

# Electromagnetic Design of Accelerator Magnets and ROXIE User's Course

Dynamic effects

Stephan Russenschuck, CERN, 2022



# Timetable

Week	Mo	Tu	We	Th	Fr
15 (03.04)		Introduction, lumped circuits	Vector fields, field harmonics	14:00 Line currents and coil design	Magnet X- sections
16 (10.04)	Coil-ends (Brookhaven session)				
17 (17.04)			Optimization techniques	X-sec optimization	Numerical field comp., BEM- FEM
18 (24.04)	Yoke design	Integrated quant./ Dynamic effects / Computations		15:30 Diff. geom. /Coil ends, CCT / Cos theta ends	
19 (01.05)		Quench simulation	Demands and future plans/ Quench simulation (TBC)		

Perhaps a Master Class later in the year

Faraday paradoxes, coil magnetometers, stretched-wire measurements, CCT, Tori, ROXIE 22



$$W = \frac{1}{2} \int_V \mathbf{H} \cdot \mathbf{B} dV$$

$$\operatorname{div}(\mathbf{A} \times \mathbf{H}) = \mathbf{H} \cdot \operatorname{curl} \mathbf{A} - \mathbf{A} \cdot (\operatorname{curl} \mathbf{H})$$

$$\begin{aligned} W &= \frac{1}{2} \int_V \mathbf{H} \cdot \operatorname{curl} \mathbf{A} dV \\ &= \frac{1}{2} \int_V \operatorname{div}(\mathbf{A} \times \mathbf{H}) dV + \frac{1}{2} \int_V \mathbf{A} \cdot \operatorname{curl} \mathbf{H} dV \\ &= \frac{1}{2} \int_{\partial V} (\mathbf{A} \times \mathbf{H}) \cdot d\mathbf{a} + \frac{1}{2} \int_V \mathbf{A} \cdot \operatorname{curl} \mathbf{H} dV \end{aligned}$$

$$W = \frac{1}{2} \int_V \mathbf{A} \cdot \operatorname{curl} \mathbf{H} dV = \frac{1}{2} \int_V \mathbf{A} \cdot \mathbf{J} dV$$

$$\begin{aligned}\int_{\partial a} \mathbf{H} \cdot d\mathbf{s} &= \int_a \mathbf{J} \cdot d\mathbf{a} \\ H 2\pi r &= \frac{I}{\pi r_0^2} \pi r^2 \\ H &= \frac{I r}{2\pi r_0^2}\end{aligned}$$

$$dW = \frac{1}{2} B H 2\pi r l dr = \mu_0 H^2 \pi r l dr = \frac{\mu_0 l I^2}{4\pi r_0^4} r^3 dr$$

Therefore the total energy in the wire is

$$\frac{W}{l} = \frac{\mu_0 I^2}{4\pi r_0^4} \int_0^{r_0} r^3 dr = \frac{\mu_0 I^2}{16\pi}$$

For one aperture of the LHC main dipole the stored energy at 8.33 T is 237 kJ/m. The energy stored in the strands is 4.3 J/m.

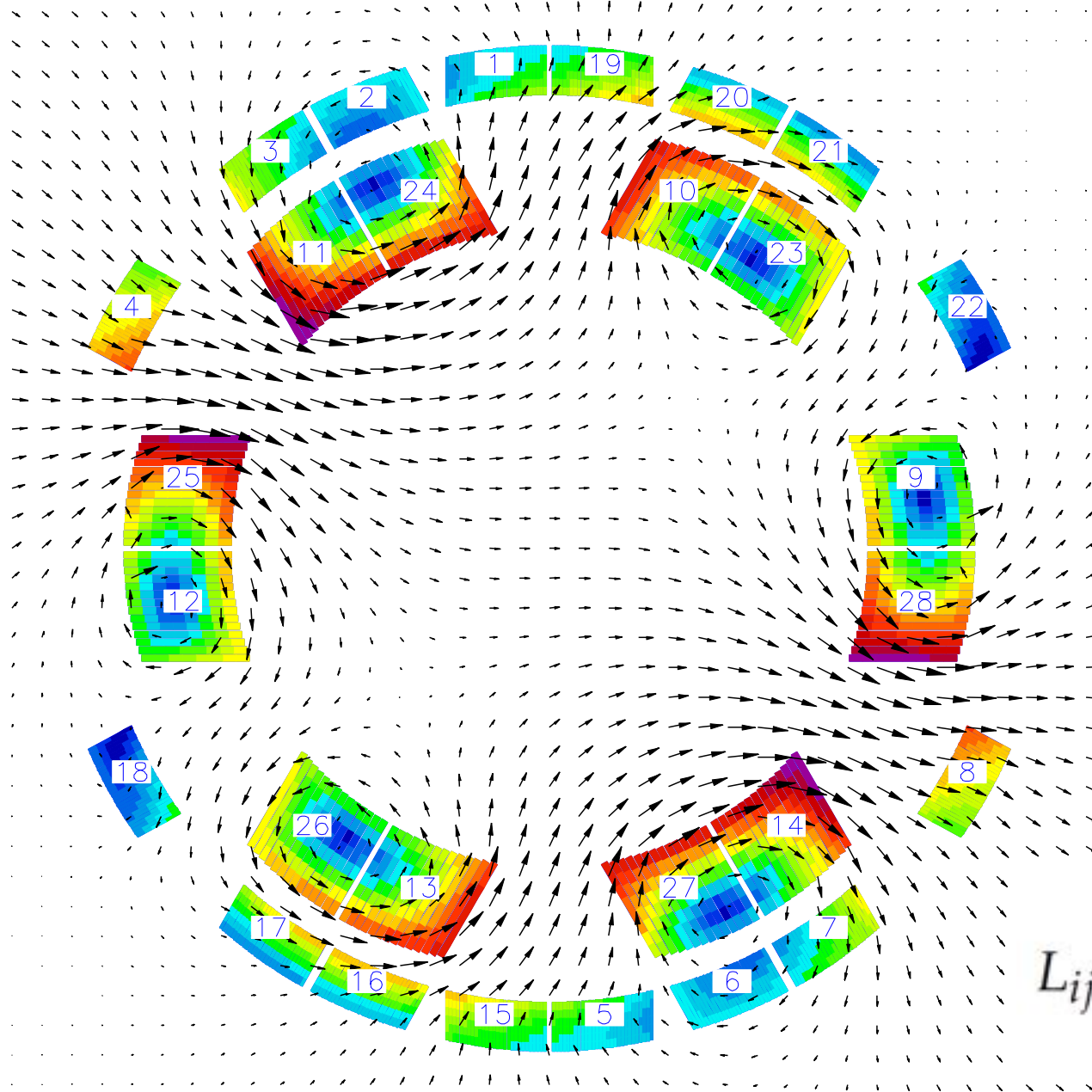


$$W = \frac{\mu_0}{8\pi} \int_V \int_{V'} \frac{\mathbf{J}(\mathbf{r}) \cdot \mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' dV$$

$$\begin{aligned} W &= \sum_{i=1}^n \sum_{j=1}^n W_{ij} = \frac{\mu_0}{8\pi} \sum_{i=1}^n \sum_{j=1}^n \int_V \int_{V'} \frac{\mathbf{J}_i(\mathbf{r}) \cdot \mathbf{J}_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' dV \\ &= \frac{\mu_0}{8\pi} \sum_{i=1}^n \sum_{j=1}^n I_i I_j \int_V \int_{V'} \frac{\mathbf{J}_i(\mathbf{r}) \cdot \mathbf{J}_j(\mathbf{r}')}{I_i I_j |\mathbf{r} - \mathbf{r}'|} dV' dV \end{aligned}$$

$$L_{ij} = \frac{\mu_0}{4\pi I_i I_j} \int_V \int_{V'} \frac{\mathbf{J}_i(\mathbf{r}) \cdot \mathbf{J}_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' dV$$

# Combined Dipole Sextupole Corrector



$$L_{ii} = \frac{2W_{ii}}{I^2}$$

$$L_{ij} = \frac{1}{2} \left( \frac{2W_{ij}}{I^2} - L_{ii} - L_{jj} \right)$$

# Mutual Inductance Matrix

Coil	1	2	3	4	5	6	7	8
1	12.601	6.517	-0.245	0.252	0.478	-0.478	-0.252	0.245
2	6.517	12.601	-0.478	-0.252	0.245	-0.245	0.252	0.478
3	-0.245	-0.478	0.136	0.027	-0.010	0.009	-0.010	0.027
4	0.252	-0.252	0.027	0.136	0.027	-0.010	0.009	-0.010
5	0.478	0.245	-0.010	0.027	0.136	0.027	-0.010	0.009
6	-0.478	-0.245	0.009	-0.010	0.027	0.136	0.027	-0.010
7	-0.252	0.252	-0.010	0.009	-0.010	0.027	0.136	0.027
8	0.245	0.478	0.027	-0.010	0.009	-0.010	0.027	0.136

A coil of multipole order  $N$  does not couple into one of order  $K$



## Nonlinear Circuits (Differential Inductance)

$$U(t) = \frac{d\Phi}{dt} = \frac{d(LI)}{dt} = L \frac{dI}{dt} + I \frac{dL}{dt}$$

$$U = L^d \frac{dI}{dt}$$

$$dL = \frac{\partial L}{\partial I} dI + \frac{\partial L}{\partial t} dt$$

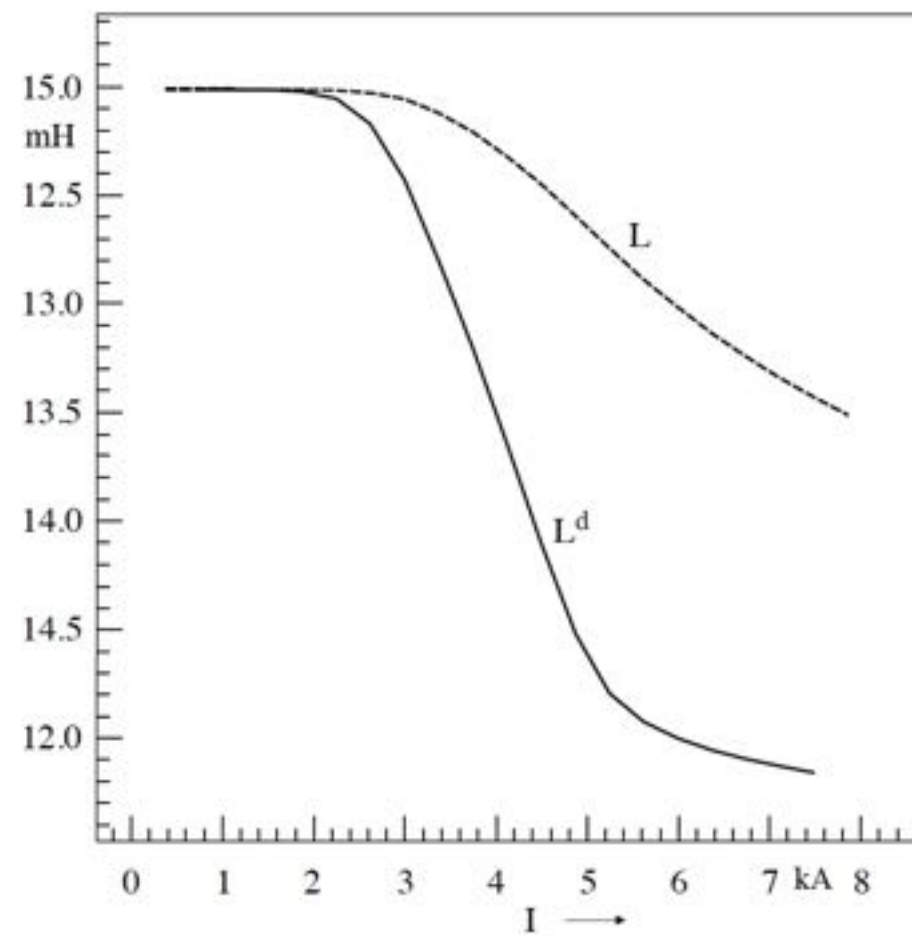
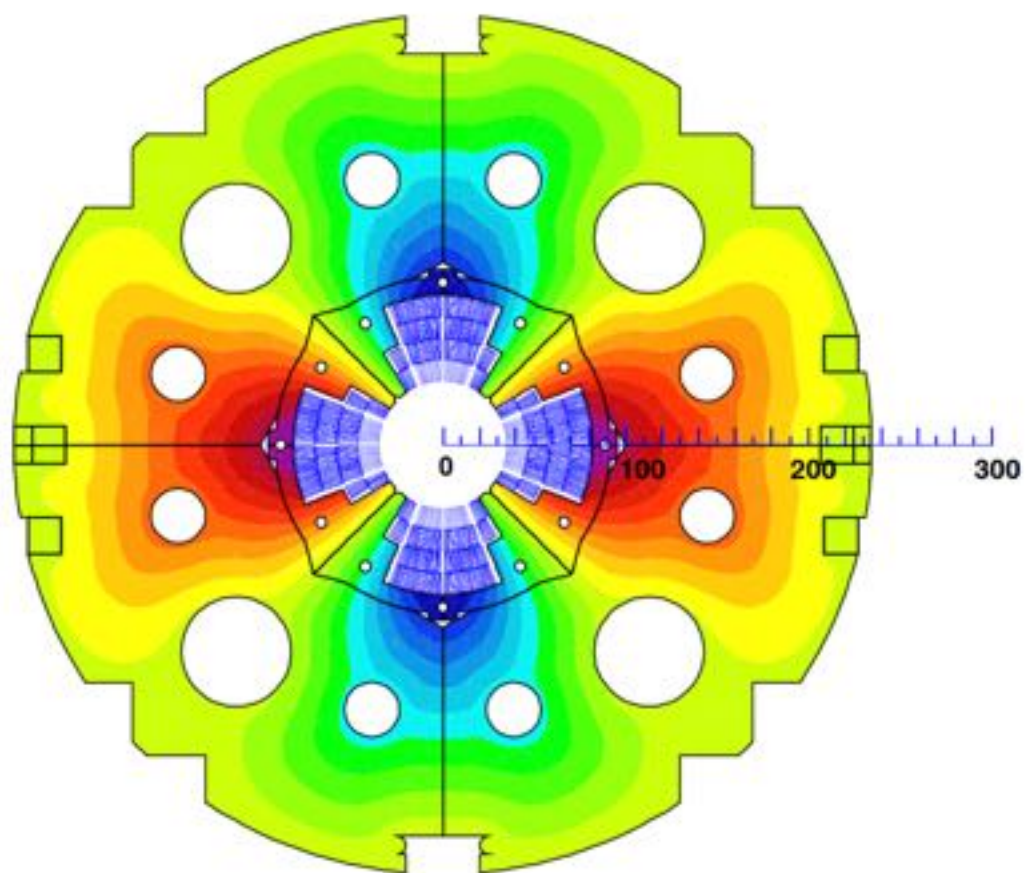
$$U(t) = \left( \frac{\partial L}{\partial I} I + L \right) \frac{dI}{dt} + I \frac{\partial L}{\partial t}$$

$$L^d = L + I \frac{\partial L}{\partial I} = \frac{d\Phi}{dI}$$

For example,  
machine rotor motion



# Differential Inductance for the MQXY



$$\begin{aligned}\mathbf{F}_m &= \int_{\partial\mathcal{V}} \left( \frac{1}{\mu_0} (\mathbf{B} \cdot \mathbf{n}) \mathbf{B} - \frac{1}{2\mu_0} B^2 \mathbf{n} \right) da \\ &= \int_{\partial\mathcal{V}} \left( \left( \frac{1}{2\mu_0} B_n^2 - \frac{\mu_0}{2} H_t^2 \right) \mathbf{n} + B_n H_t \mathbf{t} \right) da.\end{aligned}$$

$$\mathbf{F}_m = \int_{\mathcal{V}} \frac{1}{\mu_0} \operatorname{div} \mathbf{T}_m dV = \int_{\partial\mathcal{V}} \frac{1}{\mu_0} \mathbf{T}_m \cdot \mathbf{n} da,$$

$$\mathbf{T}_m = (T_{ij}) := \begin{pmatrix} B_1^2 - \frac{1}{2} B^2 & B_1 B_2 & B_1 B_3 \\ B_2 B_1 & B_2^2 - \frac{1}{2} B^2 & B_2 B_3 \\ B_3 B_1 & B_3 B_2 & B_3^2 - \frac{1}{2} B^2 \end{pmatrix}$$

$$\mathbf{F} = \int_{\mathcal{V}} (\rho \mathbf{E} + \rho \mathbf{v} \times \mathbf{B}) dV = \int_{\mathcal{V}} (\rho \mathbf{E} + \mathbf{J} \times \mathbf{B}) dV.$$

$$\mathbf{F} = \int_{\mathcal{V}} \left( (\operatorname{div} \mathbf{D}) \mathbf{E} + (\operatorname{curl} \mathbf{H}) \times \mathbf{B} + (\operatorname{curl} \mathbf{E}) \times \mathbf{D} - \frac{1}{c^2} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{H}) \right) dV$$

$$(\operatorname{curl} \mathbf{H}) \times \mathbf{B} = \frac{1}{\mu_0} (\mathbf{B} \cdot \operatorname{grad}) \mathbf{B} - \frac{1}{2\mu_0} \operatorname{grad} (B^2),$$

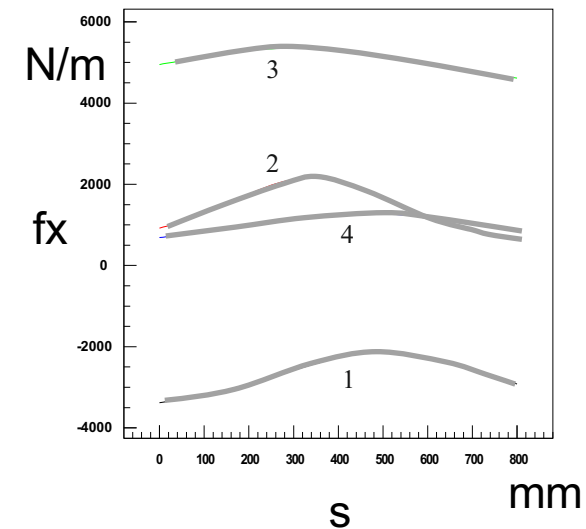
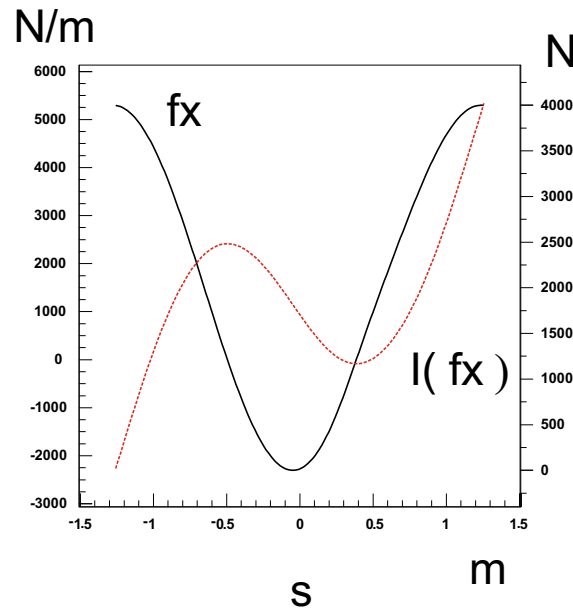
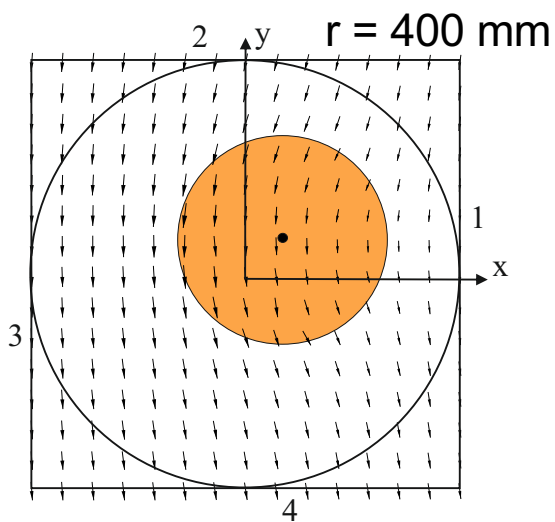
$$(\operatorname{curl} \mathbf{E}) \times \mathbf{D} = \epsilon_0 (\mathbf{E} \cdot \operatorname{grad}) \mathbf{E} - \frac{1}{2} \epsilon_0 \operatorname{grad} (E^2),$$

$$\mathbf{F} = \int_{\mathcal{V}} \left( (\cancel{\operatorname{div} \mathbf{D}}) \mathbf{E} + \epsilon_0 (\cancel{\mathbf{E} \cdot \operatorname{grad}}) \mathbf{E} + (\cancel{\operatorname{div} \mathbf{B}}) \mathbf{H} + \frac{1}{\mu_0} (\mathbf{B} \cdot \operatorname{grad}) \mathbf{B} - \frac{1}{2} \operatorname{grad} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) - \frac{1}{c^2} \frac{\partial}{\partial t} (\cancel{\mathbf{E} \times \mathbf{H}}) \right) dV.$$

$$\mathbf{T}_m = (T_{ij}) := \begin{pmatrix} B_1^2 - \frac{1}{2} B^2 & B_1 B_2 & B_1 B_3 \\ B_2 B_1 & B_2^2 - \frac{1}{2} B^2 & B_2 B_3 \\ B_3 B_1 & B_3 B_2 & B_3^2 - \frac{1}{2} B^2 \end{pmatrix}$$

# Testcase: Conductor in homogeneous field

$$\mathbf{F}_m = \int_{\partial\mathcal{V}} \left( \left( \frac{1}{2\mu_0} B_n^2 - \frac{\mu_0}{2} H_t^2 \right) \mathbf{n} + B_n H_t \mathbf{t} \right) da.$$



1: -2107  
 2: 1176  
 3: 4075  
 4: 856  
 Σ: 4000

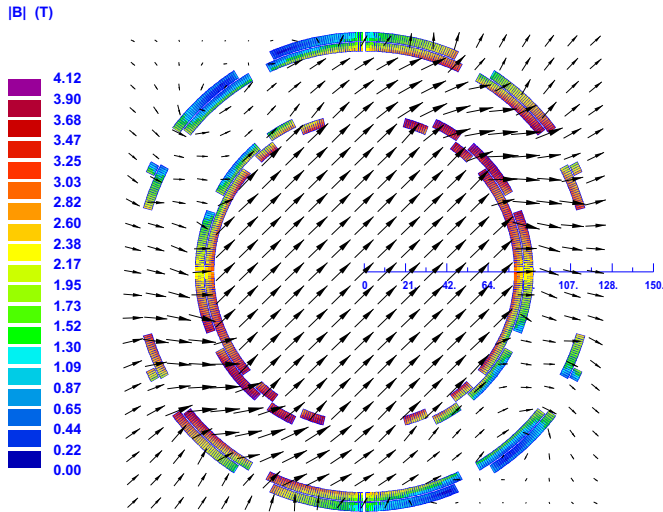
40000 A in 0.1 T dipole field  
 = 4000 N per meter length

Along the circle of 400 mm

Along the sides of the box

# MCBXF

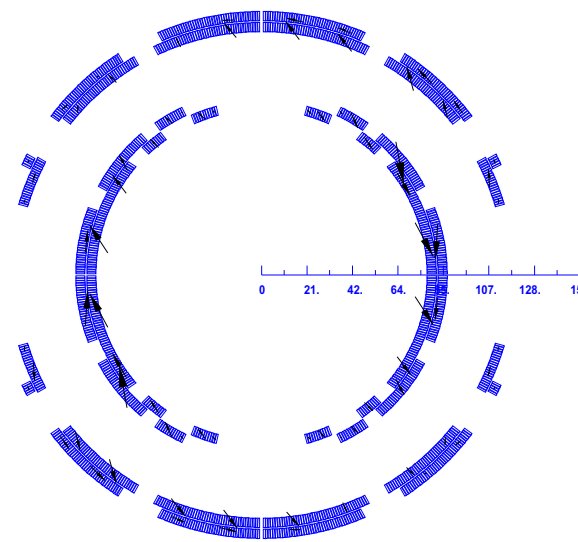
R = 100 mm



Nominal current inner 1584  
outer 1402

B1 = 2.177 T  
A1 = 2.254 T

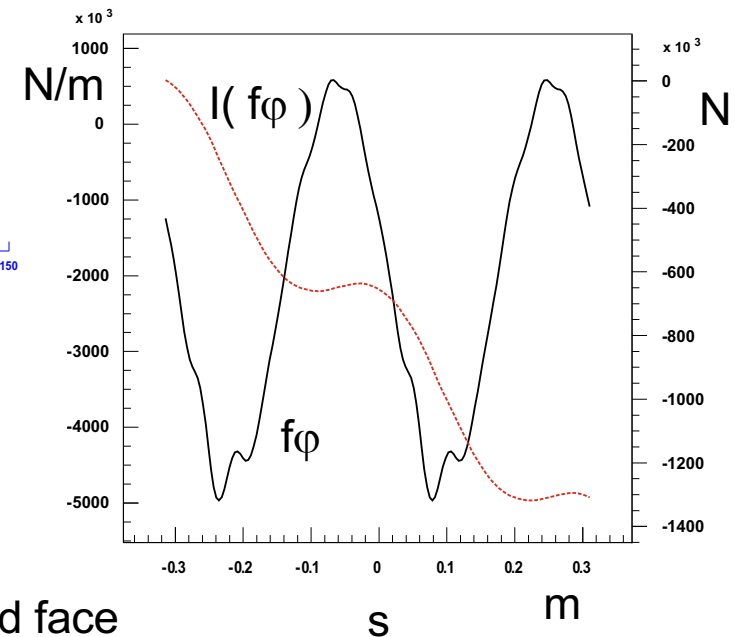
Max LL: 49.84 %



Sum of forces normal to broad face

-  $1.621 \cdot 10^6$  N per meter length  
= - 162.2 kNm torque per meter

Integral:  $-1.311 \cdot 10^6$  N



Torque per m length  
-131.57 kNm per meter



## Results when using $T = m \times B$

$$\mathbf{m} = I \mathbf{a},$$

$n$	$K_n^{\text{rad}}$	$K_n^{\text{tan}}$
1	$N L w$	$N L w$
2	$N L w r$	$N L w r_c \cos(\frac{\delta}{2})$
3	$N L w (\frac{w^2}{12} + r^2)$	$\frac{1}{3} N L w r_c^2 (1 + 2 \cos(\delta))$
4	$N L w r (\frac{w^2}{4} + r^2)$	$N L w r_c^3 \cos(\delta) \cos(\frac{\delta}{2})$
5	$N L w (\frac{w^4}{80} + \frac{w^2 r^2}{2} + r^4)$	$\frac{1}{5} N L w r_c^4 (1 + 2 \cos(\delta) + 2 \cos(2\delta))$
6	$N L w r (\frac{w^4}{16} + \frac{5w^2 r^2}{6} + r^4)$	$\frac{1}{3} N L w r_c^5 (4 \cos^2(\delta) - 1) \cos(\frac{\delta}{2})$

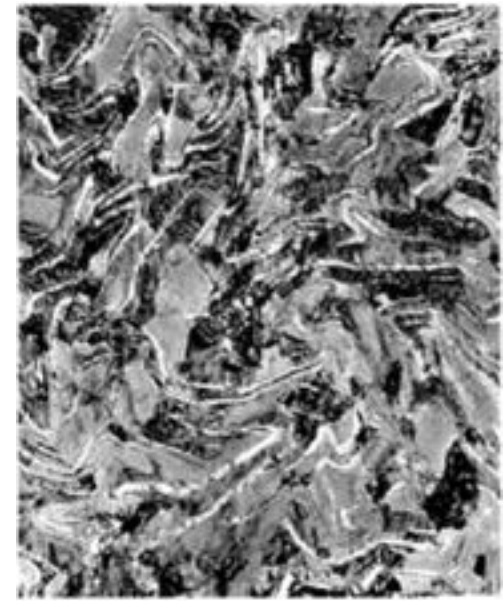
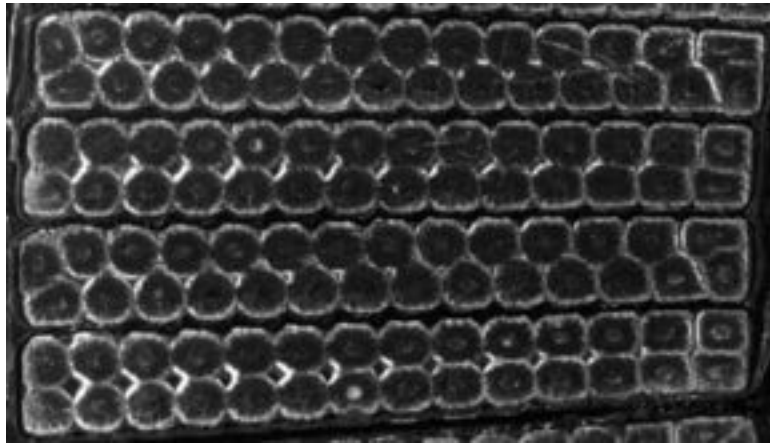
In ROXIE: use the routine for the K-values of search coils.  $K_1 = a$

The magnetic moment of the inner coil of MCBXF at nominal excitation of 1584 A is 58668 Am<sup>2</sup> per meter length. The outer coil creates a homogeneous dipole field of 2.254 T, which yields 132.23 kNm per meter when m and B are at a right angle. I1 = current in inner coil, I2 current in outer coil

$$\{T\}_N = \{I1\}_A * 37.04 \times \{I2\}_A * 2.254/1402 = \{I1\} \times \{I2\} * 0.059546$$



# Rutherford (Roebel) Kabel, Strand, Nb-Ti Filament



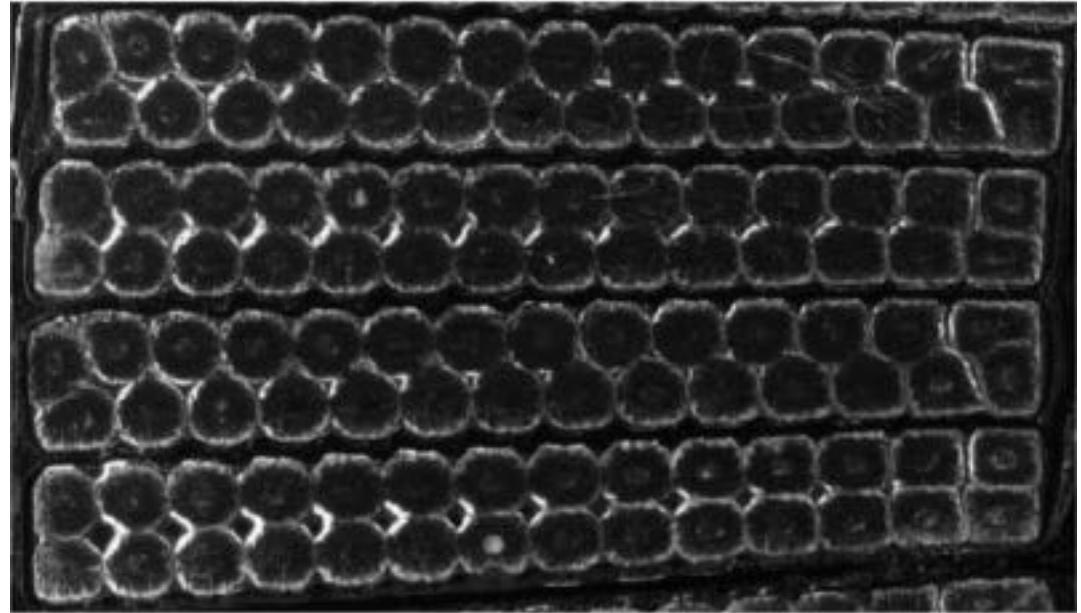
200 nm 

# Coil Block, Pinning Centers in Filaments

## Magnetization



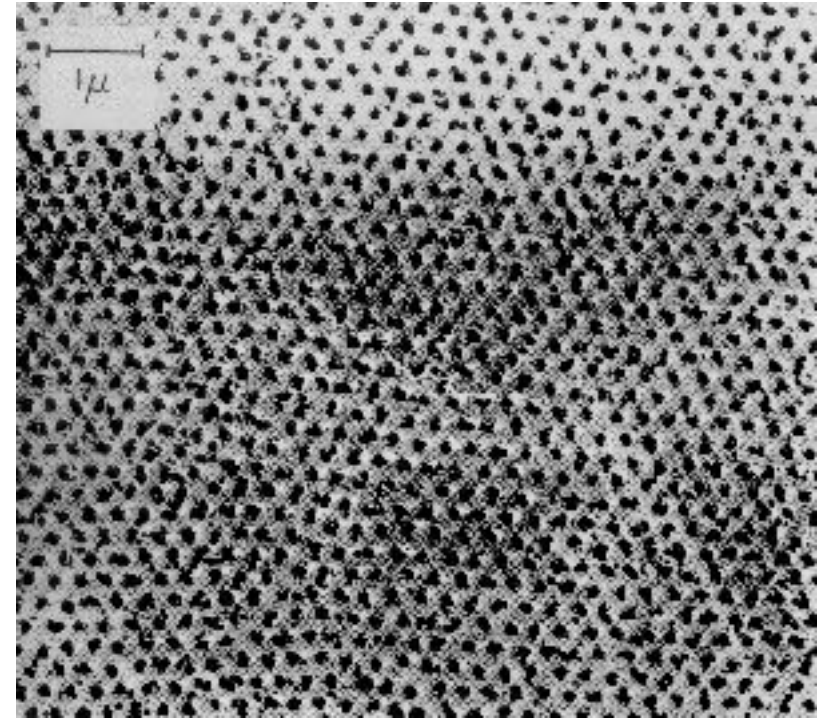
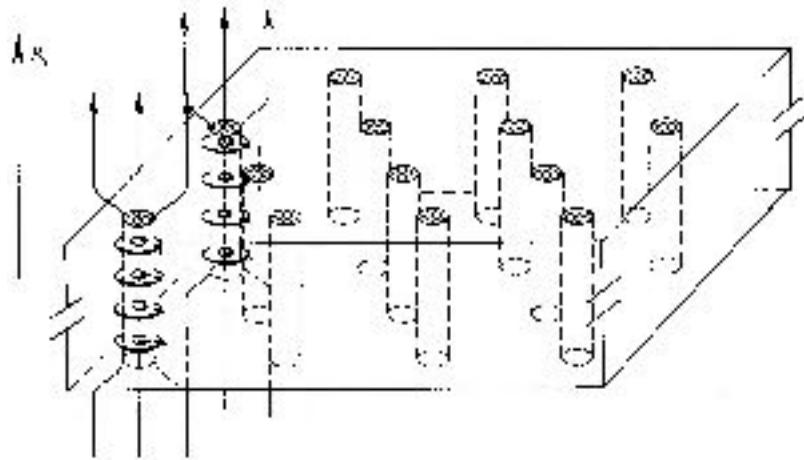
200 nm 



Grading of current density

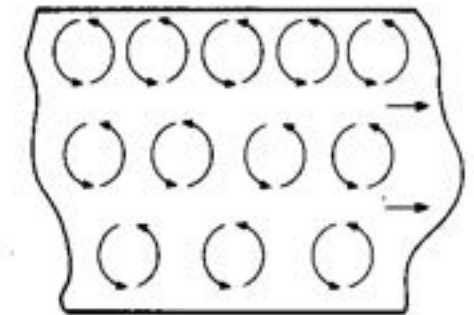
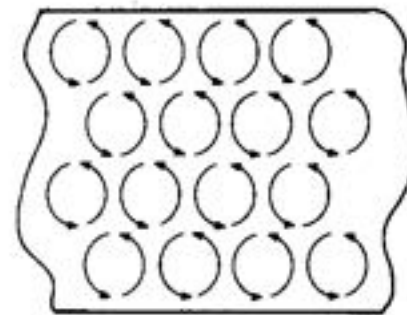


# Type II Superconductors and Fluxoids



200 nm

$$\text{curl } \mathbf{H} = \frac{\partial H_y}{\partial x} \mathbf{e}_z = J_z \mathbf{e}_z = \mathbf{J}_c$$



# Superconductor Properties

## → Hard Superconductors (Type 2)

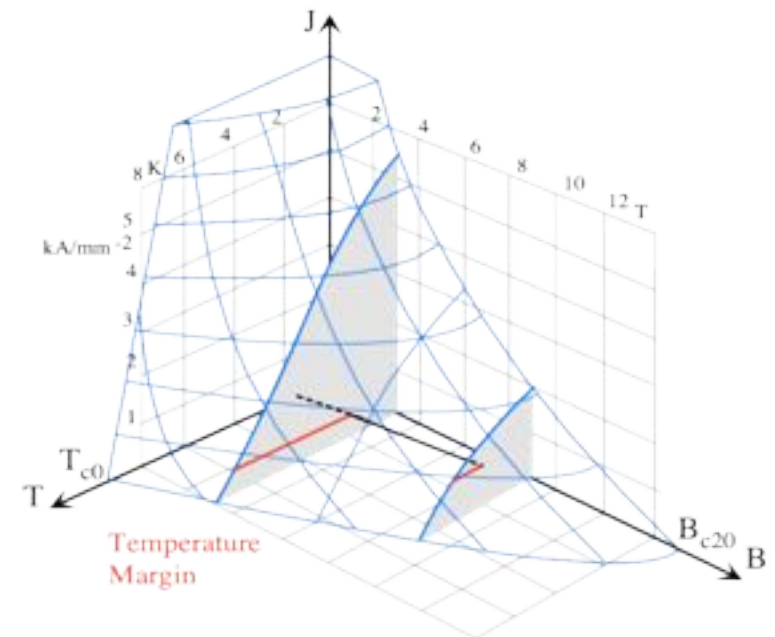
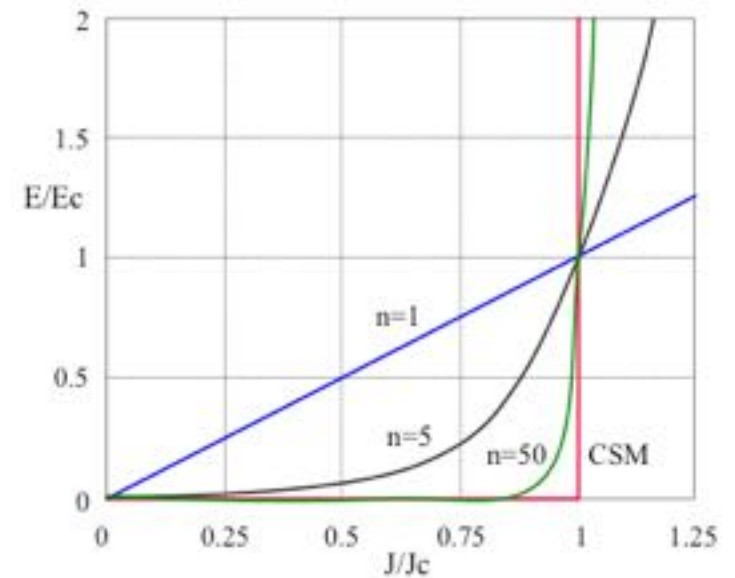
- Magnetic field can penetrate
- Magnetization with hysteresis

## → Critical current density $J_c$

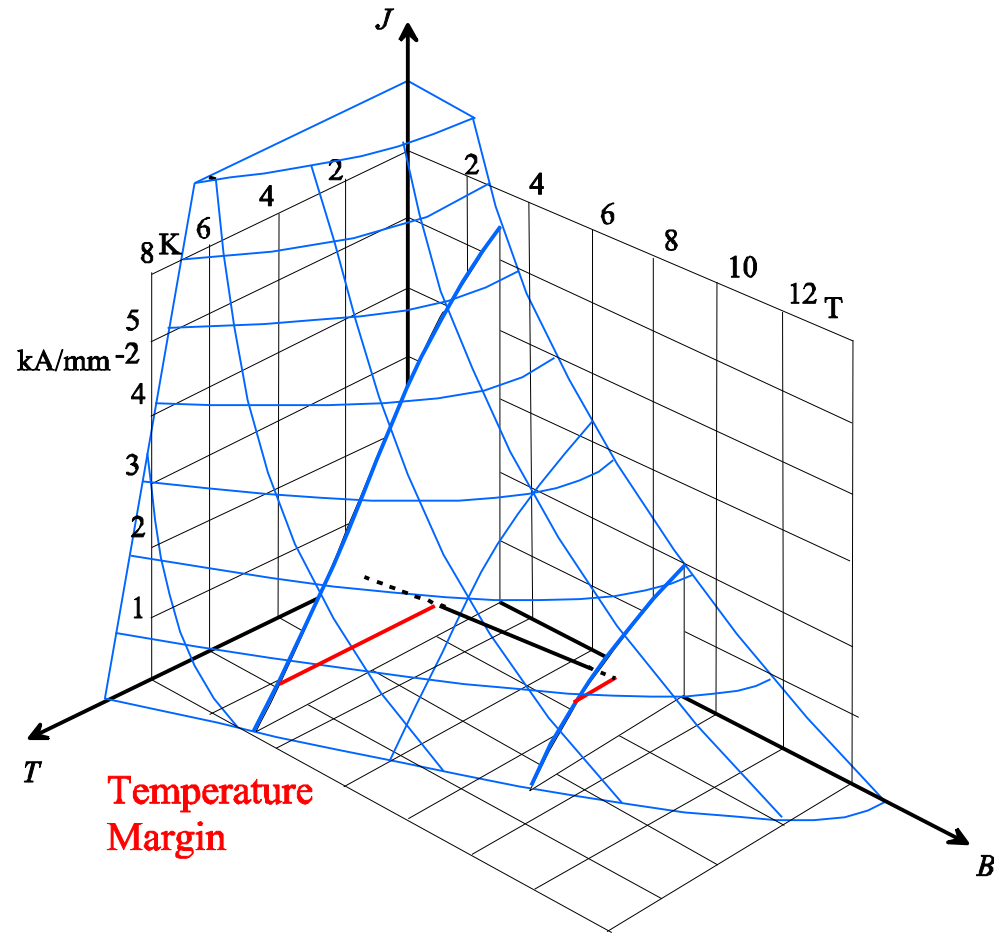
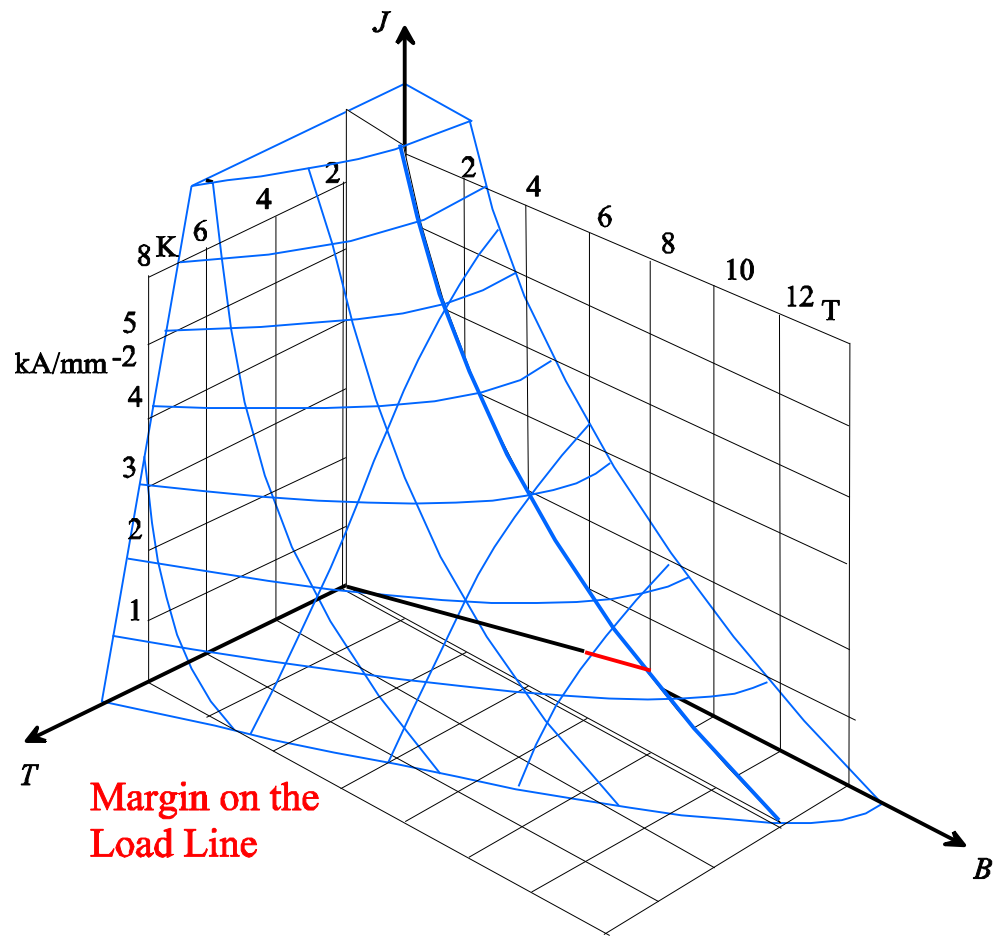
- Current density at spec. electric field ( $E_c = 1 \mu\text{V}/\text{cm}$ )

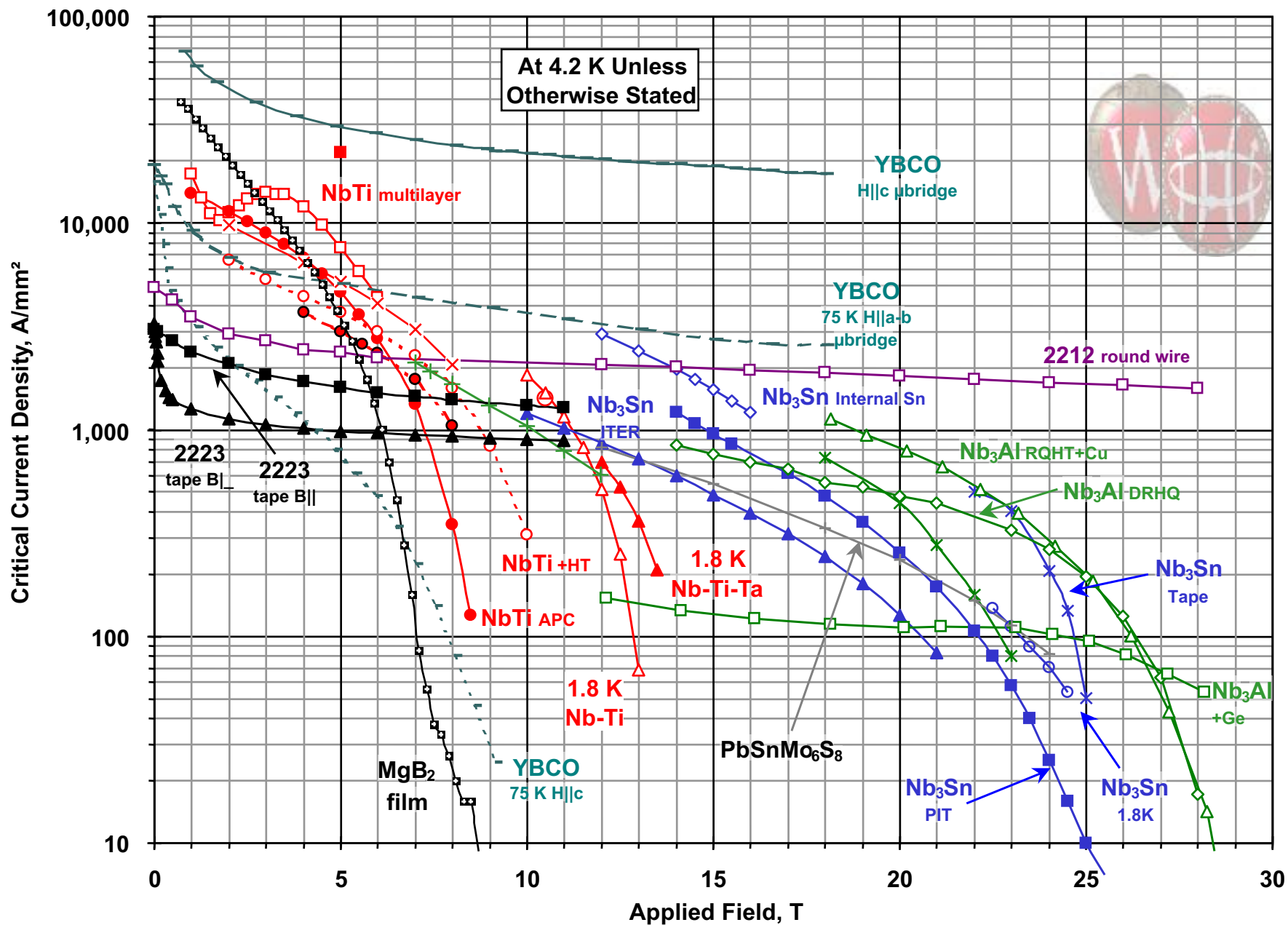
## → Critical surface

- Dependence of  $J_c$  on  $T$  and  $B$

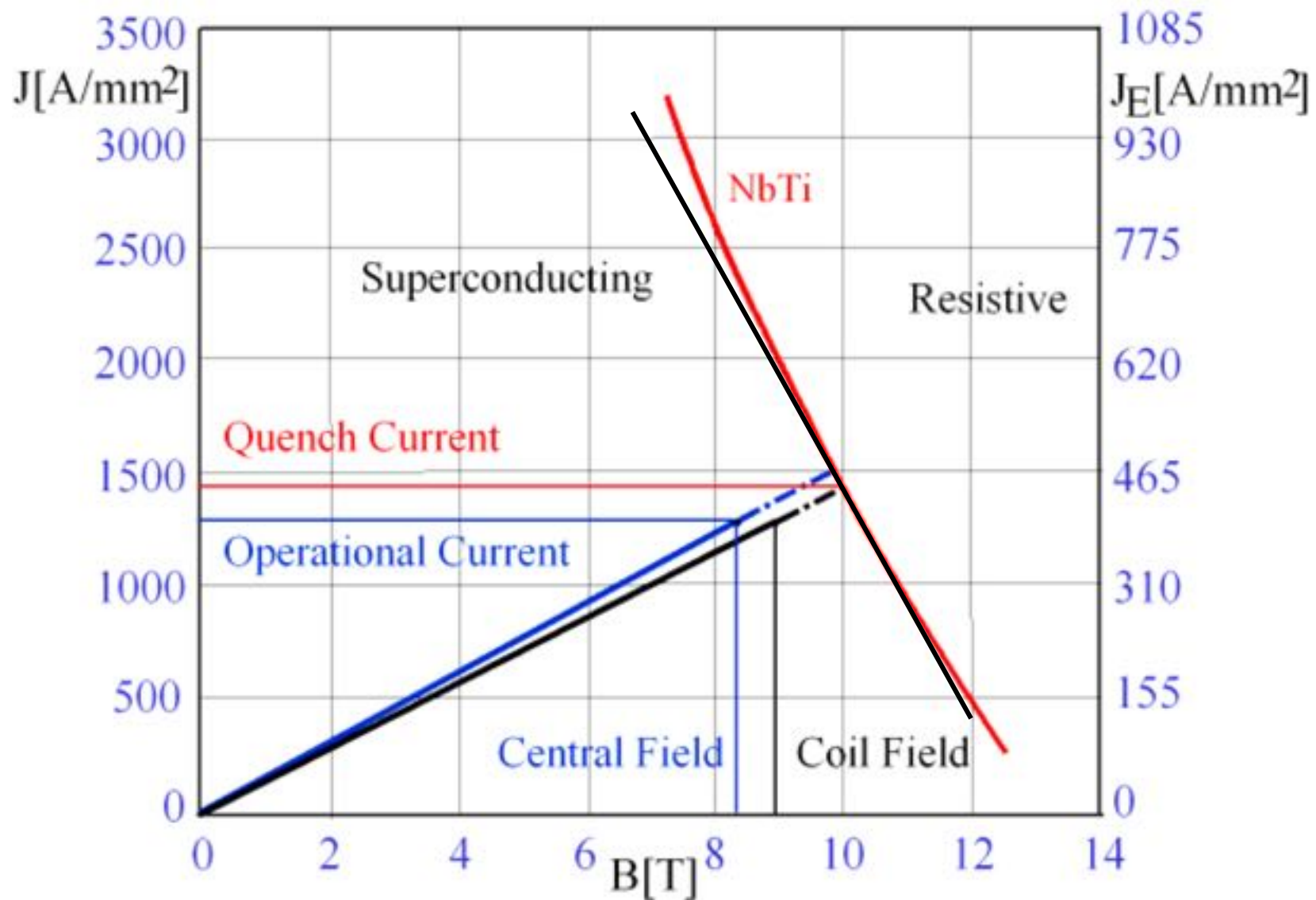


# Critical Surface of NbTi

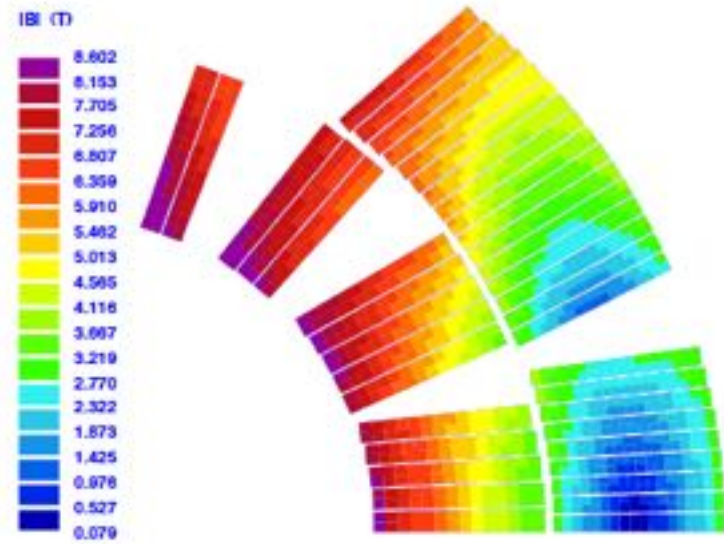
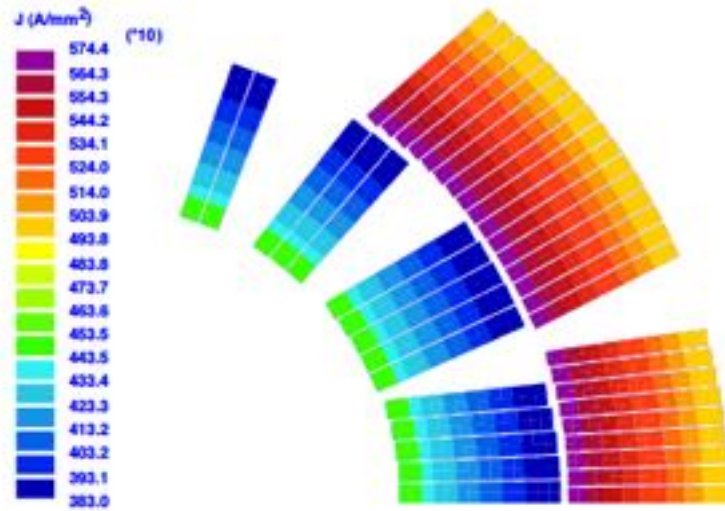
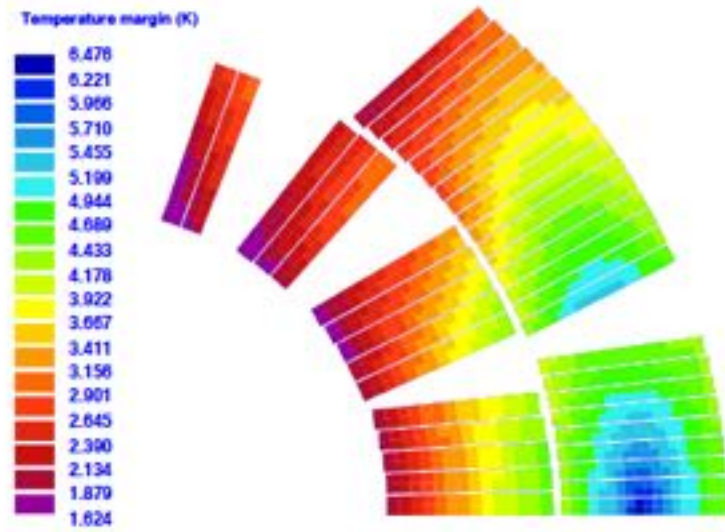
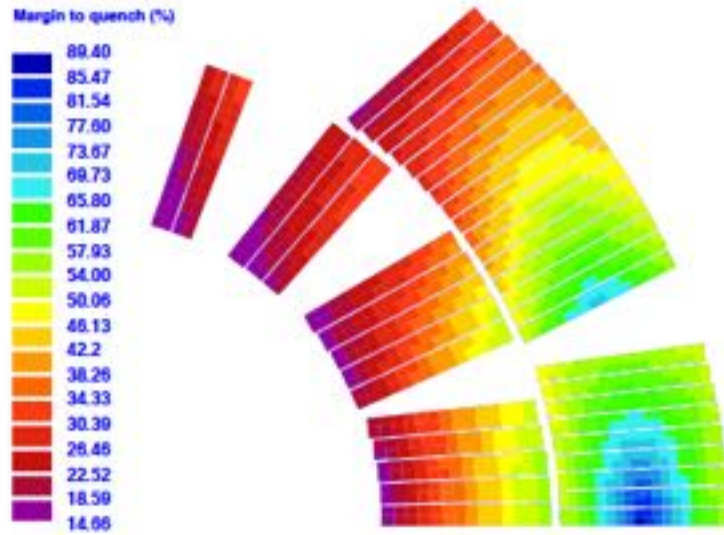




# Working Point of LHC Dipole Magnets

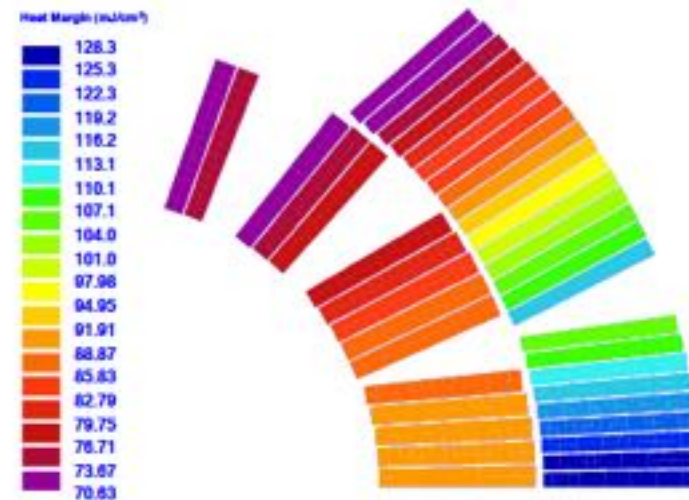
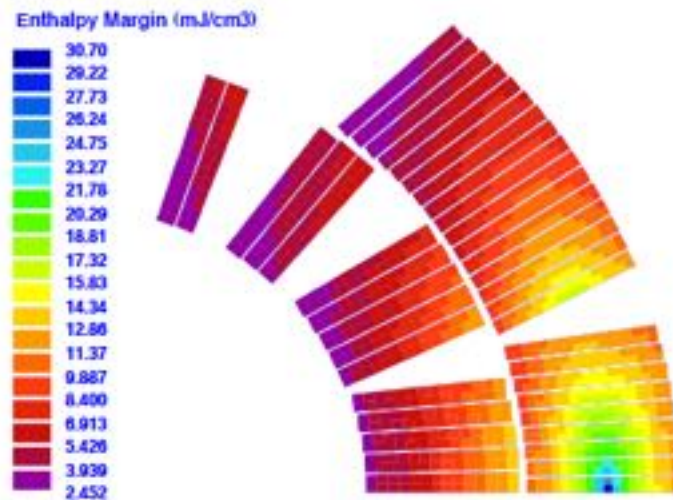


# Margins



# Margins

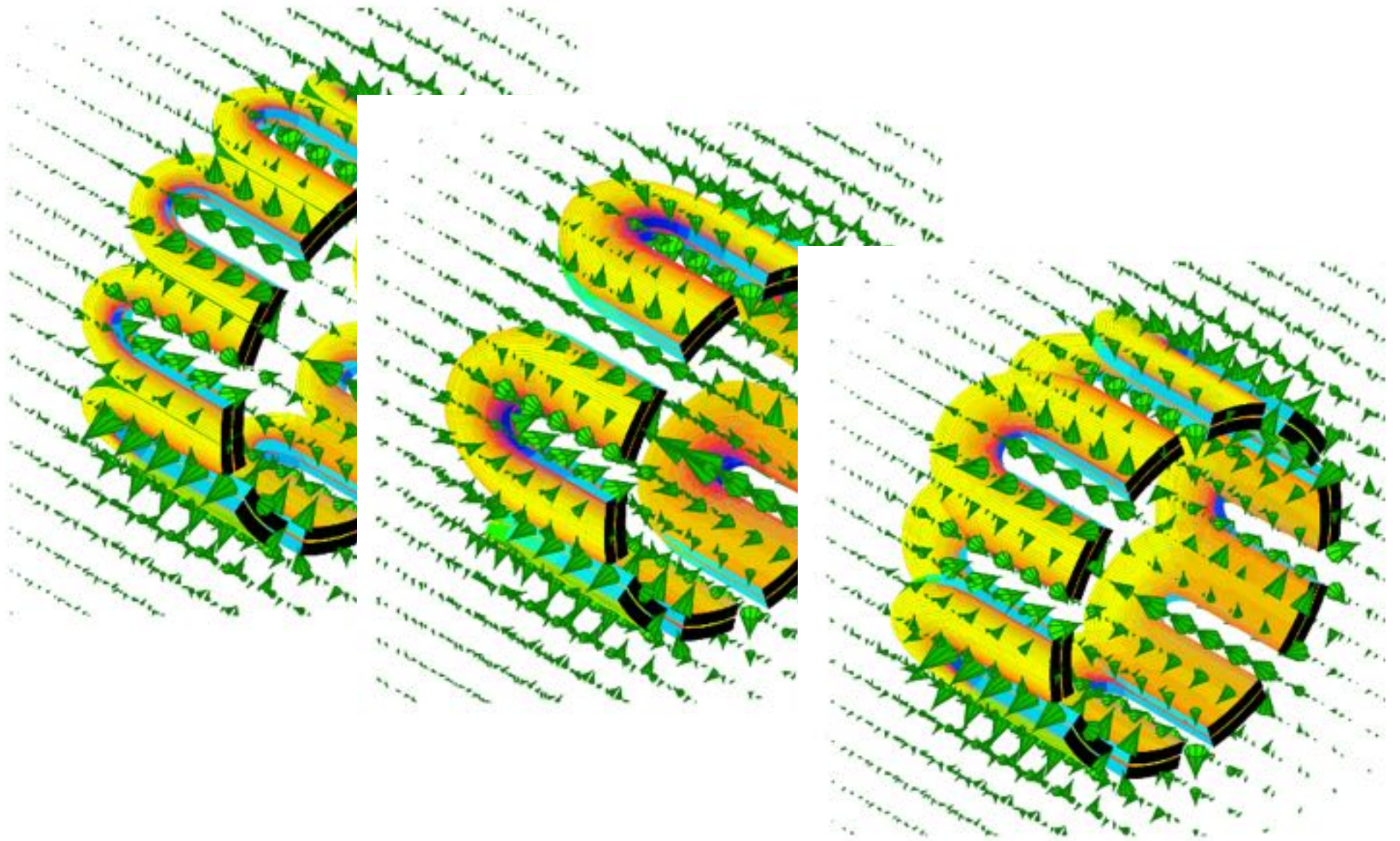
$$\Delta h := \int_{T_b}^{T_c(J,B)} \rho c_p(T) dT,$$



Enthalpy reserve  
Heat capacity of the  
copper/SC strand until  
quench

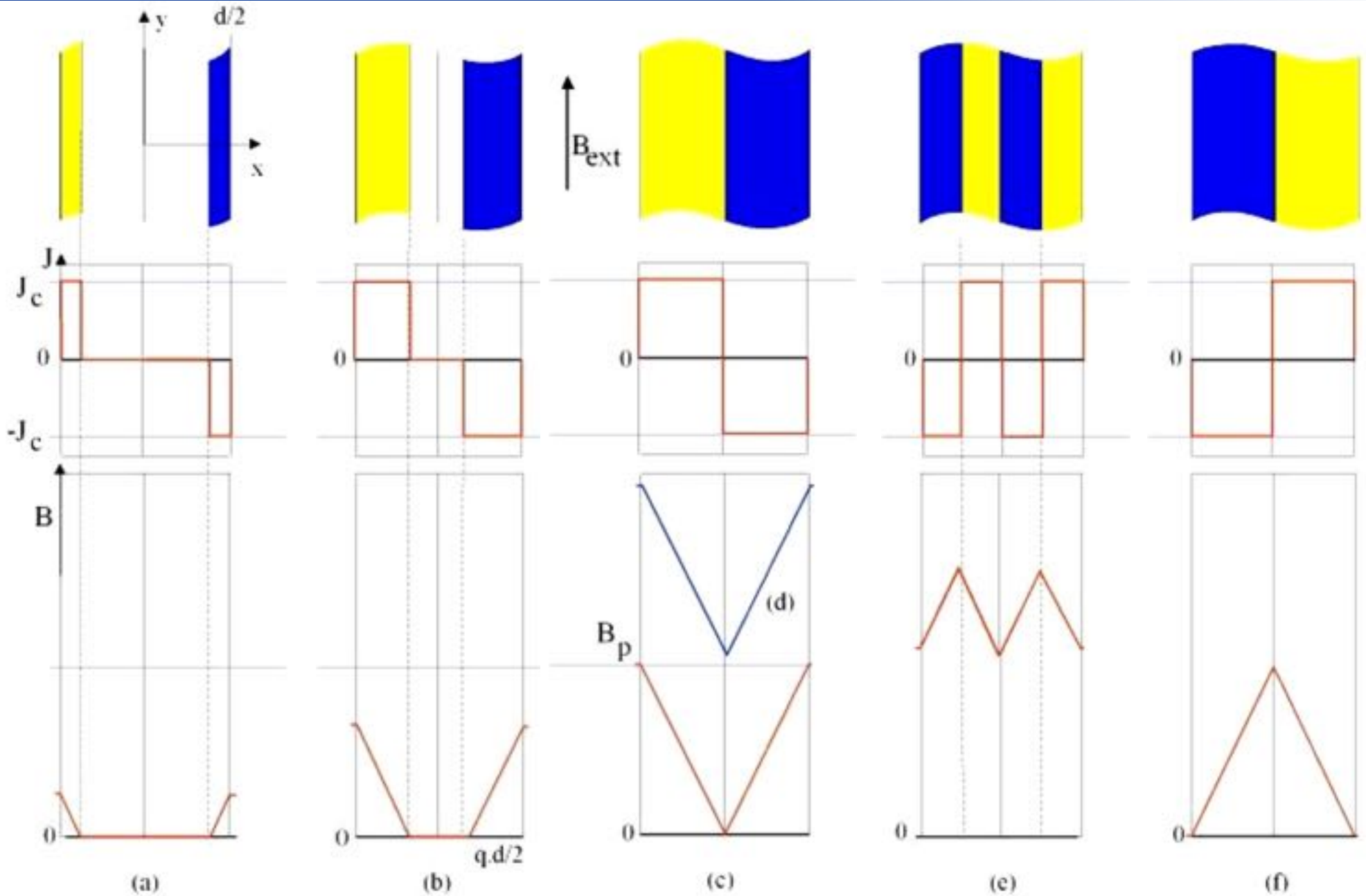
Average heat reserve (copper, SC and  
helium) in the cable (slow losses). No  
heat transfer.

# Margins

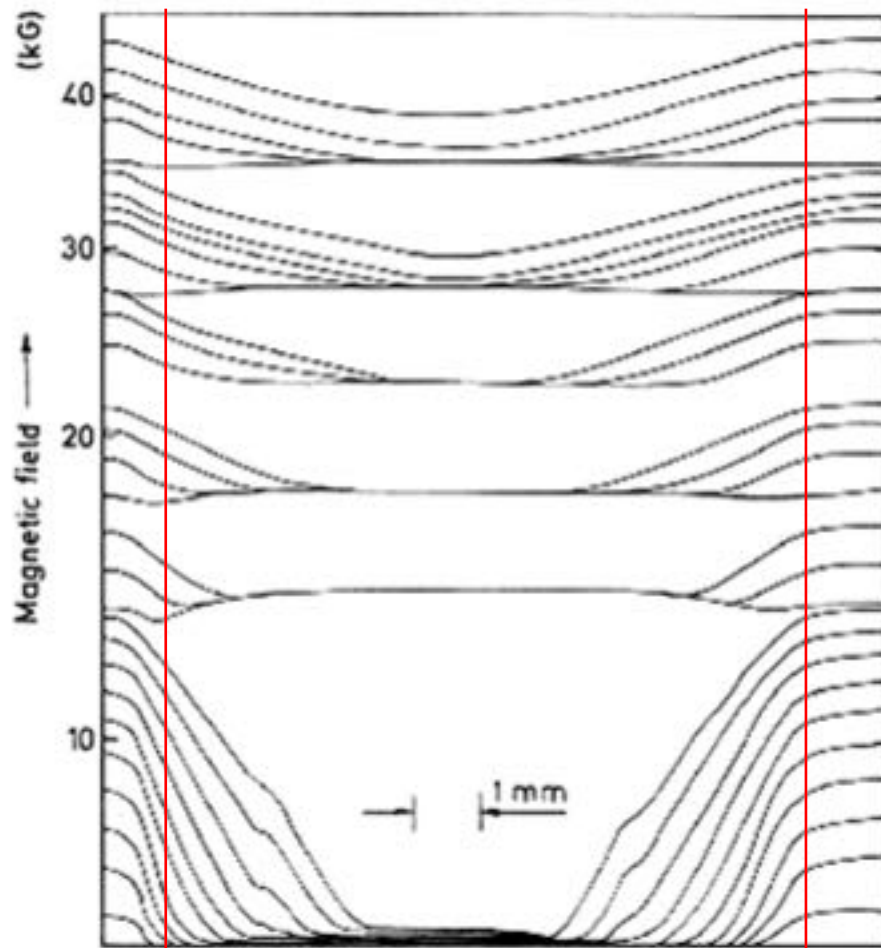




# Bean's Critical State Model (CSM)



# Screening Field in a Slab



$q = 1$  center of filament  
 $t$  = modulus of shielding field

$$q^* := \frac{B_{\text{ext}}}{\mu_0 J_c} \frac{2}{d}$$

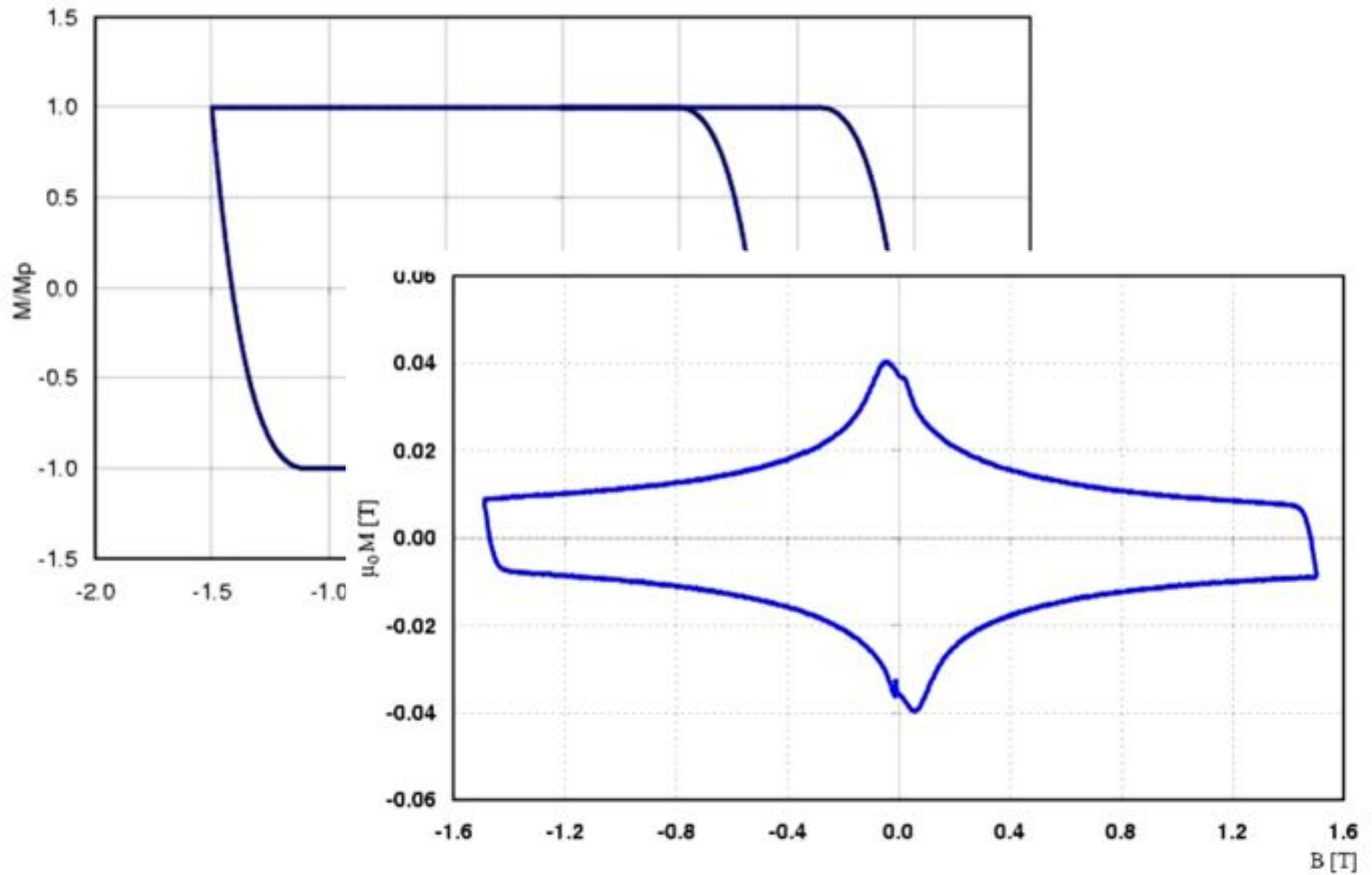
$$B_p = \mu_0 J_c \frac{d}{2}$$

$$M = \int_{\frac{d}{2}(1-q^*)}^{\frac{d}{2}} -\frac{J_c 2x}{d} dx$$

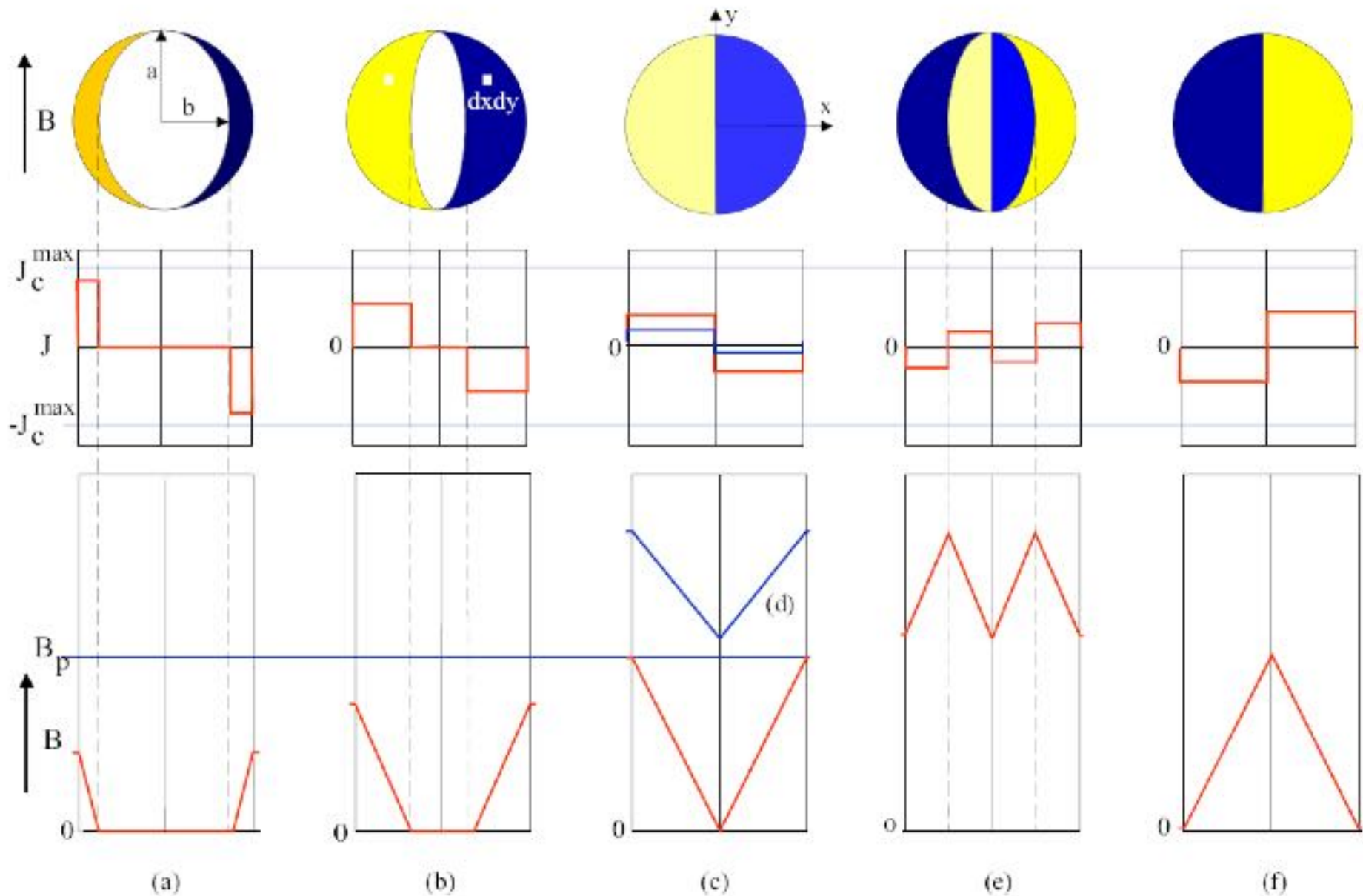
$$M_p = -J_c \frac{d}{4}$$

$$B_y = B_{\text{ext}} - t = B_{\text{ext}} - \mu_0 J_c \frac{d}{2} q$$

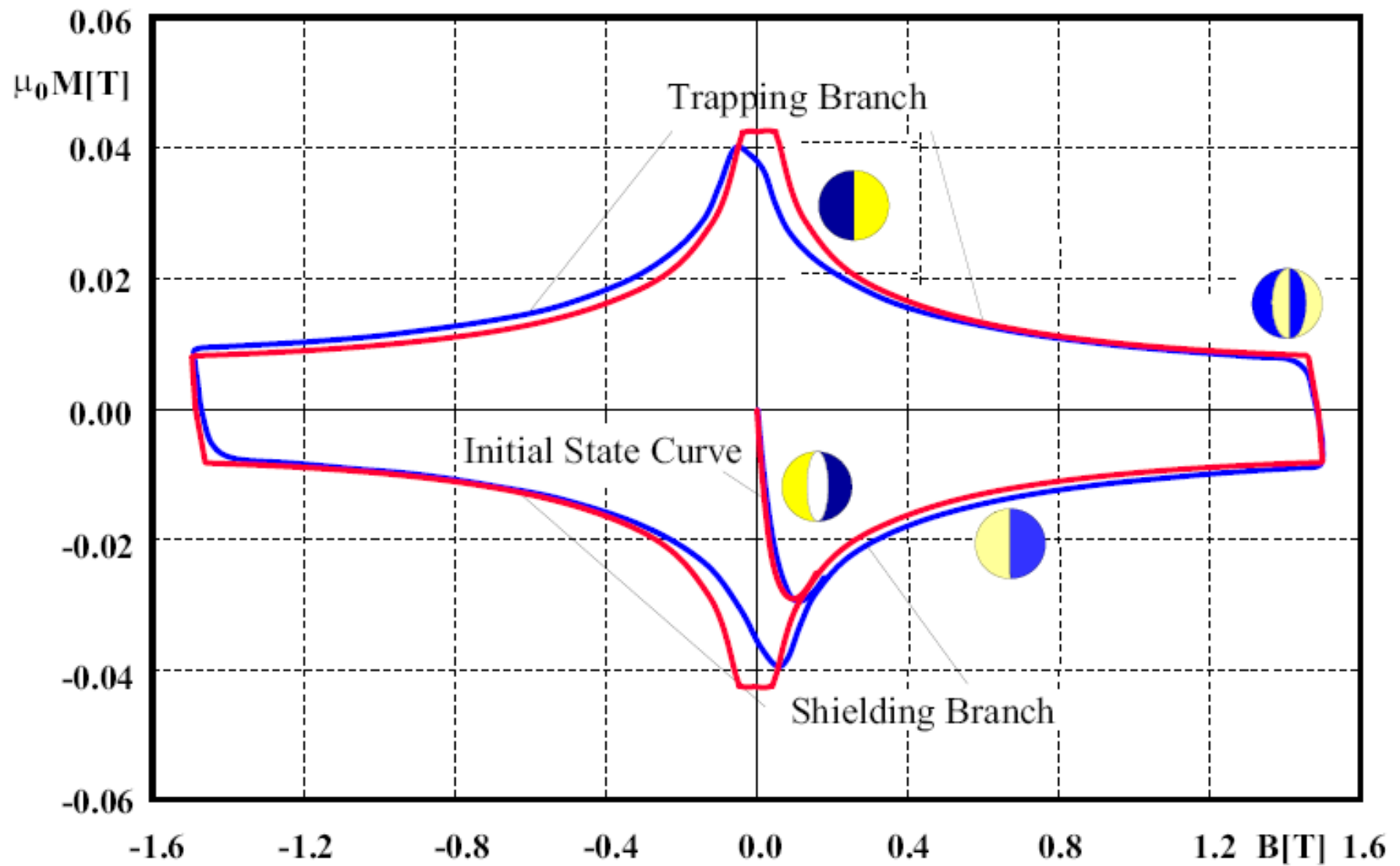
# Magnetization in SC Slab, Measured in LHC Strands



# The Bean, Wilson Model



# Bean-Wilson Model



# Screening Field of Intersecting Circles

$$a_i = U - (2i - 1) \frac{c}{2}$$

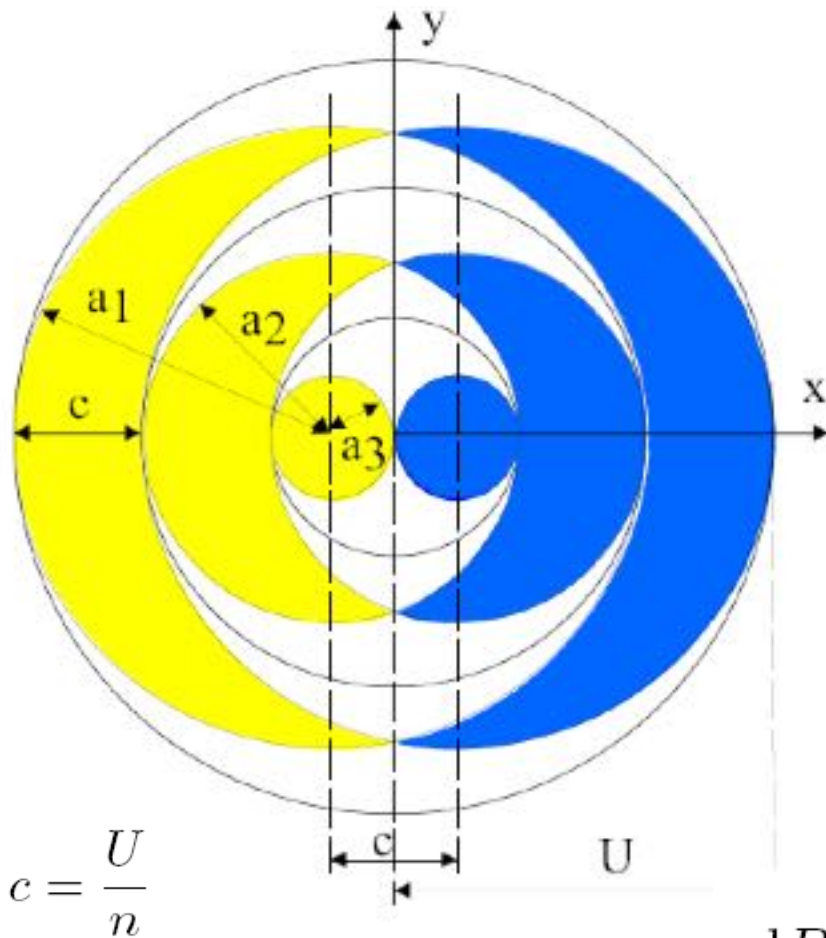
$$J_c(B, T) = \frac{J_c^{\text{ref}} C_0 B^{\alpha-1}}{(B_{c2})^\alpha} \left(1 - \frac{B}{B_{c2}}\right)^\beta \left(1 - \left(\frac{T}{T_{c0}}\right)^{1.7}\right)^\gamma$$

Approximation for small fields and constant temperature

$$J_c(B) \approx \frac{\mathcal{F}(B_{\text{ext}})}{\sqrt{B}} := J_c(B_{\text{ext}}) \frac{\sqrt{B_{\text{ext}}}}{\sqrt{B}}$$

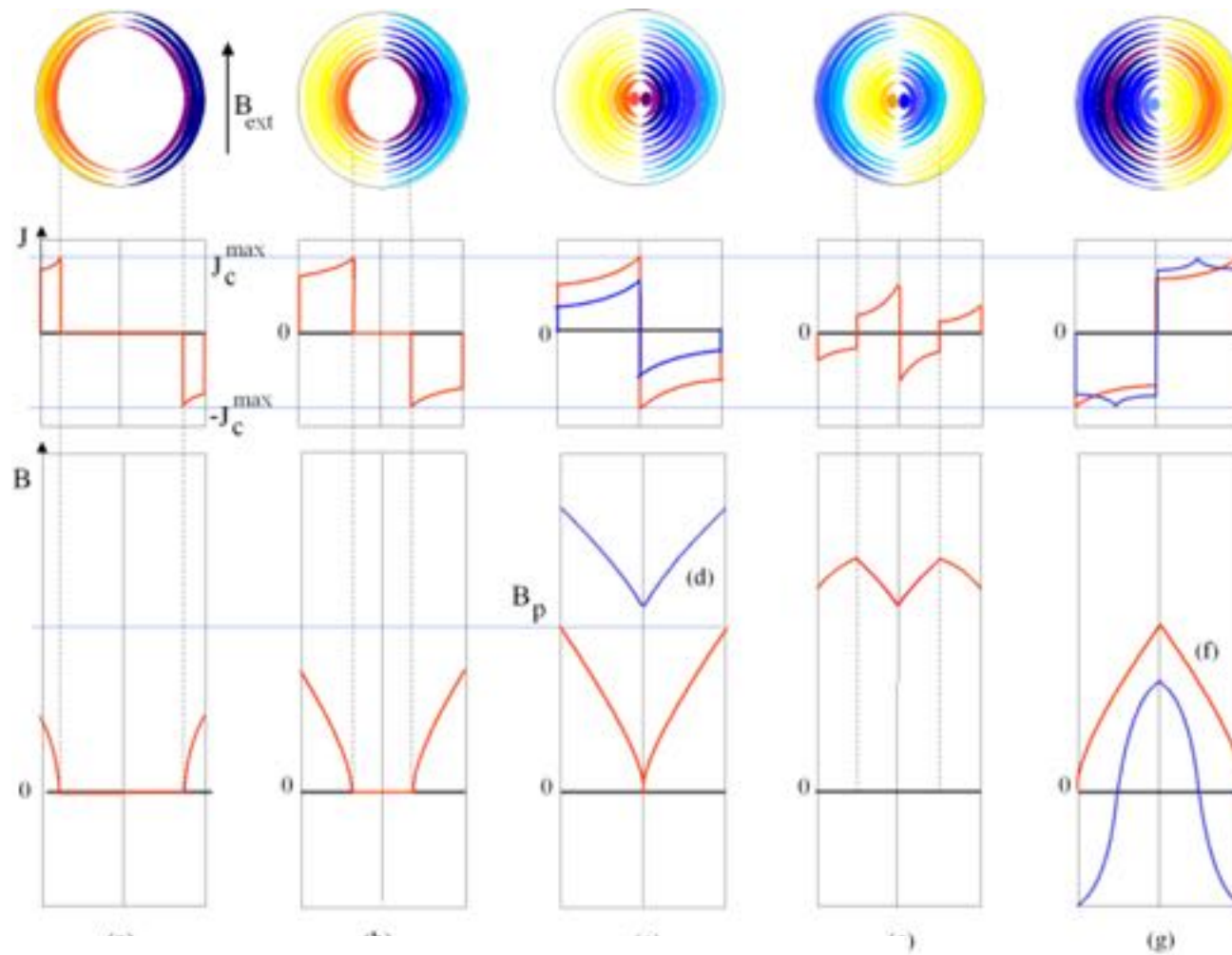
$$|\Delta t| = \frac{\mu_0 r}{2} \int_{q_1}^{q_2} J_c(B(q)) dq$$

$$dB(q) = \xi \mu_0 \mathcal{H} J_c(B(q)) dq = \frac{\xi \mu_0 \mathcal{H} \mathcal{F}(B_{\text{ext}}) dq}{\sqrt{B(q)}}$$



$$c = \frac{U}{n}$$

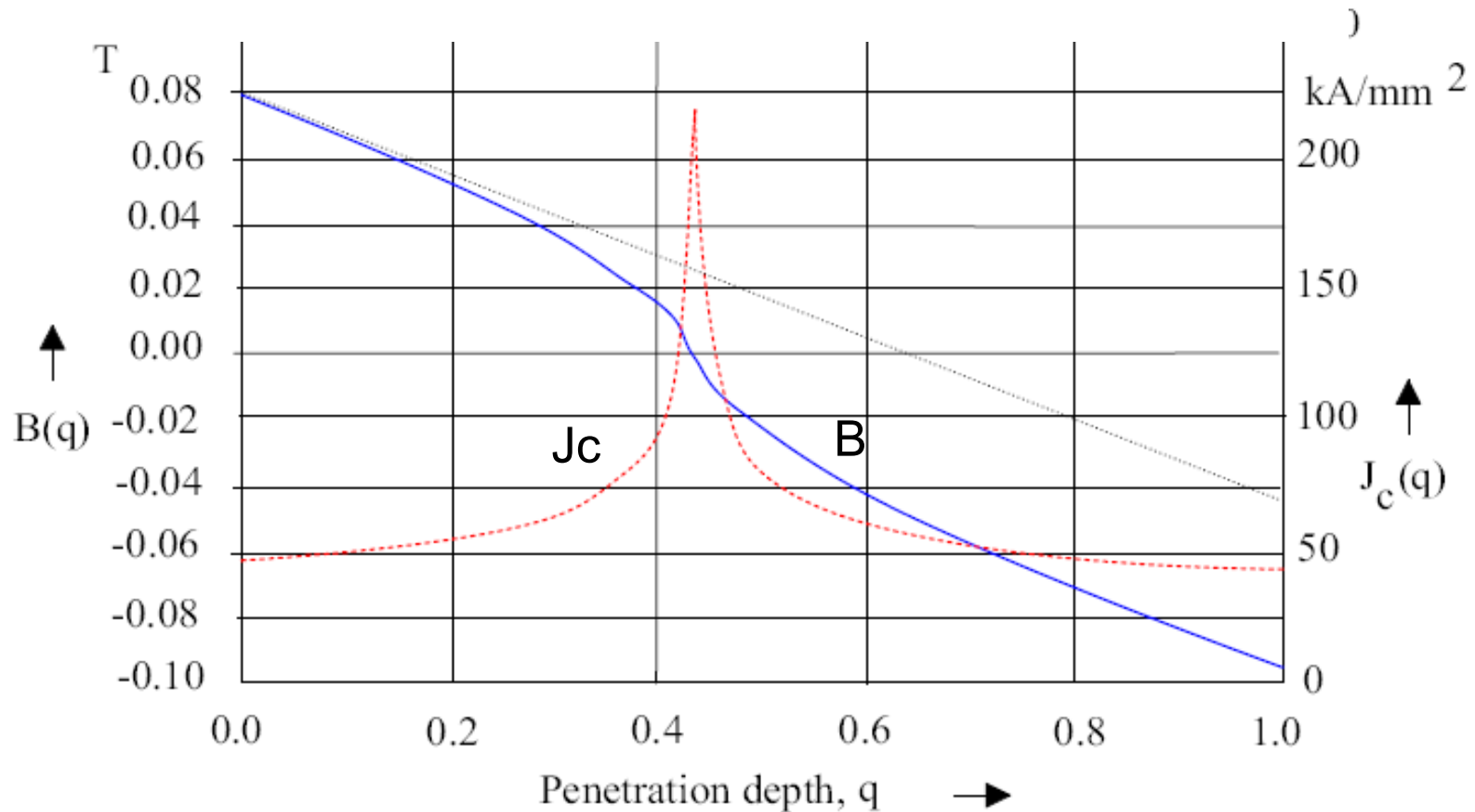
# Modified CSM Model



$$dB(q) = \zeta \mu_0 \frac{r}{2} J_c(B(q)) dq = \frac{\zeta \mu_0 r dq}{2 \sqrt{B(q)}} \mathcal{F}(B_a) \quad \mathcal{F}(B_a) := J_c(B_a) \sqrt{B_a}$$

# B(q)

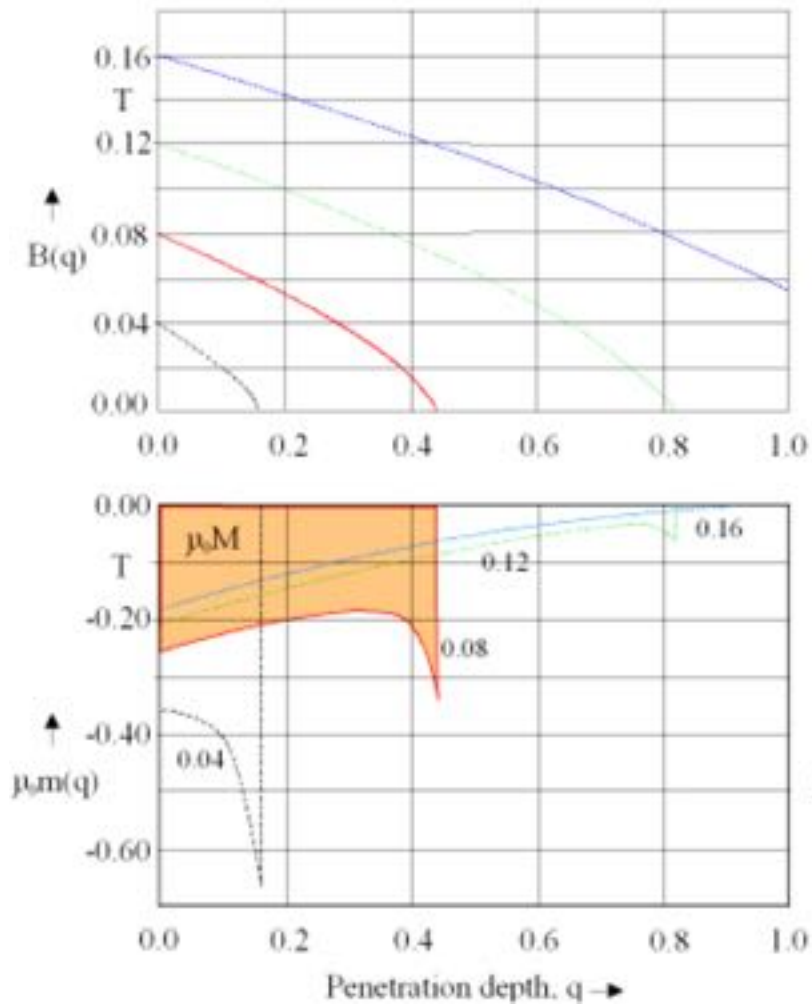
$$B(q) = \left( B_{\text{ext}}^{3/2} + \frac{3}{2} \xi \mathcal{H} \mathcal{F}(B_{\text{ext}}) \mu_0 q \right)^{2/3}$$



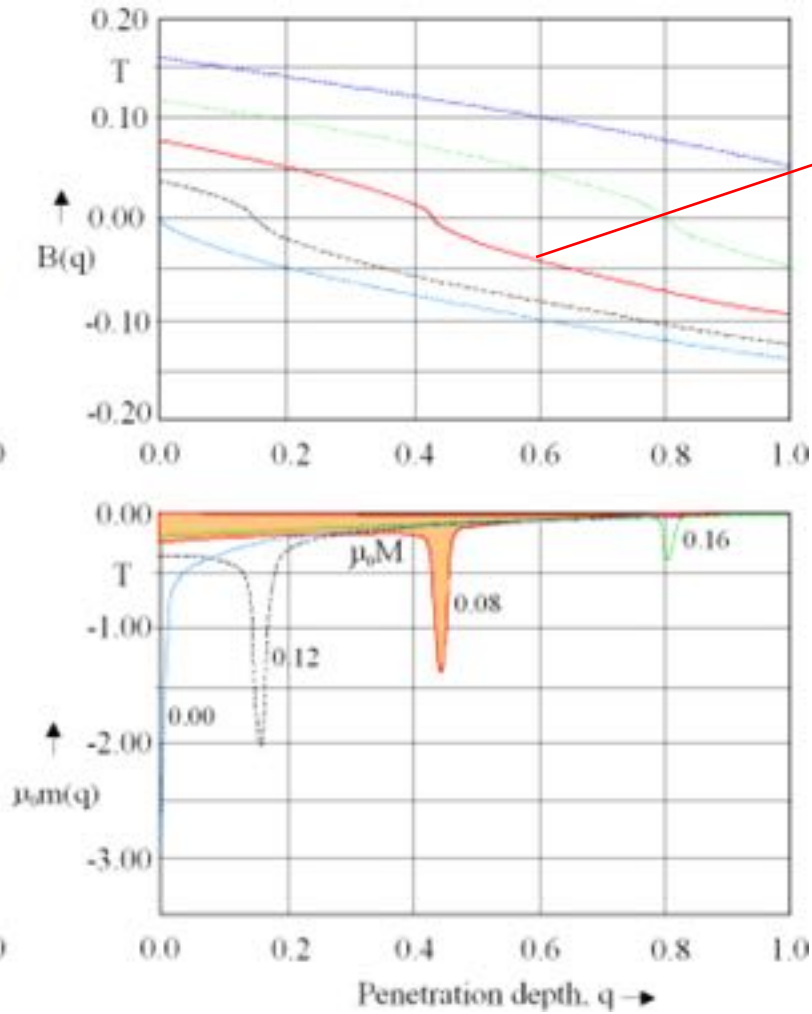
Seq:  $\{ 0, B_{\text{max}}, -B_{\text{max}}, B_a = 0.08 \text{ T} \}$



# Peak Shifting

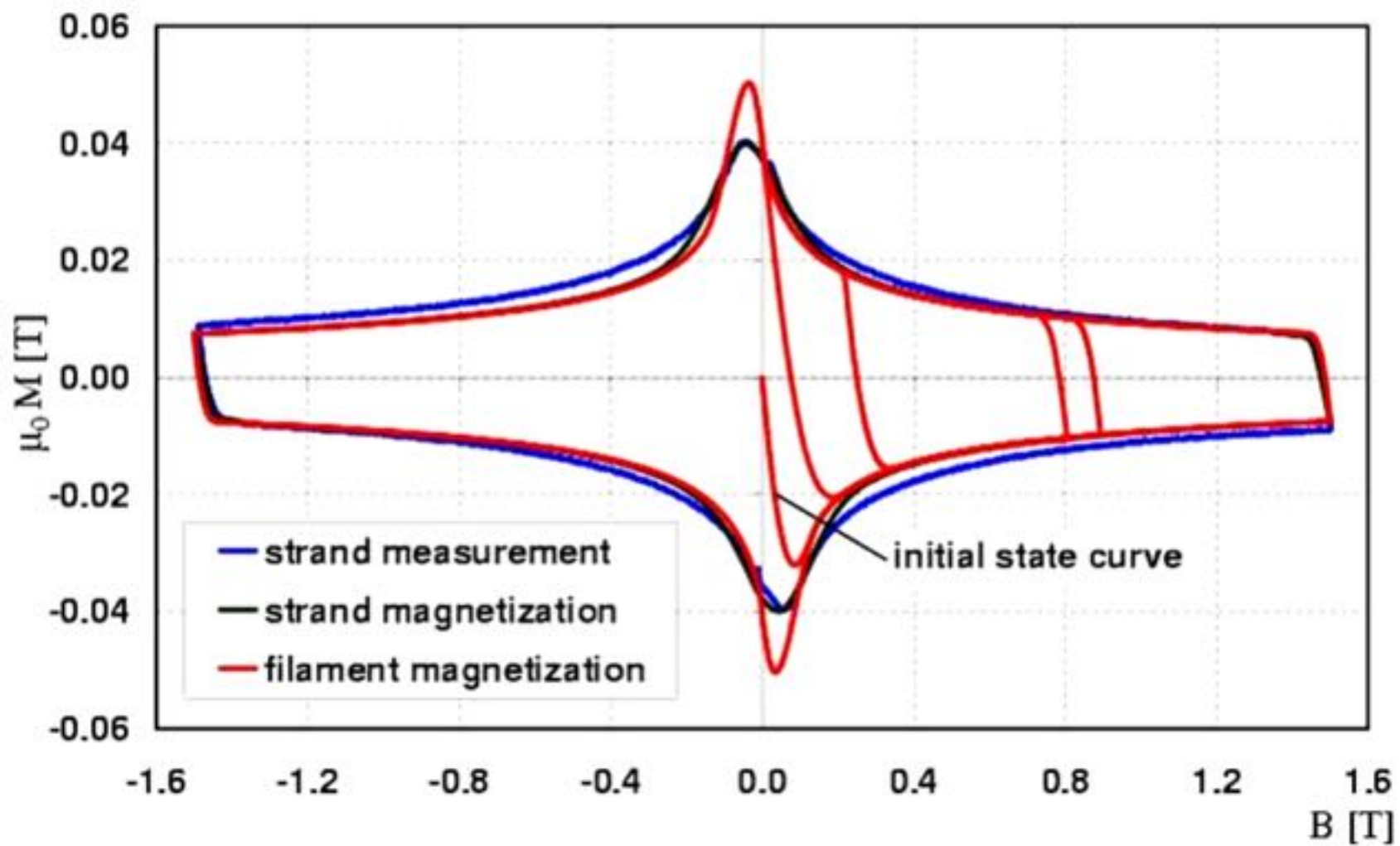


Virgin state

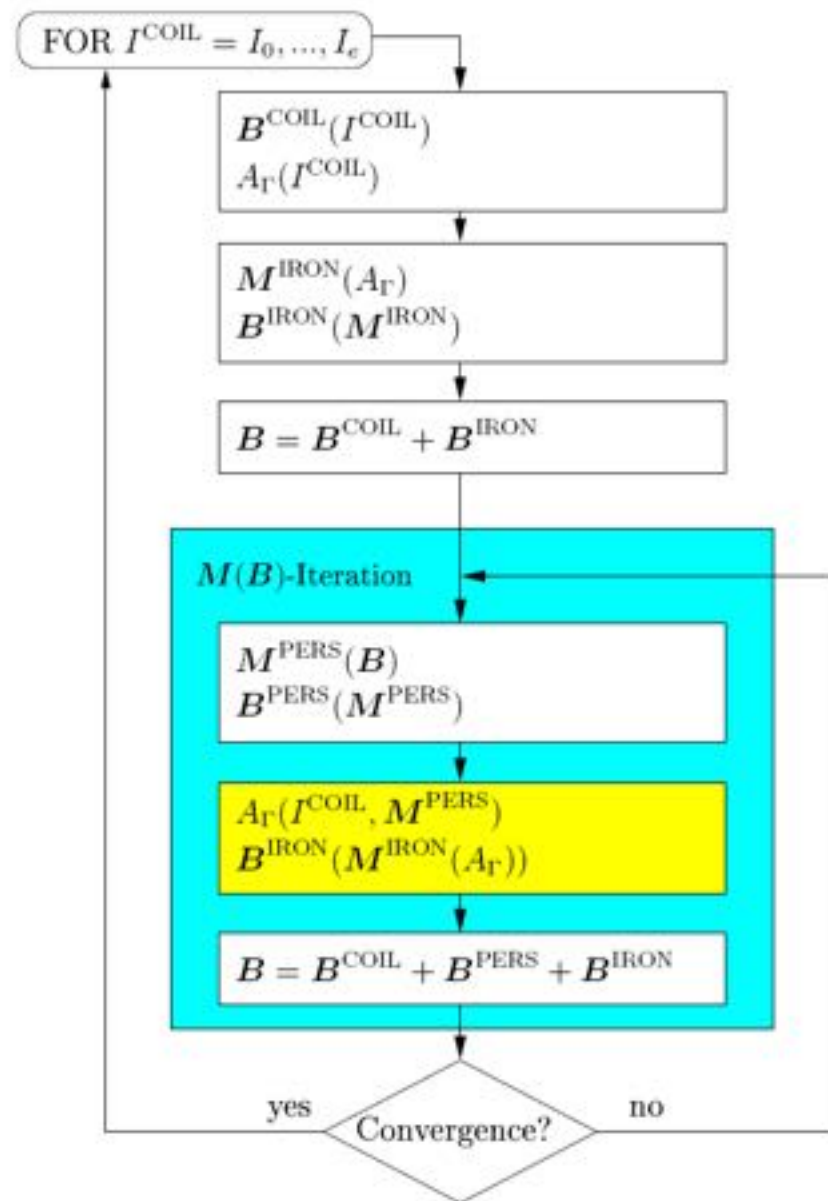


Seq:  $\{ 0, B_{\max}, -B_{\max}, B_a > 0 \}$

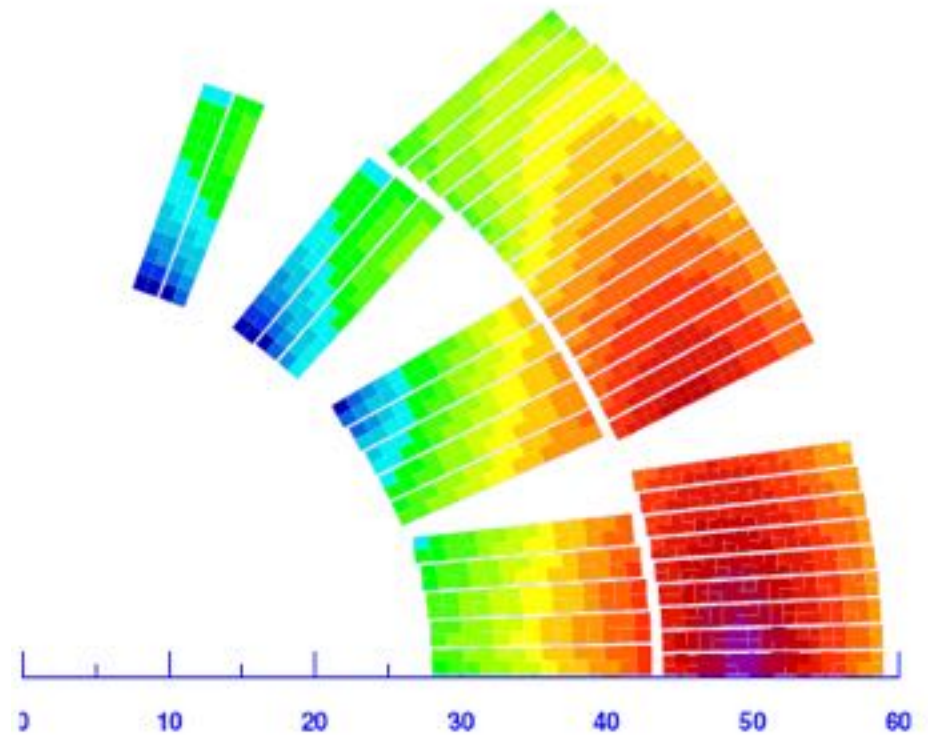
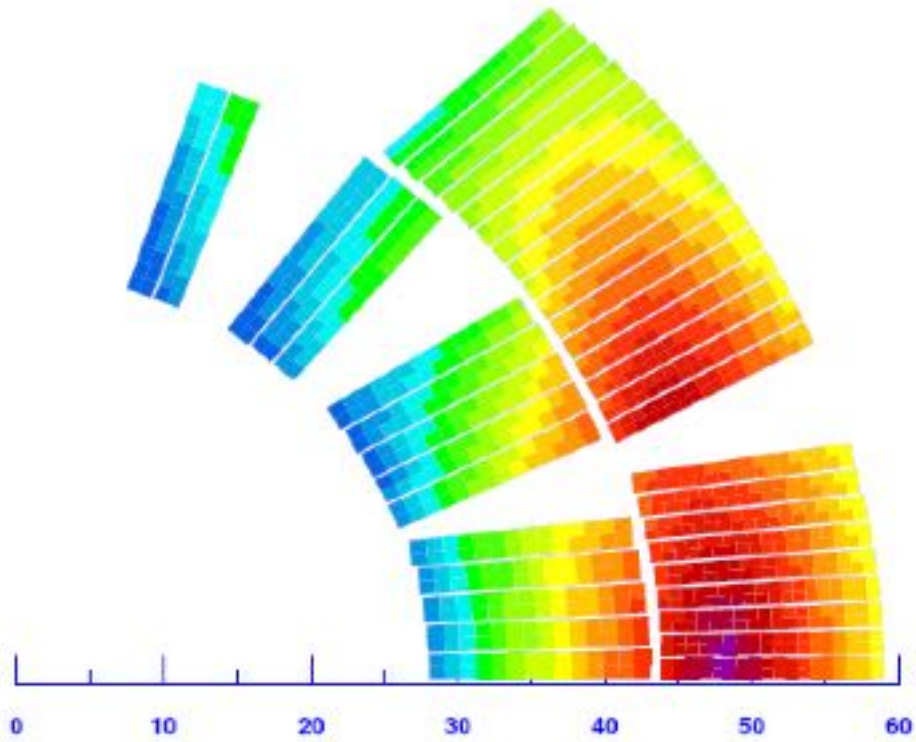
# Magnetization in LHC Strands



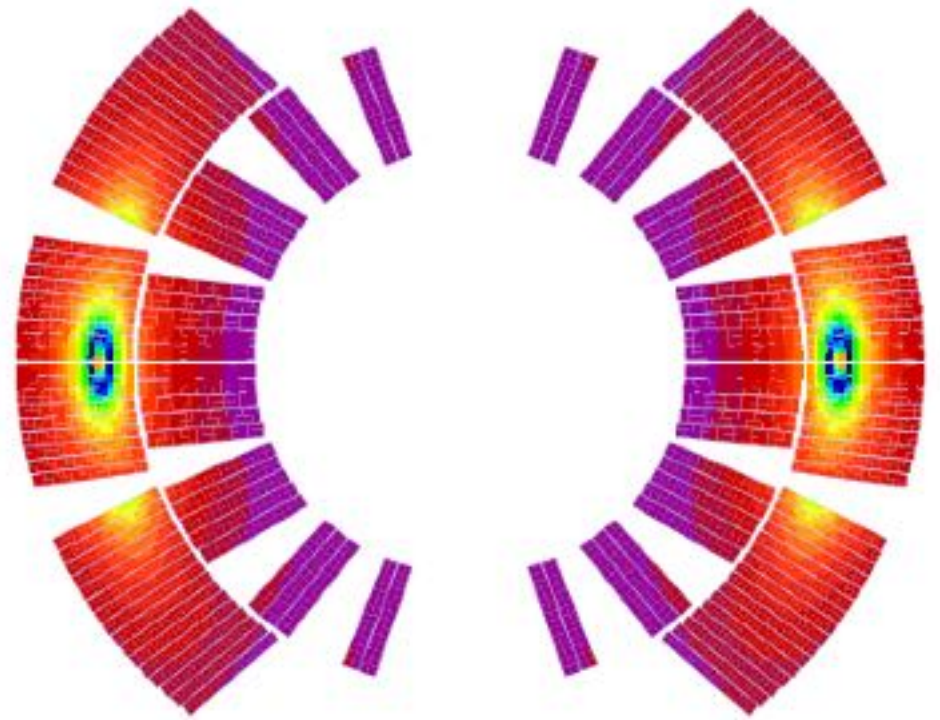
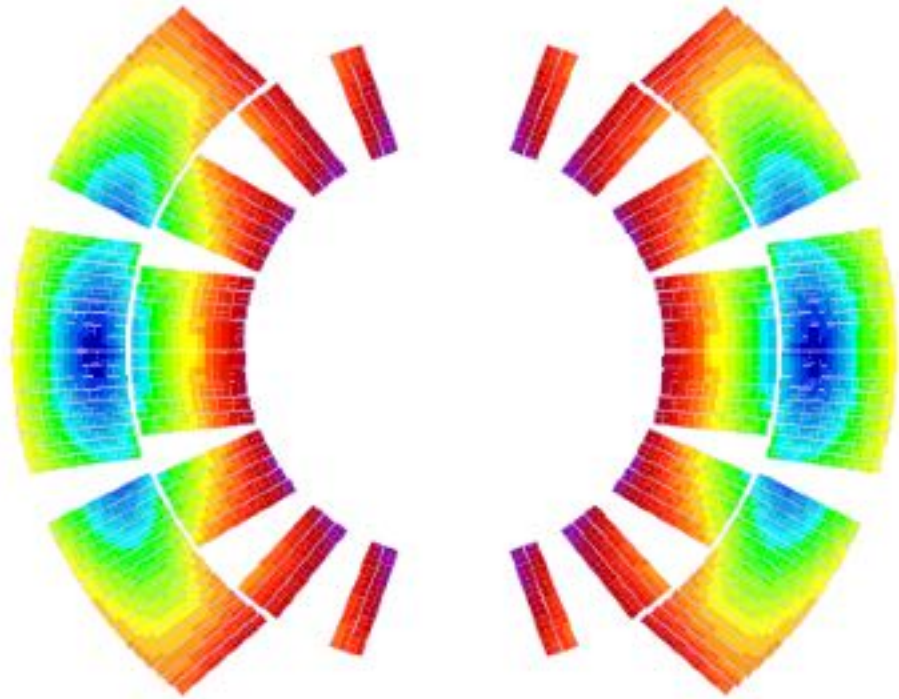
# M(B) Iteration Scheme



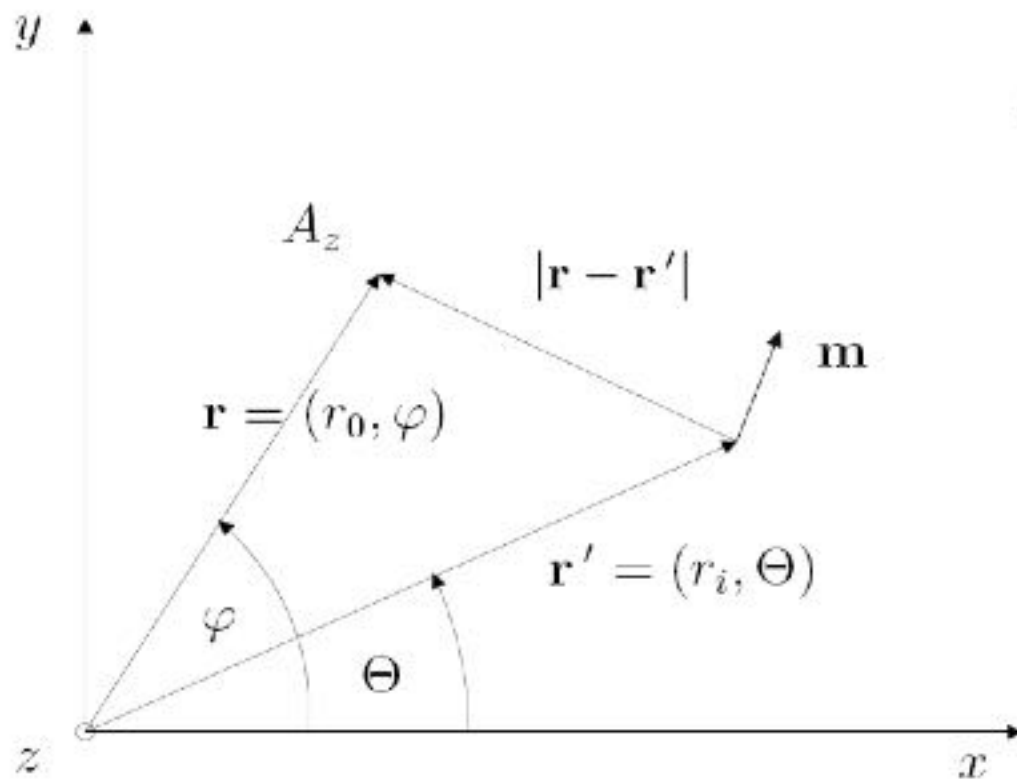
# Magnetization without and with Iteration



# Field and SC Magnetization



# Field Quality Calculation from Magnetization

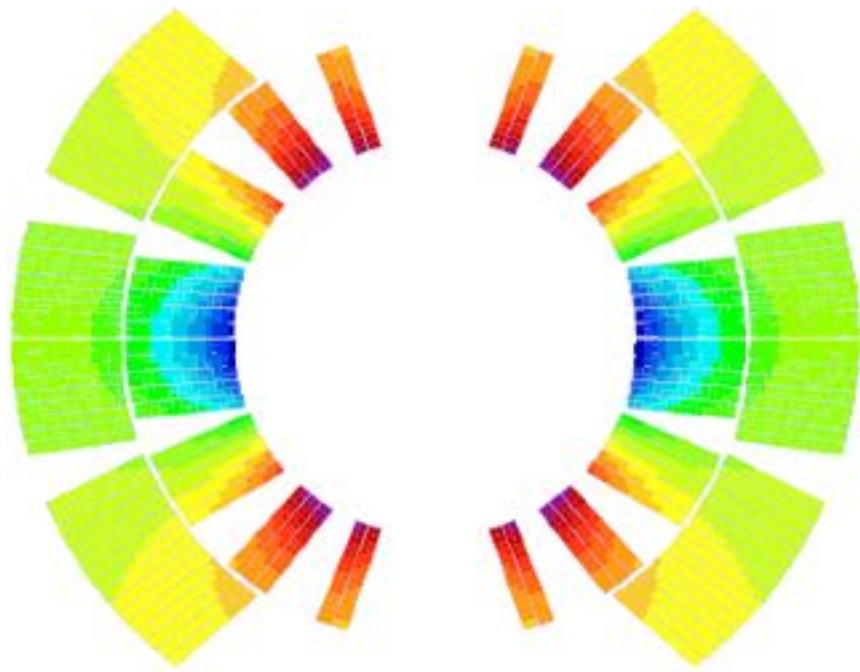


$$A_z(\mathbf{r}) = \frac{\mu_0 \mathbf{m}}{2\pi} \times \text{grad}_{\mathbf{r}'} \ln \left( \frac{|\mathbf{r} - \mathbf{r}'|}{R_{\text{ref}}} \right)$$

$$A_n = \frac{\mu_0}{2\pi} \frac{r_0^{n-1}}{r_i^{n+1}} n (m_{\mathbf{r}'} \cos n\theta - m_\theta \sin n\theta)$$

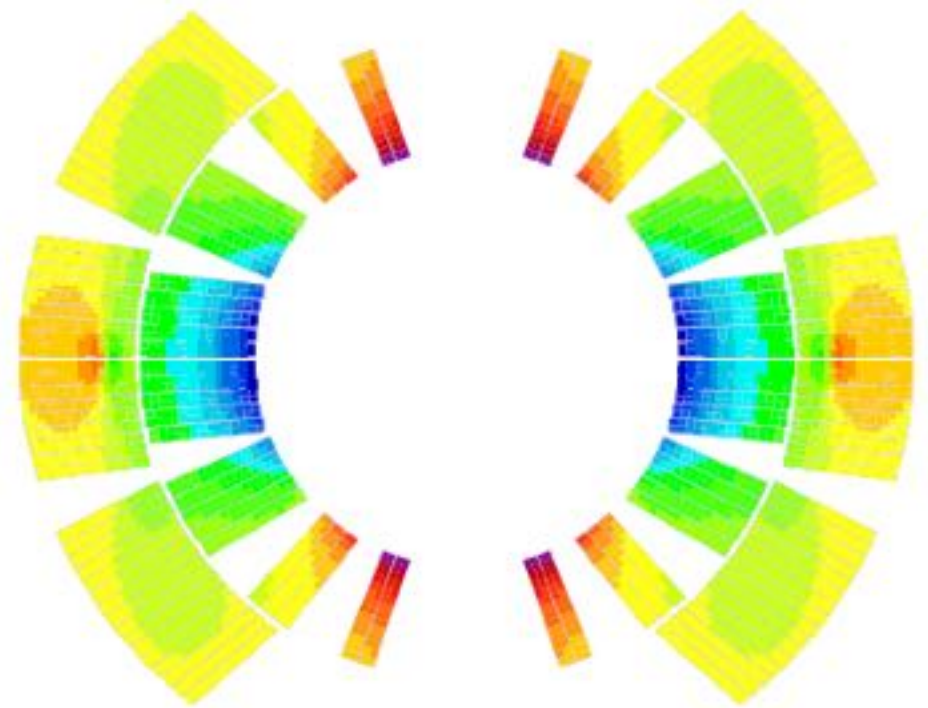
$$B_n = \frac{\mu_0}{2\pi} \frac{r_0^{n-1}}{r_i^{n+1}} n (m_{\mathbf{r}'} \sin n\theta + m_\theta \cos n\theta)$$

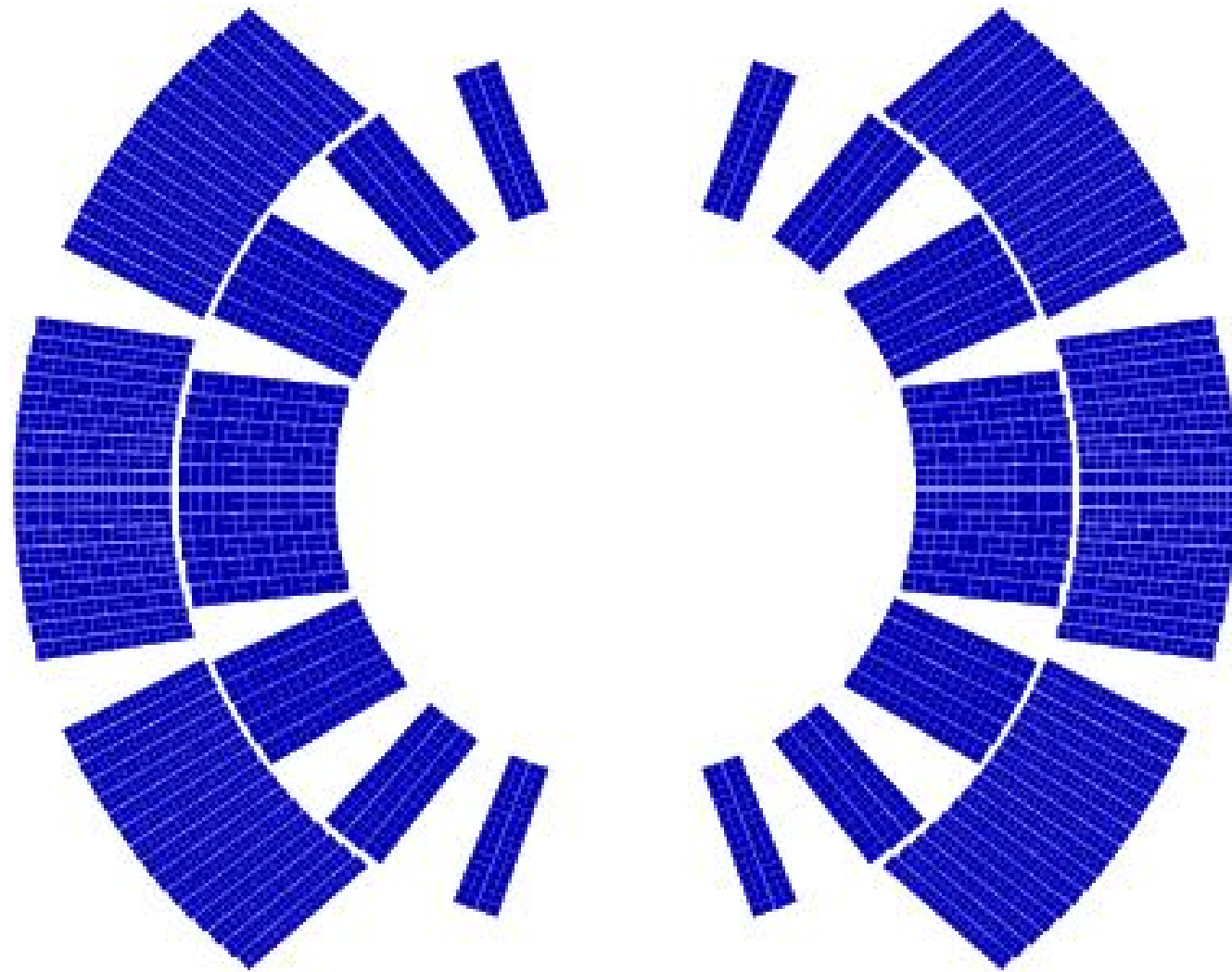
# $B_3$ Contribution from Transport Current and Magnetization



$B_3(I)$

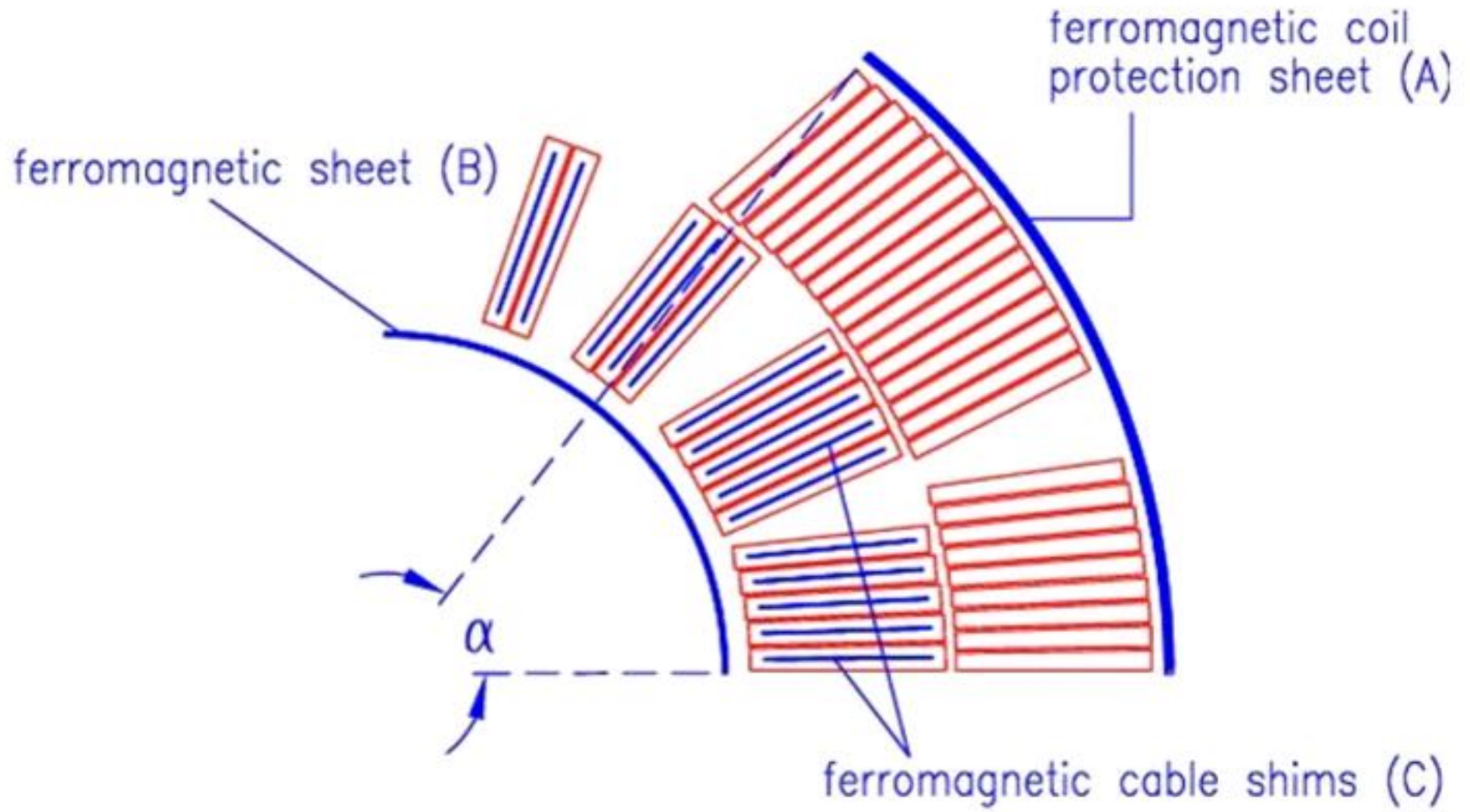
$B_3(M)$







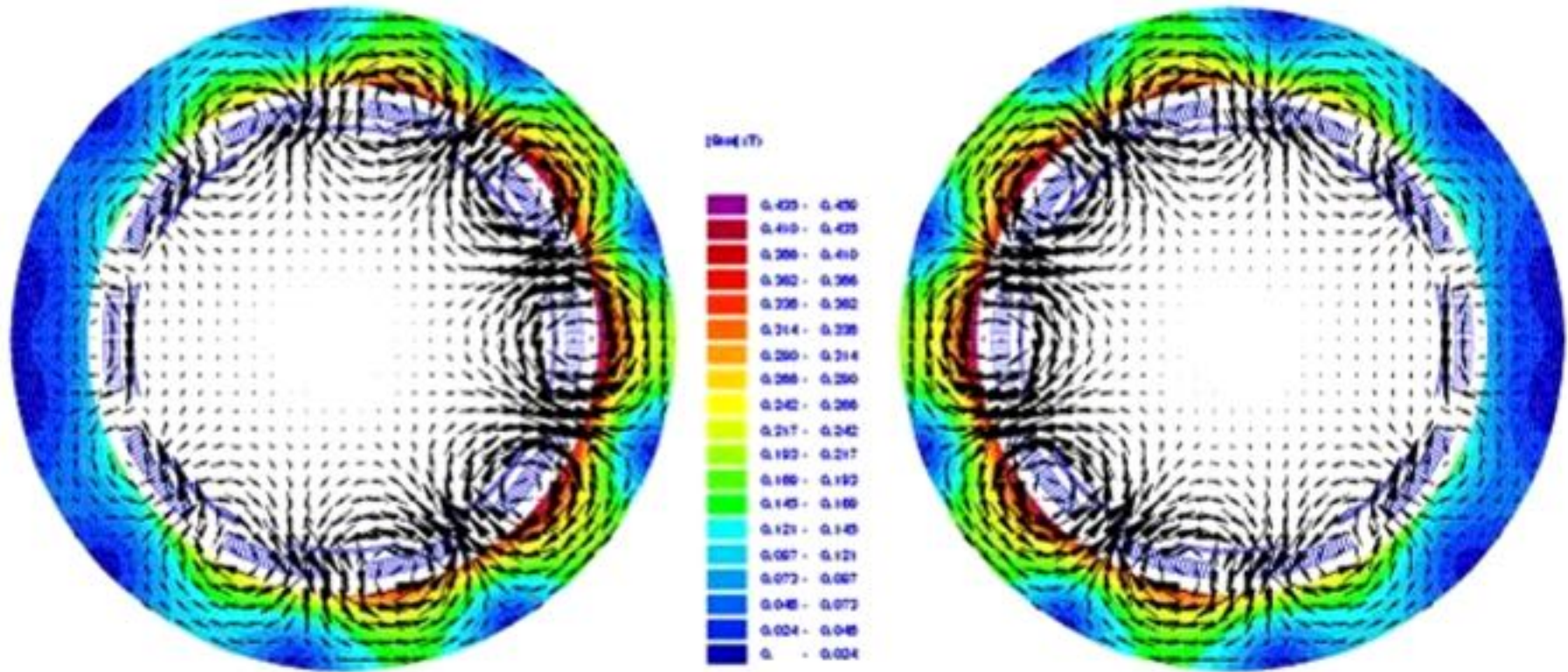
# Compensation Schemes



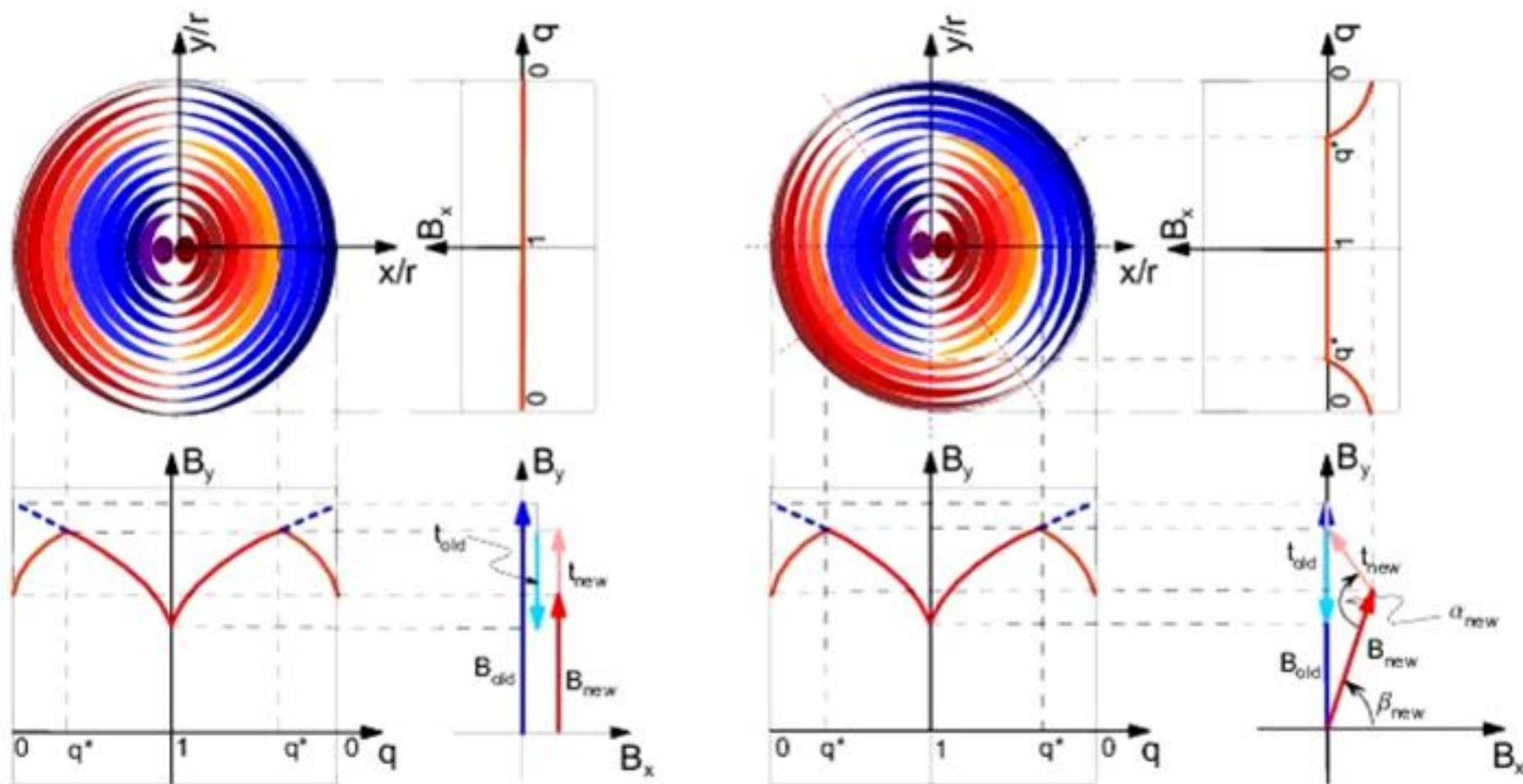
# Coil Protection Sheet



# Excitation of Spool Piece Correctors



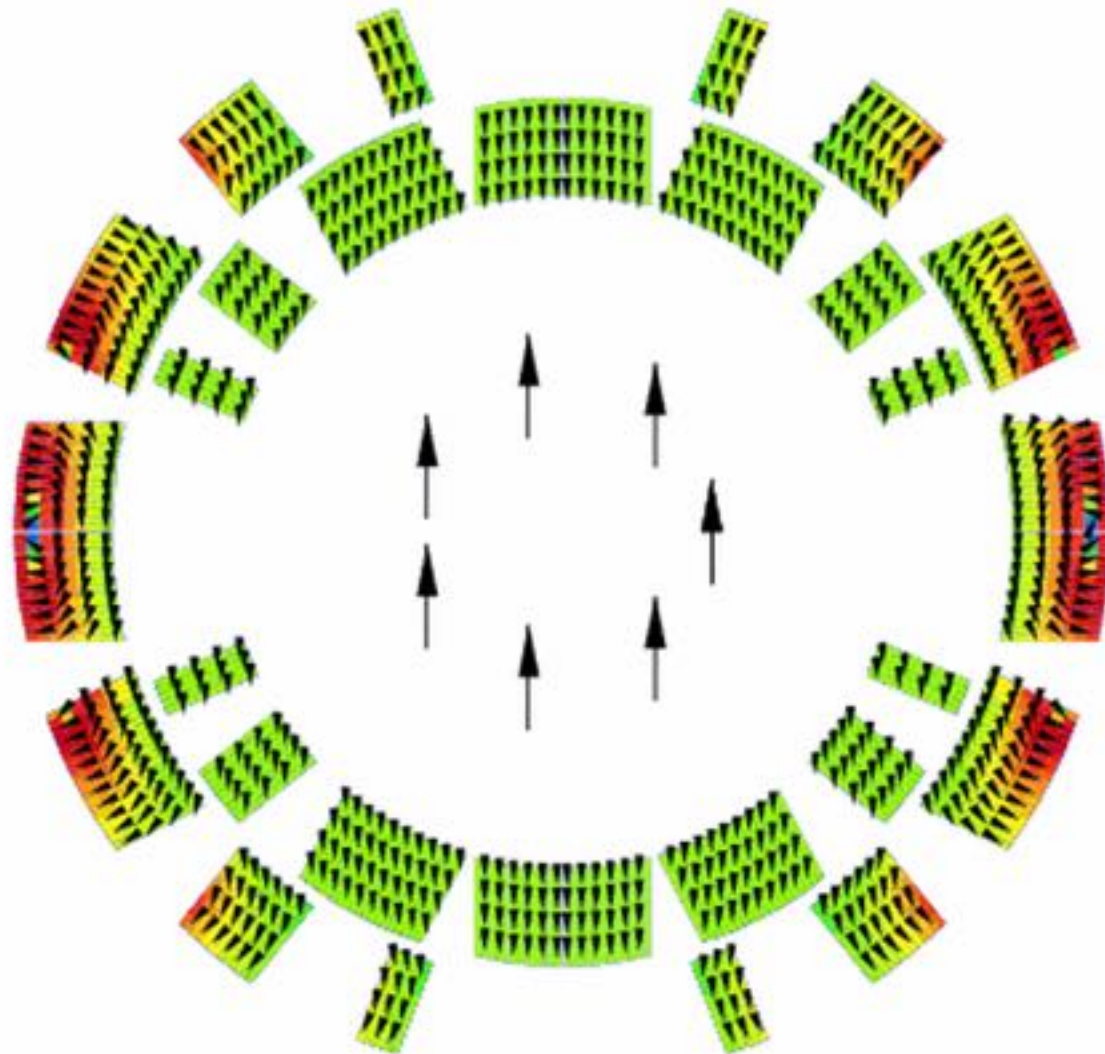
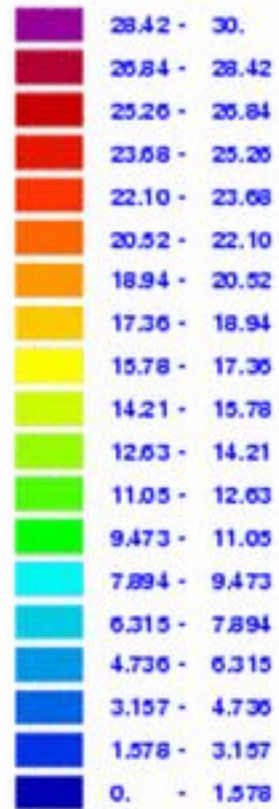
# Vector-Hysteresis Model

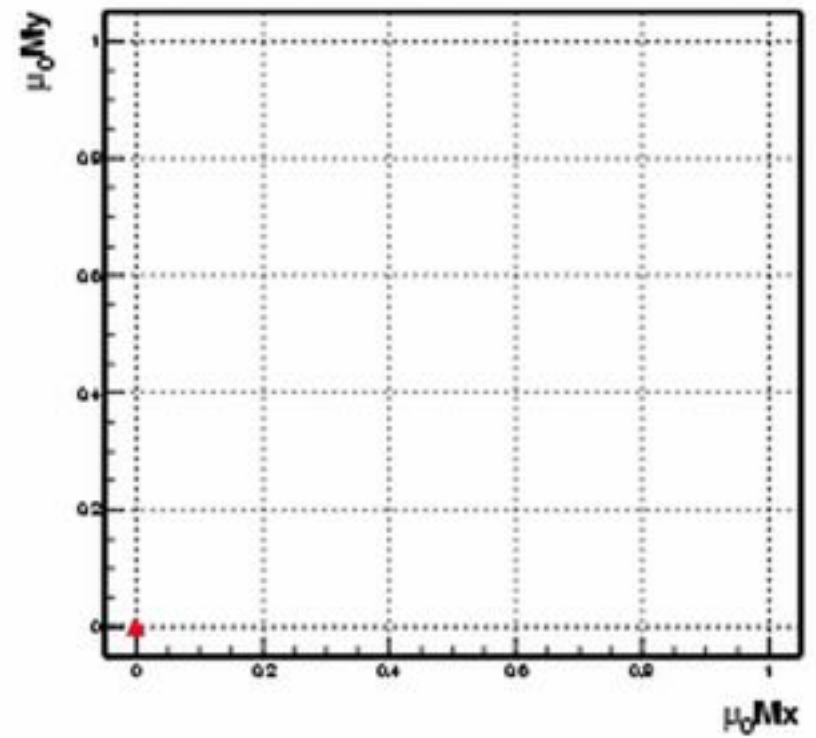
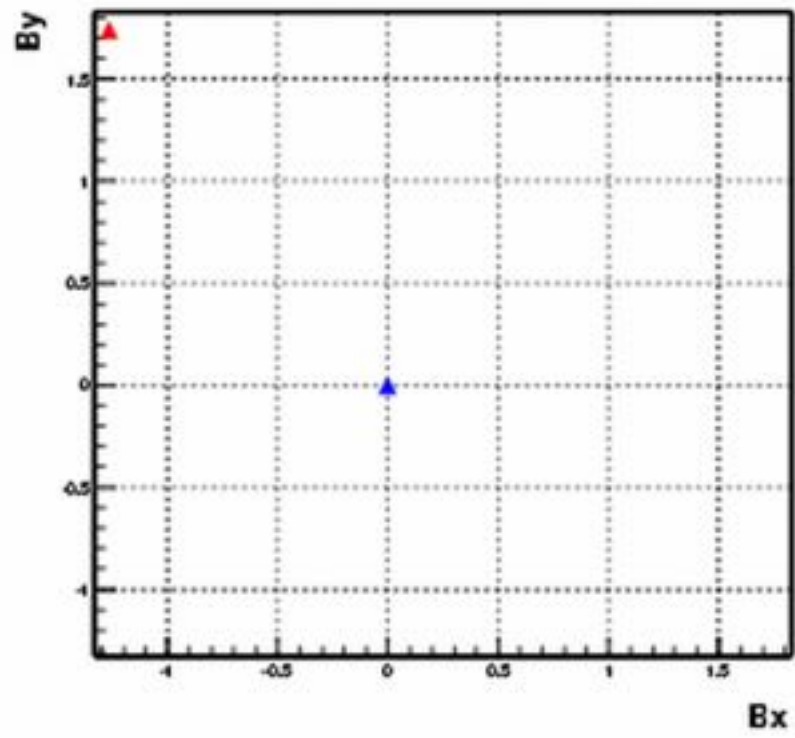


Time (s) : 1.

[M] (A.m)

( $\cdot 10^3$ )

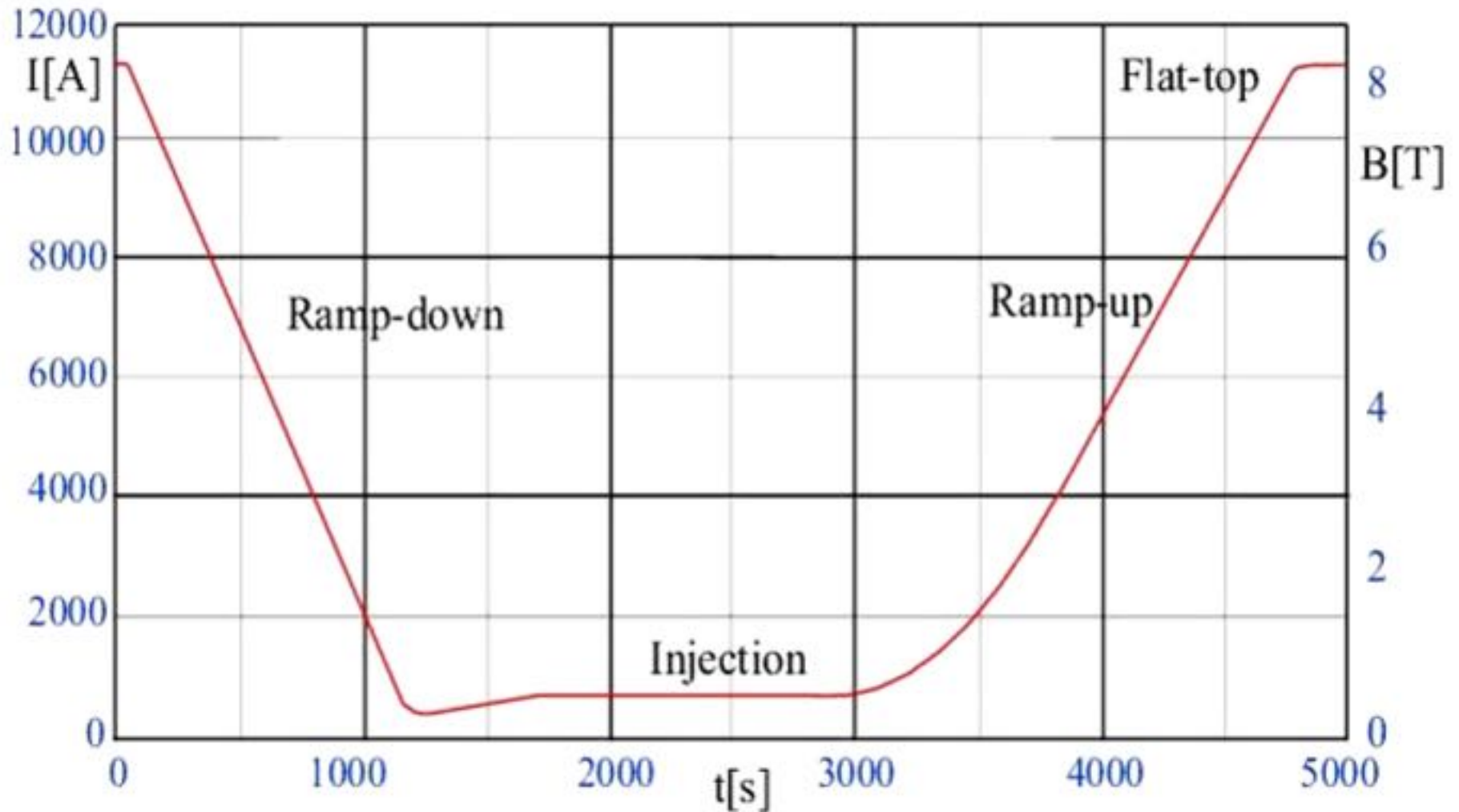




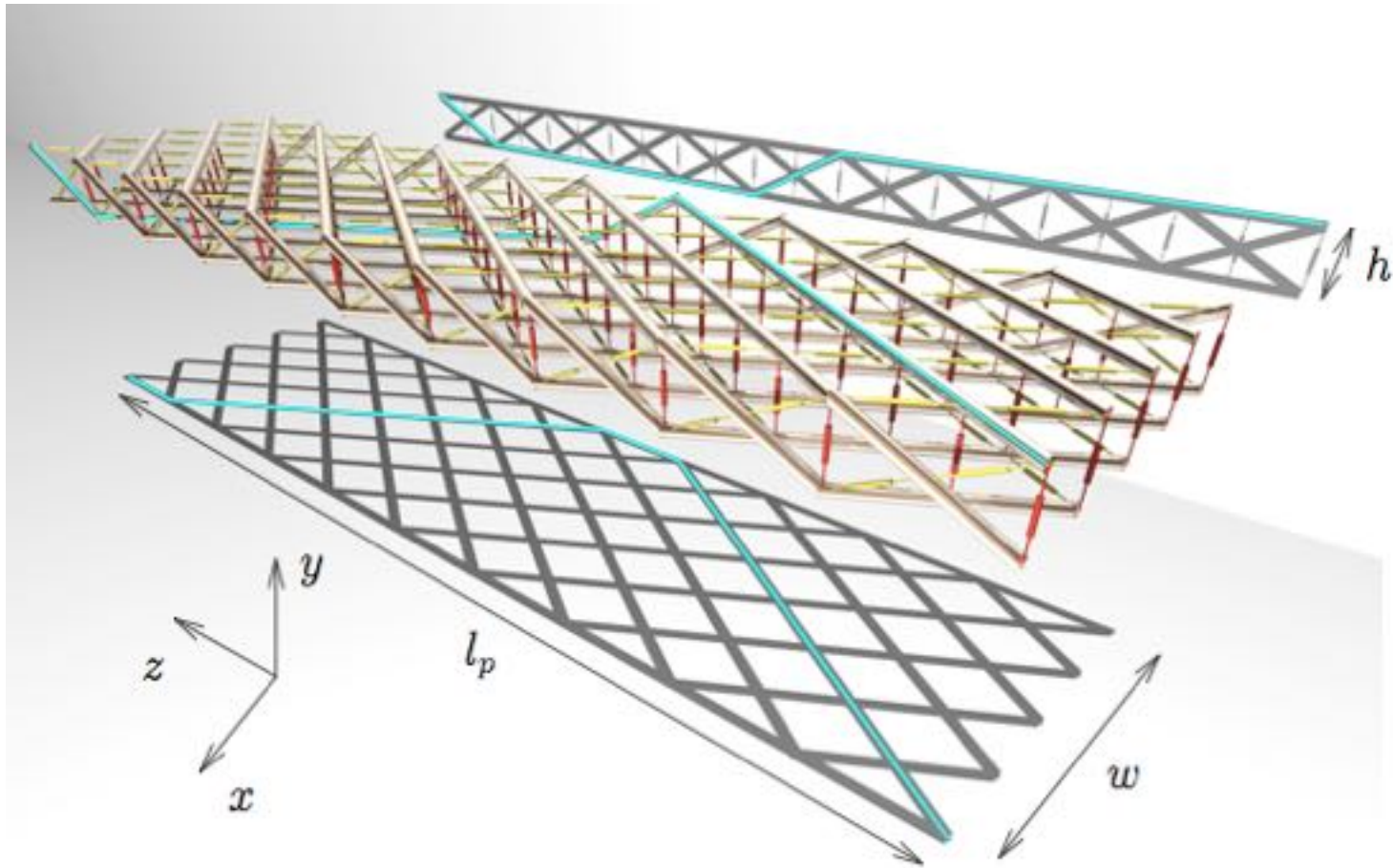
# The LHC Excitation Cycle

$$V \approx 2 E / I t$$

$E = 1.15 \text{ TJ}$  (320 kWh),  $I = 11800 \text{ A}$ , Ramp rate  $10 \text{ A/s}$ ,  $155 \text{ V}$

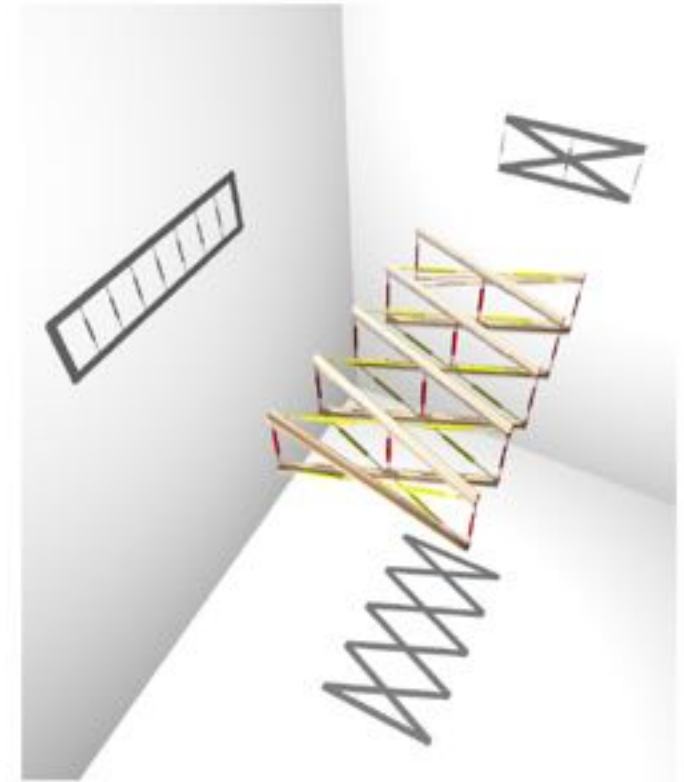
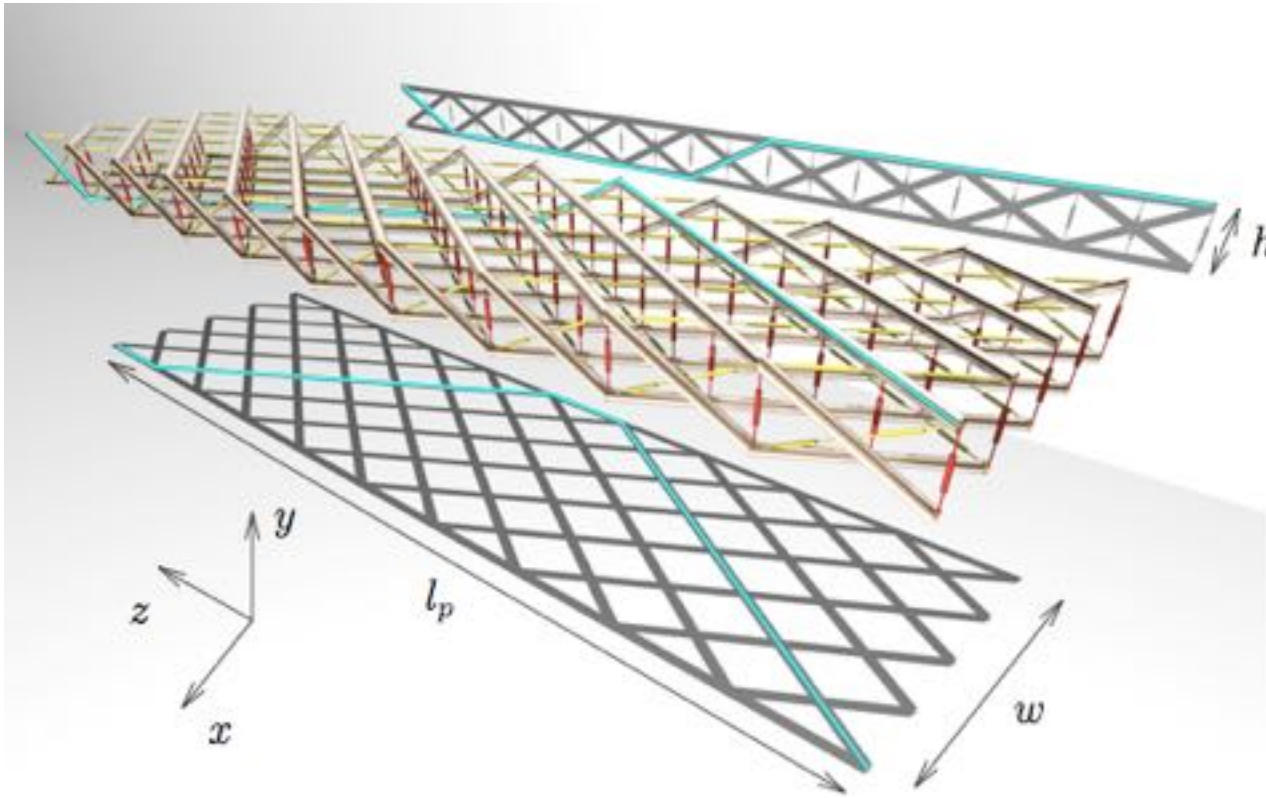


# Eddy Currents in Rutherford Cables





# Eddy Currents in Rutherford Cables (Algebraic Topology)

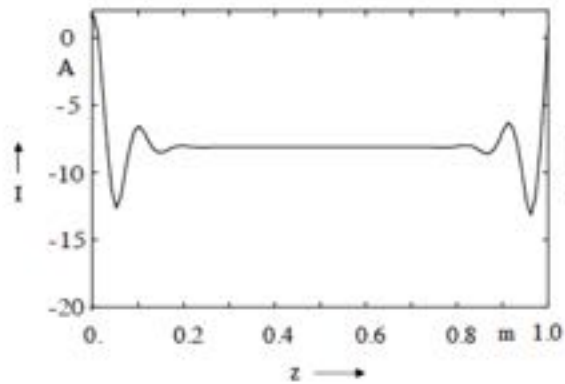


Nodes:  $2 N_s * N_b + N_s$   
 $N_b = L / \text{pitch} * N_s$

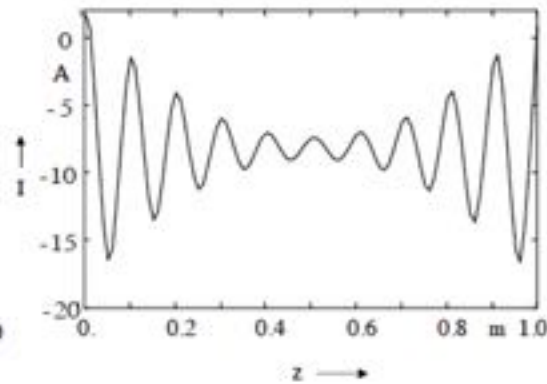
Pitch = 100 mm  
 $N_s = 36$   
Nodes = 30 000 / meter  
LHC main dipole = 4.2 km cable  
126 Million Nodes

# Boundary Induced Coupling Currents

@ 1 sec

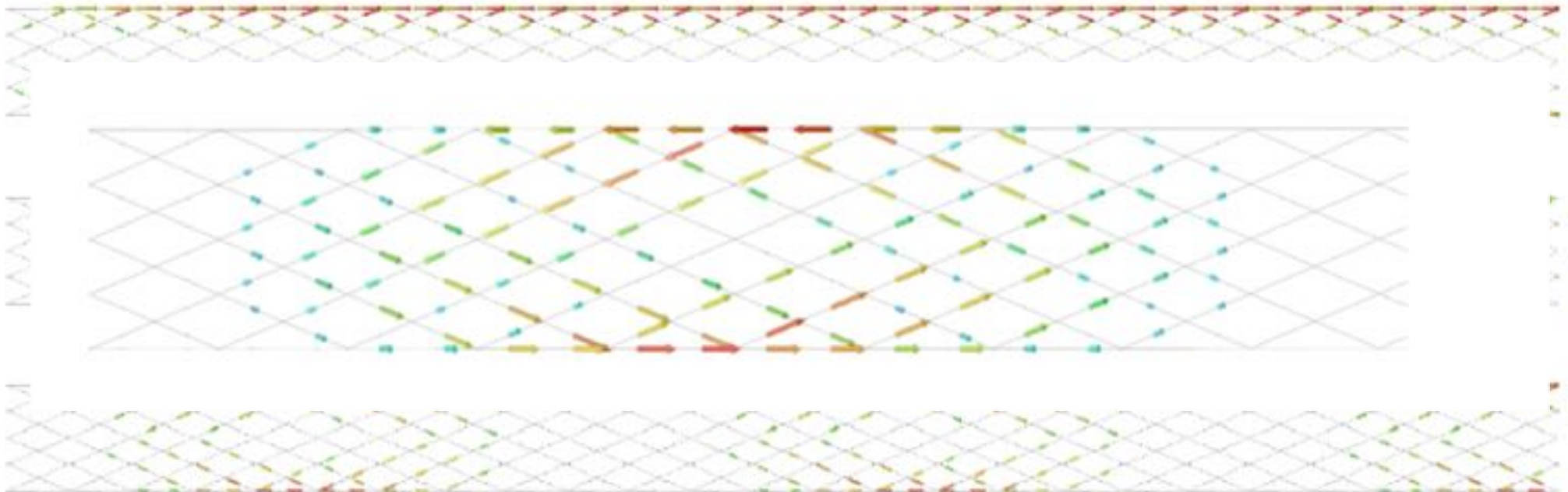


@ 10 sec

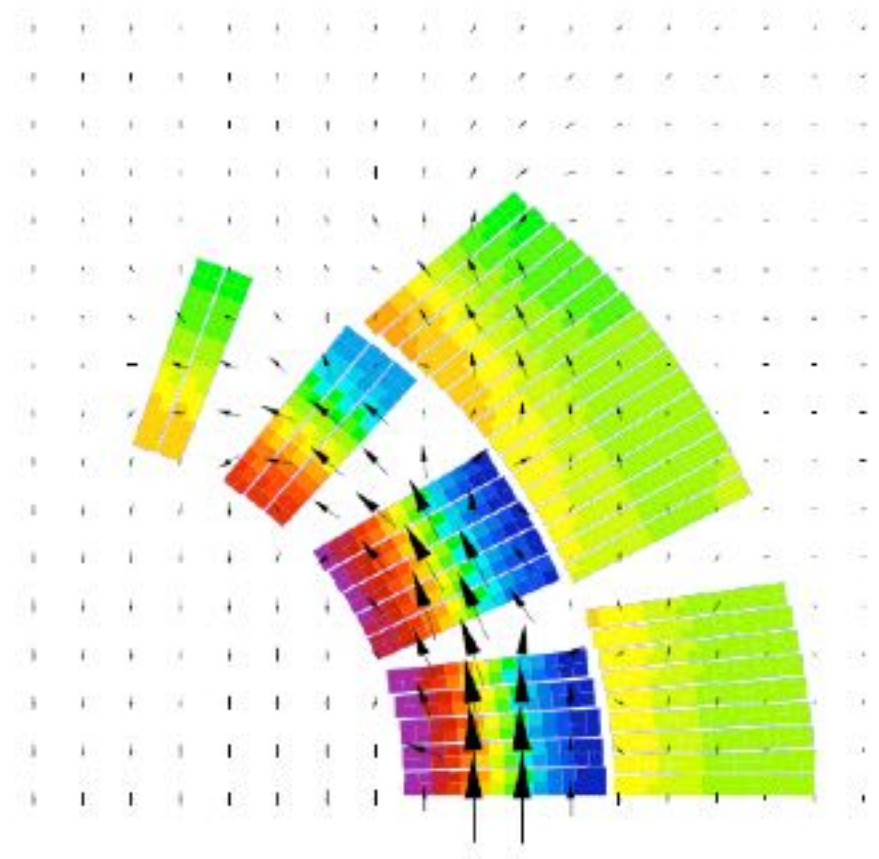
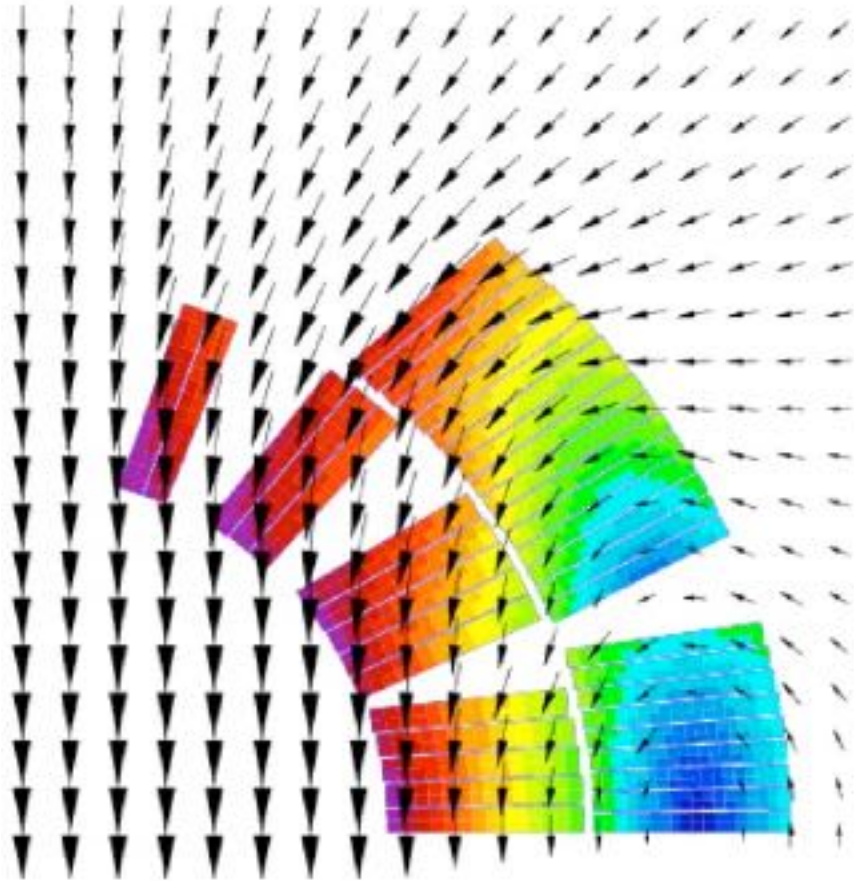


R&D Challenge

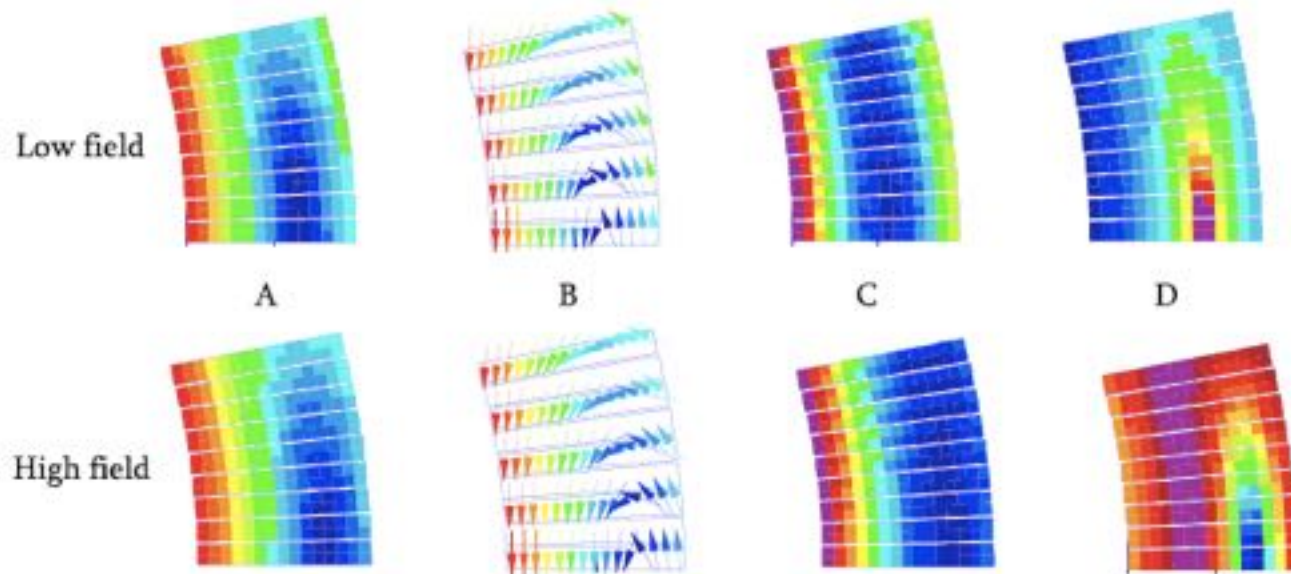
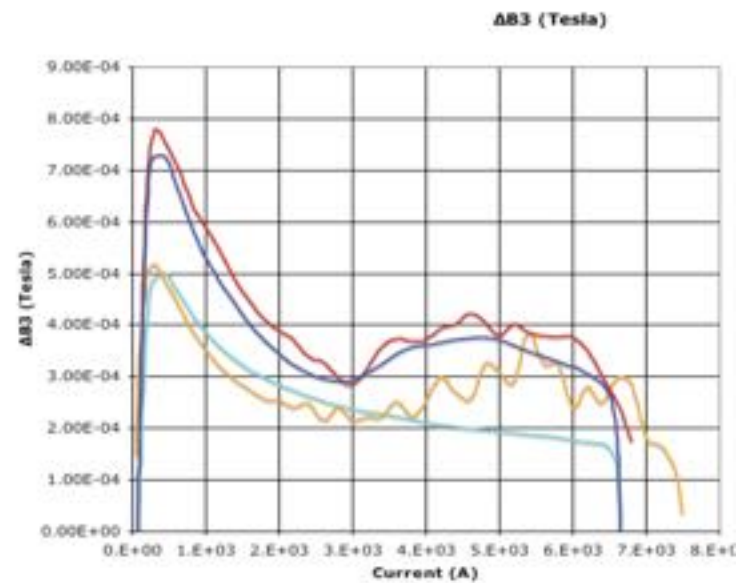
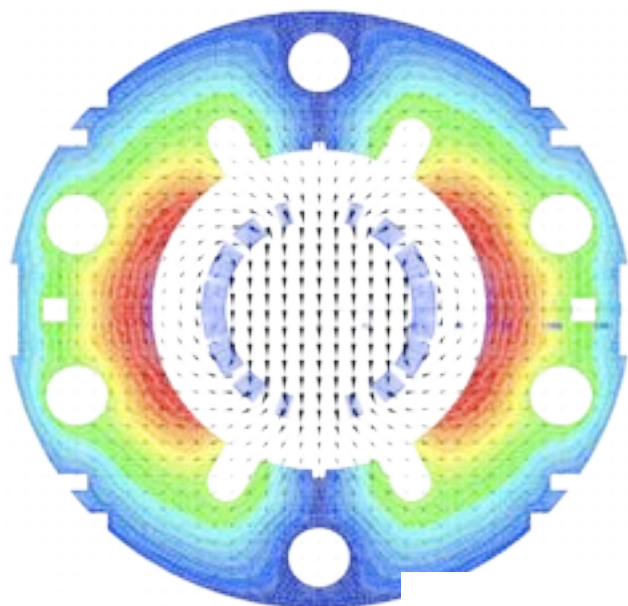
Dimensional reduction



# Field Generated by ISCC

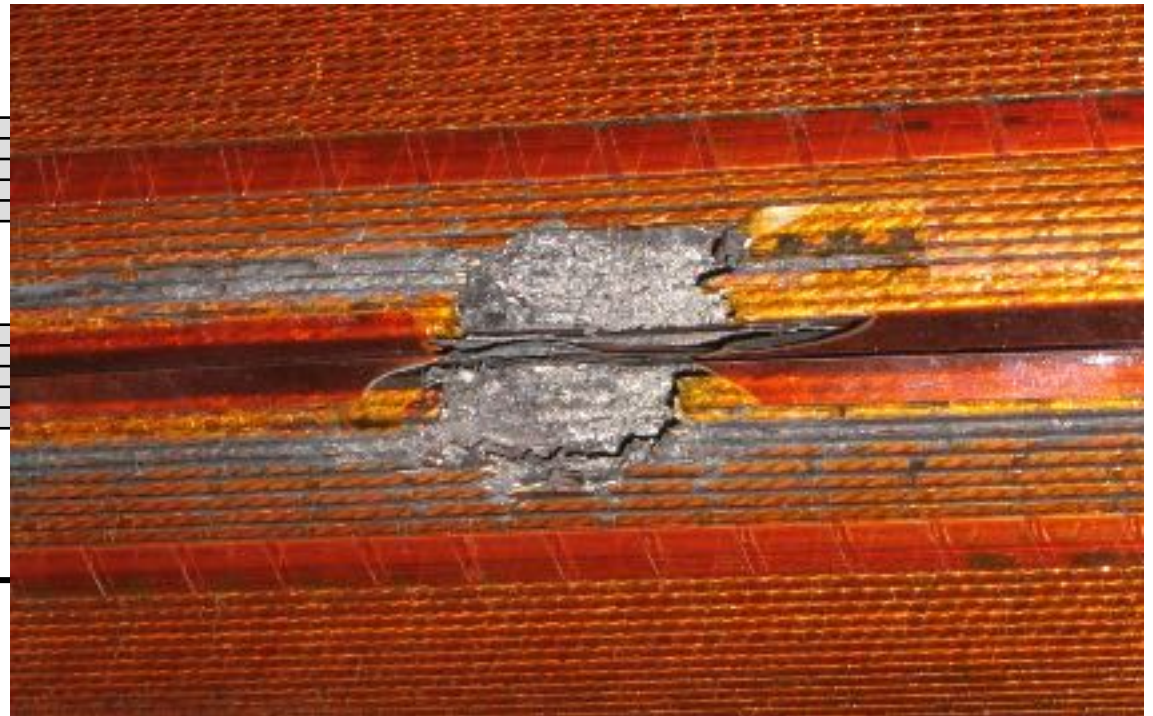
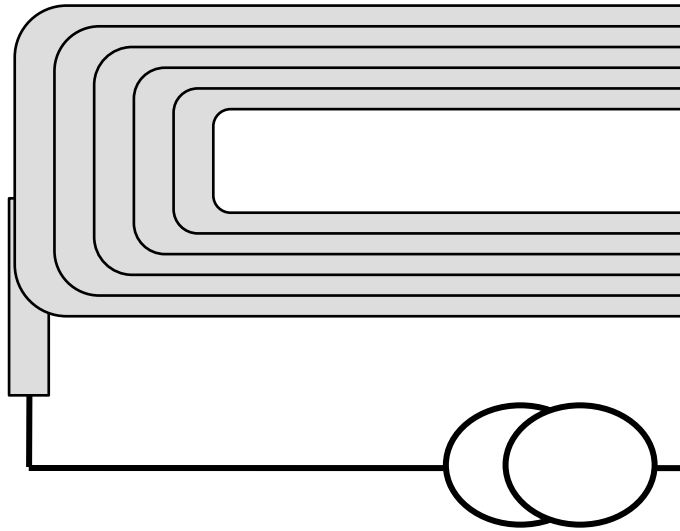


# 2-D Transient Field Computation for GSI-001

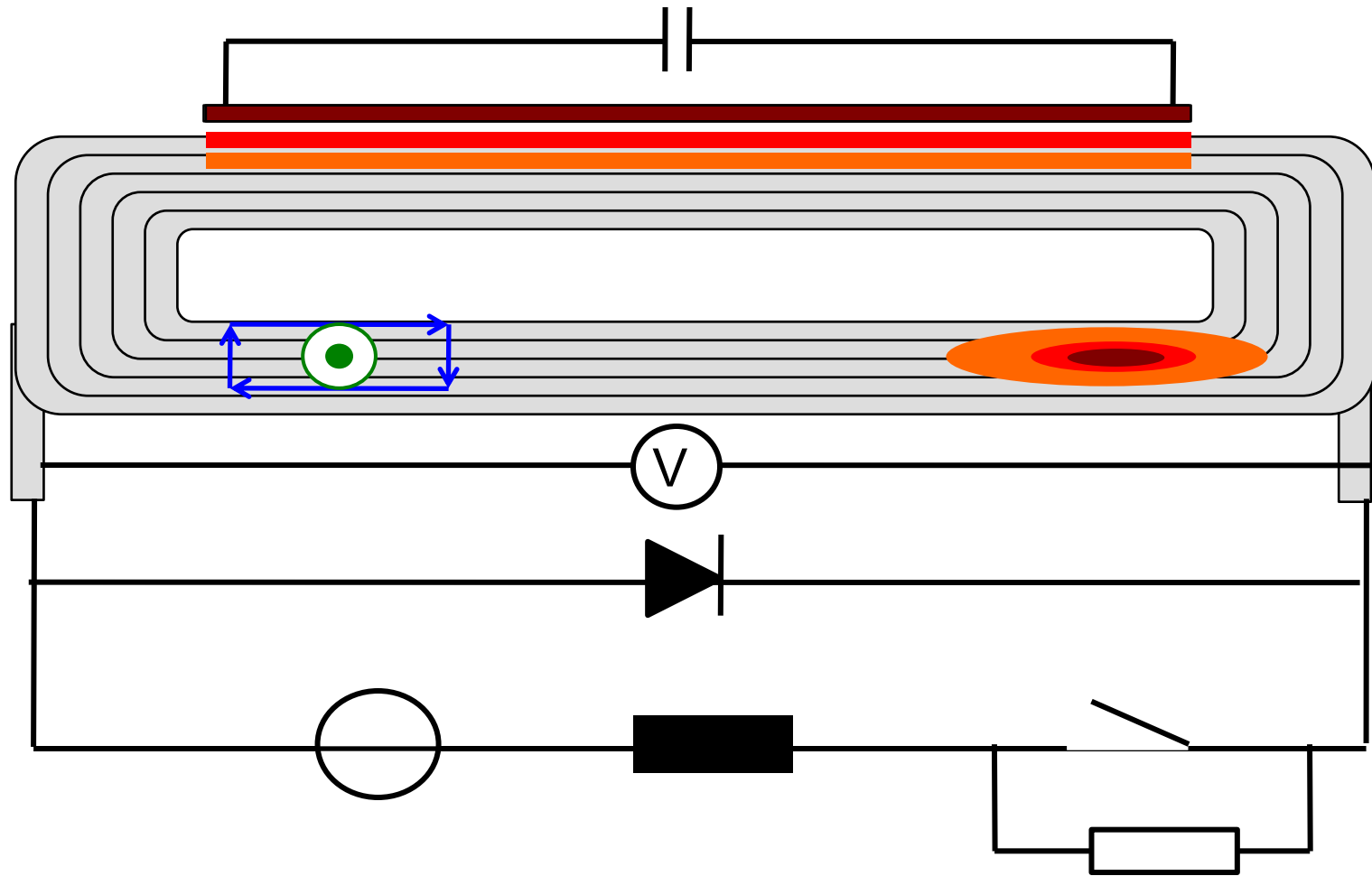


# Quench

- **Quench:** Transition from SC to normal conducting state caused by beam losses, conductor movement, eddy currents etc.
- **Propagation:**
  - Normal conducting zone generates Ohmic heat
  - Quench- und temperature distribution determined by loss-mechanisms and cooling capacity



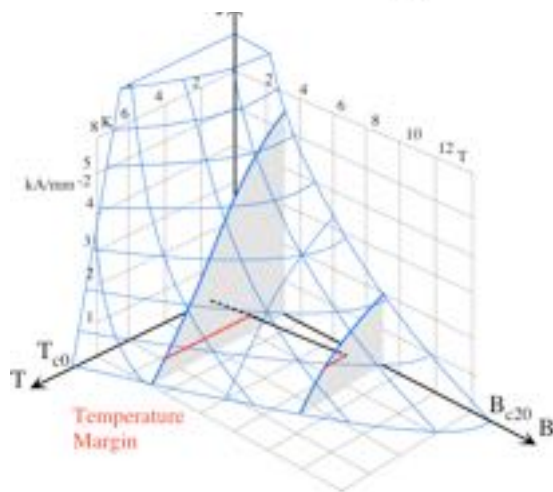
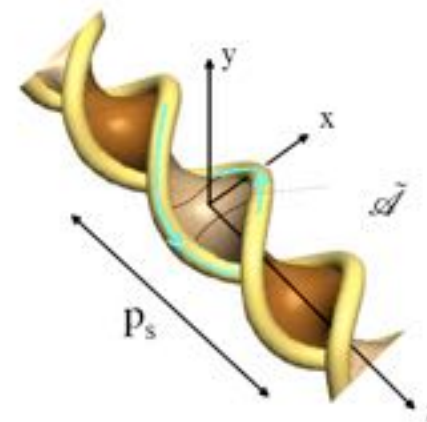
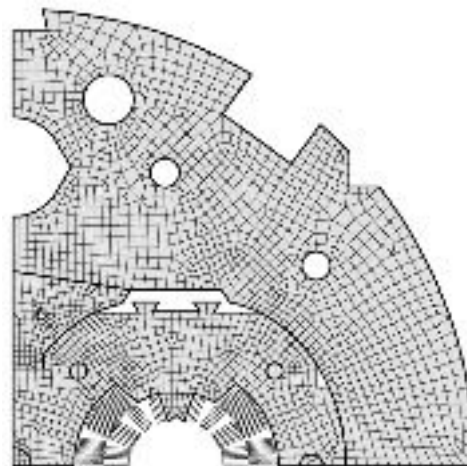
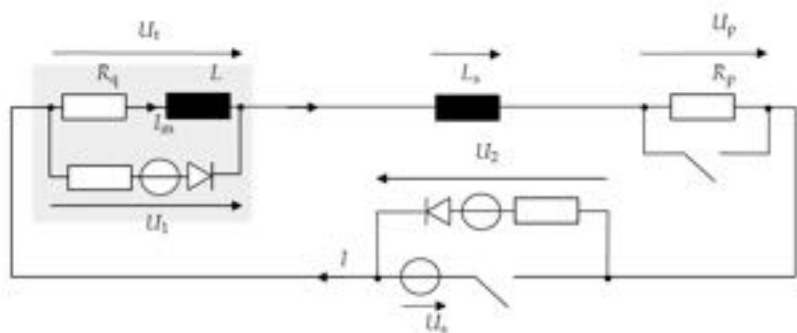
# Quench Mechanism and Magnet Protection



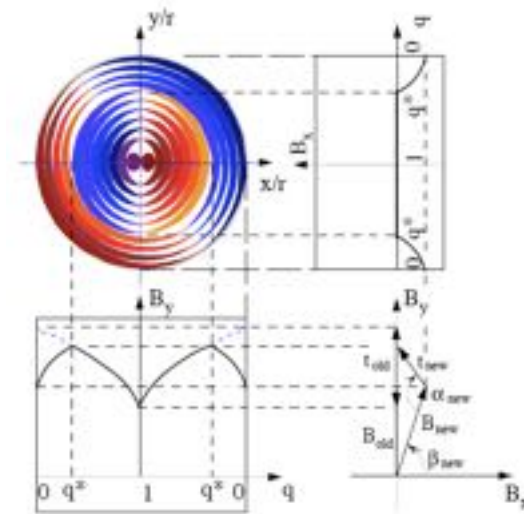
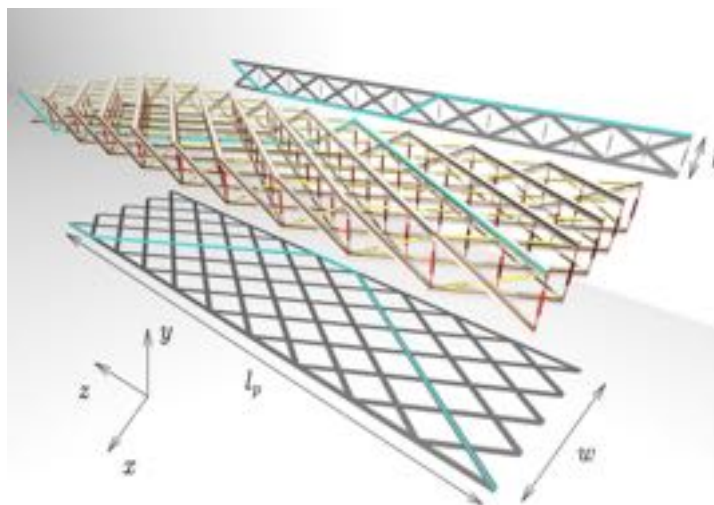
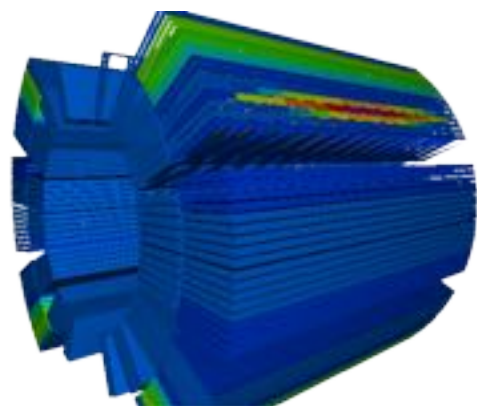
# Switches and Dump Resistors



# Quench Simulation (Multi-Physics, Multi-Scale)



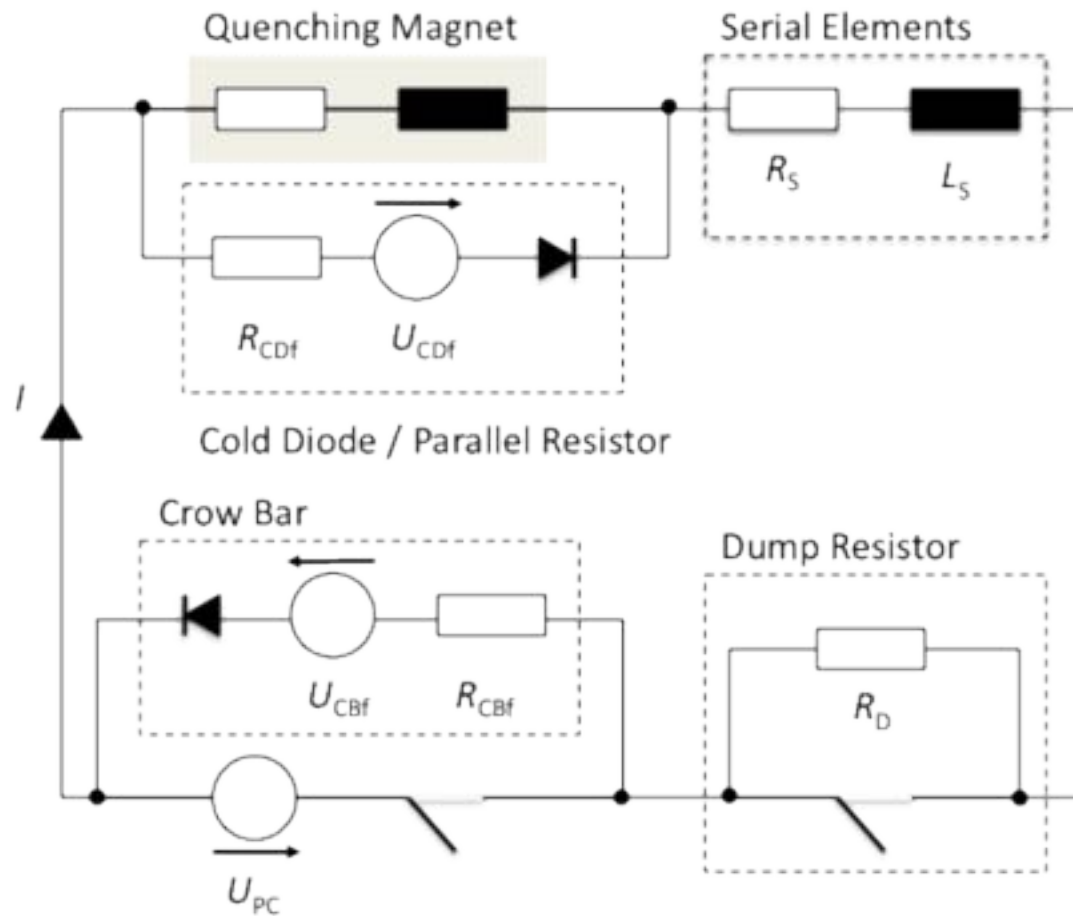
## Quench Simulation in ROXIE





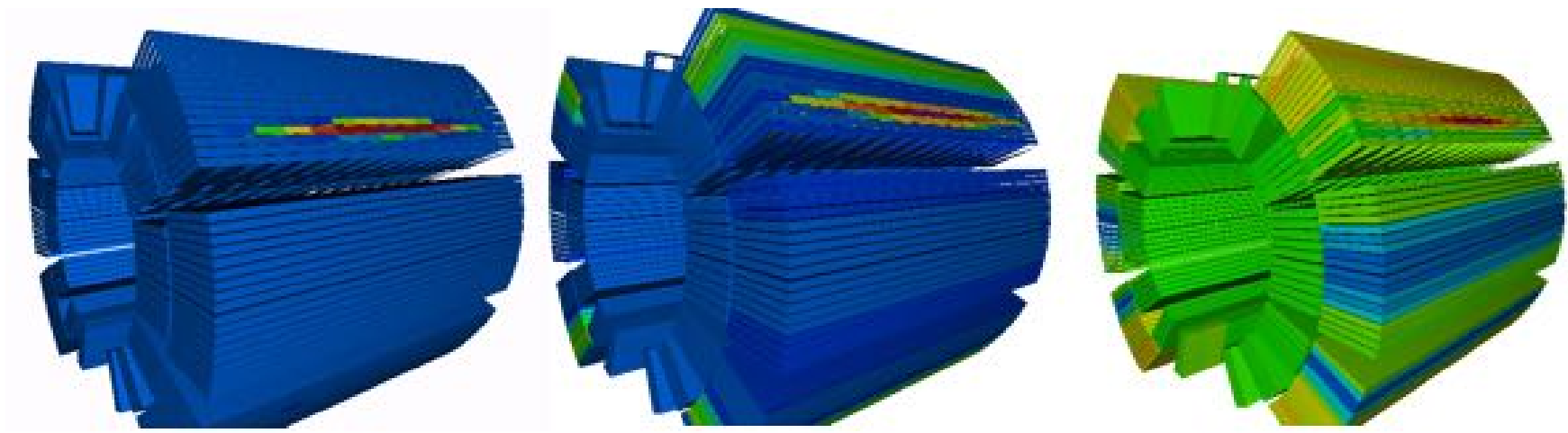
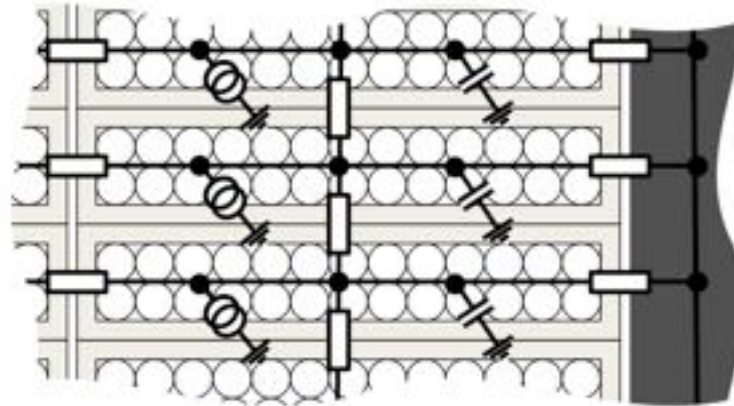
# Electrical (External) Network

$$L_d(B) \frac{dI}{dt} = U_{\text{Diode}} - (R_Q(B, T) + R_P(t))I$$

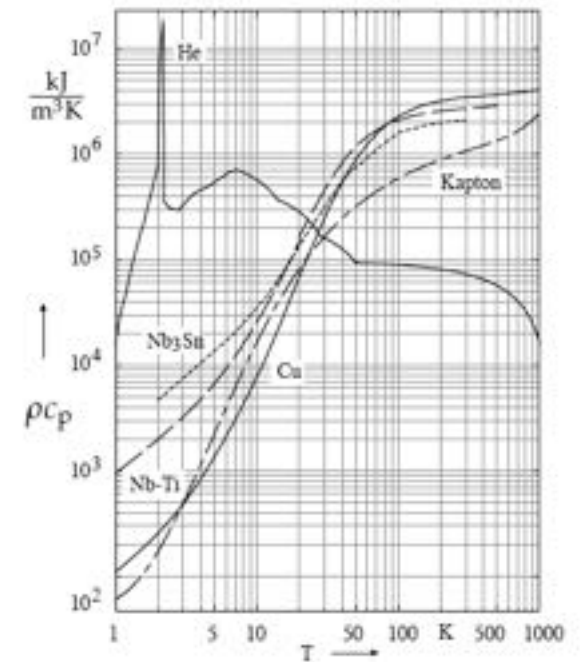
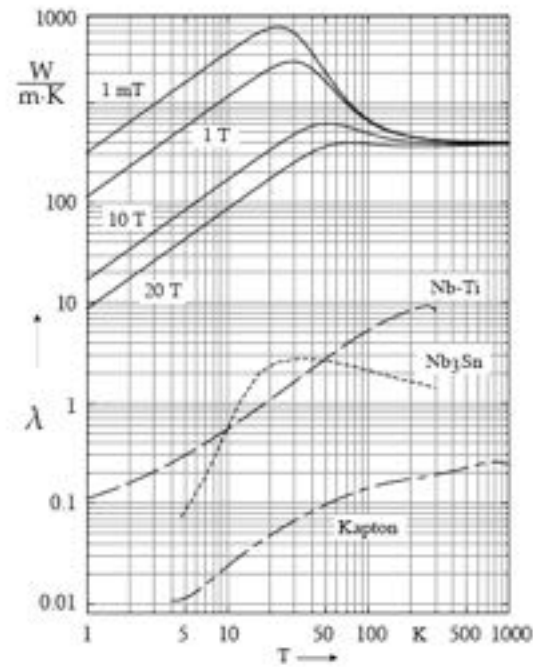
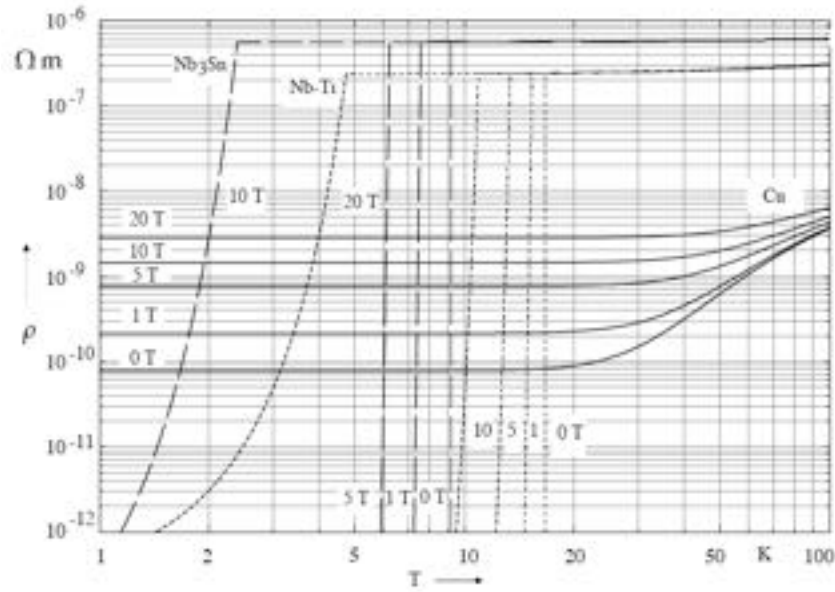


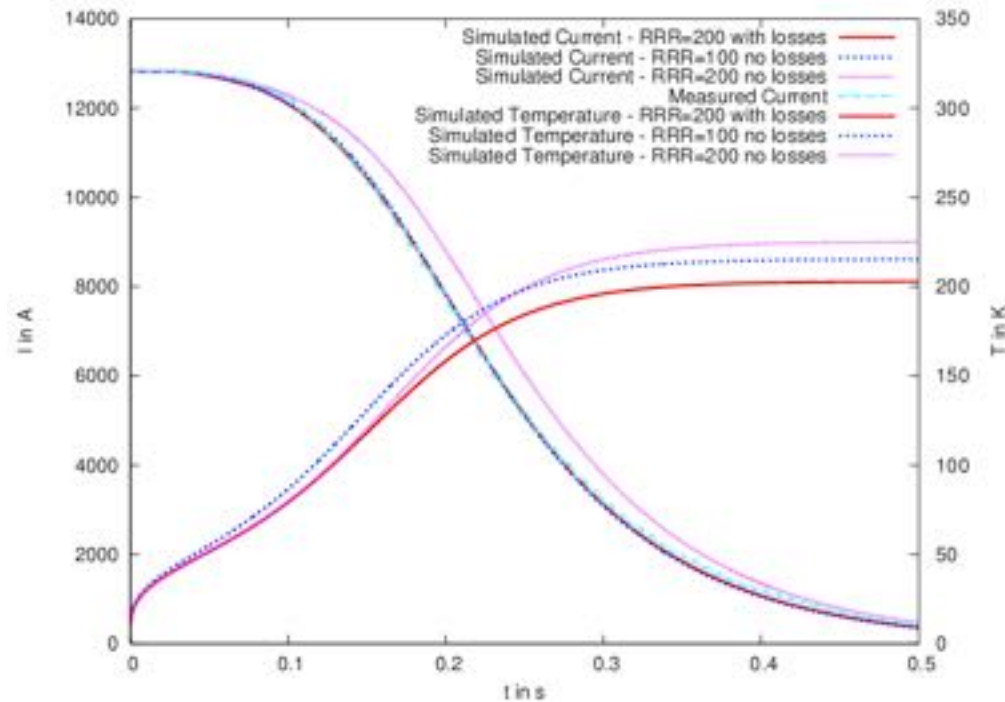
# Thermal Model

$$c(T) \frac{dT}{dt} = p - \vec{\nabla} \cdot \left( \kappa(T, B) \vec{\nabla} T \right)$$



# Material Parameters





Empirical parameters:

- RRR
- $R_a/R_c$
- IFCC effective res.
- heat conductivity
- heat capacity

- ➔ Different families of parameters yield exactly the same observable  $I(t)$ .
- ➔ More than one solution exists.
- ➔ Great care must be taken to model
  - all relevant phenomena,
  - using realistic material parameters.

The challenge of quench simulation:

Model **all relevant physical phenomena** with **adequate accuracy** so that we can be confident to **simulate internal states** of a quenching magnet and understand its behavior.

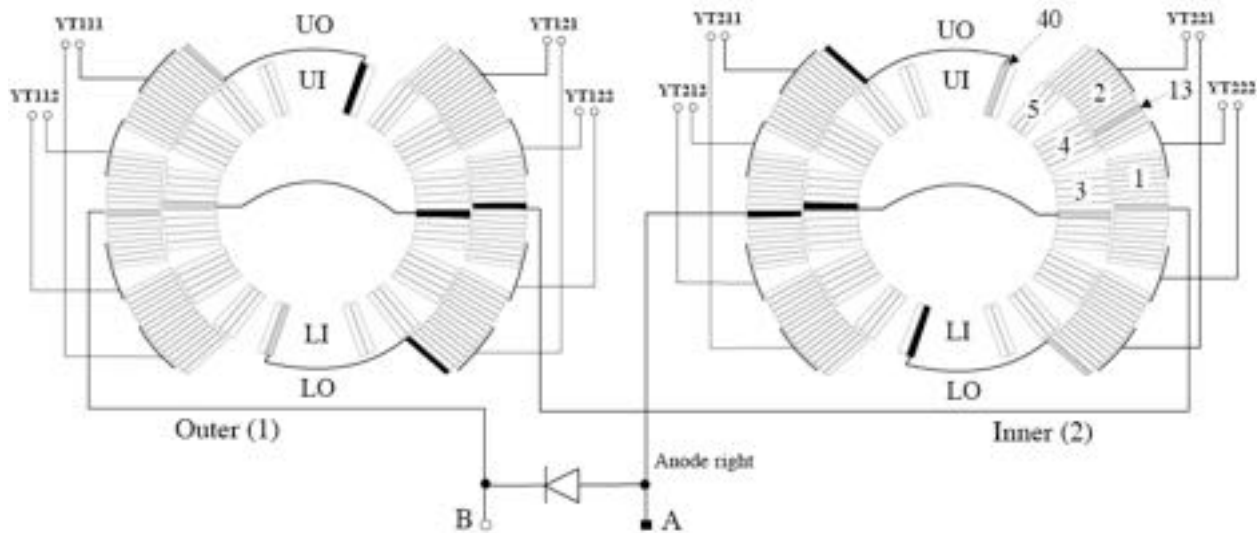
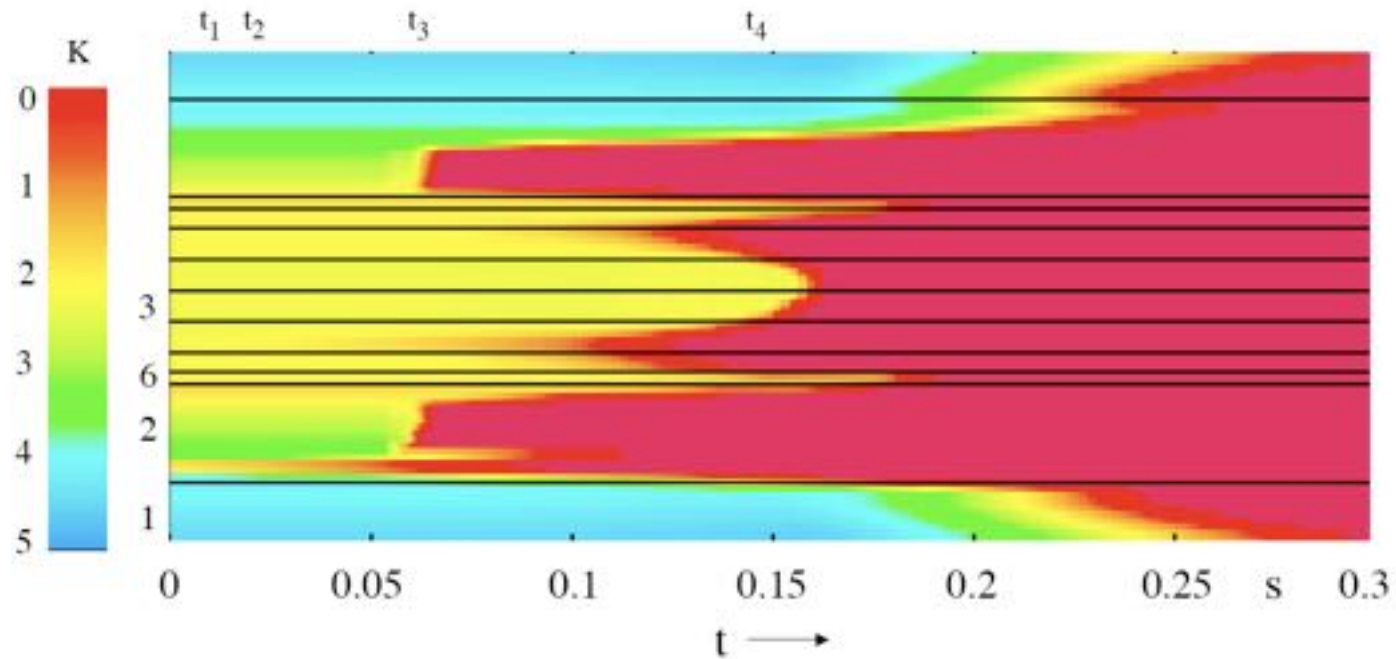
Validation:

**Measured quantities** can be **reproduced** with all **material- and model-parameters** **within their range of uncertainty**,

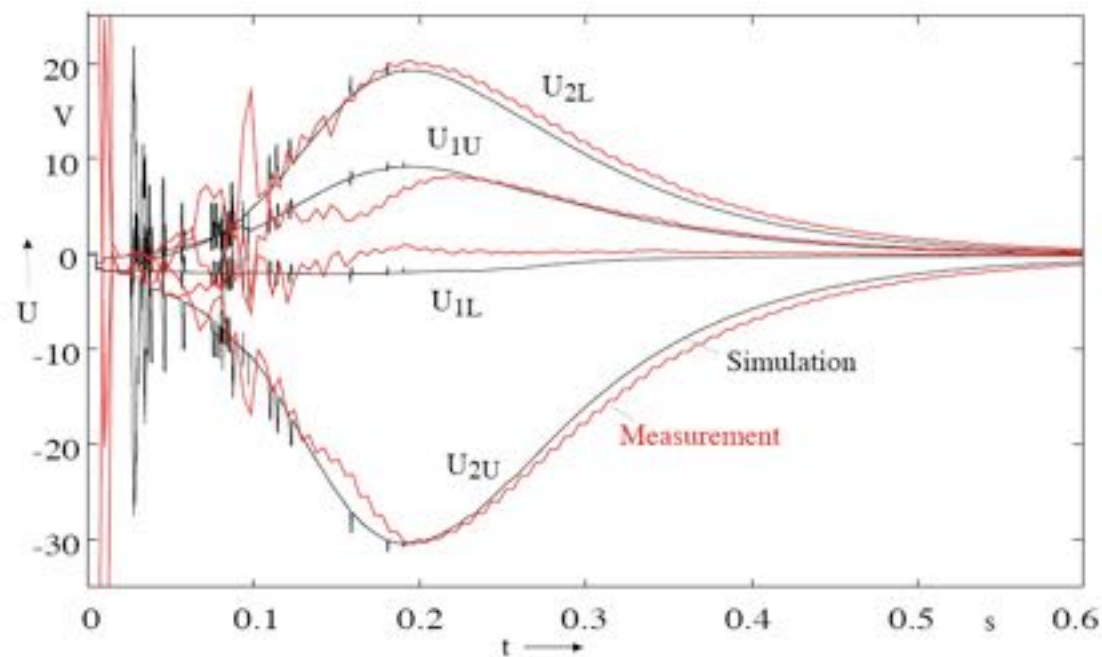
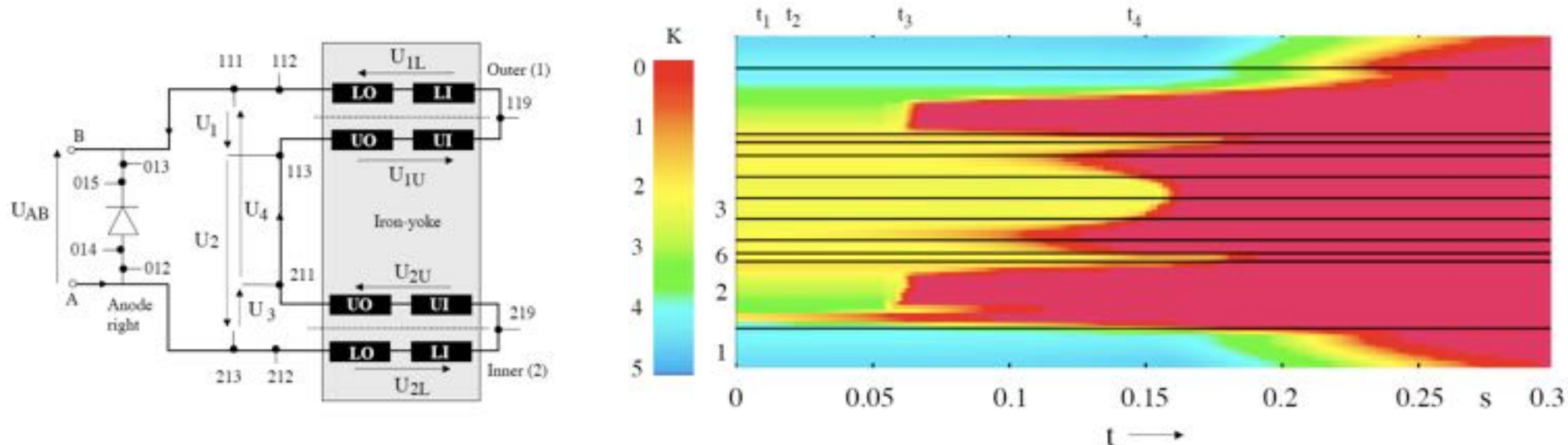
Extrapolation and Introspection

If the above criteria are reached, **extrapolated** results will match measurements **without adaptation of material- and model parameters**. It is then also possible to **simulate internal states** of the magnet that escape measurements.

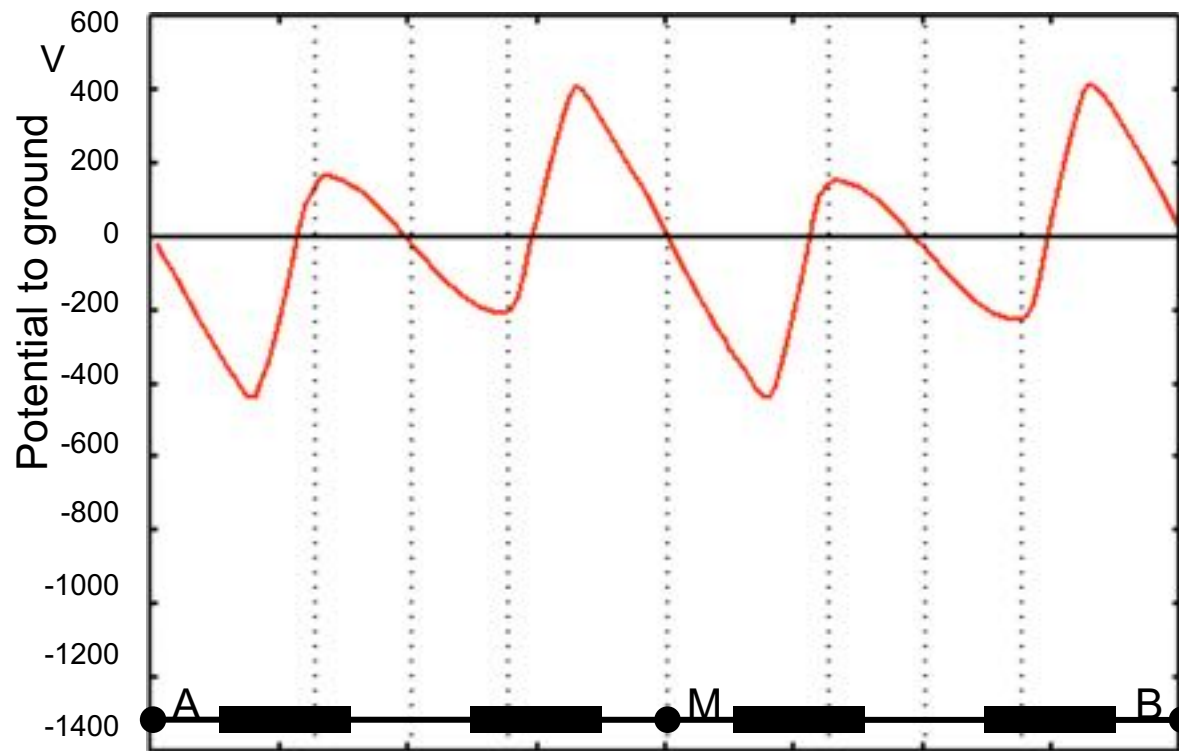
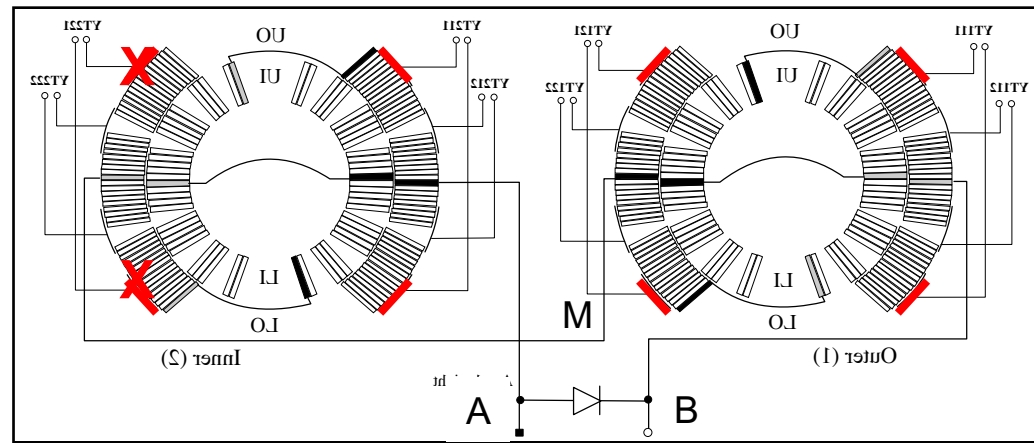
# Introspection (Quench Margin)



# Introspection (Voltage Ripples)

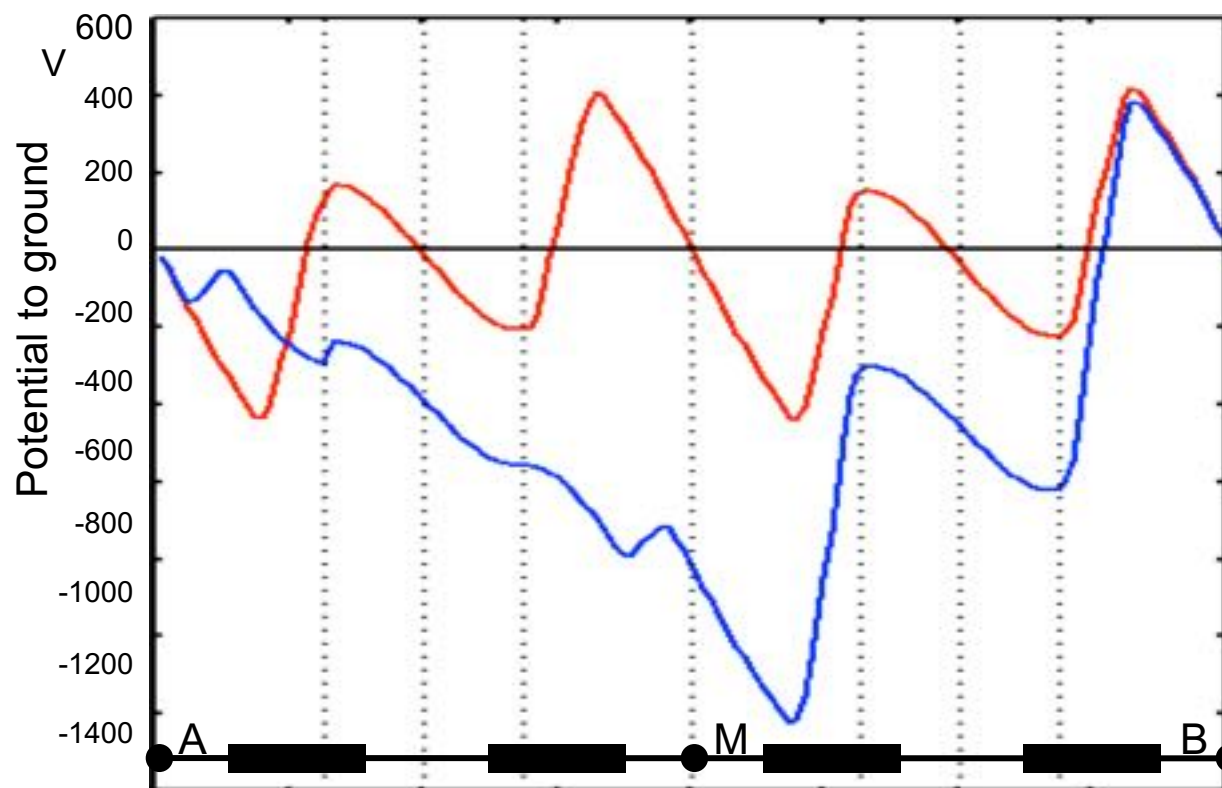
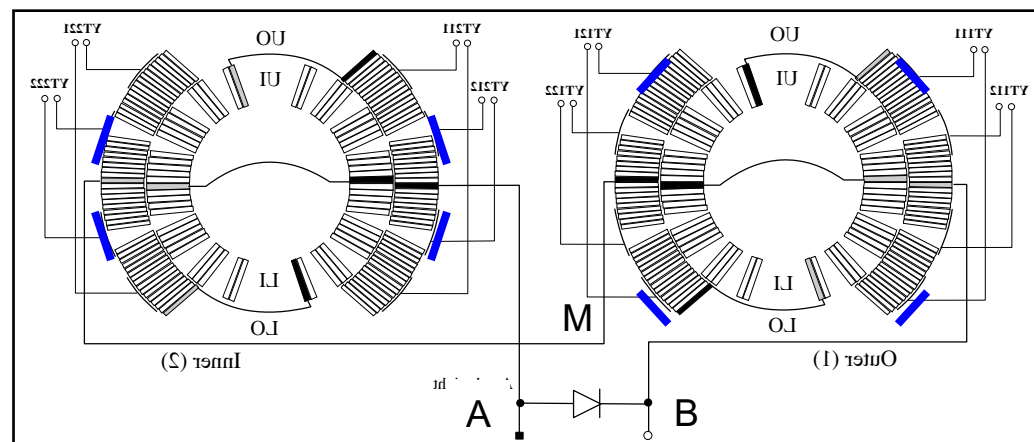


# Defect on Quench Heater Circuit

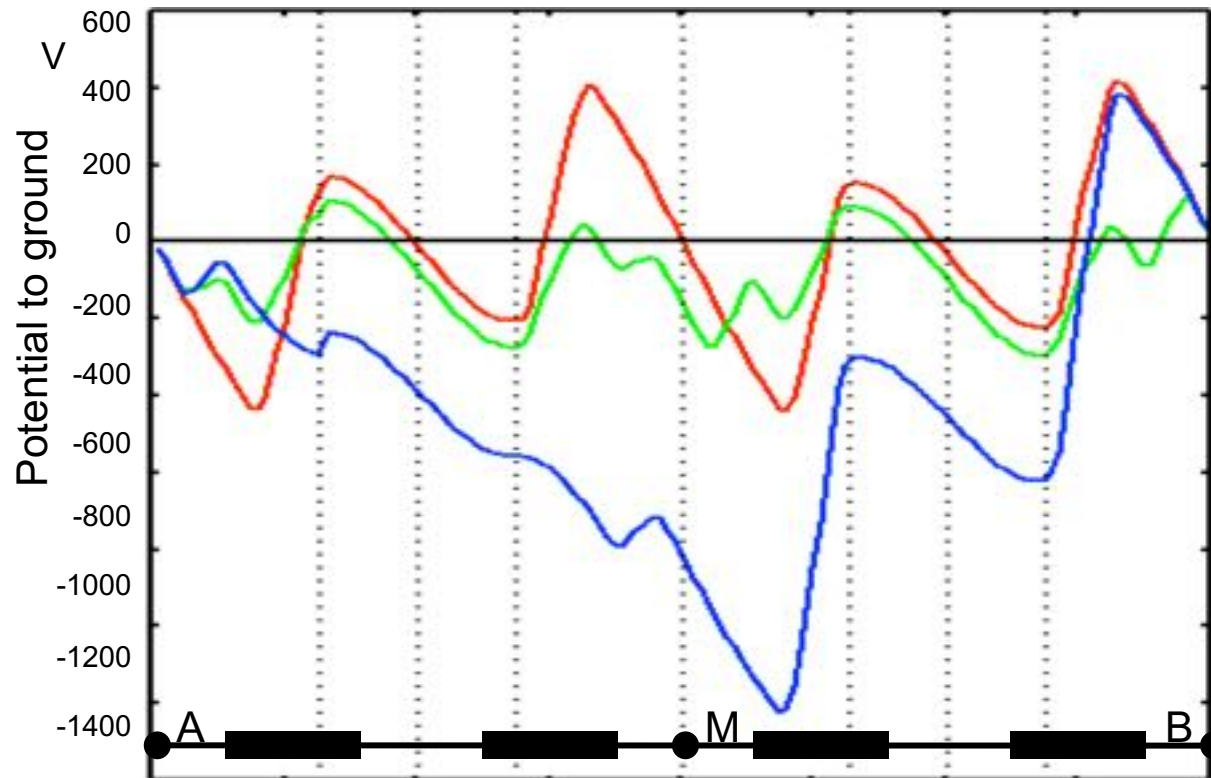
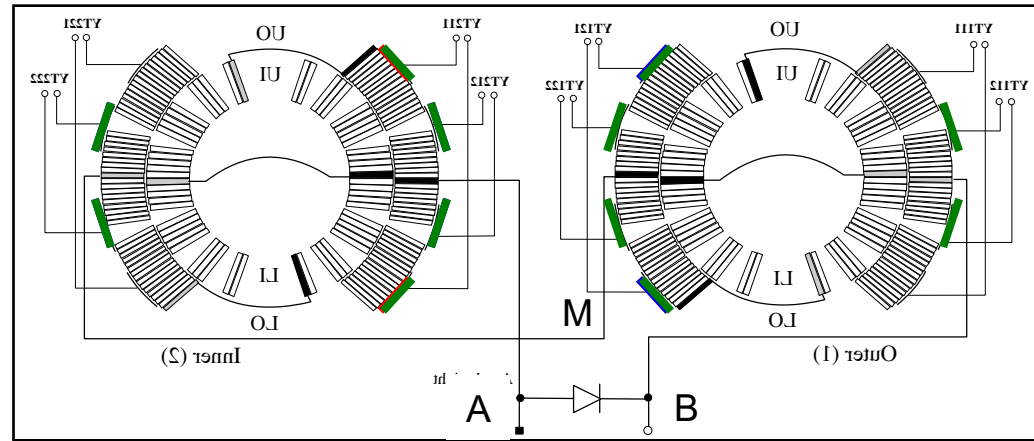




# Defect on Quench Heater Circuit



# Defect on Quench Heater Circuit



# Challenges

## → Multiscale

- Filaments  $6\ \mu\text{m}$
- Strands  $1\ \text{mm}$
- Cable  $0.1\ \text{m}$
- Magnet  $10\ \text{m}$
- String  $3.2\ \text{km}$

## → Multiphysics

- The smallest time constant determines the Runge-Kutta step

## → More advanced methods

- New software framework

