_R^4 \quad \Longrightarrow \quad M > 3 \cdot 10^4 \,\text{GeV} \left(\frac{q\,T}{\text{GeV}}\right)^{1/2}$$

The value of λ depends upon the relation between κ and M_R:

- $\kappa \sim M_R \Longrightarrow$ the frequency of oscillations is of the order of M_R, $|\lambda| \sim M_R$.
- $\kappa \gg M_R \Longrightarrow$ 2 possible solutions: $|\lambda| \sim M_R$ and $|\lambda| \sim M_R(\kappa/M_R)^2 \gg M_R$.
- High frequency oscillations of R would lead to efficient gravitational particle production and, as a result, to damping of the oscillations.

Stabilization: Fermionic Case

$$S_{tot}[Q,L] = -\int d^4x\,\sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}\left(R - \frac{R^2}{6M_R^2}\right) + \frac{d}{M^2}(\partial_\mu R)J_Q^\mu - \mathcal{L}[Q,L]\right]$$

The equation for the curvature evolution:

$$\begin{aligned} - \quad & \frac{m_{Pl}^2}{8\pi}\left(R + \frac{1}{M_R^2}D^2 R\right) = m_Q \bar{Q}Q + m_L \bar{L}L + \\ + \quad & \frac{2g}{m_X^2}\left[(\bar{Q}\,Q^c)(\bar{Q}L) + (\bar{Q}^c Q)(\bar{L}Q)\right] - \frac{2d}{M^2}(R + 3D^2)D_\alpha J_Q^\alpha + T_{matt} \end{aligned}$$

- In the early universe, when various species are relativistic $T_{matt} = 0$.
- Higher order equation for R originates after we substitute the current divergence $D_\alpha J^\alpha$ calculated from kinetic equation in external field R.

In complete analogy with the previous cases we obtain:

$$\frac{d^4R}{dt^4} + \frac{\kappa_f^4}{M_R^2}\frac{d^2R}{dt^2} + \kappa_f^4 R = 0, \quad \kappa_f^4 = \frac{5m_{Pl}^2 M^4}{36\pi g_s B_Q^2 T^2}$$

The value of κ_f is only slightly numerically different from κ in the scalar case.

Conclusions

- The derivative coupling of baryonic current to the curvature scalar in GBG scenarios leads to higher (4th) order equations for gravitational field.
- These equations are unstable with respect to small perturbations of the FRW background and such instability leads to an exponential rise of the curvature.
- For a large range of cosmological temperatures the development of the instability is much faster than the universe expansion rate.
- Simple versions of GBG based on the coupling, $\sim (\partial_\mu R) J^\mu_B$, would be incompatible with observations and a stabilization mechanism is desirable.
- The problem of stability can be solved by adding to the Hilbert-Einstein action the quadratic in curvature term.
- Stablilization is achieved at a very high value of the curvature scalar.

THE END

THANK YOU FOR ATTENTION