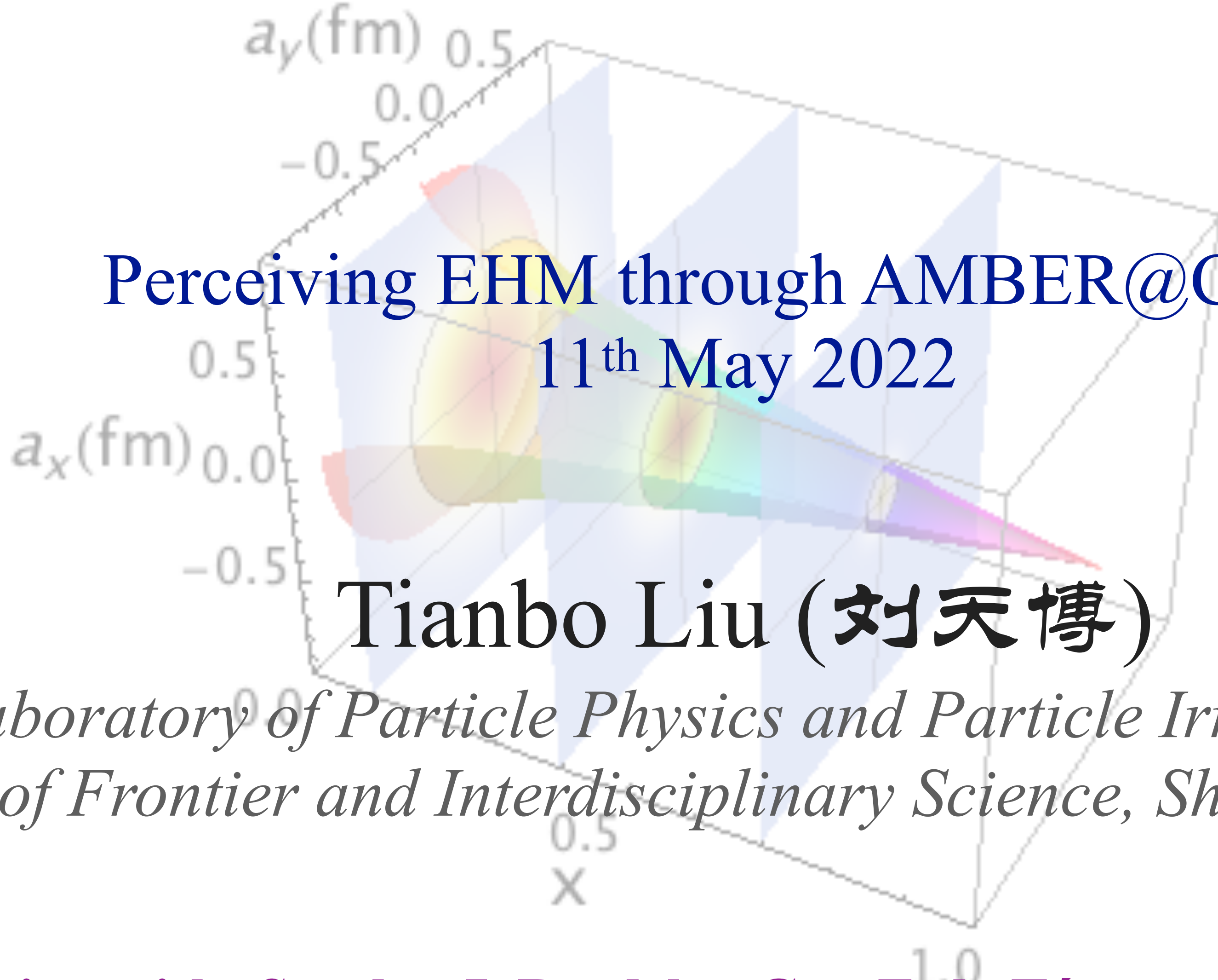
AdS/QCD and Gluon Matter Distribution in the Proton and Pion



Perceiving EHM through AMBER@CERN
11th May 2022

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粒子科学技术研究中心
Research Center for Particle Science and Technology

Gauge/Gravity Duality and LF holography

Maldacena's conjecture

J.M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999).

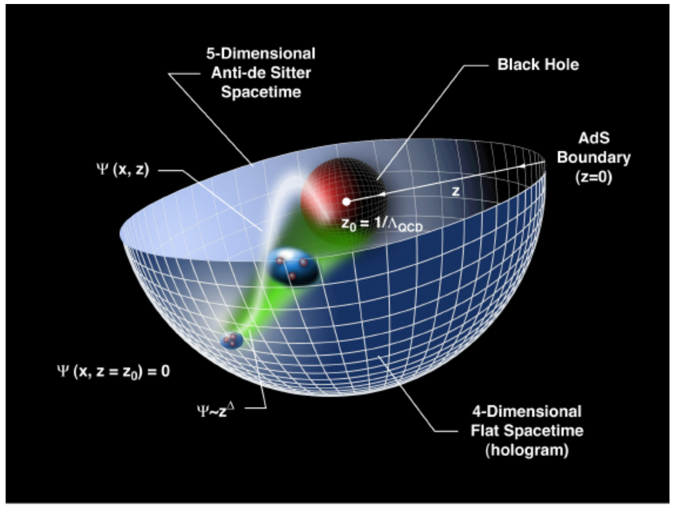
Gauge field theory in *d*-dim

⇕

Gravitational theory in *d+1*-dim

A realization: AdS / CFT

semiclassical gravity approximation to strongly coupled QFTs



[Brodsky & de Téramond]

Light-front holographic QCD

QCD: conformal symmetry is broken by quark masses and quantum effects

Asymptotic freedom

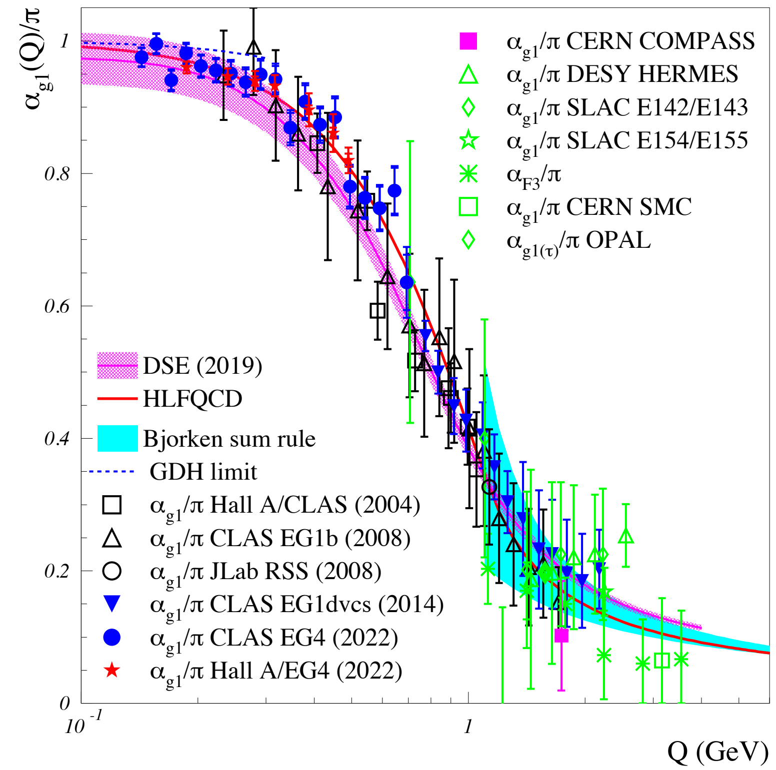
Confinement and an infrared fixed point

"bottom-up" approach: modify the background AdS space

impact LF variable-$\zeta \Leftrightarrow z$ holographic variable in AdS

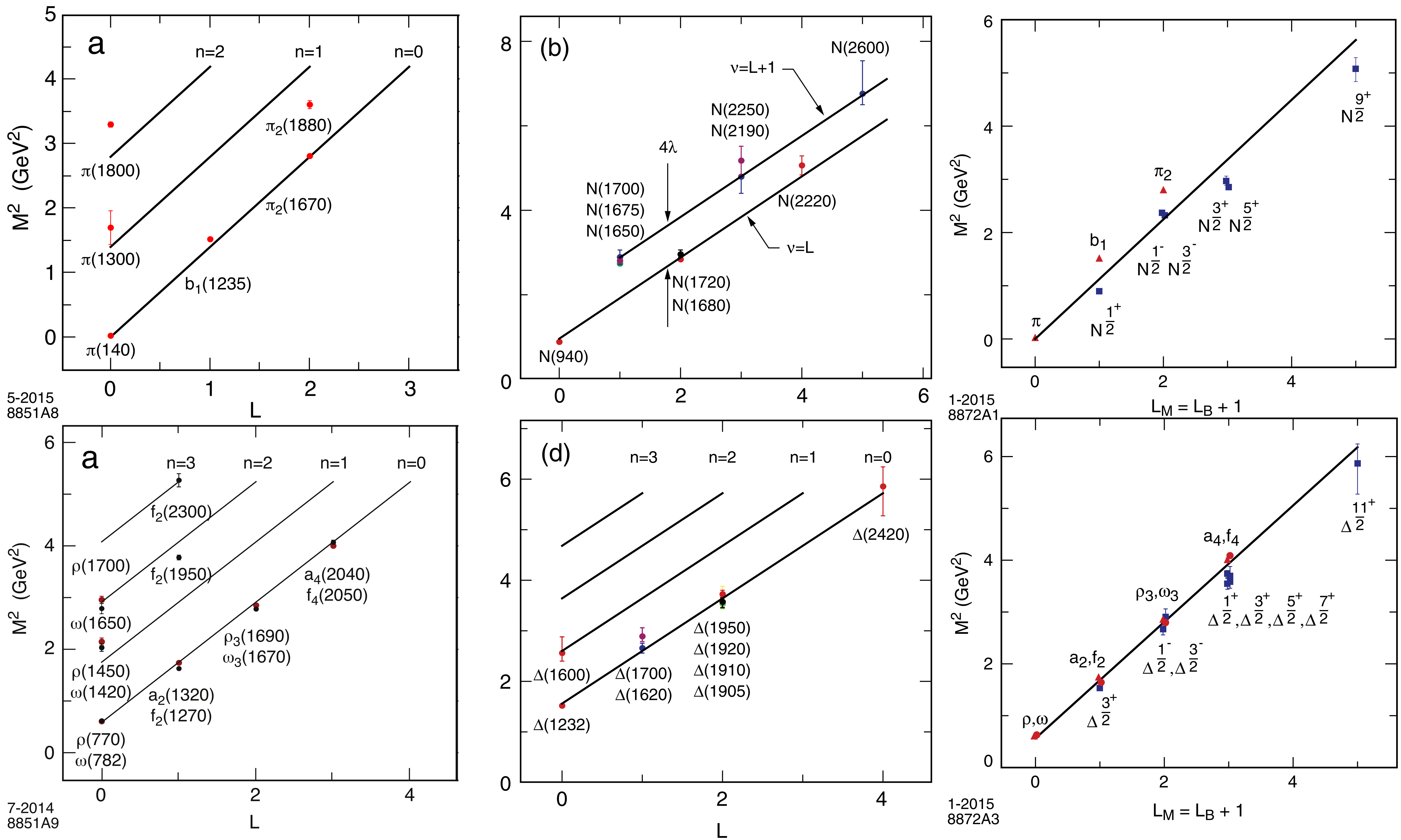
measuring the separation of partons in a hadron

G.F. de Téramond and S.J. Brodsky, Phys. Rev. Lett. 102, 081601 (2009);
S.J. Brodsky and G.F. de Téramond, Phys. Rev. Lett. 96, 201601 (2006);
Phys. Rev. D 77, 056007 (2008); Phys. Rev. D 78, 025032 (2008).



A. Deur, V. Burkert, J.P. Chen, W. Korsch, arXiv:2205.01169 [hep-ph].

Hadron Spectrum in LF Holography



S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015);
H.G. Dosch, G.F. de Téramond, S.J. Brodsky, Phys. Rev. D 91, 045040 (2015); D91, 085016 (2015).

EM Form Factors in Holographic QCD

Form factor of a spinless hadron

$$\int d^4x dz \sqrt{g} \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_M \Phi_P(x,z) A^M(x,z)$$

→ The coupling of an external EM field propagating in AdS space to a hadron mode

$$\sim (2\pi)^4 \delta^4(P' - P - q) \epsilon_\mu (P + P')^\mu F(q^2)$$

→ EM form factor in physical spacetime

$$x^M = (x^\mu, z) \qquad \sqrt{g} = (R/z)^5$$

J. Polchinski and M.J. Strassler, JHEP 05 (2003) 012.

$$\Phi_P(x,z) = e^{iP\cdot x}\Phi(z) \qquad A_\mu(x,z) = e^{iq\cdot x}V(q^2,z)\epsilon_\mu(q), \qquad A_z = 0$$

Extracting the momentum conservation factor

$$F(Q^2) = \int \frac{dz}{z^3} V(Q^2, z) \Phi^2_\tau(z)$$

For hadron modes scale as $\Phi_\tau \sim z^\tau$ at small $z \sim 1/Q$

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$$

recover the hard-scattering power scaling

S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. 31, 1153 (1973).

V.A. Matveev, R.M. Muradian, A.N. Tevkhelidze, Lett. Nuovo Cimento 7, 719 (1973).

EM Form Factors in Holographic QCD

Hadron wave function of twist-τ (soft-wall)

$$\Phi_\tau(z) = \sqrt{\frac{2}{\Gamma(\tau-1)}}\kappa^{\tau-1} z^\tau e^{-\kappa^2 z^2/2}$$

Vector current

$$V(Q^2, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/4\kappa^2} e^{-\kappa^2 z^2 x/(1-x)} = 4\kappa^4 z^2 \sum_{n=0}^{\infty} \frac{L_n^1(\kappa^2 z^2)}{M_n^2 + Q^2}$$

have poles at $-Q^2 = M_n^2 = 4\kappa^2(n+1)$

H.R. Grigoryan and A.V. Radyushkin, Phys. Rev. D 76, 095007 (2007).

compare with LFHQCD spectral formula $M_n^2 = 4\kappa^2\left(n + \frac{J+L}{2}\right)$

corresponding to the Regge trajectory $J = L = 1$

ρ meson trajectory is $J = L + 1 = 1$ shift poles to $-Q^2 = M_{\rho_n}^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$

S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015).

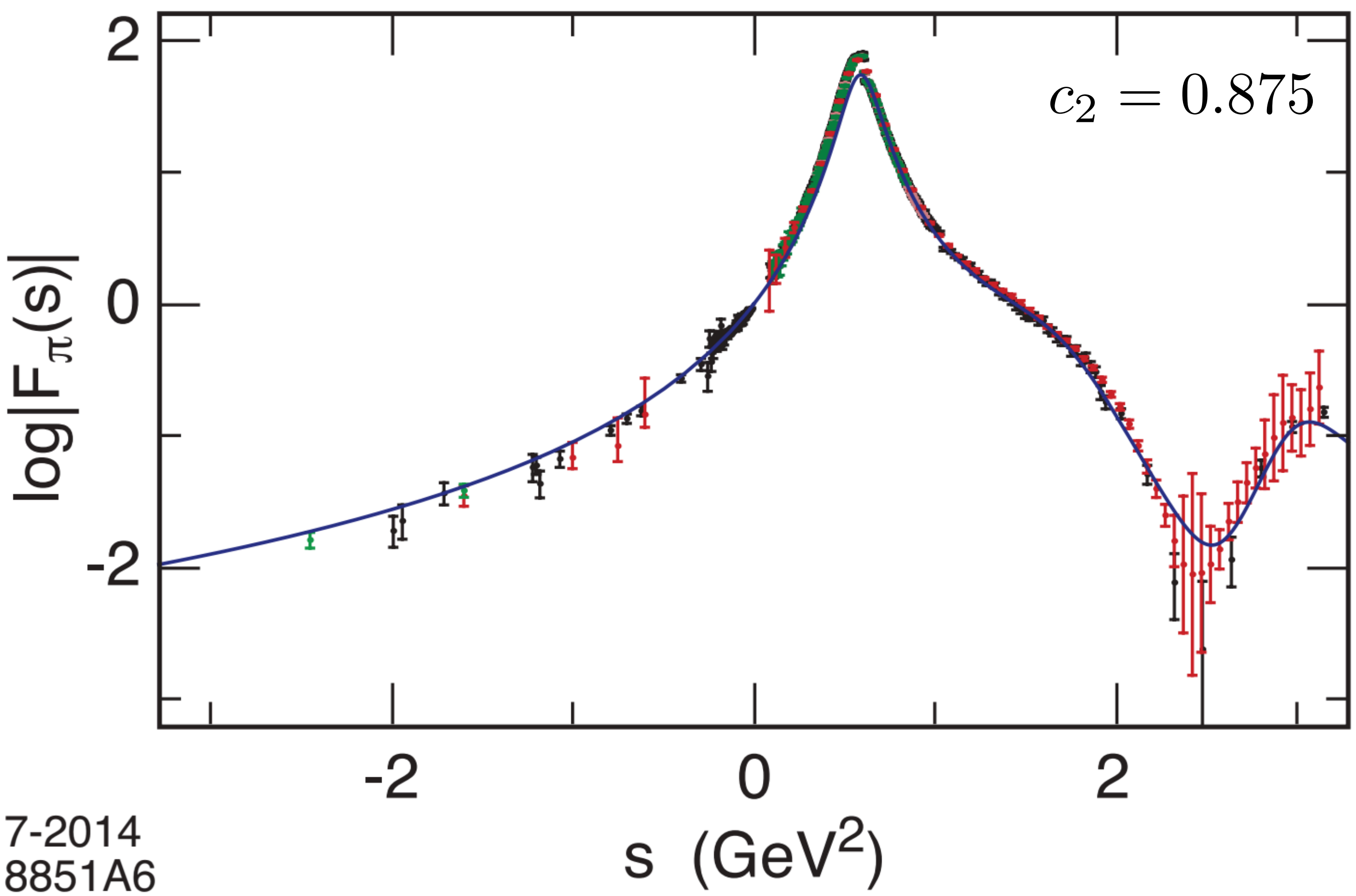
Form factor

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M^2_{\rho_{n=0}}}\right)\left(1 + \frac{Q^2}{M^2_{\rho_{n=1}}}\right)\cdots\left(1 + \frac{Q^2}{M^2_{\rho_{n=\tau-2}}}\right)}$$

S.J. Brodsky and G.F. de Téramond, Phys. Rev. D 77, 056007 (2008).

Pion EM Form Factor

Pion form factor compared with data



$$F_\pi(t) = \sum_\tau P_\tau F_\tau(t) \qquad \sum_\tau P_\tau = 1$$

Truncated at twist-$\tau = 4$

$$F_\pi(t) = c_2 F_{\tau=2}(t) + (1 - c_2) F_{\tau=4}(t)$$

G.F. de Téramond and S.J. Brodsky, Proc. Sci. LC2010 (2010) 029.
S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015). [Sec. 6.1.5]

EM Form Factors in Holographic QCD

Nucleon EM form factor: spin-nonflip (Dirac form factor)

$$\int d^4x dz \sqrt{g}\bar{\Psi}_{P'}(x,z) e_A^M \Gamma^A A_M(x,z) \Psi_P(x,z)$$

→ The coupling of an external EM field propagating in AdS space to a nucleon mode

$$\sim (2\pi)^4 \delta^4(P' - P - q)\epsilon_\mu \bar{u}(P')\gamma^\mu F_1(q^2) u(P)$$

→ Dirac form factor in physical spacetime

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB} \qquad \Gamma^A = (\gamma^\mu, -i\gamma^5) \qquad e_A^M = \left(\frac{z}{R}\right)\delta_A^M$$

Nucleon wave function

$$\Psi_+(z) \sim z^{\tau+1/2} e^{-\kappa^2 z^2/2}, \qquad \Psi_-(z) \sim z^{\tau+3/2} e^{-\kappa^2 z^2/2}$$

G.F. de Téramond, H.G. Dosch, S.J. Brodsky, Phys. Rev. D 87, 075005 (2013).

Dirac form factor

$$F_1^N(Q^2) = \int \frac{dz}{z^4} V(Q^2, z)[g_+ \Psi_+^2(z) + g_- \Psi_-^2(z)]$$

EM Form Factors in Holographic QCD

Form factor for arbitrary twist-τ state

S.J. Brodsky and G.F. de Téramond, Phys. Rev. D 77, 056007 (2008);

S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015).

$$F_1(t) = c_\tau F_\tau(t) + c_{\tau+1} F_{\tau+1}(t)$$

$$F_\tau = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right) \qquad N_\tau = \frac{\sqrt{\pi}\Gamma(\tau-1)}{\Gamma(\tau - \frac{1}{2})}$$

For integer τ, it gives the pole structure: $M_n^2 = 4\lambda(n + \frac{1}{2}), n = 0, 1, 2, \ldots, \tau - 2$

It has the same structure of Veneziano amplitude $B(1 - \alpha(s), 1 - \alpha(t))$ with the s-channel dependence replaced by a fixed pole $1 - \alpha(s) \to \tau - 1$

$$\alpha(t) = \frac{t}{4\lambda} + \frac{1}{2}$$

One can fix the mass scale λ with spectroscopy, *e.g.*, ρ/ω trajectory: $\sqrt{\lambda} = \kappa = 0.534\,\mathrm{GeV}$

For large $Q^2 = -t$, it has the scaling behavior $F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$

Nucleon Dirac Form Factor

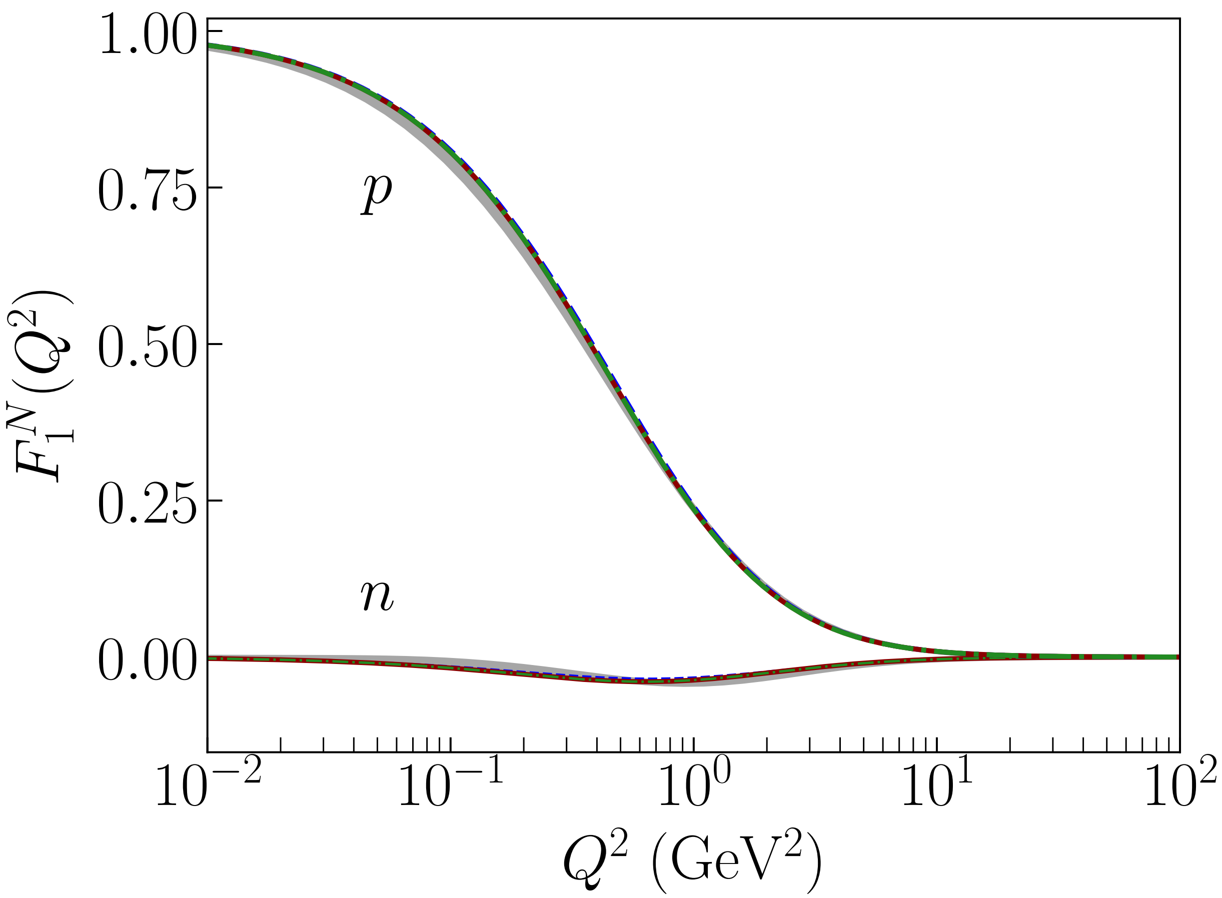
$$F_1(t) = \sum_{q,\tau} e_q [c_{\tau,q} F_\tau(t) + c_{\tau+1,q} F_{\tau+1}(t)]$$

We fit the coefficients for three cases:

i) Only the valence (τ=3) state contribution

ii) Truncate at τ=5, including a pair of $u\bar{u}$ or $d\bar{d}$

iii) Truncate at τ=5, including a pair of $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$



Match recent extraction of nucleon EM form factors:
Z. Ye, J. Arrington, R.J. Hill, G. Lee, Phys. Lett. B 777, 8 (2018).

From Form Factors to Parton Distributions

Writing the form factor in terms of GPD at zero skewness

$$F_1^q(t) = \int_0^1 dx\; H_v^q(x,t)$$

Express the form factor with the Euler integral representation

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right)$$

$$B(u,v) = \int_0^1 dy\, y^{u-1}(1-y)^{v-1} \qquad y = w(x)$$

$w(x)$ is a reparametrization function

$$H^q(x,t) = \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w(x)^{-\frac{1}{2}} w'(x) e^{(t/4\lambda)\log[1/w(x)]}$$
$$= q_\tau(x)\exp[tf(x)],$$

$$q_\tau(x) = \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w(x)^{-\frac{1}{2}} w'(x) \qquad f(x) = \frac{1}{4\lambda}\log\left(\frac{1}{w(x)}\right)$$

The collinear distribution $q(x)$ and the profile function $f(x)$ are related by a universal τ-*independent* reparametrization function $w(x)$.

Constraints on w(x)

Mathematical constraints:

$$B(u,v) = \int_0^1 dy\, y^{u-1}(1-y)^{v-1} \qquad\qquad y = w(x)$$

$$w(0) = 0, \qquad w(1) = 1, \qquad w'(x) \geq 0 \quad \text{for} \quad x \in [0,1]$$

Physical requirements:

Small-x behavior: $H_v^q(x,t) \sim x^{-t/4\lambda} q_v(x)$ Regge theory motivated ansatz

$$w(x) \sim x$$

Large-x behavior: $q_\tau(x) \sim (1-x)^{2\tau-3}$ Drell-Yan inclusive counting rule

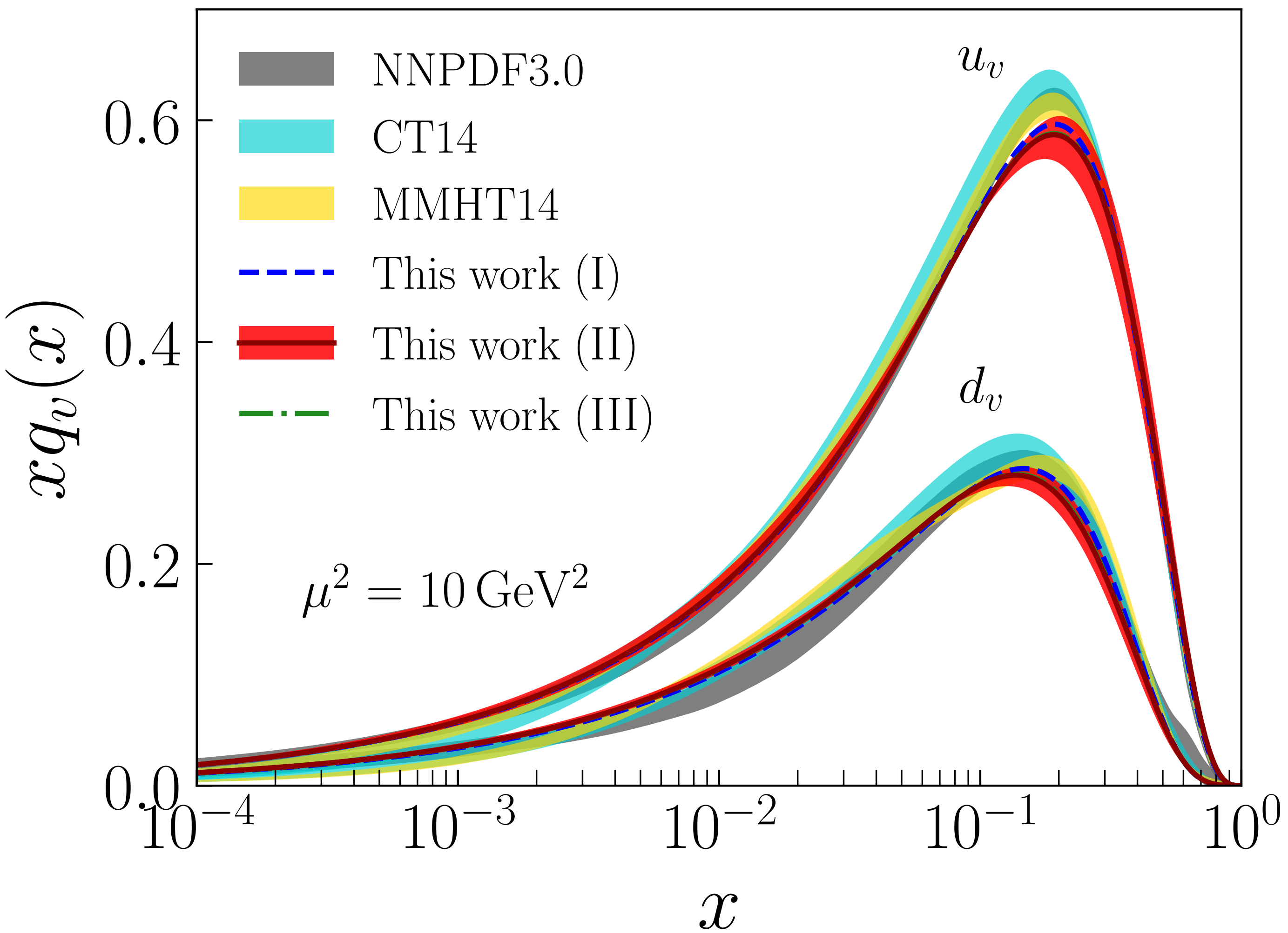
$$w(x) = 1 - (1-x)w'(1) + \frac{1}{2}(1-x)^2 w''(1) + \cdots$$

$$w'(1) = 0 \quad \text{and} \quad w''(1) \neq 0$$

$$f(x) = \frac{1}{4\lambda}\log\left(\frac{1}{w(x)}\right) \qquad\qquad f'(1) = 0 \;\; \text{and} \;\; f''(1) \neq 0$$

Quark Distribution in the Proton

Nucleon PDFs in comparison with global fits



A parameterization form for w(x):

$$w(x) = x^{1-x} e^{-a(1-x)^2}$$

$$a = 0.48 \pm 0.04$$

one can choose other forms, but *universal* for all distributions

Evolved from the matching scale 1.06 ± 0.15 GeV

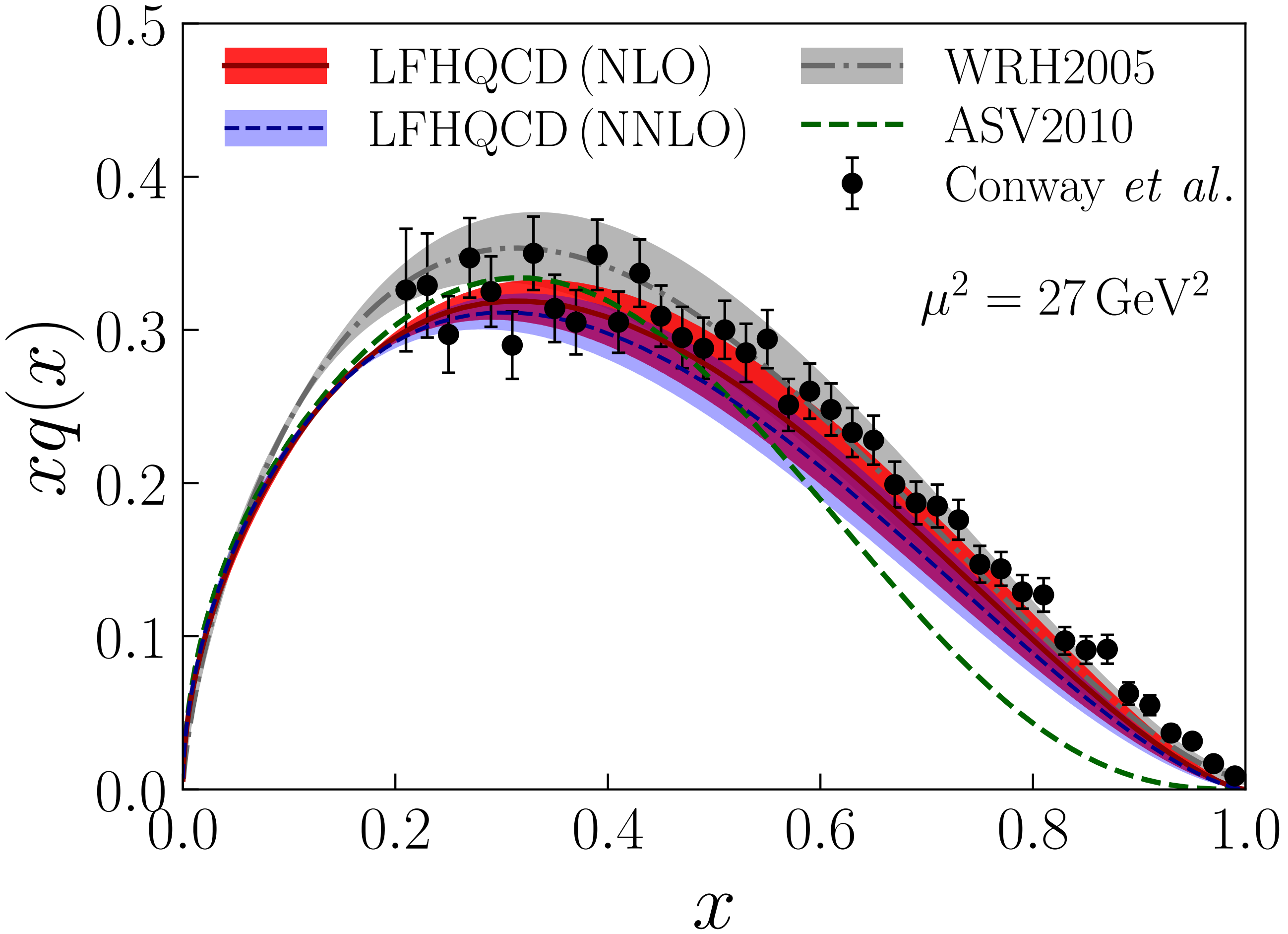
Red bands: the uncertainties of the matching scale.

G.F. de Téramond, TL, R.S. Sufian, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 120, 182001 (2018).

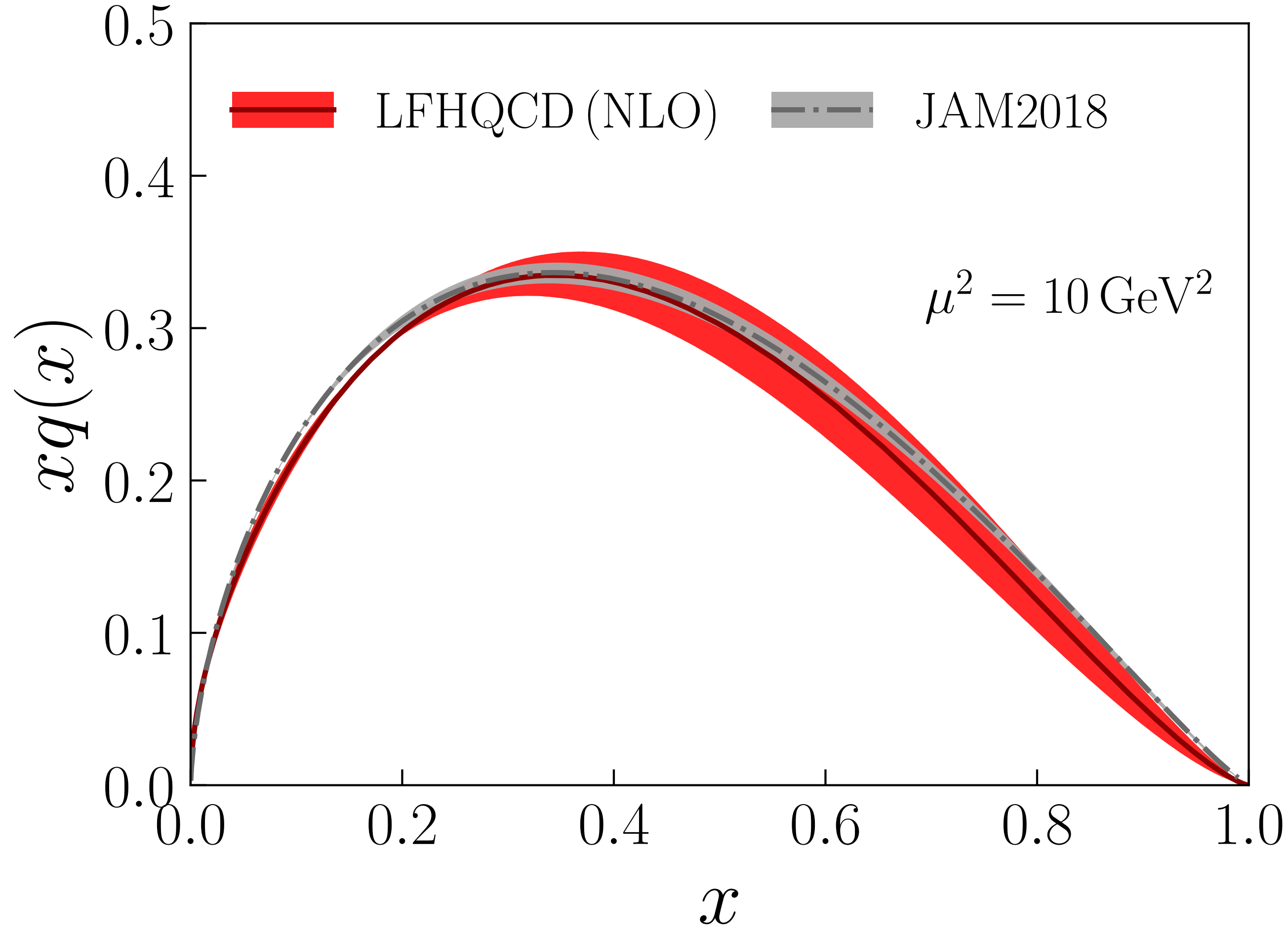
Quark Distribution in the Pion

Prediction for pion PDF with the same $w(x)$

$$w(x) = x^{1-x} e^{-a(1-x)^2}$$



Evolved from the matching scale.
Using the same w(x).



Comparison with a new global fit

de Téramond, TL, Sufian, Dosch, Brodsky, Deur, Phys. Rev. Lett. 120, 182001 (2018).

Barry, Sato, Melnitchouk, Ji, Phys. Rev. Lett. 121, 152001 (2018).

Quark Mass Correction

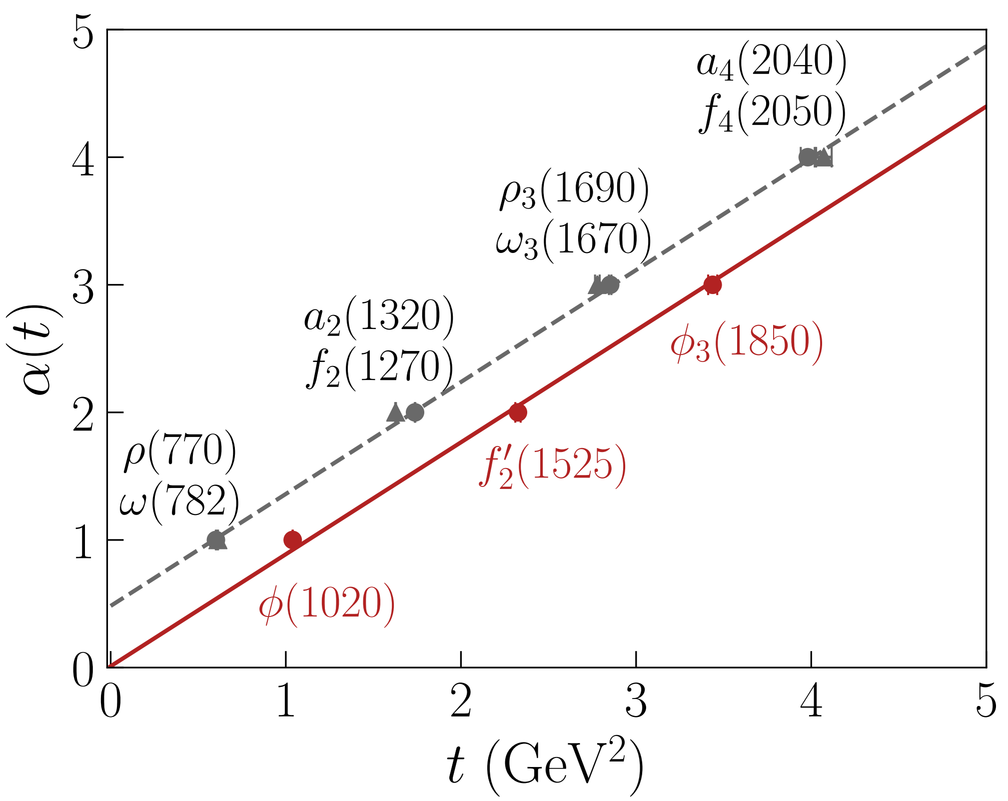
FF with twist-τ: $F_\tau(t) = \frac{1}{N_\tau} B(\tau - 1, 1 - \alpha(t))$ $\alpha(t) = \alpha(0) + \alpha' t = \frac{1}{2} + \frac{t}{4\lambda} - \frac{\Delta M^2}{4\lambda}$

Calculate the mass shift with effective quark mass:

$$M^2 = 4\lambda\left(n + \frac{L+J}{2}\right) + \Delta M^2[m_1, m_2] \qquad \Delta M^2[m_1, m_2] = \frac{1}{N}\int_0^1 dx \left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right) e^{-\frac{1}{\lambda}\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015).

Determine the intercept $\alpha(0)$, or mass shift, from Regge trajectory



$$\lim_{Q^2\to\infty} F_\tau(Q^2) = \Gamma(\tau - 1)\left(\frac{1}{\alpha' Q^2}\right)^{\tau-1}$$

Scaling behavior unchanged

$$\alpha_\rho(0) = \alpha_\omega(0) = 0.496$$

$$\alpha_\phi(0) = 0.010$$

R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

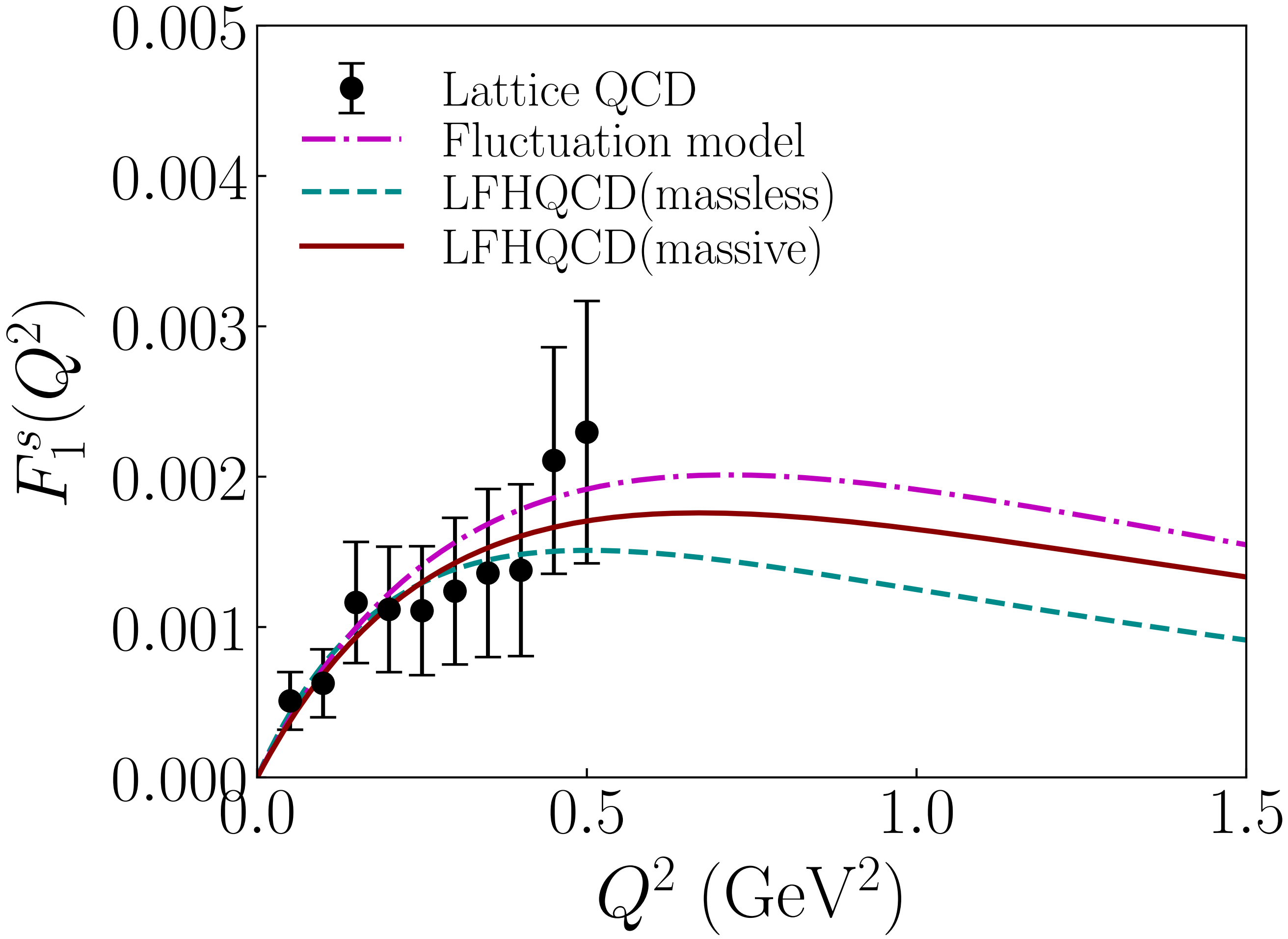
Strange Form Factor in LFHQCD

Strange form factor with nonzero quark mass:

$$F_1^s(Q^2) = (1-\eta)N_s[F^{\phi}_{\tau=5}(Q^2) - F^{\phi}_{\tau=6}(Q^2)] + \eta N_s[F^{\omega}_{\tau=5}(Q^2) - F^{\omega}_{\tau=6}(Q^2)],$$

ϕ is nearly a pure $s\bar{s}$ state

$\eta \approx 0$



$$Q^8 F_1^s(Q^2) \to \text{Const}$$

$$\text{Const} = 1680 N_s \lambda^4 \simeq 0.5 \text{ GeV}^8$$

Lattice data from:

R.S. Sufian, Y.-B. Yang, A. Alexandru, T. Draper, J. Liang, and K.-F. Liu, Phys. Rev. Lett. 118, 042001 (2017).

R.S. Sufian, Y.-B. Yang, J. Liang, T. Draper, and K.-F. Liu, Phys. Rev. D 96, 114504 (2017).

R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

PDF in LFHQCD with Quark Mass

Express the form factor with the Euler integral representation:

$$F_\tau(t) = \frac{1}{N_\tau} \int_0^1 dx\, w'(x) w(x)^{-\frac{t}{4\lambda}-\frac{1}{2}} [1 - w(x)]^{\tau-2} e^{-\frac{\Delta M^2}{4\lambda} \log\left(\frac{1}{w(x)}\right)}.$$

PDF:

$$q_\tau(x) = \frac{1}{N_\tau} [1 - w(x)]^{\tau-2} w(x)^{-\frac{1}{2}} w'(x) e^{-\frac{\Delta M^2}{4\lambda} \log\left(\frac{1}{w(x)}\right)}$$

with the same $w(x)$ that satisfies

$$w(0) = 0, \qquad w(1) = 1, \qquad w'(x) \geq 0 \qquad \text{for} \quad x \in [0,1]$$

$$w(x) \sim x \quad \text{at small-}x, \quad \text{and} \quad w'(1) = 0 \quad \text{and} \quad w''(1) \neq 0$$

Large-x: $q_\tau(x) \sim (1-x)^{2\tau-3}$ — the counting rule is unchanged

Small-x: $q_\tau(x) \sim x^{-\alpha(0)} \sim x^{-\frac{1}{2}+\frac{\Delta M^2}{4\lambda}}$ — softened by a factor $x^{\Delta M^2/4\lambda}$

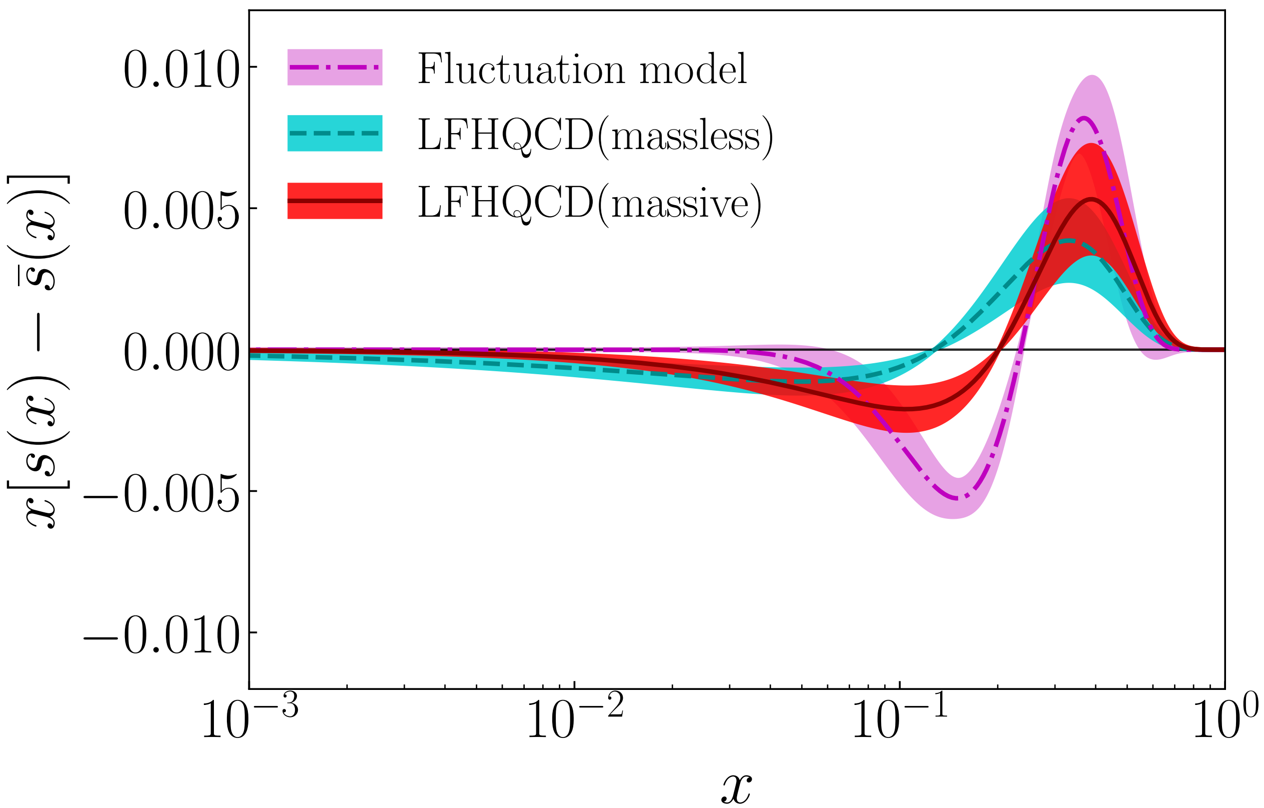
R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

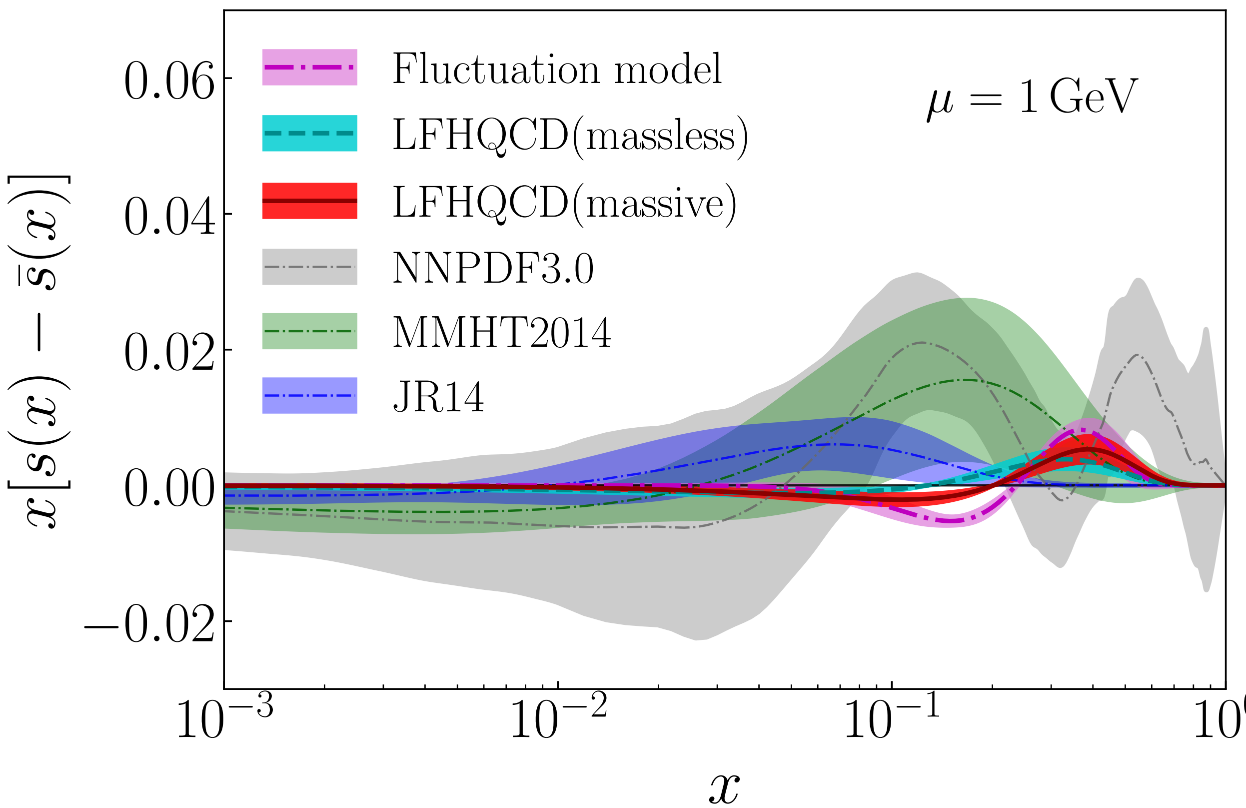
Strange-antistrange PDF in LFHQCD

$$s(x) - \bar{s}(x) = (1-\eta)N_s[q^{\phi}_{\tau=5}(x) - q^{\phi}_{\tau=6}(x)] + \eta N_s[q^{\omega}_{\tau=5}(x) - q^{\omega}_{\tau=6}(x)],$$

ϕ is nearly a pure $s\bar{s}$ state

$\eta \approx 0$





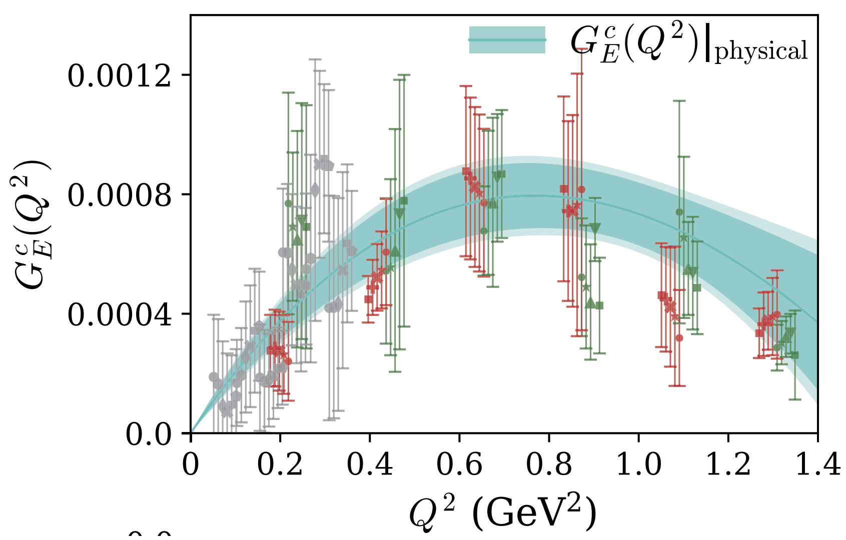
R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

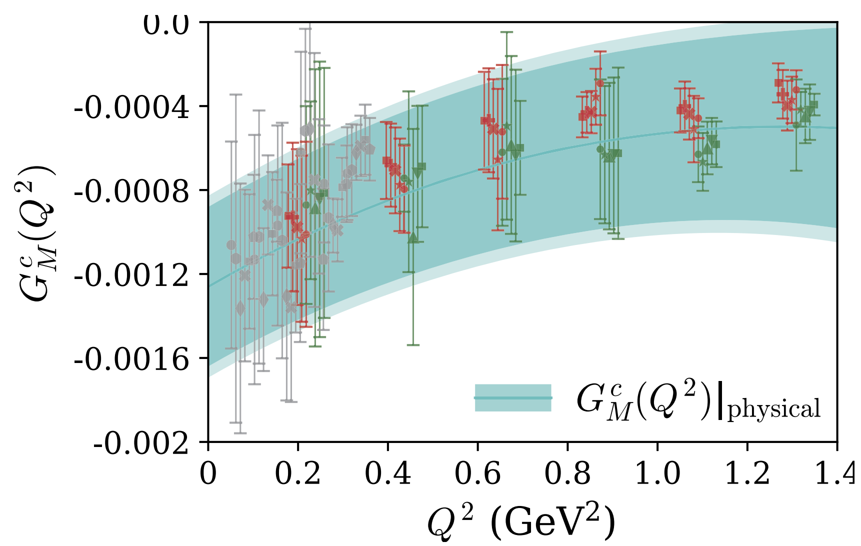
Intrinsic Charm in the Proton

Original proposal of intrinsic charm

S.J. Brodsky, P. Hoyer, C. Peterson, N. Sakai, Phys. Lett. B 93, 451-455 (1980).

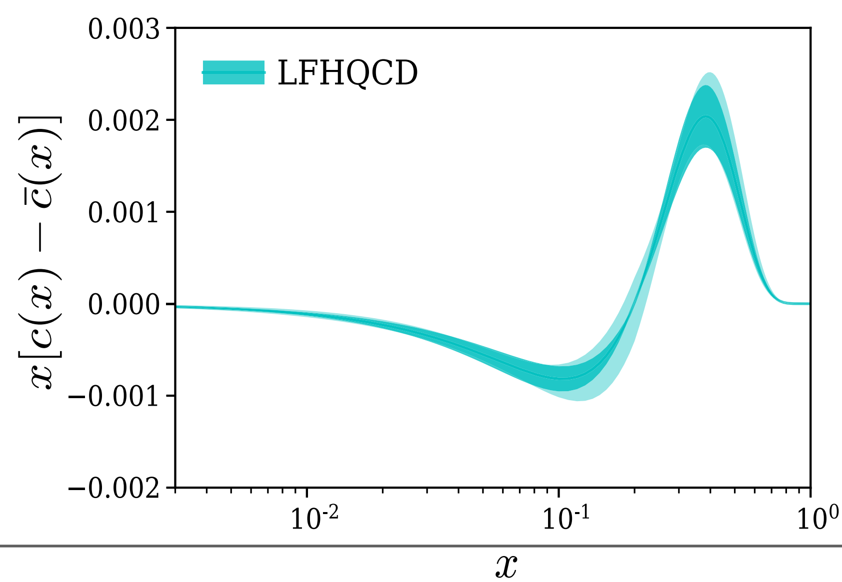
Lattice QCD exploration





First LQCD computation of charm quark EM form factors with three ensembles (one at physical pion mass)

LFHQCD analysis



Intrinsic charm-anticharm asymmetry with normalization constrained by LQCD data

R.S. Sufian, TL, A. Alexandru, S.J. Brodsky, G.F. de Téramond, H.G. Dosch, T. Draper, K.-F. Liu, Y.-B. Yang, Phys. Lett. B 808, 135633 (2020).

Axial Form Factor

Axial form factor:

$$F_A(Q^2) = \int \frac{dz}{z^4} A(Q^2, z)[g_+ \Psi_+^2(z) - g_- \Psi_-^2(z)]$$

The "–" sign for the second term is due to the γ_5

Compare the vector and axial FFs:

$$F_1(t) = c_{V,\tau} F_{V,\tau}(t) + c_{V,\tau+1} F_{V,\tau+1}(t)$$

$$F_A(t) = c_{A,\tau} F_{A,\tau}(t) - c_{A,\tau+1} F_{A,\tau+1}(t)$$

$$F_{V,\tau}(t) = \frac{1}{N_{V,\tau}} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right)$$

$$F_{A,\tau}(t) = \frac{1}{N_{A,\tau}} B\left(\tau - 1, 1 - \frac{t}{4\lambda}\right)$$

It just replaces the trajectory by the axial one: $\alpha_A(t) = \frac{t}{4\lambda}$

Coefficients are related: $$\frac{c_{V,\tau}}{N_{V,\tau}} = \frac{c_{A,\tau}}{N_{A,\tau}}$$

A convenient convention for this work: $N_{V,\tau} = N_{A,\tau} = N_\tau$ → $c_{V,\tau} = c_{A,\tau} = c_\tau$

One can also choose other normalization conventions.

TL, R.S. Sufian, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 124, 082003 (2020).

Polarized PDFs

For a twist-τ state:

$$\Delta q(x) = c_\tau \Delta q_\tau(x) - c_{\tau+1}\Delta q_{\tau+1}(x)$$

$$\Delta q_\tau(x) = \frac{1}{N_\tau}[1-w(x)]^{\tau-2} w'(x)$$

Large x:

$$\Delta q_\tau(x) = \frac{[-w''(1)]^{\tau-1}}{2^{\tau-2}N_\tau}(1-x)^{2\tau-3} + \cdots$$

$$\frac{\Delta q(x)}{q(x)} = 1 + \left(\frac{1}{4} + \frac{c_{\tau+1}N_\tau}{c_\tau N_{\tau+1}}\right) w''(1)(1-x)^2 + \cdots$$

$$\lim_{x\to 1}\frac{\Delta q(x)}{q(x)} = 1$$

helicity retention prediction by pQCD

Small x:

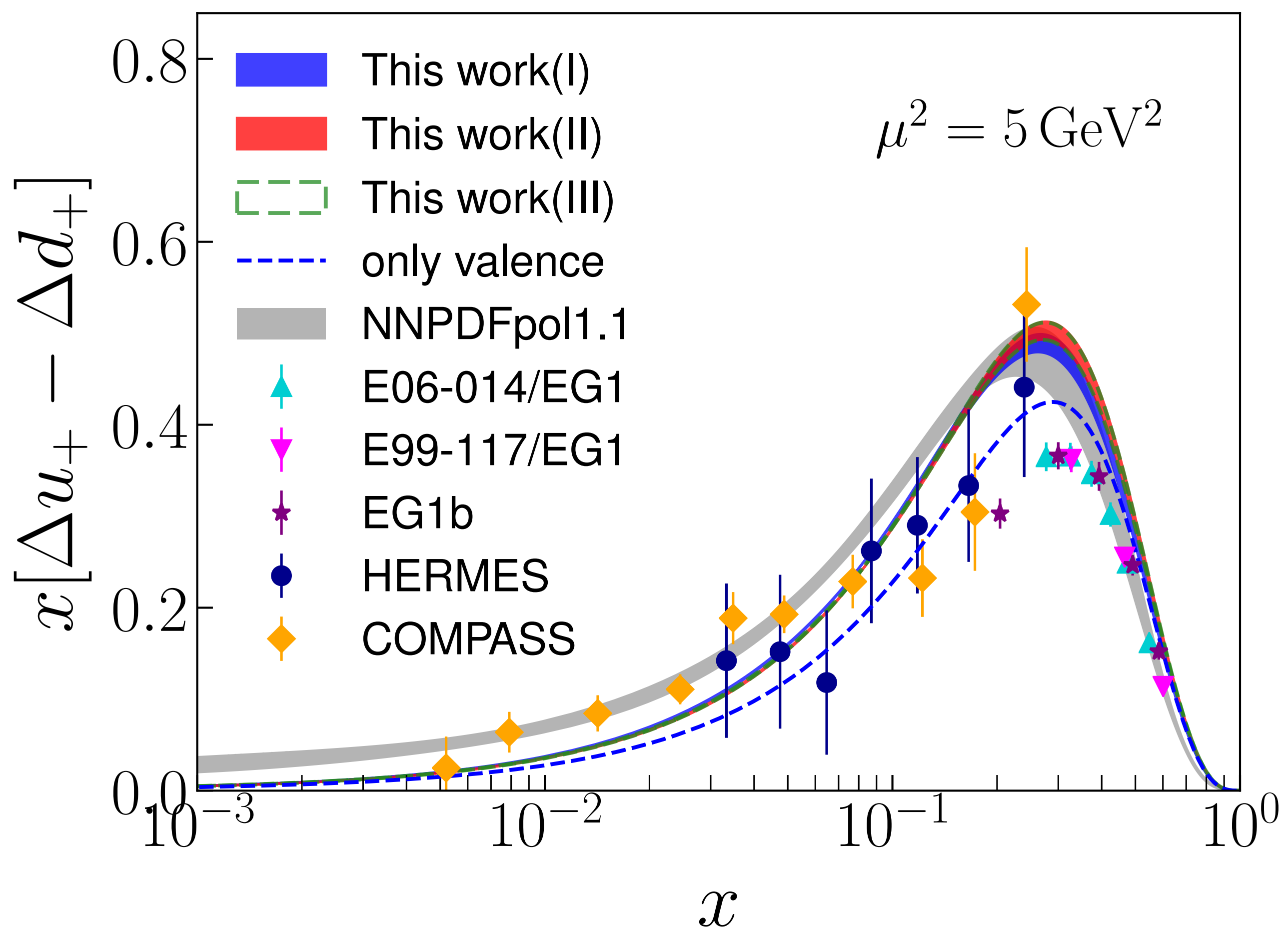
$$\frac{\Delta q(x)}{q(x)} \sim x^{\frac{1}{2}}$$

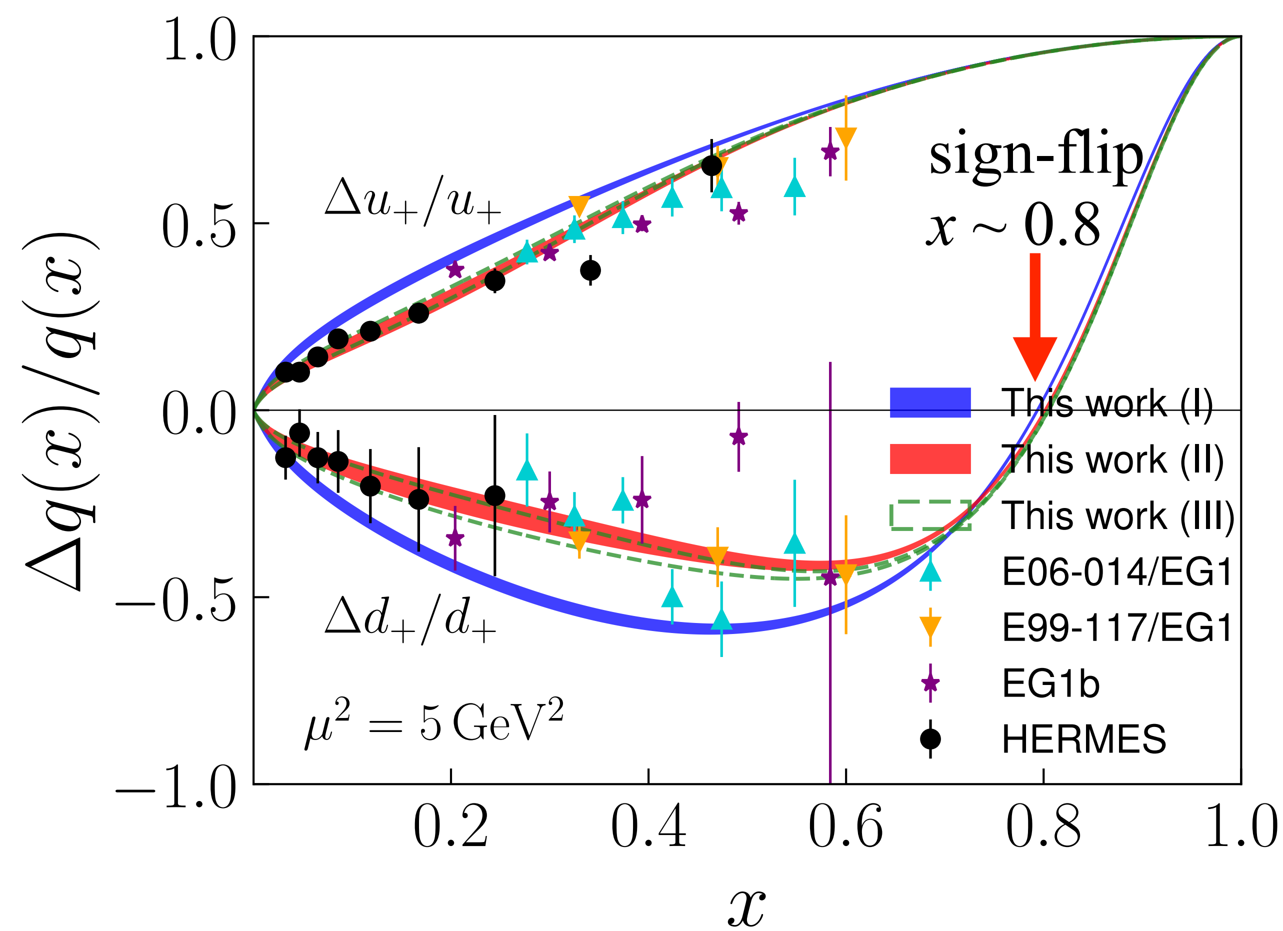
$$\lim_{x\to 0}\frac{\Delta q(x)}{q(x)} = 0$$

helicity correlation disappears at $x \sim 0$

TL, R.S. Sufian, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 124, 082003 (2020).

Numerical Results of Quark Polarized PDFs





Dashed curve: only valence state, without saturating the axial charge.

Bands: different ways to saturate the axial charge.

The same w(x) as in the unpolarized distributions.

TL, R.S. Sufian, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 124, 082003 (2020).

Graviton and Pomeron

AdS gravity action

$$S_G[g] = -\frac{1}{4}\int d^5x\sqrt{g}e^{\varphi_g(z)}(\mathcal{R}-\Lambda)$$

performing a small deformation: $g_{MN} \to g_{MN} + h_{MN}$

effective action: $S_{\text{eff}}[h,\Phi] = S_g[h] + S_i[h,\Phi]$

$$S_g[h] = -\frac{1}{4}\int d^5x\sqrt{g}e^{\varphi_g(z)}\left(\partial_L h^{MN}\partial^L h_{MN} - \frac{1}{2}\partial_L h\partial^L h\right)$$

$$S_i[h,\Phi] = \frac{1}{2}\int d^5x\sqrt{g}h_{MN}T^{MN}(\Phi)$$

in the harmonic gauge

$$\partial_L h^L_M = \frac{1}{2}\partial_M h, \quad h \equiv h^L_L$$

identify the gravity probe in AdS with the Pomeron, $J^{PC}=2^{++}$ bound state of gluons

effective trajectory: $\alpha_P(t) = \alpha_P(0) + \alpha'_P t$

$$\alpha_P(0) \simeq 1.08$$
$$\alpha'_P \simeq 0.25\text{GeV}^{-2}$$

Gravitational Form Factor

The propagation of h_{MN} in Minkowski coordinates

$$-\frac{z^3}{e^{\varphi_g(z)}}\partial_z\left(\frac{e^{\varphi_g(z)}}{z^3}\partial_z h_\mu^\nu\right)+\partial_\rho\partial^\rho h_\mu^\nu=0$$

soft-wall profile: $\varphi_g(z)=-\lambda_g z^2$

plane wave along the physical coordinates: $h_\mu^\nu(x,z)=\epsilon_\mu^\nu e^{-iq\cdot x}H\left(q^2,z\right)$

boundary conditions: $H\left(q^2=0,z\right)=H\left(q^2,z=0\right)=1$

solution: $H(a,\xi)=\Gamma(2+a)U(a,-1,\xi)$ $a=Q^2/4\lambda_g,\quad \xi=\lambda_g z^2$

The coupling with EMT

A scalar field, e.g. pion, $S_q[\Phi]=\int d^5x\sqrt{g}e^{\varphi_q(z)}\left(g^{MN}\partial_M\Phi^*\partial_N\Phi-\mu^2\Phi^*\Phi\right)$

$$T_{MN}=\partial_M\Phi^*\partial_N\Phi+\partial_N\Phi^*\partial_M\Phi$$

transition amplitude: $\int d^5x\sqrt{g}h_{MN}\left(\partial^M\Phi_{P'}^*\partial^N\Phi_P+\partial^N\Phi_{P'}^*\partial^M\Phi_P\right)$

Gravitational Form Factor

Gravitational Form Factor

hadronic matrix element: $\langle P' \left| T_\mu^\nu \right| P \rangle = \left(P^\nu P'_\mu + P_\mu P'^\nu \right) A\left(Q^2\right)$

GFF: $A_\tau\left(Q^2\right) = \int_0^\infty \frac{dz}{z^3} H\left(Q^2, z\right) \Phi_\tau^2(z)$ $\qquad \Phi_\tau^g(z) \sim z^\tau e^{-\lambda_g z^2/2}$

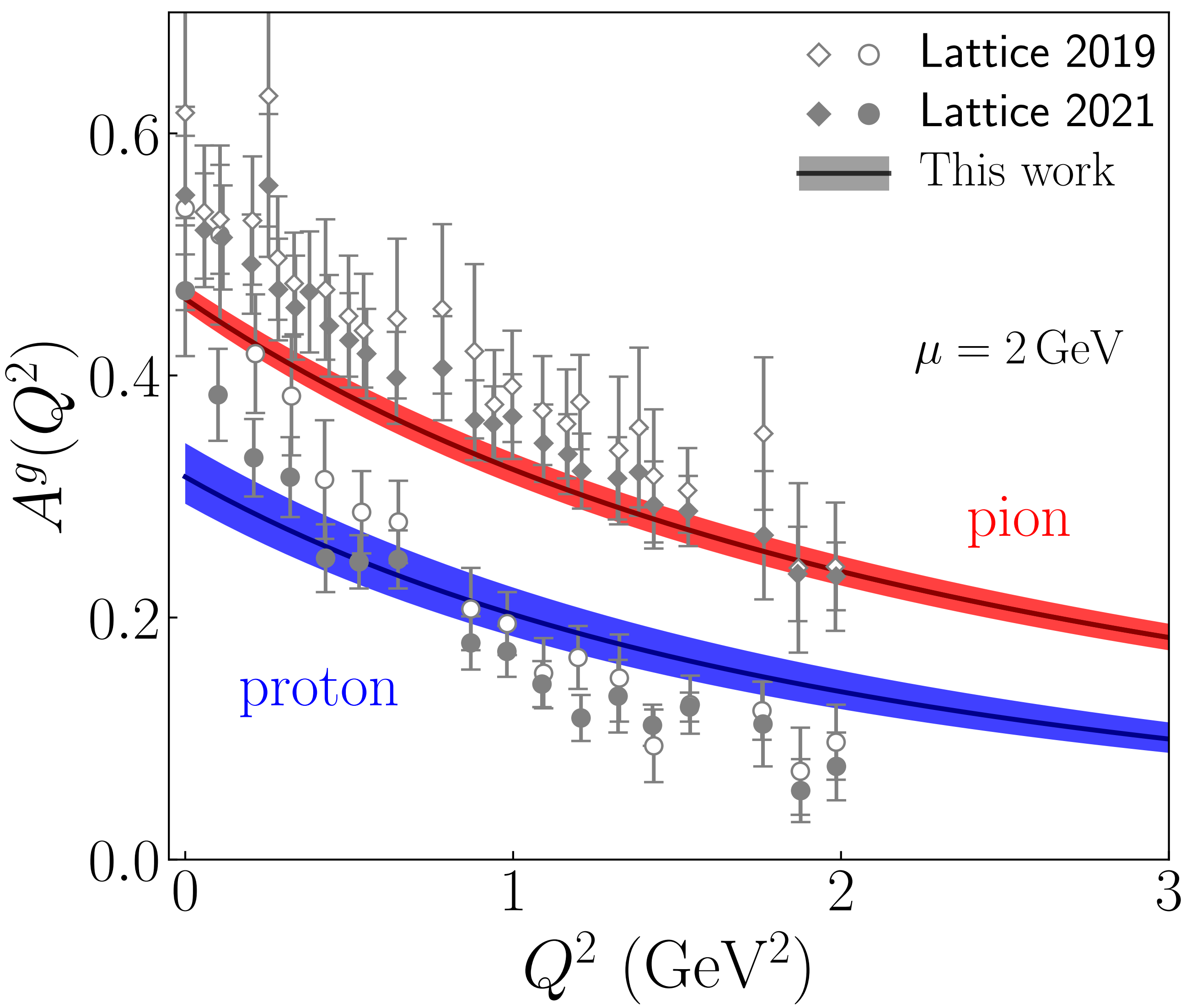
$$A_\tau^g\left(Q^2\right) = \frac{1}{N_\tau} B\left(\tau - 1, 2 - \alpha_P\left(Q^2\right)\right)$$

Pomeron coupling to the constituent gluon

Proton: the lowest Fock state $\quad |uudg\rangle \quad \tau = 4$

Pion: the lowest Fock state $\quad |u\bar{d}g\rangle \quad \tau = 3$

Numerical Results of GFF



$$\langle r_g^2 \rangle = \frac{6}{A^g(0)} \frac{dA^g(t)}{dt}\bigg|_{t=0}$$

$$\langle r_g^2 \rangle_p = 2.93/\lambda_g = (0.34\mathrm{fm})^2$$

$$\langle r_g^2 \rangle_\pi = 2.41/\lambda_g = (0.31\mathrm{fm})^2$$

G.F. de Téramond, H.G. Dosch, TL, R.S. Sufian, S.J. Brodsky, A. Deur, Phys. Rev. D 104, 114005 (2021).

Gluon Distributions

From GFF to gluon distributions

follow the procedure in quark distribution, but start from gluon gravitational form factor

$$A_\tau^g(t) = \frac{1}{N_\tau} \int_0^1 dx w'(x) w(x)^{1-\alpha_P(t)} [1 - w(x)]^{\tau-2}$$

the same w(x) as in quark distributions

intrinsic gluon distribution:

$$g_\tau(x) = \frac{1}{N_\tau} \frac{w'(x)}{x} [1 - w(x)]^{\tau-2} w(x)^{1-\alpha_P(0)}$$

$$g(x) = \sum_\tau c_\tau g_\tau(x)$$

large x behavior $g_\tau(x) \sim (1-x)^{2\tau-3}$

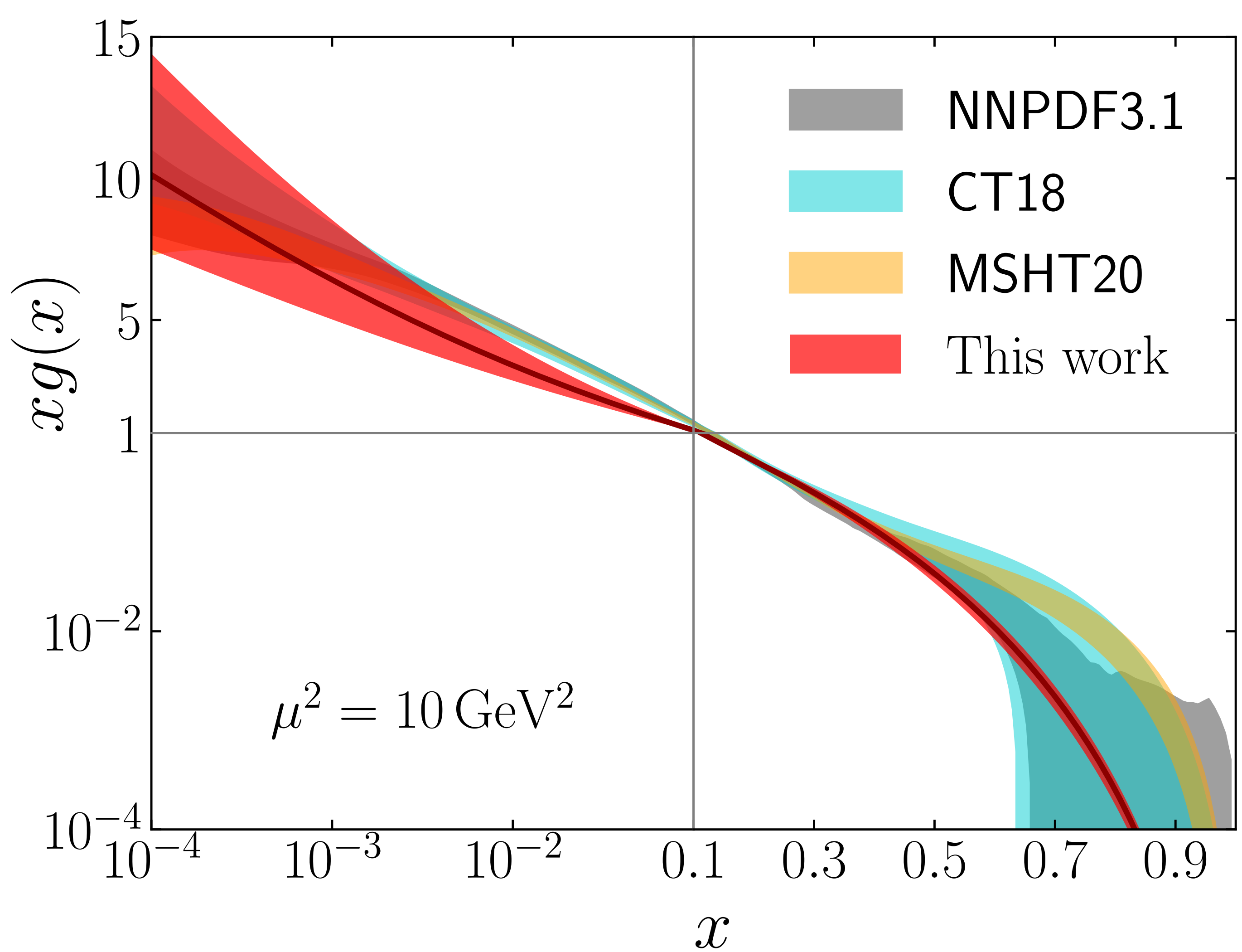
normalization from the momentum sum rule:

$$\int_0^1 dx x \left[g(x) + \sum_q q(x) \right] = 1$$

G.F. de Téramond, H.G. Dosch, TL, R.S. Sufian, S.J. Brodsky, A. Deur, Phys. Rev. D 104, 114005 (2021).

Numerical Results of Gluon Distribution

Gluon distribution in the proton



only keep the leading twist component
i.e., $\tau = 4$

$$c_{\tau=4} = 0.225 \pm 0.014$$

quark distribution from previous result in PRL 124, 082003 (2020).

the same choice of w(x)

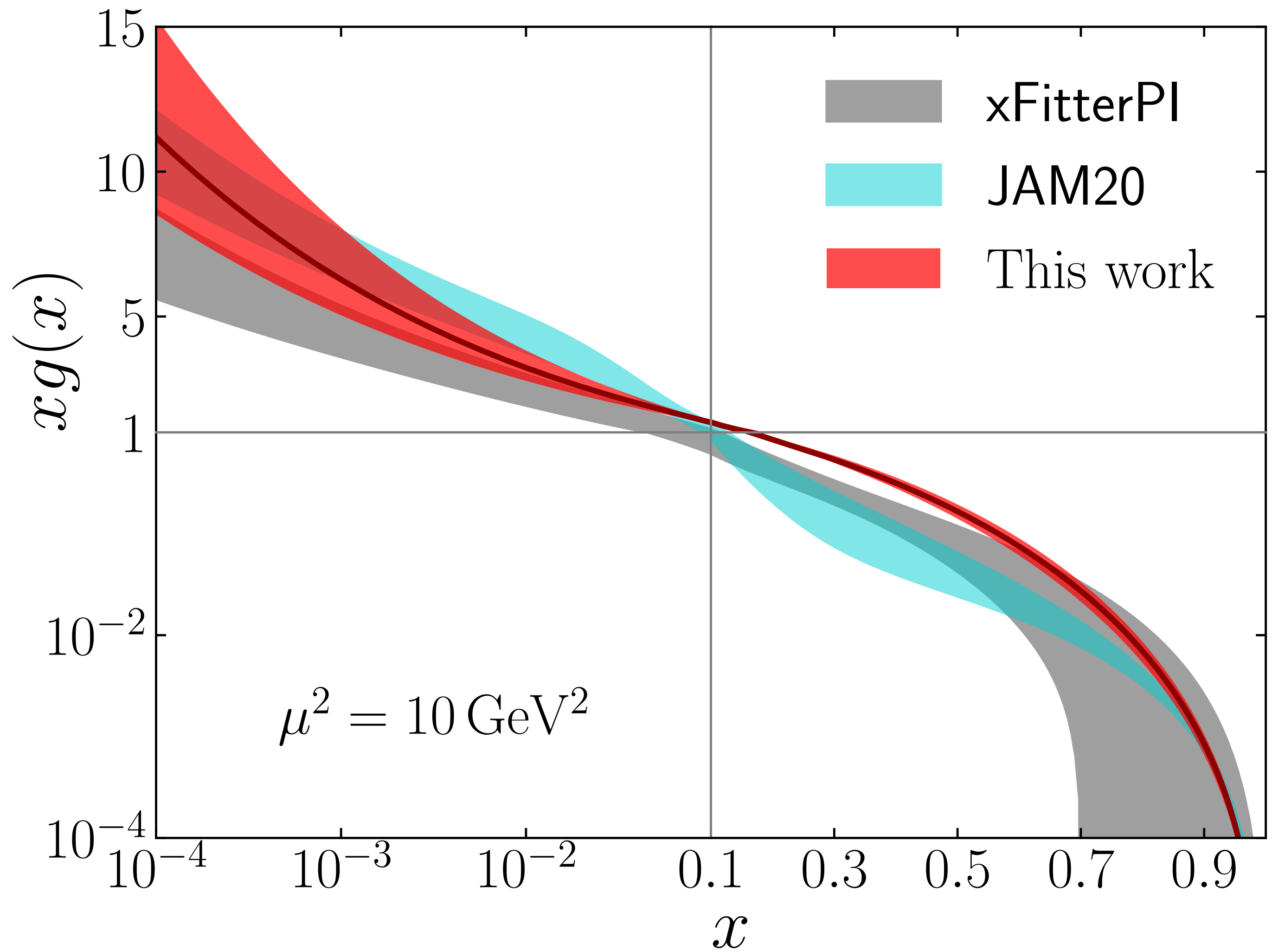
$$w(x) = x^{1-x} \exp\left[-b(1-x)^2\right]$$

$$b = 0.48 \pm 0.04$$

G.F. de Téramond, H.G. Dosch, TL, R.S. Sufian, S.J. Brodsky, A. Deur, Phys. Rev. D 104, 114005 (2021).

Numerical Results of Gluon Distribution

Gluon distribution in the pion



only keep the leading twist component *i.e.*, $\tau = 3$

$$c_{\tau=3} = 0.429 \pm 0.007$$

quark distribution from previous result in PRL 120, 182001 (2018).

the same choice of w(x)

$$w(x) = x^{1-x} \exp\left[-b(1-x)^2\right]$$

$$b = 0.48 \pm 0.04$$

G.F. de Téramond, H.G. Dosch, TL, R.S. Sufian, S.J. Brodsky, A. Deur, Phys. Rev. D 104, 114005 (2021).

Summary

In LF holographic QCD, we determine the structure of GPDs up to a universal reparametrization function w(x), incorporating Regge behavior at small x and counting rules at large x.

It connects parton distributions in the proton and those in the pion. Including quark mass correction, this approach can be applied to intrinsic strange and intrinsic charm.

Given the unpolarized quark distributions, the polarized distributions are uniquely determined, consistent with the helicity retention at $x \to 1$ predicted by pQCD.

With the holographic coupling of the spin-two soft Pomeron to hadron EMT, we provide simultaneous description of intrinsic gluon GFF and distribution within a unified framework for both nucleon and pion.

With quark distributions determined from previous studies, the gluon distributions are predicted using only the leading Fock component with no additional parameters.

Thanks!

Backup

Form Factors in Holographic QCD

Nucleon form factor: spin-flip

$$\int d^4x dz \sqrt{g} \bar{\Psi}_{P'}(x,z) e_A^M e_B^N [\Gamma^A, \Gamma^B] F_{MN}(x,z) \Psi_P(x,z)$$ → Effective spin-flip amplitude in AdS space of an external EM field coupling to a nucleon

$$\sim (2\pi)^4 \delta^4(P' - P - q) \epsilon_\mu \bar{u}(P') \frac{\sigma^{\mu\nu} q_\nu}{2M_N} F_2(q^2) u(P),$$ → Pauli form factor in physical spacetime

Z. Abidin and C.E. Carlson, Phys. Rev. D 79, 115003 (2009).

Pauli form factor

$$F_2^N(Q^2) = \chi_N \int \frac{dz}{z^3} \Psi_+(z) V(Q^2, z) \Psi_-(z)$$

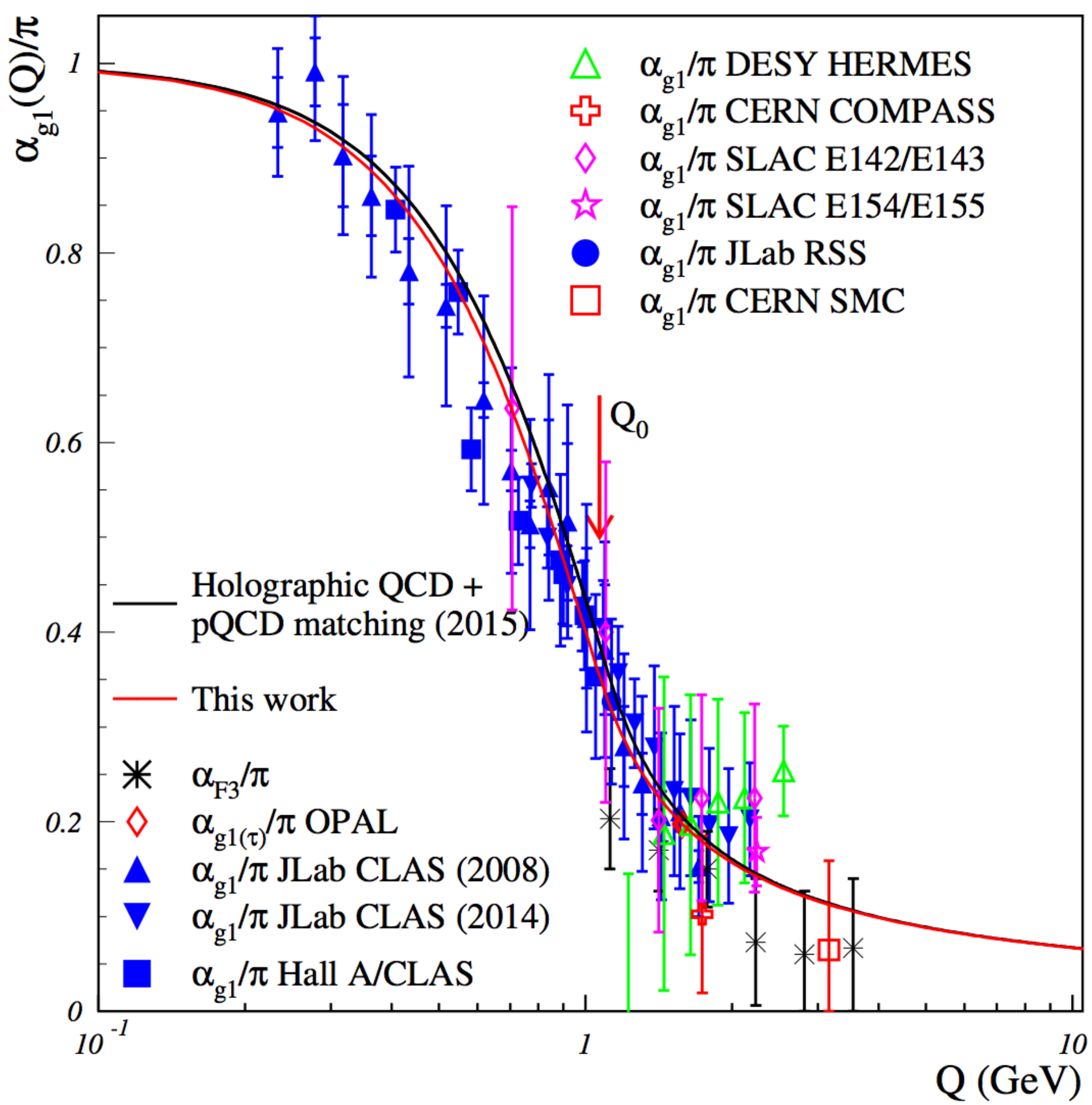
normalized to anomalous magnetic moments

Scaling: additional power of z in the wave function product of Ψ_+ and Ψ_-

the leading scaling of the Pauli form factor has additional power of $1/Q^2$

Matching Scale

Matching the couplings from LFHQCD and pQCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD (valid at low-Q^2)

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp\left(-Q^2/4\kappa^2\right)$$

Imposing continuity for α and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

Strange-antistrange Distributions

Nucleon valence component: $|uud\rangle$

Nonvalence nature of the strange distribution:

PDF: $$\langle s - \bar{s} \rangle = \int_0^1 dx[s(x) - \bar{s}(x)] = 0$$

Form factor: $$F_1^s(Q^2 = 0) = 0$$

Extrinsic and intrinsic strange sea:

Extrinsic: from gluon splitting, $g \to s\bar{s}$, triggered by a hard probe

can be calculated perturbatively

Intrinsic: encoded in nucleon nonvalence LF Fock state wave functions

can in principle be obtained by solving the Hamiltonian eigenvalue problem

Strange-antistrange Asymmetry

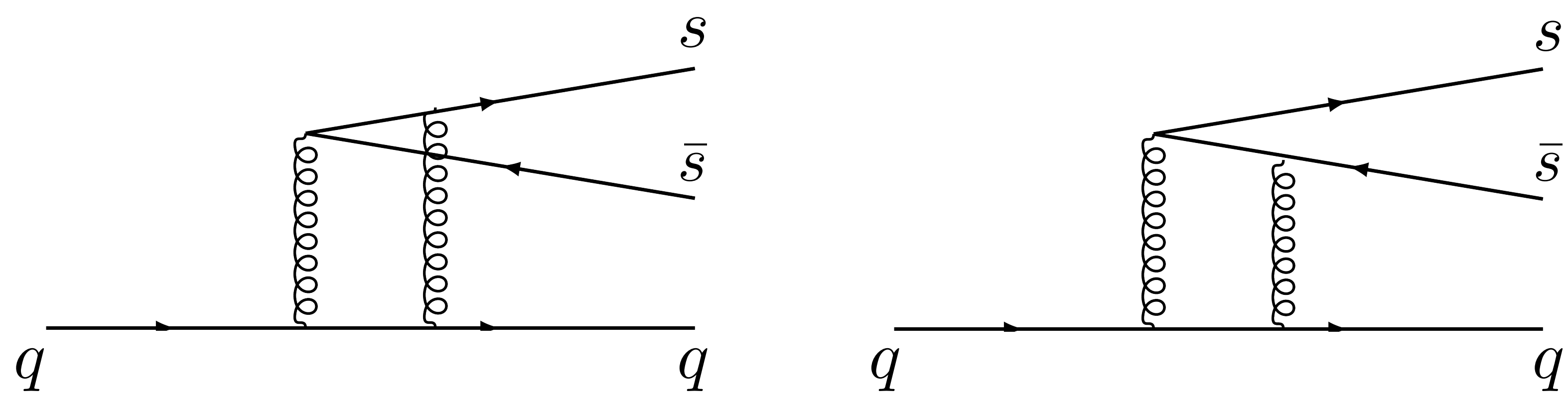
Can strange and antistrange distributions be different?

Perturbative QCD calculation

Splitting functions $q \rightarrow s$ and $q \rightarrow \bar{s}$ are different at NNLO

extrinsic and very small asymmetry $s(x) - \bar{s}(x)$

S. Catani, D. de Florian, G. Rodrigo, and W. Vogelsang, Phys, Rev. Lett. 93, 152003 (2004).



Strange FF and PDF with LFWFs

Hadrons are eigenstates of QCD LF Hamiltonian $H_{\rm LF}^{\rm QCD}|\Psi\rangle = M^2|\Psi\rangle$

A nucleon state:

$$|N; P^+, \mathbf{P}_\perp, S^z\rangle = \sum_{n,\{\lambda_i\}} \int [dx][d^2\mathbf{k}_\perp]\psi_{n/N}(x_i, \mathbf{k}_{i\perp}, \lambda_i)|n; x_i P^+, x_i\mathbf{P}_\perp + \mathbf{k}_{i\perp}, \lambda_i\rangle$$

PDF:

$$s(x) = \sum_{\lambda_s}\int \frac{d^2\mathbf{k}_{s\perp}}{16\pi^3}|\psi_{s/N}(x_s, \mathbf{k}_{s\perp}, \lambda_s)|^2, \quad \bar{s}(x) = \sum_{\lambda_{\bar{s}}}\int \frac{d^2\mathbf{k}_{\bar{s}\perp}}{16\pi^3}|\psi_{\bar{s}/N}(x_{\bar{s}}, \mathbf{k}_{\bar{s}\perp}, \lambda_{\bar{s}})|^2.$$

normalization: $\sum_{\lambda_s}\int \frac{dx_s d^2\mathbf{k}_{s\perp}}{16\pi^3}|\psi_{s/N}(x_s, \mathbf{k}_{s\perp}, \lambda_s)|^2 = \sum_{\lambda_{\bar{s}}}\int \frac{dx_{\bar{s}} d^2\mathbf{k}_{\bar{s}\perp}}{16\pi^3}|\psi_{\bar{s}/N}(x_{\bar{s}}, \mathbf{k}_{\bar{s}\perp}, \lambda_{\bar{s}})|^2 = I_s$

Form factor:

$$\begin{aligned} F_1^s(Q^2 = \mathbf{q}_\perp^2) &= \sum_{\lambda_s}\int \frac{dx_s d^2\mathbf{k}_{s\perp}}{16\pi^3}\psi^*_{s/N}(x_s, \mathbf{k}_{s\perp} + (1-x_s)\mathbf{q}_\perp, \lambda_s)\psi_{s/N}(x_s, \mathbf{k}_{s\perp}, \lambda_s) \\ &\quad - \sum_{\lambda_{\bar{s}}}\int \frac{dx_{\bar{s}} d^2\mathbf{k}_{\bar{s}\perp}}{16\pi^3}\psi^*_{\bar{s}/N}(x_{\bar{s}}, \mathbf{k}_{\bar{s}\perp} + (1-x_{\bar{s}})\mathbf{q}_\perp, \lambda_{\bar{s}})\psi_{\bar{s}/N}(x_{\bar{s}}, \mathbf{k}_{\bar{s}\perp}, \lambda_{\bar{s}}) \\ &= \rho_s(\mathbf{q}_\perp) - \rho_{\bar{s}}(\mathbf{q}_\perp), \end{aligned}$$

Asymmetries in FF and PDF: Qualitative

Coordinate space distribution:

$$\rho_{s/\bar{s}}(\mathbf{q}_\perp) = \int \frac{d^2\mathbf{a}_\perp}{(2\pi)^2} e^{i\mathbf{q}_\perp\cdot\mathbf{a}_\perp} \tilde{\rho}_{s/\bar{s}}(\mathbf{a}_\perp).$$

normalization: $\int d^2\mathbf{a}_\perp \tilde{\rho}_s(\mathbf{a}_\perp) = \int d^2\mathbf{a}_\perp \tilde{\rho}_{\bar{s}}(\mathbf{a}_\perp) = I_s$

$$\tilde{\rho}_{s/\bar{s}}(\mathbf{a}_\perp) = \int d^2\mathbf{q}_\perp e^{-i\mathbf{q}_\perp\cdot\mathbf{a}_\perp} \rho_{s/\bar{s}}(\mathbf{q}_\perp)$$

$$= \sum_{\lambda_{s/\bar{s}}} \int \frac{dx_{s/\bar{s}}}{(1-x_{s/\bar{s}})^2} \left| \tilde{\psi}_{s/\bar{s}}\left(x_{s/\bar{s}}, \frac{\mathbf{a}_\perp}{1-x_{s/\bar{s}}}, \lambda_{s/\bar{s}}\right)\right|^2.$$

$F_1^s(Q^2) \neq 0$ → $\tilde{\rho}_s(\mathbf{a}_\perp) \neq \tilde{\rho}_{\bar{s}}(\mathbf{a}_\perp)$ → $|\tilde{\psi}_s(x, \mathbf{b}_\perp)|^2 \neq |\tilde{\psi}_{\bar{s}}(x, \mathbf{b}_\perp)|^2$

→ $|\psi_s(x, \mathbf{k}_\perp)|^2 \neq |\psi_{\bar{s}}(x, \mathbf{k}_\perp)|^2$ → (no privileged direction) $s(x) \neq \bar{s}(x)$

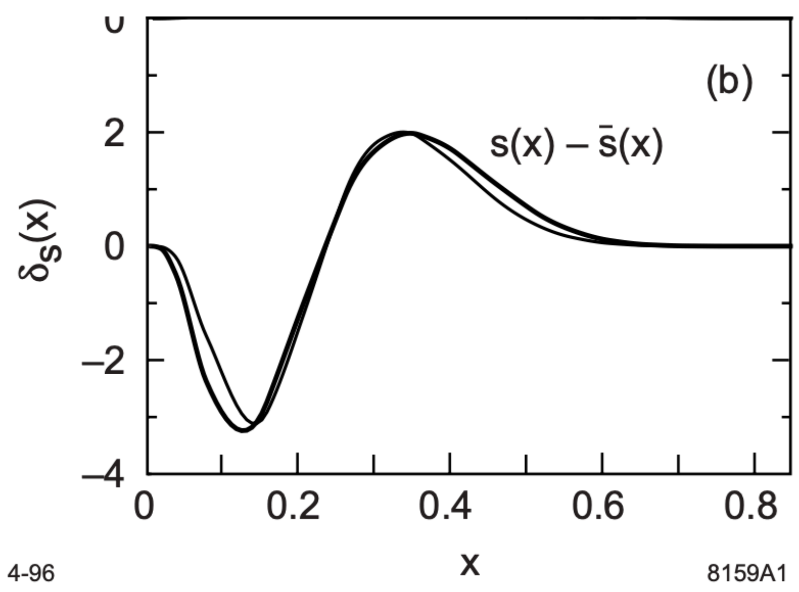
$F_1^s(Q^2) > 0$ → *s* quark is more centralized in coordinate space and more spread out in momentum space

It favors a positive $s(x) - \bar{s}(x)$ at large-*x*, and a negative value at small-*x*.

Strange FF and PDF in Fluctuation Model

Model: consider a nucleon fluctuating to a kaon-hyperon state, *e.g.*, KΛ

PDF:

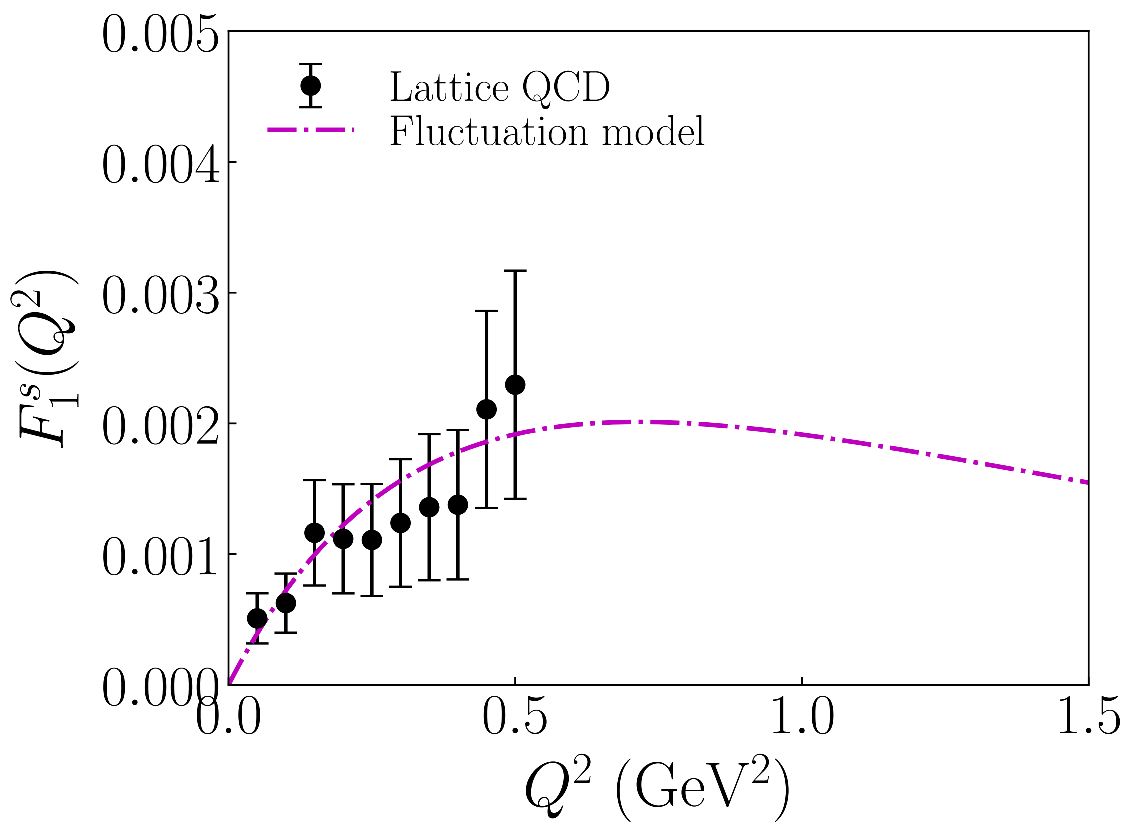


S.J. Brodsky and B.-Q. Ma, Phys. Lett. B 381, 317 (1996).

This is reproduced in
A. Vega, I. Schimidt, T. Gutsche, and V.E. Lyubovitskij,
Phys. Rev. D 93, 056001 (2016).

Form factor:

use original model parameters in
Brodsky&Ma paper



R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

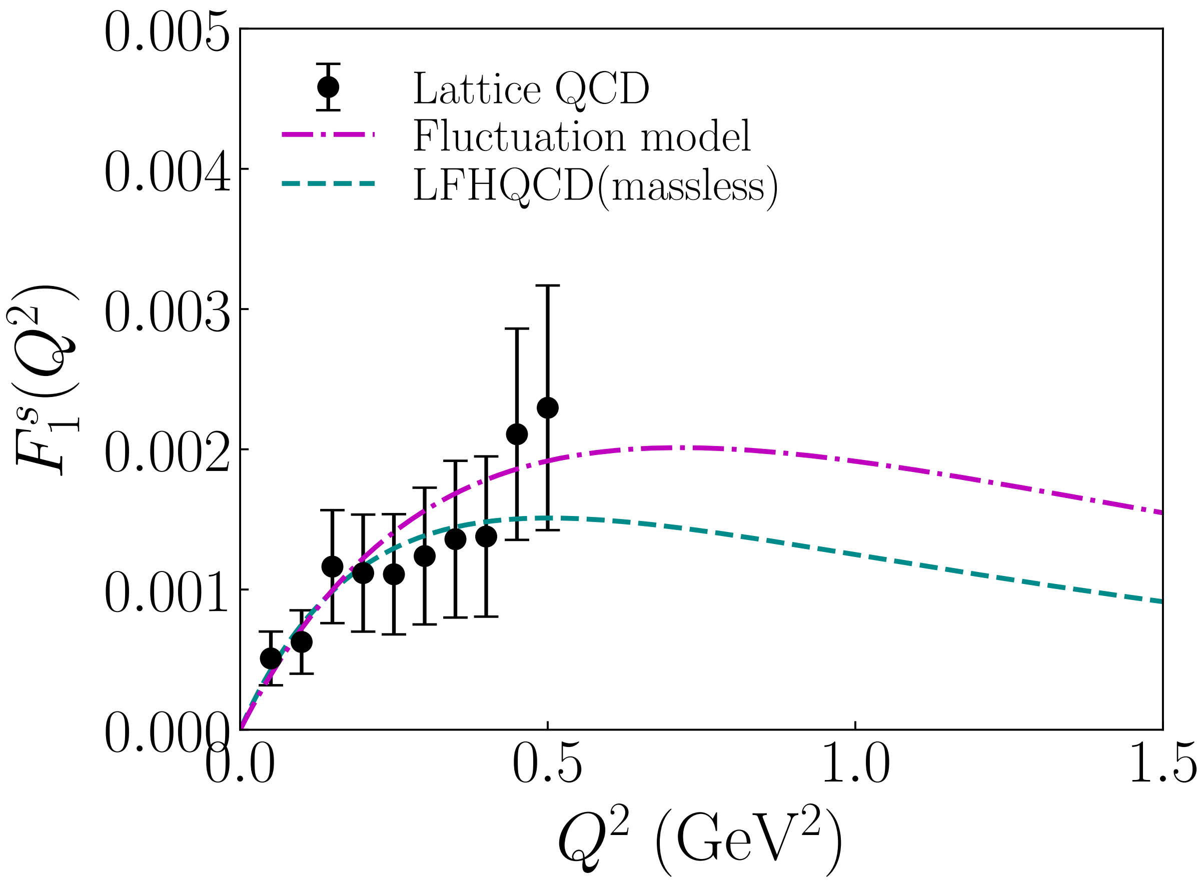
Strange Form Factor in LFHQCD

FF with twist-τ: $F_\tau(t) = \frac{1}{N_\tau} B(\tau - 1, 1 - \alpha(t))$ $\alpha(t) = \frac{1}{2} + \frac{t}{4\lambda}$

Leading Fock state with strange-antistrange for a nucleon: $|uuds\bar{s}\rangle$

$$F_1^s(Q^2) = N_s[F_{\tau=5}(Q^2) - F_{\tau=6}(Q^2)]$$

Strange form factor in LFHQCD with massless quarks:



R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

Separation of Strange and Antistrange

Expand the distribution into twist-5 and twist-6 components

$$s(x) = \alpha q_{\tau=5}(x) + \beta q_{\tau=6}(x) \qquad \bar{s}(x) = \gamma q_{\tau=5}(x) + \delta q_{\tau=6}(x)$$

The coefficients satisfy: $\alpha + \beta = I_s, \quad \gamma + \delta = I_s, \quad \alpha - \gamma = N_s, \quad \delta - \beta = N_s$

where $\int dx\, s(x) = \int dx\, \bar{s}(x) = I_s$ and $s(x) - \bar{s}(x) = N_s[q^{\phi}_{\tau=5}(x) - q^{\phi}_{\tau=6}(x)]$

General solution: $\beta = I_s - \alpha, \quad \gamma = \alpha - N_s, \quad \delta = I_s - \alpha + N_s.$

$$s(x) = \alpha q_{\tau=5}(x) + (I_s - \alpha) q_{\tau=6}(x), \qquad \bar{s}(x) = (\alpha - N_s) q_{\tau=5}(x) + (I_s - \alpha + N_s) q_{\tau=6}(x)$$

Physical constraint: $s(x) \geq 0$ and $\bar{s}(x) \geq 0$

twist-5 term dominates at large-x: $\alpha \geq 0$ and $\gamma \geq 0$

examine the small-x: $I_s \geq (1 - R)\alpha$ $\qquad \lim_{x\to 0} \frac{q_{\tau=5}(x)}{q_{\tau=6}(x)} = \frac{N_{\tau=6}}{N_{\tau=5}} \equiv R$

R.S. Sufian, TL, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, M.T. Islam, and B.-Q. Ma Phys. Rev. D 98, 114004 (2018).

Polarized PDFs

Spin-aligned and spin-antialigned distributions:

$$q_{\uparrow/\downarrow}(x) = \frac{1}{2}[q(x) \pm \Delta q(x)]$$

Large x limit:

$$q_{\uparrow}(x) \to c_\tau q_\tau(x) \sim (1-x)^{2\tau-3}$$

$$q_{\downarrow}(x) \to c_{\tau+1} q_{\tau+1}(x) \sim (1-x)^{2\tau-1}$$

Two helicity states tend to a pure contribution from one component.

E.g.: for valence state, $\tau=3$

$$q_{\uparrow}(x) \sim (1-x)^3 \qquad q_{\downarrow}(x) \sim (1-x)^5$$

Consistent with pQCD up to logarithmic corrections.

TL, R.S. Sufian, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 124, 082003 (2020).

Numerical Results of Polarized PDFs

Coefficients c_τ by fitting EM form factors do not separate quark and antiquark

E.g.: $$c_{5,u} = u_{\tau=5} - \bar{u}_{\tau=5} \qquad c_{6,u} = u_{\tau=6} - \bar{u}_{\tau=6}$$

similar for down quark

Adding equal terms to u and ū, or d and d̄, does not change EM form factors.

Axial charge:

$$g_A = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d}) \approx 1.2732(23)$$

precisely measured via neutron weak decay.

Saturate the axial charge by a shift:

$$u_{\tau=5} \to u_{\tau=5} + \delta_u \qquad \bar{u}_{\tau=5} \to \bar{u}_{\tau=5} + \delta_u$$
$$d_{\tau=6} \to d_{\tau=6} + \delta_d \qquad \bar{d}_{\tau=6} \to \bar{d}_{\tau=6} + \delta_d$$

Variation due to different ways to saturate g_A is taken as part of our uncertainty.

TL, R.S. Sufian, G.F. de Téramond, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 124, 082003 (2020).