New Boundary Conditions for Cosmology

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with Latham Boyle, see recent preprints 2109.06204, 2110.06258, 2201.07279

extending Boyle, Finn, NT Phys. Rev. Lett. 121 (2018) 251301
Annals of Physics 438 (2022) 168767

Large Scale Universe

remarkably simple fit

just 5 parameters

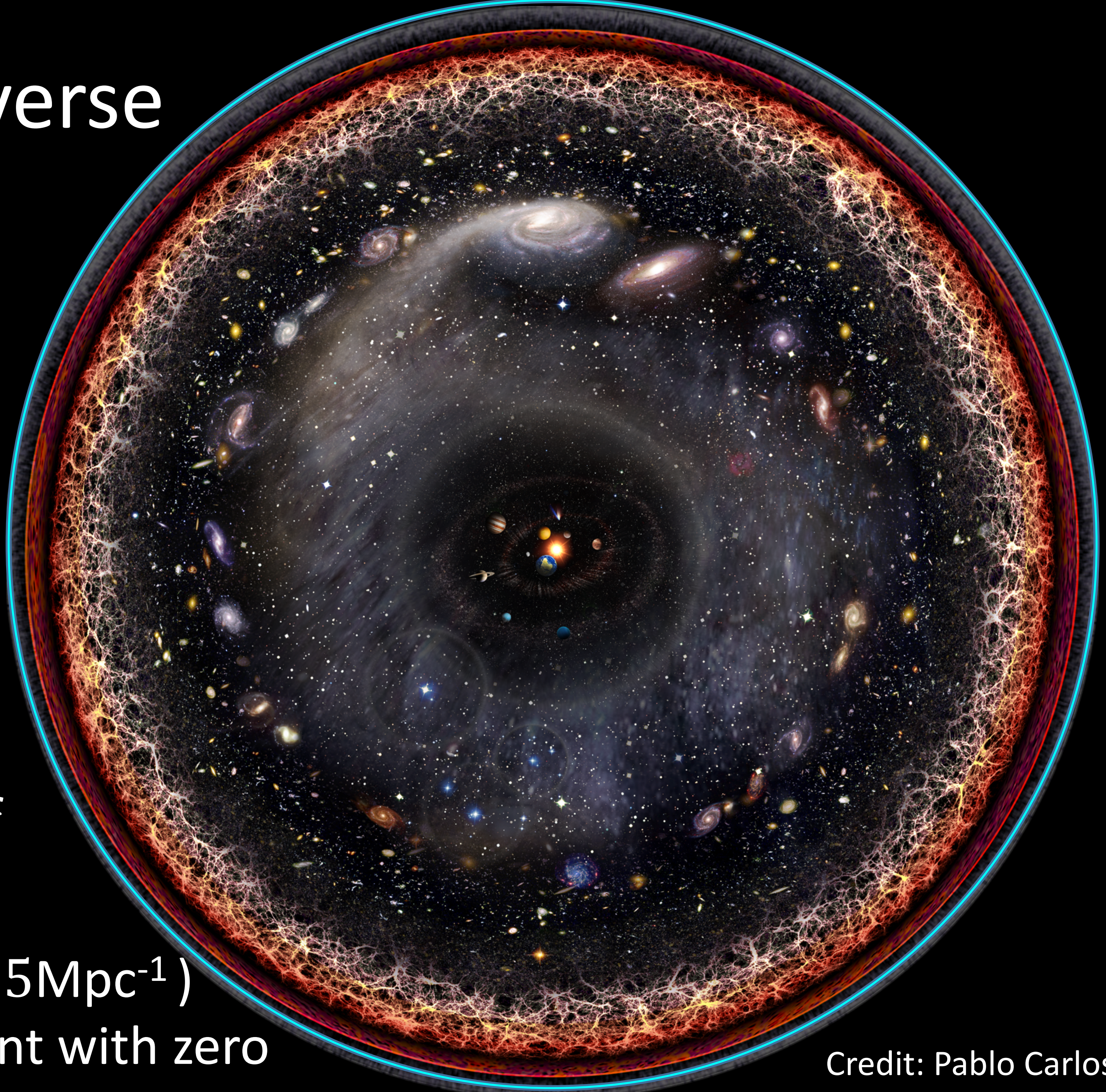
energy content

- n_B/n_γ
- ρ_{DM}/ρ_B
- ρ_Λ

comoving perturbations

- $rms\ R_k^{(3)} \sim k^2 A(\frac{k}{k_s})^{-\varepsilon}$
- $A \sim 7.0 \times 10^{-5}$
- $\varepsilon \sim 0.015 \quad (k_s \equiv 0.05\text{Mpc}^{-1})$

many quantities consistent with zero



Credit: Pablo Carlos Budassi

let’s explore the possibility that
the universe gets simpler as you look back to the bang

the puzzling large-scale geometry of the cosmos

Penrose

flatness closer to home…

one explanation

a better explanation

- the earth is large
  ($\sim 10^{50}$ atoms)
- gravity, dissipation, thermodynamics
  (entropy)

measures

any "solution" of cosmological puzzles, *e.g.,* horizon, flatness, homogeneity, Lambda … rests on some (perhaps intuitive) notion of a measure

in physics, measures arise from quantum statistical mechanics

is there a quantum statistical mechanics for cosmology?

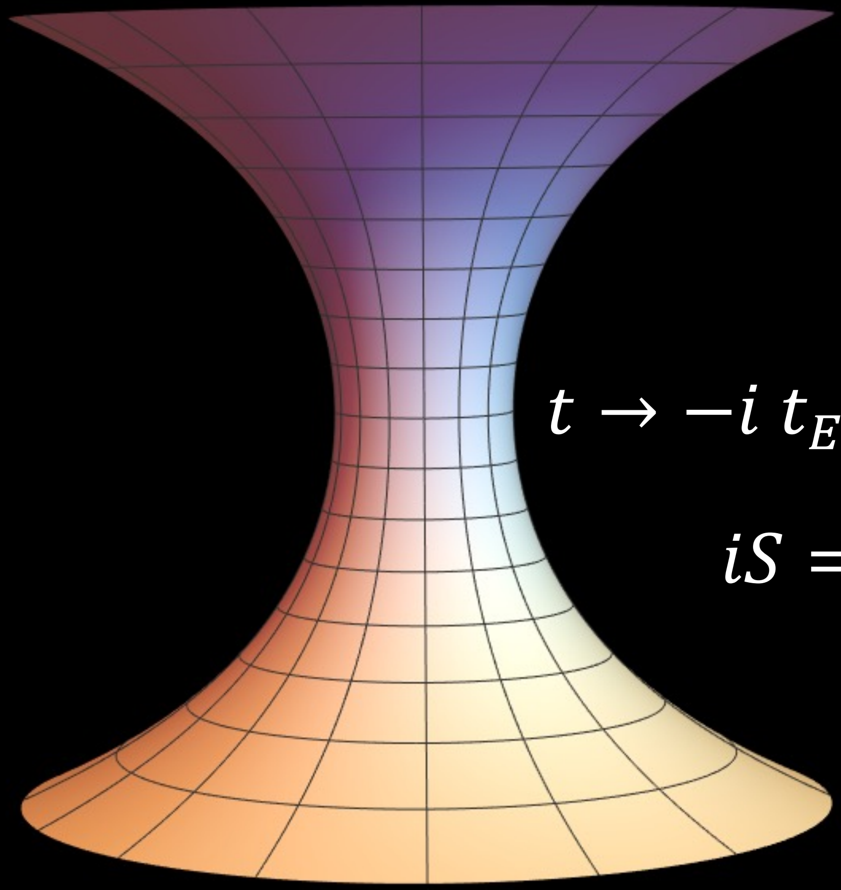
gravitational entropy:
black hole thermodynamics

Hawking
Bekenstein
Bardeen
Geroch
Gibbons
Hartle
Unruh
Wald

$$T_H = \frac{M_P^2}{M}; \quad S = \frac{A_{hor}}{4G} = \frac{M^2}{2M_P^2}; \; M_P^2 \equiv \frac{1}{8\,\pi G}; \; L_P^2 = 8\pi G$$

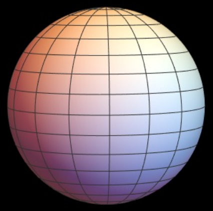
(in units where $c = \hbar = k_B = 1$)

de Sitter



Gravitational entropy calculated from the Euclidean path integral

S^4



$t \to -i\, t_E$

trace of Einstein eqs

$$R = \frac{4\rho_\Lambda}{M_P^2}$$

$$iS = -S_E = \int \left(\tfrac{1}{2} M_P^2 R - \rho_\Lambda\right) = \rho_\Lambda \,\mathrm{Vol} = \frac{24\pi^2 M_P^4}{\rho_\Lambda}$$

$$\equiv S_\Lambda \approx 3.26\times 10^{122} \ \text{ for measured } \rho_\Lambda$$

De Sitter Entropy

(this derivation hides an important sign!)

Cosmology with radiation plus Lambda – with compact space

$$\rho = \rho_r + \rho_\Lambda; \qquad \rho_r = \frac{r}{a^4 L_{Pl}^2}, \ \rho_\Lambda = \frac{\lambda}{L_{Pl}^2}; \ \ L_{Pl}^2 = 8\pi G$$

$$ds^2 = a(t)^2\left(-dt^2 + \gamma_{ij}dx^i dx^j\right) \ \ R^{(3)} = 6\kappa$$

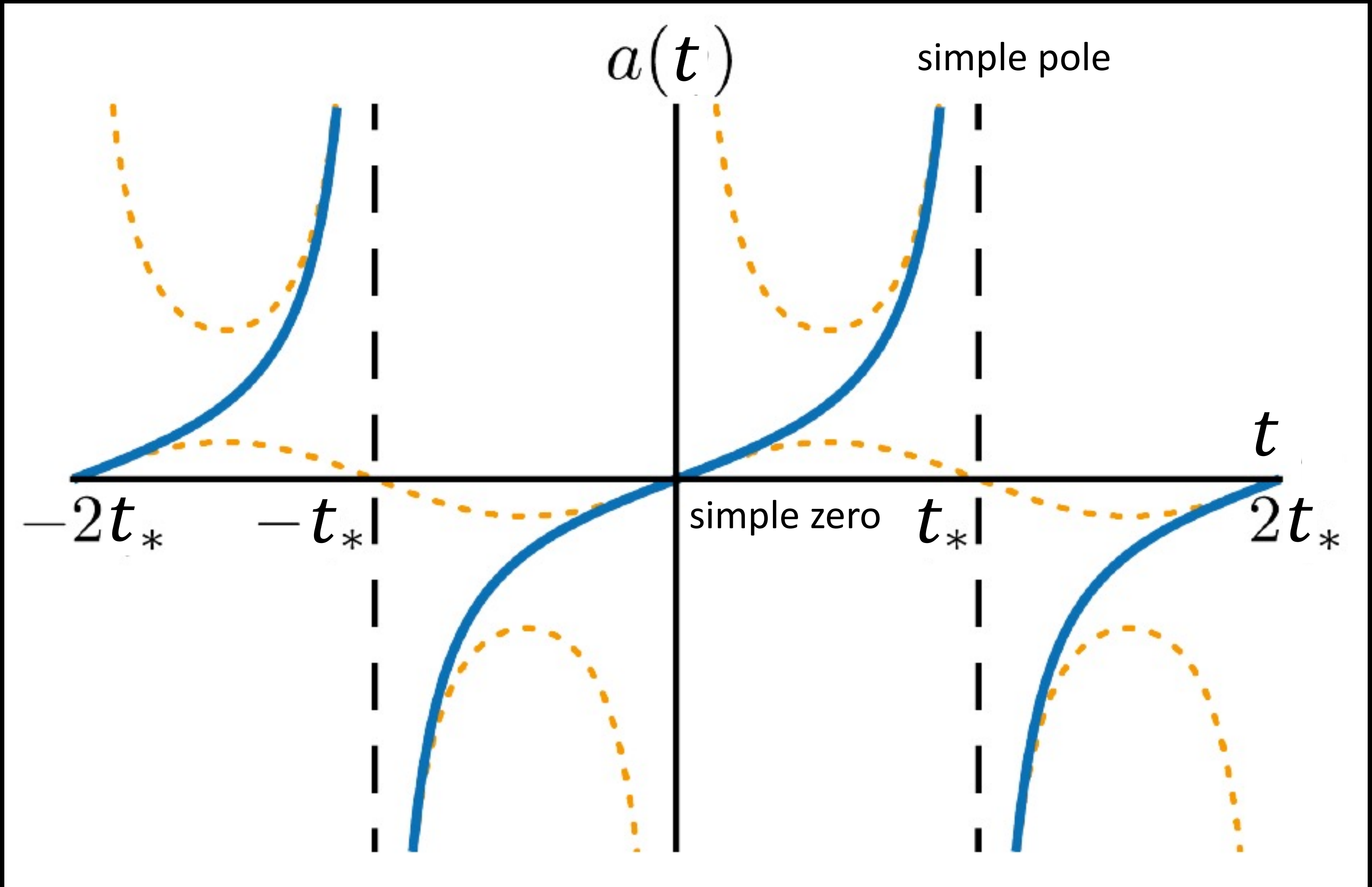
$[a] = L$
$[r] = L^2$
$[\lambda] = L^{-2}$
$[\kappa] = L^0$

Friedmann equation $\quad a'^2 = \frac{1}{3}\left(r - 3\kappa a^2 + \lambda a^4\right)$

Jacobi elliptic function

Solution $\ a(t) = \alpha \, \mathrm{sn}(\beta t, m); \quad \frac{m}{(1+m)^2} = \frac{\lambda r}{(3\kappa)^2}; \ \alpha = \sqrt{\frac{r(1+m)}{3\kappa}}; \ \beta = \sqrt{\frac{\kappa}{(1+m)}}$

has remarkable global analytical properties



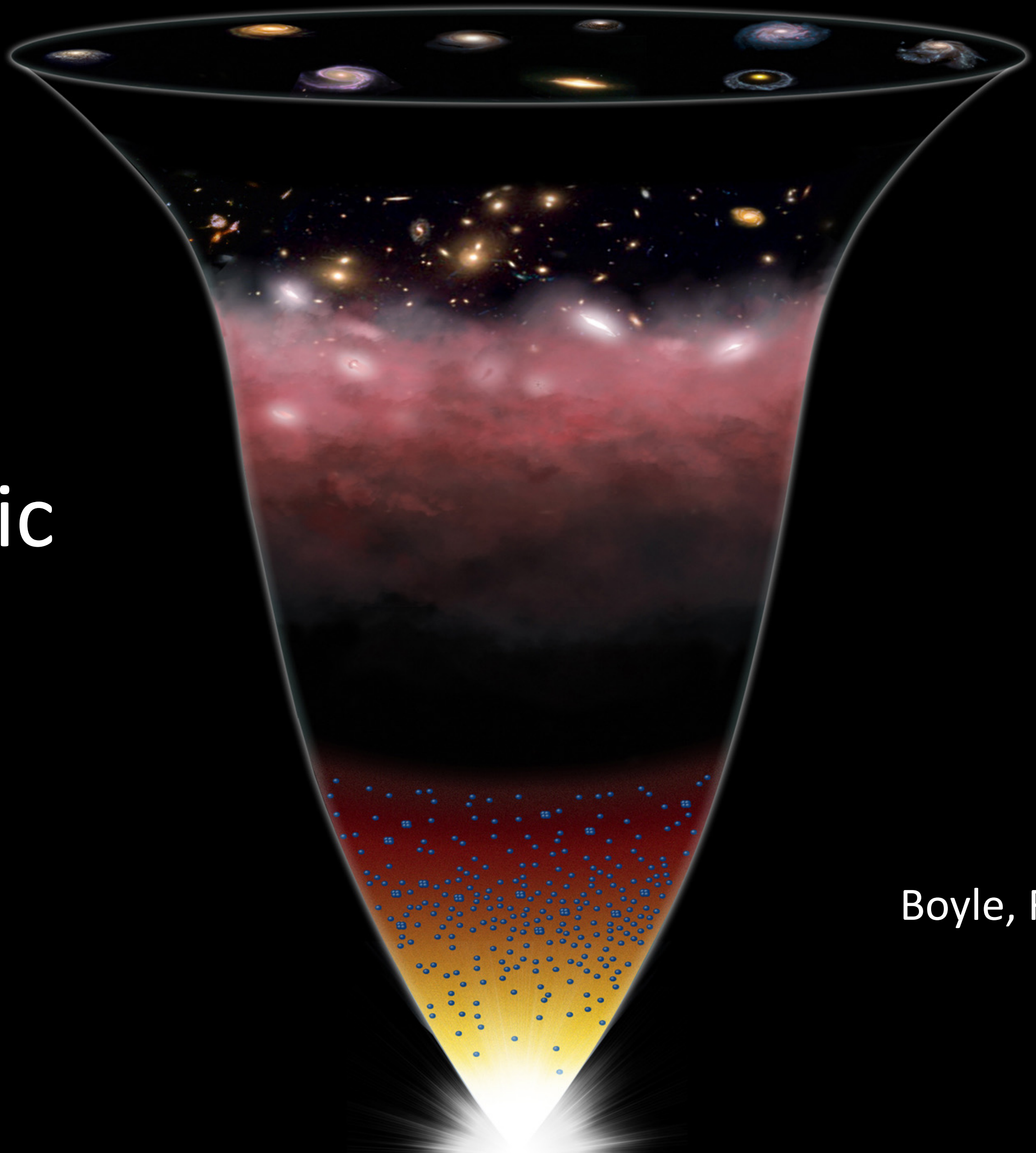
$a(t)$ is single-valued in the complex t-plane

Its only singularities are simple poles and it is doubly-periodic

Periodicity in imaginary time implies a Hawking temperature

L. Boyle and NT, hep-th/2109.06204

CPT symmetric universe



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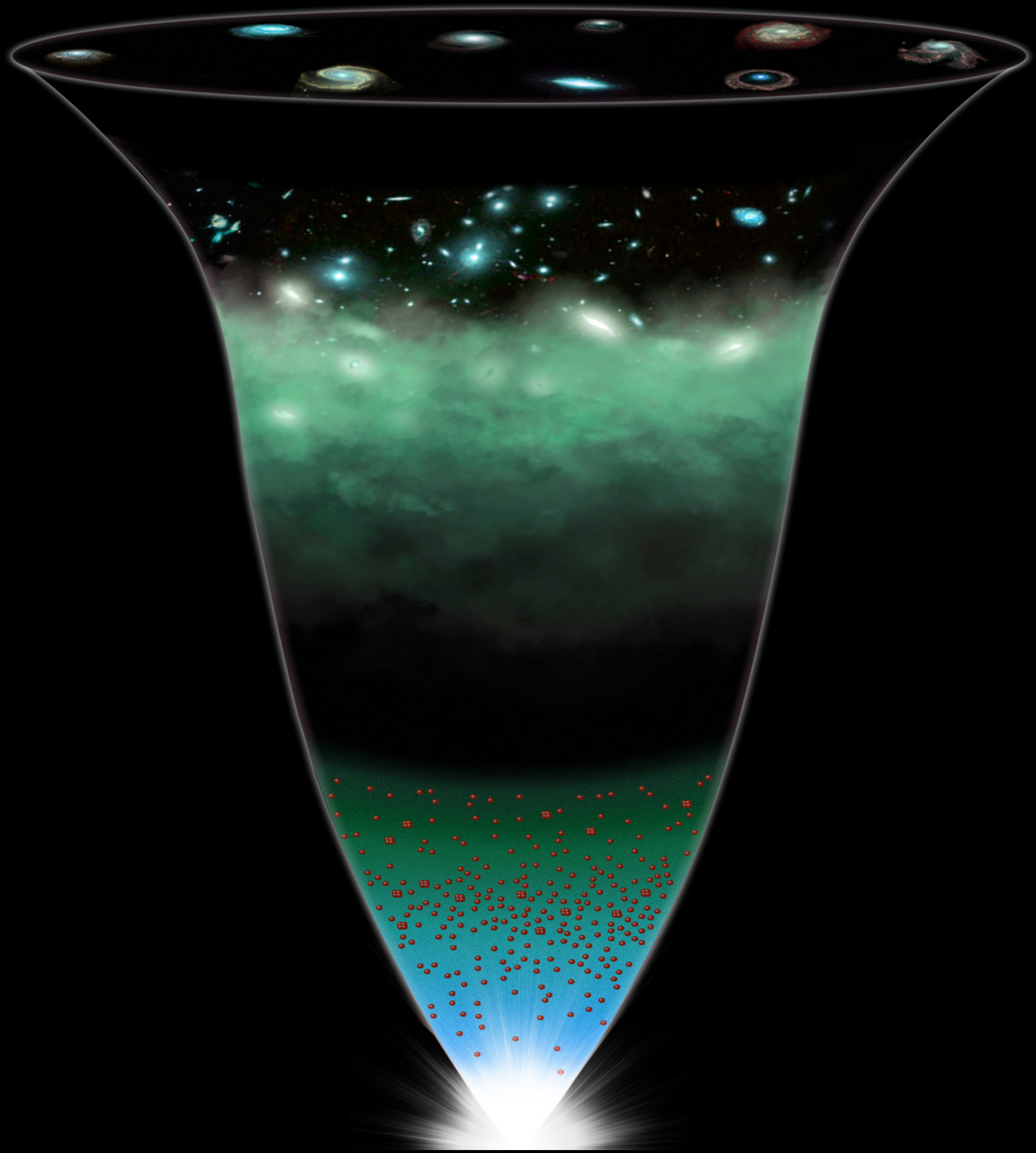
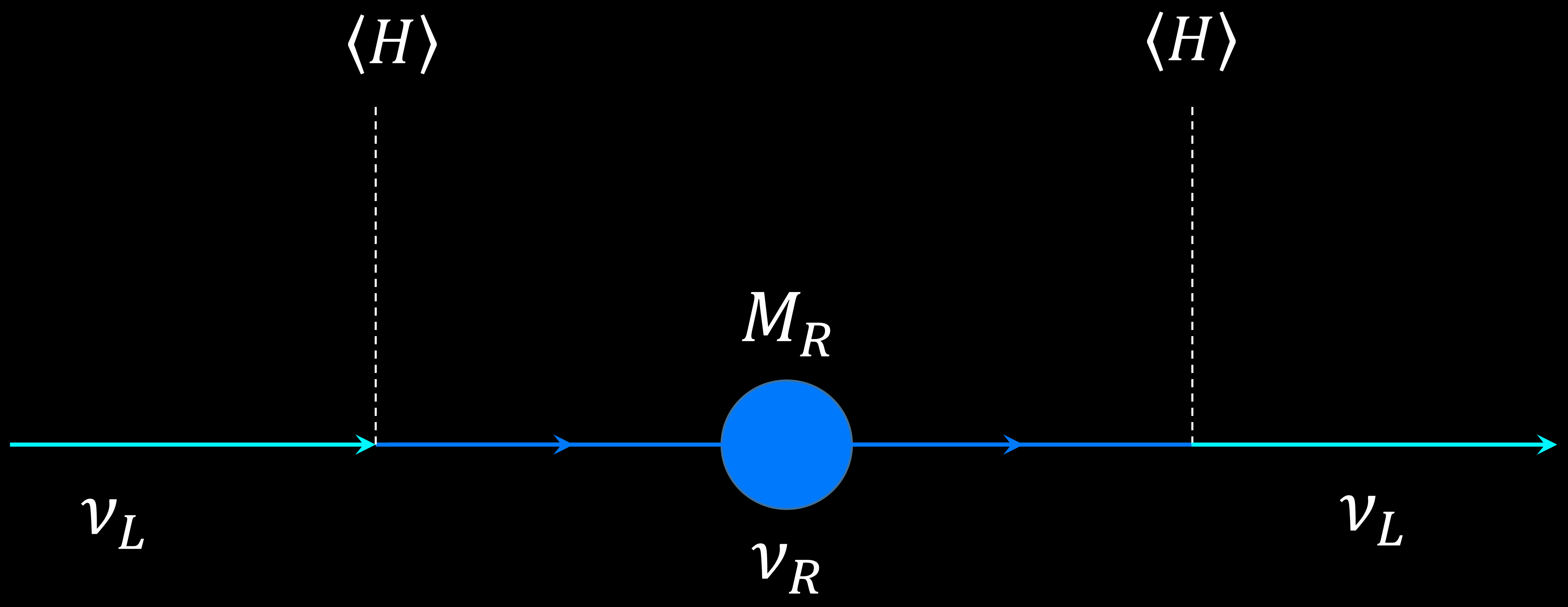


the big bang is a
``perfect CPT mirror”

“Conformal zero”
R. Newman (1993)
GR+conformal fluid

regular conformal 3-metric
provides complete Cauchy
data at the bang

seesaw mechanism and neutrino masses



By imposing perfect CPT symmetry, we predict the abundance of RH neutrinos. They account for the dark matter if their mass is $4.8\ 10^8$ GeV.

statistical mechanics for cosmology

1) out of equilibrium $T_r \gg T_{dS} \approx 10^{-39} T_{0,CMB}$

2) the Hamiltonian is zero – no canonical ensemble

3) Nevertheless, one can still define a statistical ensemble

4) Expansion is adiabatic $H \sim T_r^2 / M_P \ll T_r$

5) The radiation is in local thermal equilibrium: treat as a perfect fluid.

Path Integrals: amplitudes and partition functions

1) Amplitudes: pure state to pure state $\int Dx\ e^{iS}$

P-L theorem: for any relevant saddle, $|e^{iS}| \leq 1$ $\Rightarrow$ semiclassical exponents *cannot* be positive

(series of papers by Feldbrugge+Lehners+NT)

(forthcoming paper on real time path integrals by Feldbrugge+NT)

2) Partition functions: $Z = Tr\left(e^{-\beta H}\right) = e^{S-E/T}$

S is often exponentially large

For gravity, H annihilates physical states so $Z =$ number of physical states

$Z = e^S > 1 \Longrightarrow S > 0$ semiclassical exponents *must* be positive

$Z = \mathrm{Tr}(\int dn e^{-iHn})$; $H = H_g + H_r$

Trace over radiation as conformal-invariant matter at temperature $T = \beta^{-1}$

Hence $\mathrm{Tr}_r\left(e^{-\beta H_r}\right) = e^{S_r - \beta U_r}$; S_r is the (adiabatically conserved) total entropy in radiation, $U_r(S_r)$ is the associated internal energy

Set $\beta = in$ to obtain $Z = e^{S_r}\,\mathrm{Tr}_g\left(\int dn\, e^{-i(H_g+U_r)n}\right)$; path integral for cosmology with

$$iS = i\frac{V}{L_{Pl}^2}\int dt\, n(-3\frac{\dot{a}^2}{n^2} + 3\kappa a^2 - \lambda a^4 - r)$$

where $U_r = r\,V/L_{Pl}^2$; $L_{Pl}^2 \equiv 8\pi G$

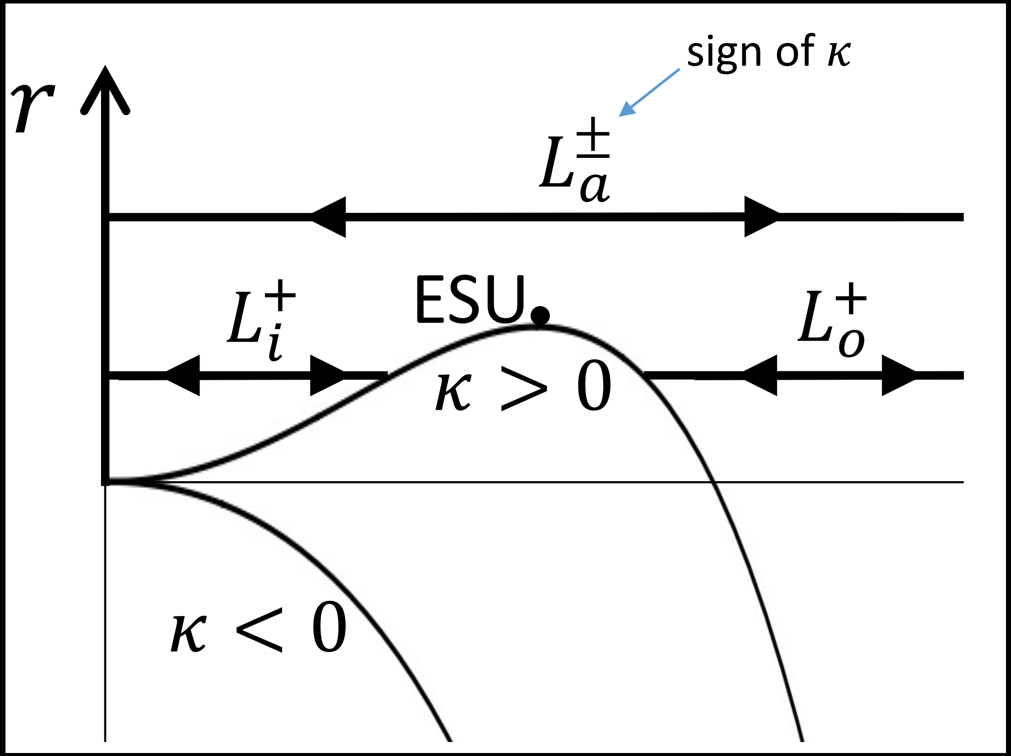
Note: odd in n
- this is the sign

Cosmology with Lambda, radiation & space curvature

Friedmann $3a'^2 + \underbrace{3\kappa a^2 - \lambda a^4}_{V_{eff}(a)} = r;$ $\quad \rho_r = \frac{r}{a^4 L_{Pl}^2};\ \rho_\Lambda = \frac{\lambda}{L_{Pl}^2};$ $\quad$ for $\kappa > 0$, $r_{ESU} = \frac{9\,\kappa^2}{4\lambda}$

Einstein static universe
(no horizon, no gravitational entropy)

Real time



Two ways to go Euclidean

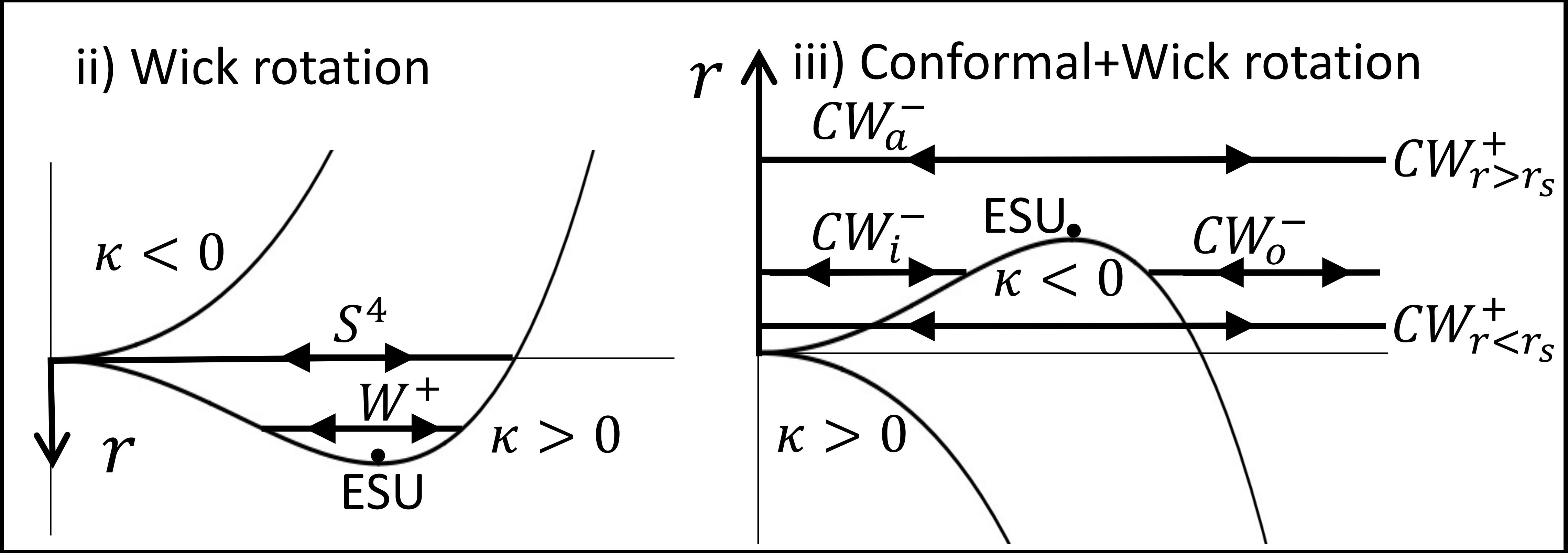
$$ds^2 = a^2(nt)(-n^2 dt^2 + \gamma_{ij} dx^i dx^j)$$

$a(nt)$ even Wick $n = -iN, N$ real

$a(nt)$ odd Wick plus Conformal $a(nt) = -ib(Nt), b(t)$ real

(“conformal rotation” C;
Gibbons+Hawking+Perry)

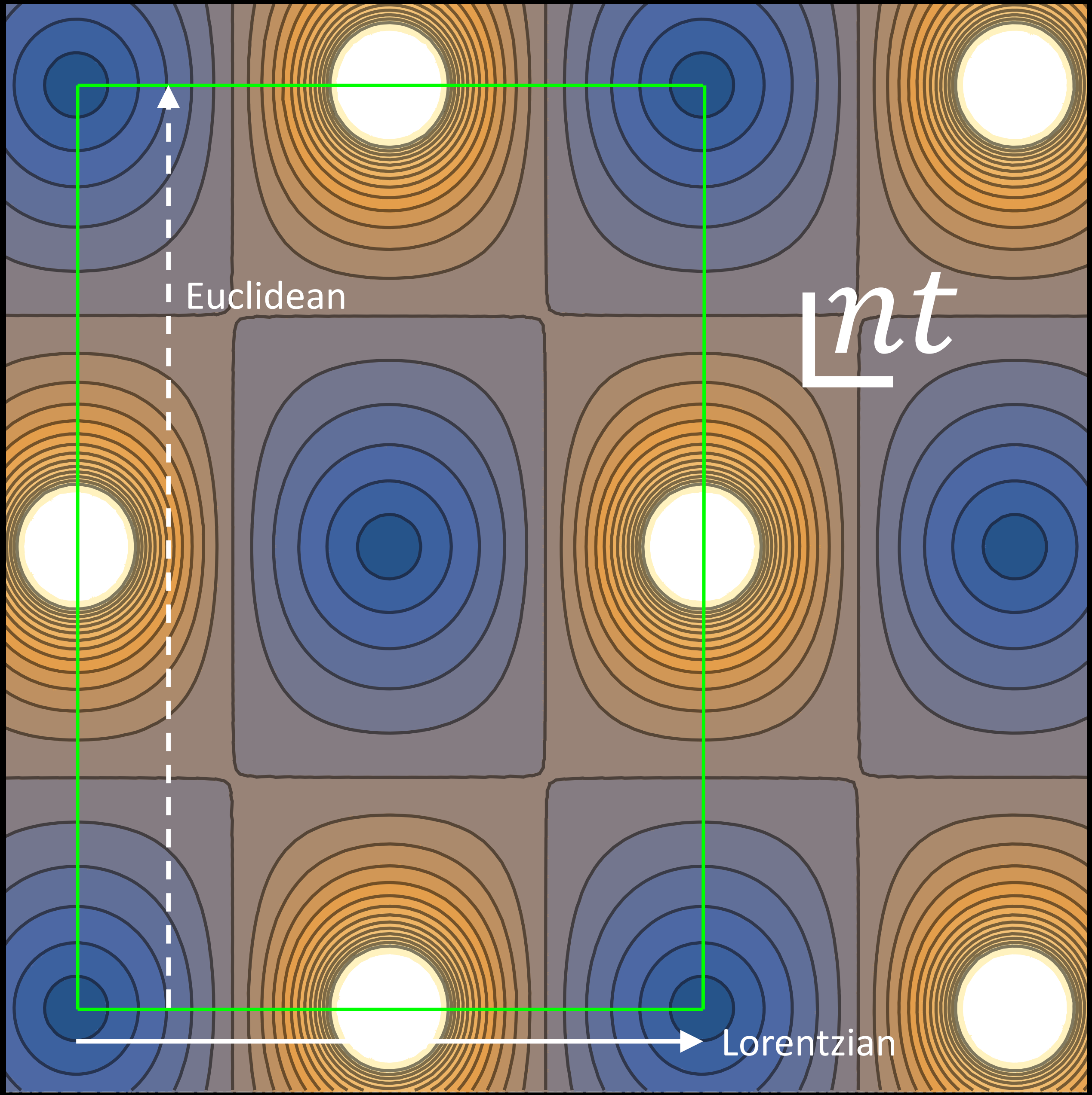
Imaginary time



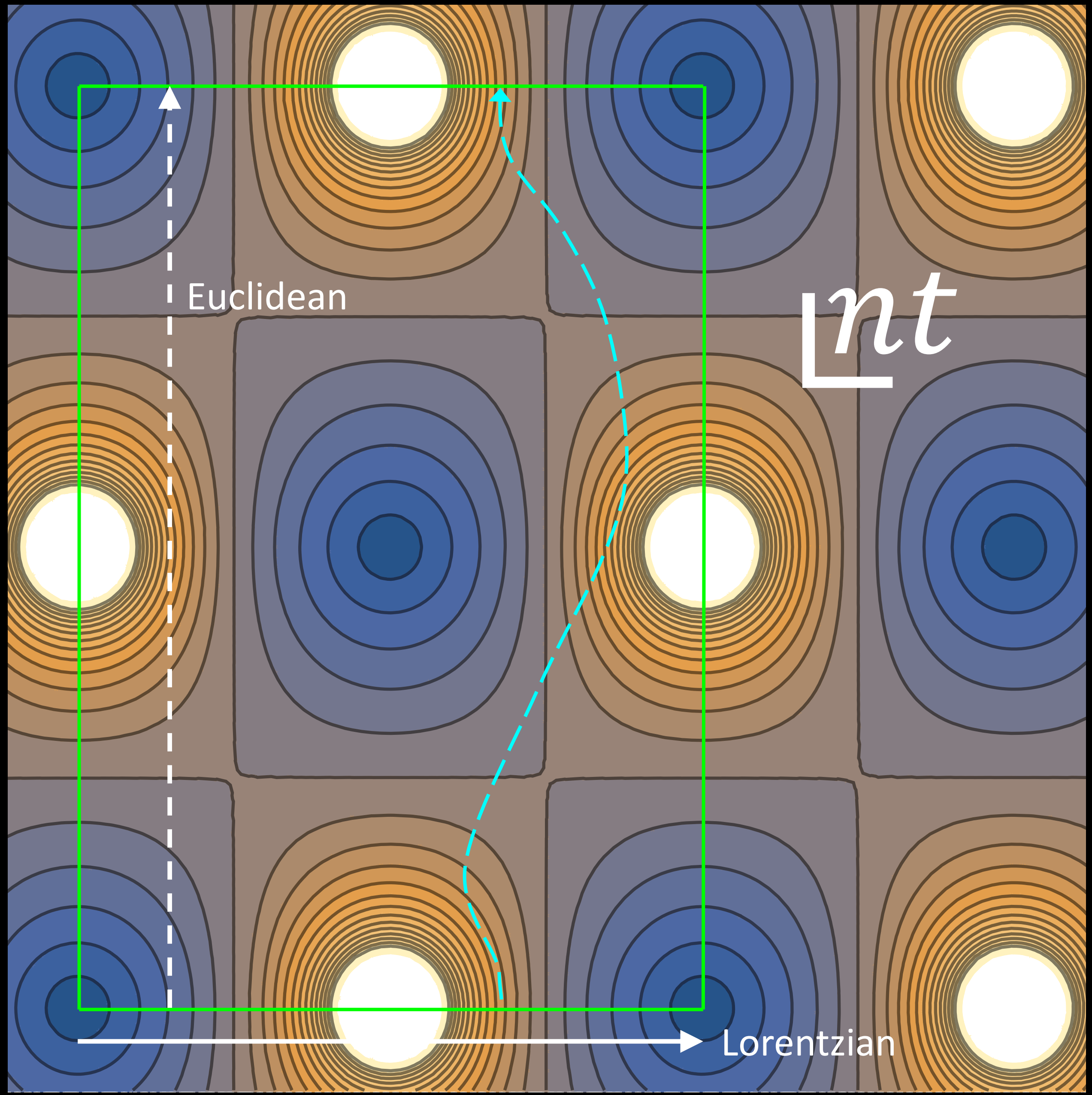
yields instantons only for $\kappa > 0, r \leq r_{ESU}$

yields instantons for all values of $\kappa, \lambda > 0, r > 0$

$$|a(nt)|$$



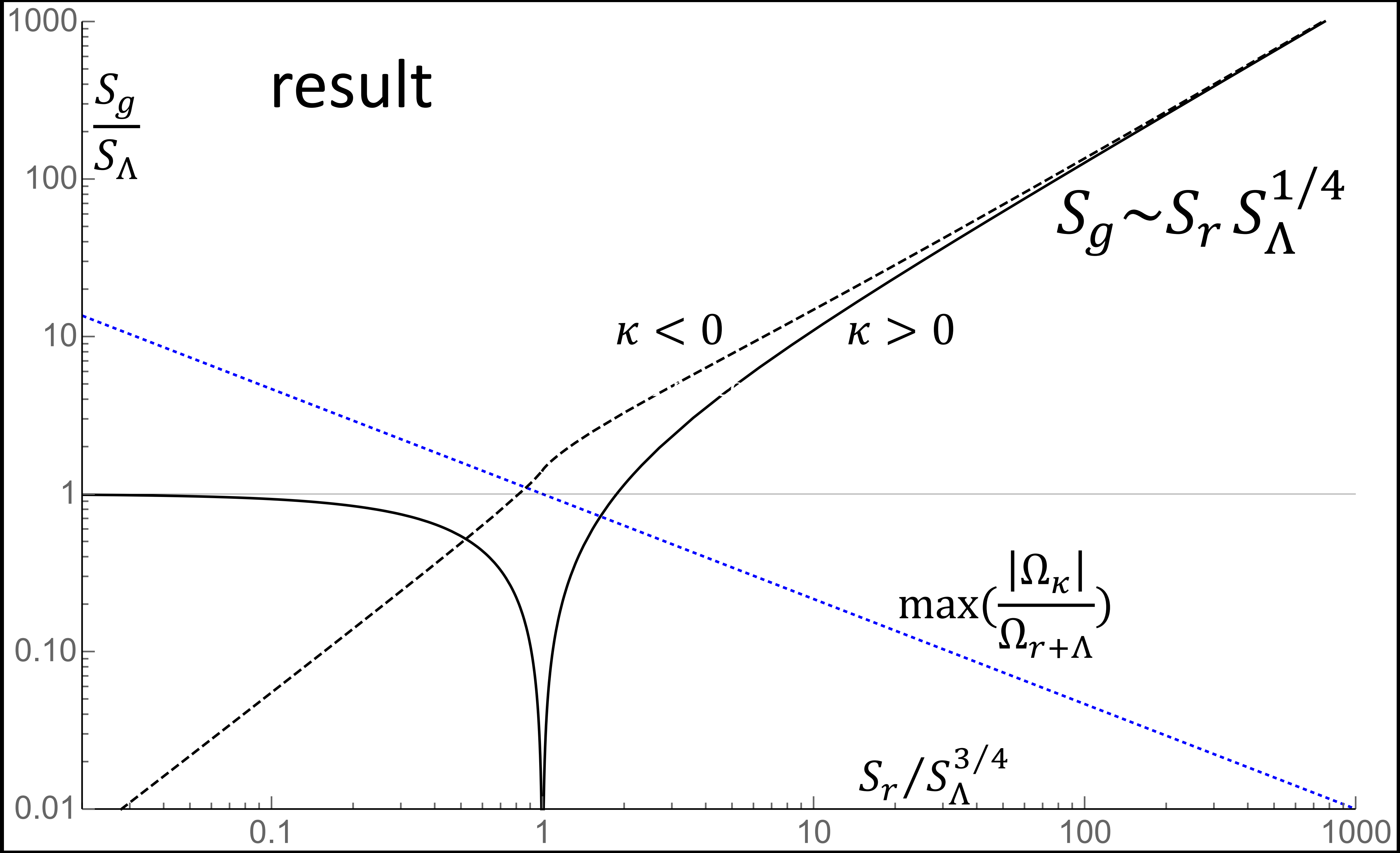
$$|a(nt)|$$



action is topological: invariant under contour deformations

Case	scale factor $a(t)$ or $b(t)$	N	S_g/S_Λ	m	$z_\pm$
W^+	$\sqrt{\frac{3\kappa}{\lambda(1+m)}}\mathrm{dn}(\sqrt{\frac{\kappa}{1+m}}Nt, 1-m)$	$+\sqrt{\frac{1+m}{\kappa}}2K(1-m)$	$+\frac{(1+m)E(1-m)-2mK(1-m)}{(1+m)^{3/2}}$	$e^{-\alpha}$	$e^{\pm\alpha/4}$
$CW^+_{r<r_s}$	$i\sqrt{\frac{3\kappa m}{\lambda(1+m)}}\mathrm{sn}(i\sqrt{\frac{\kappa}{1+m}}Nt, m)$	$+\sqrt{\frac{1+m}{\kappa}}2K(1-m)$	$+\frac{(1+m)E(1-m)-2mK(1-m)}{(1+m)^{3/2}}$	$e^{-\alpha}$	$e^{\pm\alpha/4}$
$CW^+_{r>r_s}$	$i\sqrt{\frac{3\kappa m}{\lambda(1+m)}}\mathrm{sn}(i\sqrt{\frac{\kappa}{1+m}}Nt, m)$	$-\sqrt{\frac{1+m}{\kappa}}2K(1-m)$	$-\frac{(1+m)E(1-m)-2mK(1-m)}{(1+m)^{3/2}}$	$e^{i\theta}$	$e^{\pm i\theta/4}$
CW^-_i	$\sqrt{\frac{3\|\kappa\|m}{\lambda(1+m)}}\mathrm{sn}(\sqrt{\frac{\|\kappa\|}{1+m}}Nt, m)$	$-\sqrt{\frac{1+m}{\|\kappa\|}}2K(m)$	$2\frac{(1+m)E(m)-(1-m)K(m)}{(1+m)^{3/2}}$	$e^{-\alpha}$	$\pm ie^{-\alpha/4}$
CW^-_a	$\sqrt{\frac{3\|\kappa\|m}{\lambda(1+m)}}\mathrm{sn}(\sqrt{\frac{\|\kappa\|}{1+m}}Nt, m)$	$-\mathrm{Re}\left(\sqrt{\frac{1+m}{\|\kappa\|}}2K(m)\right)$	$\mathrm{Re}\left(2\frac{(1+m)E(m)-(1-m)K(m)}{(1+m)^{3/2}}\right)$	$e^{i\theta}$	$\pm ie^{\mp i\theta/4}$

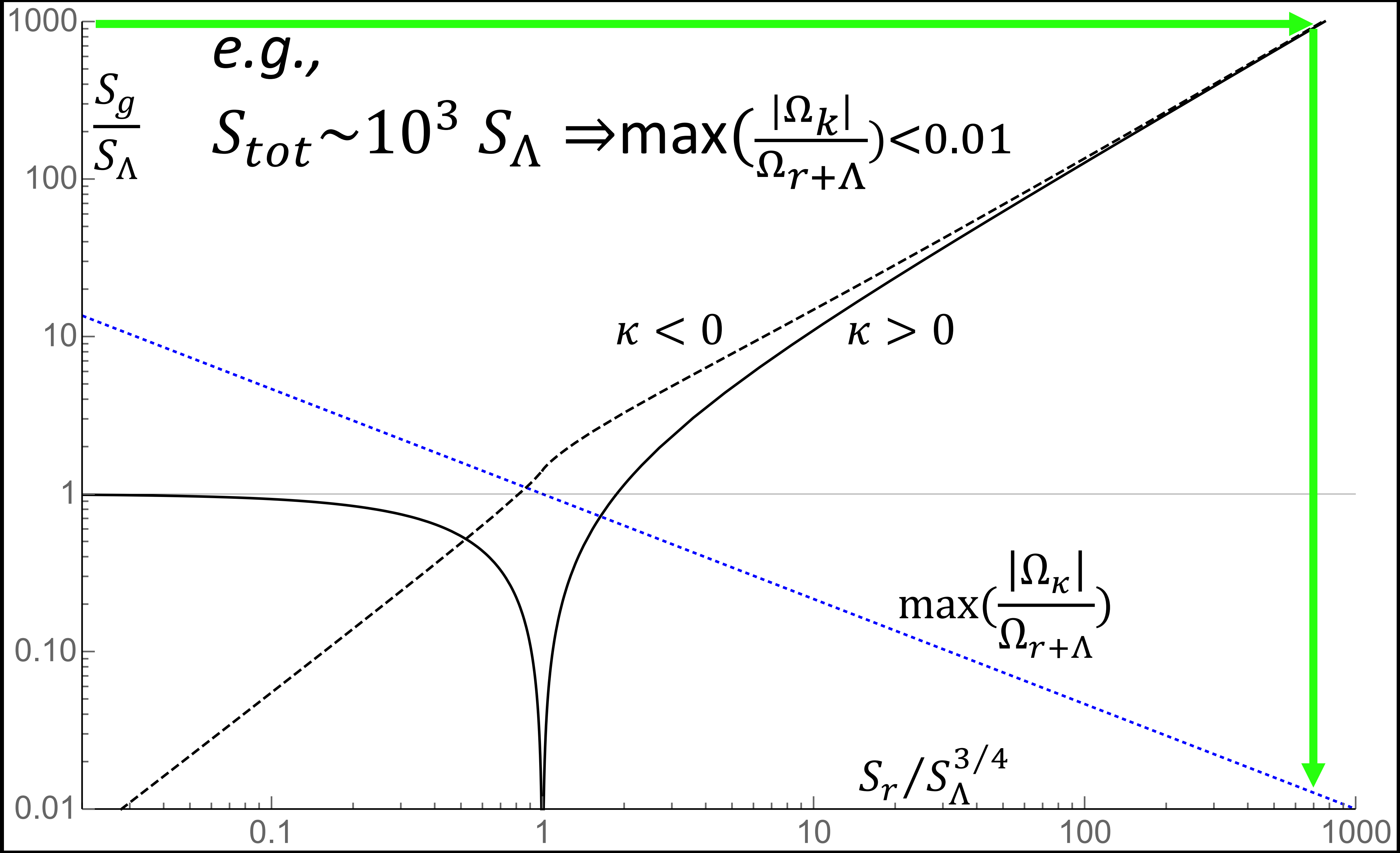
Table I: Analytic forms for Euclidean instantons (Fig. 1, ii) and iii)), periodic in t with period unity, from which the Lorentzian solutions (Fig. 1, i)) are obtained by analytic continuation. Here, $\mathrm{sn}(z,m)$, $\mathrm{dn}(z,m)$ are Jacobi elliptic functions and $K(m)$, $E(m)$ are complete elliptic integrals [23]. W denotes Wick rotation and CW combined conformal and Wick rotations. Superscript $\pm$ indicates $\kappa \lessgtr 0$; m satisfies $m/(1+m)^2 = \lambda r/(3\kappa)^2$. For $0 < r \leq r_s$, $m = e^{-\alpha}$ with $0 \leq \alpha < \infty$, and for $r \geq r_s$, $m = e^{i\theta}$ with $0 \leq \theta < \pi$



result
$\frac{S_g}{S_\Lambda}$
$S_g \sim S_r\, S_\Lambda^{1/4}$
$\kappa < 0$
$\kappa > 0$
$\max(\frac{|\Omega_\kappa|}{\Omega_{r+\Lambda}})$
$S_r / S_\Lambda^{3/4}$
1000
100
10
1
0.10
0.01
0.1
1
10
100
1000

can understand large S_g scaling by counting

dS horizons at Lambda-radiation equality



e.g.,
$S_{tot} \sim 10^3\ S_\Lambda \Rightarrow \max(\frac{|\Omega_k|}{\Omega_{r+\Lambda}}) < 0.01$
$\frac{S_g}{S_\Lambda}$
$\kappa < 0$
$\kappa > 0$
$\max(\frac{|\Omega_\kappa|}{\Omega_{r+\Lambda}})$
$S_r/S_\Lambda^{3/4}$
1000
100
10
1
0.10
0.01
0.1
1
10
100
1000

scalar, tensor, conformal perturbations decrease the gravitational entropy

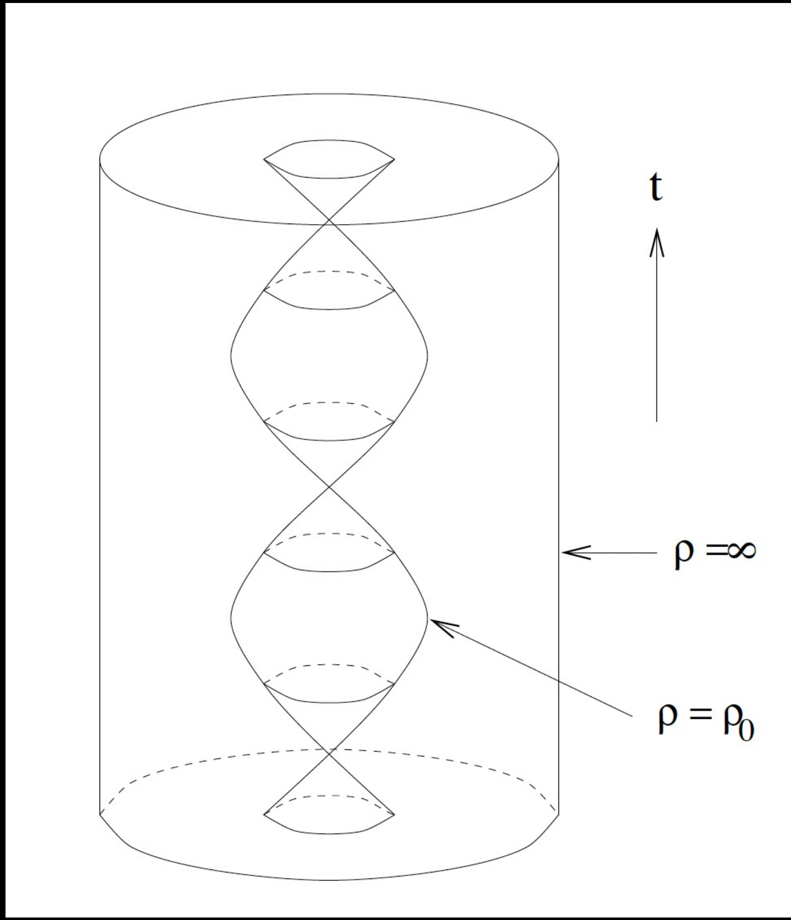
most probable universe is homogeneous, isotropic and flat on large scales

Much remains to be done:

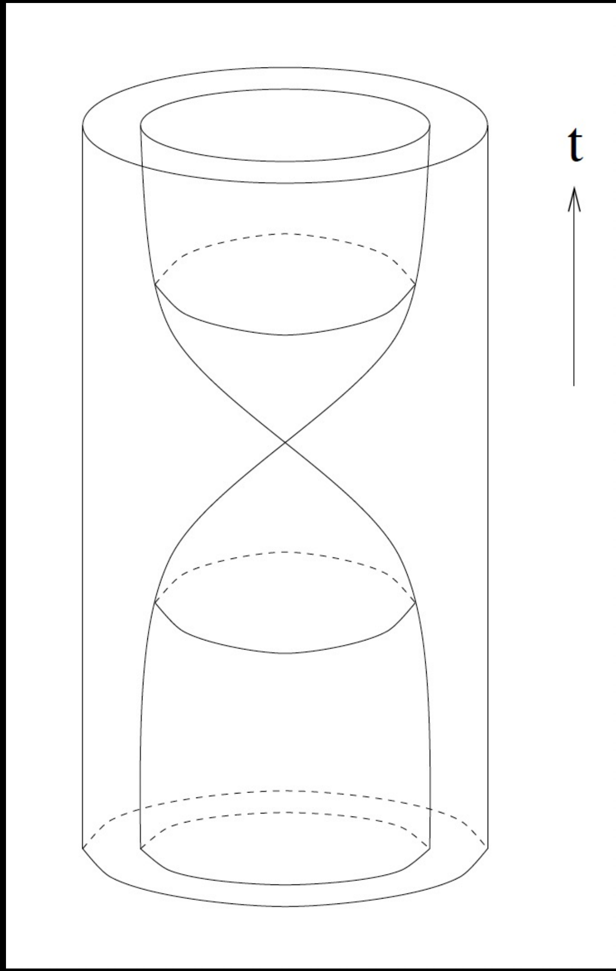
- explicitly construct the quantum ensemble, show instantons are its saddles
- generalize to non-conformal invariant matter & include black holes
- string theory as a 1+1 model cosmology with a big bang and horizons
- origin of perturbations – from conformal anomaly (Boyle+NT, in prep)

Thank You!

short strings in AdS_3



long strings



1+1 analog model: string loop in AdS_3 with constant 3-form

w/ Malcolm Perry

See, e.g., Maldacena and Ooguri, hep-th/0001053 [hep-th] and references therein

Equations of motion in orthonormal gauge

$$-\ddot{x}^{\mu} + x''^{\mu} + \Gamma^{\mu}_{\nu\lambda}\left(-\dot{x}^{\nu}\dot{x}^{\lambda} + x'^{\nu}x'^{\lambda}\right) + H^{\mu}{}_{\nu\lambda}\dot{x}^{\nu}x'^{\lambda} = 0;$$

$$H^{\mu\nu\lambda} = H\varepsilon^{\mu\nu\lambda};\ \text{constraints}\quad \dot{x}^2 + x'^2 = 0,\ \dot{x}x' = 0$$

$$x^{\mu} = (T(\tau), R(\tau)\cos\sigma, R(\tau)\sin\sigma)\,;\quad ds_2^2 = R^2(\tau)(-d\tau^2 + d\sigma^2)$$

circular loop

$$\Rightarrow\qquad -\dot{T}^2(1+R^2) + \dot{R}^2(1+R^2)^{-1} + R^2 = 0\ \ (1)$$

$$-\ddot{T} - \frac{2R}{1+R^2}\dot{R}\dot{T} - \frac{4R}{1+R^2}\dot{R} = 0\ (2)$$

$$(2)\Rightarrow\ \ \dot{T} = -\frac{(2R^2+d)}{1+R^2}\qquad (1)\Rightarrow\ \dot{R}^2 - 3R^4 - R^2(4d-1) = d^2$$

$$R(\tau) = A\,sn\!\left(\frac{d}{A}t, m\right);\ A = \frac{d\sqrt{1+m}}{4d-1}$$

Jacobi elliptic solutions, again

Vacuum energy and conformal anomalies

dim-one scalars, dim-zero scalars, Weyl or Majorana fermions, gauge fields

$$E_{\boldsymbol{k}} = \frac{1}{2}\hbar k(n_0 + 2n_{0'} - 2\,n_{1/2} + 2n_1)$$

$$C^2 = C^{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta};$$

$$E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$$

adjustable

$$\langle T^{\mu}_{\mu}\rangle = c\,C^2 - a\,E + \xi\Box R:$$

$$a = \frac{1}{360(4\,\pi)^2}\left[n_0 - 28\,n_{0'} + \frac{11}{2}n_{1/2} + 62\,n_1\right]$$

$$c = \frac{1}{120(4\,\pi)^2}\left[n_0 - 8\,n_{0'} + 3\,n_{1/2} + 12\,n_1\right]$$

Unique solution assuming the gauge group is SU3xSU2xU1:

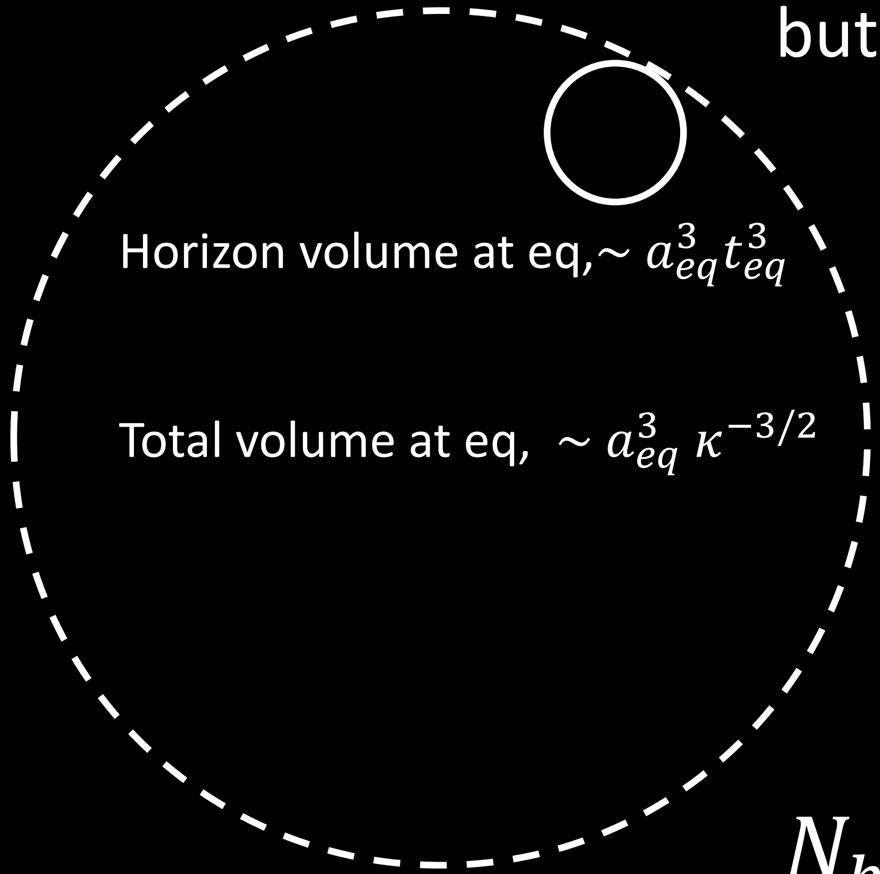
$$n_{1/2} = 4n_1 = 48;\ \ n_{0'} = 3n_1 = 36;\ \ n_0 = 0.$$

Requires precisely three generations, each containing a RH neutrino

understanding in terms of horizons

$3\dot{a}^2 = -3\kappa a^2 + r + \lambda a^4$; equal Λ, radiation density at $a_{eq} = (r/\lambda)^{1/4}$

but for $a < a_{eq}$, $a \sim r^{1/2} t$ so $t_{eq} \sim 1/(r\lambda)^{1/4}$



number of horizon volumes at equality

$$N_{hor} \sim (\lambda r/\kappa^2)^{3/4}$$

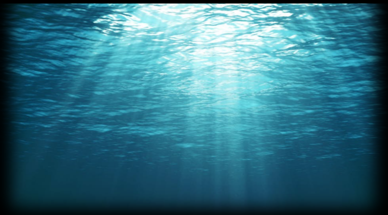
multiply by de Sitter entropy λ^{-1}

$$N_{hor}\lambda^{-1} \sim (r/\kappa^2)^{3/4}\ \lambda^{-1/4} \sim S_r S_\Lambda^{1/4}$$

Can we explain nature's observed (and surprising) simplicity with simple and highly predictive principles?

gauge fields

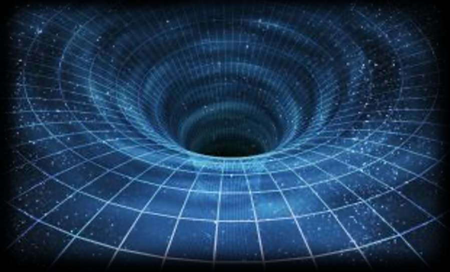
Higgs



$$\Psi = \int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi} i \not{D}\psi - \lambda H\overline{\psi}\psi + |DH|^2 - V(H)\right)}$$

dark energy

particles



gravity

a program of foundational research is called for, *e.g.*,

Do (real-time) path integrals exist? Feldbrugge and NT, forthcoming preprint

Large Scale Universe

remarkably simple fit,
with just 5 parameters

energy content

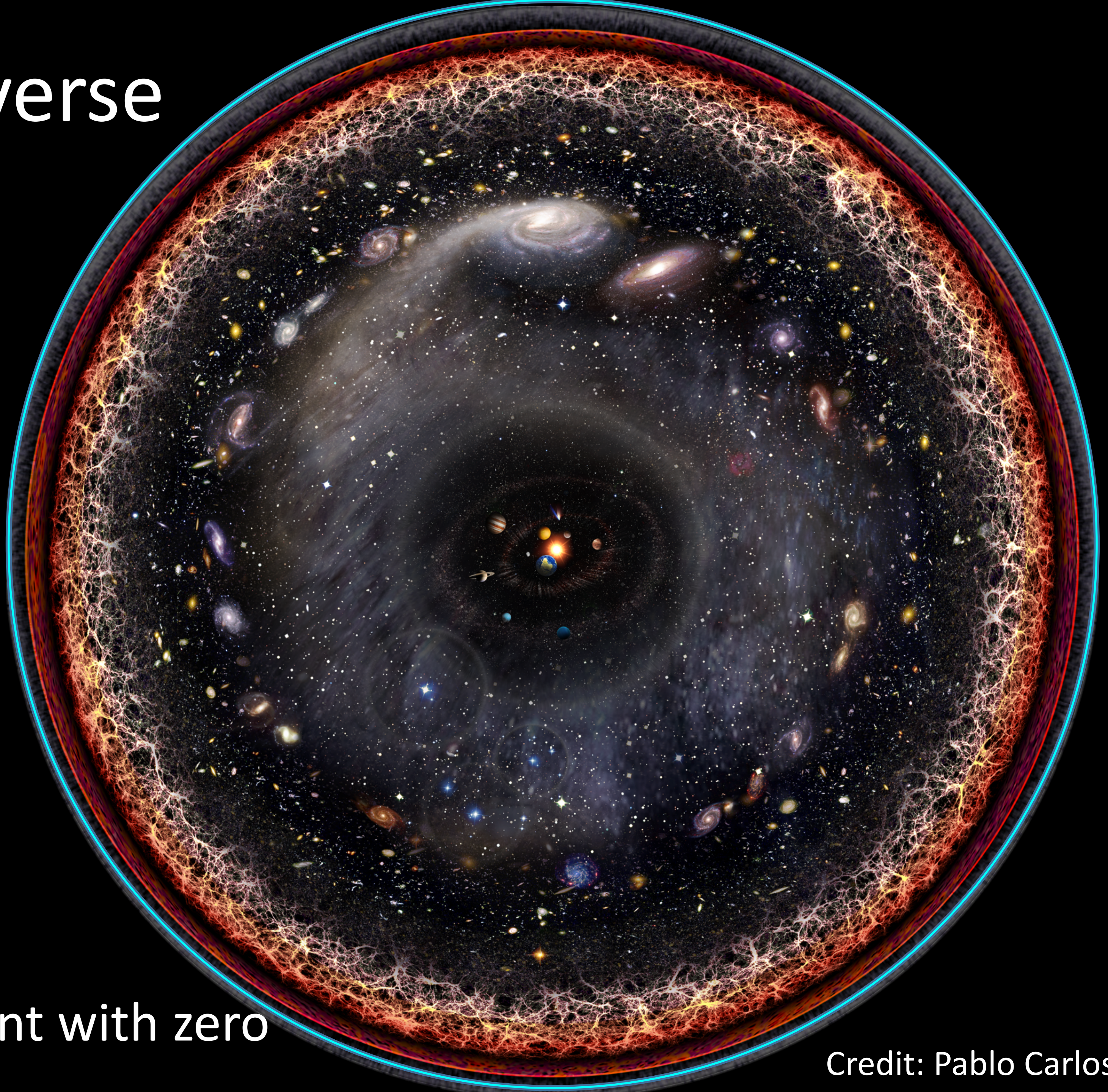
- n_B/n_γ
- ρ_{DM}/ρ_B
- ρ_Λ

comoving curvature

$k^{-2}R_k^{(3)} \sim A_k k^{-\varepsilon}$

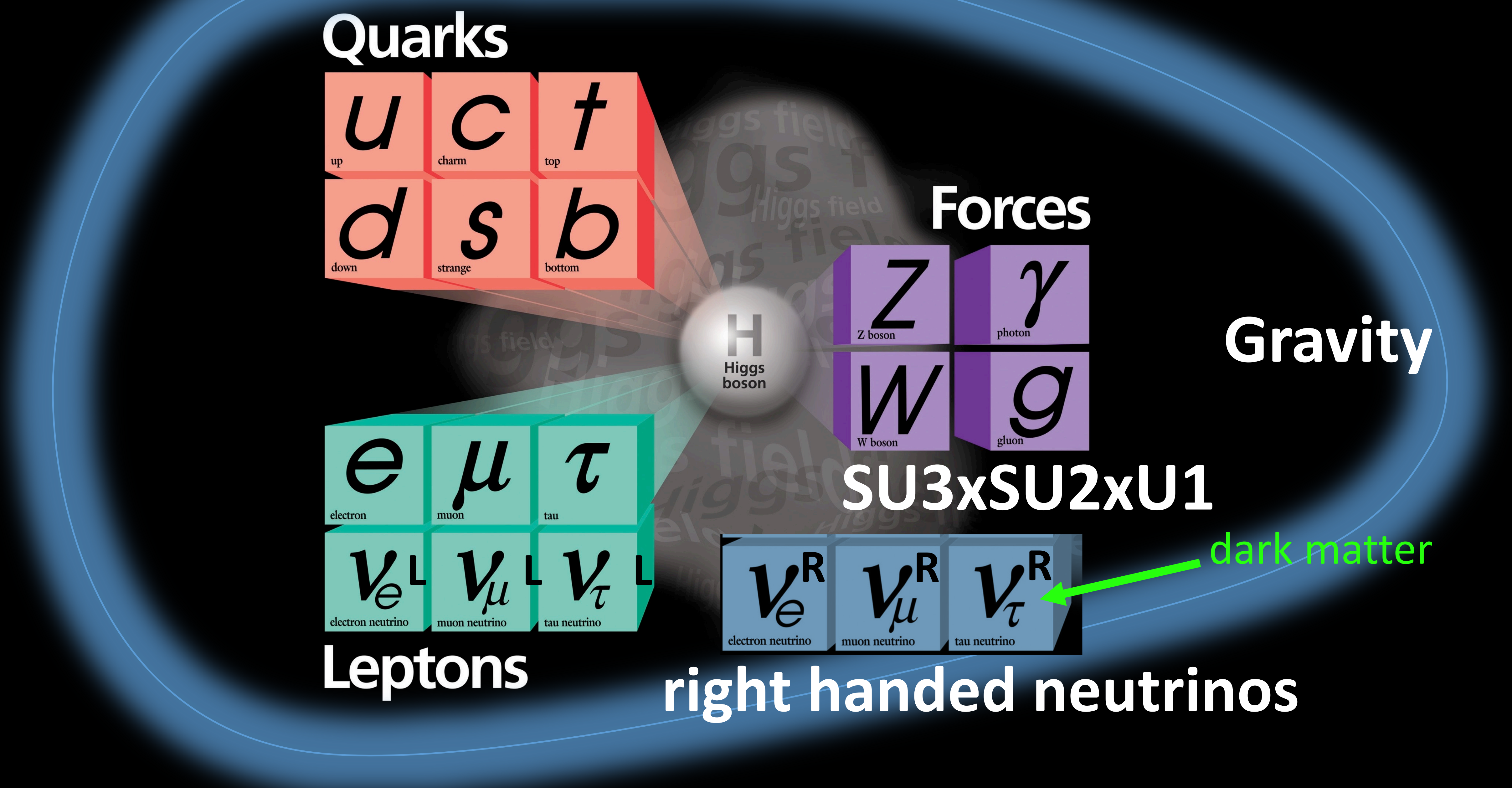
- $A_{k,rms} \sim 7.0\times10^{-5}$
- $\varepsilon \sim 0.017$

many quantities consistent with zero

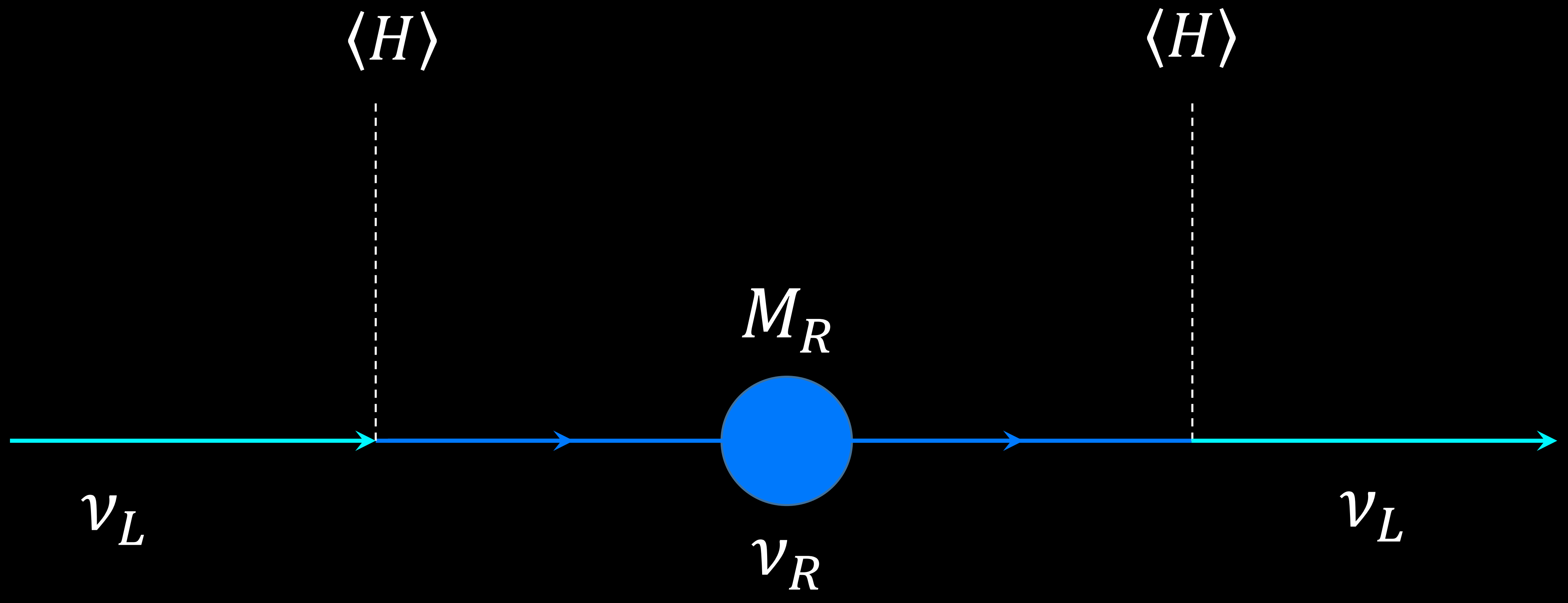


Credit: Pablo Carlos Budassi

Small Scale Universe



seesaw mechanism and neutrino masses



Trace anomaly: QFT in curved spacetime

Hilbert

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}};$$

Weyl

$$\delta g_{\mu\nu} = \varepsilon g_{\mu\nu} \Rightarrow \frac{\delta S}{\delta \varepsilon} = -\frac{\sqrt{-g}}{2} T^{\mu}_{\mu}$$

However, the quantum stress tensor is divergent (products of fields at the same point). After renormalization, one finds Weyl symmetry is broken

$$\langle T^{\mu}_{\mu} \rangle = c\, C^2 - aE + \xi \Box R;$$

$C^2 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 2R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{3}R^2$ Weyl curvature squared

$E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$ Euler density

c, a depend on the matter content, cannot be altered by local counterterms

ξ may be altered by adding a local (R^2) counterterm to the Lagrangian

(there are also contributions from the running of couplings)

Scalar Lagrangians

Dimension-one scalars have a two-derivative, Weyl invariant action

$$S_2 = \frac{1}{2}\int d^4x\sqrt{g}\, g^{\mu\nu}\partial_\mu H\partial_\nu H + \cdots \qquad H(x) \longrightarrow \Omega(x)^{-1}H(x)$$

Dimension-zero scalars have a four-derivative Weyl-invariant action

$$S_4 = \frac{1}{2}\int d^4x\sqrt{g}(\Box\varphi)^2 + \cdots \qquad \varphi(x) \longrightarrow \varphi(x)$$

These were considered in higher-derivative UV finite theories, thought to be plagued by ghosts.

However, their Euclidean action is positive. We have recently shown them to possess a well-defined partition function, suggesting the quantum theory is healthy.

The theory is unusual: the vacuum is the only energy eigenstate. There are no particle-like states.