Electromagnetic Design of Accelerator Magnets and ROXIE User's Course

Theory 1

Stephan Russenschuck, CERN, 2022

Timetable

Week	Mo	Tu	We	Th	Fr
15 (03.04)		Introduction, lumped circuits	Vector fields, field harmonics	14:00 Line currents and coil design	Magnet X-sections
16 (10.04)	Coil-ends (Brookhaven session)				
17 (17.04)			Optimization techniques	X-sec optimization	Numerical field comp., BEM-FEM
18 (24.04)	Yoke design	Integrated quant./ Dynamic effects / Computations		14:00 Diff. geom. /Coil ends, CCT / Cos theta ends	
19 (01.05)		Quench simulation	Demands and future plans/ Quench simulation (TBC)		

Perhaps a Master Class later in the year
Faraday paradoxes, coil magnetometers, stretched-wire measurements, CCT, Tori, ROXIE 22

Magnet Types

Iron dominated

Coil dominated

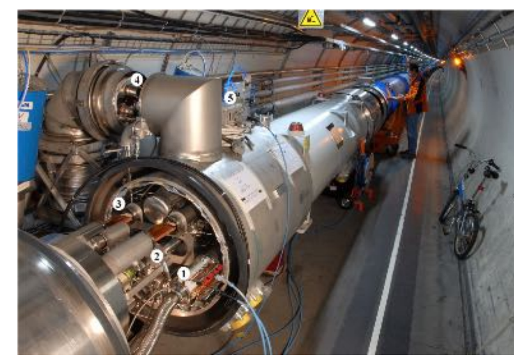
Normal conducting

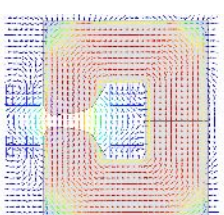
Superferric

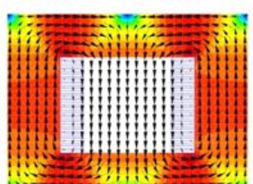
Permanent magnet

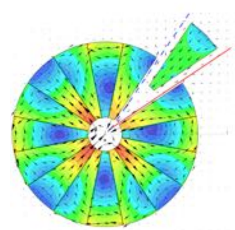
Class 1 large area, "medium" field

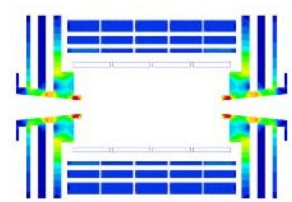
Class 2 Small area high field high current density

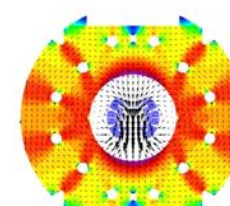






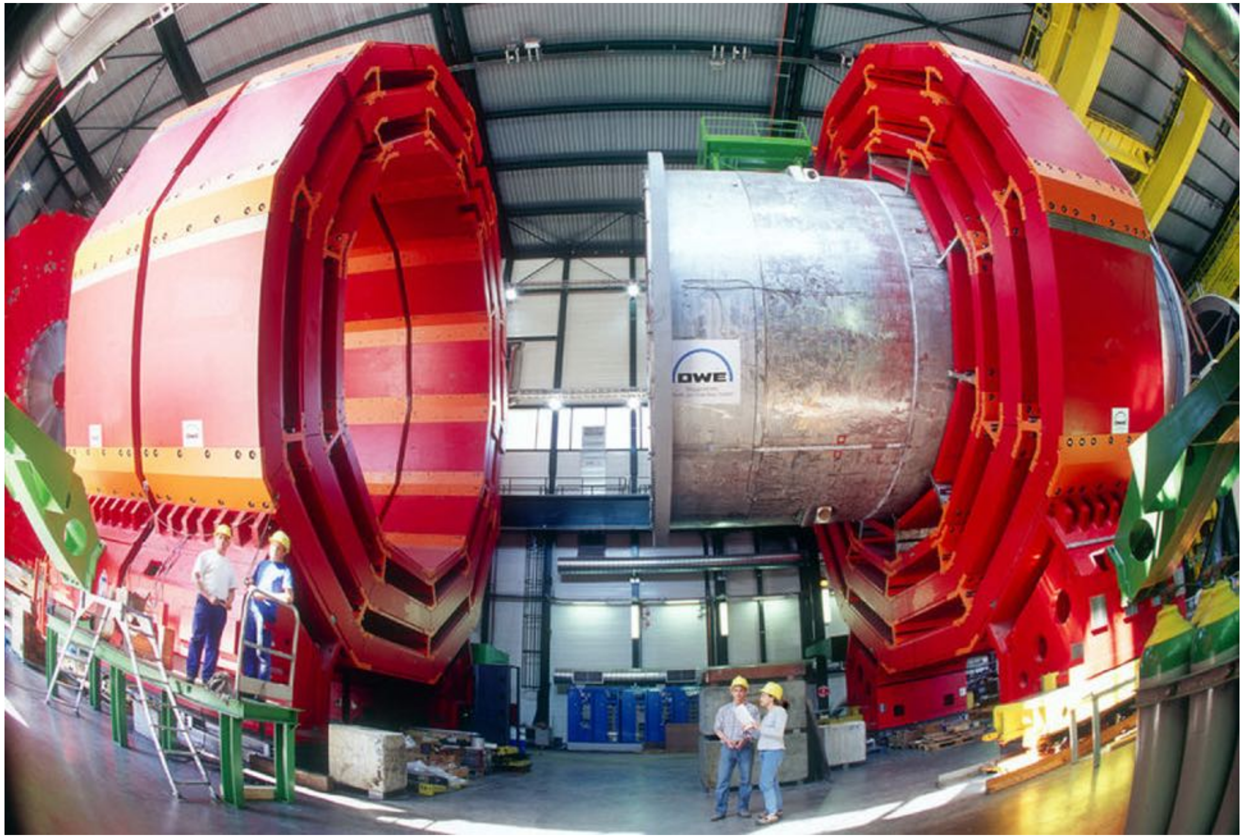






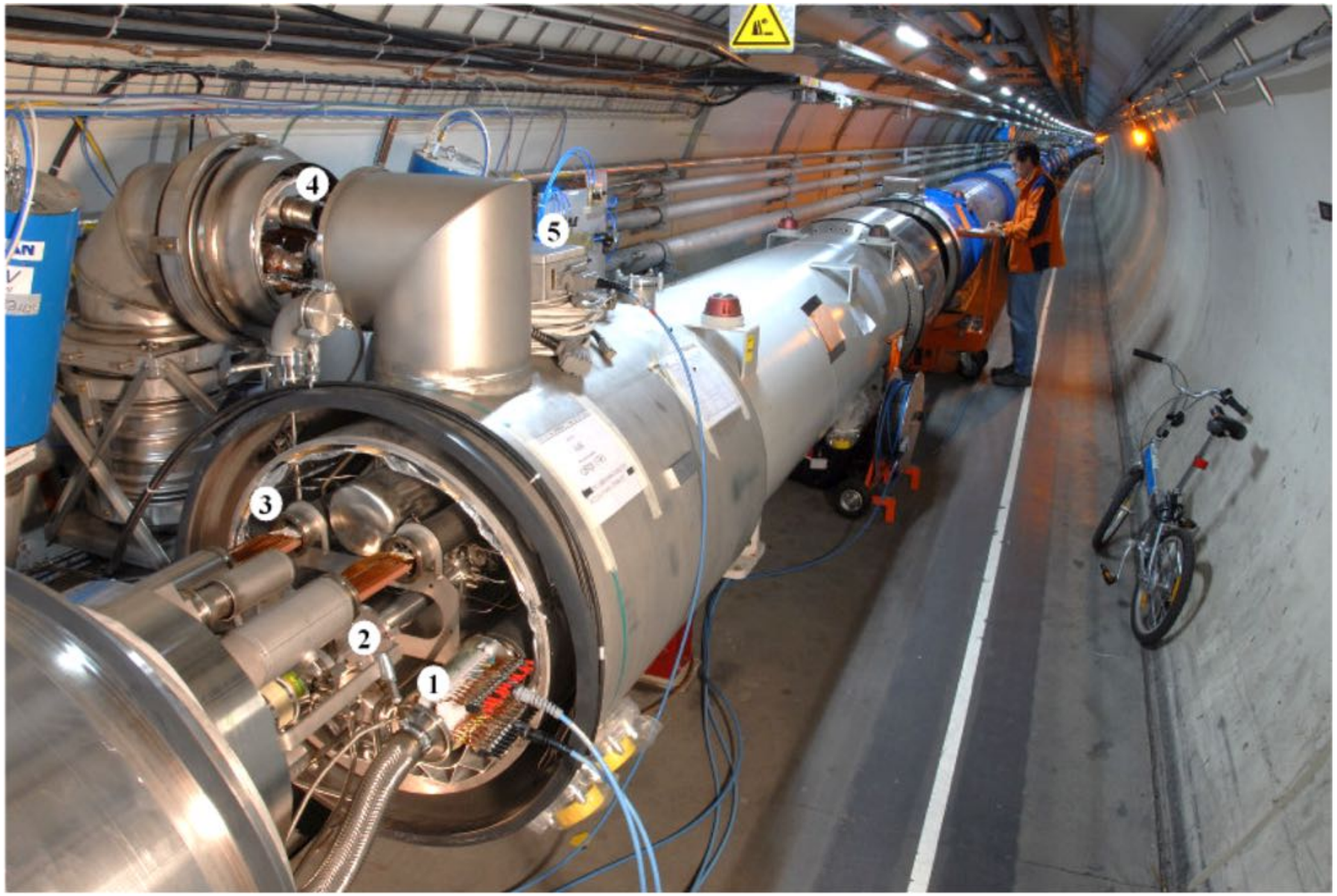
Solenoidal Magnet System for CMS (Superconducting Class 1 Magnet)

$$S = R\left[1 - \cos\left(\frac{\phi}{2}\right)\right] \approx \frac{R\phi^2}{8} = \frac{L^2}{8R} = \frac{eB_0L^2}{8p}$$



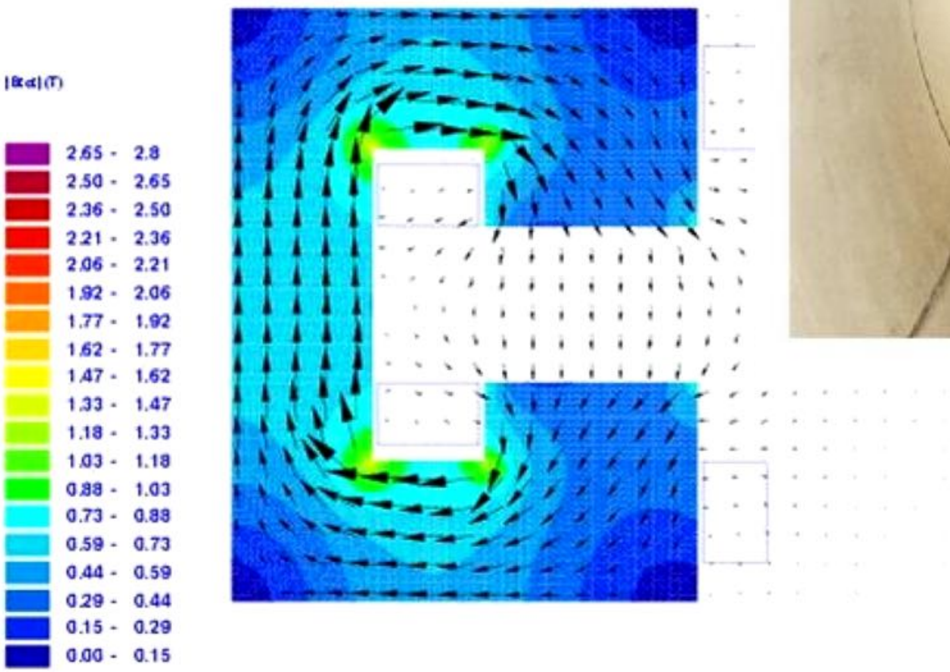
String of LHC Magnets in the Tunnel (Class 2 Magnets)

$$\{p\}_{\mathrm{GeV/c}} \approx 0.3\{Q\}_e\{R\}_{\mathrm{m}}\{B_0\}_{\mathrm{T}}$$



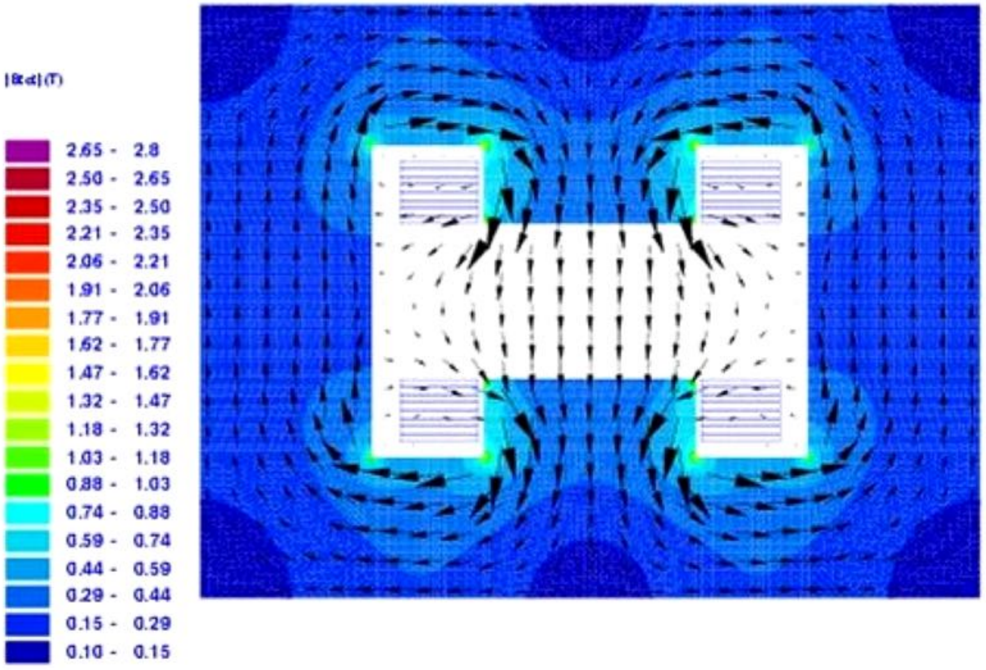
High field and high current density

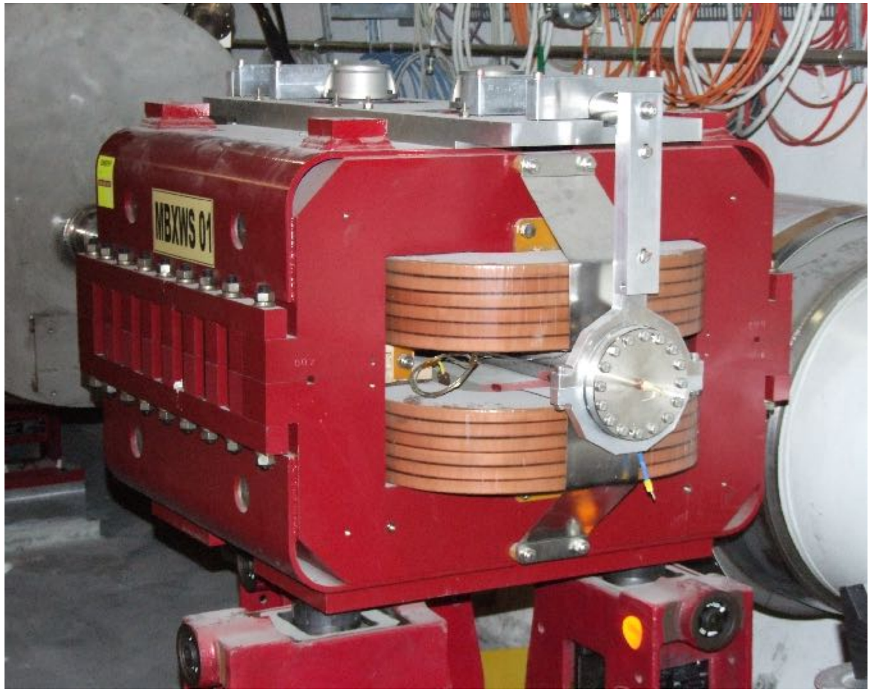
LEP Dipole (Iron Dominated Magnet)



$N \cdot I = 4480$ A $\quad B_1 = 0.13$ T $\quad B_S = 0.042$ T $\quad$ Fill.fac. 0.27

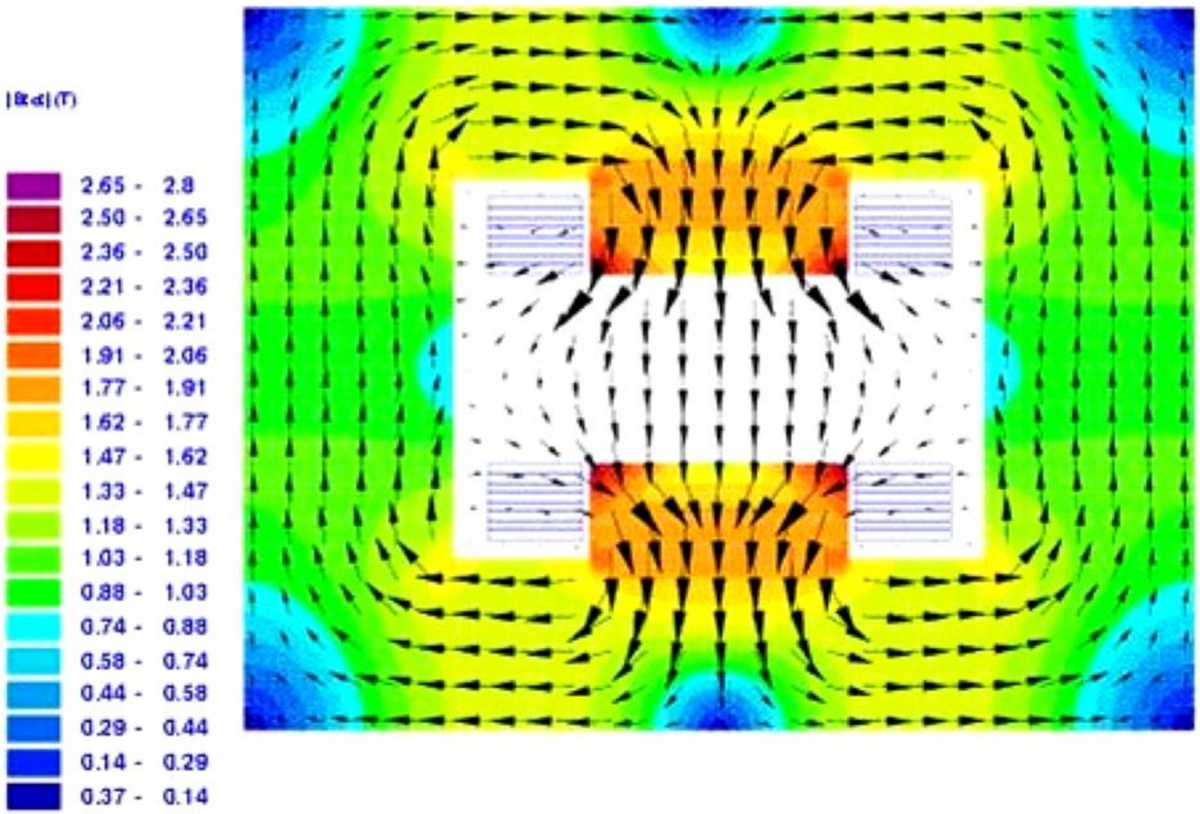
H Magnet (LHC transfer line)





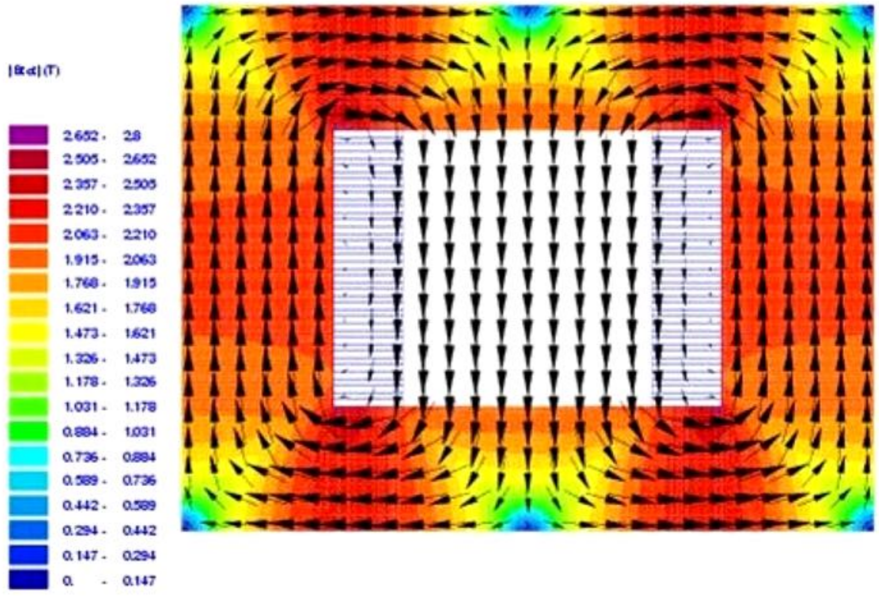
$N \cdot I = 24000$ A $\quad B_1 = 0.3$ T $\quad B_s = 0.065$ T $\quad$ Fill.fac. 0.98

Super-Ferric H Magnet

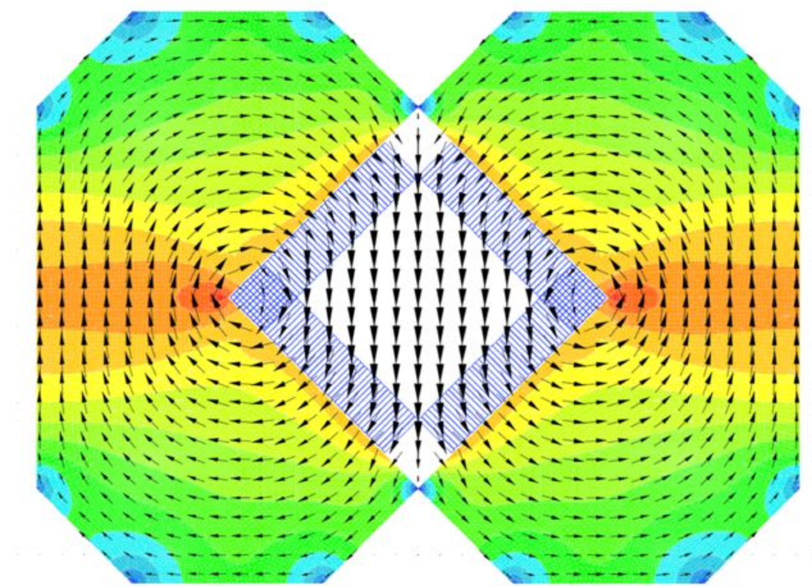


$N \cdot I = 96000\ \mathbf{A} \qquad B_1 = 1.18\ \mathbf{T} \qquad B_S = 0.26\ \mathbf{T}$

Window Frame Magnet

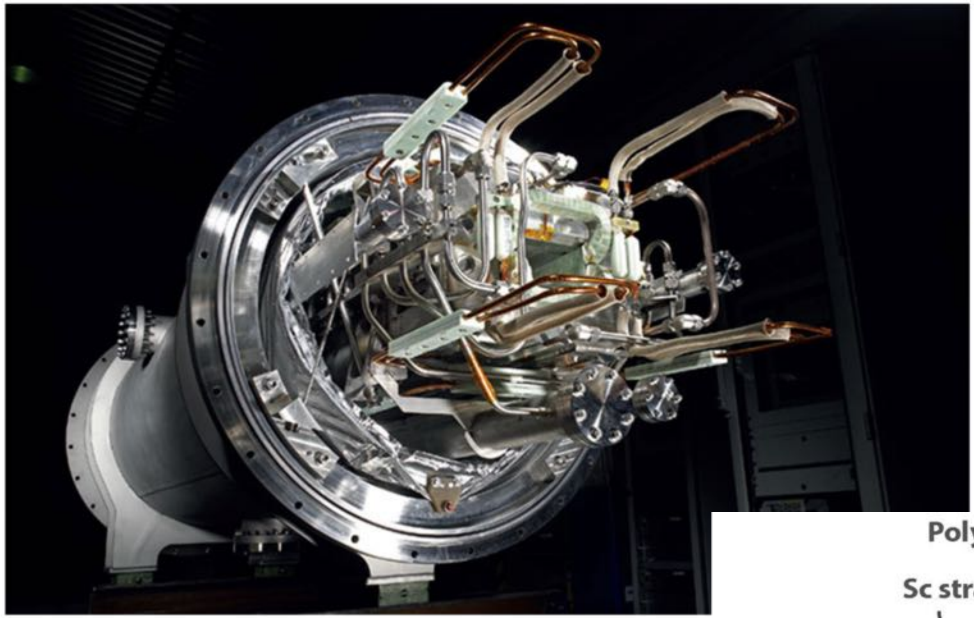


$N{\cdot}I = 360$ kA, $B_t = 2.0$ T, $B_s = 1.04$ T



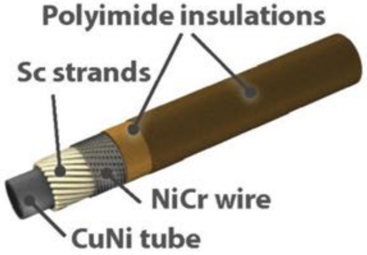
$N{\cdot}I = 625$ kA, $B_t = 2.38$ T, $B_s = 1.36$ T

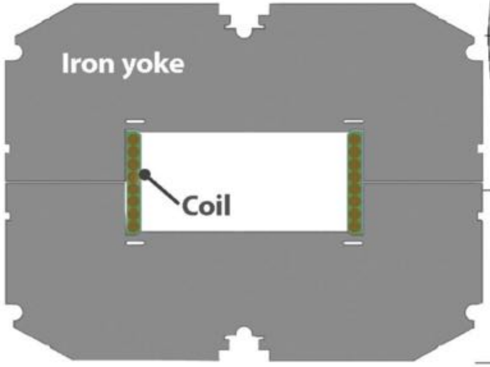
Example: SIS 100 Magnets

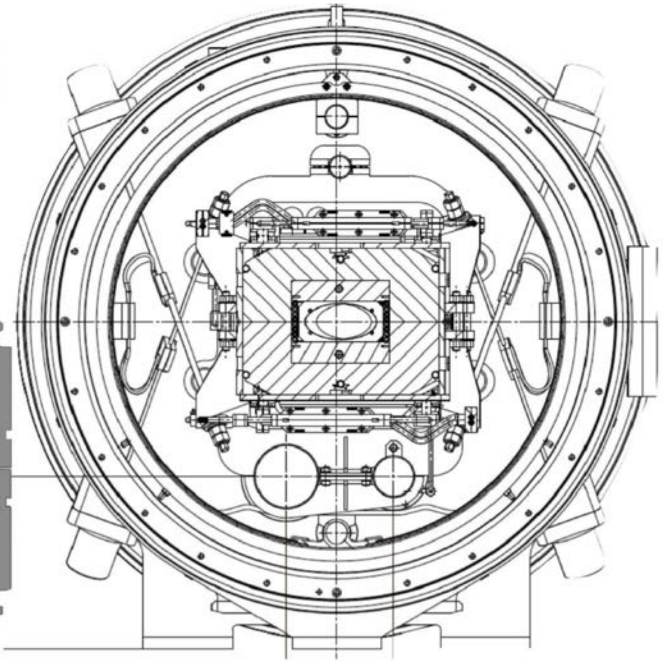


The newly developed fast-cycling superconductiong dipole magnet for FAIR's SIS100 synchrotron.
Image credit: Babcock Noell GmbH.

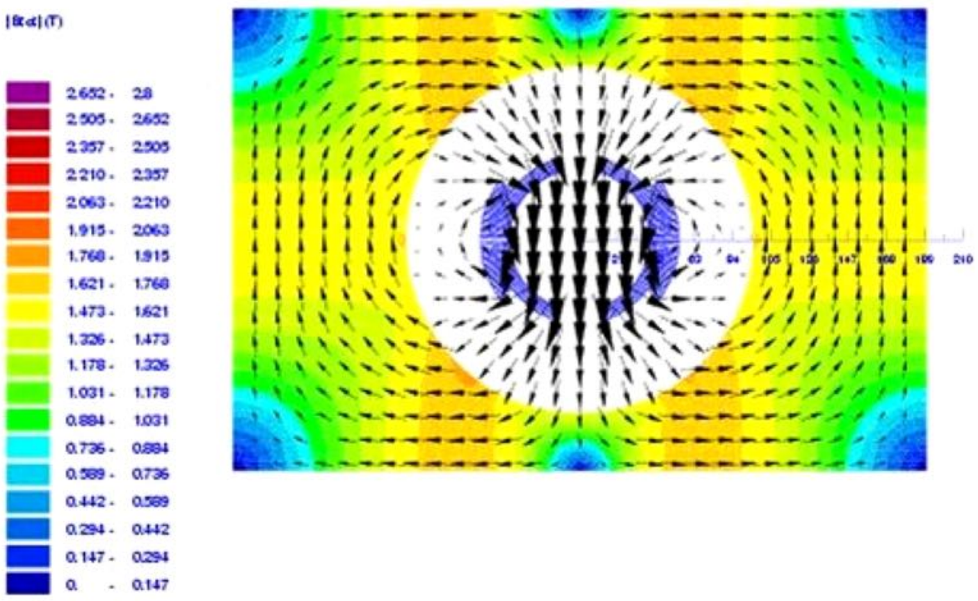
Parameter	Unit	Value
Max. field strength B_1	T	1.9
Max. current	kA	13.1
Ramp rate	T/s	4
Magnetic field quality		$\pm 6 \times 10^{-4}$

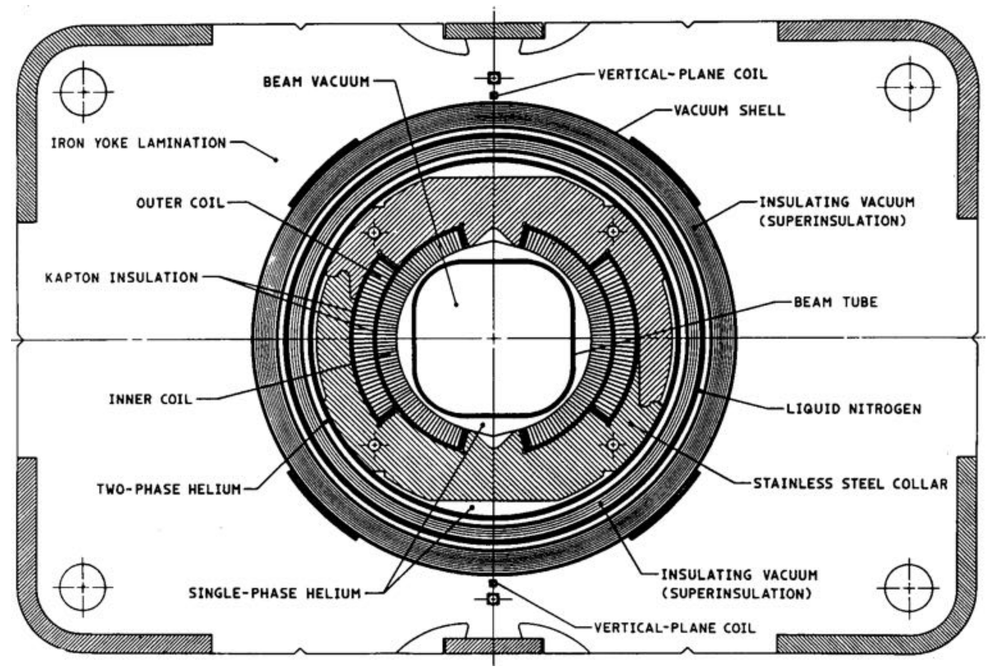






Cos θ (Warm iron yoke) - Tevatron Dipole (Coil Dominated Magnet)

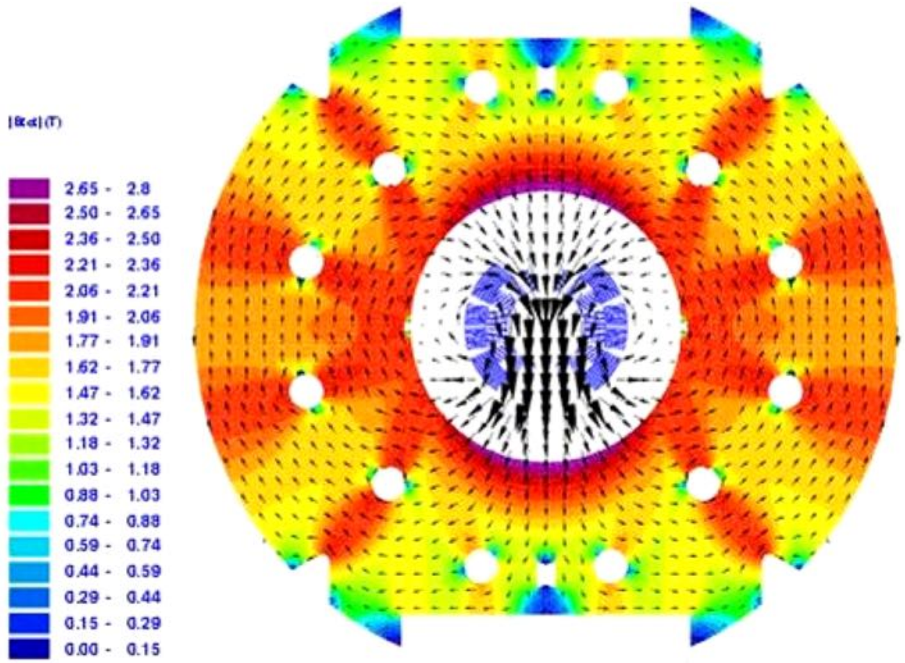




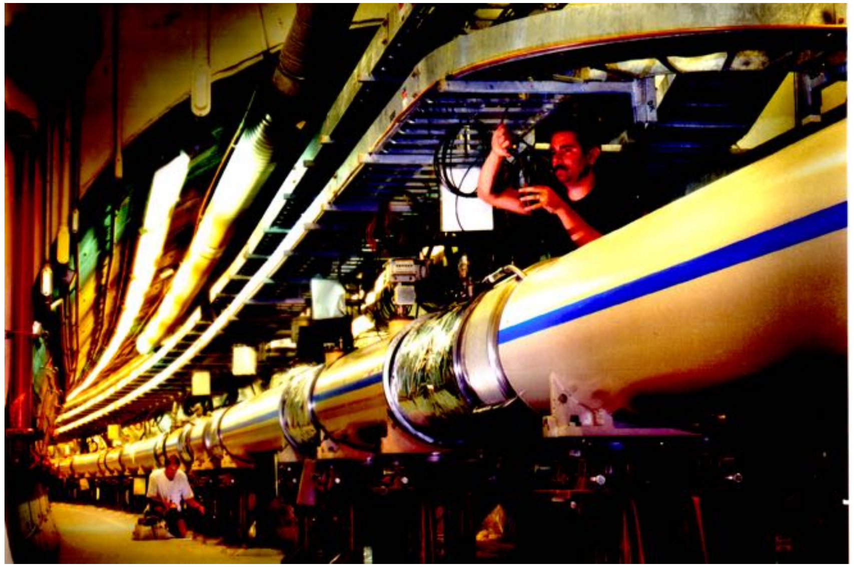
$N \cdot I = 471000$ A $\quad B_1 = 4.16$ T $\quad B_S = 3.39$ T

Notice the lower field in the iron yoke compared to the window frame

LHC Coil Test Facility for LHC (Based on HERA/RHIC Magnet Technology)

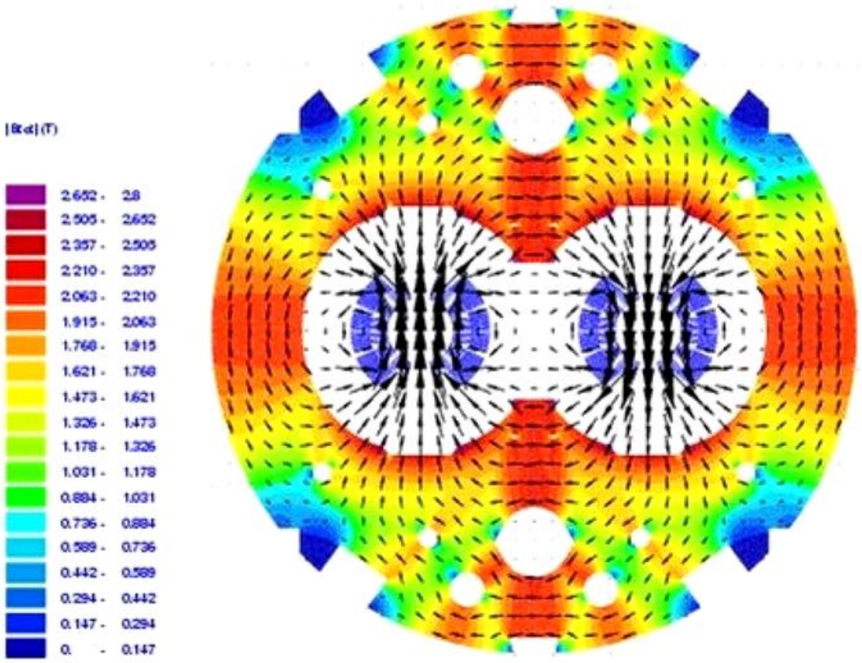


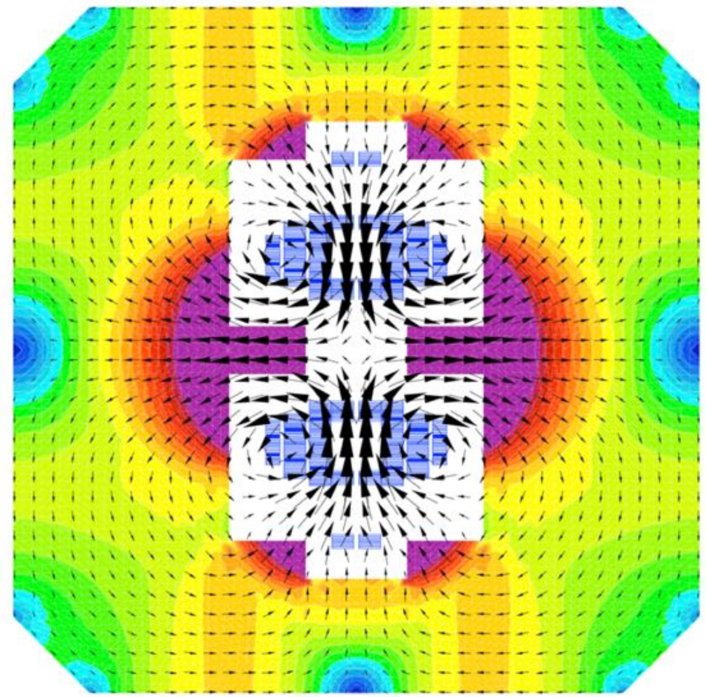
$N \cdot I = 960000$ A $\quad B_1 = 8.33$ T $\quad B_s = 7.77$ T



Two-In-One Dipoles

$$N \cdot I = 2 \cdot 944 \text{ kA}, B_t = 8.32 \text{ T}, B_s = 7.44 \text{ T}$$





$$N \cdot I = 2 \cdot 1034 \text{ kA}, B_t = 8.34 \text{ T}, B_s = 7.35 \text{ T}$$

Conventional and Superconducting Magnets

- **Normal conducting magnets**
  - Important ohmic losses require water cooling
  - Field is defined by the iron pole shape (max 1.5 T)
  - Easy electrical and beam-vacuum interconnections
  - Voltage drop over one coil of the MBW magnets = 22 V

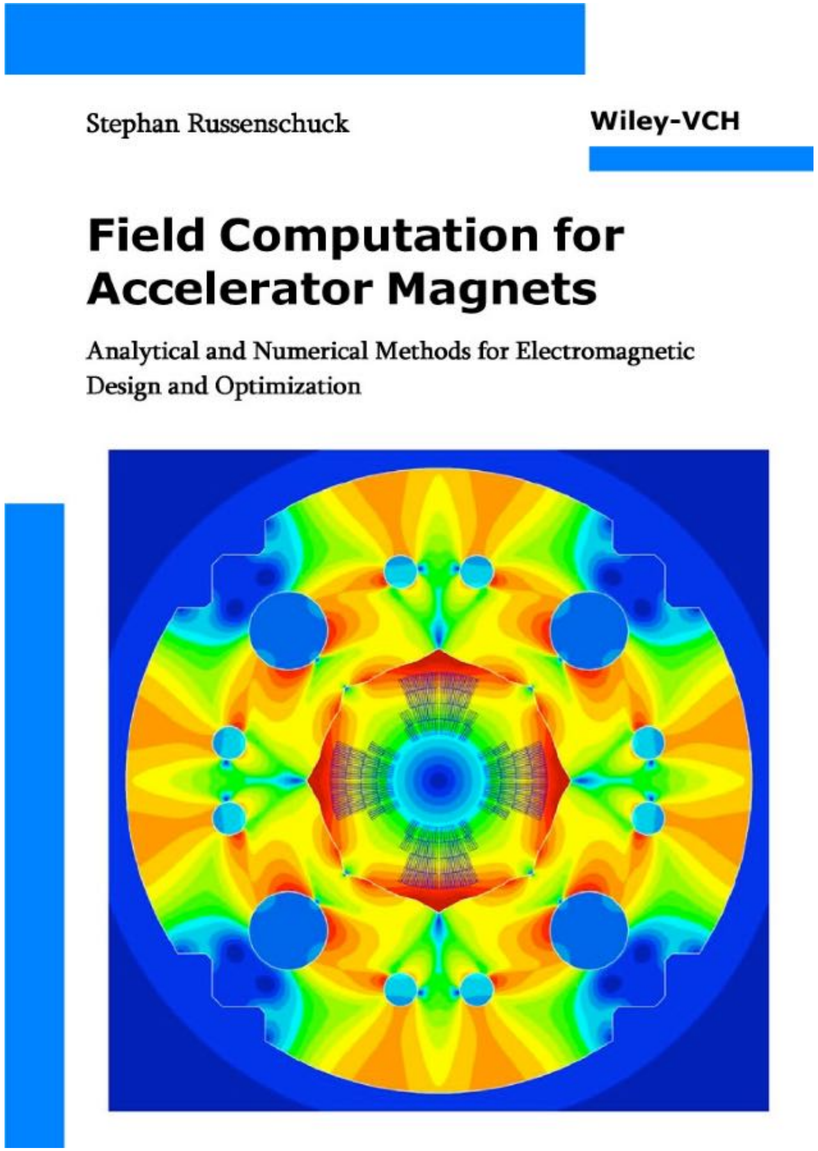
- **Superconducting magnets**
  - Field is defined by the coil layout
  - Maximum field limited to 10 T (NbTi), 14 T (Nb_3Sn)
  - Enormous electromagnetic forces (400 tons/m in MB for LHC)
  - Quench detection and magnet protection system required
  - Cryogenic installation (1.8 K)
  - Electrical interconnections in cryo-lines
  - Voltage drop on LHC magnet string (154 MB) 155 V

A Multiphysics Problem

- Beam physics
- Material science: Superconducting cable, Steel, Insulation
- Mechanics and large-scale mechanical engineering
- Vacuum technology
- Cryogenics (Superfluid helium)
- Metrology and alignment
- Field measurements
- Electrical engineering (Power supplies, leads, buswork, quench detection and magnet protection
- Analytical and numerical field computation

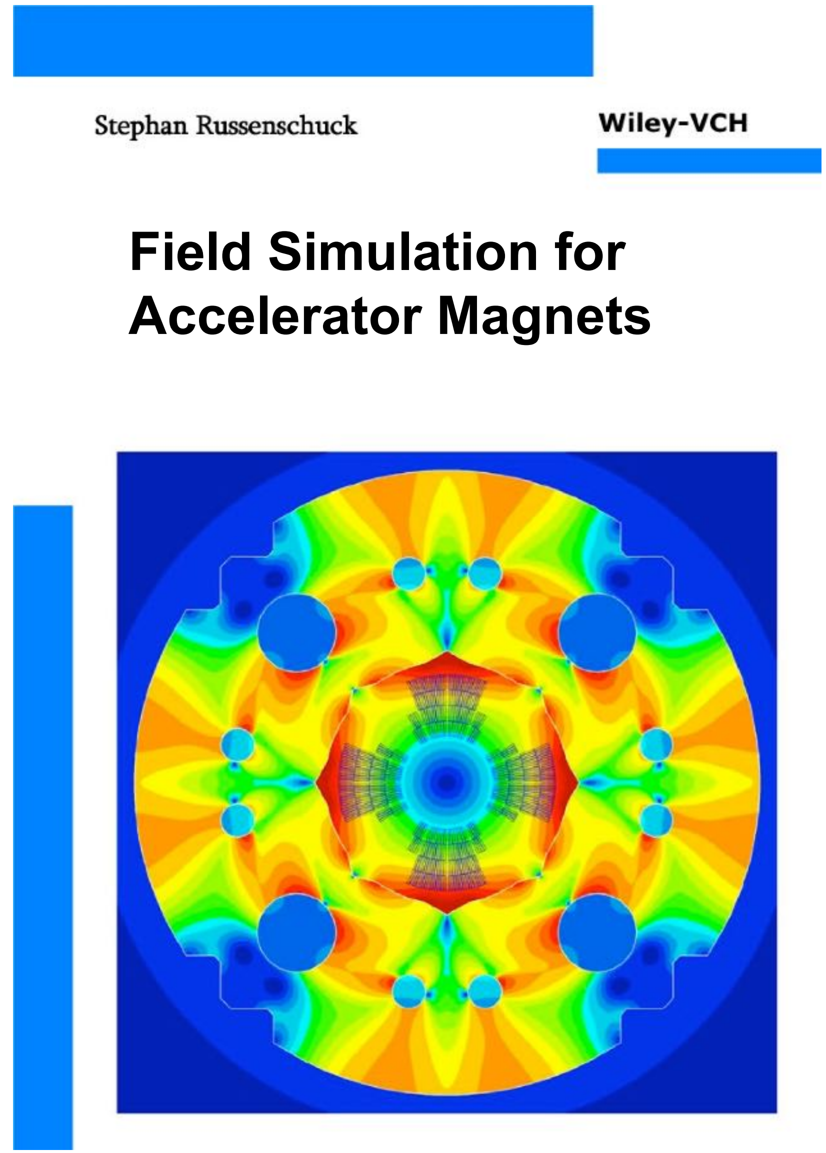
Analytical and Numerical Field Computation

- ➔ Linear algebra
- ➔ Vector analysis
- ➔ Harmonic fields
- ➔ Green's functions and the method of images
- ➔ Complex analysis
- ➔ Differential geometry
- ➔ Numerical field computation
- ➔ Hysteresis modeling
- ➔ Coupled (thermo, magnetic, electric) systems
- ➔ Mathematical optimization



New Edition, Autumn 2023

- Field harmonics
  - Toroidal harmonics
  - Pseudo-multipoles
- Coil Magnetometers
- Stretched-Wire Measurements
- Synchrotron Radiation
- Faraday Paradoxes
- Iron-dominated magnets
  - Wigglers and Undulators
- Coil-dominated magnets
  - CCT Magnets



Comparison NC and SC Magnets (EM-Design)

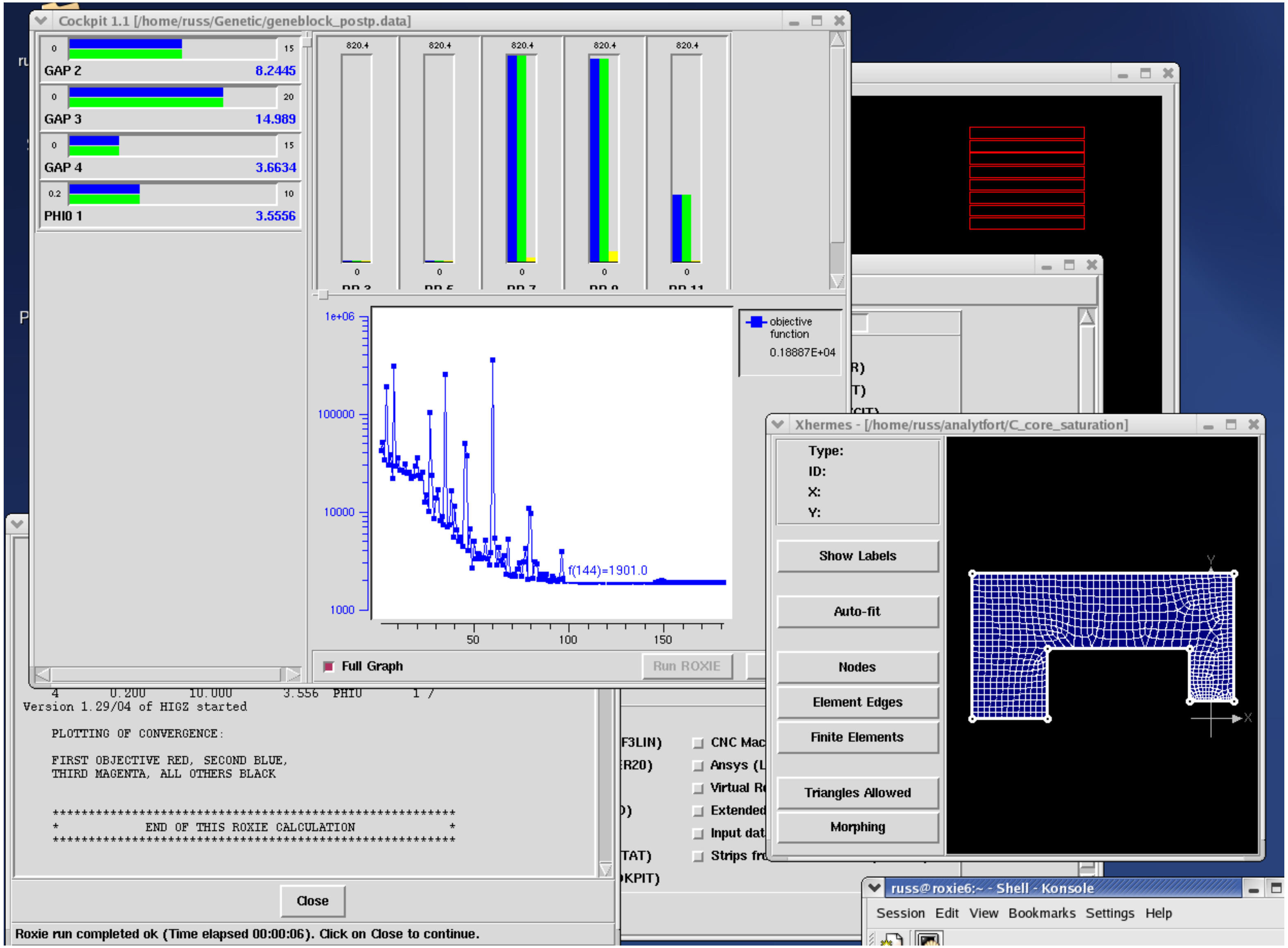
➔ Normal conducting (iron dominated) magnets

- Ideal pole shape known from potential theory
- One-dimensional (analytical) field computation for main field
- Commercial FEM software can be used as a black box (hysteresis modeling)

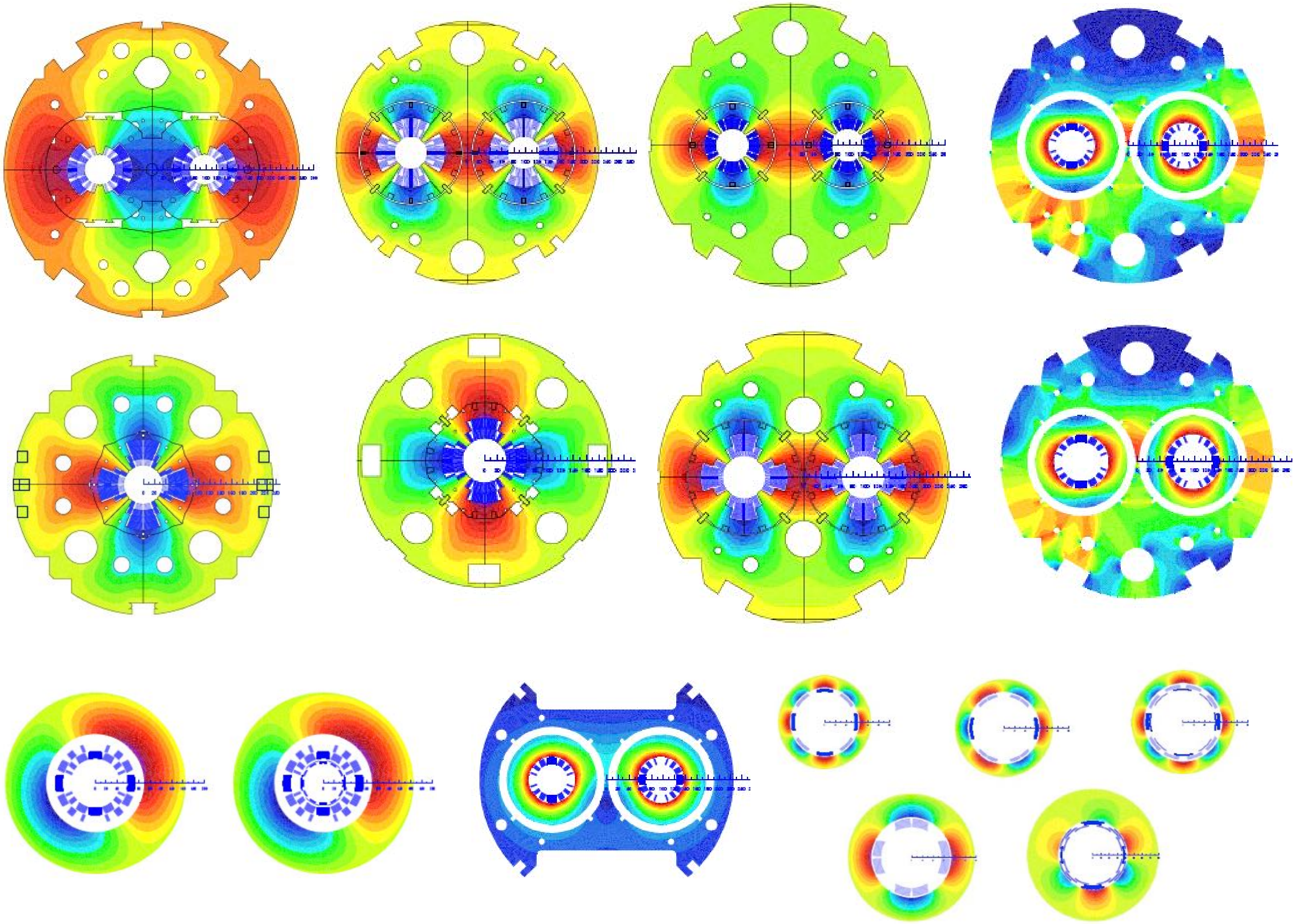
➔ Superconducting (coil dominated) magnets

- Decoupling of coil and yoke optimization
- Accuracy of the field solution
- Modeling of the coils
- Filament magnetization
- Quench simulations

The CERN Field Computation Program ROXIE



The LHC Magnet Zoo



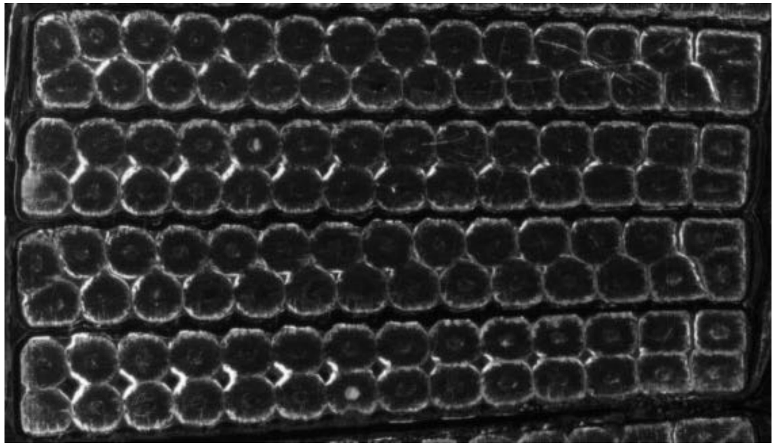
Objectives for the ROXIE Development

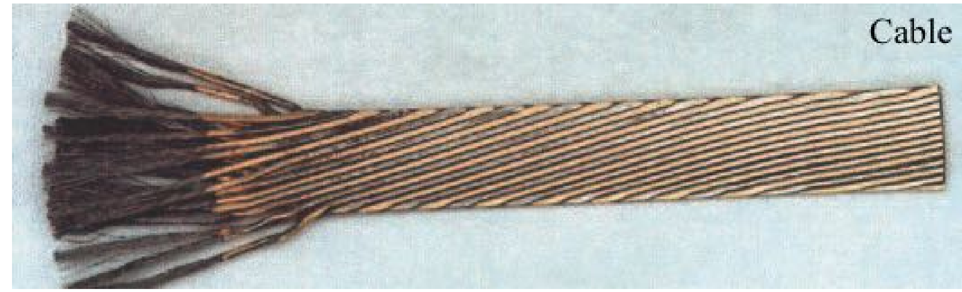
- ➔ Automatic generation of coil and yoke geometries
  - – Features: Layers, coil-blocks, conductors, strands, holes, keys
- ➔ Field computation specially suited for magnet design (BEM-FEM)
  - – No meshing of the coil
  - – No artificial boundary conditions
  - – Higher order quadrilateral meshes, Parametric mesh generator
  - – Dynamic effects (SC magnetization, quench)
- ➔ Mathematical optimization techniques
  - – Genetic optimization, Pareto optimization, Search algorithms
- ➔ CAD/CAM interfaces
  - – Drawings, End-spacer design and manufacture

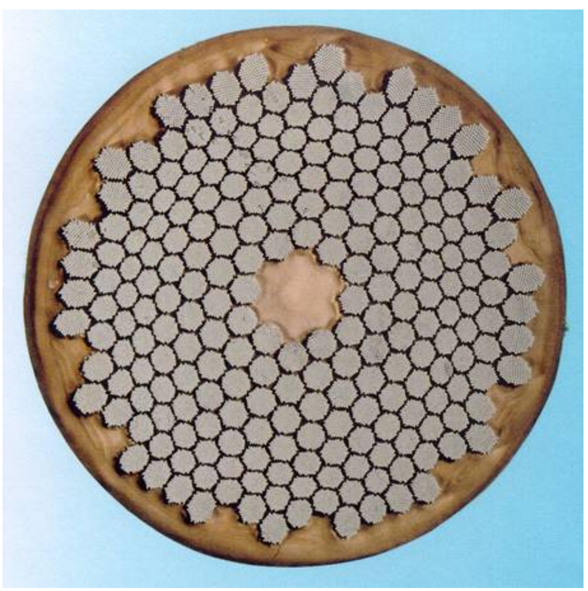
Preview ROXIE 22 (Automn 2022)

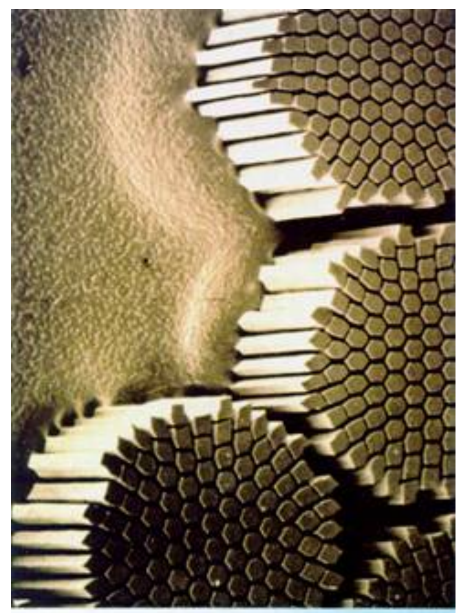
- Bug fixes
- Dynamic memory allocation
- Zonal harmonics for solenoid design
- K-values of search coils
- CCT magnets
- External HMO files (HyperMesh Interface)
- Wigglers and Undulators
- Platform-independent version
- Quench simulation update
- Python interface (post-processing, multiphysics, traceability)
- Material databases

Rutherford (Roebel) Kabel, Strand, Nb-Ti Filament

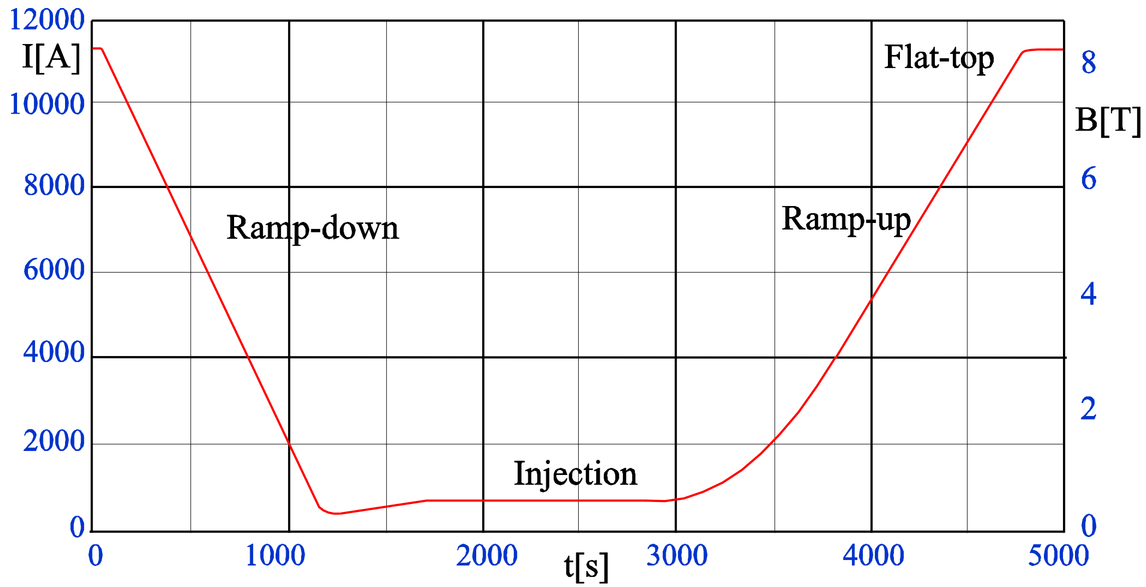


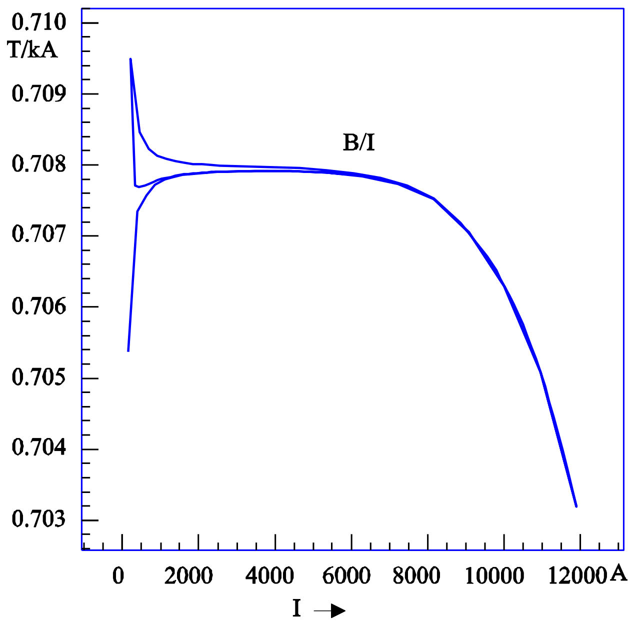


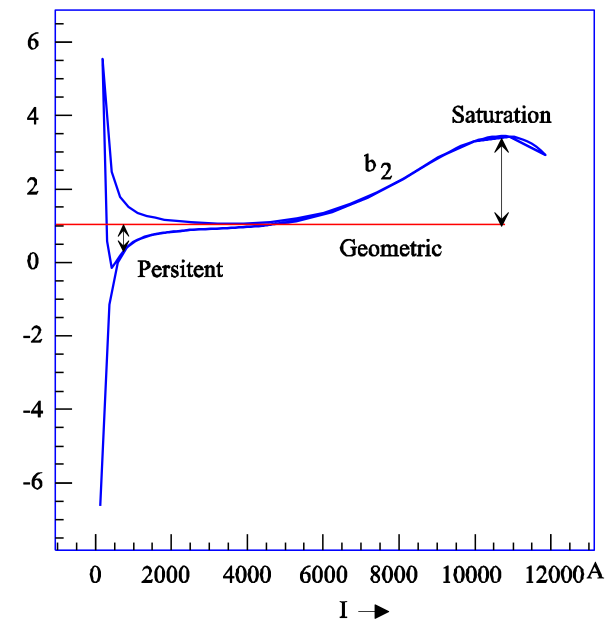




Excitation Cycle

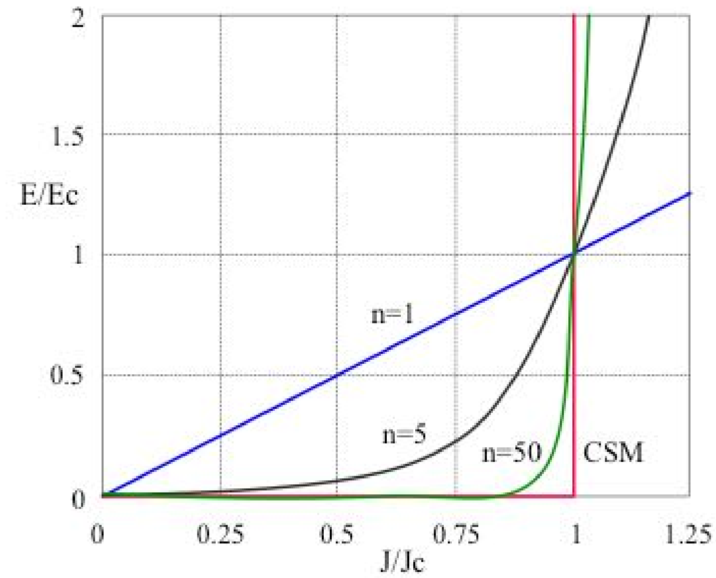


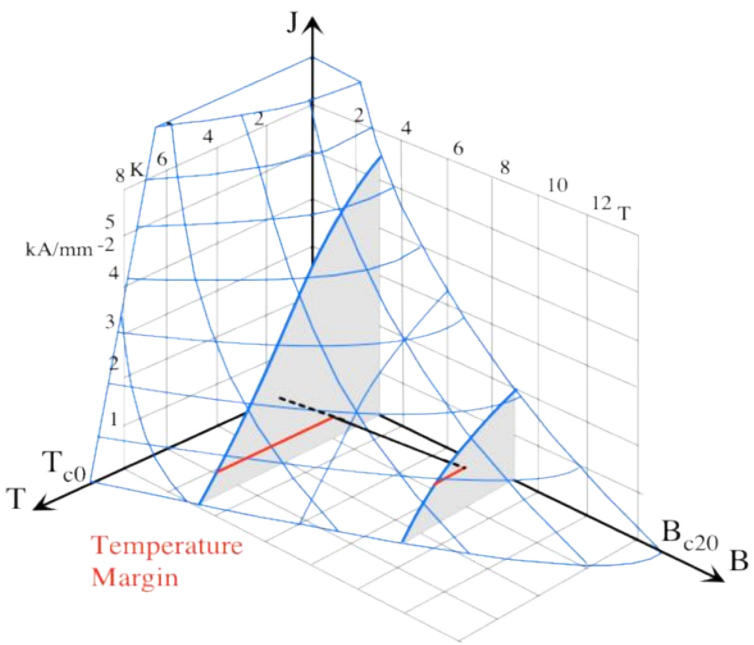




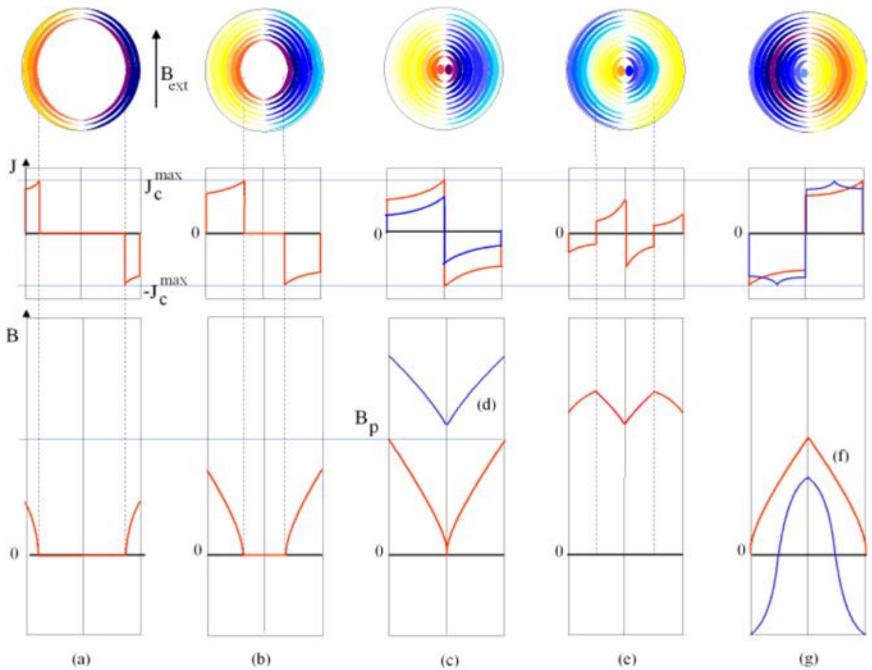
Superconductor Properties

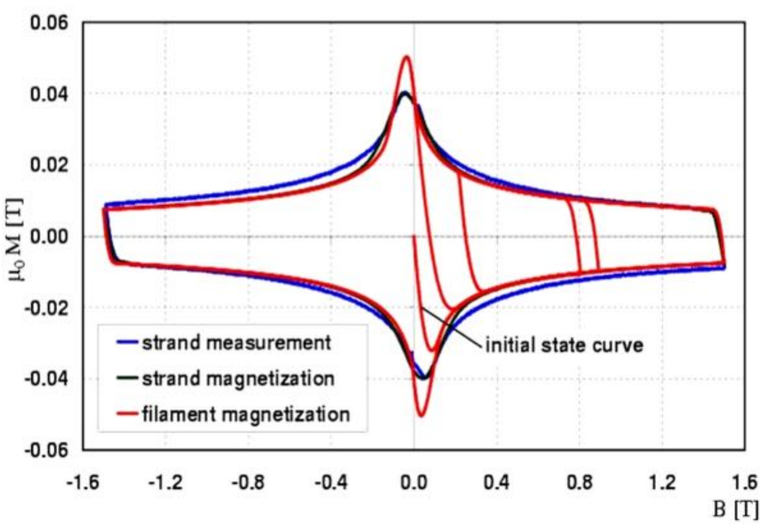
- Hard Superconductors (Type 2)
  - Magnetic field can penetrate
  - Transport current -> non-uniform flux distr.
  - Magnetization with hysteresis
- Critical current density J_c
  - De-pinning creates electric field
  - Current density at spec. electric field (E_c = 1 µV/cm)
- Critical surface
  - Dependence of J_c on T and B

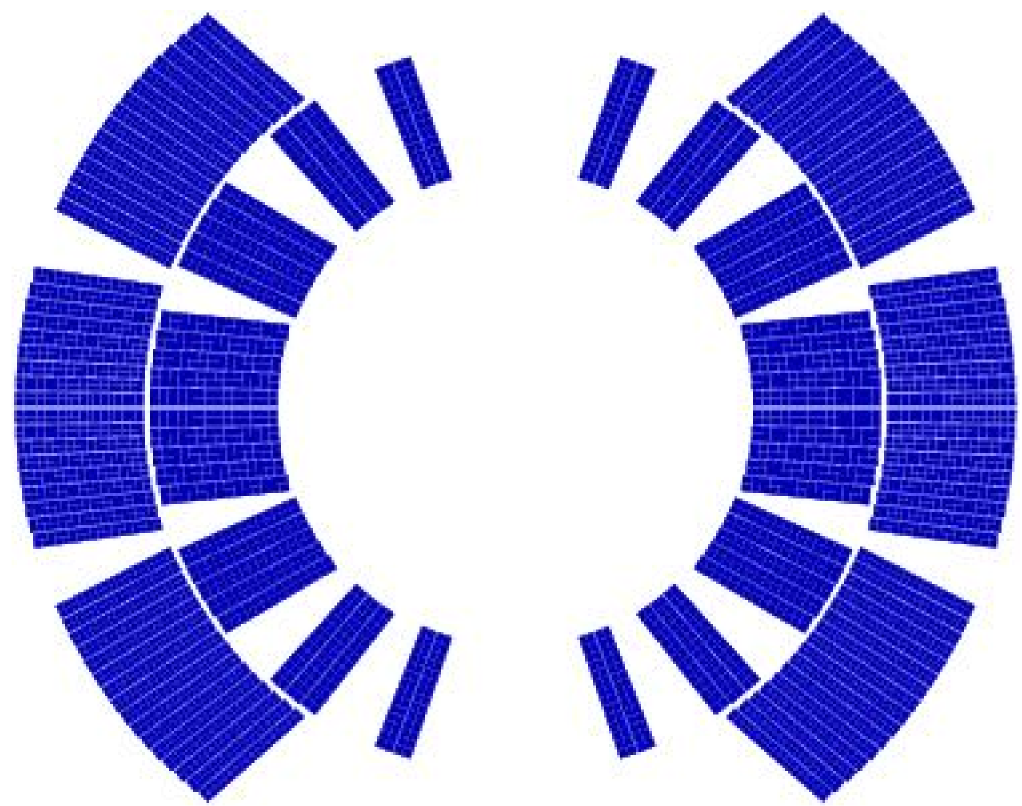




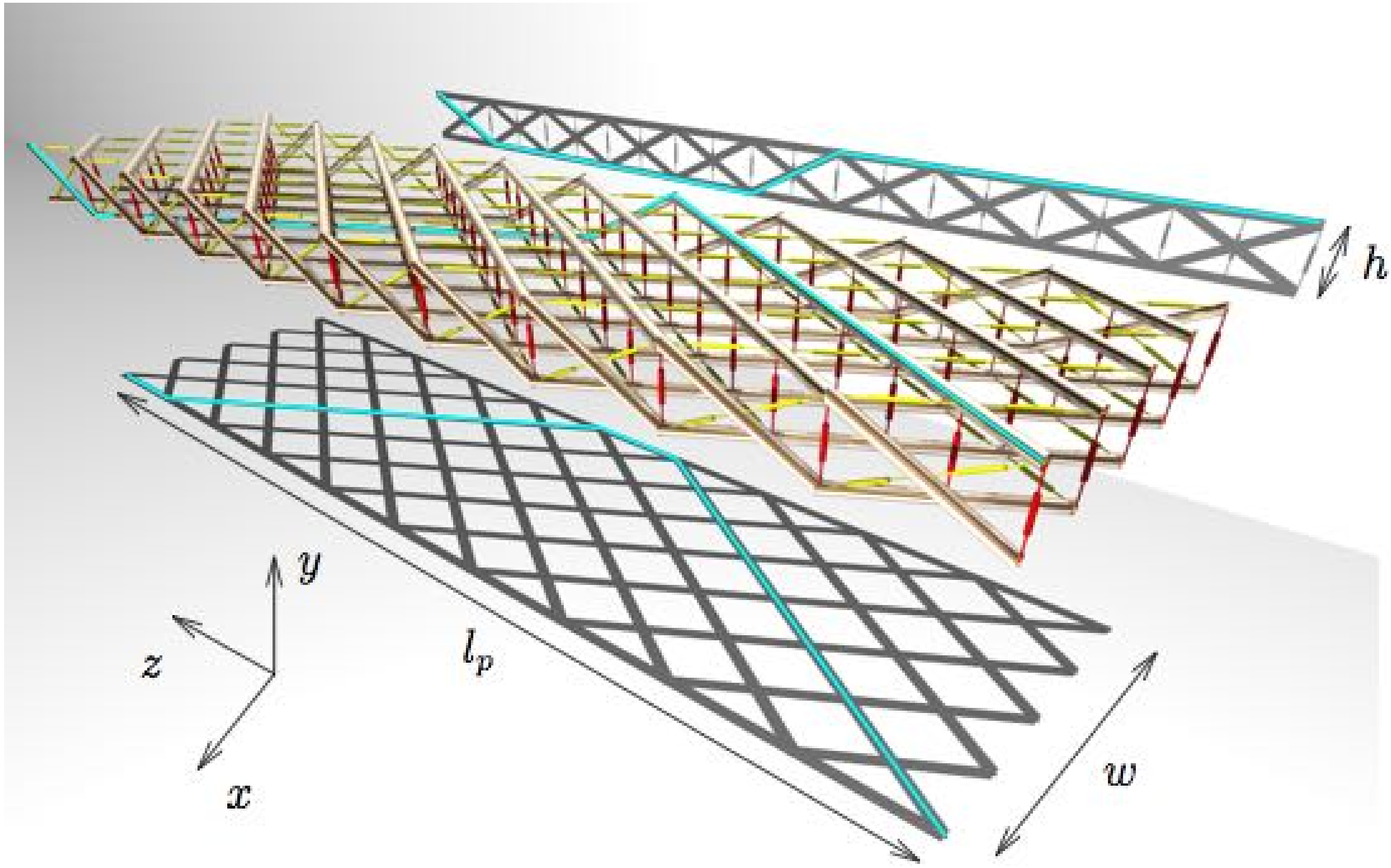
Superconducting Magnetization (Hysteresis Model)



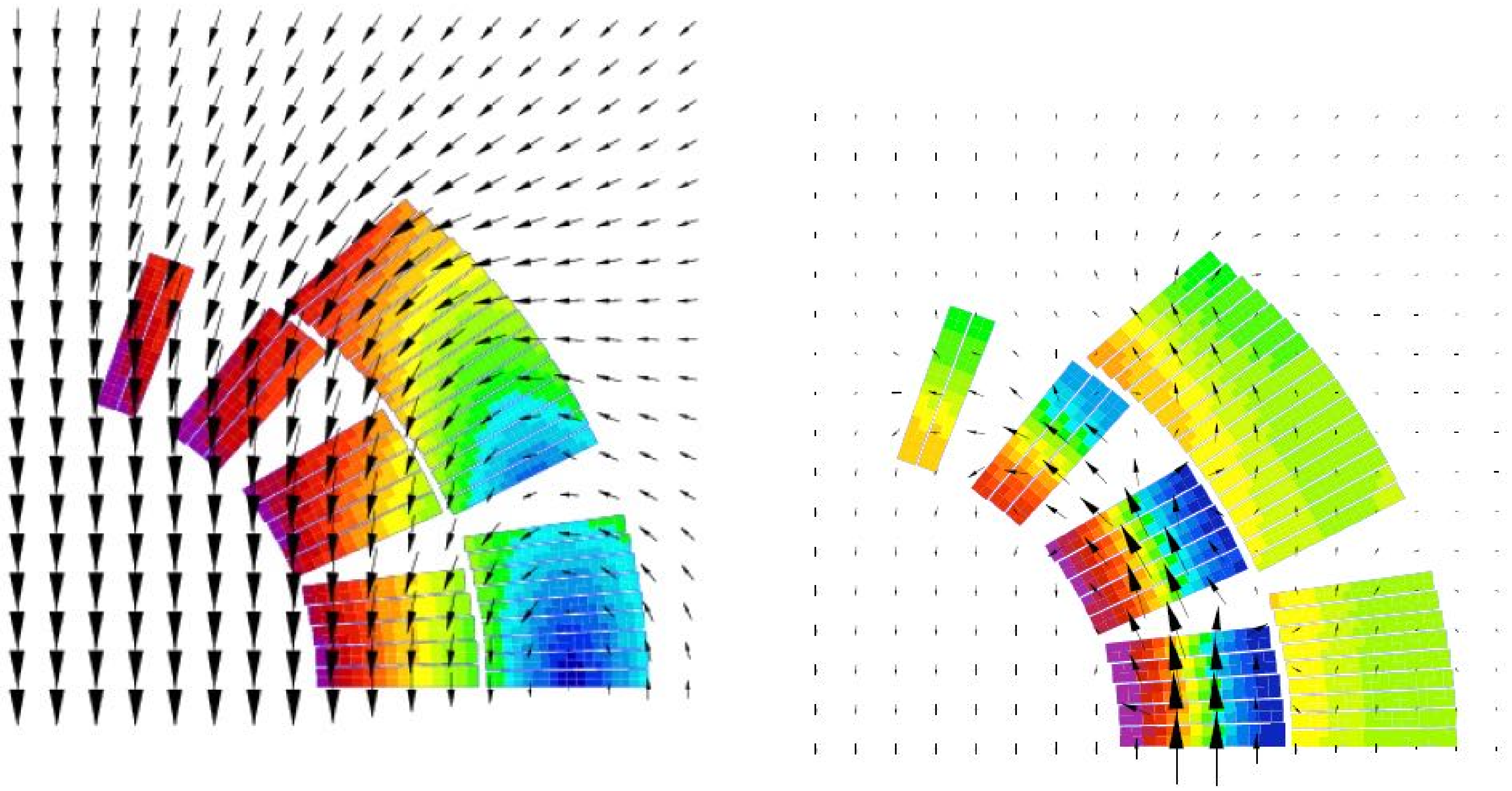




Eddy Currents in Rutherford Cables

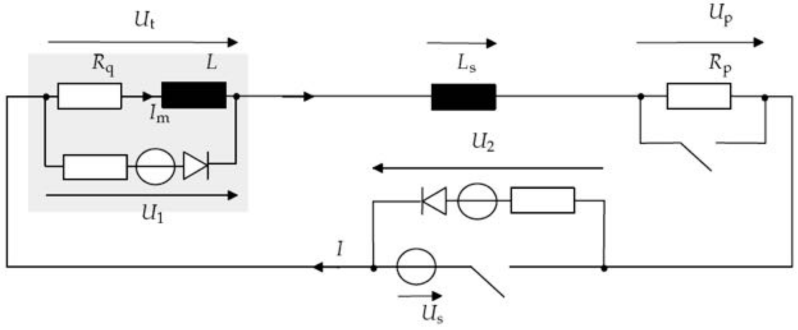


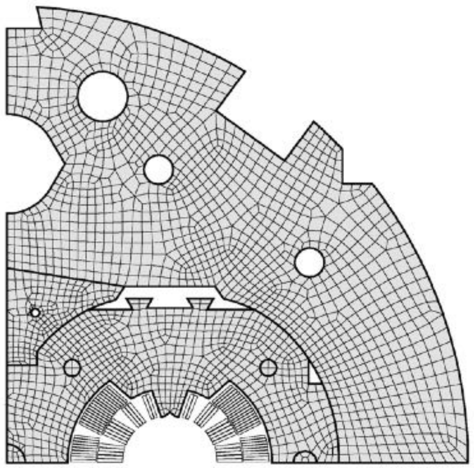
Field Generated by ISCC

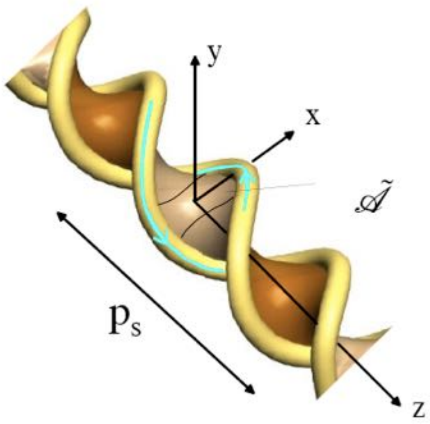


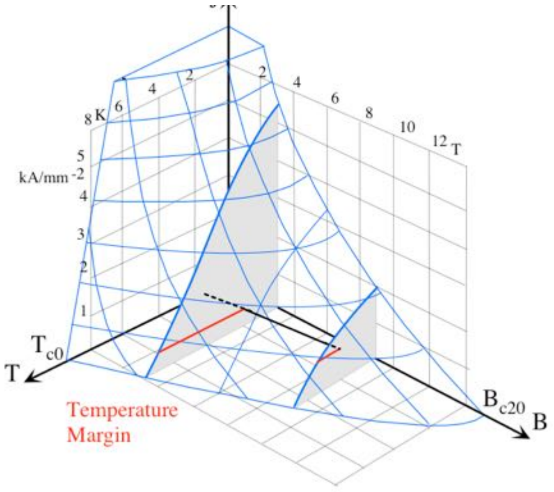
Computation relying on empirical parameters such as
RRR, and adjacent/transversal contact resistances in the cable

Quench Simulation (Multi-Physics, Multi-Scale)

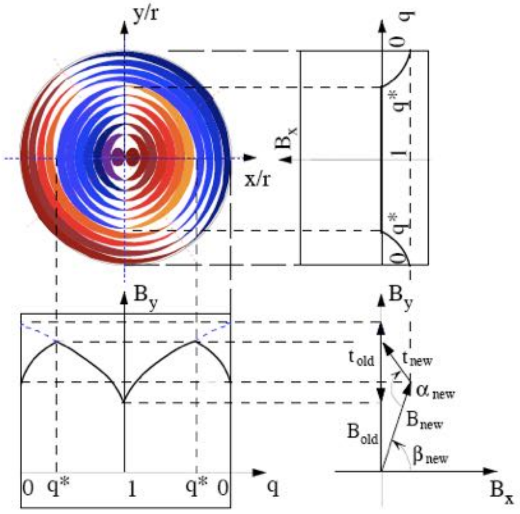


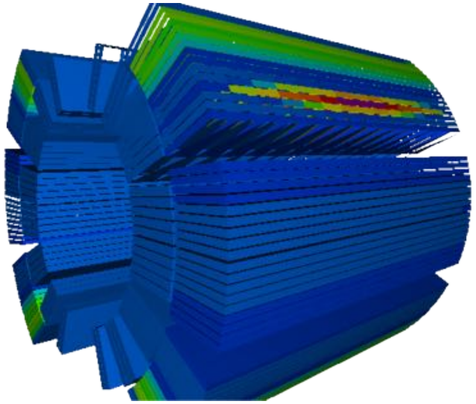


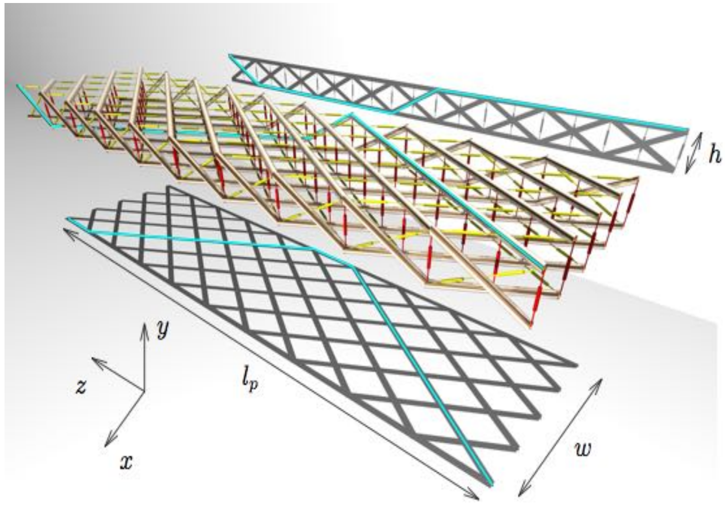




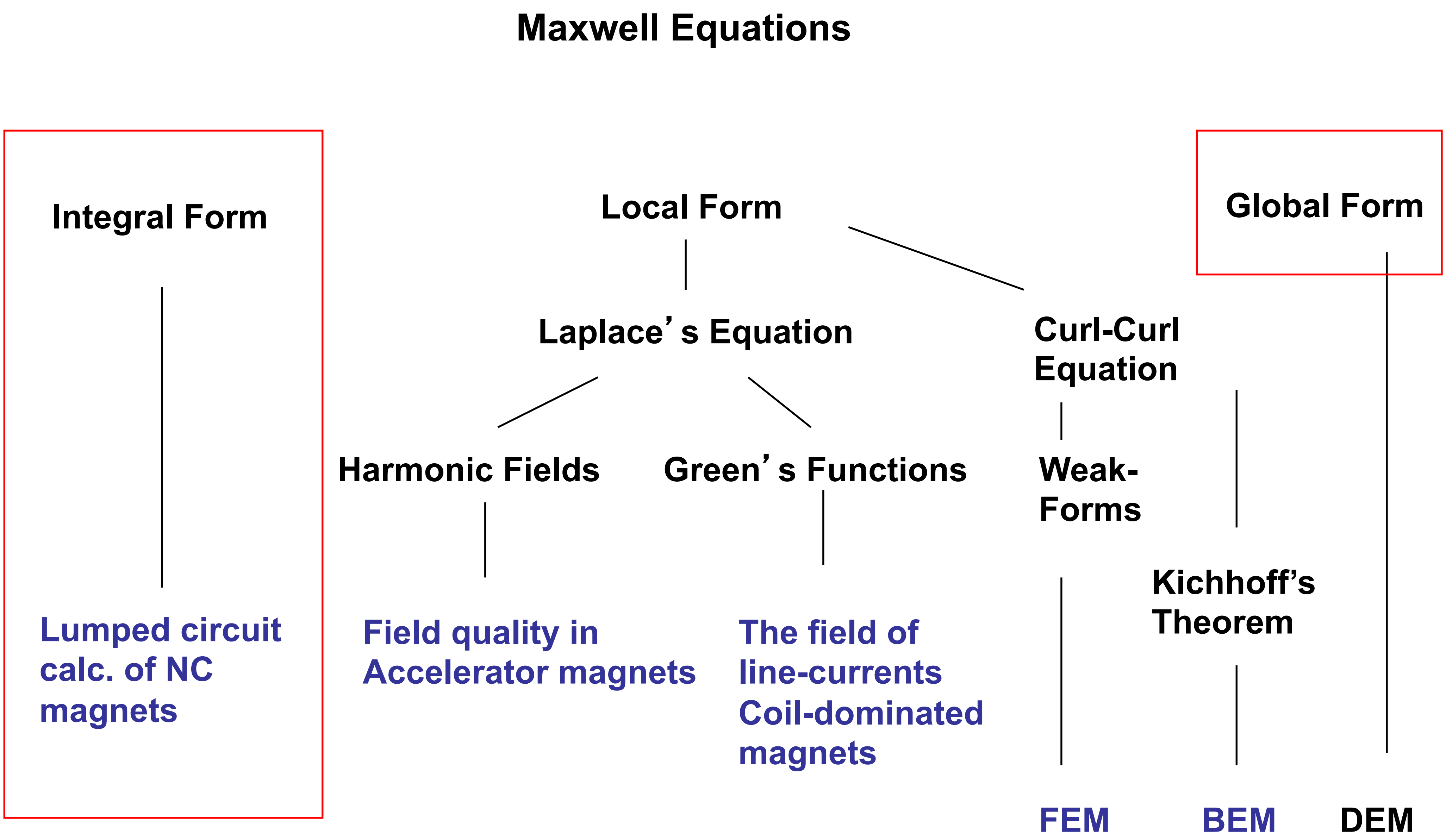
Quench Simulation in ROXIE



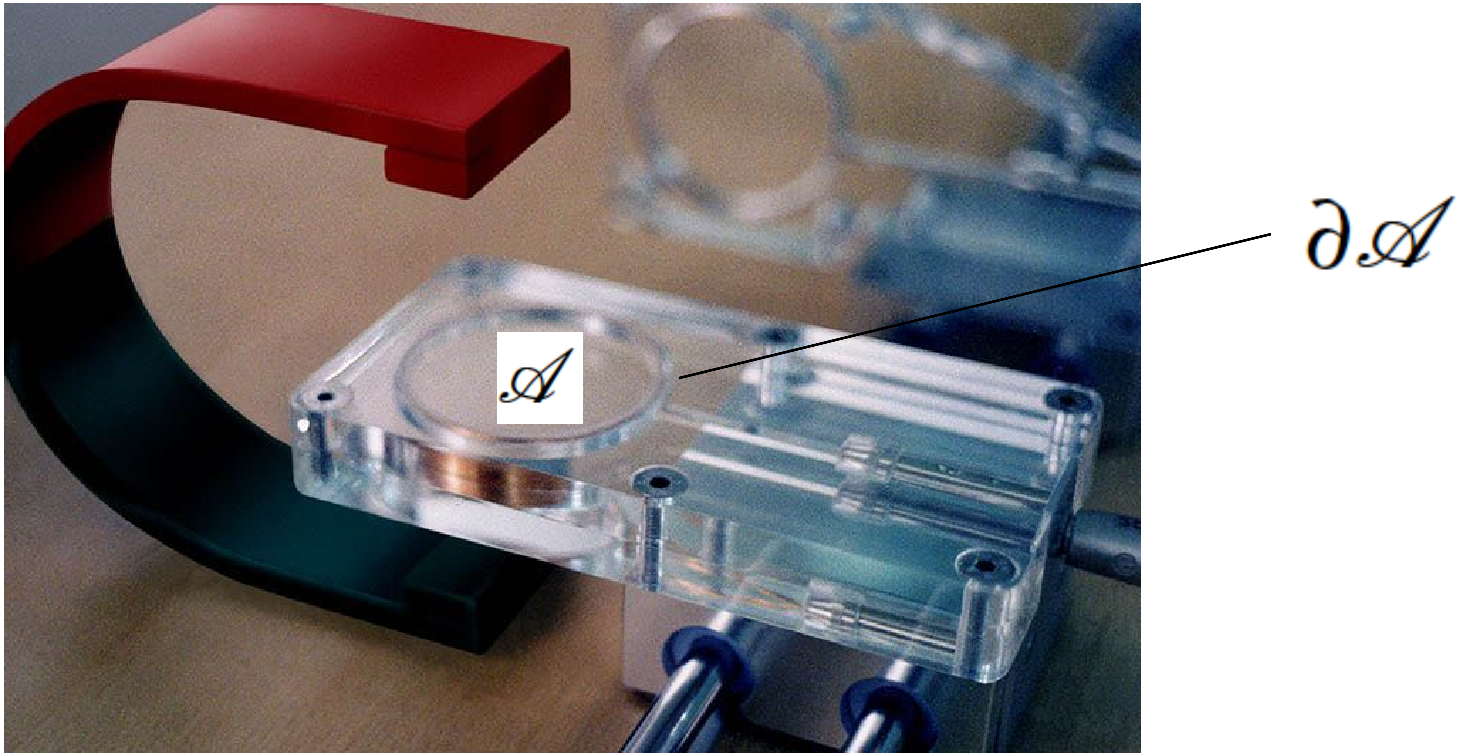




Mathematical Foundations of Magnet Design



Faraday's Law (Inner Oriented Surface, Voltage along its Rim)



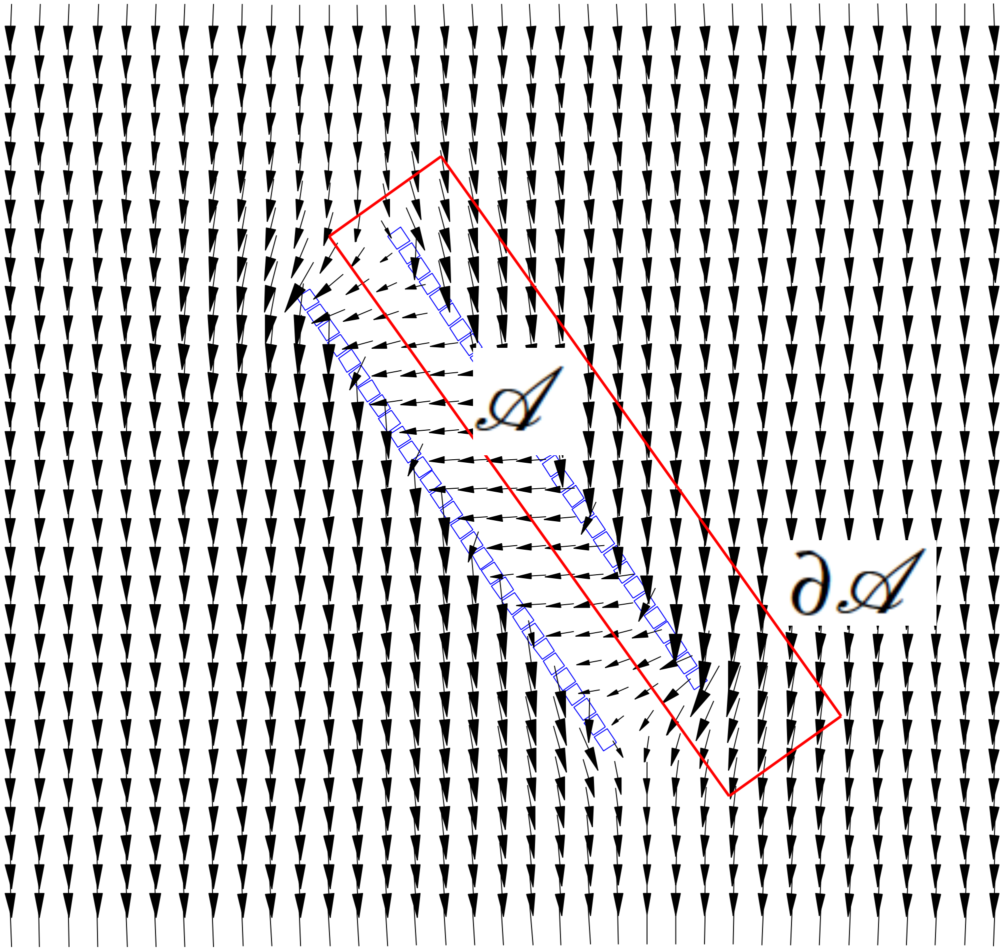
$$U(\partial\mathscr{A}) = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi(\mathscr{A})$$

The potential to induce a voltage

B. Auchmann, S. Kurz and S. Russenschuck, "A Note on Faraday Paradoxes," in *IEEE Transactions on Magnetics*, vol. 50, no. 2, Feb. 2014

Ampere's Law (Outer Oriented Surface; Current crossing)

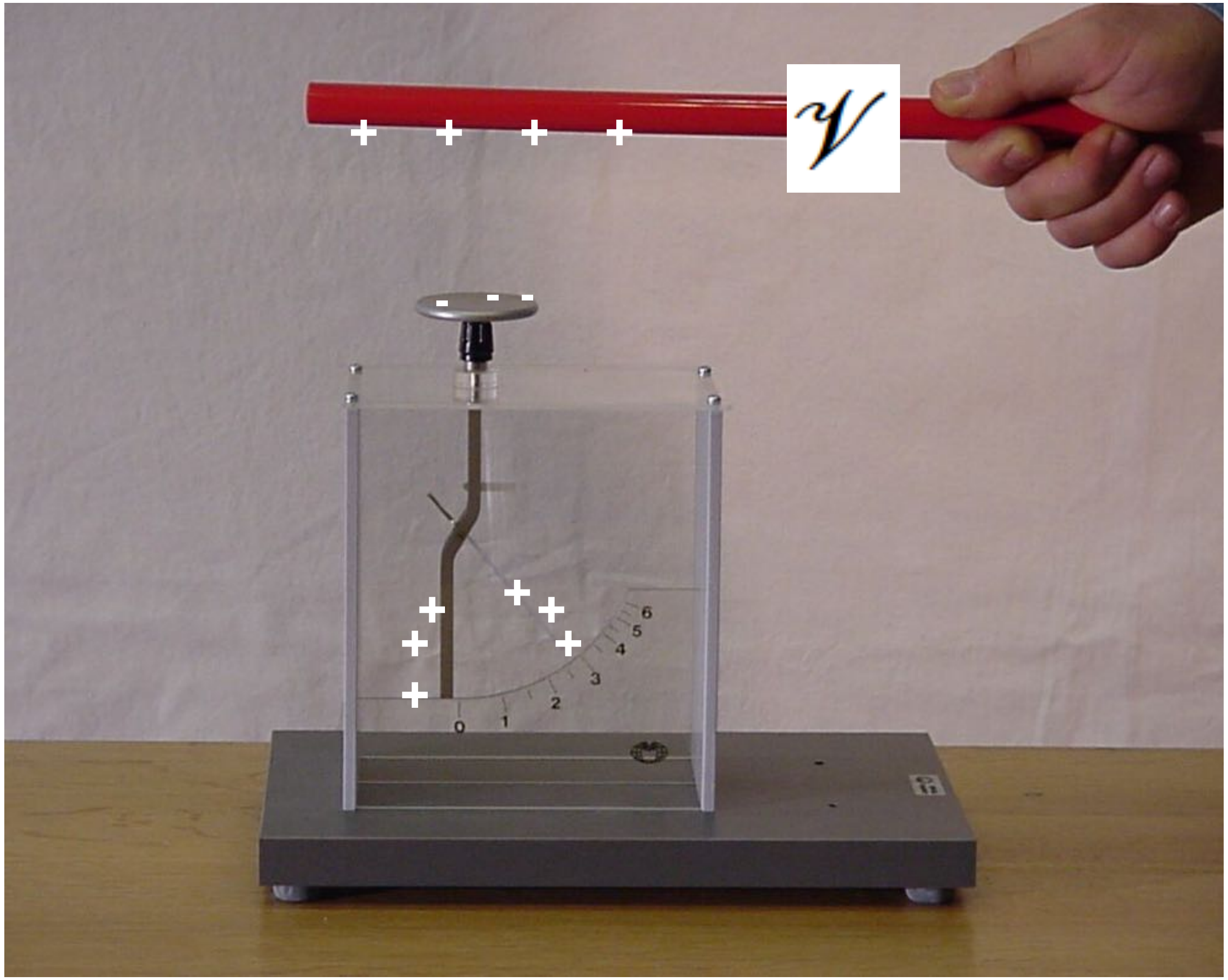
$$V_{\mathrm{m}}(\partial\mathscr{A}) = I(\mathscr{A}) .$$



The current needed to cancel the longitudinal field component (magneto-motive force)

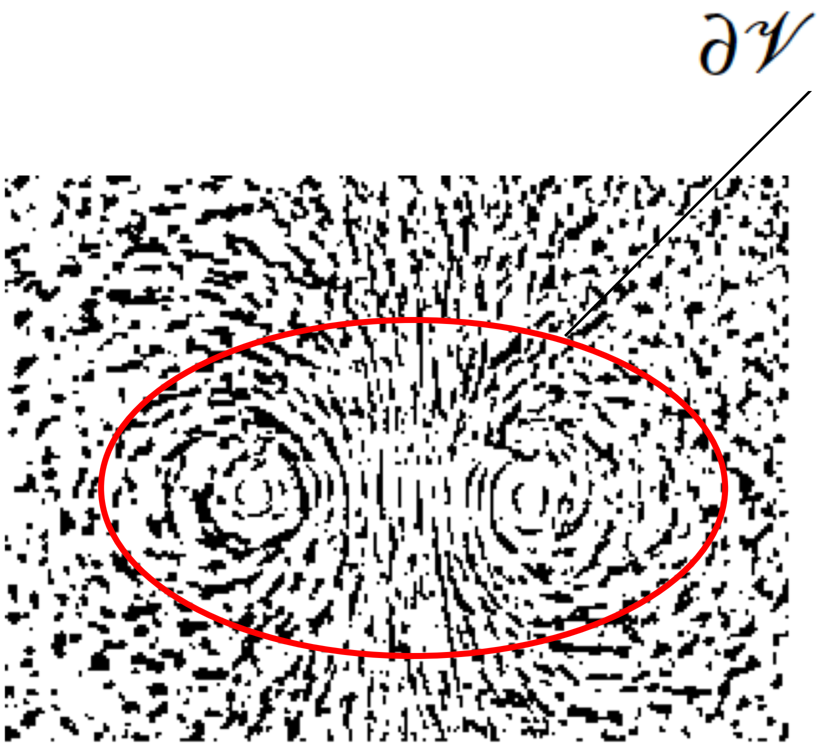
Gauss Law (Outer Oriented Volume; Electric Charge that can be influenced)

$$\Psi(\partial\mathscr{V}) = Q(\mathscr{V})$$



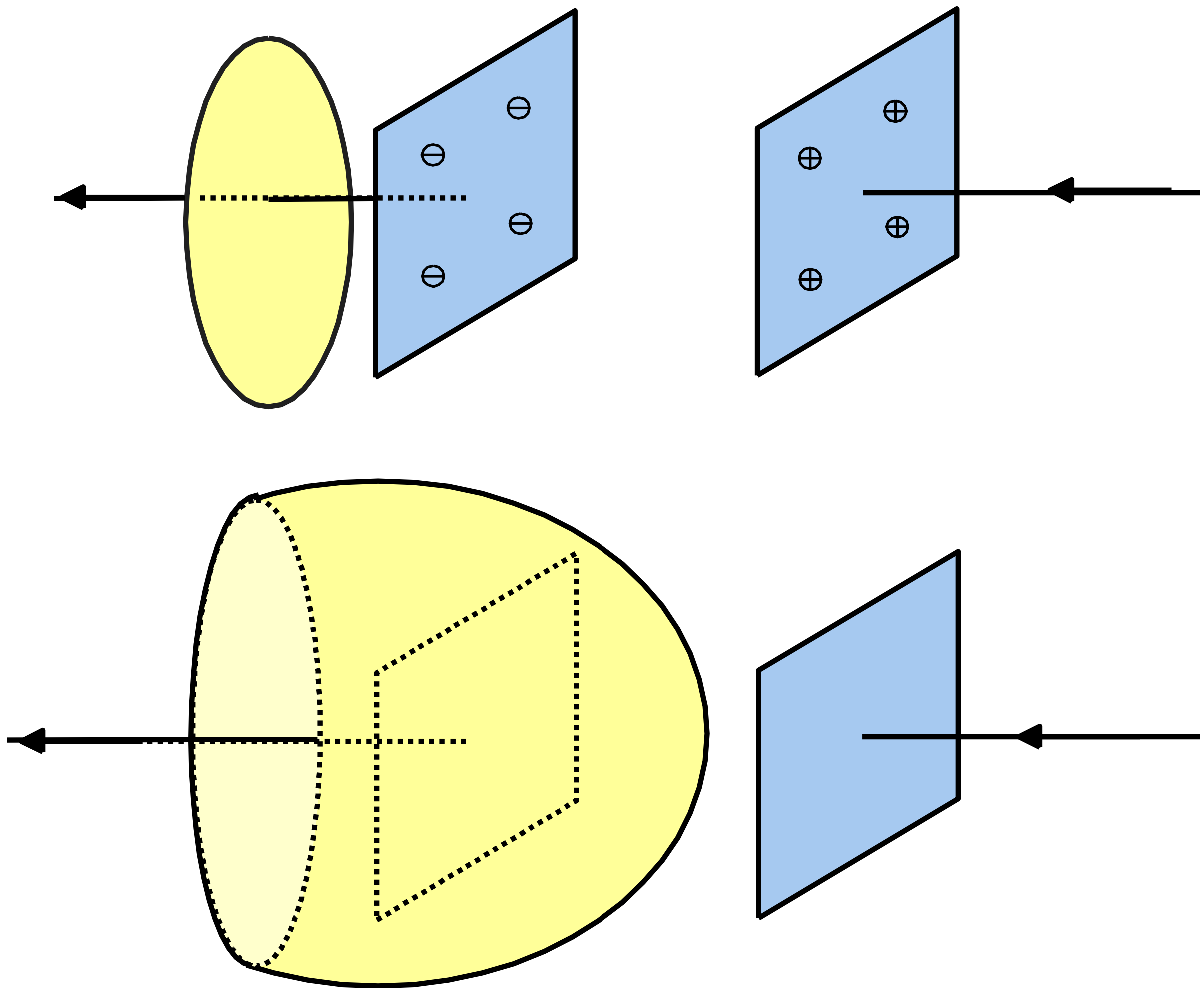
The capacity to induce charge

Conservation of flux



$$\Phi(\partial\mathscr{V}) = 0$$

Maxwell's Extension



Ampere

$$V_{\mathrm{m}}(\partial\mathscr{A}) = I(\mathscr{A}) + \frac{\mathrm{d}}{\mathrm{d}t}\Psi(\mathscr{A})$$

Rate of change of charge

Maxwell's Equations in Global Form

Ampere	$V_{\mathrm{m}}(\partial a) = I(a) + \frac{\mathrm{d}}{\mathrm{d}t}\Psi(a)$
Faraday	$U(\partial a) = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi(a)$
Flux conservation	$\Phi(\partial V) = 0$
Gauss	$\Psi(\partial V) = Q(V)$

Conservation of charge / Kirchhoff law

$$V_{\mathrm{m}}(\partial(\partial V)) = 0 = I(\partial V) + \frac{\mathrm{d}}{\mathrm{d}t}Q(V)$$

In words: The current exiting a volume is equal to the negative rate of the charge in that volume

Maxwell's Equations in Integral Form

$$\int_{\partial \mathscr{A}} \mathbf{H} \cdot \mathrm{d}\mathbf{r} = \int_{\mathscr{A}} \mathbf{J} \cdot \mathrm{d}\mathbf{a} + \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathscr{A}} \mathbf{D} \cdot \mathrm{d}\mathbf{a},$$

$$\int_{\partial \mathscr{A}} \mathbf{E} \cdot \mathrm{d}\mathbf{r} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathscr{A}} \mathbf{B} \cdot \mathrm{d}\mathbf{a},$$

$$\int_{\partial \mathscr{V}} \mathbf{B} \cdot \mathrm{d}\mathbf{a} = 0,$$

$$\int_{\partial \mathscr{V}} \mathbf{D} \cdot \mathrm{d}\mathbf{a} = \int_{\mathscr{V}} \rho \, \mathrm{d}V.$$

$$V_{\mathrm{m}}(\partial \mathscr{A}) = I(\mathscr{A}) + \frac{\mathrm{d}}{\mathrm{d}t}\Psi(\mathscr{A}),$$

$$U(\partial \mathscr{A}) = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi(\mathscr{A}),$$

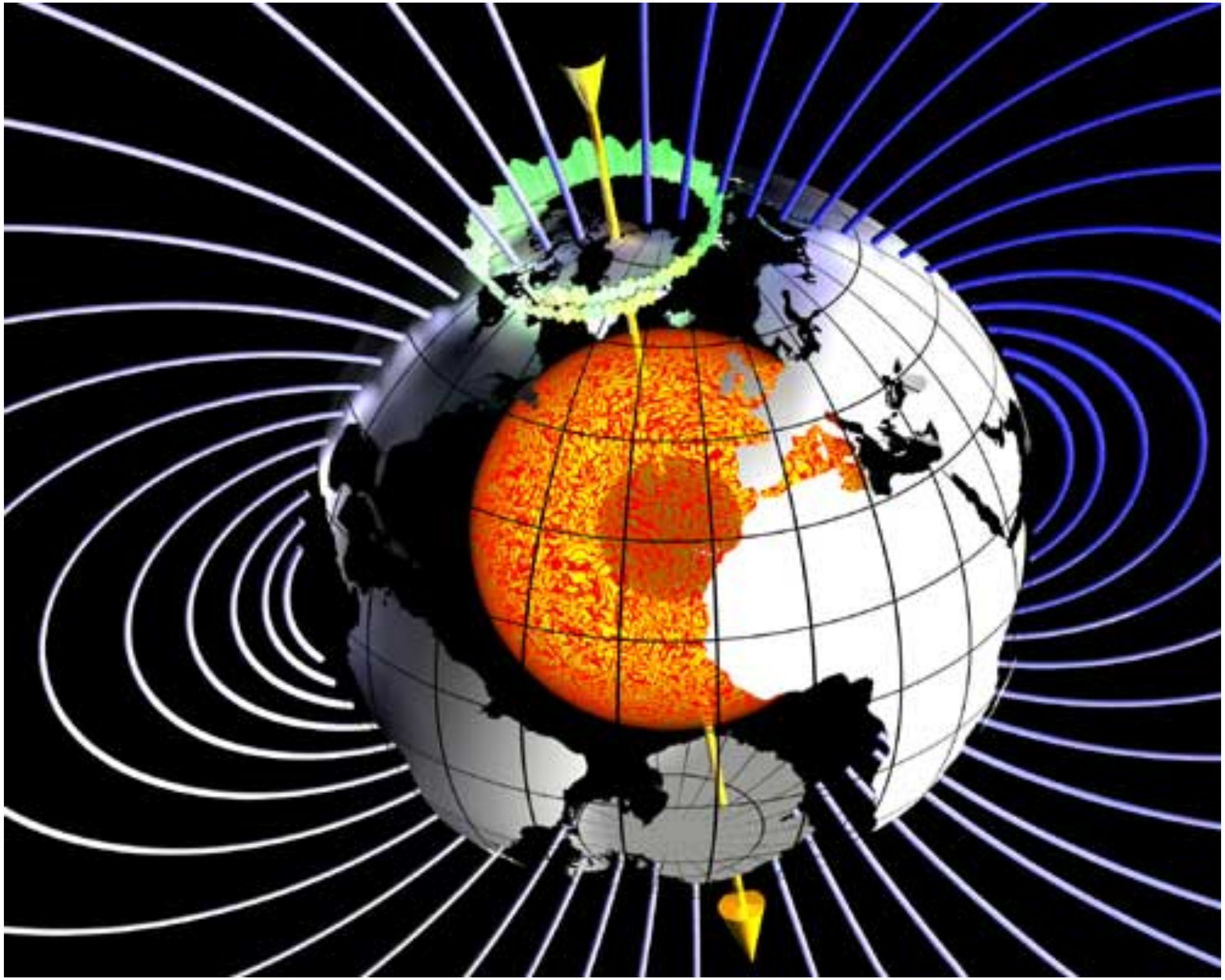
$$\Phi(\partial \mathscr{V}) = 0,$$

$$\Psi(\partial \mathscr{V}) = Q(\mathscr{V}).$$

Electromagnetic Fields

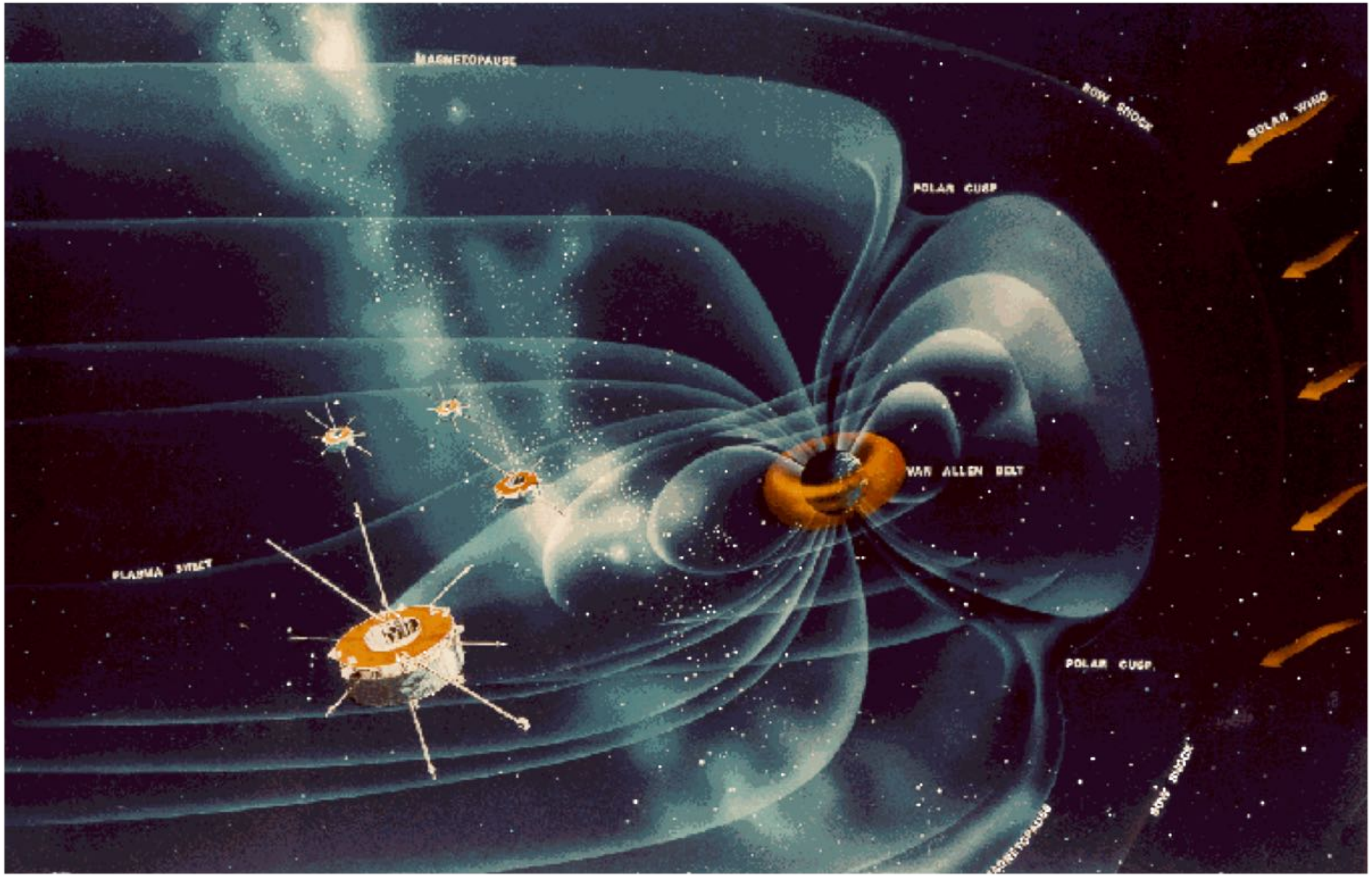
Global quantity	SI unit	Relation			SI unit	Field
MMF	$1\,\mathrm{A}$	$V_{\mathrm{m}}(\mathscr{S})$	$=$	$\int_{\mathscr{S}} \mathbf{H} \cdot \mathrm{d}\mathbf{r}$	$1\,\mathrm{A\,m^{-1}}$	Magnetic field
Electric voltage	$1\,\mathrm{V}$	$U(\mathscr{S})$	$=$	$\int_{\mathscr{S}} \mathbf{E} \cdot \mathrm{d}\mathbf{r}$	$1\,\mathrm{V\,m^{-1}}$	Electric field
Magnetic flux	$1\,\mathrm{V\,s}$	$\Phi(\mathscr{A})$	$=$	$\int_{\mathscr{A}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$	$1\,\mathrm{V\,s\,m^{-2}}$	Magnetic flux density
Electric flux	$1\,\mathrm{A\,s}$	$\Psi(\mathscr{A})$	$=$	$\int_{\mathscr{A}} \mathbf{D} \cdot \mathrm{d}\mathbf{a}$	$1\,\mathrm{A\,s\,m^{-2}}$	Electric flux density
Electric current	$1\,\mathrm{A}$	$I(\mathscr{A})$	$=$	$\int_{\mathscr{A}} \mathbf{J} \cdot \mathrm{d}\mathbf{a}$	$1\,\mathrm{A\,m^{-2}}$	Electric current density
Electric charge	$1\,\mathrm{A\,s}$	$Q(\mathscr{V})$	$=$	$\int_{\mathscr{V}} \rho \cdot \mathrm{d}V$	$1\,\mathrm{A\,s\,m^{-3}}$	Electric charge density

Flux Tubes of Mother Earth (or what is a magnetic field)

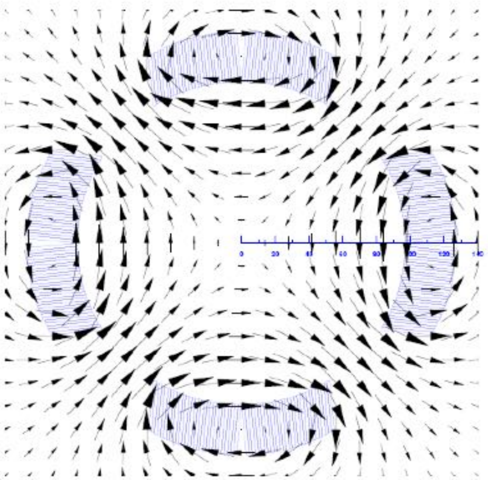


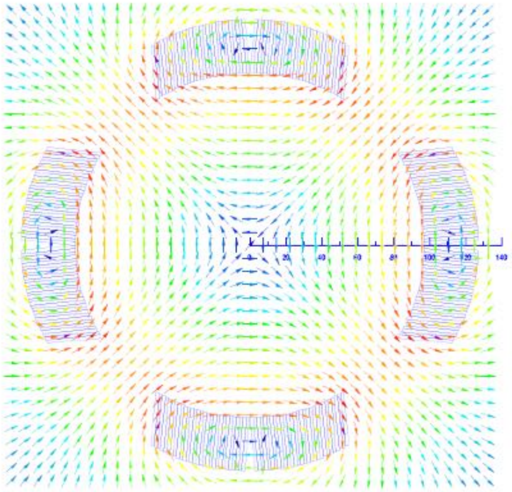
Earth Magnetic Field (better representation)

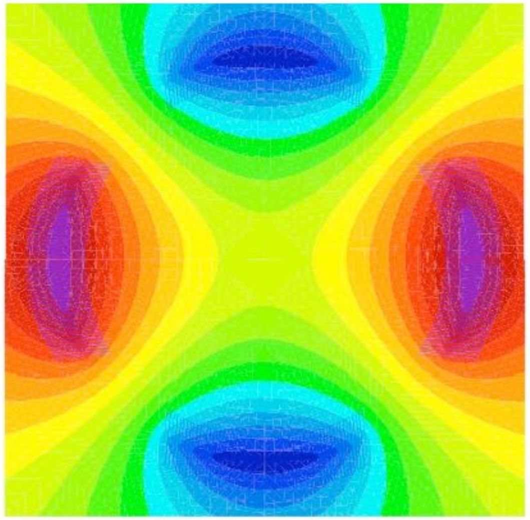
Erdmagnetfeld

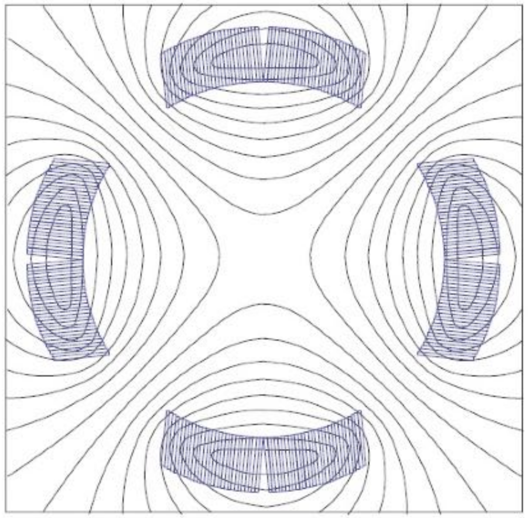


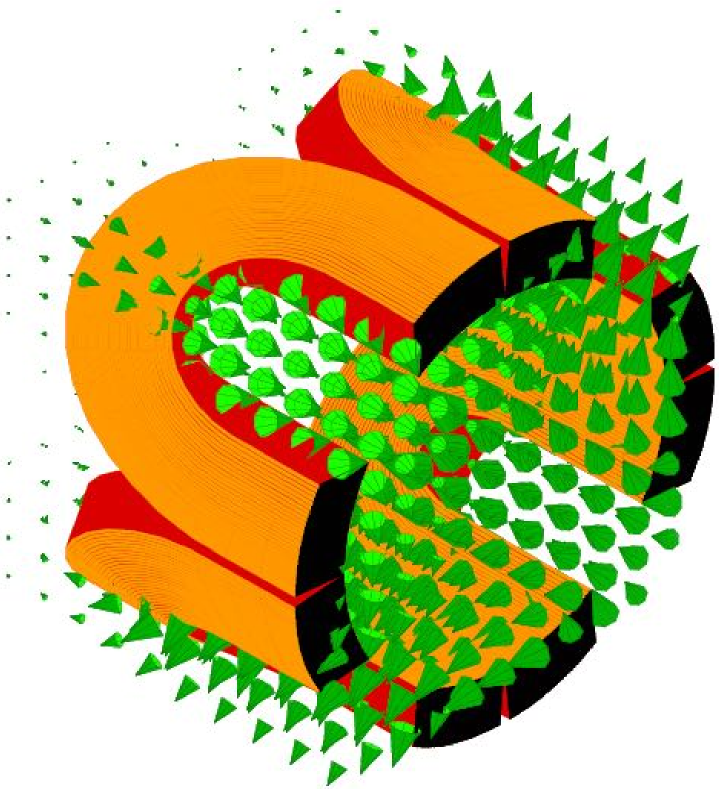
Different Renderings of the Same Vector Field







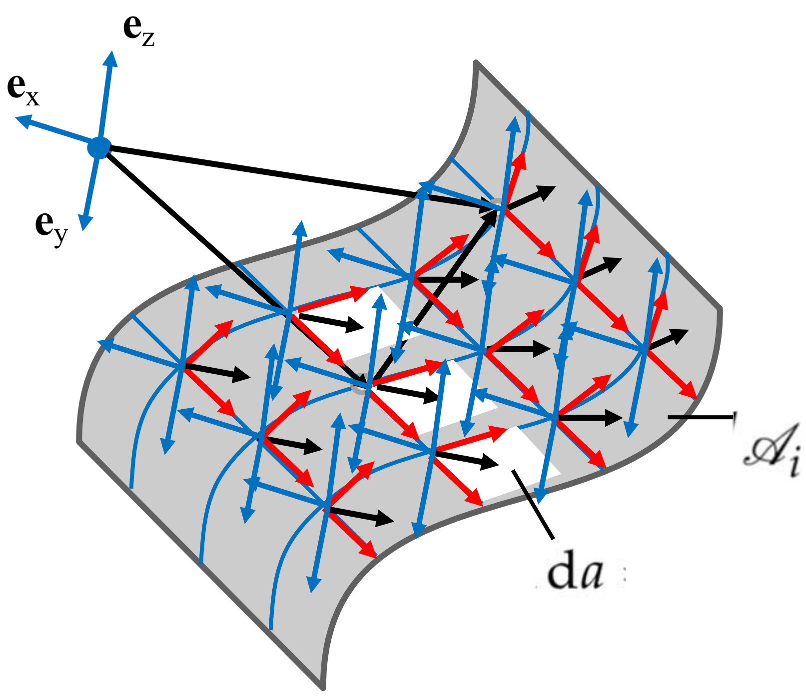




Vector and Scalar Fields

$$\mathbf{a} : \Omega \to \mathbb{R}^3 : \mathbf{r} \mapsto \mathbf{a}(\mathbf{r}) : \mathbf{a}(\mathbf{r}) = (a^1(\mathbf{r}), a^2(\mathbf{r}), a^3(\mathbf{r}))$$

$$\mathbf{x} : \Omega \to \bigcup_{\mathscr{P} \in \Omega} T_{\mathscr{P}}\Omega : \mathscr{P} \mapsto \mathbf{x}(\mathscr{P})$$



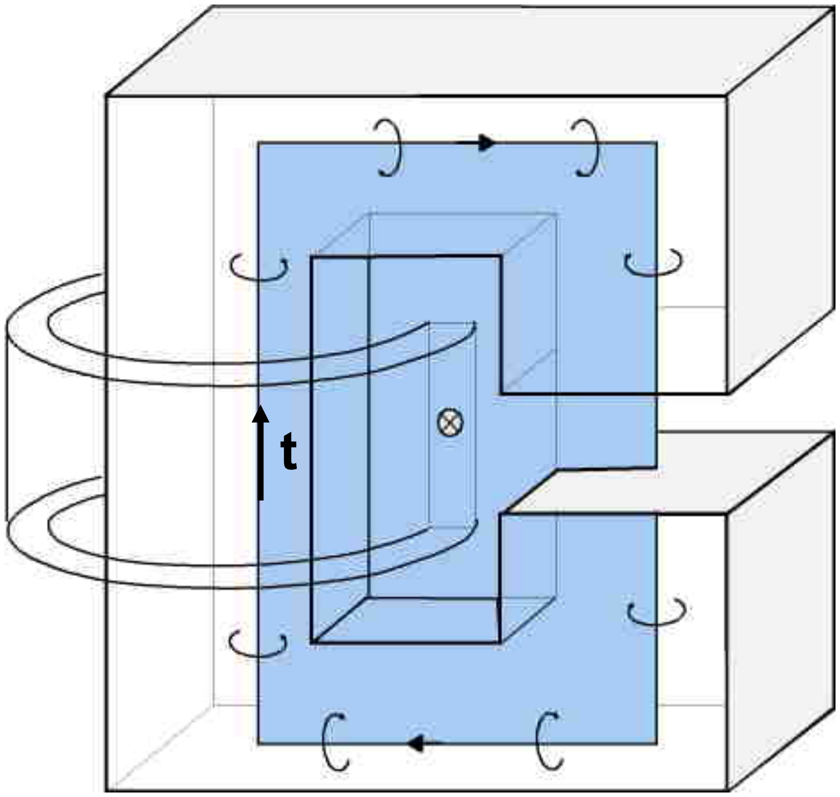
- Linear (vector) space structure
- Metric space (distance and angles)
- Origin and basis -> coordinate representation
- Basis field by translation
- Field components are projections on this basis field

$$\phi : \Omega \to \mathbb{R} : \phi \mapsto \phi(\mathbf{r})$$

$$\Phi_i = \int_{\mathscr{A}_i} \mathbf{B} \cdot \mathbf{n} \, \mathrm{d}a$$

Inner and Outer Oriented Surfaces

$$\int_{\partial\tilde{\mathscr{A}}} \mathbf{H}\cdot \mathrm{d}\mathbf{r} = \int_{\partial\tilde{\mathscr{A}}} \mathbf{H}\cdot \mathbf{t}\,\mathrm{d}s = \int_{\tilde{\mathscr{A}}} \mathbf{J}\cdot \mathbf{n}\,\mathrm{d}a$$

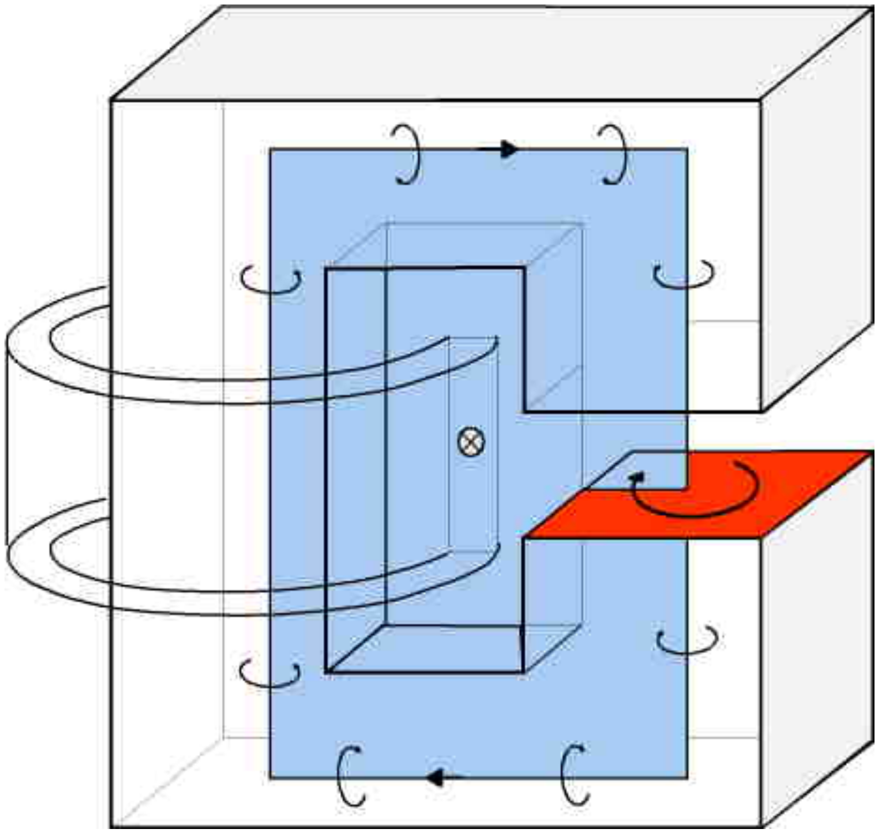


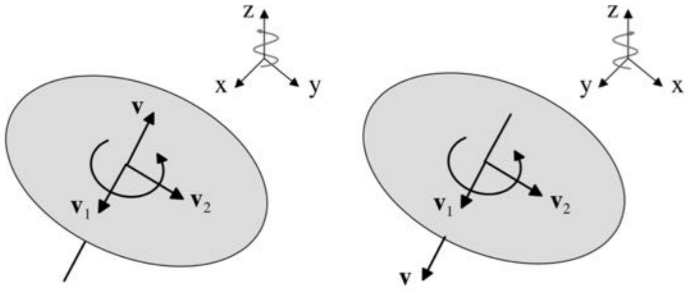
Outer oriented
by the current

$$\Phi_i = \int_{\mathscr{A}_i} \mathbf{B}\cdot \mathbf{n}\,\mathrm{d}a$$

Inner and Outer Oriented Surfaces

$$\int_{\partial \tilde{\mathscr{A}}} \mathbf{H} \cdot \mathrm{d}\mathbf{r} = \int_{\partial \tilde{\mathscr{A}}} \mathbf{H} \cdot \mathbf{t} \, \mathrm{d}s = \int_{\tilde{\mathscr{A}}} \mathbf{J} \cdot \mathbf{n} \, \mathrm{d}a$$



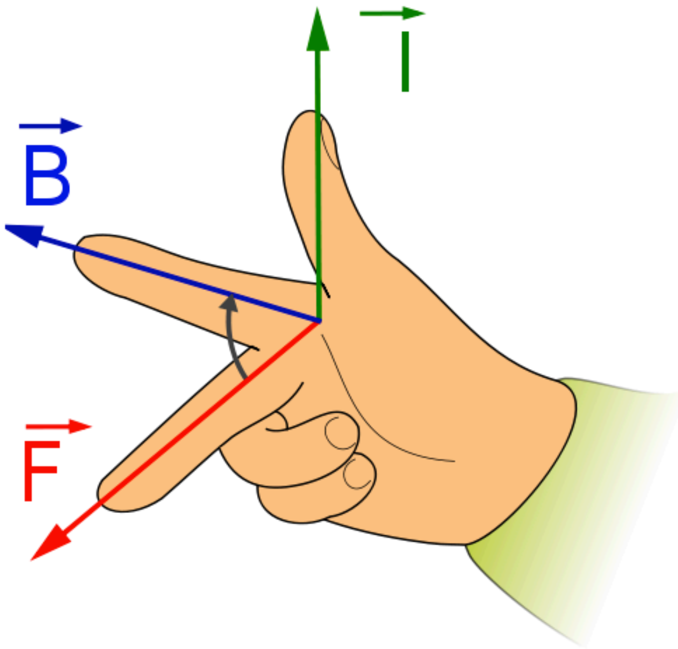


Inner oriented, because flux is a measure for the voltage that can be generated on the rim

Embedding into oriented ambient space (Origin, coordinates)

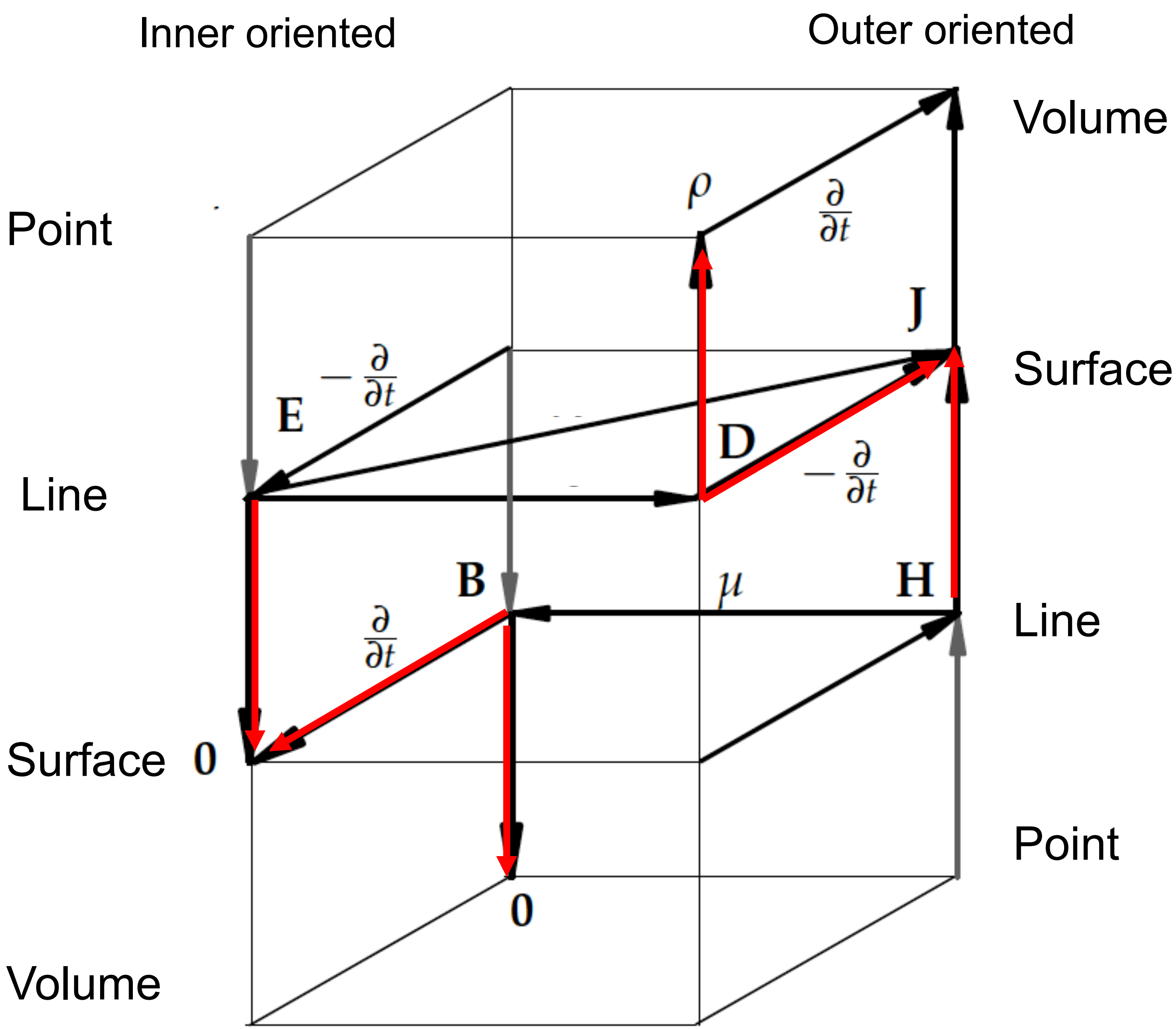
$$\Phi_i = \int_{\mathscr{A}_i} \mathbf{B} \cdot \mathbf{n} \, \mathrm{d}a$$

The Right-Hand Rule or "Magnetic Discussion"



Bruno Touschek (1921-1978)

Maxwell’s House



Constitutive Equations

$$\mathbf{B} = \mu \mathbf{H}, \qquad \mathbf{D} = \varepsilon \mathbf{E}, \qquad \mathbf{J} = \varkappa \mathbf{E},$$

Permeability: $[\mu] = 1\,\mathrm{V\,s\,A^{-1}\,m^{-1}} = 1\,\mathrm{H\,m^{-1}}$,

Permittivity: $[\varepsilon] = 1\,\mathrm{A\,s\,V^{-1}\,m^{-1}}$,

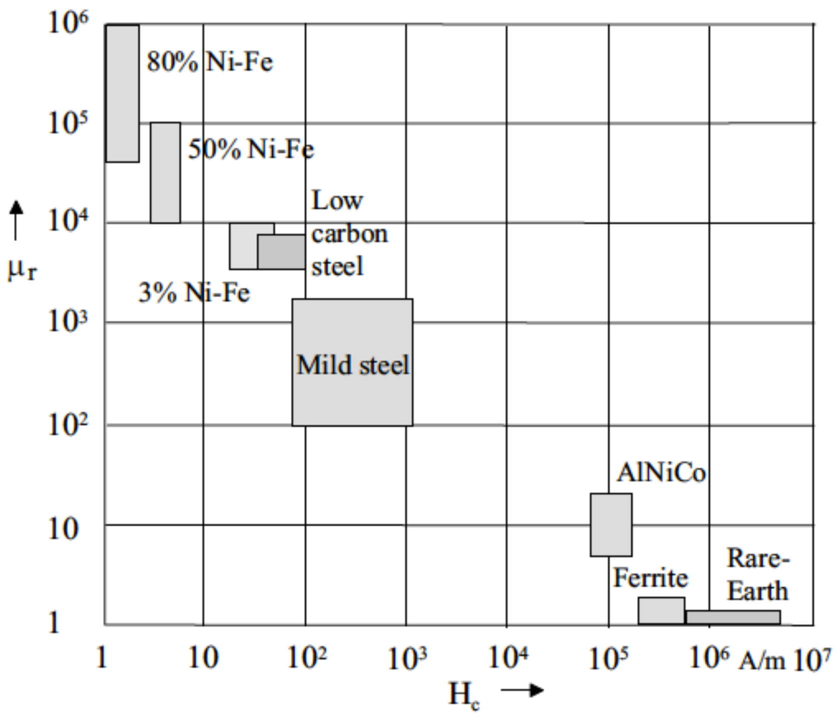
Conductivity: $[\varkappa] = 1\,\mathrm{A\,V^{-1} m^{-1}} = 1\,\Omega^{-1}\,\mathrm{m^{-1}}$.

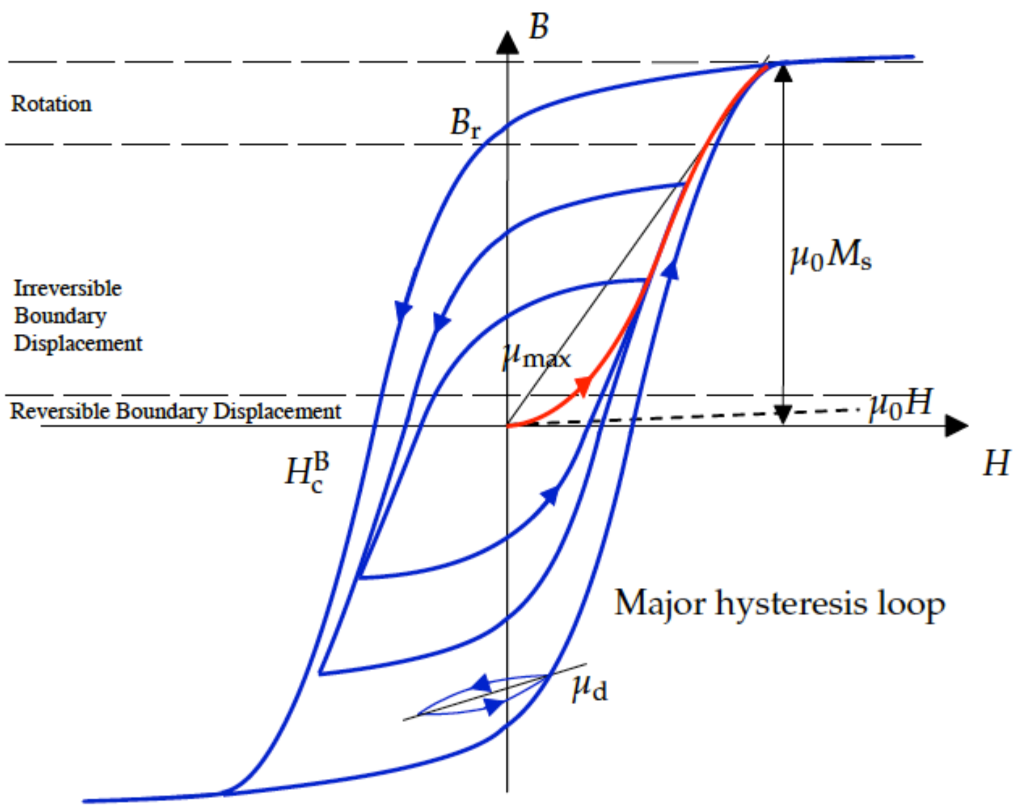
Linear (field independent, homogeneous (position independent), lossless, isotropic (direction independent), stationary

$$\mu = \mu_\mathrm{r} \mu_0, \qquad \varepsilon = \varepsilon_r \varepsilon_0,$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}, \qquad \varepsilon_0 = 8.8542\ldots \times 10^{-12}\,\mathrm{F\,m^{-1}},$$

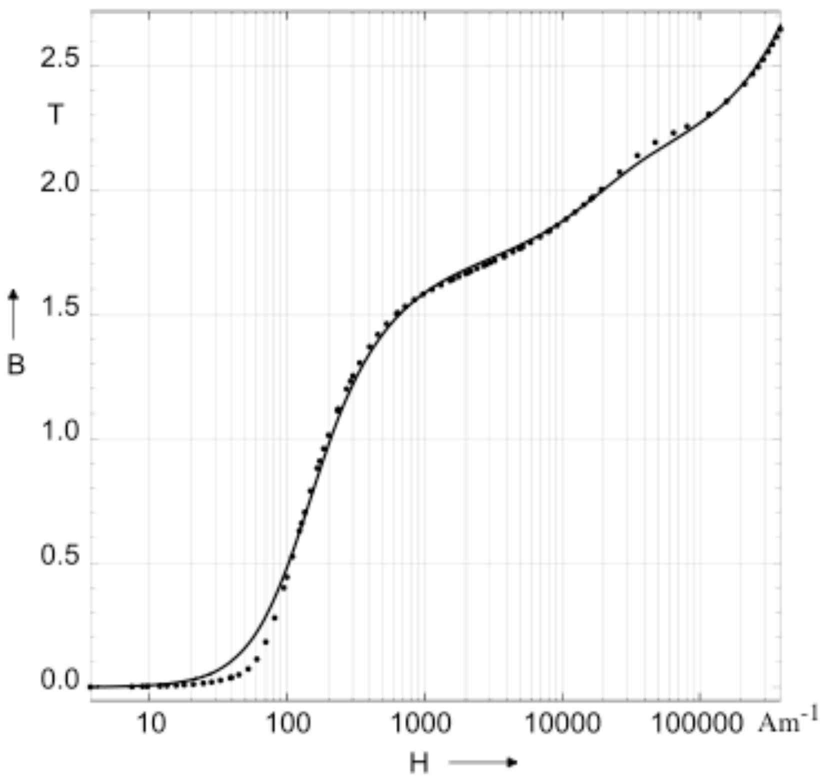
Hysteresis





$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{P}_m(\mathbf{H}) = \mu_0(\mathbf{H} + \mathbf{M}(\mathbf{H})),$$

Nonlinear Iron Magnetization



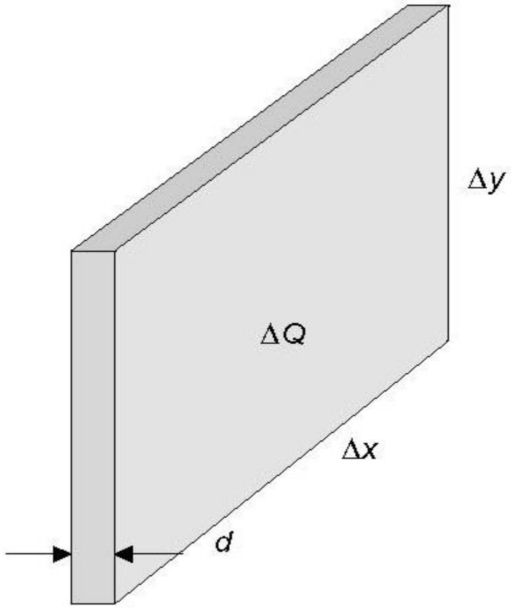
$$L\left(\frac{H}{a}\right) := \coth\left(\frac{H}{a}\right) - \left(\frac{a}{H}\right)$$

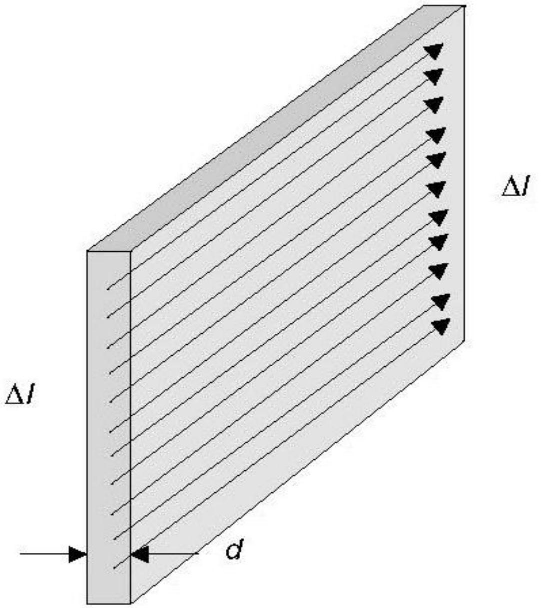
$$M(H) = M_a L\left(\frac{H}{a}\right) + M_b \tanh\left(\frac{|H|}{b}\right) L\left(\frac{H}{b}\right)$$

Wlodarski: Analytical description of magnetization curves, Physica B, Elsevier, 2005

Measured curve does not fulfill the smoothness requirements for M(B) and Newton-Raphson iterative solvers

Surface Charge and (Fictitious) Surface Current





Thin layer with ρ_{mag}

$\Delta Q = \Delta x \Delta y d \rho_{\text{mag}}$

$\rho_{\text{mag}} \to \infty$ and $d \to 0$

$\sigma_{\text{mag}} = d\rho_{\text{mag}}$

$[\sigma_{\text{mag}}] = 1$ V·s/m^2

Thin layer with J

$\Delta I = J d \Delta l$

$J \to \infty$ and $d \to 0$

$\boldsymbol{\alpha} = \mathbf{J} d$

$[\boldsymbol{\alpha}] = 1$ A · m^{-1}

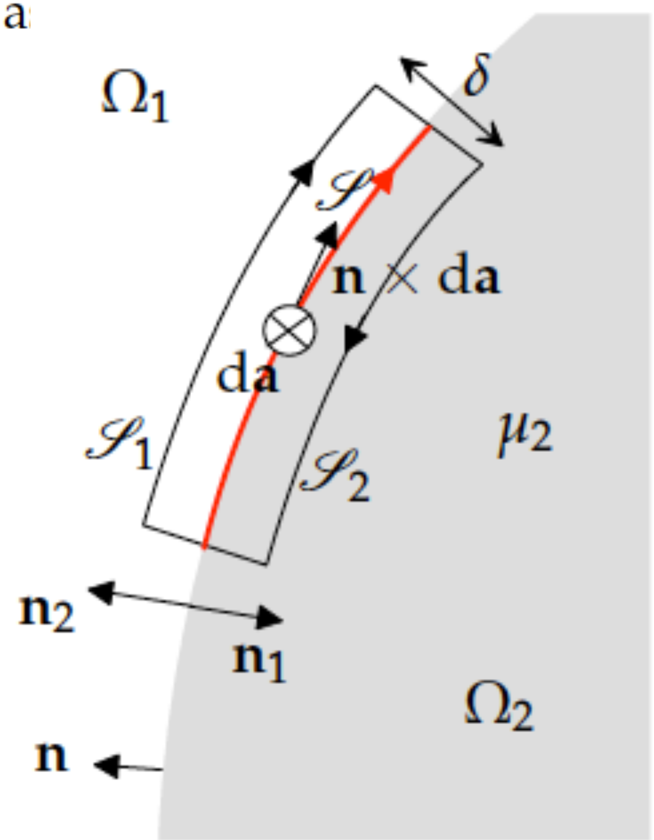
Fictitious quantities to define boundary values

Continuity Conditions (1)

Applying Ampère's law $\int_{\partial \mathscr{A}} \mathbf{H} \cdot \mathrm{d}\mathbf{r} = \int_{\mathscr{A}} \mathbf{J} \cdot \mathrm{d}\mathbf{a}$ to the rectangular loop, yields for $\delta \to 0$

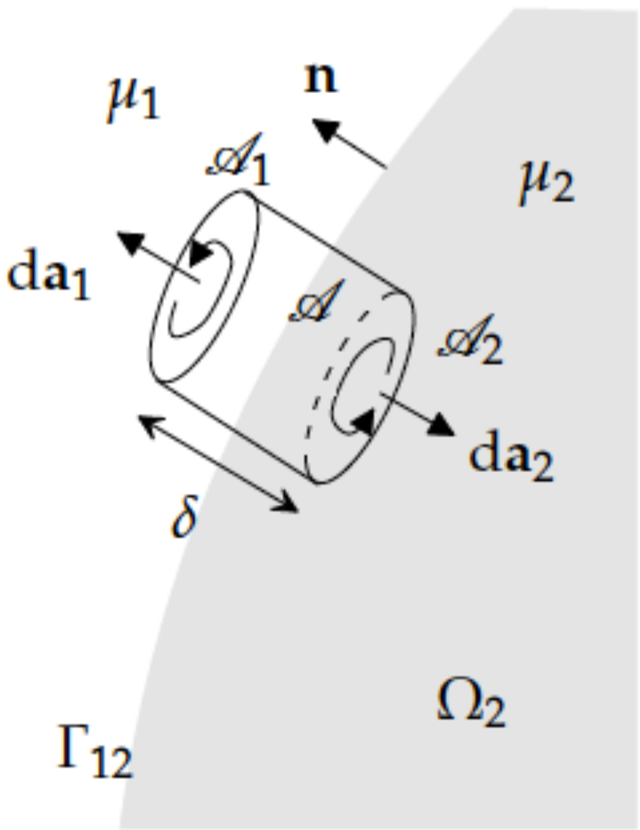
$$\int_{\mathscr{S}_2} \mathbf{H}_2 \cdot \mathrm{d}\mathbf{r} + \int_{\mathscr{S}_1} \mathbf{H}_1 \cdot \mathrm{d}\mathbf{r} = \int_{\mathscr{S}} (\mathbf{H}_1 - \mathbf{H}_2) \cdot \mathrm{d}\mathbf{r} = - \int_{\mathscr{S}} (\mathbf{n} \times \boldsymbol{\alpha}) \cdot \mathrm{d}\mathbf{r},$$

where the surface normal vector $\mathbf{n}$ points from Ω_2 to Ω_1 a



$$H_{t1} = H_{t2} \quad \equiv \quad \mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}$$

Continuity Conditions (2)



$$\int_{\partial V} \mathbf{B} \cdot \mathsf{da} = 0 \qquad \delta \to 0$$

$$\int_a \sigma_{\mathsf{mag}} \mathsf{d}a = \int_a \mathbf{B}_1 \cdot \mathsf{da}_1 + \mathbf{B}_2 \cdot \mathsf{da}_2$$

$$= \int_a (\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{n}_1 \mathsf{d}a$$

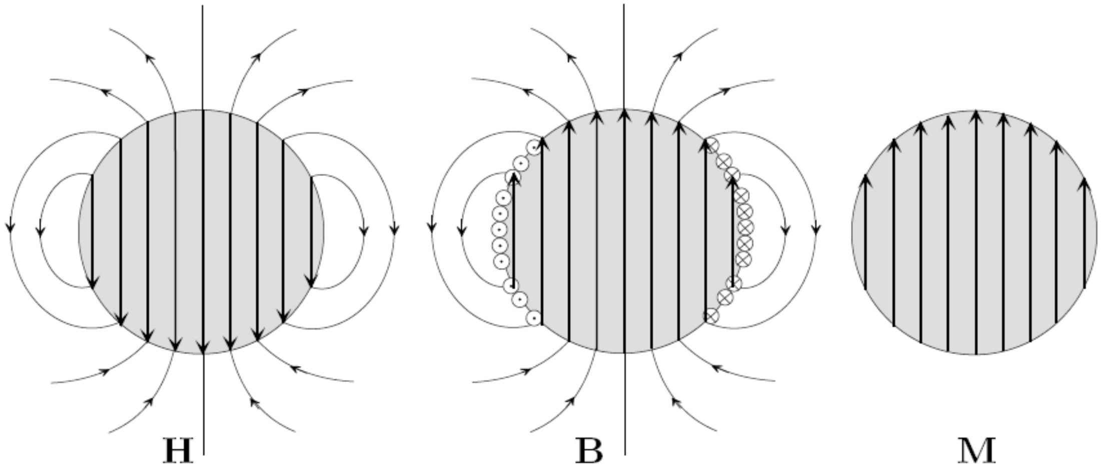
Holds for any surface a if

$$\sigma_{\mathsf{mag}} = (\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{n}$$

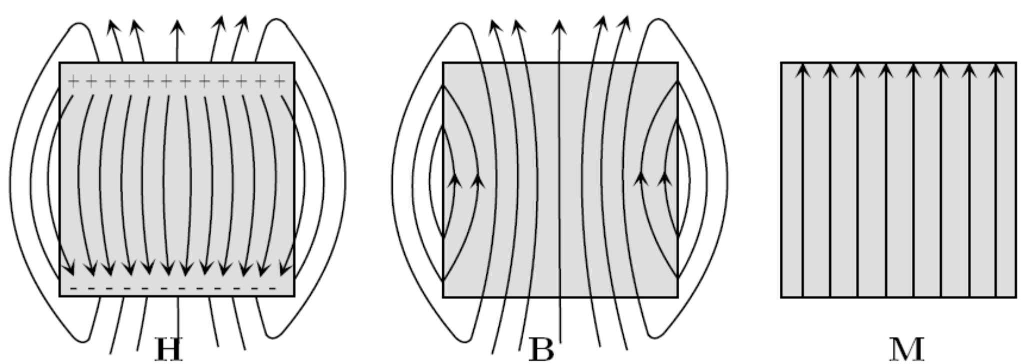
$$= [\mathbf{B} \cdot \mathbf{n}]_{12}$$

$$B_{\mathsf{n}1} = B_{\mathsf{n}2} \equiv (\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{n} = 0 \equiv [\mathbf{B} \cdot \mathbf{n}]_{12} = 0$$

Surface Current and Surface Charge



$$\lim_{c\to 0}\frac{\int_c \mathbf{H}\cdot \mathrm{ds}}{c} = (\mathbf{H}_1 - \mathbf{H}_2)\times \mathbf{n} = -\boldsymbol{\alpha}$$

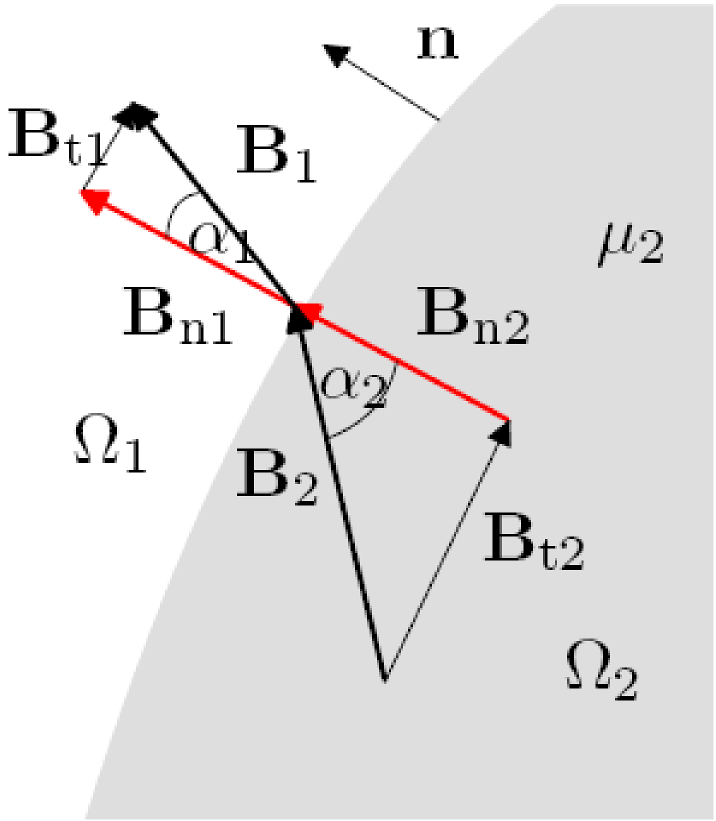


$$\lim_{a\to 0}\frac{\int_a \mathbf{B}\cdot \mathrm{da}}{a} = (\mathbf{B}_1 - \mathbf{B}_2)\cdot \mathbf{n} = \sigma_{\mathrm{mag}}$$

Continuity Conditions (3)

No surface currents:

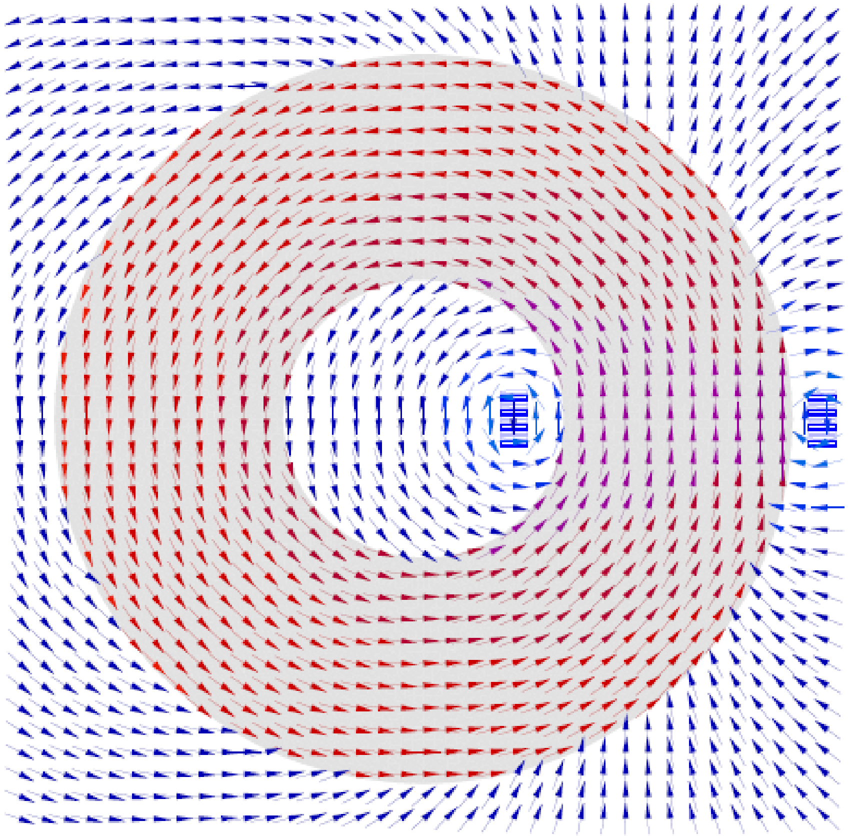
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\frac{B_{t1}}{B_{n1}}}{\frac{B_{t2}}{B_{n2}}} = \frac{\mu_1 \frac{H_{t1}}{B_{n1}}}{\mu_2 \frac{H_{t2}}{B_{n2}}} = \frac{\mu_1 H_{t1}}{\mu_2 H_{t2}} = \frac{\mu_1}{\mu_2}$$

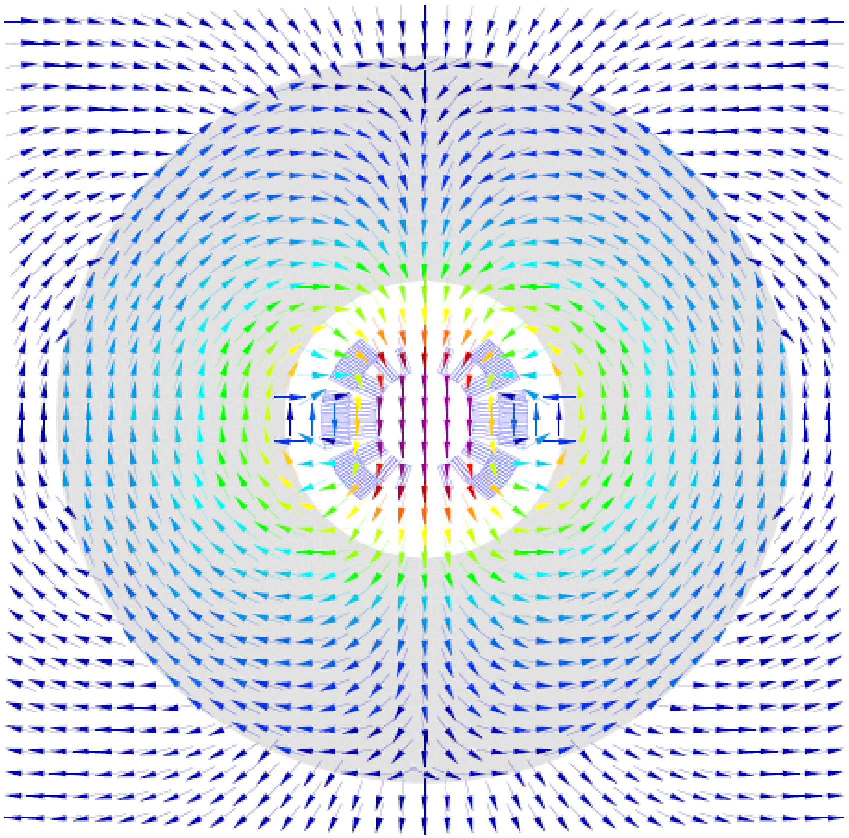


$$\mu_2 \gg \mu_1$$

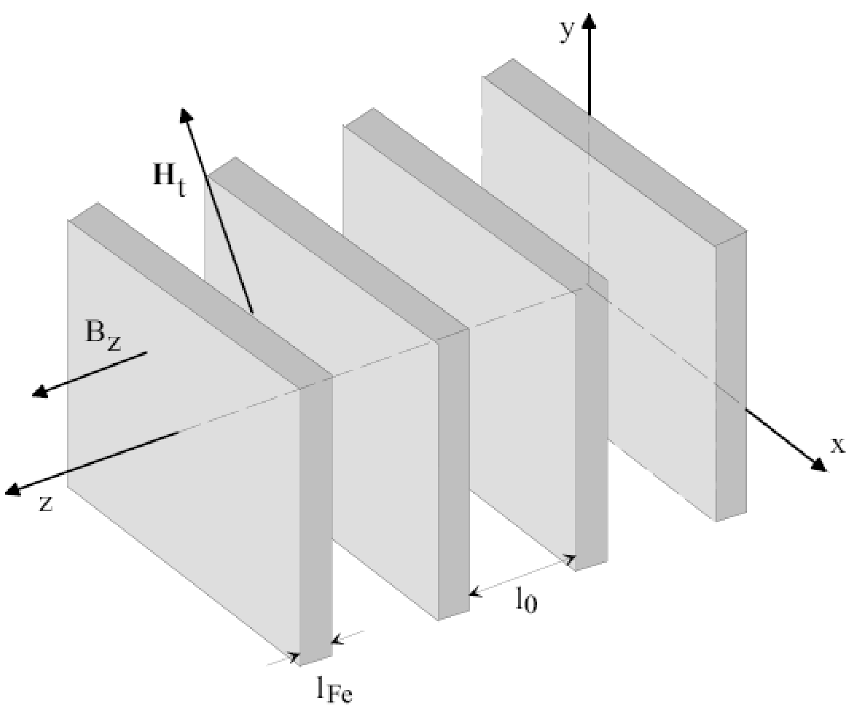
$$\alpha_1 \approx 0, \quad \text{or} \quad \alpha_2 \approx \pi/2,$$

Continuity at Iron Boundaries





Stacking Factor for Yoke Laminations



$$\mathbf{H}_{\mathrm{t}}^{0} = \mathbf{H}_{\mathrm{t}}^{\mathrm{Fe}} = \overline{\mathbf{H}}_{\mathrm{t}}$$

$$\overline{\mathbf{B}}_{\mathrm{t}} = \frac{1}{l_{\mathrm{Fe}} + l_0} \left(l_{\mathrm{Fe}} \mu \overline{\mathbf{H}}_{\mathrm{t}} + l_0 \mu_0 \overline{\mathbf{H}}_{\mathrm{t}} \right)$$

$$B_z^0 = B_z^{\mathrm{Fe}} = \overline{B}_z$$

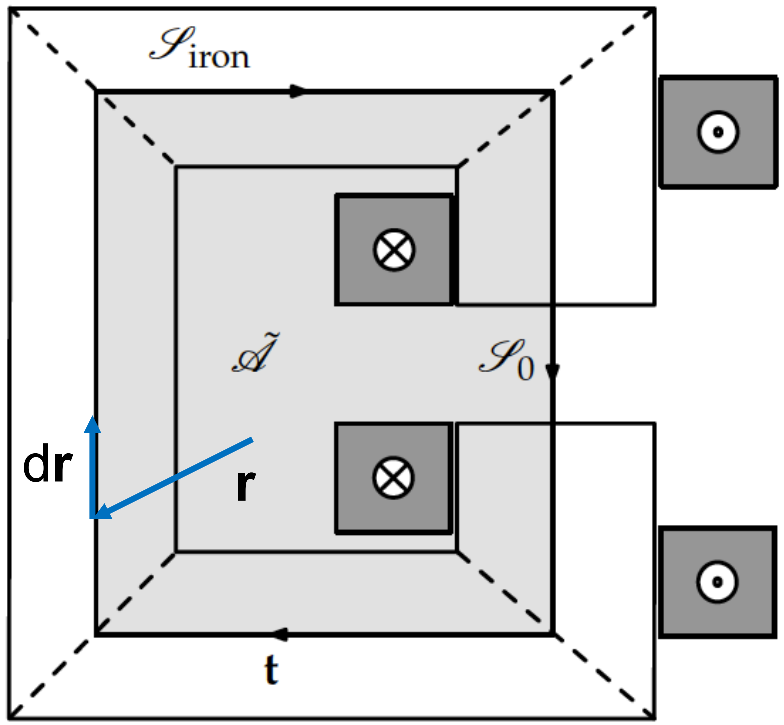
$$\overline{H}_z = \frac{1}{l_{\mathrm{Fe}} + l_0} \left(l_{\mathrm{Fe}} \frac{\overline{B}_z}{\mu} + l_0 \frac{\overline{B}_z}{\mu_0} \right)$$

$$\lambda = \frac{l_{\mathrm{Fe}}}{l_{\mathrm{Fe}} + l_0}$$

$$\overline{\mu}_{\mathrm{t}} = \lambda \mu + (1 - \lambda) \mu_0$$

$$\overline{\mu}_z = \left(\frac{\lambda}{\mu} + \frac{1 - \lambda}{\mu_0} \right)^{-1}$$

Main Field in Normal Conducting Dipole



$$\int_{\partial\tilde{\mathscr{A}}} \mathbf{H}\cdot d\mathbf{r} = \int_{\tilde{\mathscr{A}}} \mathbf{J}\cdot\mathbf{n}\, da\,,$$

$$\int_{\mathscr{S}_{\text{iron}}} \mathbf{H}\cdot d\mathbf{r} + \int_{\mathscr{S}_0} \mathbf{H}\cdot d\mathbf{r} = \int_{\tilde{\mathscr{A}}_{\text{coil}}} \mathbf{J}\cdot\mathbf{n}\, da\,,$$

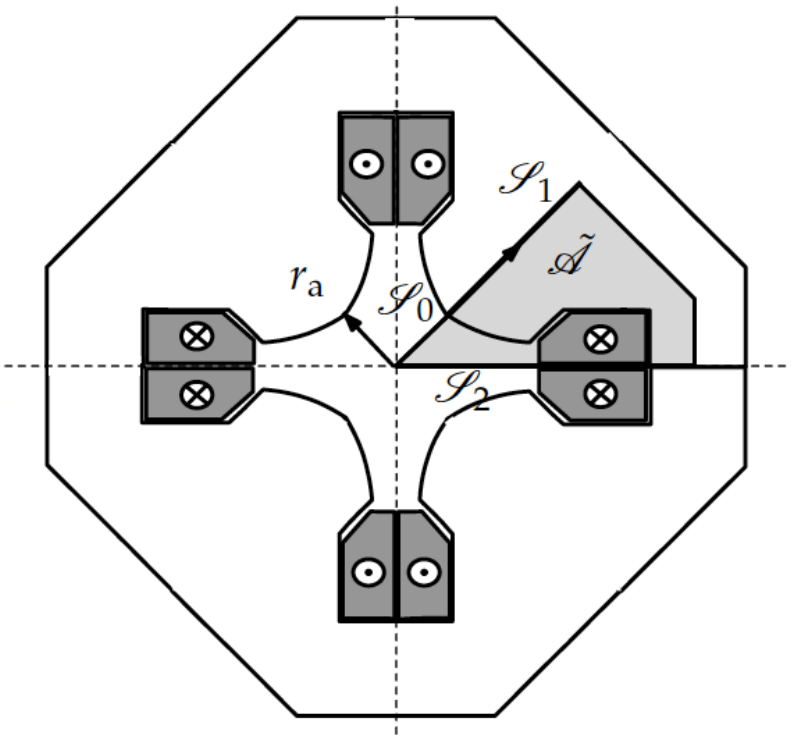
$$H_{\text{iron}}\, s_{\text{iron}} + H_0\, s_0 = N\, I\,,$$

$$\frac{1}{\mu_0\mu_r} B_{\text{iron}}\, s_{\text{iron}} + \frac{1}{\mu_0} B_0\, s_0 = N\, I\,,$$

$$B_0 = \frac{\mu_0 N\, I}{s_0}\,.$$

Gradient in Normal Conducting Quadrupole

$$\int_{\partial \tilde{\mathscr{A}}} \mathbf{H} \cdot \mathrm{d}\mathbf{r} = \int_{\mathscr{S}_0} \mathbf{H}_0 \cdot \mathrm{d}\mathbf{r} + \int_{\mathscr{S}_1} \mathbf{H}_1 \cdot \mathrm{d}\mathbf{r} + \int_{\mathscr{S}_2} \mathbf{H}_2 \cdot \mathrm{d}\mathbf{r} = N I .$$



$$B_x = gy, \qquad B_y = gx;$$

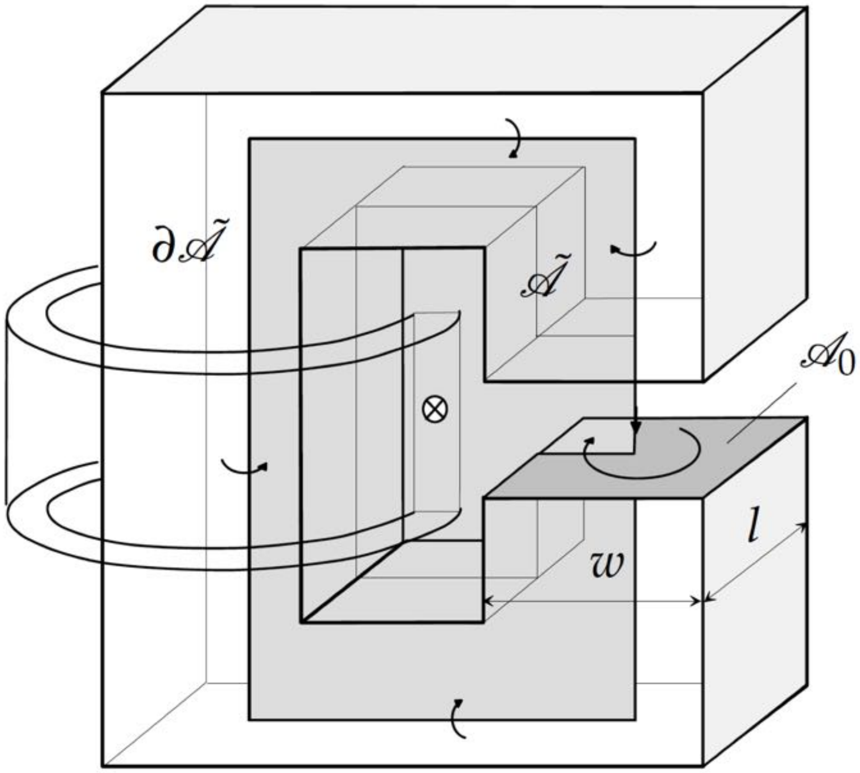
$$H = \frac{g}{\mu_0}\sqrt{x^2+y^2} = \frac{g}{\mu_0} r.$$

$$\int_0^{r_a} H \mathrm{d}r = \frac{g}{\mu_0}\int_0^{r_a} r \mathrm{d}r = \frac{g}{\mu_0}\frac{r_a^2}{2} = N I,$$

or

$$g = \frac{2\mu_0 N I}{r_a^2}.$$

Dipole with Varying Cut-Section



$$\sum_{i=0}^{n} H_i s_i = N\, I$$

$$H_i = \frac{B_i}{\mu_i} = \frac{\Phi}{a_i\, \mu_i}$$

$$\Phi \sum_{i=0}^{n} \frac{s_i}{a_i\, \mu_i} = N\, I = V_\mathrm{m}$$

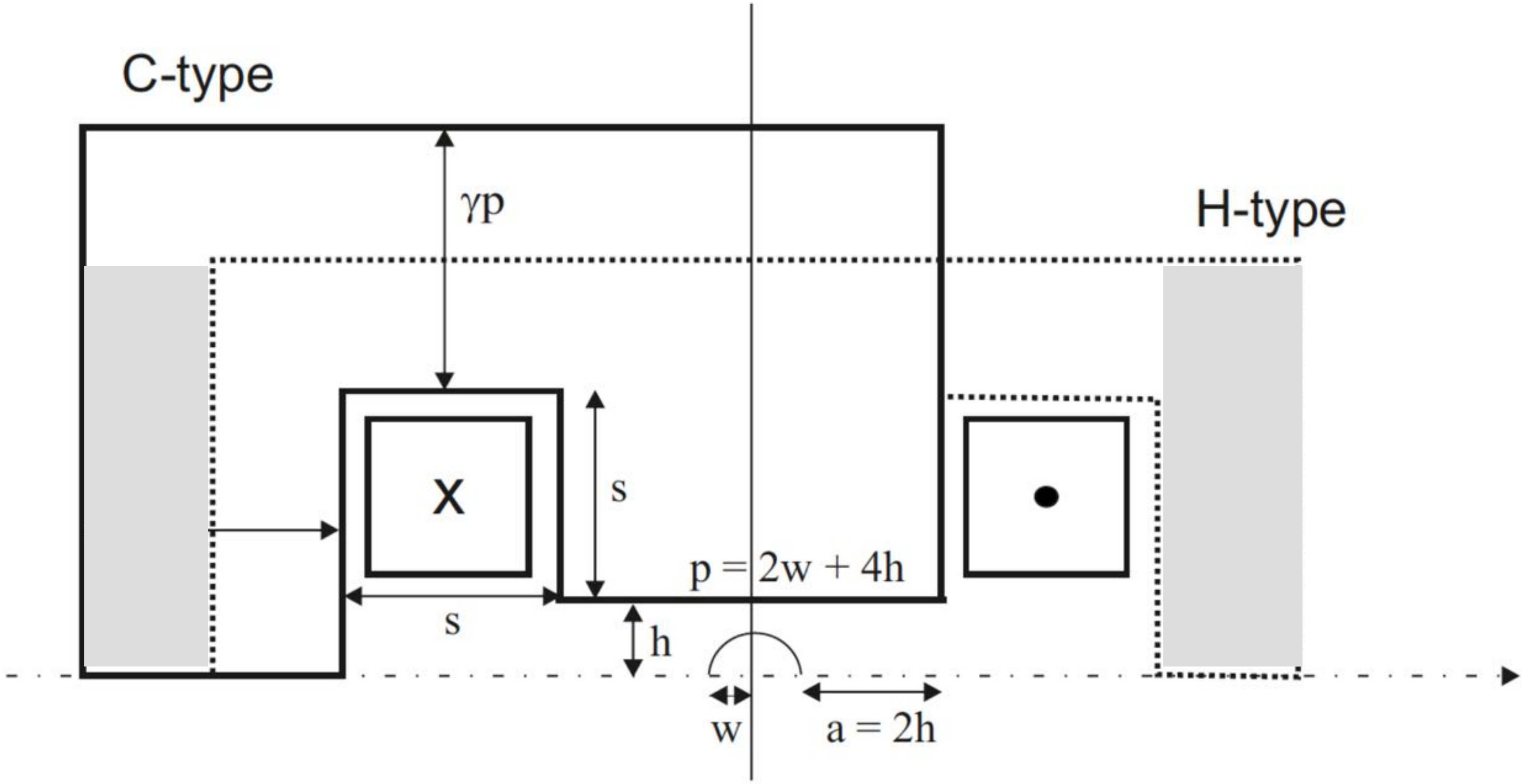
Ohm's law: $I \sum_{i=0}^{n} \frac{s_i}{a_i\, \kappa_i} = U$

$$N\, I = \Phi \sum_{i=0}^{n} \frac{s_i}{a_i\, \mu_i} = \Phi \left(\frac{s_0}{a_0\, \mu_0} + \sum_{i=1}^{n} \frac{s_i}{a_i\, \mu_i} \right)$$

Conclusion: Magnet with large air-gap is stabilized against variations in permeability

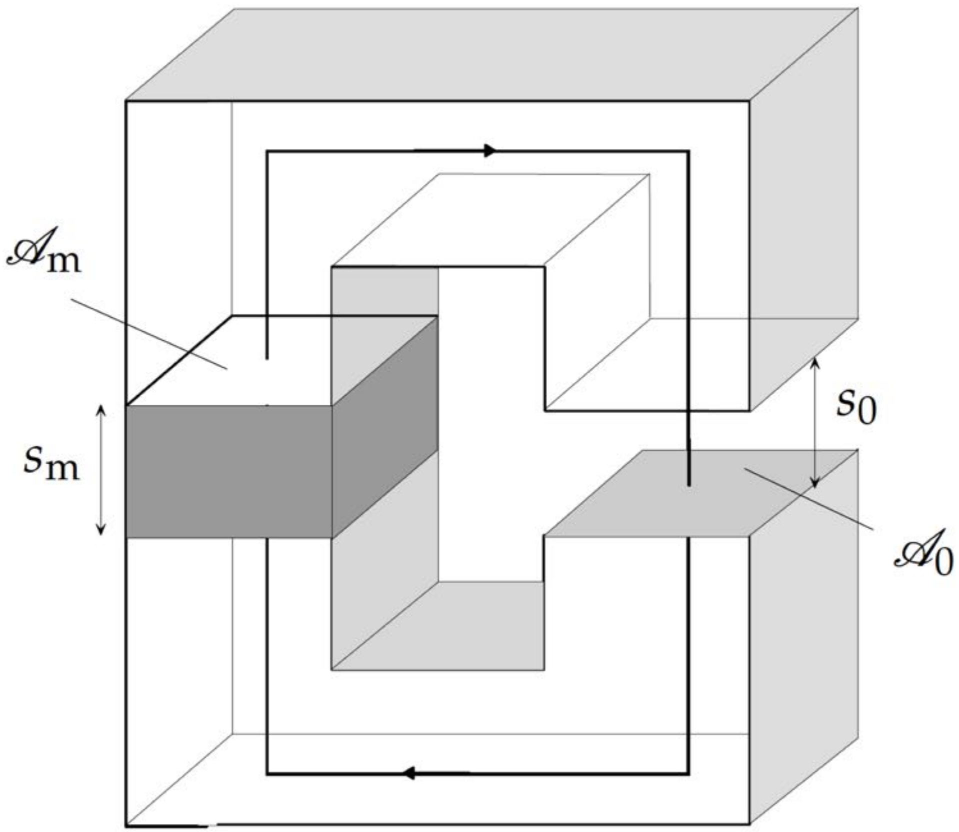
The Mass of the Iron Yoke

$$A = p^2(2\gamma + \gamma^2) + p(2s + \gamma(s + 2h + 2s)) + s^2$$



$$A = 2(h + s + 0.5\gamma p)\,(p + 2s + \gamma p) - 2h\,(p + 2s)$$

Permanent Magnet Excitation



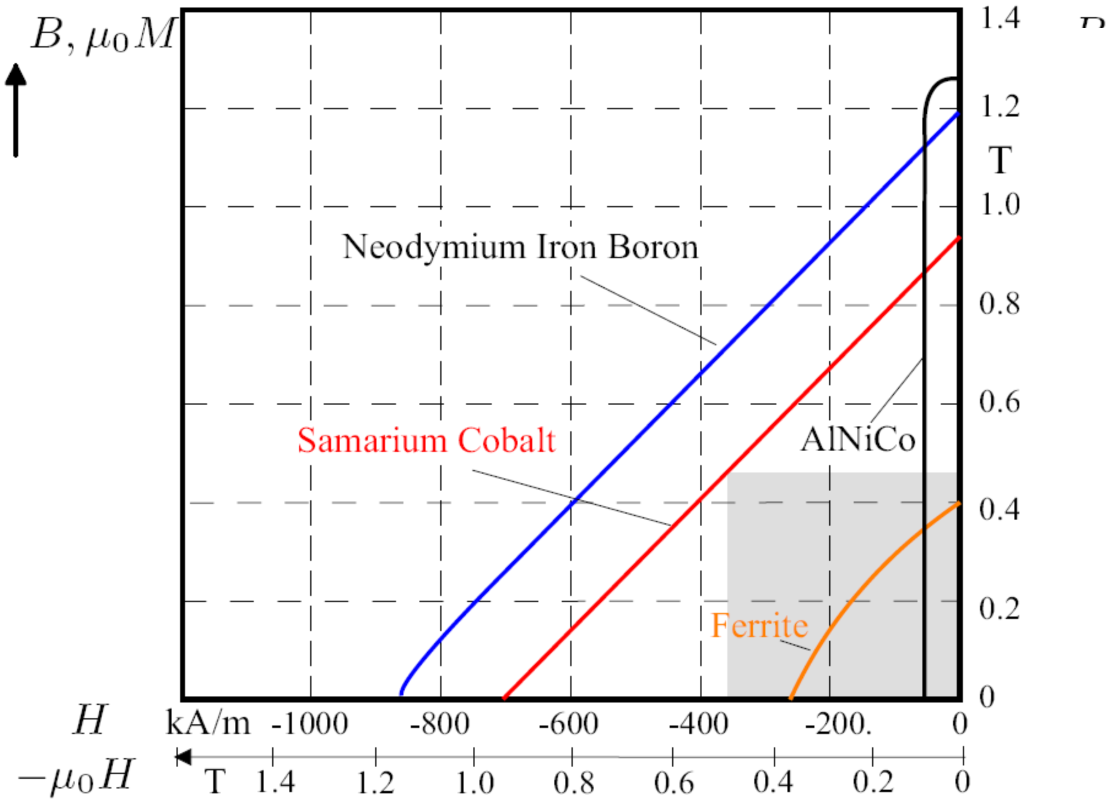
$$B_m a_m = B_0 a_0 = \mu_0 H_0 a_0$$

$$H_0 s_0 = -H_m s_m$$

$$B_m a_m s_m = \mu_0 H_0 a_0 \frac{-H_0 s_0}{H_m}$$

$$H_0 = \sqrt{\frac{(a_m s_m)(-B_m H_m)}{\mu_0 (a_0 s_0)}} = \sqrt{\frac{V_m(-B_m H_m)}{\mu_0 V_0}}$$

BH Maximum



$$H_0\, s_0 + H_m\, s_m = 0$$

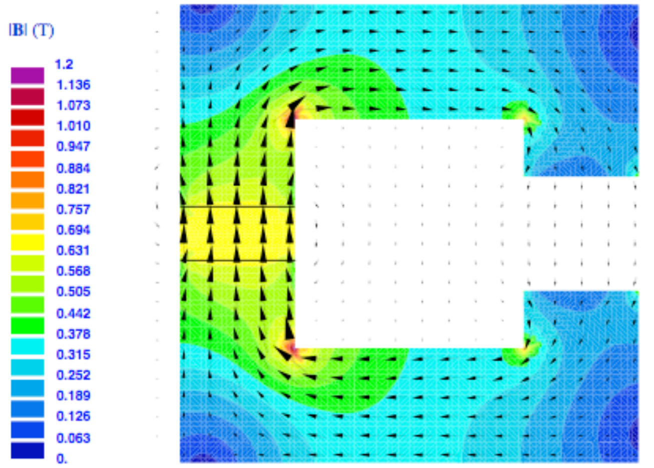
$$B_m a_m = B_0 a_0 = \mu_0 H_0 a_0$$

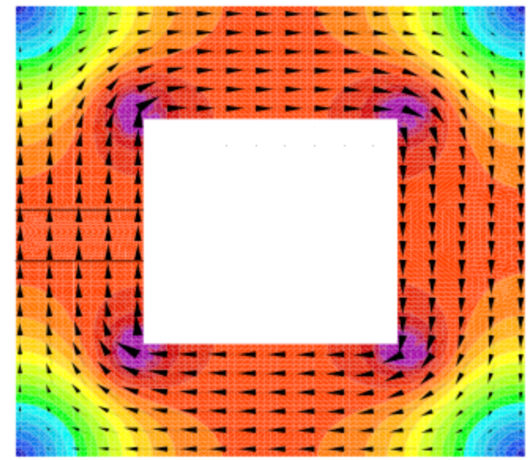
$$\begin{aligned} H_0 s_0 &= -H_m s_m, \\ \frac{1}{\mu_0} B_m \frac{a_m}{a_0} s_0 &= -H_m s_m, \\ B_m &= -\mu_0 \frac{s_m}{s_0}\frac{a_0}{a_m} H_m, \\ \frac{B_m}{\mu_0 H_m} &= -\frac{s_m}{s_0}\frac{a_0}{a_m} = P \end{aligned}$$

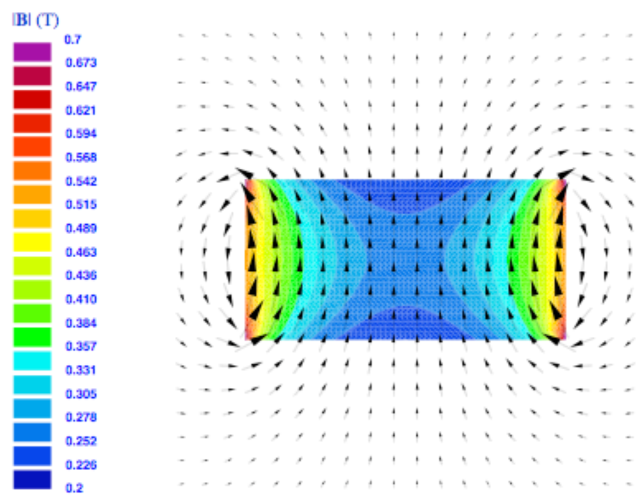
Permeance P, Slope s

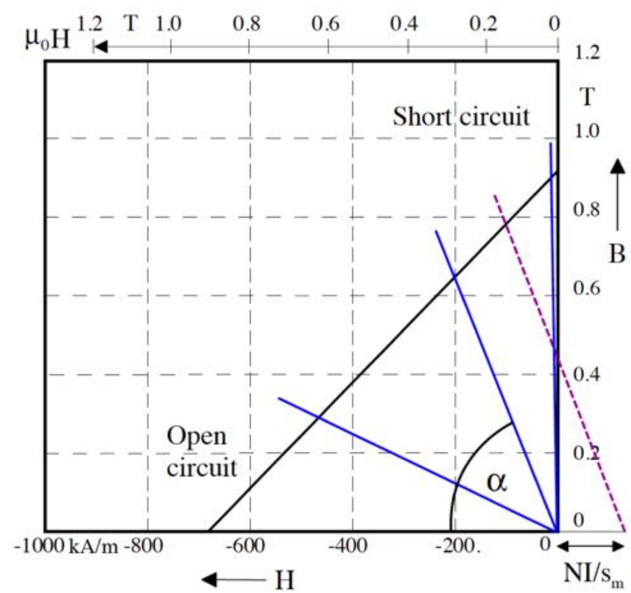
$$(BH)^{\mathrm{id}}_{\max} := \frac{B_r^2}{4\mu_0},$$

$$s = \frac{B_m}{H_m} = \mu_0 P = -\mu_0 \frac{s_m}{s_0}\frac{a_0}{a_m} = \mu_0 \frac{M(1-N)}{H_m - N M}$$





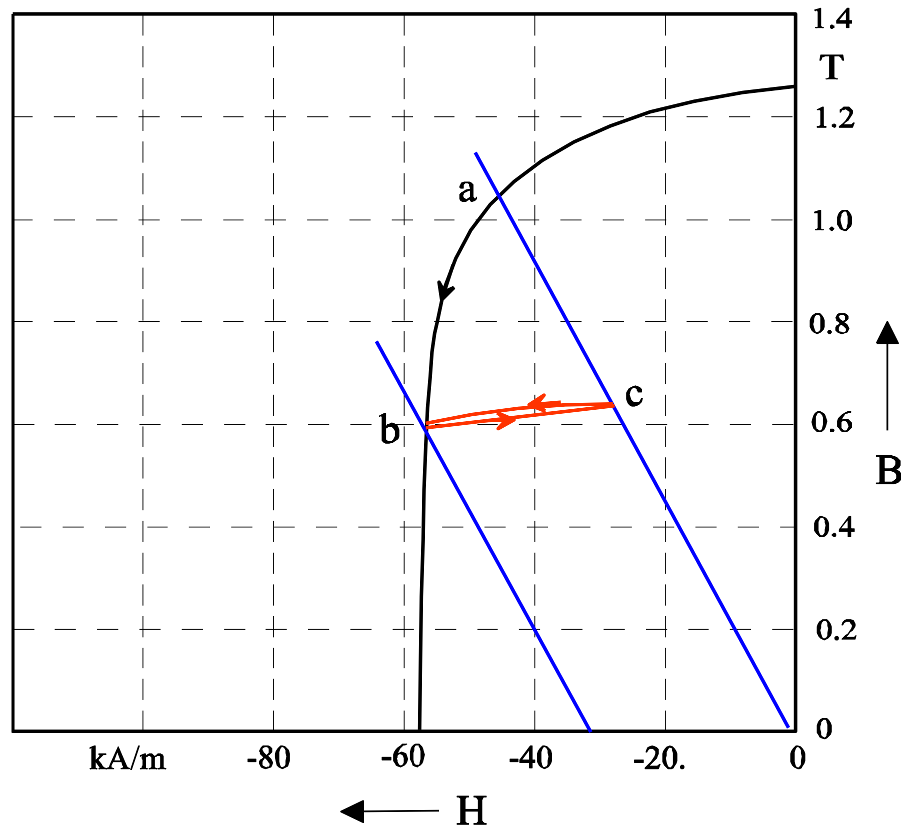




s = - tan α

$$s = \frac{B_{\mathrm{m}}}{H_{\mathrm{m}}} = \mu_0 P = -\mu_0 \frac{s_{\mathrm{m}}}{s_0} \frac{a_0}{a_{\mathrm{m}}}$$

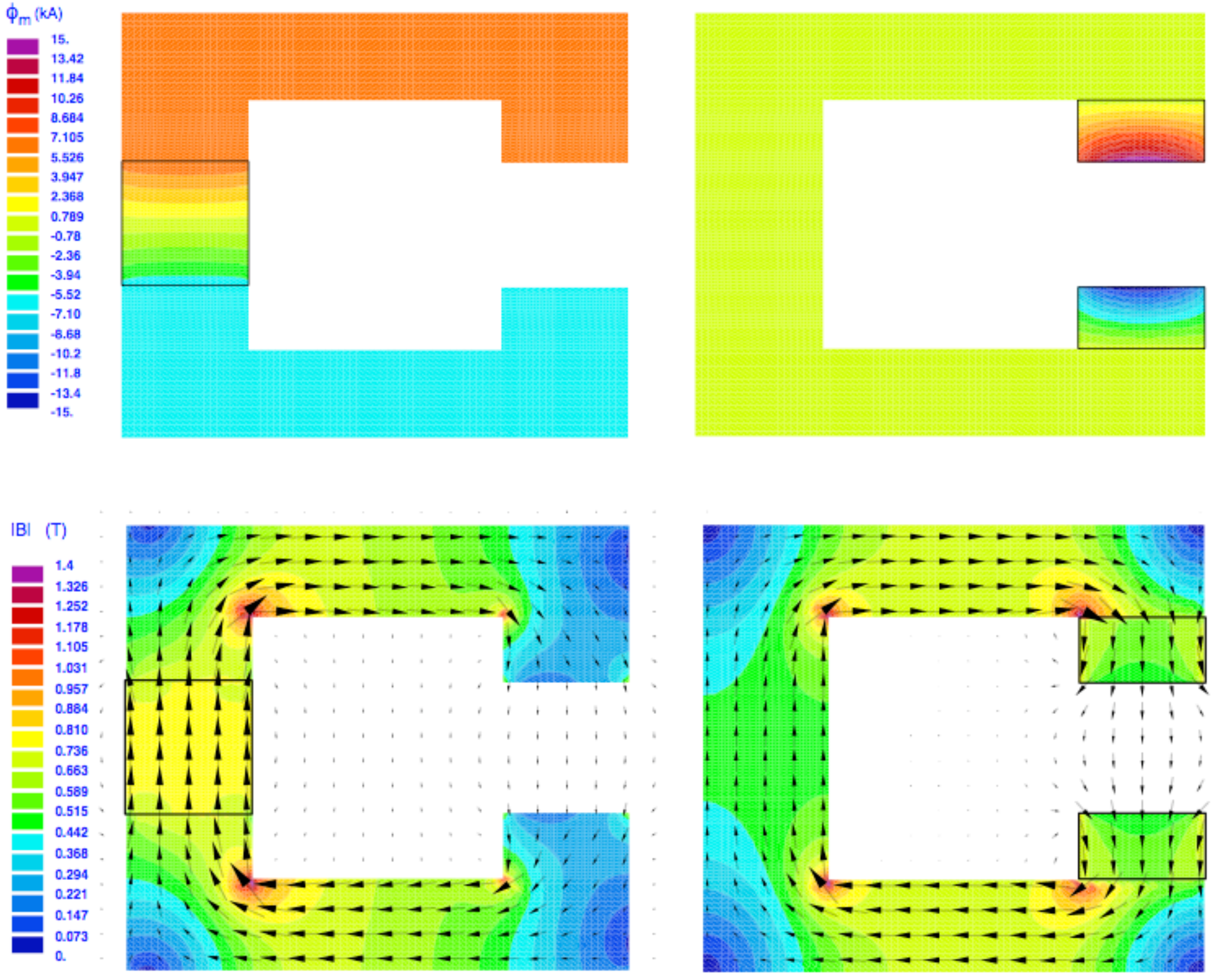
Dynamic Operation (Flux is Reduced)



$$H_0\, s_0 + H_m\, s_m = -NI$$

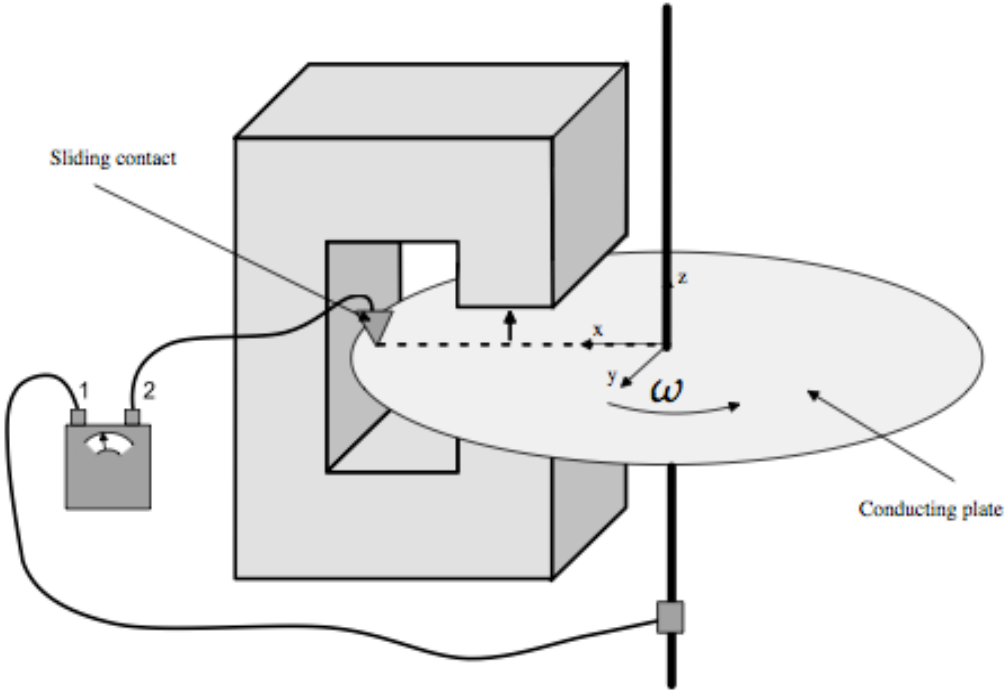
$$B_m = -\mu_0 \frac{s_m}{s_0} \frac{a_0}{a_m} \left(H_m + \frac{NI}{s_m} \right)$$

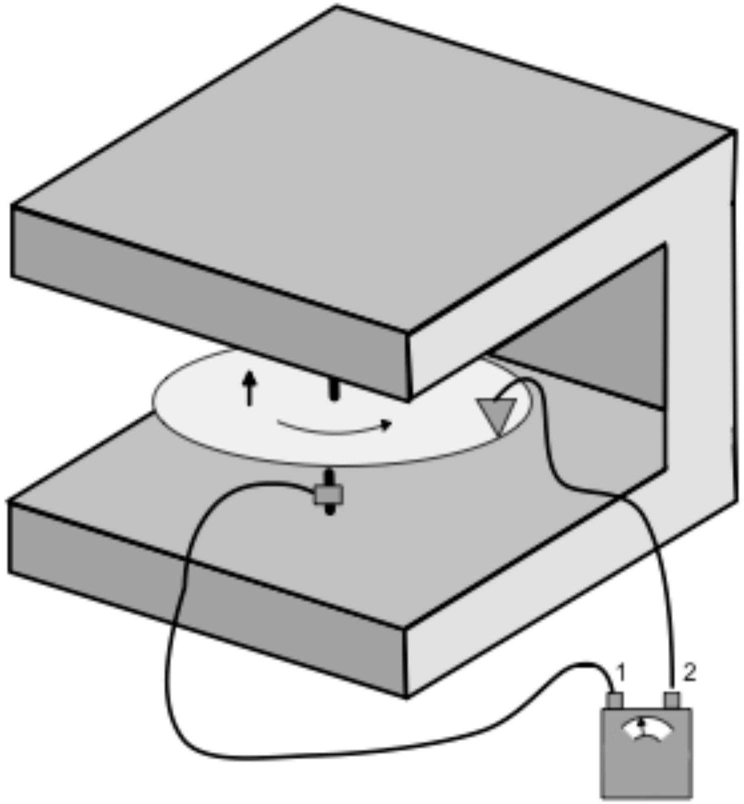
Optimal Position of Permanent Magnets



The Homopolar Generator

$$d\mathbf{F} = I\, d\mathbf{r} \times \mathbf{B}$$





Einstein: All physics is local