NNLL resummation of groomed additive observables

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Outline

Groomed jet shape observables are a good choice for direct comparison between perturbative QCD and measurements:

- less susceptible pileup and underlying event
- hadronisation corrections are smaller
- Resummations are easier due to lack of non global logs

Introduction

- pQCD calculations have been compared to measurements for several angularities and the jet mass (CMS, ATLAS)

- Calculations compared for the groomed jet mass are either NLL + NLO (Marzani, Schunk, Soyez) or NNLL in the small $z_{\rm cut}$ limit (Frye, Larkoski, Schwartz, Yan).

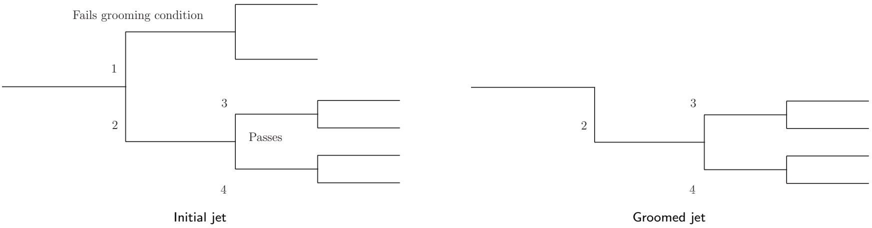
- Other groomed angularities are calculated to NLL accuracy with NLO matching (Caletti, Fedkevych, Marzani, Reichelt, Schumann, Soyez).

- Aim is to get NNLL at small $z_{\rm cut}$ with finite $z_{\rm cut}$ at NLL for any additive observable and demonstrate NLO matching for a selection of observables.

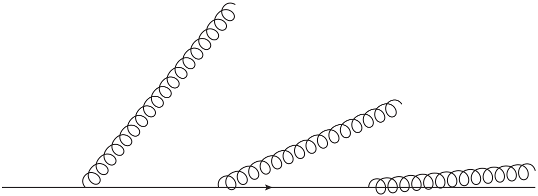
- This talk: $e^+e^- \to q\bar{q}$ split into two hemispheres, both of which are groomed with mMDTDasgupta, Fregoso, Marzani, Salam (Soft Drop Larkoski, Marzani, Soyez, Thaler with $\beta = 0$).

mMDT Grooming procedure

1. Start with a hemisphere (jet) clustered with the Cambridge Aachen (C/A) algorithm.
2. Undo the last clustering in the sequence to obtain two branches, i and j.
3. If the softer branch does not satisfy $\frac{\min(E_j,E_i)}{E_i+E_j} > z_{\mathrm{cut}}$ then it is discarded and the groomer returns to to step 2.
4. If $\frac{\min(E_i,E_j)}{E_i+E_j} > z_{\mathrm{cut}}$ is satisfied, the groomer stops and the groomed hemisphere contains all of the particles in both i and j.



Can consider strongly ordered primary emissions inclusive of branchings



$$\Sigma^{\mathrm{NLL}'}(v; z_{\mathrm{cut}}) = \frac{1}{\sigma}\int_0^v \frac{\mathrm{d}\sigma}{\mathrm{d}v'}\mathrm{d}v' = \left(1 + \frac{C_F\alpha_s}{\pi}C_1\right)\exp[-R^{\mathrm{NLL}'}(v, z_{\mathrm{cut}})]$$

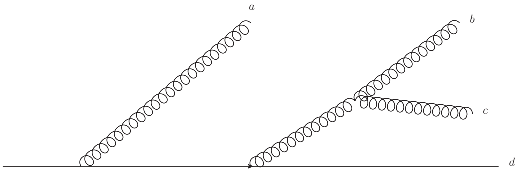
$$R^{\mathrm{NLL}'}(v, z_{\mathrm{cut}}) =$$

$$\int_{z_{\mathrm{cut}}}^1 \int^{z^2Q^2} \frac{C_F\alpha_s^{\mathrm{CMW}}(k_t^2)}{\pi}\left(\frac{2}{z} + \delta(1-z)\gamma_0^{\mathrm{h.c.}}\right)\frac{\mathrm{d}k_t^2}{k_t^2}\mathrm{d}z\Theta(V_{\mathrm{s.c}}(z, k_t) - v)$$

$$\gamma_0^{\mathrm{h.c.}} = -\frac{3}{2}$$

The resummation can be structured as an inclusive piece and a clustering correction:

$$\Sigma^{\mathrm{NNLL}}(v; z_{\mathrm{cut}}) = \Sigma_{\mathrm{inc.}}(v; z_{\mathrm{cut}}) + \Sigma_{\mathrm{clust.}}(v; z_{\mathrm{cut}})$$



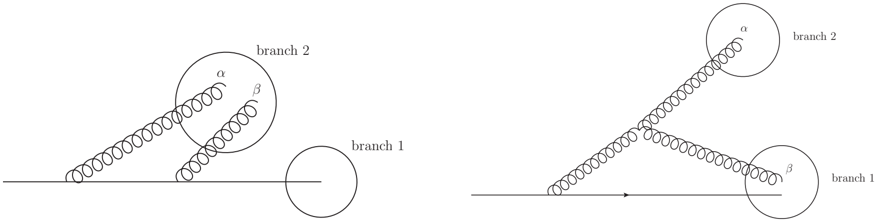
- hard collinear part proceeds the same as for ungroomed observables, e.g ARES (Banfi et al.)
- Can distil what we need into a modification of C_1 and evaluating the Sudakov factor up to NNLL accuracy.

$$\Sigma_{\text{inc.}}(v, z_{\text{cut}}) = \frac{1}{\sigma}\int_0^v \frac{\mathrm{d}\sigma}{\mathrm{d}v'}\mathrm{d}v' = \left(1 + \frac{C_F\alpha_s}{\pi}C^{\text{r.c}}(v)\right)\exp[-R^{\text{NNLL}}(v, z_{\text{cut}})]$$

$$R^{\text{NNLL}}(v, z_{\text{cut}}) = \int_{z_{\text{cut}}}^1 \int^{z^2Q^2} \frac{C_F\alpha_s(k_t^2)}{\pi}\left(\frac{2}{z}\left(1 + \frac{\alpha_s(k_t^2)K^{\text{CMW}}}{2\pi}\right)\right. \\ \left. + \delta(1-z)\left(\gamma_0^{\text{h.c.}} + \frac{\alpha_s(k_t^2)}{2\pi}\gamma_1^{\text{h.c.}}\right)\right)\frac{\mathrm{d}k_t^2}{k_t^2}\mathrm{d}z\Theta(V_{\text{s.c}}(z, k_t) - v)$$

- soft part is essentially the same as at NLL as soft logs are logs of z_{cut}
- Multiple emission effects turn out to be N^{3}LL

NNLL resummation: Clustering Correction



$$z_\alpha < z_{\text{cut}}, \qquad z_\beta < z_{\text{cut}}, \qquad z_{\text{cut}} < z_\alpha + z_\beta$$

$$\Sigma^{\text{clust}}(v, z_{\text{cut}}) = \mathcal{F}_{\text{clust.}}(v)\exp[-R^{\text{NLL}}(v, z_{\text{cut}})]$$

$$\mathcal{F}_{\textsf{clust.}}(v) = \left(\frac{\alpha_s}{2\pi}\right)^2 \times$$

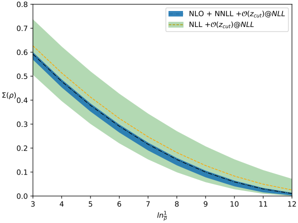
$$\left(C_F^2 \frac{4\pi}{3}\mathsf{Cl}_2\left(\frac{\pi}{3}\right) - C_F C_A 1.161 - C_F T_R n_f 1.754\right)\frac{\ln\left(v^{\frac{2}{a+b}}\right)}{1+\beta_0\alpha_s(Q^2)\ln(v^{\frac{2}{a+b}})}$$

- NLL terms suppressed by powers of $z_{\rm cut}$ could be of a similar size to NNLL terms.

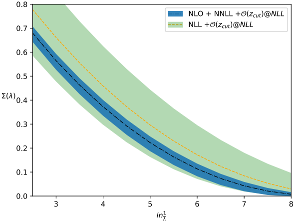
- Can match NNLL small $z_{\rm cut}$ calculation to finite $z_{\rm cut}$ NLL calculation (Dasgupta, Fregoso, Marzani, Salam)

$$\Sigma(v, z_{\rm cut}) = \big(1 + C^{\rm r.c}(v) + \mathcal{F}_{\rm clust.} \quad , 1\big)$$
$$\exp\begin{pmatrix} -R_q^{\rm NNLL} - R_{q\ z_{\rm cut}} - R_{q\to g} & R_{g\to q} \\ R_{q\to g} & -R_g^{\rm NLL} - R_{g\ z_{\rm cut}} - R_{g\to q} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}\ , \quad (1)$$

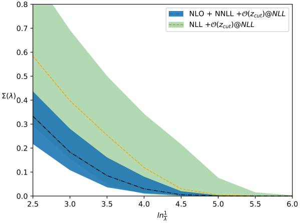
- Method applied to three observables: heavy hemisphere mass, Width (λ_1^1) and Les Houches angularity ($\lambda_{0.5}^1$)
- NLO matching using multiplicative scheme
- Uncertainty from varying resummation and renormalisation scales to introduce variation beyond the stated accuracy.



Mass

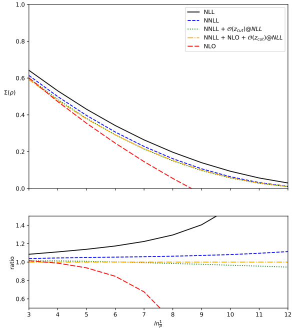


Width

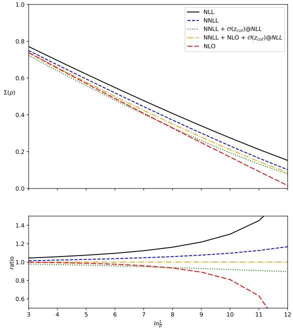


Les Houches Angularity

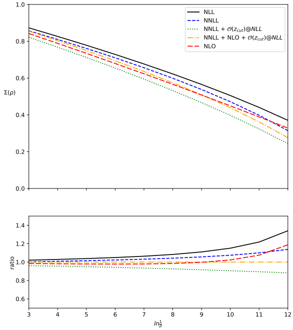
Heavy hemisphere mass



$z_{\rm cut} = 0.1$



$z_{\rm cut} = 0.2$



$z_{\rm cut} = 0.3$

Summary and Outlook

- General method for NNLL resummation of additive observables computed on jets groomed with mMDT
- First results with small z_{cut} at NNLL + $\mathcal{O}(z_{\mathrm{cut}})$ at NLL + NLO matching
- future work: extension to gluon jets
- Other extensions: non-additive observables, Soft drop with $\beta \neq 0$

Backup Slides

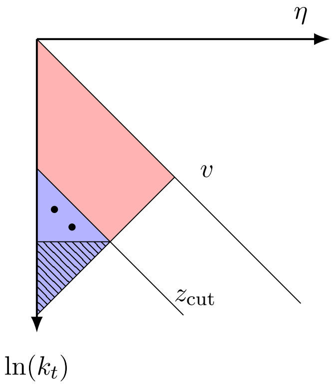
Additive Observables

$$V_{\rm s.c}(k_1, ...k_n) = \sum_{i}^{n} V_{\rm s.c}(k_i)$$

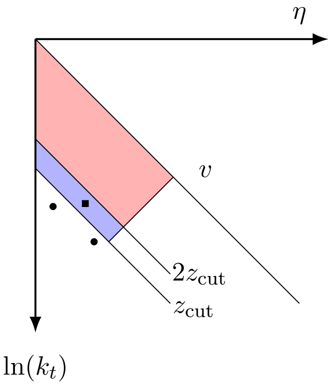
Can ignore the contribution of all emissions softer then z_{cut} to the observable

$$\to \Sigma^{\mathrm{NLL}}(v, z_{\mathrm{cut}}) = \frac{\exp[R(v, z_{\mathrm{cut}}) - \gamma_{\mathrm{E}} R'(v, z_{\mathrm{cut}})]}{\Gamma[1 + R'(v, z_{\mathrm{cut}})]} = \exp[-R(v, z_{\mathrm{cut}})] + \mathrm{N}^3\mathrm{LL} \tag{2}$$

Clustering Lund Planes



Independent emission



Correlated emission

Matching

$$\Sigma(v) = \Sigma_{\text{NNLL}}(v)\left[1 + \left(\Sigma^{(1)}(v) - \Sigma^{(1)}_{\text{NNLL}}(v)\right) + \left(\Sigma^{(2)}(v) - \Sigma^{(2)}_{\text{NNLL}}(v)\right) - \Sigma^{(1)}_{\text{NNLL}}(v)\left(\Sigma^{(1)}(v) - \Sigma^{(1)}_{\text{NNLL}}(v)\right)\right].$$

$\Sigma^{(n)}$ is the fixed order result up to α_s^n.

$$R(v, z_{\mathrm{cut}}) \to R^{\mathrm{NLL}}(v, z_{\mathrm{cut}}) + R^{\mathrm{remainder}}(xv, z_{\mathrm{cut}})$$

where

$$R^{\mathrm{remainder}}(v, z_{\mathrm{cut}}) = R^{\mathrm{NNLL}}(v, z_{\mathrm{cut}}) - R^{\mathrm{NLL}}(v, z_{\mathrm{cut}})$$

and

$$\alpha_s(Q) \to \alpha_s(xQ) + \alpha_s^2(Q)\beta_0 \ln(x)$$