Dispersive Analysis of the Primakoff Reaction $\gamma K \to K\pi$

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HISKP (Theorie)
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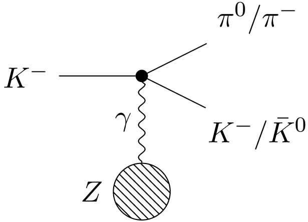
Motivation

- pion production in the Coulomb field of a heavy nucleus
- $\gamma^{(*)}\pi \to \pi\pi$ investigated [Hoferichter et al., 2012, 2017], [Niehus et al., 2021]
- measurable at AMBER/COMPASS++
- upgrade from pion beam to kaon beam



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- measurable at AMBER/COMPASS++
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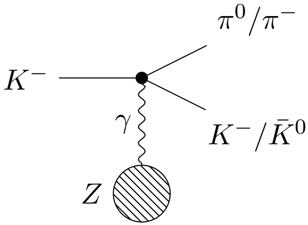
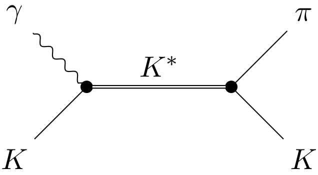
- combine knowledge about
  - chiral anomaly at $s = t = u = 0$

$$F_{KK\pi\gamma} = \frac{e}{4\pi^2 F_\pi^3}$$

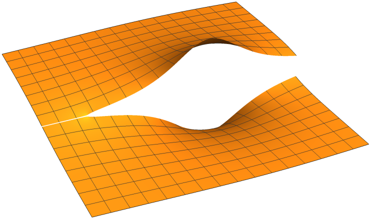
  - resonances ($K^*(892)$) at higher energies (radiative couplings)





- amplitude $\mathcal{M}(s,t,u)$ in the complex plane
- branch cuts at thresholds
- multiple Riemann sheet structure

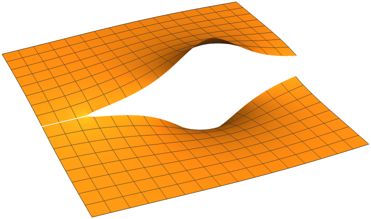
first Riemann sheet



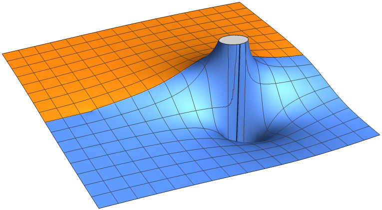
[adapted from Stefan Ropertz]

- amplitude $\mathcal{M}(s,t,u)$ in the complex plane
- branch cuts at thresholds
- multiple Riemann sheet structure
- hadronic resonances appear as poles on the second Riemann sheet
- pole position determines mass and width: $s_{\text{pole}} = (M - i \cdot \Gamma/2)^2$

first Riemann sheet



first and second Riemann sheet



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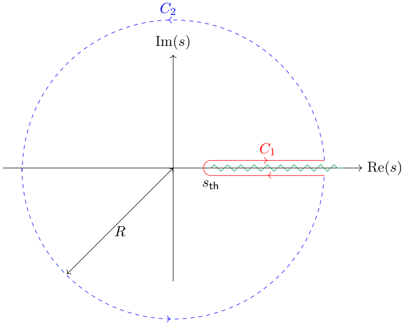
Dispersive formalism

- analyticity ($\simeq$ causality) & Cauchy's integral formula

$$f(s) = \frac{1}{2\pi i} \oint_{\partial U} \frac{f(s')}{s' - s} \mathrm{d}s'$$



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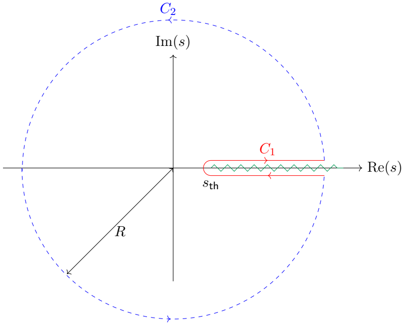

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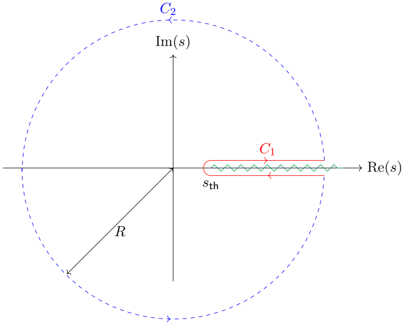

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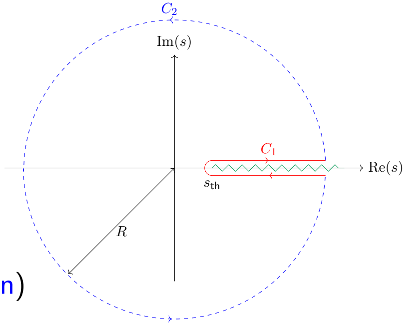
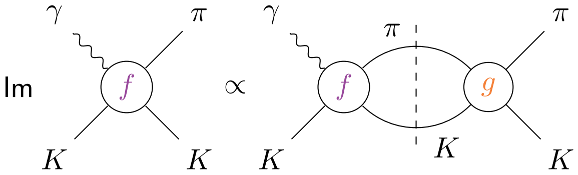


- unitarity relation ($\simeq$ prob. conservation)

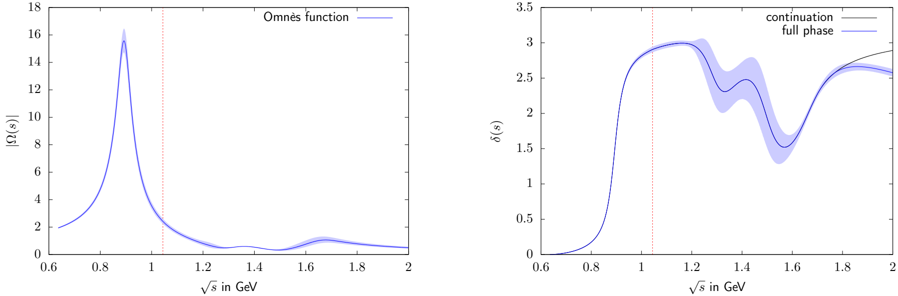
$$\operatorname{Im} f(s) \propto f(s)\cdot g^*(s)$$



- obeys Watson's final state theorem [Watson, 1954]

Homogeneous Omnès problem

- $K\pi$ P-wave phase shift from [Peláez and Rodas, 2016]
- $I = 1/2$ phase shift contains $K^*(892)$, $K^*(1410)$ and $K^*(1680)$
- very well constrained up to the $K\eta$ threshold
- Omnès function: $\Omega(s)$ [Omnès, 1958]



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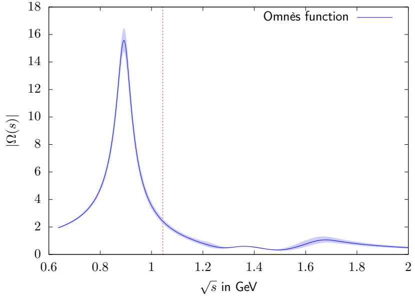

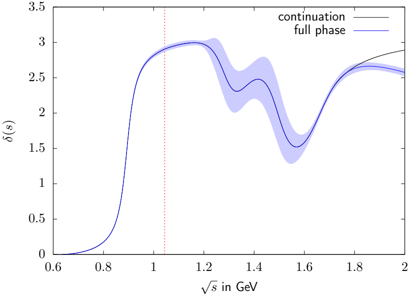

- $I = 3/2$ phase shift is $|\delta(s)| < 3°$ for $s < (1.74\,\text{GeV})^2$
- approximate it with $\delta(s) = 0 \Rightarrow \Omega(s) = 1$

Reconstruction theorem

- separate kinematic prefactor $\mathcal{M} = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}p_1^{\nu}p_2^{\alpha}p_0^{\beta}\mathcal{F}(s,t,u)$
- decompose scalar amplitude $\mathcal{F}(s,t,u)$ using isospin and $s \leftrightarrow u$ symmetry into single variable amplitudes

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$\mathcal{G}^{(0)}$	/	1	/	0	$+$	$-$	$\rho(770)$	P

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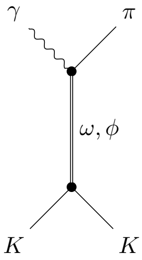
Fixed t-channel resonances

- narrow resonances ω, ϕ fixed by VMD exchange
- $V\Phi\Phi$ Lagrangian from [Klingl et al., 1996]
- $V\Phi\gamma$ Lagrangian from [Terschlüsen and Leupold, 2010]



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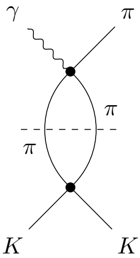

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- dispersion integral containing $\gamma\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ amplitudes
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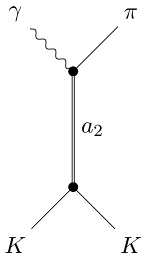

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- D-wave a_2 resonance via tensor meson dominance

Inhomogeneous Omnès problem

- the P-wave projection of the scalar amplitude is given by

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- solve this with a separation ansatz

$$\mathcal{F}(s) = \Omega(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'^n} \frac{\widehat{\mathcal{F}}(s')\sin(\delta(s'))}{|\Omega(s')|(s'-s)}\right)$$

Basis functions

- solution depends on subtraction polynomials linearly
- construct basis functions that correspond to one subtraction constant

$$\mathcal{F}(s) = \sum_{i=0}^{N} \alpha_i \mathcal{F}_i(s)$$

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- reduces computational effort dramatically
- fit/matching can be done using the basis functions

Subtraction schemes

- using $\delta^{(3/2)} = 0$ and $\delta^{(1/2)} = \delta$ we find

$$\mathcal{F}^{(0,1/2)}(s) = \Omega(s) \left(P_{n-1}^{(0,1/2)}(s) + \frac{s^n}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'^n} \frac{\widehat{\mathcal{F}}^{(0,1/2)}(s') \sin(\delta(s'))}{|\Omega(s')|(s'-s)} \right)$$

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- remove the $I = 3/2$ component with the ambiguity
- two/four free parameters

Matching: anomaly

- calculate chiral Wess–Zumino–Witten [1971,1983] anomaly

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- insert the reconstruction theorems

$$a_n^{(1/2)} = \frac{F_{KK\pi\gamma} - \mathcal{G}^{(+)}(0)}{2}$$
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$$a_n^{(1/2)} = 1.0(1.3)\,\text{GeV}^{-3}$$
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Matching: coupling

Analytic continuation to the K^* pole

- starting from the unitarity condition we can connect the amplitudes on the first (I) and second (II) Riemann sheet

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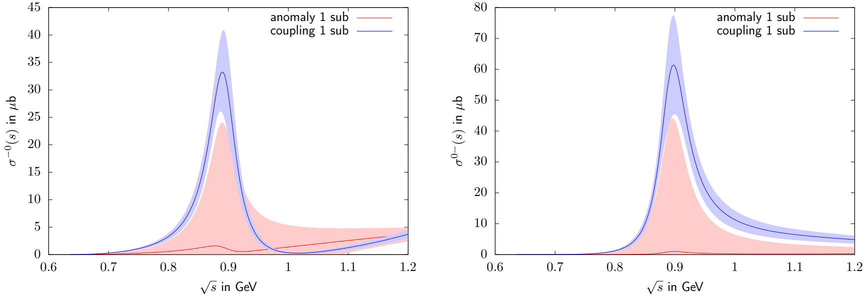
- $K\pi$ coupling and K^* pole position from Roy–Steiner analysis [Ruiz de Elvira, 2019]
- $f(s_{K^*})$ on the first Riemann sheet is calculated depending on the subtraction constants via the kernel method
- do this for both isospin components separately

Matching: anomaly or coupling

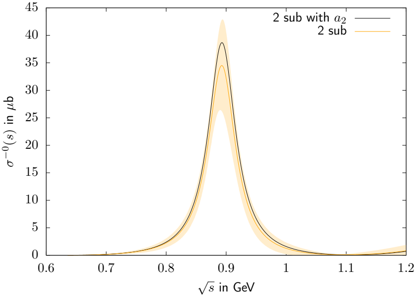
- minimal subtraction scheme
- fully determined by the anomaly or coupling



$\gamma K^- \to K^- \pi^0$ $\qquad$ $\gamma K^- \to \bar{K}^0 \pi^-$

Matching: anomaly and coupling

- twice subtracted scheme with and without a_2 resonance



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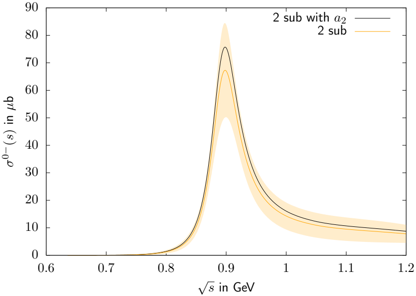
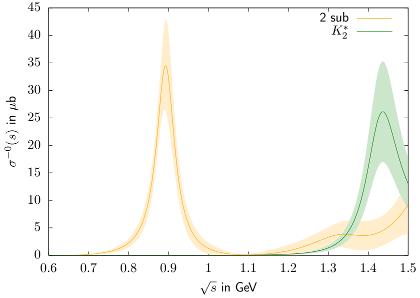


$\gamma K^- \to \bar{K}^0 \pi^-$

D-wave $K_2^*(1430)$

- use Lagrangians [Ecker and Zauner, 2007], [Plenter and Kubis, 2015]
- for neutral channel the radiative coupling is only an upper limit $\Gamma_{K_2^* \to K^0 \gamma} < 5.4\,$keV [Alavi-Harati et al. (KTeV), 2001]



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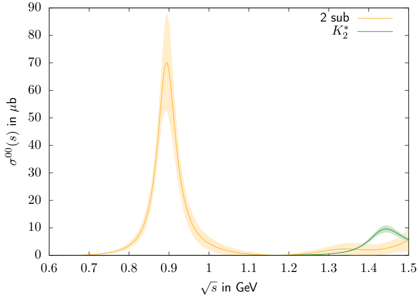

$\gamma \bar{K}^0 \to \bar{K}^0 \pi^0$

- D-wave relevant in charged channels above $1.35\,$GeV [Bacho, 2021]

Summary and outlook

- constructed a dispersive solution for the Primakoff reaction $\gamma K \to K\pi$ for all charge configurations
- input: fixed t-channels and $K\pi$ phase shift
- using the basis functions a fit to COMPASS++ (or OKA) data is possible to determine the free parameters
- matching: use radiative couplings and chiral anomaly to predict the free parameters (using the fit, extract these quantities)
- reduce error on $a_i^{(1/2)}$:
  - next-to-leading-order correction to the anomaly
  - $\omega \to K\bar{K}$ coupling: space-like kaon form factor [DS, Hariharan, Hoferichter, Kubis and Stoffer, 2022]

Spares

Matching: Anomaly or Coupling

- minimal subtraction scheme
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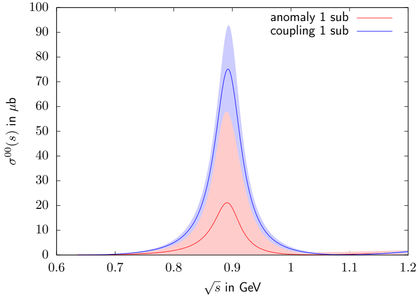
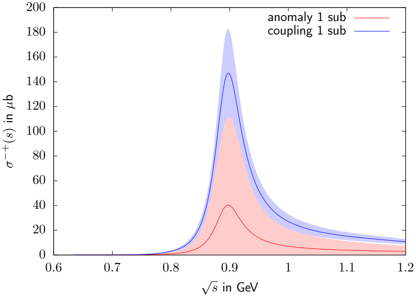


$$\gamma\bar{K}^0 \to \bar{K}^0\pi^0$$



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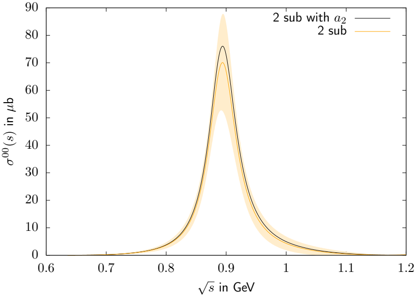

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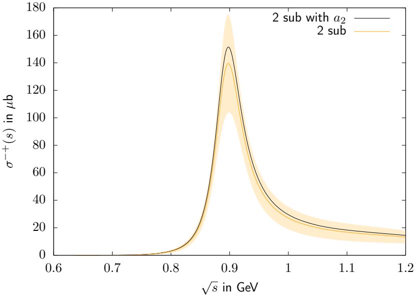

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- use linear combinations of the four reconstruction theorems

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- isovector and isoscalar part of the photon decouple

Iterative procedure

- solve KT-equations [Khuri and Treiman, 1960] iteratively

$$\mathcal{F}\left[\widehat{\mathcal{F}}\right](s) = \Omega(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\widehat{\mathcal{F}}(s')\sin(\delta(s'))}{|\Omega(s')|(s'-s)}\right)$$

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- start and end condition are defined by

$$\widehat{\mathcal{F}}^{k=0}(s) = \widehat{\mathcal{F}}_{\text{fix}}(s) \qquad |\widehat{\mathcal{F}}^k(s) - \widehat{\mathcal{F}}^{k-1}(s)| < \epsilon$$