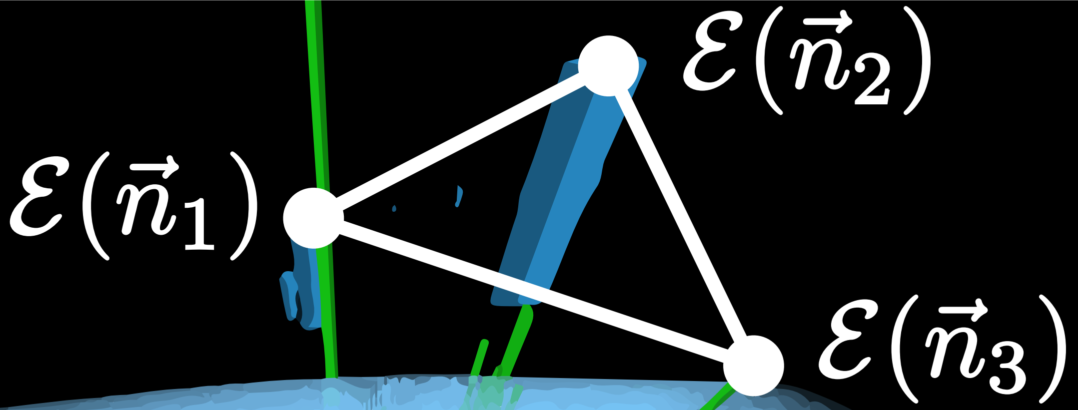


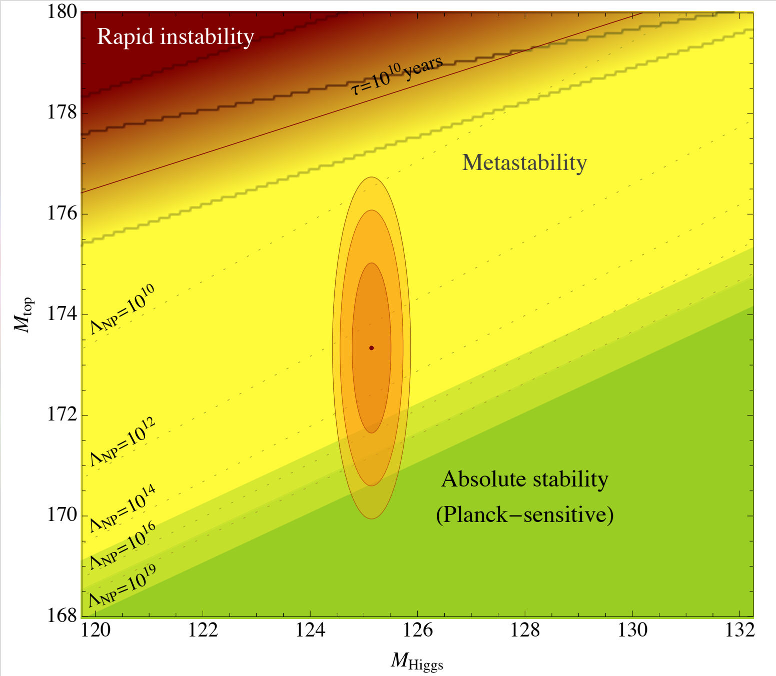
Weighing the Top with energy correlators

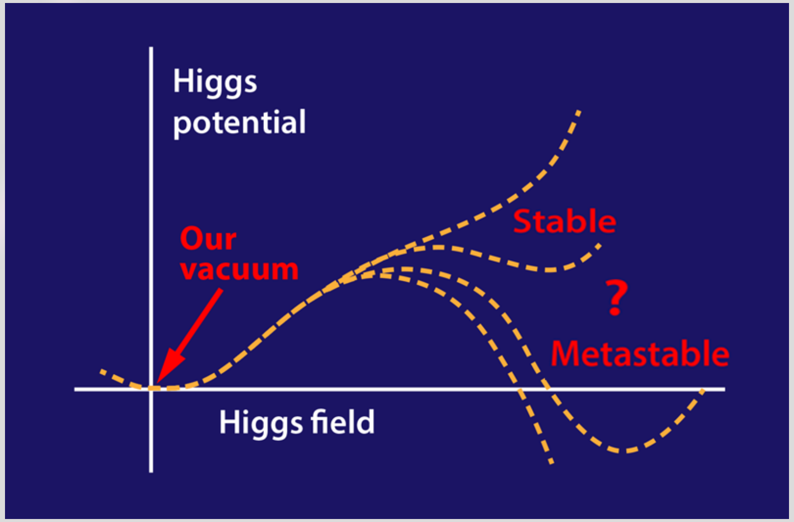
J. Holguin, I. Moult, A. Pathak, and M. Procura

arXiv:2201.08393

Why the top mass?

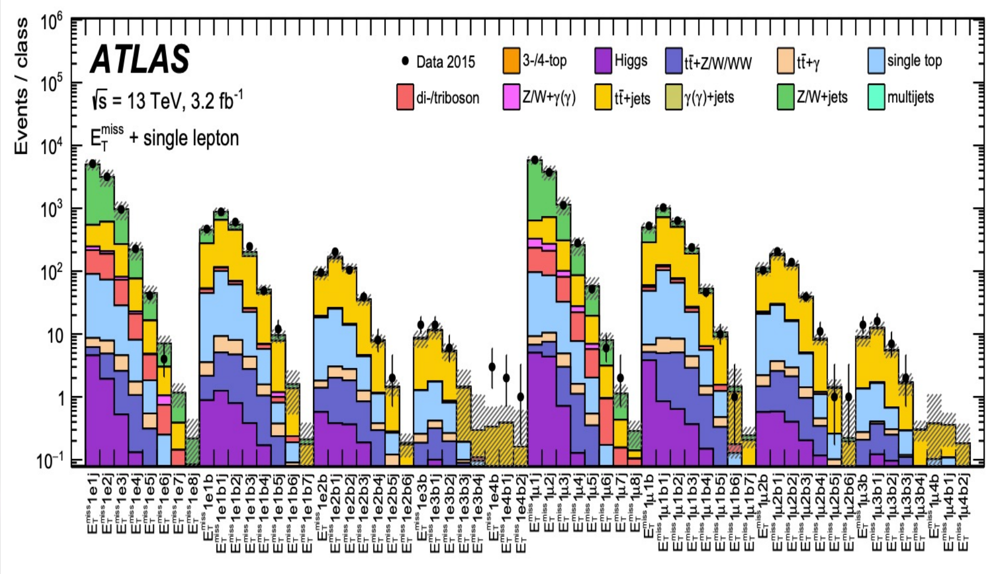
- The Top is very interesting!
  - Largest Yukawa coupling – sensitive to new physics.
  - The Top and the Higgs masses determine the stability of the electroweak vacuum. [1307.3536](1307.3536) , [1408.0292](1408.0292)





Why the top mass?

- The Top is very interesting!
  - Largest Yukawa coupling – sensitive to new physics.
  - The Top and the Higgs masses determine the stability of the electroweak vacuum.
- We are in an unprecedented era of high statistics collider physics!
  - Top measurements have transitioned from discovery to precision.



[1807.07447](https://arxiv.org/abs/1807.07447)

Current status of top mass measurements

- Current world average (HL-LHC projection ~ 200 MeV)
  - $m_t = 172.76 \pm 0.30$ GeV 10.1093/ptep/ptaa104
  - An impressive uncertainty $\sim 0.2\ \%$!
- Some of the numbers that enter this world average:
  - $m_t = 172.67 \pm 0.48$ GeV ATLAS, 1810.01772
  - $m_t = 172.26 \pm 0.61$ GeV CMS, 1812.06489
  - $m_t = 174.34\ \pm 0.64$ GeV Tevatron, 1407.2682
  - $m_t = 170.5 \pm 0.8$ GeV CMS, 1904.05237

The only quark with three masses in PDG:

Mass (direct measurements) $m = 172.76 \pm 0.30$ GeV [a,b] (S = 1.2)
Mass (from cross-section measurements) $m = 162.5^{+2.1}_{-1.5}$ GeV [a]
Mass (Pole from cross-section measurements) $m = 172.5 \pm 0.7$ GeV

A new approach

Let us re-think the problem somewhat. What do we have to work with?

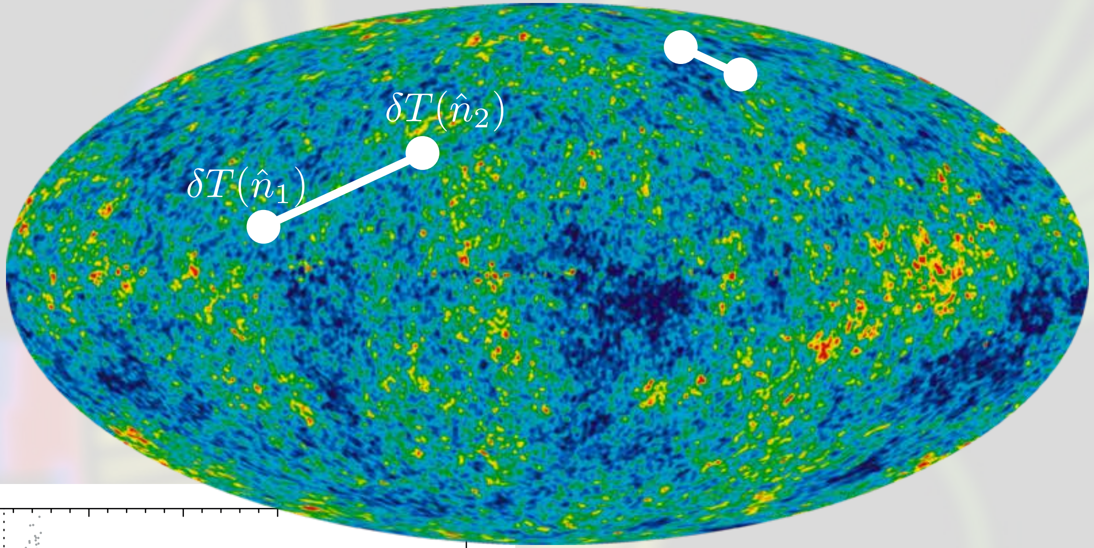
- A (partial*) kinematic breakdown of the particles in each jet.
- A lot of statistics!
  - The HL-LHC will be a top factory.
  - It is forecast that 3Billion ttbar events and 800Million mono-top events will be measured. 1902.04070

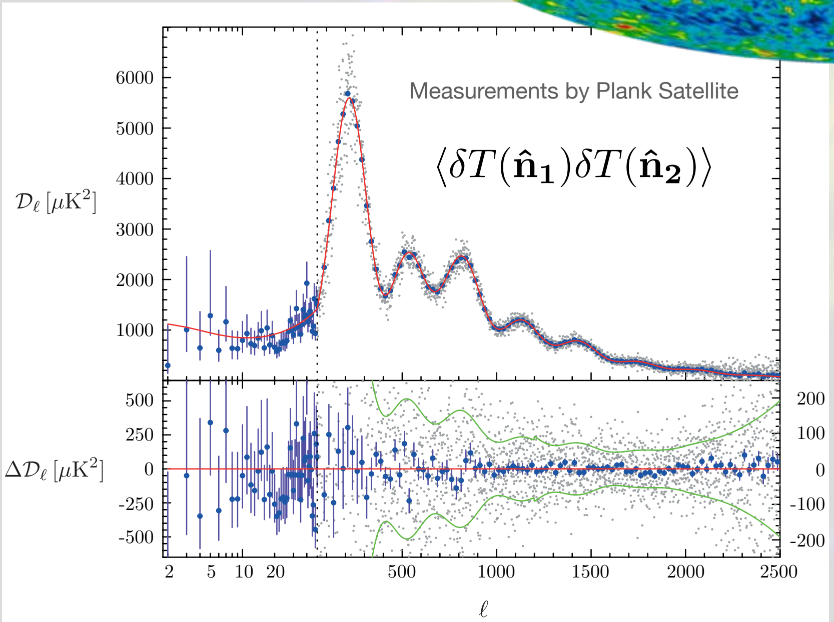
What observables do other fields of physics use when trying to extract simple properties from complicated environments with high statistics?

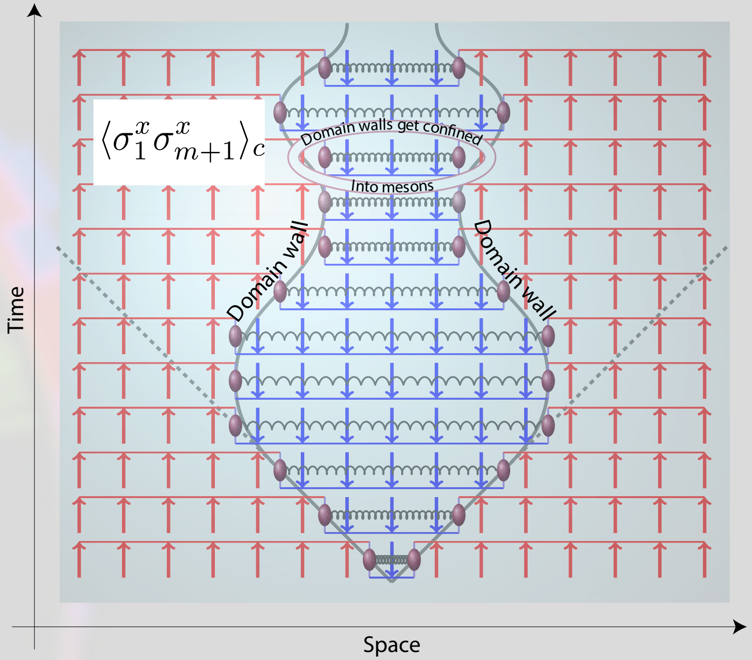
Correlation functions!

*neutrinos, pile up, fine grained detectors

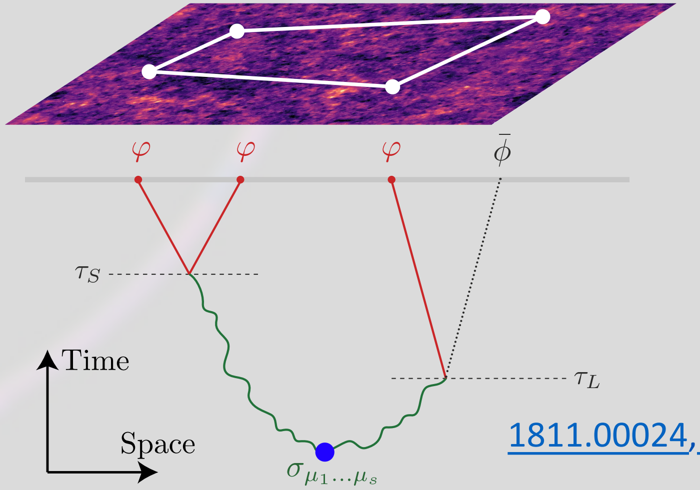
Correlation Functions







[1604.03571](1604.03571)



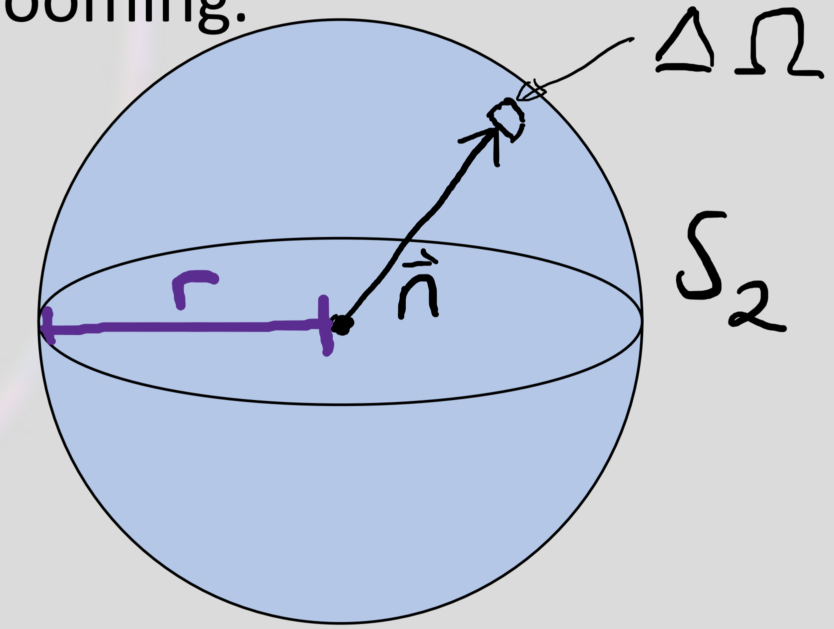
[1811.00024, 1503.08043](1811.00024, 1503.08043)

Correlation Functions

- Generally one can define correlators of any quantum charge or conserved quantity.
- For QCD, correlators of energy flux are usually of most interest – these naturally remove soft physics without grooming.

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n}) \simeq \int_0^\infty dt\ E_{\text{flux through } \Delta\Omega}(t)$$



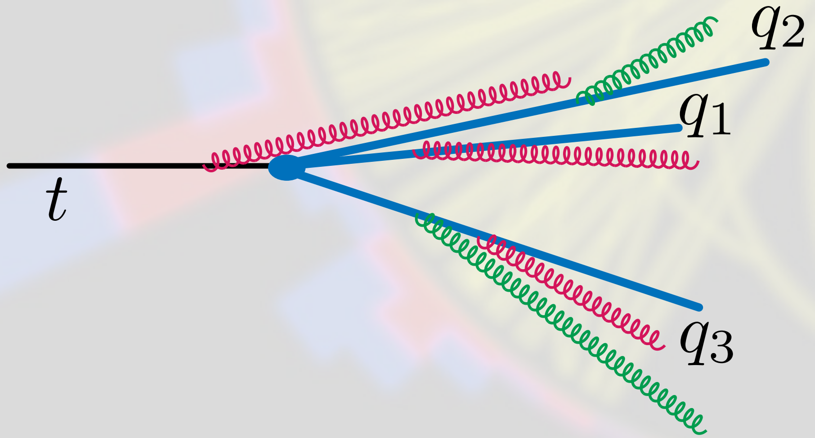
[0803.1467](0803.1467)

Correlation Functions

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\; r^2 n^i T_{0i}(t, r\vec{n})$$

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \sum_{ij} \int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}^2\vec{n}_i \mathrm{d}^2\vec{n}_j} E_i E_j\, \delta^2(\vec{n}_1 - \vec{n}_i)\delta^2(\vec{n}_2 - \vec{n}_j)$$

inclusive cross section to produce two particles, i, j, and anything else!



Correlation Functions

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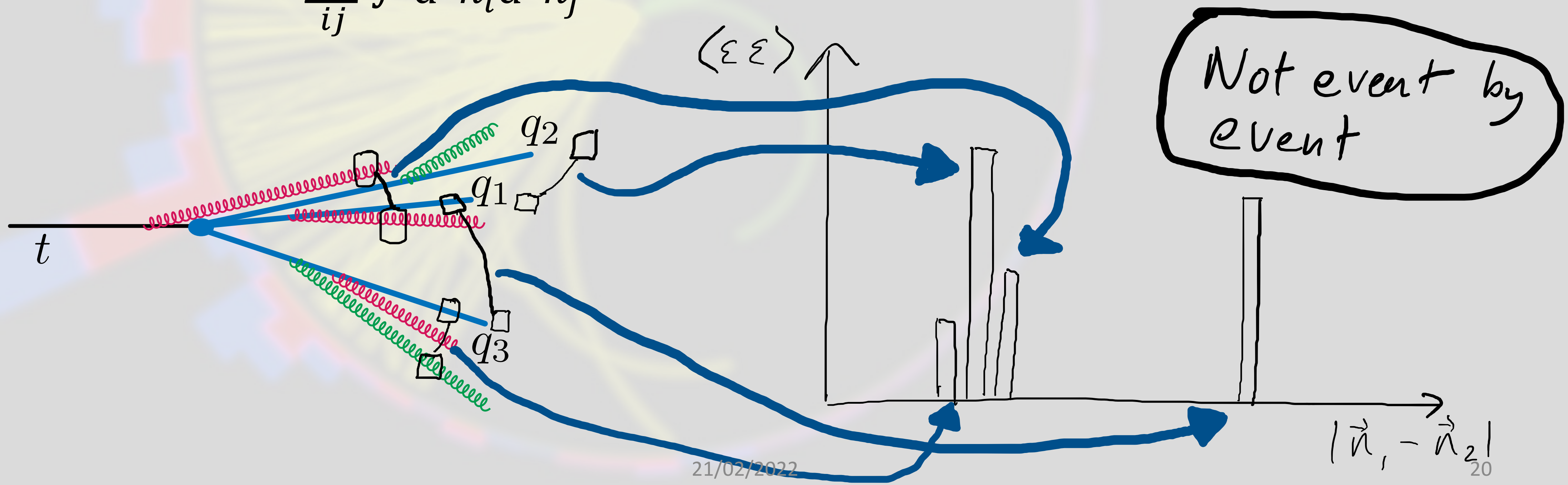


Correlation Functions

Pros:

- Defined on inclusive cross-sections and can be made insentive to soft radiation. Textbook example of where pp CSS factorisation can be used without any violation. 2109.03665

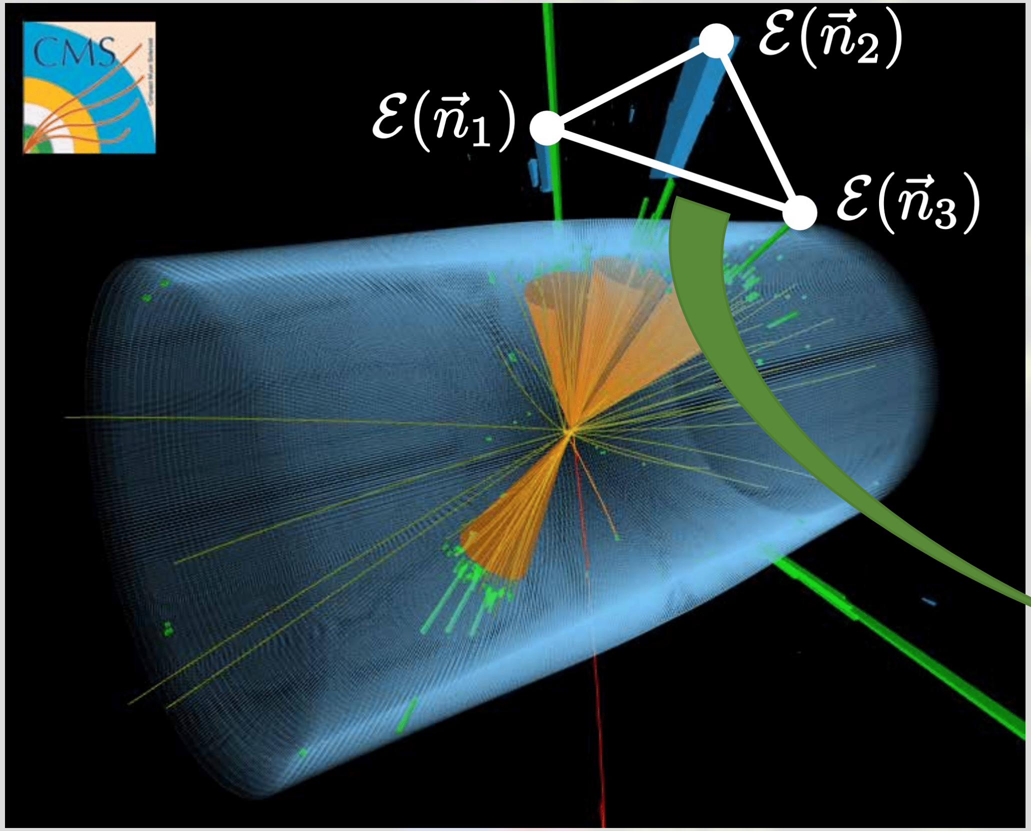
$$\frac{d\sigma}{d\zeta} = \int dE_J\, E_J^2 H(E_J) J_{\mathrm{EEC}}(\zeta, E_J) + \text{power corrections}\,,$$

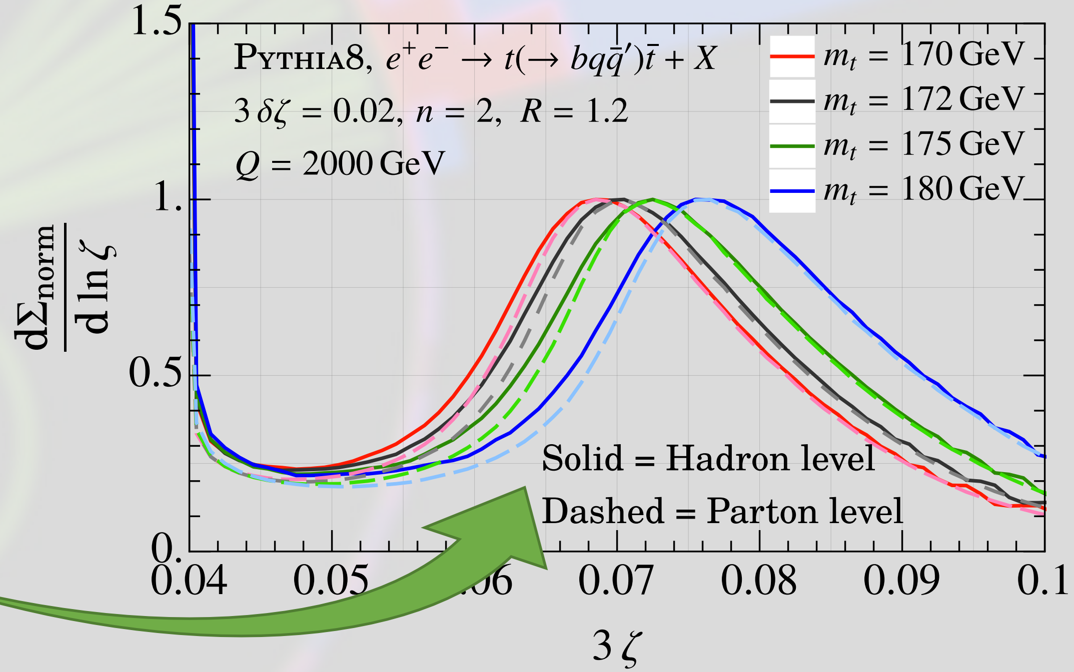
- Well studied by CFT community. Powerfull techniques exist for calculations: light-ray OPE, celestial Blocks, lorentzian inversion. 2202.04085

Cons:

- Reliant on high stats. A precission tool, not a typical discovery tool.
- Not event-by-event so cannot be directly used to tag.

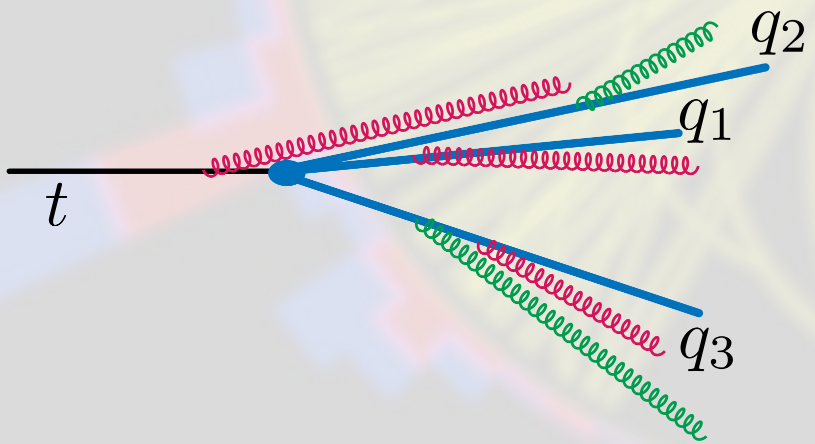
Part 3: Energy Correlators for Tops





Energy Correlators for Tops

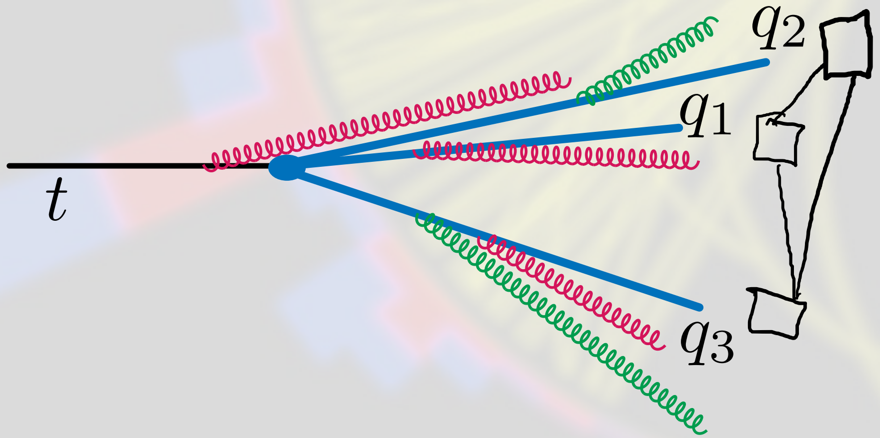
Which correlator will well characterise the top decay?



Energy Correlators for Tops

Which correlator will well characterise the top decay?

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$$
$$= \sum_{ij} \int \frac{\mathrm{d}\sigma_{ijk}}{\mathrm{d}^2\vec{n}_i \mathrm{d}^2\vec{n}_j \mathrm{d}^2\vec{n}_k}\, E_i E_j E_k\, \delta^2(\vec{n}_1 - \vec{n}_i)\delta^2(\vec{n}_2 - \vec{n}_j)\delta^2(\vec{n}_2 - \vec{n}_k)$$



Energy Correlators for Tops

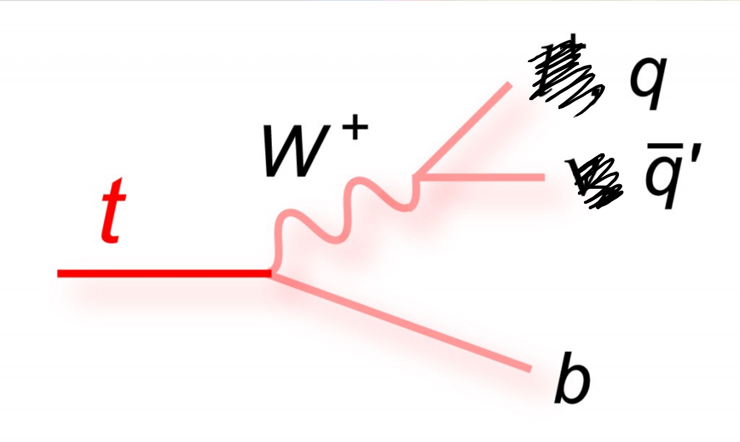
The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

I'm going to sketch a 'back of the envelope' calculation which gives intuition for the observable.

3-body kinematics

The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

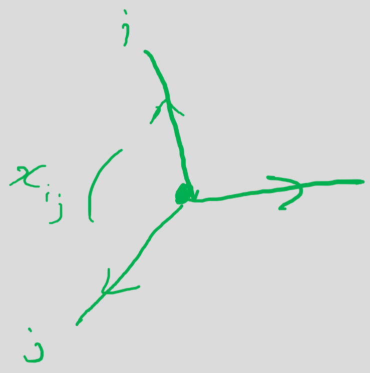
$$\zeta_{ij} = \frac{p_i \cdot p_j}{2} = \frac{(1-\cos\theta_{ij})}{2} \simeq \theta_{ij}^2/4$$



3-body kinematics

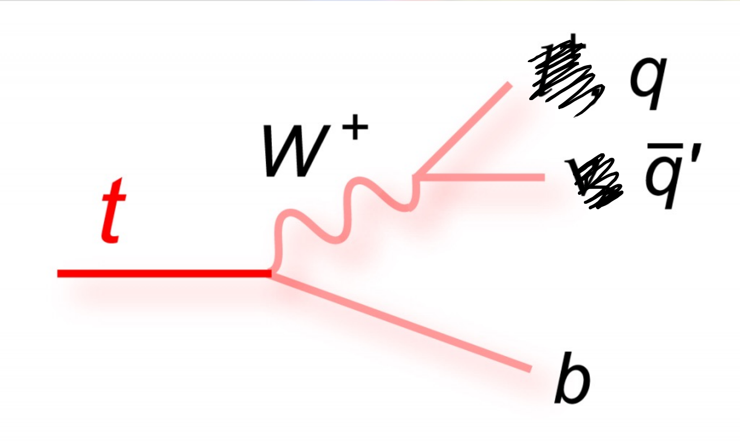
The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

$$\xi_{ij} = \frac{p_i \cdot p_j}{2} = \frac{(1-\cos\theta_{ij})}{2} \approx \theta_{ij}^2/4$$



In the Top rest frame

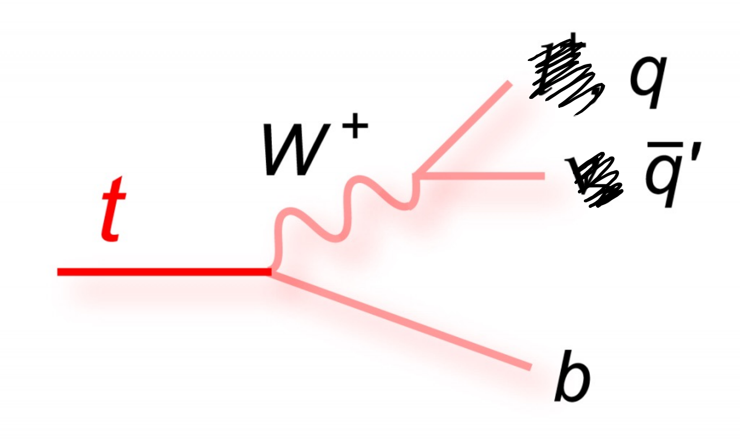
$$\xi_{12} + \xi_{23} + \xi_{31} \in [2, 2.25]$$



3-body kinematics

The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

One can find the boost from the Top rest frame to the lab frame where we measure angles $\tilde{\zeta}_{ij}$. We find that



$$\boxed{\sum_{i,j} \tilde{\zeta}_{ij} \approx \left(\frac{m_t}{Q}\right)^2 \sum_{i,j} \zeta_{ij}}$$

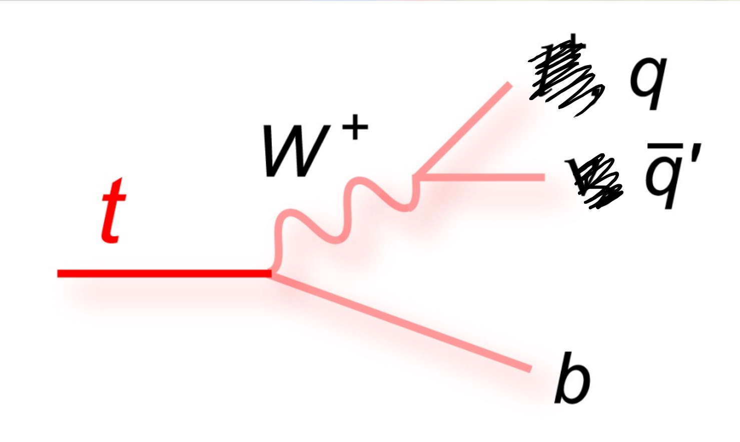
Building the observable

The correlator is sensitive to the angles between the decay products. What do we expect to see at fixed order?

Fixed order teaches us that look at

$$G(\zeta) = \int d^2\vec{n}_1\, d^2\vec{n}_2\, d^2\vec{n}_3 \,\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$$

$$\times\, \delta\left(\zeta - |\vec{n}_1-\vec{n}_2| - |\vec{n}_2-\vec{n}_3| - |\vec{n}_3-\vec{n}_1|\right).$$

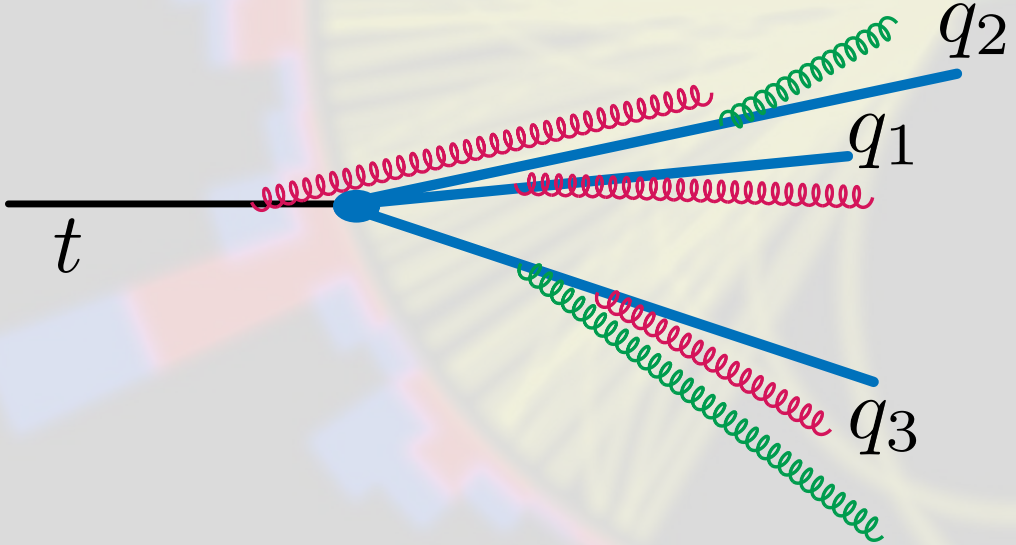


Doing the average over $\vec{n}_i$

we find $\langle\zeta\rangle \simeq 3\,\frac{m_t^2}{Q^2}$

Building the observable

What about higher order perturbative corrections?

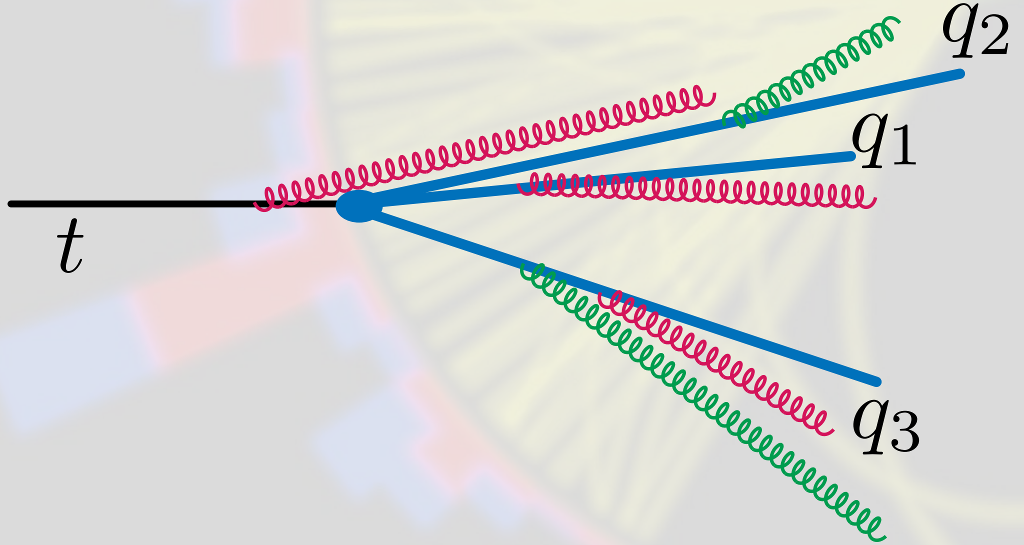


We want to preserve the $\langle \zeta \rangle = 3\frac{m_t^2}{Q^2}$ dependence!

Energy correlators not sensitive to soft physics, but will pick up collinear. How to minimise?

Building the observable

What about higher order perturbative corrections?



Solution:

require $|\vec{n}_1 - \vec{n}_2| \approx |\vec{n}_2 - \vec{n}_3| \approx |\vec{n}_3 - \vec{n}_1|$

that way we never have a small angle in the sum over ζ_{ij} around $\zeta = \frac{3m_t^2}{Q^2}$.

Building the observable

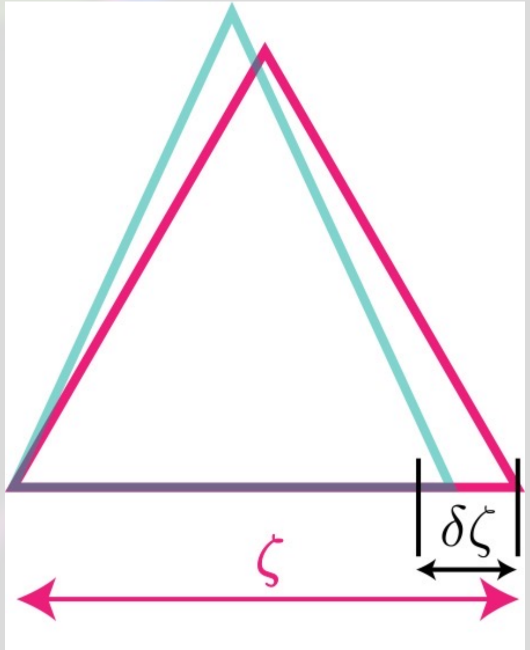
In all, we have...

$$\frac{\mathrm{d}\Sigma(\delta\zeta)}{\mathrm{d}Q\mathrm{d}\zeta} = \int \mathrm{d}\zeta_{12}\mathrm{d}\zeta_{23}\mathrm{d}\zeta_{31} \int \mathrm{d}\sigma \widehat{\mathcal{M}}_{\triangle}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta)\,, \tag{4}$$

where the measurement operator $\widehat{\mathcal{M}}_{\triangle}^{(n)}$ is

$$\widehat{\mathcal{M}}_{\triangle}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta) = \widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) \tag{5}$$
$$\times\, \delta(3\zeta - \zeta_{12} - \zeta_{23} - \zeta_{31}) \prod_{l,m,n\in\{1,2,3\}} \Theta(\delta\zeta - |\zeta_{lm} - \zeta_{mn}|)\,.$$

$$\widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \tag{2}$$
$$\sum_{i,j,k} \frac{E_i^n E_j^n E_k^n}{Q^{3n}} \delta\left(\zeta_{12} - \hat{\zeta}_{ij}\right) \delta\left(\zeta_{23} - \hat{\zeta}_{ik}\right) \delta\left(\zeta_{31} - \hat{\zeta}_{jk}\right)\,.$$

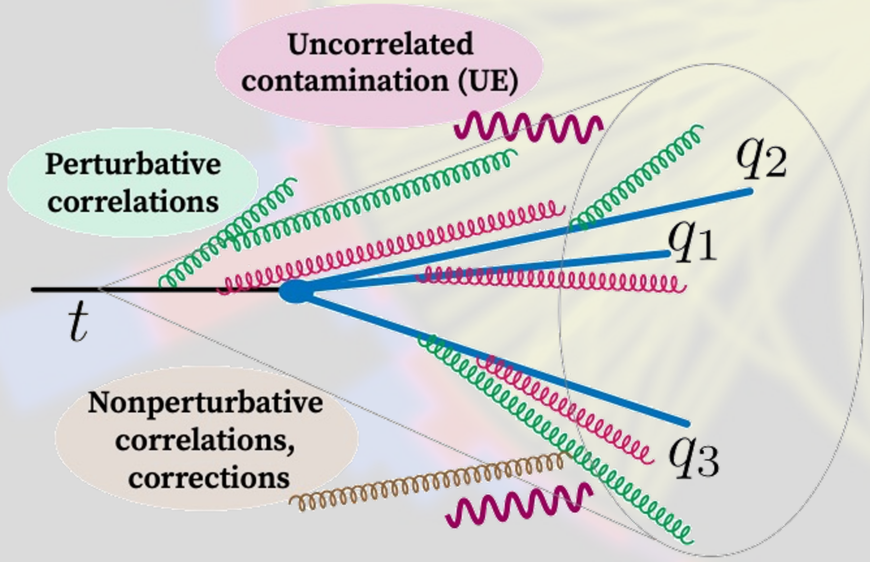


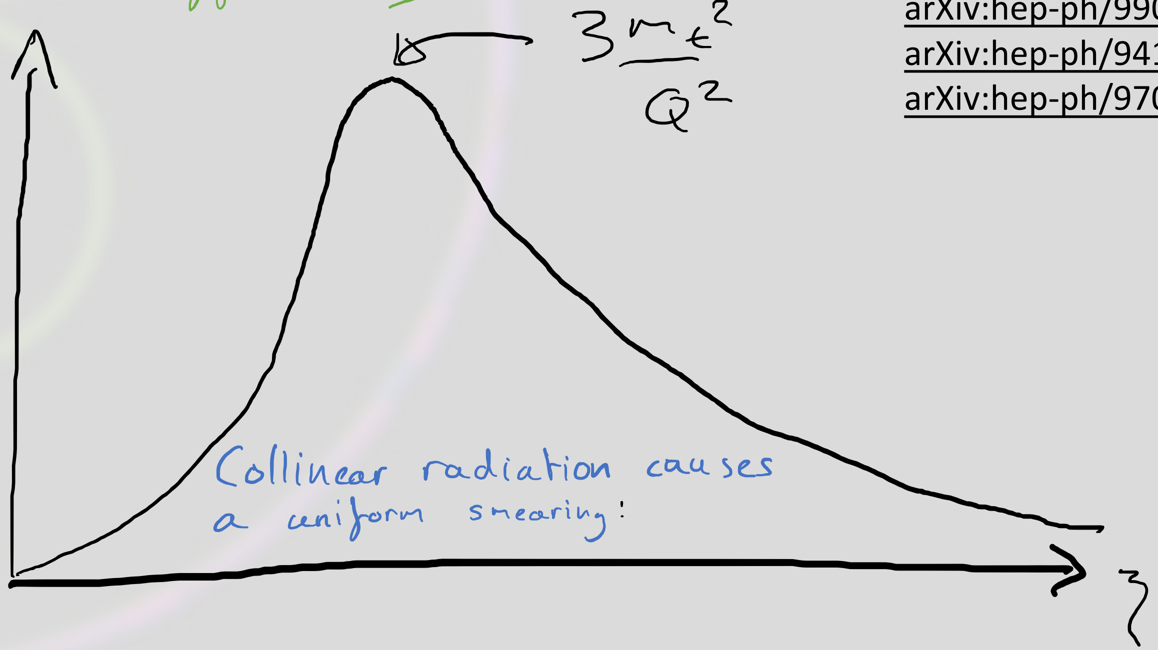
Understanding the distribution

What will the distribution look like with N.P. corrections?

We know from other studies of energy correlators that N.P. corrections are an addative power law. Effectively a change in normalisation.

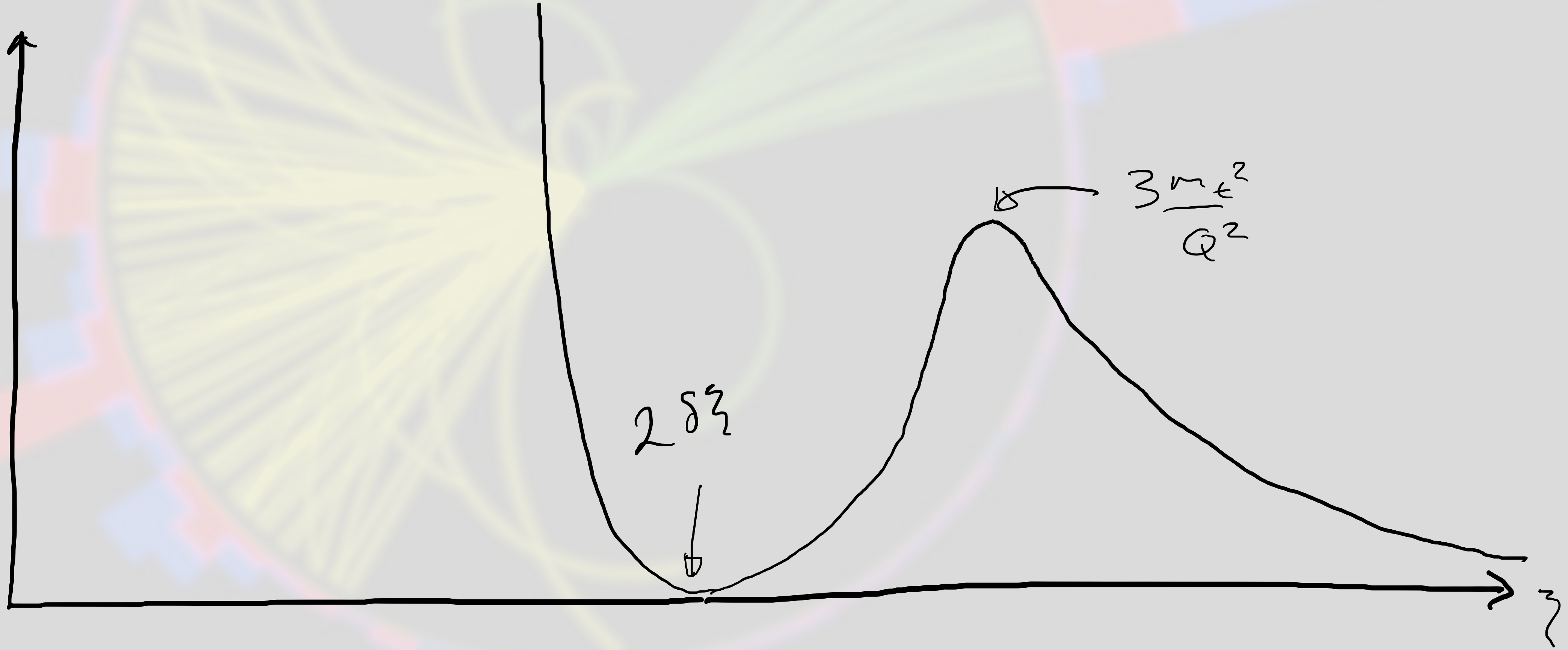
arXiv:hep-ph/9902341
arXiv:hep-ph/9411211
arXiv:hep-ph/9708346





Understanding the distribution

What is the effect of the asymmetry in the triangle?



Understanding the distribution

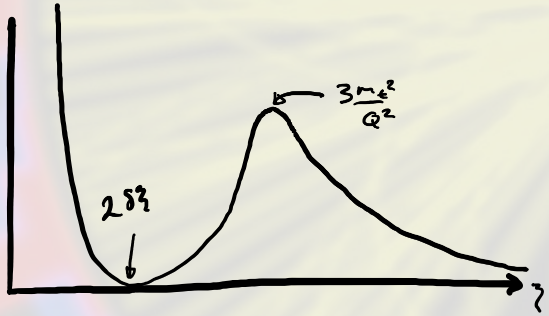
Enough sketching!

Let us simulate.

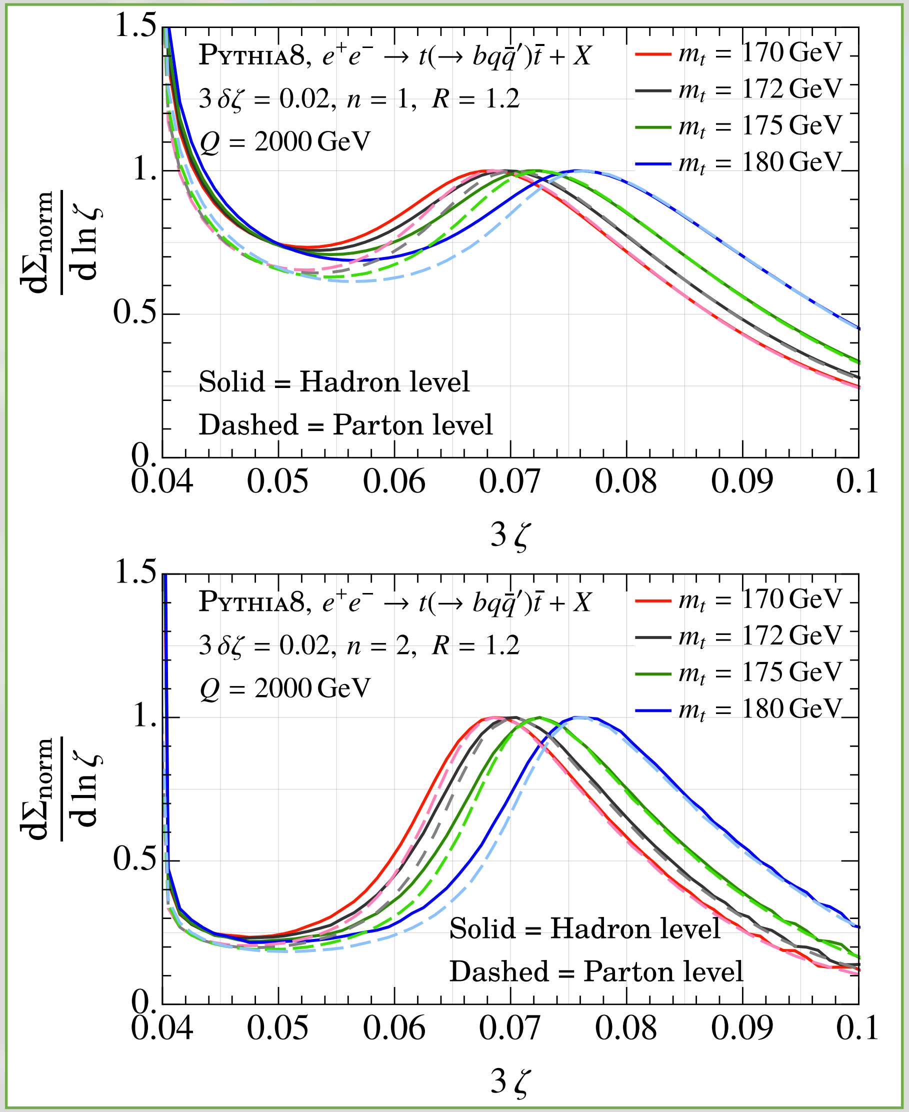
Simulation in Pythia8

We did a complete simulated pp analysis, however for this talk consider e^+e^- where the hard scale Q is just half the CoM Energy.

- Key features exactly as expected.



- Peak is sentive to Top mass.
- Very low sentivity to hadronisation. The shift is equivelant to $\Delta m_t = 150 \pm 50$MeV.



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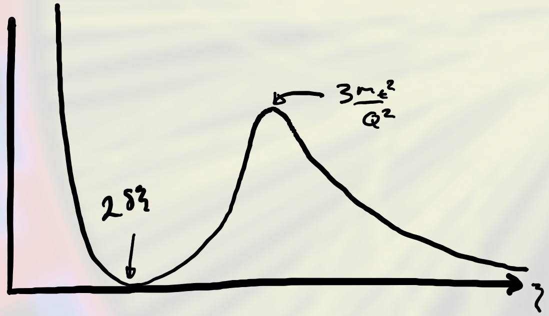

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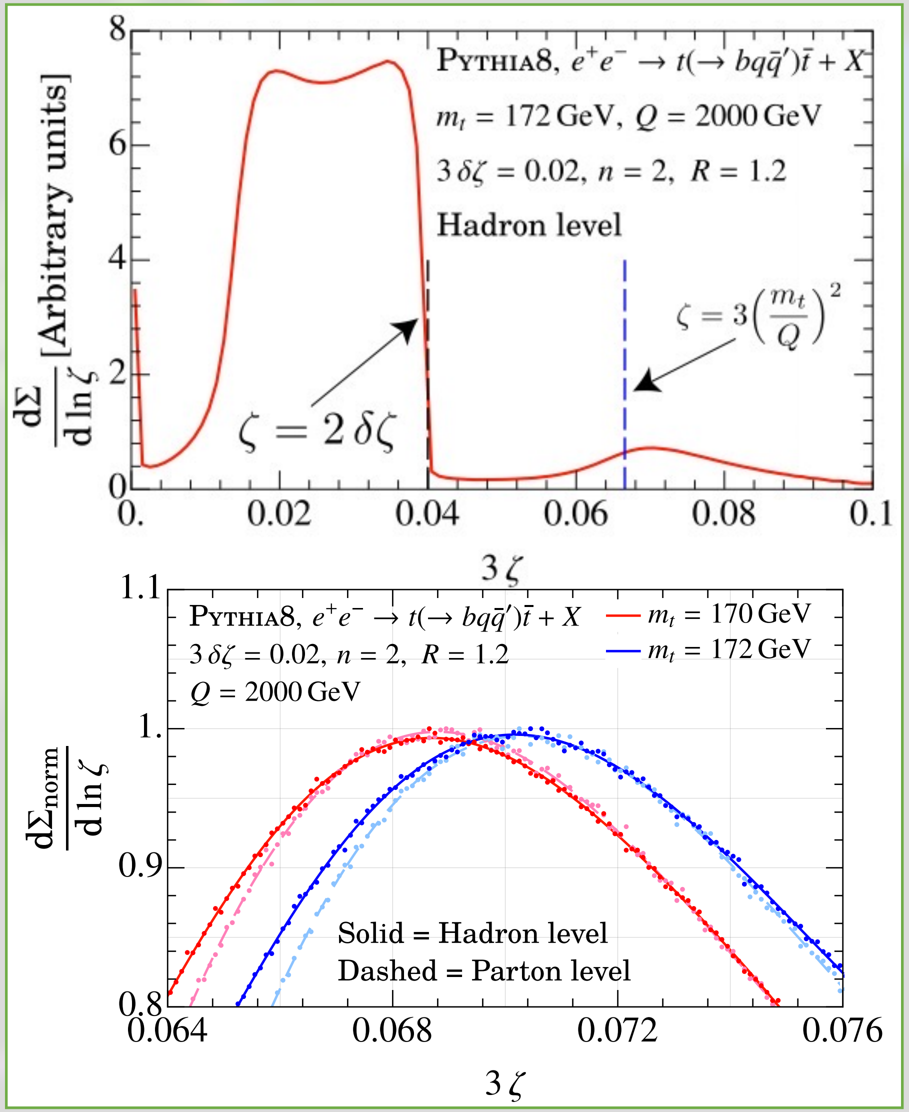


Simulation in Pythia8

Now consider hadron colliders. Must now use boost invariant quantities.

- Q is now the partonic Top p_T. This is not a measurable quantity. Instead we have the p_T of the Top jet. This adds a little complexity.

$$\frac{\mathrm{d}\Sigma(\delta\zeta)}{\mathrm{d}p_{T,\mathrm{jet}}\mathrm{d}\zeta} = \frac{\mathrm{d}\Sigma(\delta\zeta)}{\mathrm{d}p_{T,t}\mathrm{d}\zeta}\frac{\mathrm{d}p_{T,t}}{\mathrm{d}p_{T,\mathrm{jet}}}$$

- Now must consider underlying event.
- Can we measure on tracks? (Yes)

$$\widehat{\mathcal{M}}^{(n)}(\zeta_{12},\zeta_{23},\zeta_{31}) = \sum_{i,j,k}\frac{E_i^n E_j^n E_k^n}{Q^{3n}}\delta\left(\zeta_{12}-\hat{\zeta}_{ij}\right)\delta\left(\zeta_{23}-\hat{\zeta}_{ik}\right)\delta\left(\zeta_{31}-\hat{\zeta}_{jk}\right). \qquad (2)$$

$$\widehat{\mathcal{M}}^{(n)}_{(pp)}(\zeta_{12},\zeta_{23},\zeta_{31}) = \sum_{i,j,k\,\in\,\mathrm{jet}}\frac{(p_{T,i})^n(p_{T,j})^n(p_{T,k})^n}{(p_{T,\mathrm{jet}})^{3n}} \times\delta\left(\zeta_{12}-\hat{\zeta}^{(pp)}_{ij}\right)\delta\left(\zeta_{23}-\hat{\zeta}^{(pp)}_{ik}\right)\delta\left(\zeta_{31}-\hat{\zeta}^{(pp)}_{jk}\right), \qquad (7)$$

where $\hat{\zeta}^{(pp)}_{ij} = \Delta R^2_{ij} = \sqrt{\Delta\eta^2_{ij}+\Delta\phi^2_{ij}}$, with η,ϕ the standard rapidity, azimuth coordinates.

Simulation in Pythia8

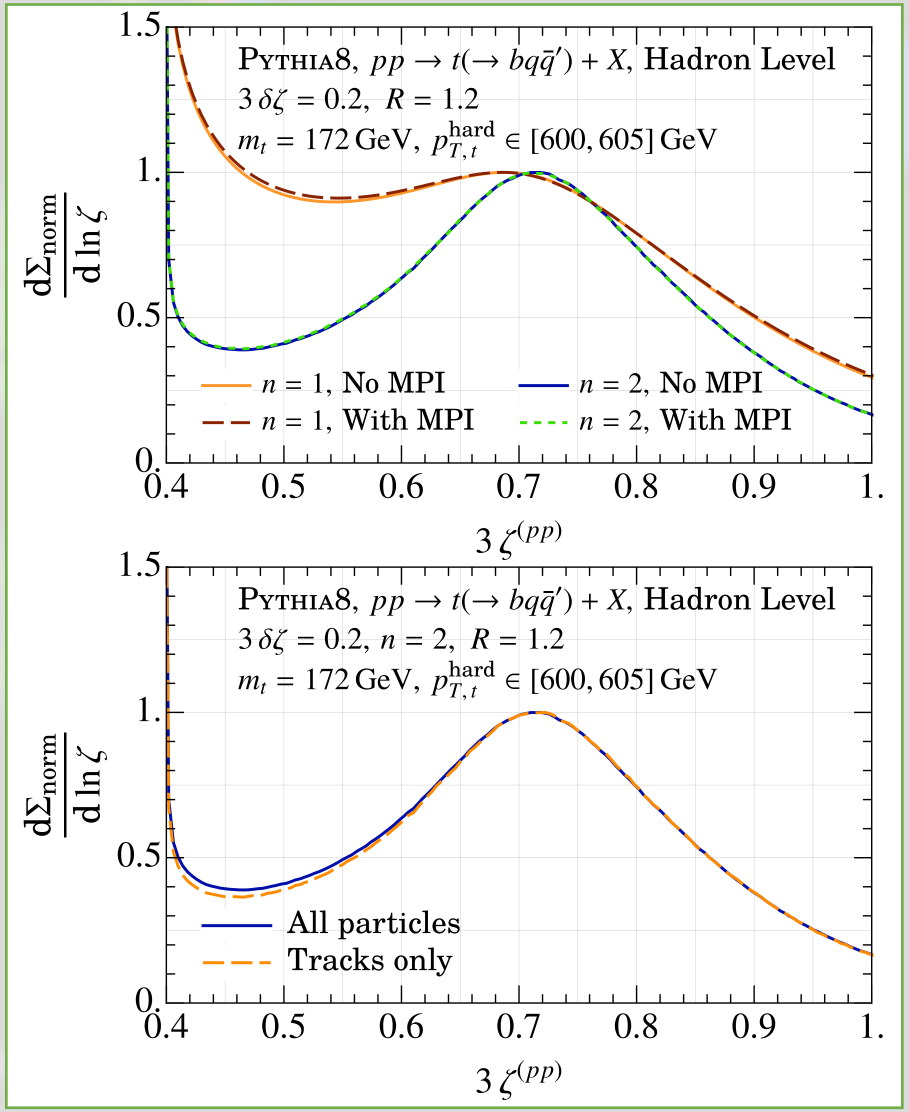
Let us study the hadron collider environment in two parts.

1. First study the observable whilst unphysically fixing the partonic Top p_T. This is to answer:

- Now must consider underlying event.
- Can we measure on tracks? (Yes)

2. Then study the physical observable and in conjunction with the p_T spectrum.

Stage 1



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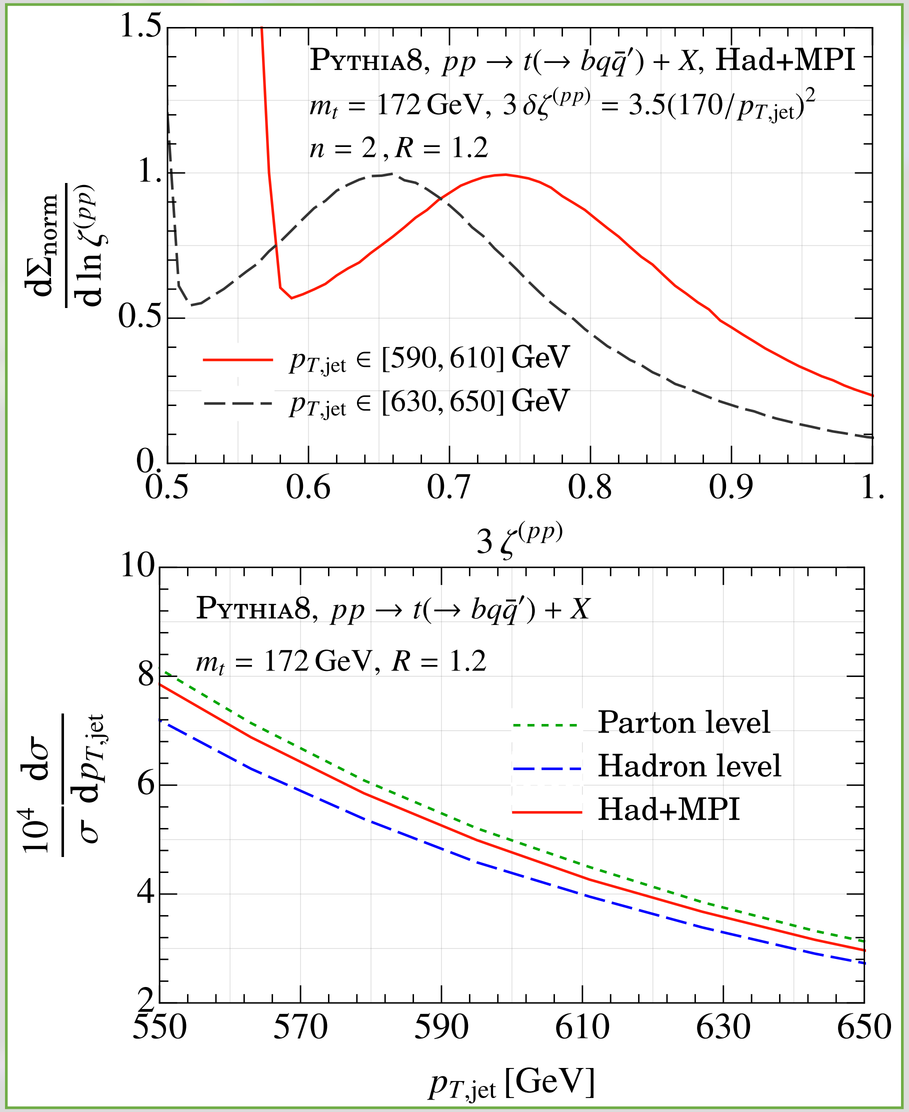
Stage 2



Simulation in Pythia8

How to handle these p_T shifts? One method:

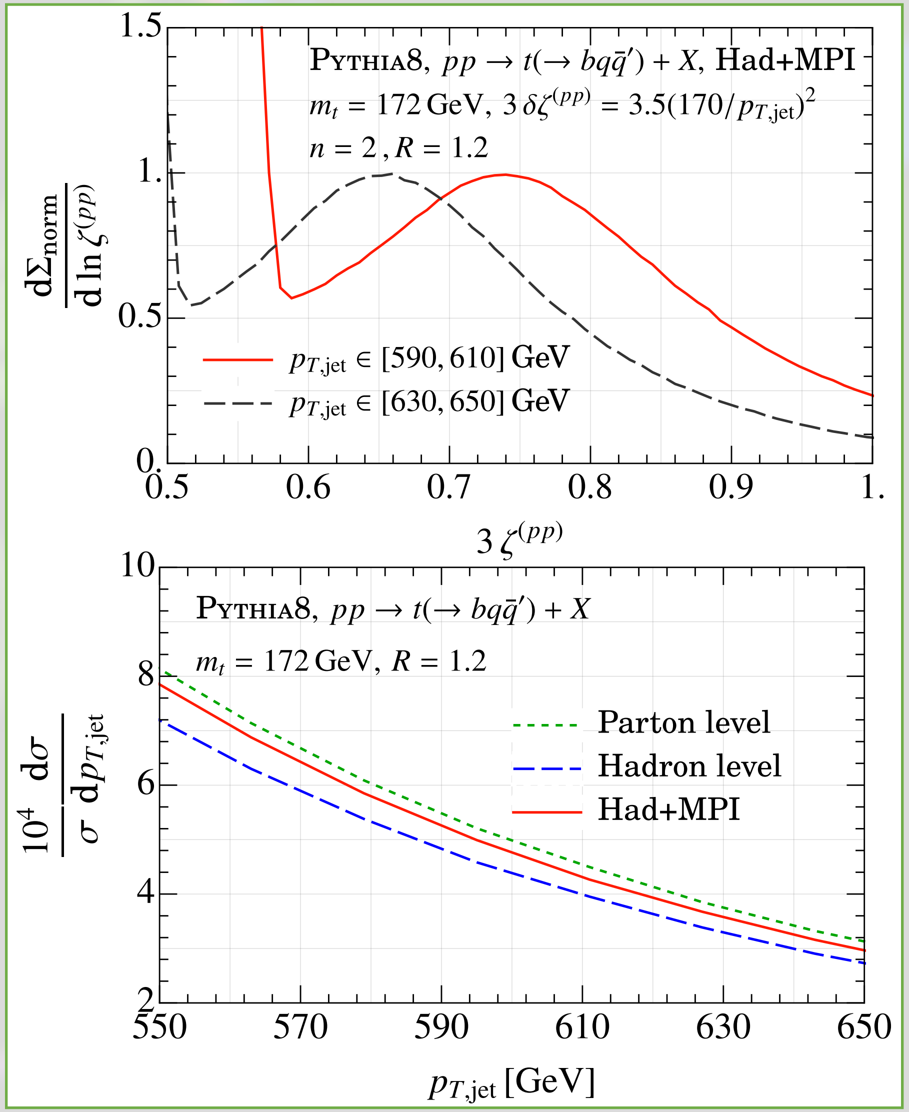
- Fixed order gives

$$\zeta_{\text{peak}}^{(pp)} \approx 3m_t^2/p_{T,t}^2$$

- From Factorisation properties of the observable,

$$\zeta_{\text{peak}}^{(pp)} = \frac{3F_{\text{pert}}(m_t, p_{T,\text{jet}}, \alpha_s, R)}{(p_{T,\text{jet}} + \Delta_{\text{NP}}(R) + \Delta_{\text{MPI}}(R))^2}\,.$$

Stage 2



Simulation in Pythia8

Stage 2

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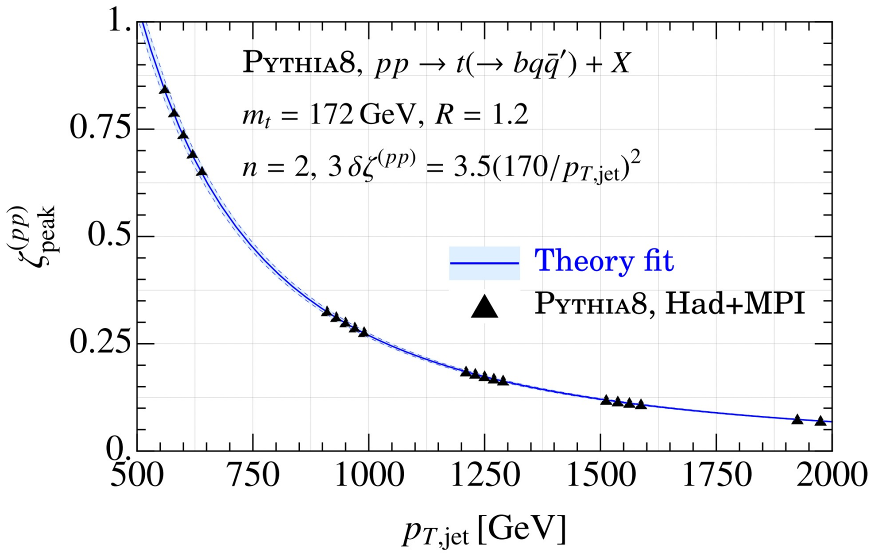
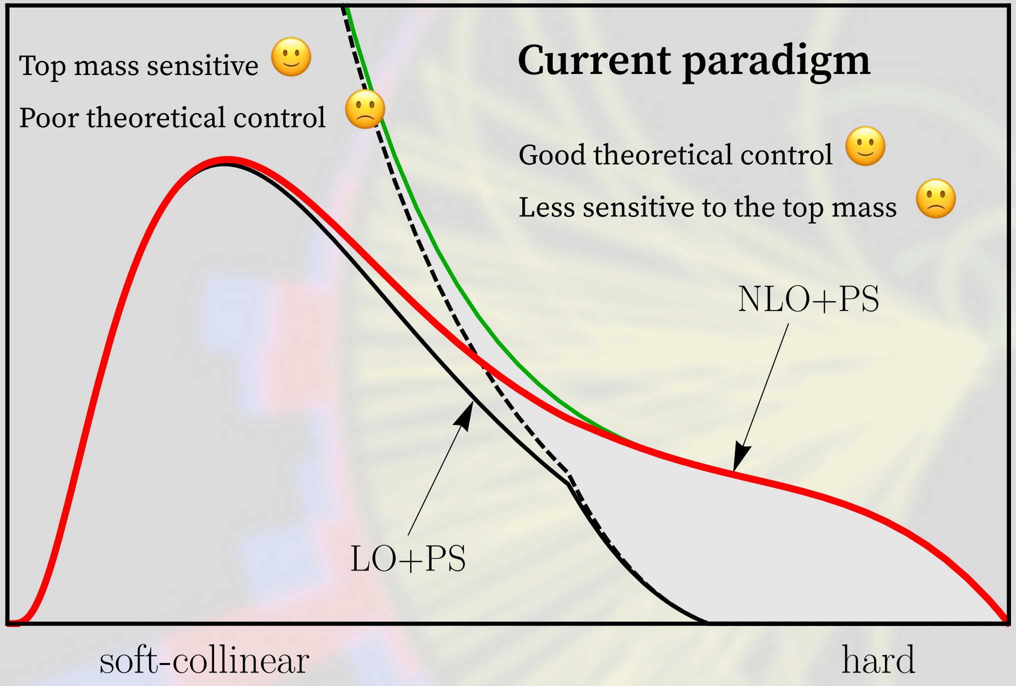
$$\zeta^{(pp)}_{\text{peak}} = \frac{3F_{\text{pert}}(m_t, p_{T,\text{jet}}, \alpha_s, R)}{\left(p_{T,\text{jet}} + \Delta_{\text{NP}}(R) + \Delta_{\text{MPI}}(R)\right)^2} \,.$$

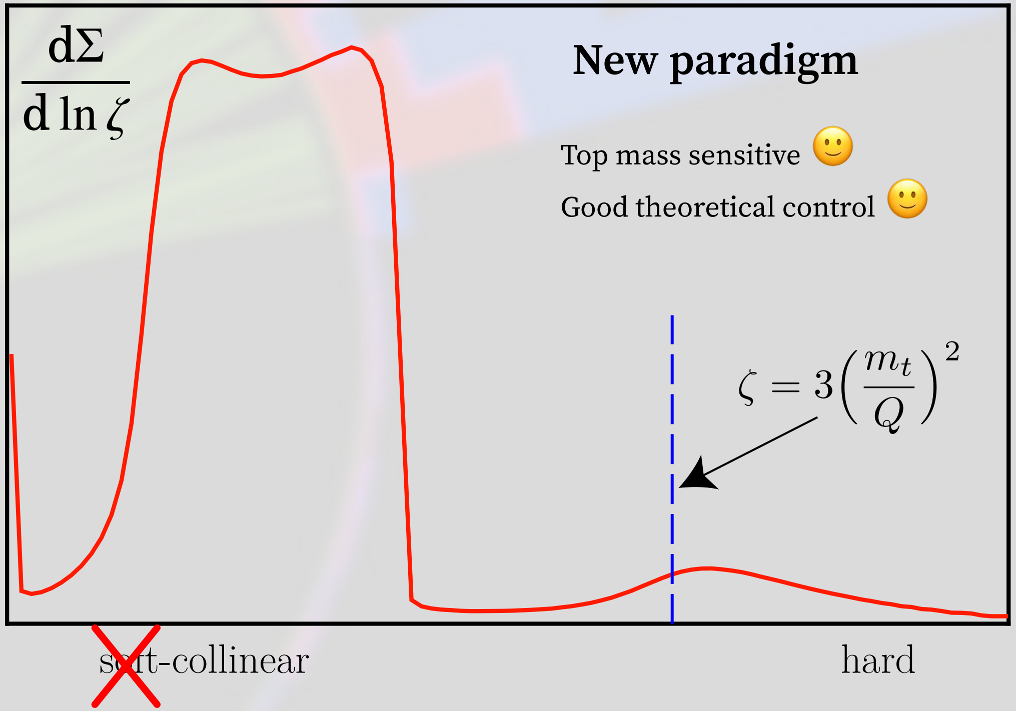
PYTHIA8 m_t	Parton $\sqrt{F_{\text{pert}}}$	Hadron + MPI $\sqrt{F_{\text{pert}}}$
172 GeV	172.6 ± 0.3 GeV	$172.3 \pm 0.2 \pm 0.4$ GeV
173 GeV	173.5 ± 0.3 GeV	$173.6 \pm 0.2 \pm 0.4$ GeV
175 GeV	175.5 ± 0.4 GeV	$175.1 \pm 0.3 \pm 0.4$ GeV
$173 - 172$	0.9 ± 0.4 GeV	1.3 ± 0.6 GeV
$175 - 172$	2.9 ± 0.5 GeV	2.8 ± 0.6 GeV

TABLE I: Values of the effective parameter $F_{\text{pert}}(m_t)$ extracted at parton level, and hadron+MPI level. The consistency of the two approaches provides a measure of our uncertainty due to non-perturbative corrections.



Conclusions





$$\frac{d\Sigma}{dp_{T,\text{jet}}\,d\eta\,d\zeta} = f_i \otimes f_j \otimes H_{i,j\to t}\left(z_J; p_{T,t} = \frac{p_{T,\text{jet}}}{z_J}, \eta\right)$$
$$\otimes J_{t\to t}(z_J, z_h; R) \otimes J^{[\text{tracks}]}_{\text{EEEC}}(n, z_h, \zeta; m_t; \Gamma_t)$$

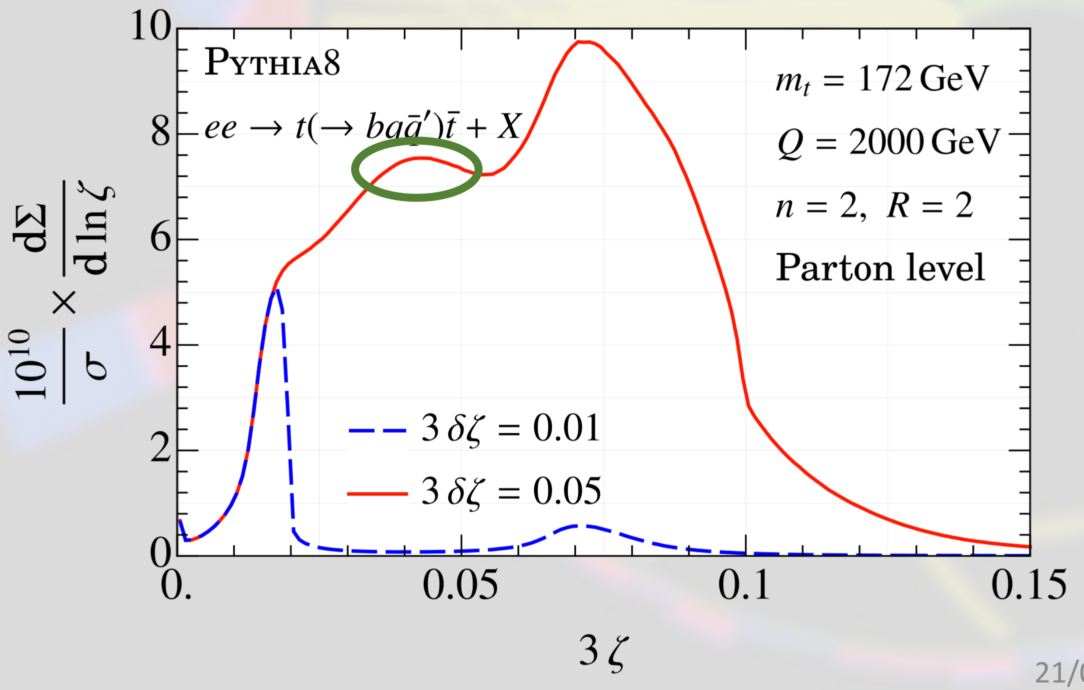
Outlook

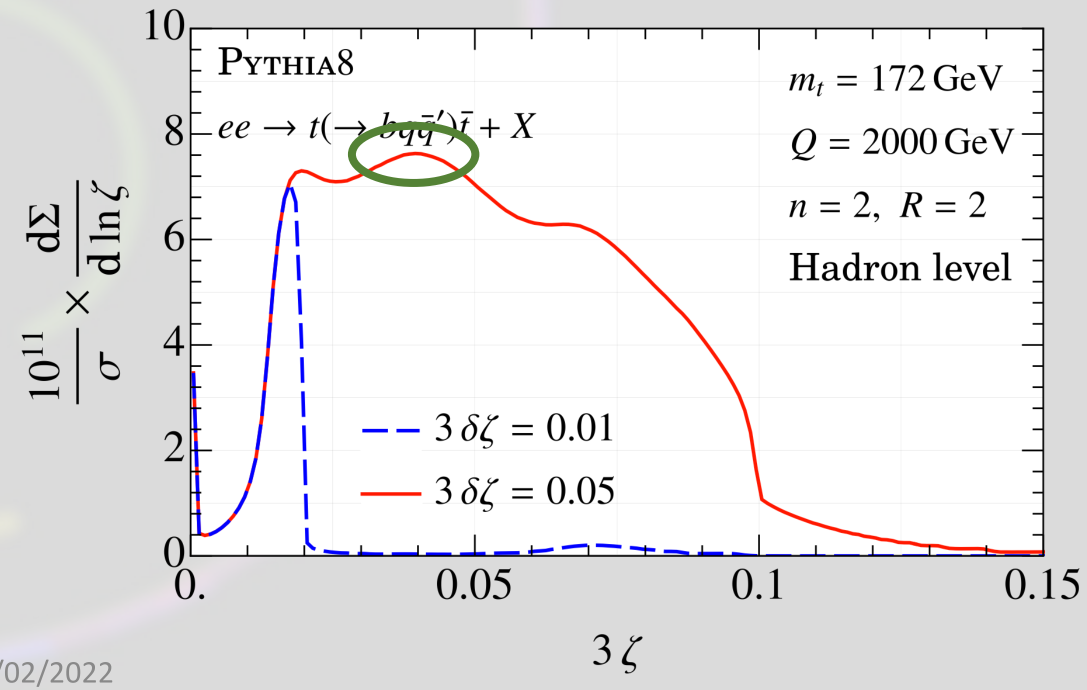
- The three-point energy correlator shows promise as a top mass sentive observable with theoretical control comparable to the current precision of direct measurements.
- So far studies have been proof-of-concept. An experimental feasibility study would be prescient.
- Much of the ingredients needed for a precission calculation already exist. Missing pieces are the EEEC jet function and a broader study of factorisation.

Further improved?

Using the equalatorial configuration we projected onto the top peak. However the W also imprints on the correlator in a diferent part of the parameter space.

The distrobution $\frac{\mathrm{d}\Sigma(\delta\zeta)}{\mathrm{d}\zeta_W \mathrm{d}\zeta}$ is independent of the pt distrobution, gives $m_t(m_W)$.





Supplementary material

1. Parameterize the all orders peak position:

$$\zeta_{\text{peak}}^{(pp)} = 3(1+\mathcal{O}(\alpha_s))\frac{m_t^2}{f(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}})^2} \equiv 3(1+\mathcal{O}(\alpha_s))\frac{m_t^2}{(p_{T,\text{jet}} + \Delta(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}}))^2}$$

2. Work with

$$\rho^2(\zeta_{\text{peak}}^{(pp)\text{v}}, p_{T,\text{jet}}^{\text{v}}) = \left(\zeta_{\text{peak}}^{(pp)\text{ref}} - \zeta_{\text{peak}}^{(pp)\text{v}}\right)\left(\frac{3(1+\mathcal{O}(\alpha_s))}{(p_{T,\text{jet}}^{\text{v}})^2} - \frac{3(1+\mathcal{O}(\alpha_s))}{(p_{T,\text{jet}}^{\text{ref}})^2}\right)^{-1},$$

3. Define

$$\Delta^{\text{ref}} \equiv \Delta(p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}), \qquad \Delta^{\text{v}}(p_{T,\text{jet}}^{\text{v}} - p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}) \equiv \Delta(p_{T,\text{jet}}^{\text{v}}, m_t, \alpha_s, \Lambda_{\text{QCD}}) - \Delta^{\text{ref}}$$

4. Solve for ρ:

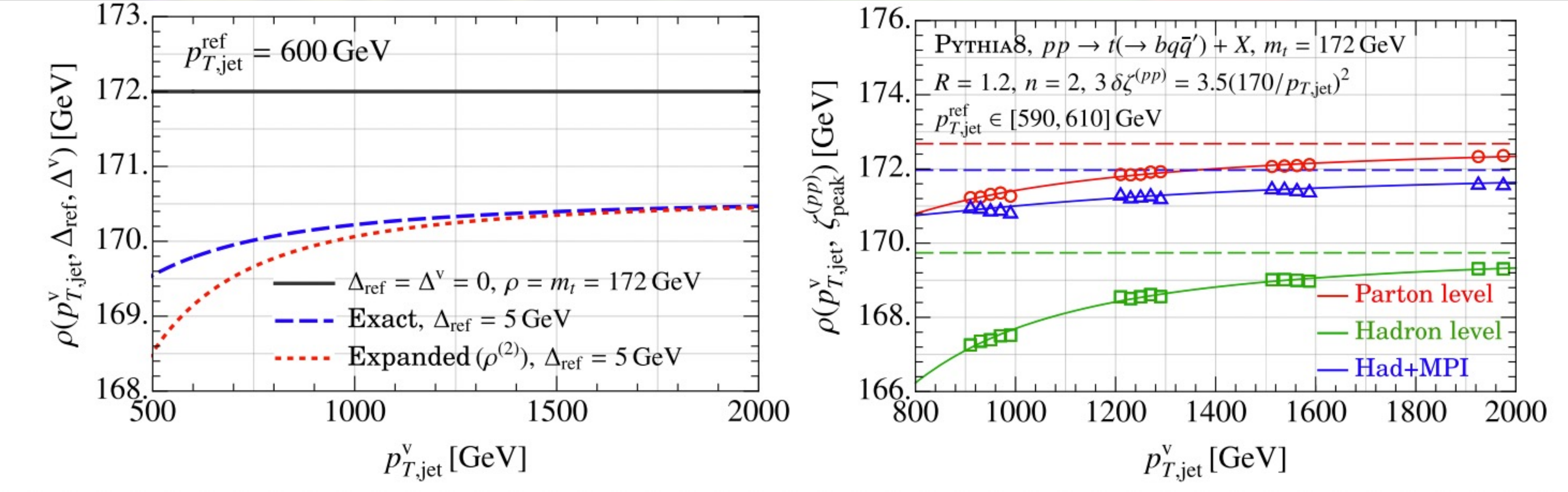
$$\rho(p_{T,\text{jet}}^{\text{v}}, \Delta^{\text{ref}}, \Delta^{\text{v}}) = \sqrt{F_{\text{pert}}}\frac{p_{T,\text{jet}}^{\text{ref}}}{p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}}}\left(1 - \frac{2p_{T,\text{jet}}^{\text{ref}}\Delta^{\text{ref}} + (\Delta^{\text{ref}})^2}{2(p_{T,\text{jet}}^{\text{v}})^2} + \frac{\left(p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}}\right)^2\left(\Delta^{\text{ref}} + \Delta^{\text{v}}\right)}{8(p_{T,\text{jet}}^{\text{v}})^3} + \mathcal{O}\left(\frac{1}{(p_{T,\text{jet}}^{\text{v}})^4}\right)\right)$$

5. The asymptotic value for $p_{T,\text{jet}}^{\text{v}}$ depends only on m_t and Δ^{ref}.

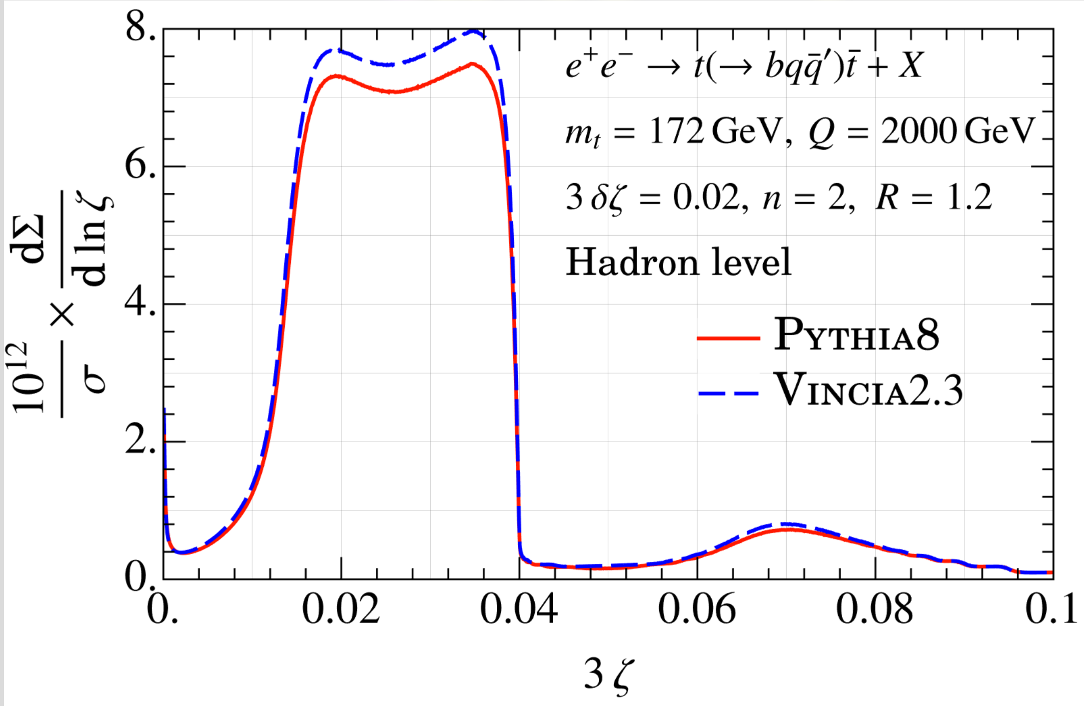
Supplementary material

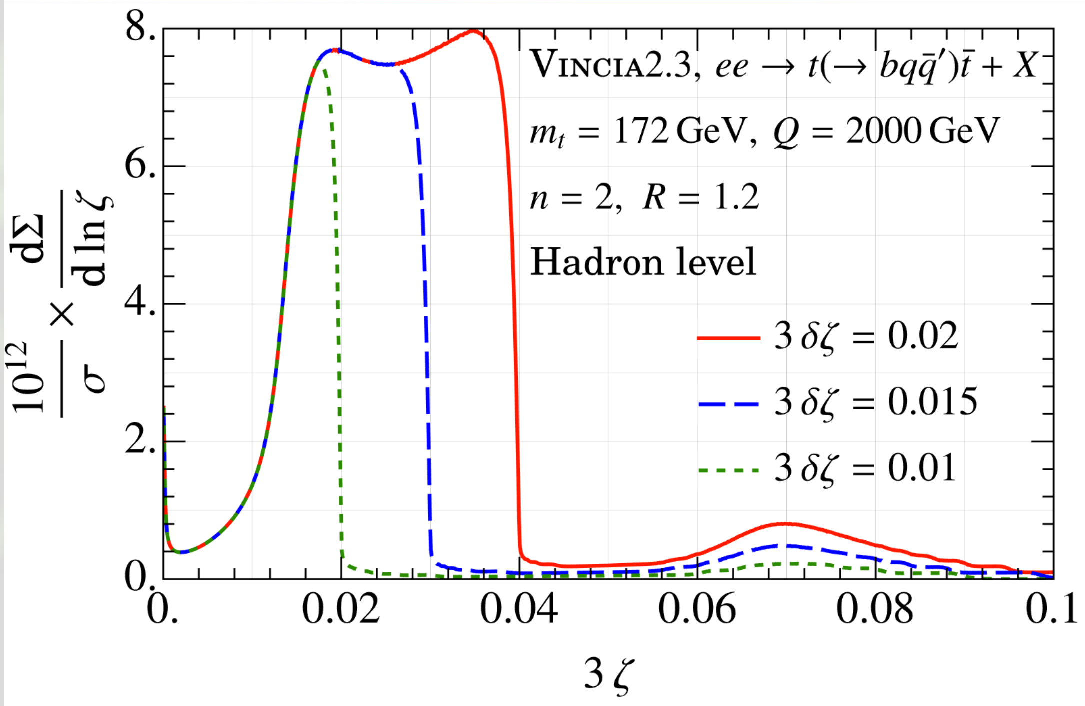
Fit function:

$$\rho = \rho_{\text{asy}} + c_2 (p_{T,\text{jet}}^{\text{v}})^{-2} + c_3 (p_{T,\text{jet}}^{\text{v}})^{-3}$$



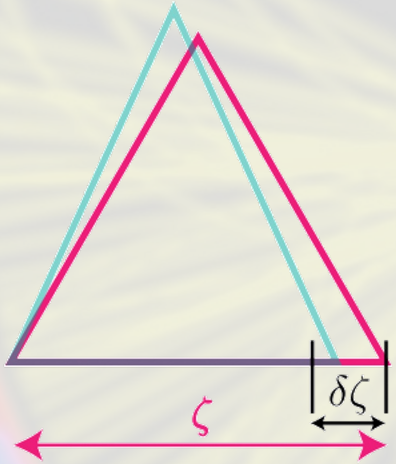
Supplementary material

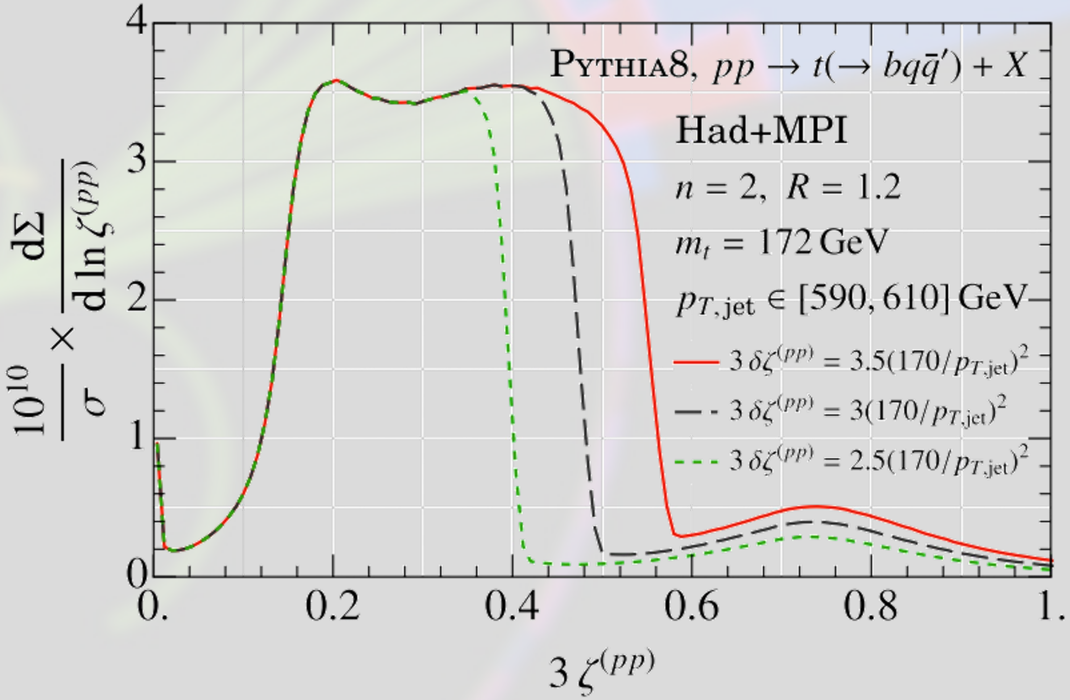




Supplementary material

Asymmetry cut $\delta\zeta$ only constrains triangles with $\zeta > 2\delta\zeta$

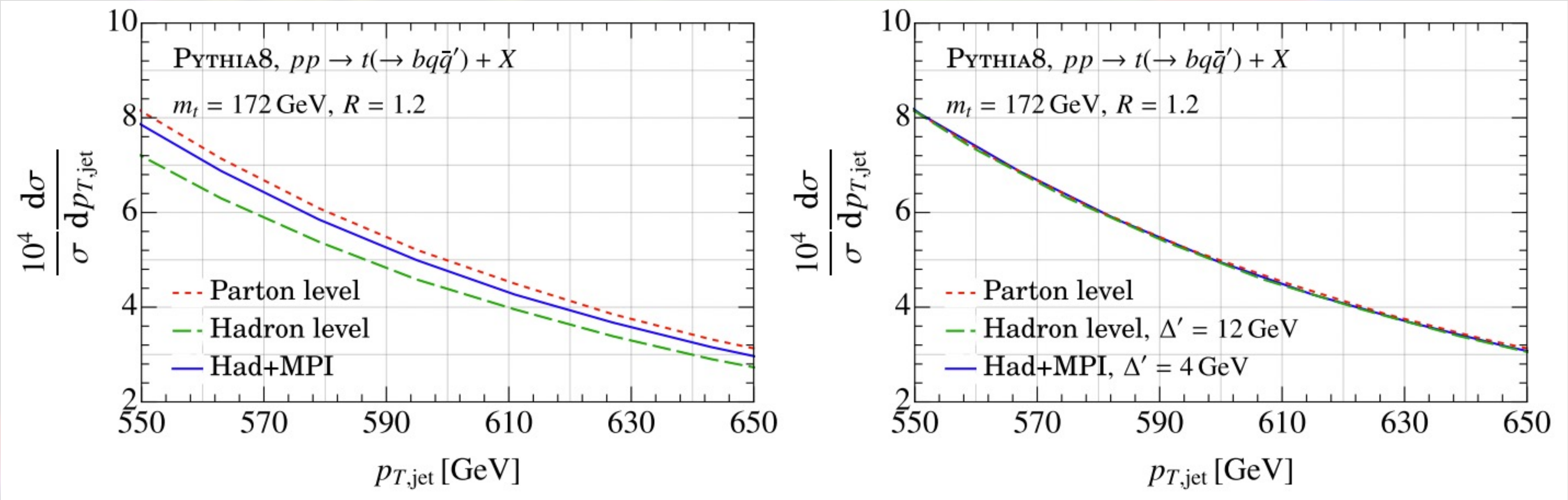




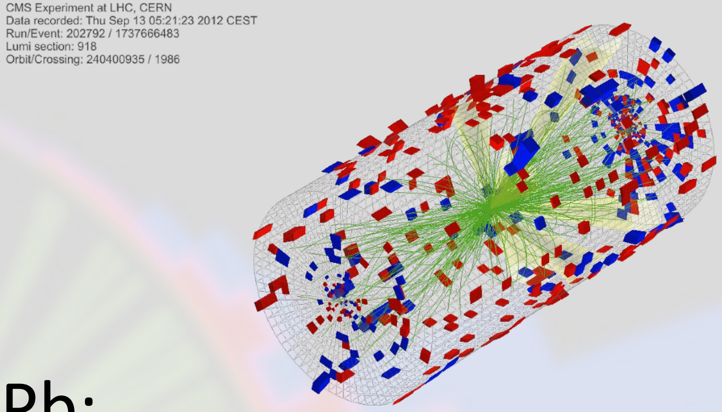
$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\zeta} \approx 4(\delta\zeta)^2\, G^{(n)}(\zeta,\zeta,\zeta;m_t)\,, \qquad \delta\zeta \ll \zeta$$

Supplementary material

Here we show $p_{T,\mathrm{jet}}$ shifts relative to parton level:



Correlation Functions



- Case study from study of the QGP in Pb-Pb:

$$\langle N(\eta_1,\phi_1)N(\eta_2,\phi_2)\rangle = N_1N_2P(\eta_1,\phi_1,\eta_2,\phi_2)$$

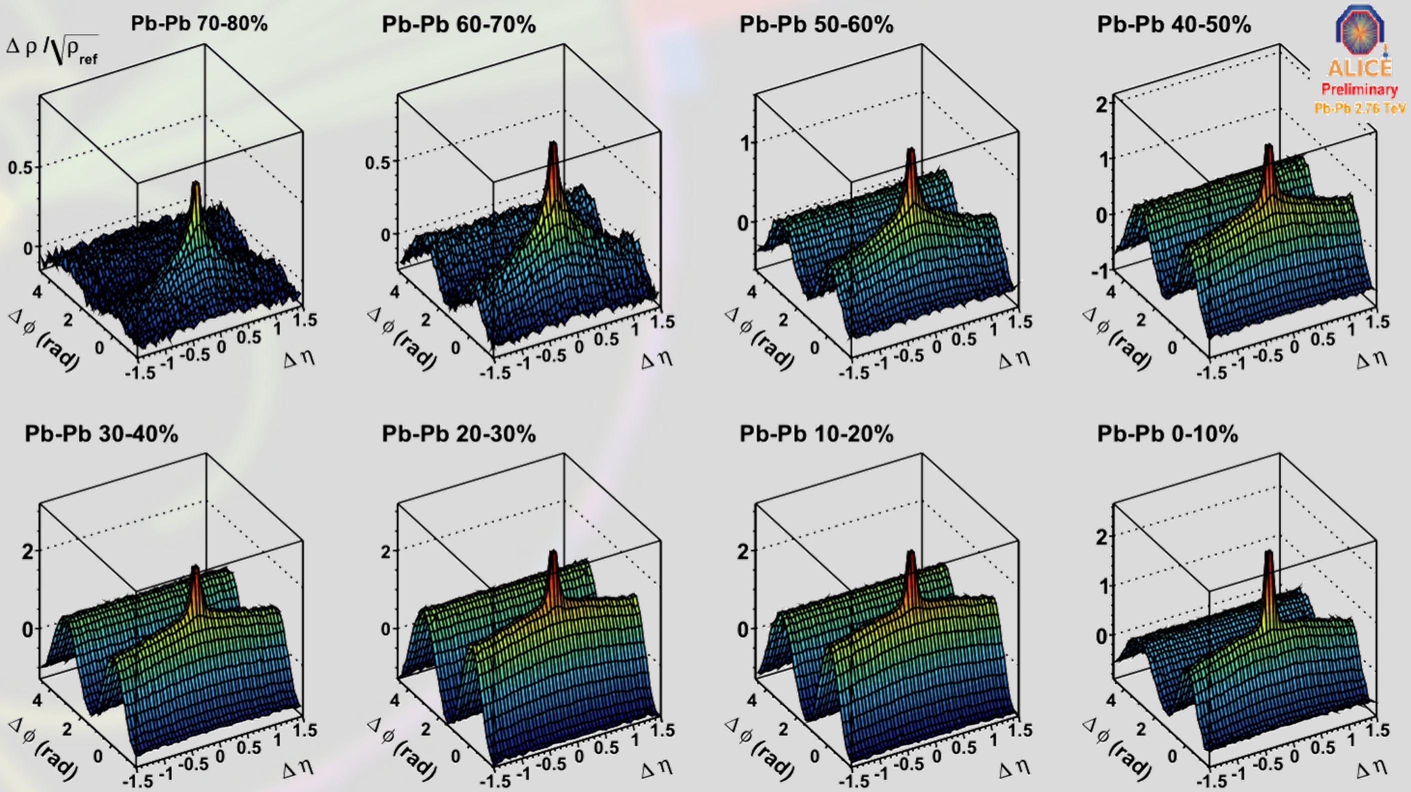
$$\sim \sum_X N_X(\eta_1,\phi_1)N_X(\eta_2,\phi_2)\langle \mathrm{Pb}-\mathrm{Pb}|X\rangle\langle X|\mathrm{Pb}-\mathrm{Pb}\rangle$$

$$= \sum_X \langle \mathrm{Pb}-\mathrm{Pb}\big|\hat{N}\,(\eta_1,\phi_1)\hat{N}(\eta_2,\phi_2)|X\rangle\langle X\big|\mathrm{Pb}-\mathrm{Pb}\rangle$$

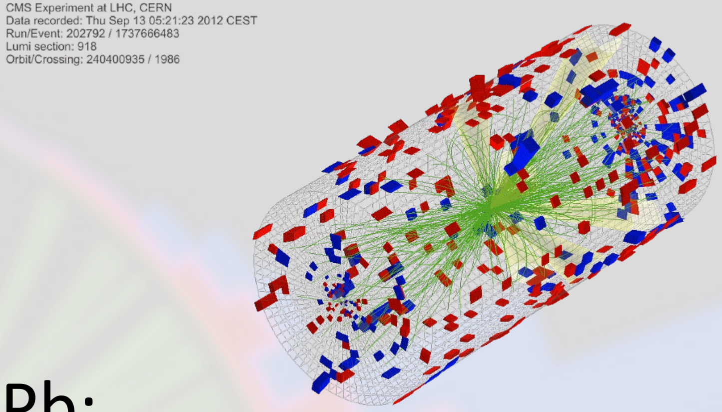
$$= \langle \mathrm{Pb}-\mathrm{Pb}\big|\hat{N}\,(\eta_1,\phi_1)\hat{N}(\eta_2,\phi_2)\,\big|\mathrm{Pb}-\mathrm{Pb}\rangle$$

$$\frac{dN}{d^2p\,d^2k\,d\eta\,d\xi} = \langle\hat{\sigma}(k)\ \hat{\sigma}(p)\rangle_{P,T}$$

1211.1928

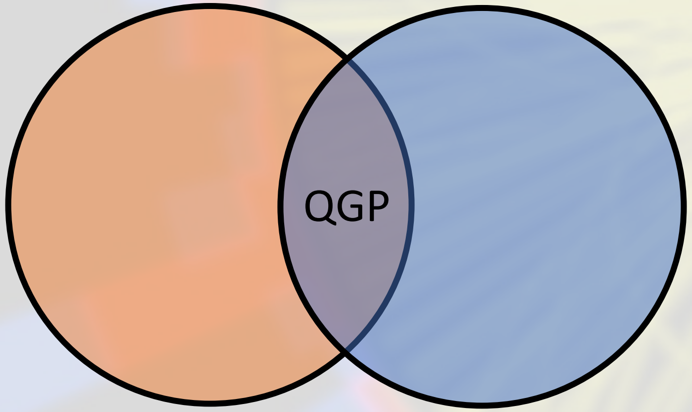


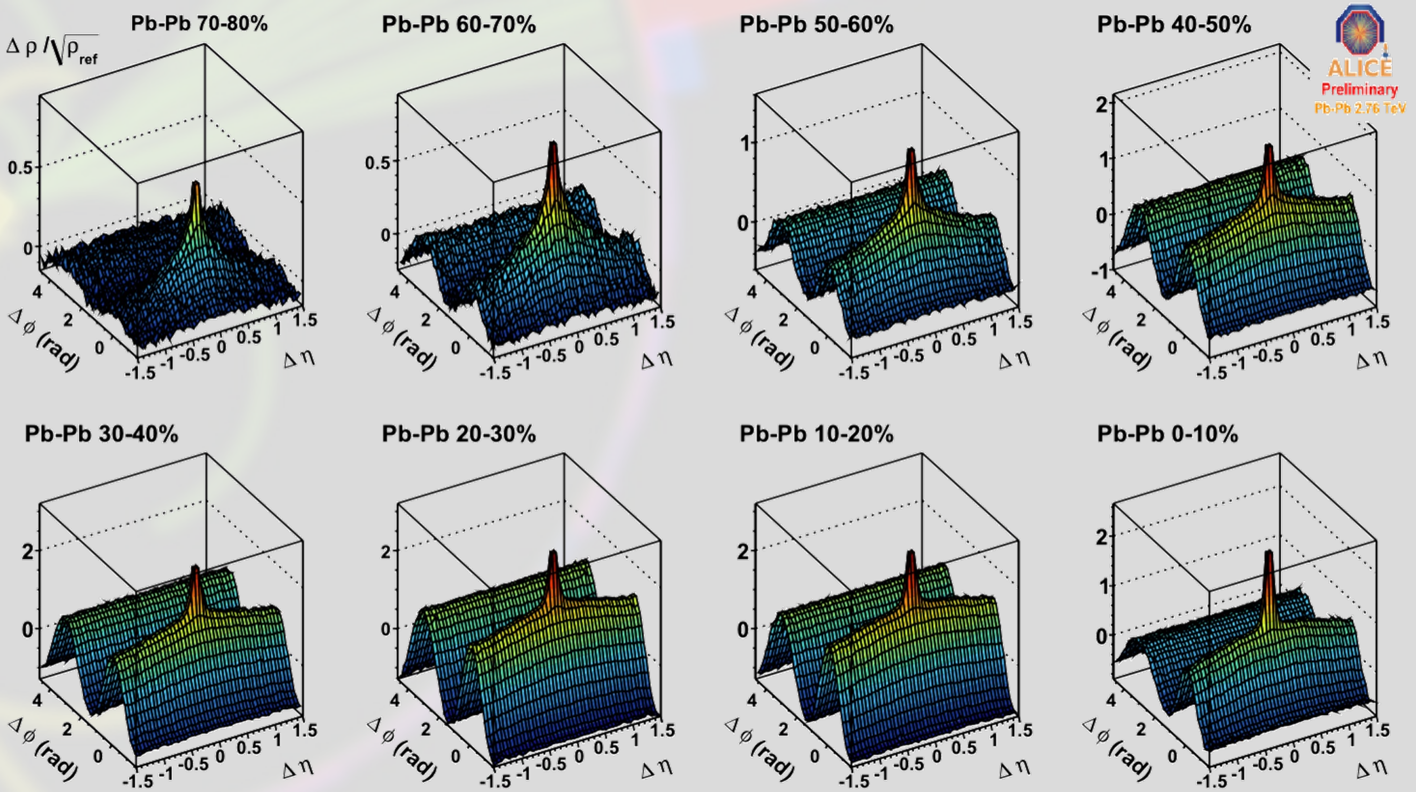
1106.6057



Correlation Functions

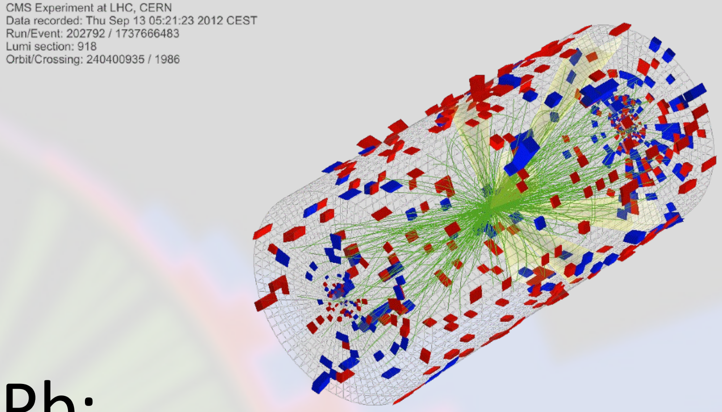
- Case study from study of the QGP in Pb-Pb:



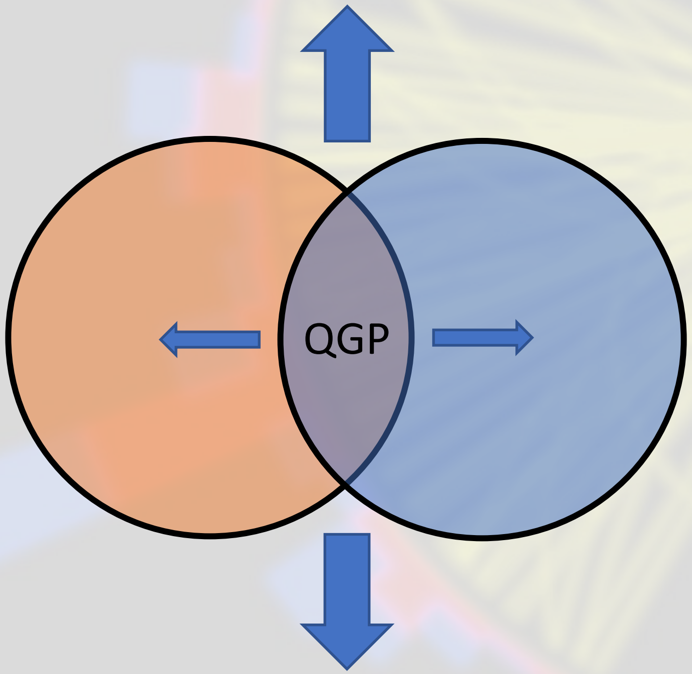


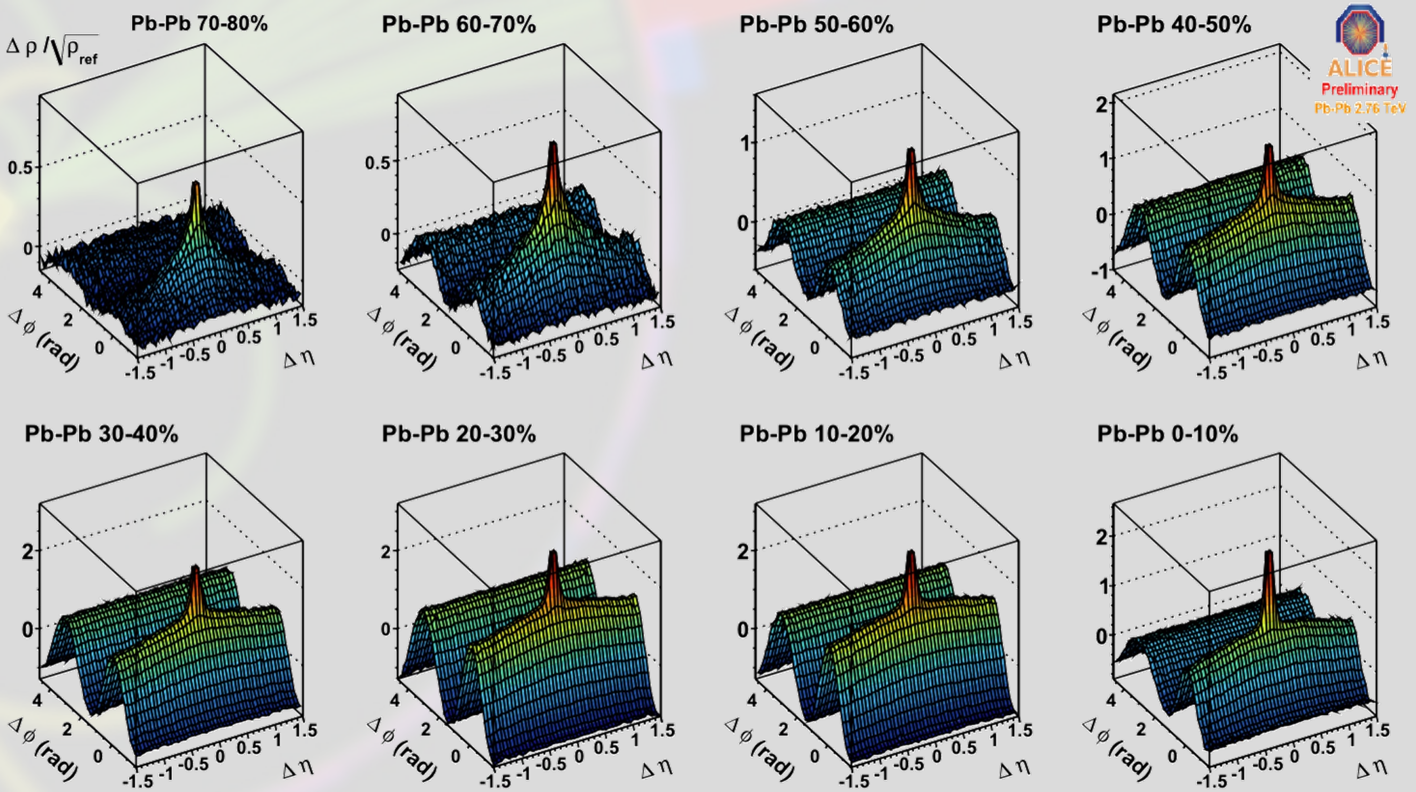
[1106.6057]

Correlation Functions



- Case study from study of the QGP in Pb-Pb:





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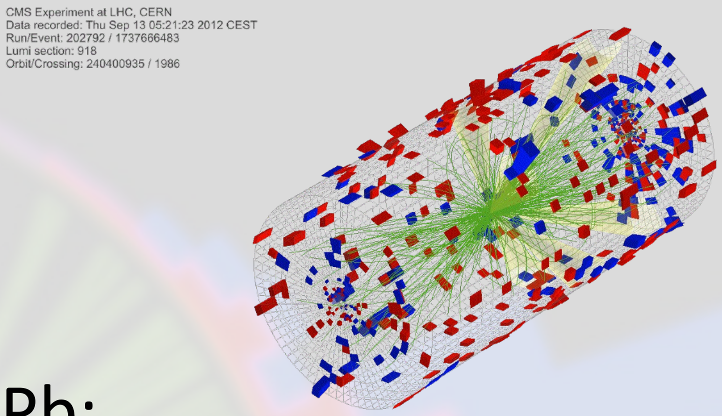
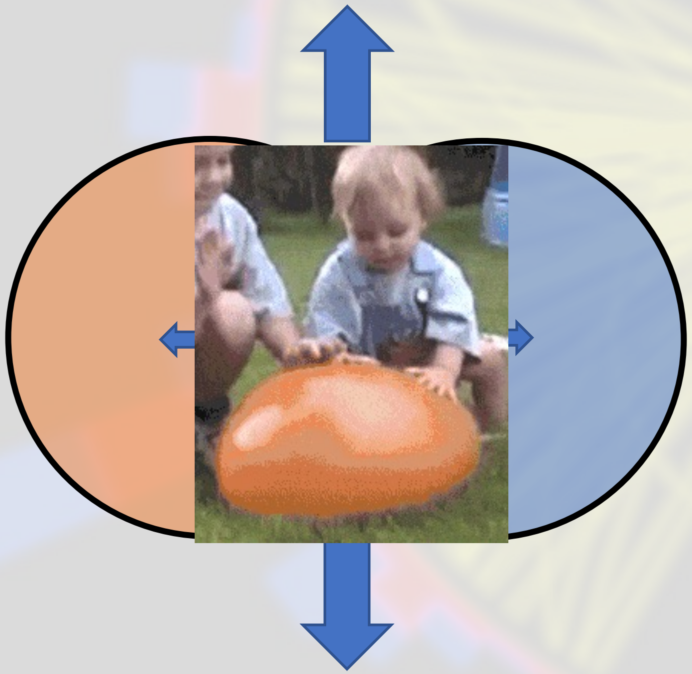
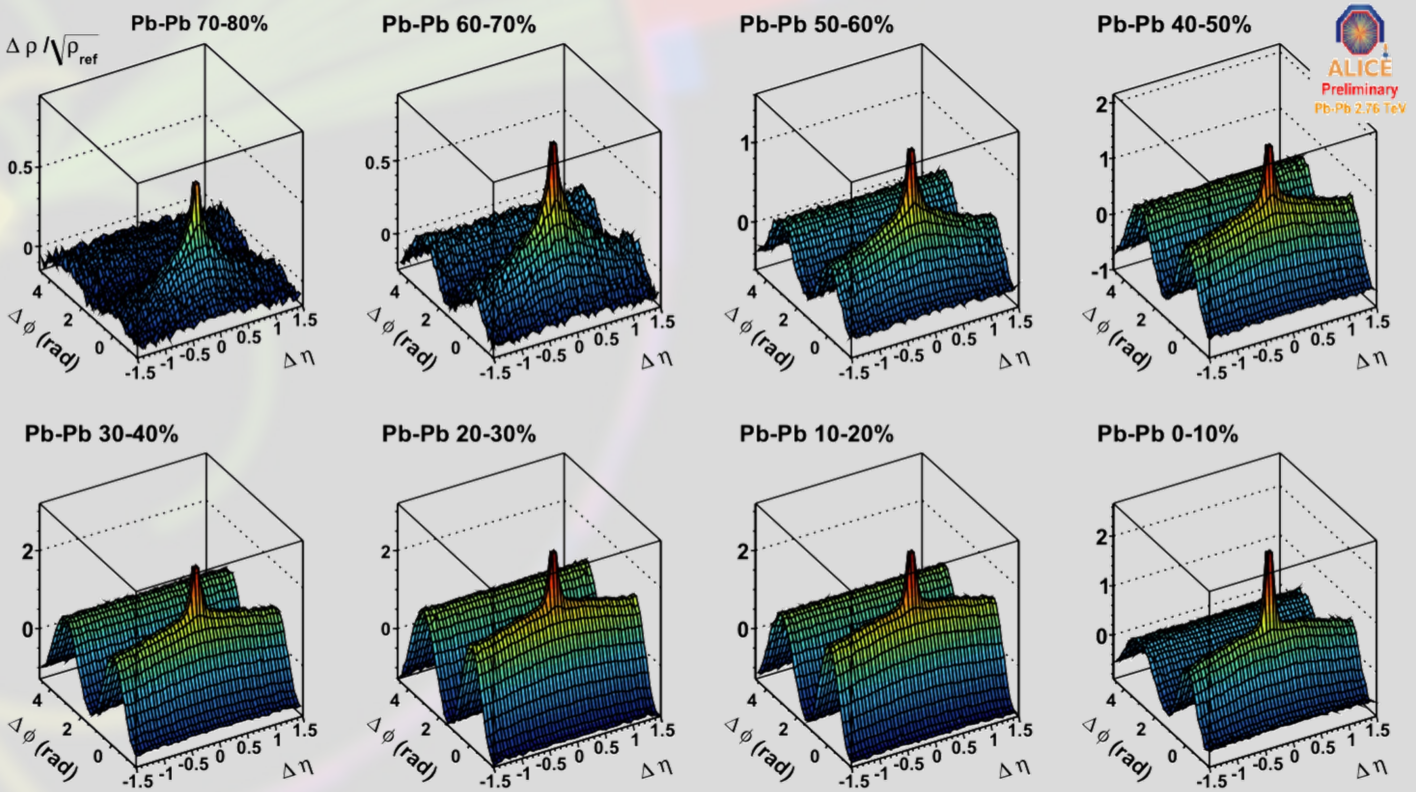
Correlation Functions



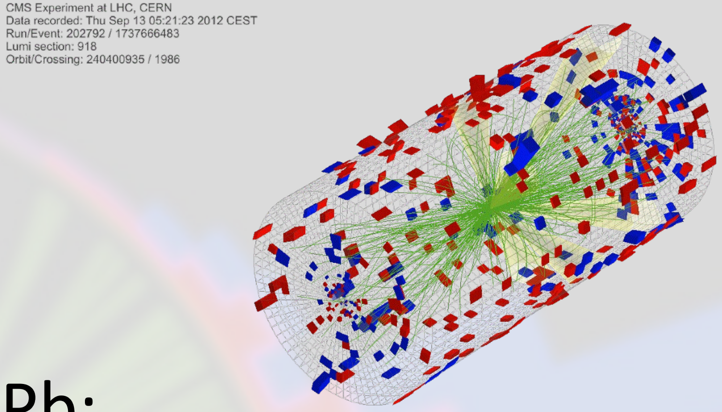
- Case study from study of the QGP in Pb-Pb:



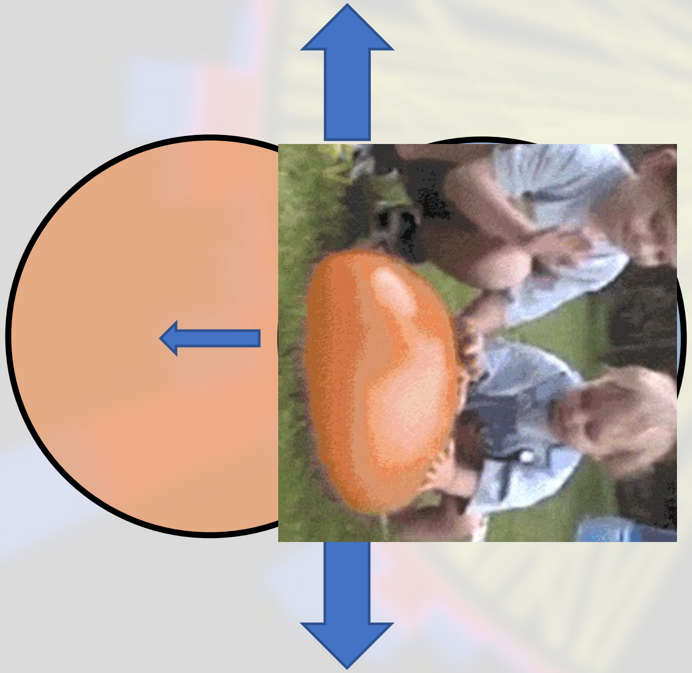


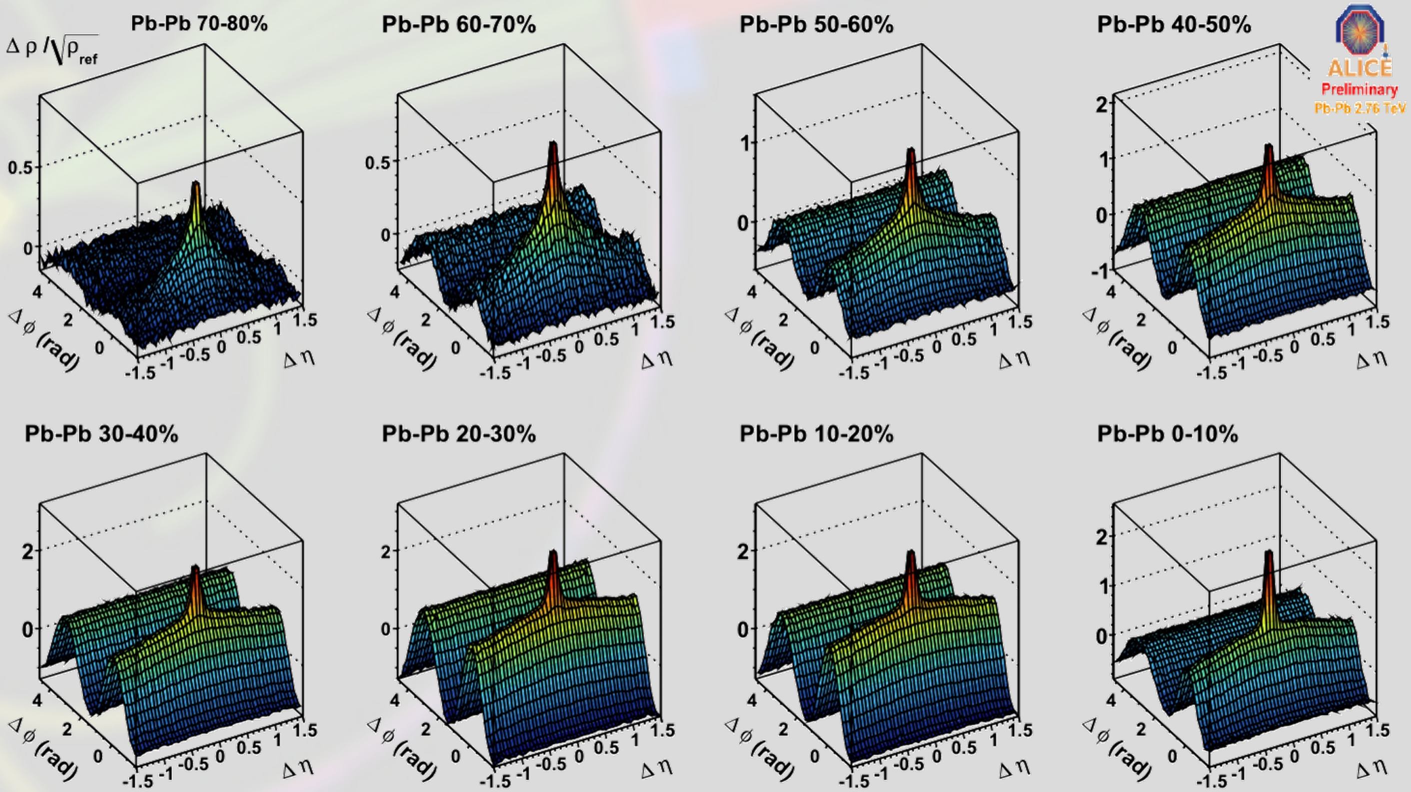
[1106.6057](1106.6057)

Correlation Functions



- Case study from study of the QGP in Pb-Pb:





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Correlation Functions

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\; r^2 n^i T_{0i}(t, r\vec{n})$$

$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle$ ← Not the QFT VEV.

Quantum mechanical expectation value, just as $\langle \Psi | \hat{p} | \Psi \rangle = \langle p \rangle$:

i.e.

$\mathcal{S}_{i\to j} = \langle j | i \rangle$ let $|j\rangle = \mathcal{O}|\Omega\rangle$

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = N \langle \Omega | \mathcal{O}^\dagger \hat{\mathcal{E}}(\vec{n}_1)\hat{\mathcal{E}}(\vec{n}_2)\mathcal{O}|\Omega\rangle$$

End