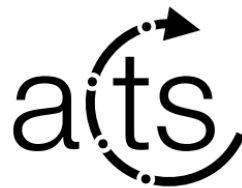


Update on Matrix Algebra

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Math/Backends of Algebra-plugins

		Backends		
		CPU	CUDA	SYCL
Math	cmath (home-made)	Backend is supported	Backend is supported	Backend is supported
	Eigen	Backend is supported	Backend is supported	Supported but not tested
	SMatrix	Backend is supported	Backend is NOT supported	Backend is NOT supported
	VC	Backend is supported	Backend is NOT supported	Backend is NOT supported

Legend:

- Backend is supported
- Supported but not tested
- Backend is NOT supported

Developed algebras before v0.8.0:

- vector algebra
- local \leftrightarrow global transform
- matrix generation and element access
 - matrix algebra had been missing

Required Matrix Operations

- Matrix Creation
 - Zero
 - Identity
- Addition and Subtraction
- Multiplication
 - Normal multiplication
 - Blocked multiplication
- Transpose
- Inverse
 - 2x2 (KF updater)
 - 4x4 (local ↔ global transform)
 - 6x6 (KF smoother)
 - Maybe more?

Eigen and SMatrix implementation

- Writing the matrix algebra for Eigen and SMatrix is just porting the existing functions.

```
/// "Matrix actor", assuming an Eigen matrix
template <typename scalar_t>
struct actor {

    /// 2D matrix type
    template <int ROWS, int COLS>
    using matrix_type = Eigen::Matrix<scalar_t, ROWS, COLS>;

    // Create transpose matrix
    template <int ROWS, int COLS>
    ALGEBRA_HOST_DEVICE inline matrix_type<COLS, ROWS>
    transpose(
        const matrix_type<ROWS, COLS> &m) {
        return m.transpose();
    }

    // Create inverse matrix
    template <int N>
    ALGEBRA_HOST_DEVICE inline matrix_type<N, N> inverse(
        const matrix_type<N, N> &m) {
        return m.inverse();
    }
};
```

cmath Implementation

- For cmath, we can try something better
 - User can decide which specific algorithm will be used for which matrix dimensions in compile time
 - Various matrix_actor can be defined to test different aggregations

```
// Define inverse algorithm
// Base algorithm is cofactor method
// For 2x2 and 4x4 matrix, hard coded method is used
// For 3x3 and 5x5 matrix, LU decomposition is used
using inverse_actor = matrix::inverse::actor<cofactor<>,
                                             hard_coded<2,4>, LU_decomposition<3,5>>

// Define matrix actor
using matrix_actor = matrix::actor<scalar, inverse_actor>

matrix<2,2> m22;
matrix<3,3> m33;
matrix<7,7> m77;

m22_inv = matrix_actor().inverse(m22) // hard-coded
m33_inv = matrix_actor().inverse(m33) // LU decomposition
m77_inv = matrix_actor().inverse(m77) // cofactor
```

Customizable algorithms in cmath matrix

- Determinant
 - cofactor
 - hard_coded for 2x2 and 4x4
- Inverse
 - cofactor
 - hard_coded for 2x2 and 4x4

```

  ✓ math
    ✓ cmath
      ✓ include / algebra / math
        ✓ algorithms
          ✓ matrix
            ✓ determinant
              ↻ cofactor.hpp
              ↻ hard_coded.hpp
            ✓ inverse
              ↻ cofactor.hpp
              ↻ hard_coded.hpp
          ✓ utils
            ↻ algorithm_finder.hpp

```

Outlooks

- Similar concept of algorithm aggregation can be applied to Eigen and SMatrix implementation
- cmath needs 6x6 hard coded inversion or LU decomposition
 - cofactor is slow!
- cmath matrix multiplication relies on * operator which does standard multiplication
 - Needs to add customizable actor for multiplication as done for inverse and determinant