



Boer-Mulders function of pion & proton and pretzelosity of proton

Bo-Qiang Ma (马伯强)

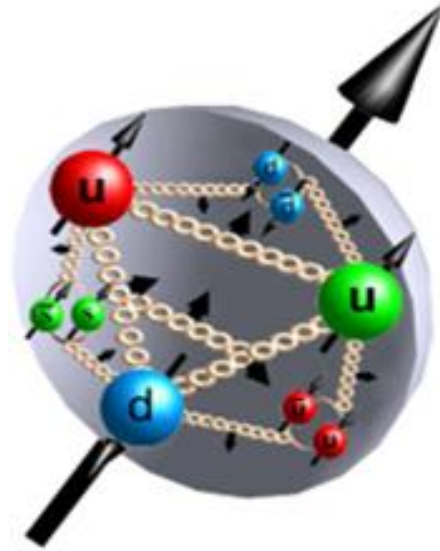
Peking Univ (北京大学)

?

Perceiving EHM through AMBER@CERN - VII
10-13 May 2022, CERN Teleworkshop

Collaborators: Enzo Barone, Stan Brodsky, Jacques Soffer, Andreas Schafer,
Ivan Schmidt, Jian-Jun Yang, Qi-Ren Zhang
and students: Bowen Xiao, Zhun Lu, Bing Zhang, Jun She, Jiakai Zhu, Xinyu Zhang,
Tianbo Liu, Xiaonan Liu

The structure of the proton



- **The most abundant piece of matter in our world.**
- **A very mysterious object with many puzzles.**

The Proton “Spin Crisis”

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

In contradiction with the naïve quark model expectation:

Naive Quark Model:

$$\Delta u = \frac{4}{3}; \quad \Delta d = -\frac{1}{3}; \quad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

How to get a clear picture of nucleon?

- PDFs are physically defined in the IMF (infinite-momentum frame) or with space-time on the light-cone.
- Whether the physical picture of a nucleon is the same in different frames?

A physical quantity defined by matrix element is frame-independent, but its physical picture is frame-dependent.

The improvement to the parton model?

- What would be the consequence by taking into account the transversal motions of partons?
- **It might be trivial in unpolarized situation. However it brings significant influences to spin dependent quantities (helicity and transversity distributions) and transversal momentum dependent quantities (TMDs or 3dPDFs).**

The Wigner Rotation

for a rest particle $(m, \vec{0}) = p^\mu$ $(0, \vec{s}) = w^\mu$

for a moving particle $L(p)p = (m, \vec{0})$ $(0, \vec{s}) = L(p)w / m$

$L(p)$ = rotationless Lorentz boost

Wigner Rotation

$$\vec{s}, p_\mu \rightarrow \vec{s}', p'_\mu$$

$$\vec{s}' = R_w(\Lambda, p)\vec{s} \quad p' = \Lambda p$$

$$R_w(\Lambda, p) = L(p')\Lambda L^{-1}(p) \quad \text{a pure rotation}$$

E. Wigner,
Ann. Math. 40(1939)149

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame
and infinite momentum frame

Or between spin states in the conventional equal time
dynamics and the light-front dynamics

$$\chi^\uparrow(T) = w[(q^- + m)\chi^\uparrow(F) - q^R\chi^\downarrow(F)];$$

$$\chi^\downarrow(T) = w[(q^- + m)\chi^\downarrow(F) + q^L\chi^\uparrow(F)].$$

H.J. Melosh, Quarks: Currents and constituents, Phys. Rev. D **9**, 1095 (1974).

F. Buccella, C.A. Savoy, and P. Sorba, Current quarks, constituent quarks and the Poincare group, Lett. Nuovo Cimento Soc. Ital. Fis. **10**, 455 (1974).

What is Δq measured in DIS

- Δq is defined by $\Delta q s_\mu = \langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle$

$$\Delta q = \langle p, s | \bar{q} \gamma^+ \gamma_5 q | p, s \rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 dx \left[q^\uparrow(x) - q^\downarrow(x) \right]$$

- Using conventional Dirac spinors

$$\Delta q = \int d^3 \vec{p} M_q \left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p}) \right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^2}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin
or quark spin in the infinite momentum frame,
not that in the rest frame of the proton

The proton spin crisis

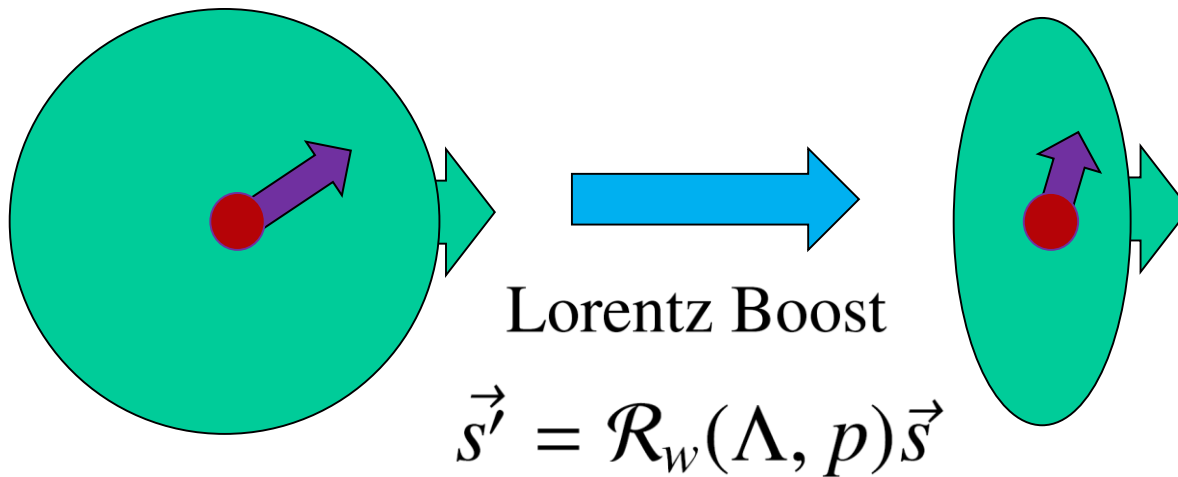
& the Melosh-Wigner rotation

- It is shown that the proton “spin crisis” or “spin puzzle” can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.

B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

An intuitive picture to understand the spin puzzle



Rest Frame

$$\sum \vec{s} = \vec{S}_p$$

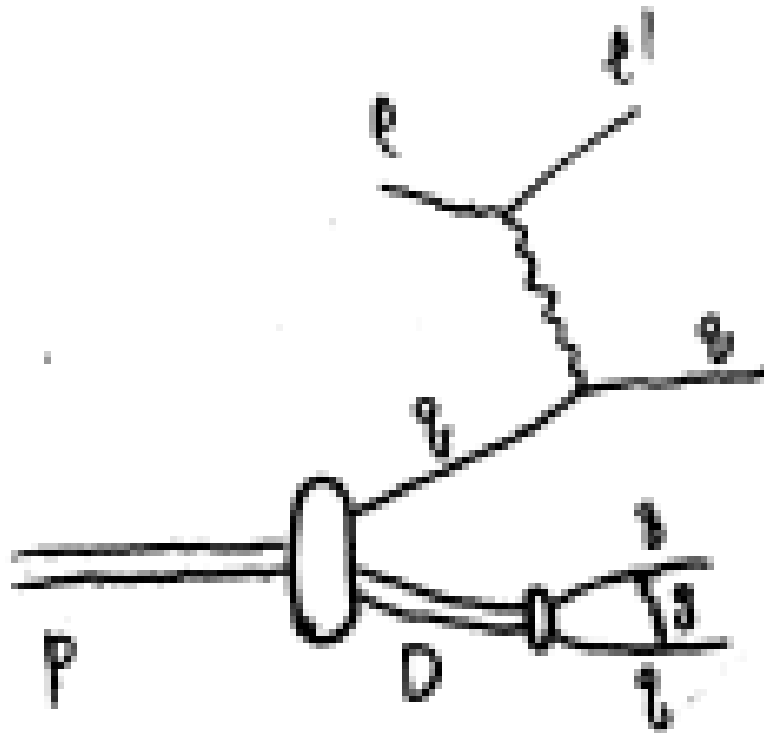
Infinite Momentum Frame

$$\sum \vec{s}' \neq \vec{S}_p$$

B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.

B.-Q. Ma, I.Schmidt, J.Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model



A relativistic quark-diquark model

- The unpolarized distribution of quark q in hadron h can be written as

$$q(x) = c_q^S a_S(x) + c_q^V a_V(x),$$

where $a_D(x)$ is

$$a_D(x) \propto \int [d^2 \mathbf{k}_\perp] |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V),$$

- BHL prescription of the light-cone momentum space wave function for quark-diquark

$$\phi(x, \mathbf{k}_\perp) = A_D \exp \left\{ -\frac{1}{8\alpha_D^2} \left[\frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x} \right] \right\},$$

A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$\Delta q(x) = \tilde{c}_q^S \tilde{a}_S(x) + \tilde{c}_q^V \tilde{a}_V(x)$$

where

$$\tilde{a}_D(x) = \int [d^2 \mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V)$$

- Melosh-Winger rotation factor

Longitudinally polarized

$$W_D(x, \mathbf{k}_\perp) = \frac{(k^+ + m_q)^2 - \mathbf{k}_\perp^2}{(k^+ + m_q)^2 + \mathbf{k}_\perp^2}$$

where $k^+ = x\mathcal{M}$, $\mathcal{M}^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$.

Different predictions in two models



Helicity distribution



SU(6) quark-diquark model:

$\Delta u(x)/u(x) \rightarrow 1$ as $x \rightarrow 1$.

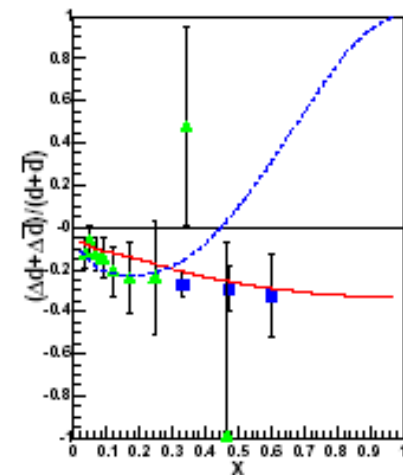
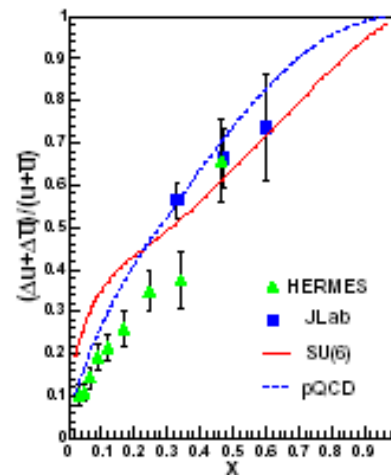
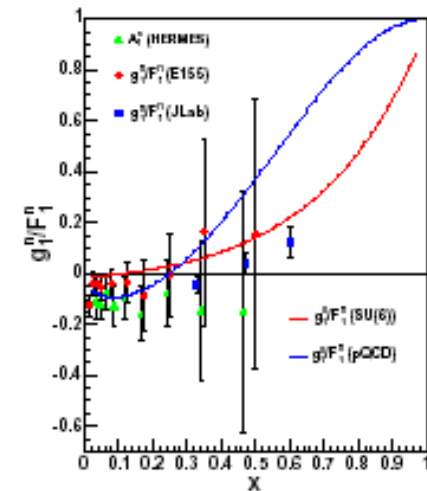
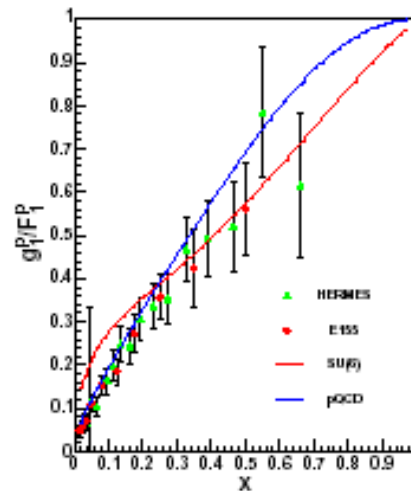
$\Delta d(x)/d(x) \rightarrow -\frac{1}{3}$ as $x \rightarrow 1$.



pQCD based counting rule analysis:

$\Delta u(x)/u(x) \rightarrow 1$ as $x \rightarrow 1$.

$\Delta d(x)/d(x) \rightarrow 1$ as $x \rightarrow 1$.



The Melosh-Wigner Rotation in Transversity

$$2\delta q = \langle p, \uparrow | \bar{q}_\lambda \gamma^\perp \gamma^+ q_{-\lambda} | p, \downarrow \rangle$$

$$\delta q(x) = \int [d^2 k_\perp] \tilde{M}_q(x, k_\perp) \Delta q_{\text{RF}}(x, k_\perp)$$

$$\tilde{M}_q(x, k_\perp) = \frac{(k^+ + m)^2}{(k^+ + m)^2 + k_\perp^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The Melosh-Wigner Rotation in Quark Orbital Angular Momentum

$$\hat{L}_q = -i \left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1} \right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

PHYSICAL REVIEW D **78**, 034025 (2008)

Transverse momentum dependent parton distributions in a light-cone quark model

B. Pasquini, S. Cazzaniga, and S. Boffi

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and Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy*

(Received 23 June 2008; published 21 August 2008)

$$\begin{aligned} h_{1T}^{\perp q}(x, \mathbf{k}_{\perp}^2) = & -P^q \int d[1]d[2]d[3] \sqrt{x_1 x_2 x_3} \\ & \times \delta(k - k_3) |\psi(\{x_i\}, \{\mathbf{k}_{i\perp}\})|^2 \\ & \times \frac{2M^2}{(m + xM_0)^2 + \mathbf{k}_{\perp}^2}, \end{aligned}$$

The Melosh-Wigner Rotation in "Pretzelocity"

B. Pasquini, S. Cazzaniga, and S. Boffi, Phys. Rev. D **78**, 034025 (2008)

The Melosh-Wigner Rotation in “Pretzelosity”

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

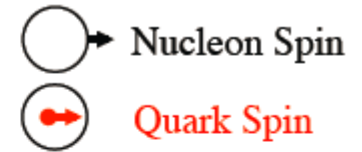






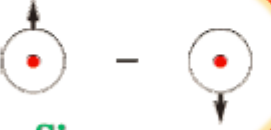



$$\text{Pretzelosity} = \Delta q - \delta q = -L_q$$

$$\text{Pretzelosity} = - \int [d^2 \mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} \Delta q_{QM}(x, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Leading-Twist TMD PDFs



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 		h_1^\perp  Boer-Mulders
	L		g_1  Helicity	h_{1L}^\perp  Long-Transversity
	T	f_{1T}^\perp  Sivers	g_{1T}  Trans-Helicity	h_1  Transversity h_{1T}^\perp  Pretzelosity

The Melosh-Wigner Rotation in five 3dPDFs

分布函数	Melosh转动因子 ($W_D(D = V, S)$)
g_{1L}	$[(x\mathcal{M}_D + m_q)^2 - p_{\perp}^2] / [(x\mathcal{M}_D + m_q)^2 + p_{\perp}^2]$
g_{1T}	$2M_N(x\mathcal{M}_D + m_q) / [(x\mathcal{M}_D + m_q)^2 + p_{\perp}^2]$
h_1	$(x\mathcal{M}_D + m_q)^2 / [(x\mathcal{M}_D + m_q)^2 + p_{\perp}^2]$
h_{1L}^{\perp}	$-2M_N(x\mathcal{M}_D + m_q) / [(x\mathcal{M}_D + m_q)^2 + p_{\perp}^2]$
h_{1T}^{\perp}	$-2M_N^2 / [(x\mathcal{M}_D + m_q)^2 + p_{\perp}^2]$

$\mathcal{M}_D^2 = \frac{m_q^2 + p_{\perp}^2}{x} + \frac{m_D^2 + p_{\perp}^2}{1-x}$ 是旁观双夸克的不变质量。

Boer-Mulders Function:

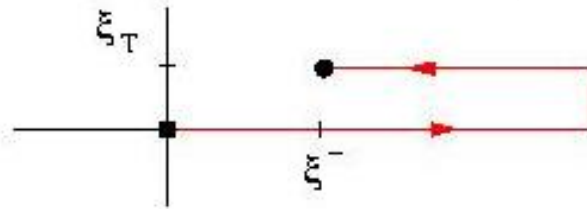
h_1^\perp



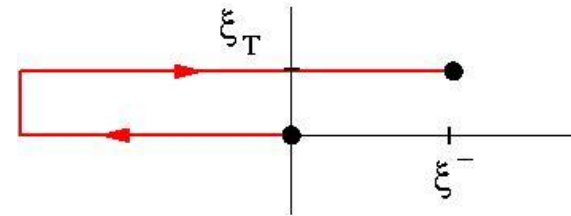
Boer-Mulders

T-odd functions with Wilson line

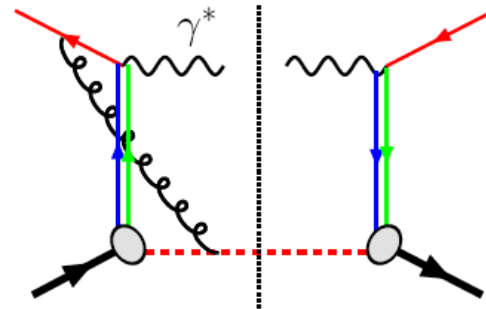
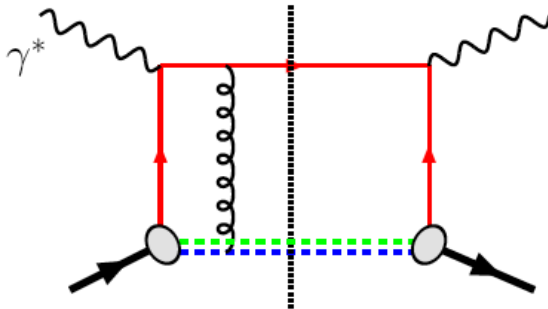
J. C. Collins, Phys.Lett.B536:43-48,2002.



SIDIS $\rightarrow \Phi^{[+]}$



Drell-Yan $\rightarrow \Phi^{[-]}$



Boer-Mulders DIS	= -	Boer-Mulders DY
Collins DIS	=	Collins DY

Boer model of BM function:

universality between pion and proton

D. Boer, Phys. Rev. D **60**, 014012 (1999).

$$h_1^{\perp a} \left(x, k_T^2 \right) = \frac{\alpha_T}{\pi} c_H^a \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$\frac{h_{1,\pi^-}^{\perp(1)\bar{u}}(x)}{h_{1,p}^{\perp(1)u}(x)} = C_u \frac{f_{1,\pi^-}^{\bar{u}}(x)}{f_{1,p}^u(x)}$$

$$C_u = \frac{M_p c_\pi^u}{M_\pi c_p^u}$$

In theory

The $\cos 2\phi$ asymmetry for Drell-Yan process can be expressed in terms of **Boer-Mulders functions**

D. Boer, P.J Mulders Phys. Rev. D 57, 5780 (1998)

$$v = \frac{2 \sum_{q=u,d} e_q^2 \mathcal{F}[\chi h_1^{\perp,q} h_1^{\perp,\bar{q}}] + (q \leftrightarrow \bar{q})}{\sum_{q=u,d} e_q^2 \mathcal{F}[f_1^q f_1^{\bar{q}}] + (q \leftrightarrow \bar{q})}$$

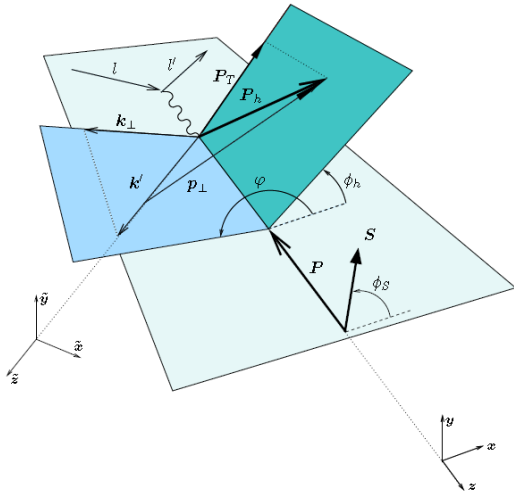
$$\mathcal{F}[\dots] = \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_\perp) \times \{\dots\} \quad \chi(\mathbf{p}_\perp, \mathbf{k}_\perp) = (2\hat{\mathbf{h}} \cdot \mathbf{p}_\perp \hat{\mathbf{h}} \cdot \mathbf{k}_\perp - \mathbf{p}_\perp \cdot \mathbf{k}_\perp) / M_p^2$$

**Parton probability interpretation
of Boer-Mulders function**



correlation between the quark transverse spin $\uparrow\downarrow$ and the quark transverse momentum \swarrow in the unpolarized hadron \bigcirc

The $\cos 2\phi$ asymmetries of unpolarized SIDIS process



$$l(\ell) + p(P) \rightarrow l'(\ell') + h(P_h) + X(P_X)$$

$$\nu = \frac{\int d\sigma \cos 2\phi}{\int d\sigma}$$

Boer-Mulders function

B. Zhang, Z. Lu, B.Q. Ma, I. Schmidt Phys.Rev.D77:054011,2008.

Collins function

Anselmino *et al.* Phys.Rev.D75:054032,2007.

$$\nu = \frac{\int \sum_a e_a^2 2x(1-y) \left\{ \frac{1}{2} \mathcal{B}[h_1^{\perp a}, H_1^{\perp a}] + \mathcal{C}[f_1^a, D_1^a] \right\}}{\int \sum_a e_a^2 x [1 + (1-y)^2] \mathcal{A}[f_1^a, D_1^a]}$$

\mathcal{B} \longrightarrow Boer-Mulders effect

\mathcal{C} \longrightarrow Cahn effect

Unpolarized distribution function

MRST2001(LO set)

Unpolarized fragmentation function

Stefan Kretzer2000

Evolution come from the unpolarized distribution function and unpolarized fragmentation function

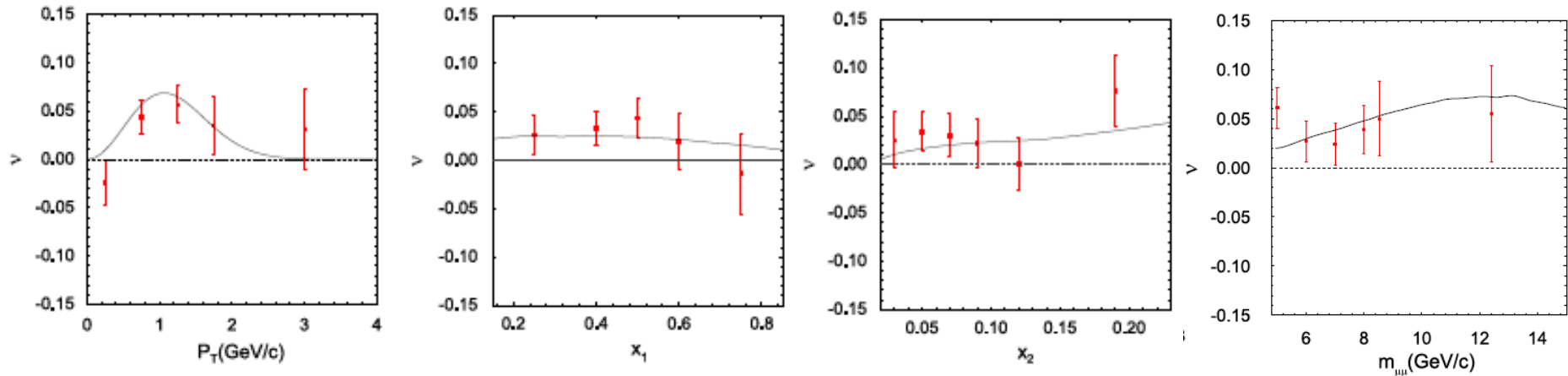
Best fit values of Boer-Mulders functions

B. Zhang, Z. Lu, B.Q. Ma, I. Schmidt Phys.Rev.D77:054011,2008.

(E866 Drell-Yan p+D Process L.Y. Zhu, J.C. Peng *et al*, Phys.Rev.Lett.99:082301,2007.)

(CERN program library MINUIT)

$4.5 \text{ GeV} < Q < 9 \text{ GeV}$ and $10.7 \text{ GeV} < Q < 15 \text{ GeV}$
 $0.15 < x_1 < 0.85$, $0.02 < x_2 < 0.24$



	Set I	Set II
H_u	3.99	4.44
H_d	3.83	-2.97
$H_{\bar{u}}$	0.91	4.68
$H_{\bar{d}}$	-0.96	4.98
p_{bm}^2	0.161	0.165
c	0.45	0.82
$\chi^2/d.o.f.$	0.79	0.79

Set I Large N_c (Pobylitsa)
 Lattice calculation (QCDSF)
 GPD approach (Buikardt, Gamberg)

$$h_1^{\perp,u} \approx h_1^{\perp,d}$$

Set II Axial-diquark model
 (Bacchetta, Schafer, Yang)

$h_1^{\perp,u}$ and $h_1^{\perp,d}$
 nearly have
 different sign

Probing Pretzelosity in pion p Drell-Yan Process

COMPASS pion p Drell-Yan process

can also measure

the pretzelosity distributions of the nucleon.

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Single spin asymmetry in πp Drell-Yan process

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Boer–Mulders function of the pion and pretzelosity distribution of the proton in the polarized pion-proton Drell–Yan process at COMPASS

Xiaonan Liu¹, Bo-Qiang Ma^{1,2,3,a}

$$\pi^- p^\uparrow \rightarrow \mu^- \mu^+ X$$

The generic q_T -weighted TSA

$$A_T^{XW_X} = \frac{\int d^2\mathbf{q}_T W_X F_T^X}{\int d^2\mathbf{q}_T F_U^1}$$

- The COMPASS Collaboration at CERN adopts a π^- beam with $P_\pi = 190$ GeV colliding on a NH_3 target which provides a great opportunity to **explore the Boer-Mulders function of the pion.**

Inputs: Boer-Mulders Function

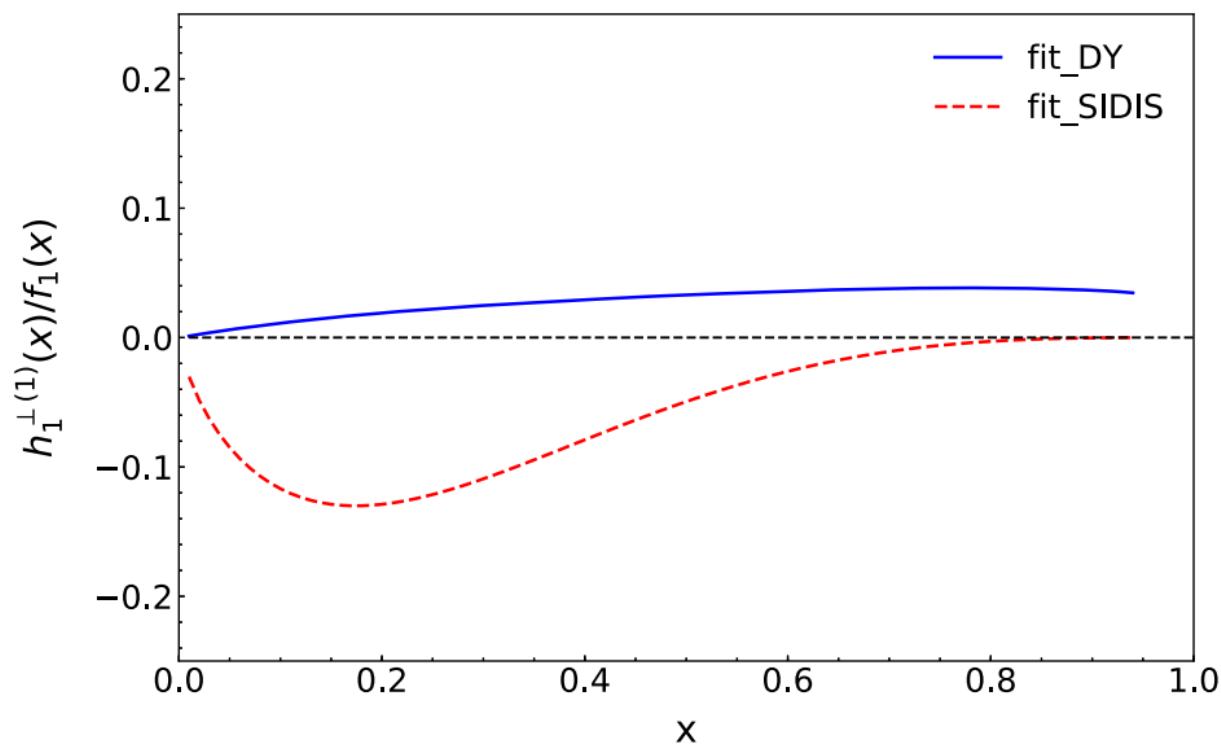


Fig. 1 The ratio $h_1^{\perp(1)}(x)/f_1(x)$ for u quark at $Q^2 = 25 \text{ GeV}^2$. The solid blue line corresponds to the DY extraction [12], and the dashed red line correspond to the SIDIS extraction [13] of the proton Boer–Mulders function

u-quark dominance assumption

$$\begin{aligned}
 & A_{\text{T}}^{\sin(2\phi - \phi_S) \frac{q_{\text{T}}}{M_{\pi}}} (x_{\pi}, x_N) \\
 &= -2 \frac{\sum_q e_q^2 \left[h_{1,\pi}^{\perp(1)\bar{q}}(x_{\pi}) h_{1,p}^q(x_N) + (q \leftrightarrow \bar{q}) \right]}{\sum_q e_q^2 \left[f_{1,\pi}^{\bar{q}}(x_{\pi}) f_{1,p}^q(x_N) + (q \leftrightarrow \bar{q}) \right]} \\
 &\approx -2 \frac{h_{1,\pi}^{\perp(1)\bar{u}}(x_{\pi}) h_{1,p}^u(x_N)}{f_{1,\pi}^{\bar{u}}(x_{\pi}) f_{1,p}^u(x_N)},
 \end{aligned}$$

$$\begin{aligned}
 & A_{\text{T}}^{\sin(2\phi + \phi_S) \frac{q_{\text{T}}^3}{2M_{\pi} M_{\text{P}}^2}} (x_{\pi}, x_N) \\
 &= -2 \frac{\sum_q e_q^2 \left[h_{1,\pi}^{\perp(1)\bar{q}}(x_{\pi}) h_{1\text{T},p}^{\perp(2)q}(x_N) + (q \leftrightarrow \bar{q}) \right]}{\sum_q e_q^2 \left[f_{1,\pi}^{\bar{q}}(x_{\pi}) f_{1,p}^q(x_N) + (q \leftrightarrow \bar{q}) \right]} \\
 &\approx -2 \frac{h_{1,\pi}^{\perp(1)\bar{u}}(x_{\pi}) h_{1\text{T},p}^{\perp(2)u}(x_N)}{f_{1,\pi}^{\bar{u}}(x_{\pi}) f_{1,p}^u(x_N)},
 \end{aligned}$$

Inputs: pretzelosity & transversity

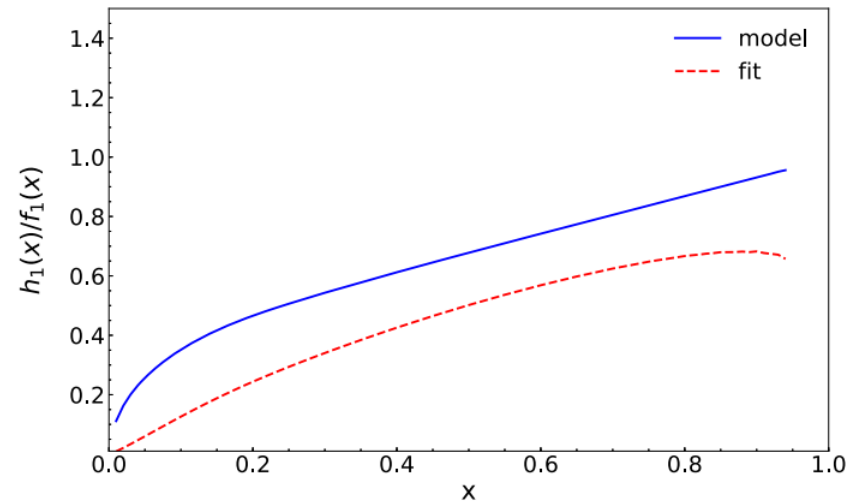
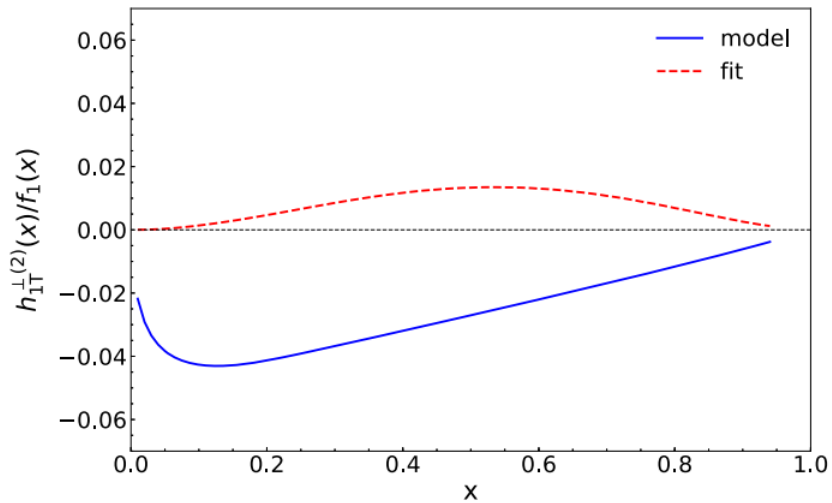


Fig. 2 Left panel: the ratio $h_{1T}^{\perp(2)}(x)/f_1(x)$ for u quark at $Q^2 = 25 \text{ GeV}^2$. Right panel: the ratio $h_1(x)/f_1(x)$ for u quark at $Q^2 = 25 \text{ GeV}^2$. The solid blue line corresponds to the pretzelosity distri-

bution [18] and the transversity distribution [19,20] calculated in the light-cone SU(6) quark-diquark model, and the dashed red line corresponds to the first extraction of the pretzelosity distribution (h_{1T}^{\perp}) [21] and the recent transversity distribution parametrizations [22]

Results: $A_T^{\sin(2\varphi - \varphi_S)q_T/M_\pi}$

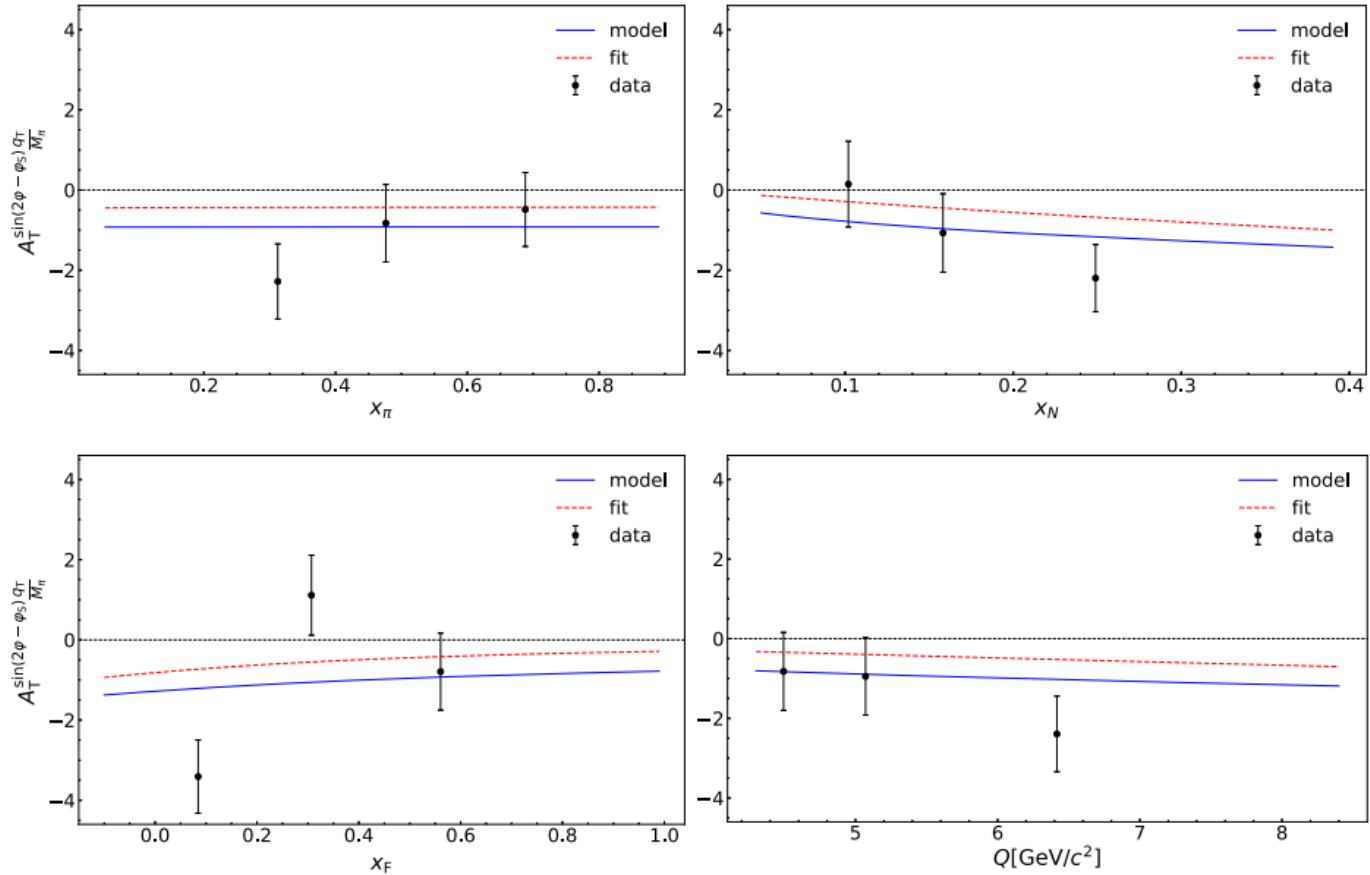


Fig. 3 Theoretical calculations and experimental statistical errors on the q_T -weighted $\sin(2\varphi - \varphi_S)$ asymmetries in various kinematic dependence for a DY measurement $\pi^- p^\uparrow \rightarrow \mu^+ \mu^- X$ with a 190 GeV/c π^- beam in the high-mass region $4 \text{ GeV}/c^2 < M_{\mu\mu} <$

$9 \text{ GeV}/c^2$ [7]. Feynman- x or x_F , is a variable of interest that sheds light on the longitudinal structure of the initial state of the interacting quark. The solid blue line represents the model calculated results, and the dashed red line represents the fitted PDF calculated results

Results: $A_T^{\sin(2\varphi+\varphi_S)q_T^3/2M_\pi M_P^2}$

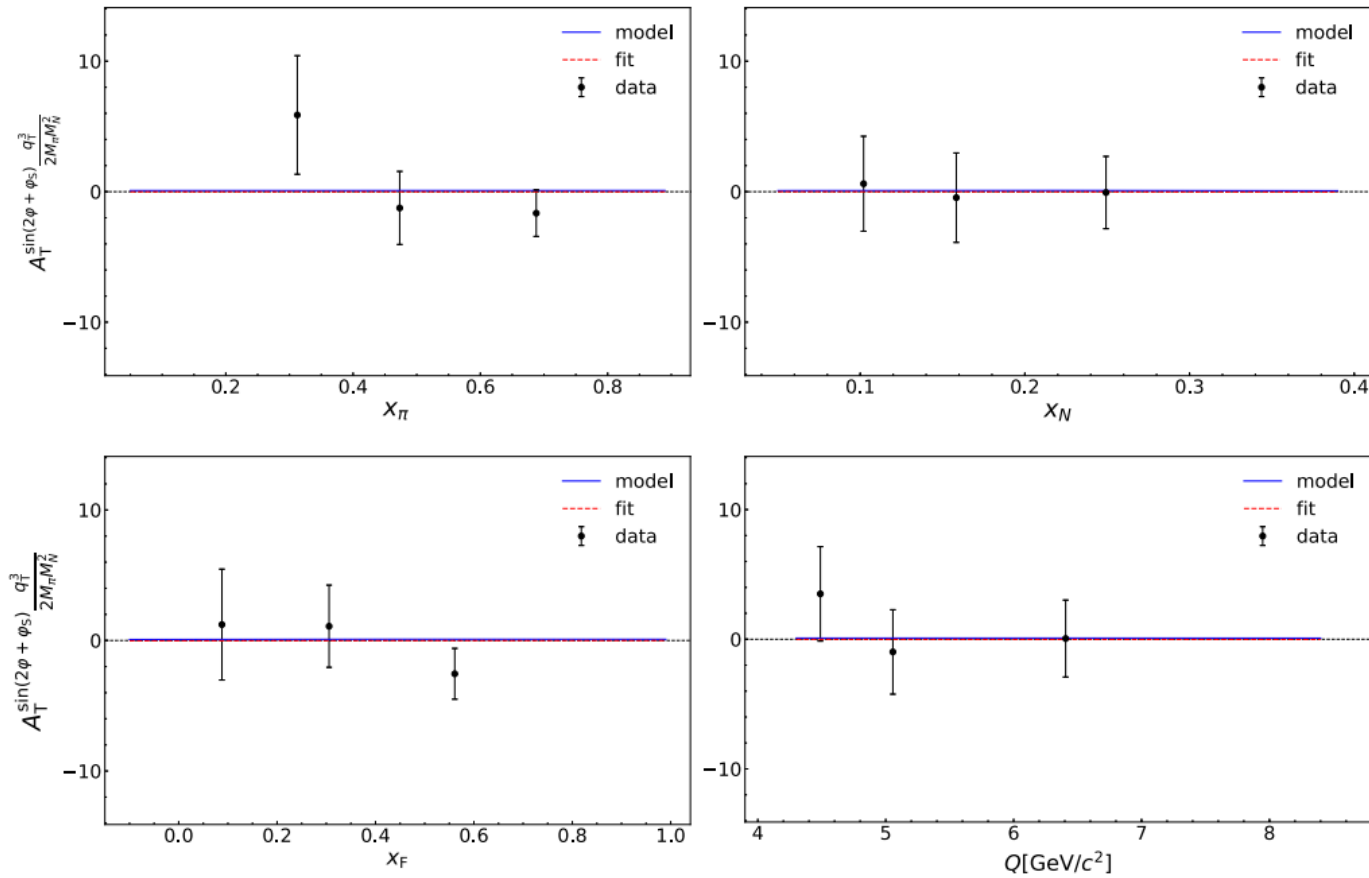


Fig. 4 Same as in Fig. 3 but for q_T -weighted $\sin(2\varphi + \varphi_S)$ asymmetries

Results: integrated over entire kinematic range

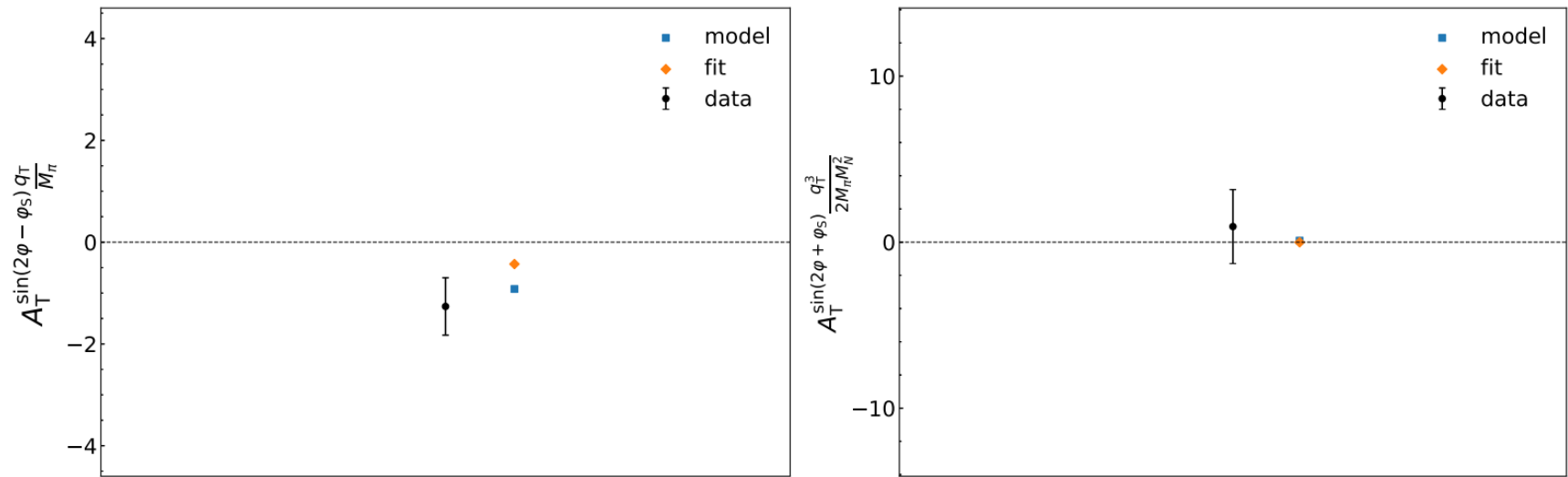


Fig. 5 q_T -weighted Drell–Yan TSAs integrated over the entire kinematic range. The blue square represents the calculated results, and the red diamond represents the fit results. The data points include the estimated corrections for systematic errors. The error bars contain statistical only

The Necessity of Polarized p \bar{p} Collider

The polarized proton antiproton Drell-Yan process

is ideal to measure

the pretzelosity distributions of the nucleon.

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Probing the leading-twist transverse-momentum-dependent parton distribution function h_{1T}^\perp via the polarized proton-antiproton Drell-Yan process

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Pretzelosity in SIDIS

- Pretzelosity can be measured through $\sin(3\phi_h - \phi_S)$ asymmetry in the SIDIS process, where the cross section can be written as

$$\frac{d^6\sigma_{UT}}{dx dy d\phi_S dz d^2\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \left\{ (1 - y + \frac{1}{2}y^2) F_{UU} + S_{\perp} \sin(3\phi_h - \phi_S) (1 - y) F_{UT}^{\sin(3\phi_h - \phi_S)} + \dots \right\}, \quad (23)$$

with $F_{UU} = \mathcal{F}[\omega_1 f_1 D_1]$, $F_{UT}^{\sin(3\phi_h - \phi_S)} = \mathcal{F}[\omega_2 h_{1T}^{\perp} H_1^{\perp}]$

- The $\sin(3\phi_h - \phi_S)$ asymmetry

$$A_{UT}^{\sin(3\phi_h - \phi_S)} = \frac{\frac{2\alpha^2}{sxy^2} (1 - y) F_{UT}^{\sin(3\phi_h - \phi_S)}}{\frac{2\alpha^2}{sxy^2} (1 - y + \frac{1}{2}y^2) F_{UU}}. \quad (24)$$

Conclusions

- The **relativistic effect** of quark transversal motions plays a significant role in spin-dependent quantities: helicity and transversity, five 3dPDFs or TMDs, Boer-Mulders Functions.
- The COMPASS measurement represents the **first** evidence for the Boer-Mulders effect in **polarized** Drell-Yan process.
- Still unable to distinguish the sign of pretzelosity due to large uncertainties.
- More precision experiments are needed to measure **new quantities** of the nucleon.