Gluon PDF from Lattice QCD Calculation

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Outline

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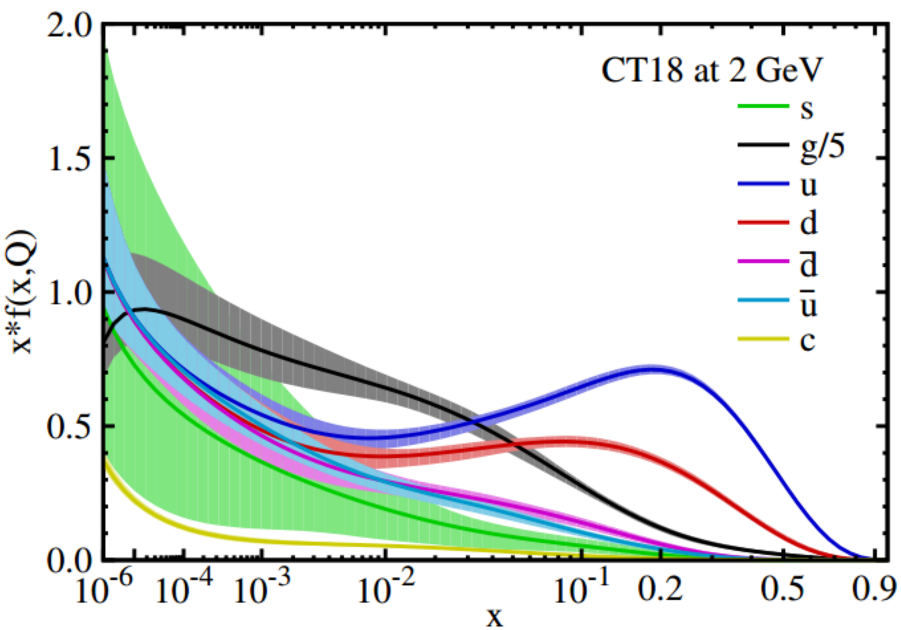
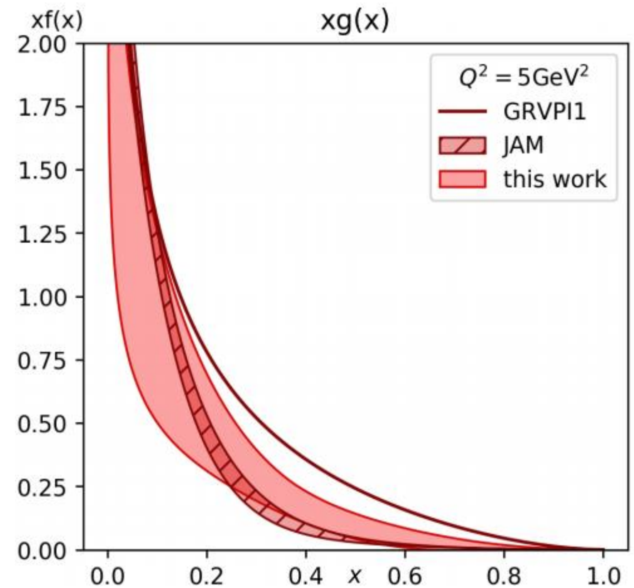
- Motivation and background
- Studies of nucleon gluon PDF using lattice QCD
  - Previous calculation of nucleon gluon PDF
  - Matrix element extraction
  - Reduced pITD fit
  - Gluon PDF reconstruction and comparison
- Studies on meson gluon PDF using lattice QCD
  - Pion gluon PDF
  - Kaon gluon PDF
- Conclusion and outlook

Studying the parton distribution functions (PDFs) is important to characterize the structure of the hadron



[CT18:1912.10053]



[xFitter: 2002.02902]

- Unpolarized gluon PDF dominates in small-x region
- The error at large x-region is large compared to the valence quark PDFs
- Current gluon PDFs from different global analyses vary by the input experiment data. For example, the Suppression of g(x) in the 0.1<x<0.4 when ATLAS and CMS jet data are included [Hou et al, RRD:2019]

Background and Motivation

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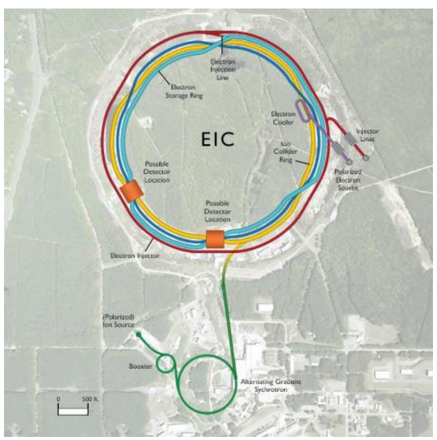
To improve the gluon PDFs,

- Experimentally, for example, there are Electron-Ion Collider (EIC) will aim at gluon PDF
- Theoretically, lattice QCD is an independent approach to calculate gluon PDF

U.S.-based EIC



[BNL News]
https://www.bnl.gov/newsroom/news.php?a=116998

China-based EicC

Figure 1.5: Accelerators in the EicC accelerator facility.

[Daniele P. A: 2102.09222]

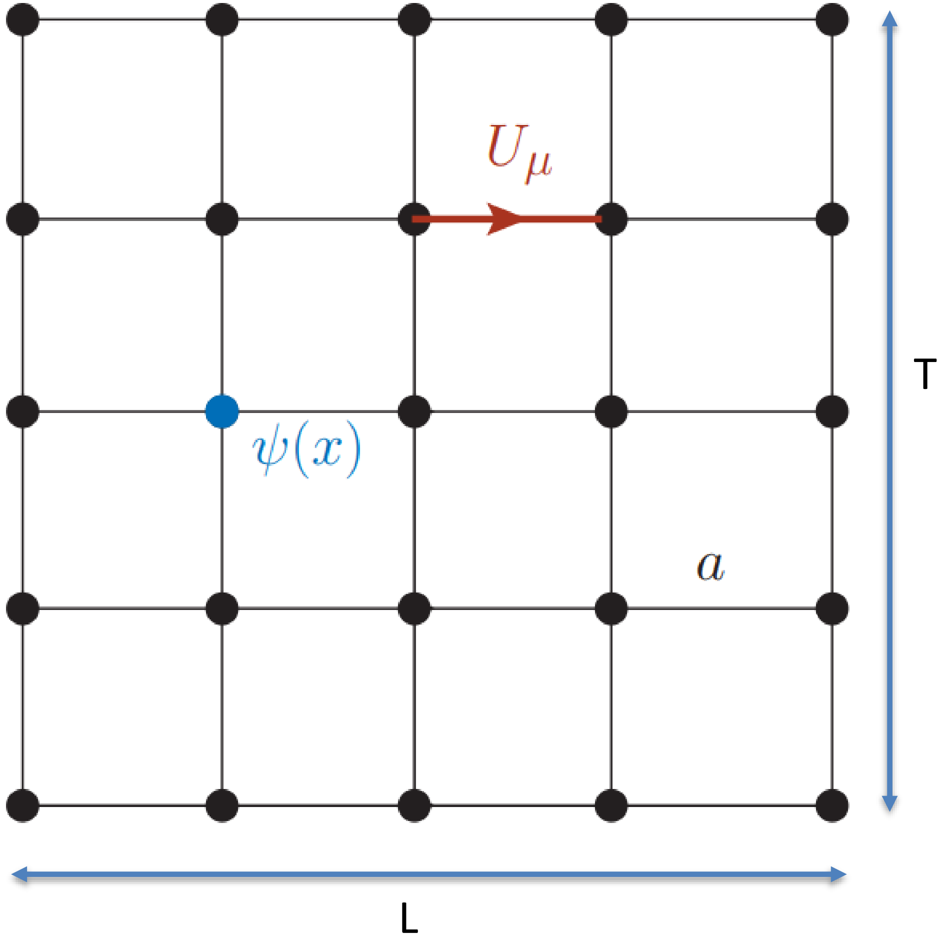
Lattice QCD in a nutshell

Lattice regularization is the only method that can describe strong interaction QCD.

Define fields on discrete Euclidean spacetime ($L^3 \times T$) with lattice spacing a

Quark fields $\psi(x), \bar{\psi}(x)$ on each site. Gauge field $U_\mu(x)$ on links.

Pion mass m_π, usually not physical pion mass (cheaper)



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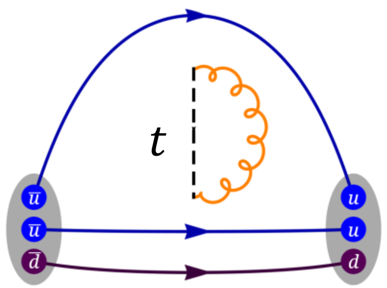
Pseudo-PDF method

The reduced Ioffe-time distribution (ITDs) definition [Orginos et al, 1706.05373],

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0,0)}$$

where $M(\nu, z^2) = \langle 0 | O_g(z) | 0 \rangle$, Ioffe-time $\nu = zP_z$.

$$O_g(z) = \sum_{i \neq z,t} O(F^{ti}, F^{ti}; z) - \sum_{i \neq z,t} O(F^{ij}, F^{ij}; z),$$



The gluon pseudo-PDF matching condition,

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x \rangle_g} (R_1(x\nu, z^2\mu^2) + R_2(x\nu))$$

[Balitsky et al, 1910.13963]

the reduce pseudo-ITD (RpITD) is matching to $xg(x, \mu^2)/\langle x \rangle_g$.

First x-dependence nucleon gluon PDF (2020)

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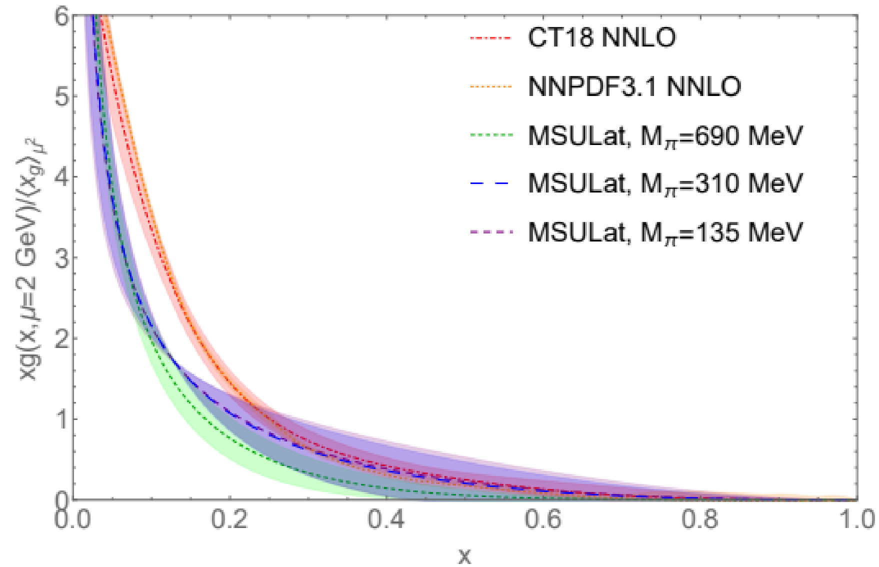
The lattice calculation [Fan et al, Int.J.Mod.Phys.A 2020] are carried out with Clover valence quarks on the MILC $N_f = 2+1+1$ HISQ fermion gauge configurations with

Symbol	$L^3 \times T$	a	m_π^{sea}	N_{cfg}	Measurements
a12m310	$24^3 \times 64$	$0.1207(11)$ fm	$306.9(5)$ MeV	898	57472

First x-dependence nucleon gluon PDF (2020)

We extracted the nucleon gluon PDF at two different pion masses and did a naïve pion mass extrapolation to pion mass 135 MeV.



moment	MSULat (690 MeV)	MSULat (310 MeV)	MSULat (extrapolated 135 MeV)	CT18	NNPDF3.1
$\langle x_g^2 \rangle_{\mu^2}$	0.040(15)(3)	0.043(26)(4)	0.045(30)(4)	0.0552(76)	0.048(13)
$\langle x_g^3 \rangle_{\mu^2}$	0.011(6)(2)	0.013(14)(3)	0.014(17)(3)	0.0154(37)	0.011(9)

- Our gluon PDF is consistent with the one from CT18 NNLO and NNPDF3.1 NNLO within one sigma in the x > 0.3 region

- In the small-x region (x < 0.3), there is a strong deviation between our lattice results and the global fits. This is likely due to the fact that the largest ν used in this calculation is less than 7, and the errors in large-ν data increase quickly as ν increases.

2022 Lattice setups

We use clover valence fermions on $N_f = 2 + 1 + 1$ highly improved staggered quarks (HISQ) lattices generated by the MILC Collaboration [MILC 1212.4768].

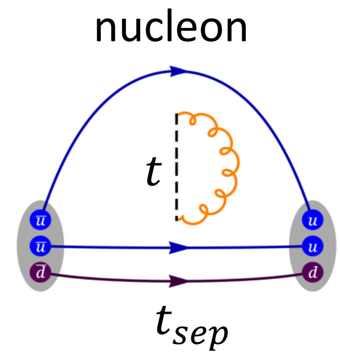
ensemble	a09m310	a12m220	a12m310	a15m310
a (fm)	0.0888(8)	0.1184(10)	0.1207(11)	0.1510(20)
M_π^{sea} (MeV)	312.7(6)	216.9(2)	305.3(4)	306.9(5)
M_π^{val} (MeV)	313.1(13)	226.6(3)	309.0(11)	319.1(31)
$M_{\eta_s}^{val}$ (MeV)	698.0(7)	696.9(2)	684.1(6)	684.1(6)
$L^3 \times T$	$32^3 \times 96$	$32^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
N_{meas}^{2pt}	387,456	1,466,944	324,160	129,600

We thank MILC Collaboration for sharing the lattices used to perform this study.

Following the work [Bhattacharya et al, 1306.5435], the correlators C_{3pt} and C_{2pt} can be decomposed as,

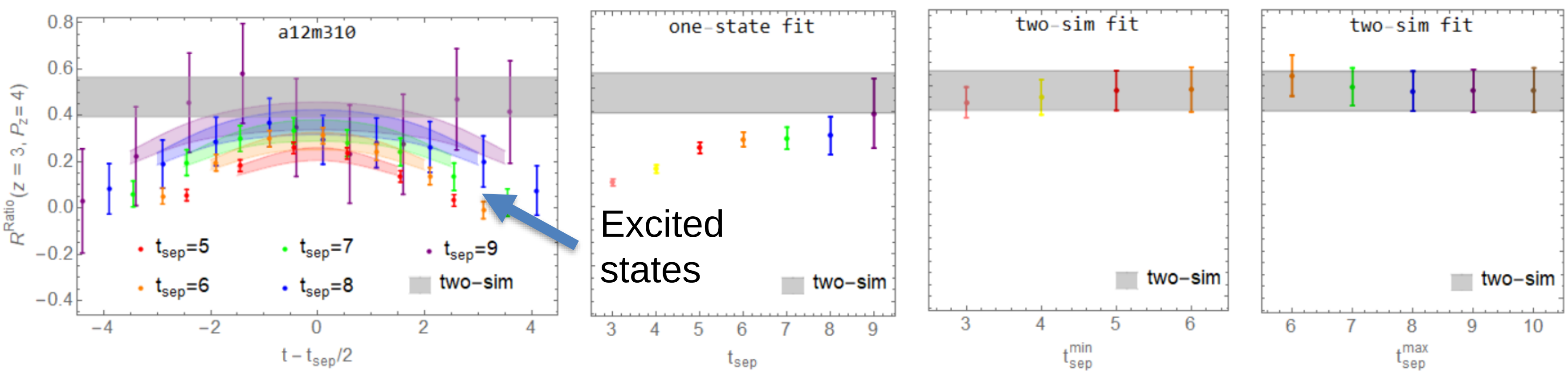
$$C_{3pt}(z,P_z;t_{sep},t) = |A_0|^2\langle 0|O|0\rangle e^{-E_0 t_{sep}} + |A_1||A_0|\langle 1|O|0\rangle e^{-E_1(t_{sep}-t)}e^{-E_0 t} + |A_0||A_1|\langle 0|O|1\rangle e^{-E_0(t_{sep}-t)}e^{-E_1 t} + \cdots$$

$$C_{2pt}(z,P_z;t_{sep}) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t} + \cdots$$



where the ground state matrix element is $\langle 0|O|0\rangle$, assuming $\langle 1|O|0\rangle = \langle 0|O|1\rangle$.

a12m310, **nucleon**



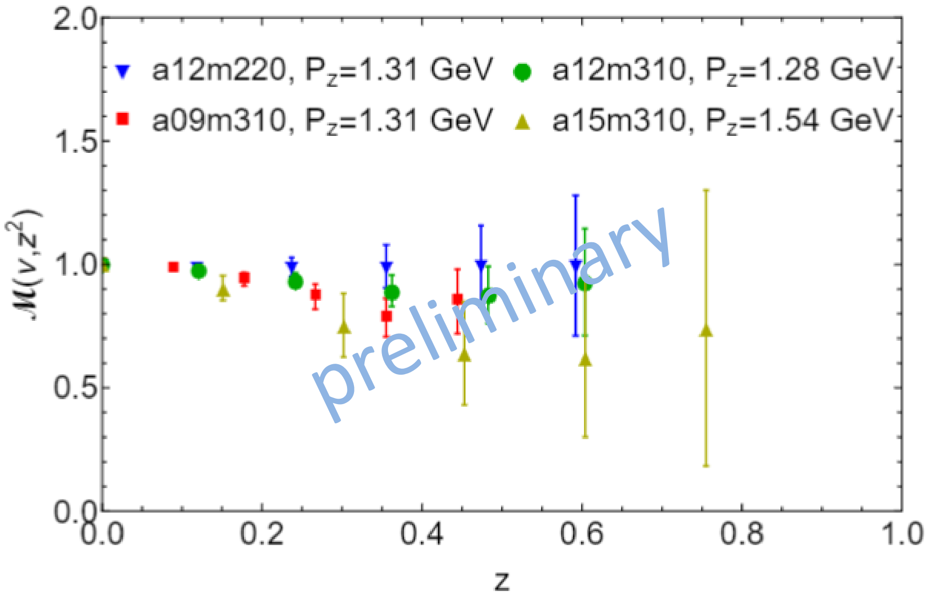
- The bare matrix element (ME) extracted from the two-sim fits are stable with the t_{sep} fit ranges choices around our final choice $t_{sep}=[5,9]$

RpITD from different ensembles

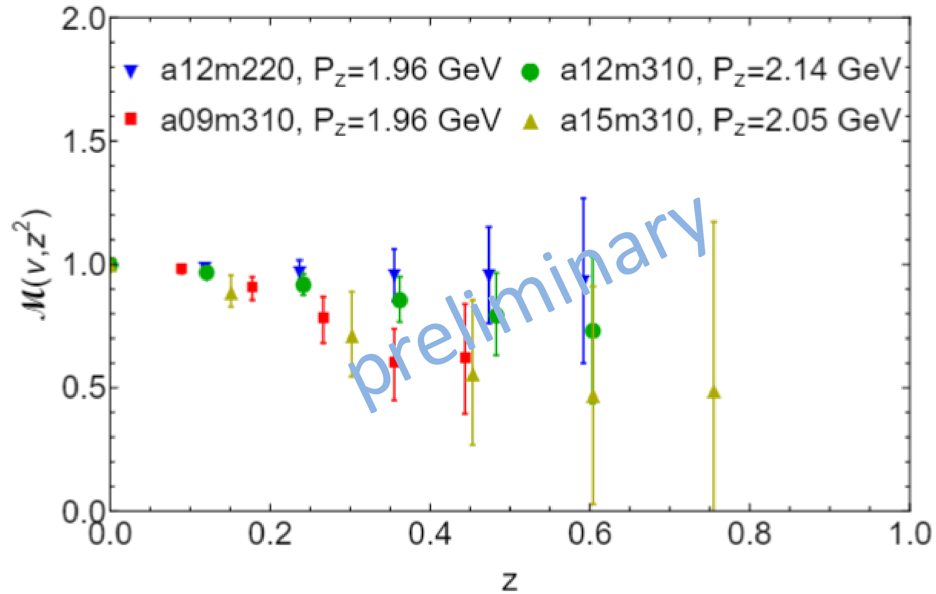
The RpITDs are obtained by taking the double-ratio of ground-state matrix element,

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0,0)}$$

$P_z \sim 1.3$ GeV



$P_z \sim 2$ GeV



- The RITDs obtained from bare matrix elements have weak lattice-spacing and pion mass dependence

Matrix element → **RpITD** → $xg(x)/\langle x \rangle$ → $xg(x)$

RpITD functional form fit

The lattice calculated RpITD can be connected with gluon PDF through the matching kernel R,

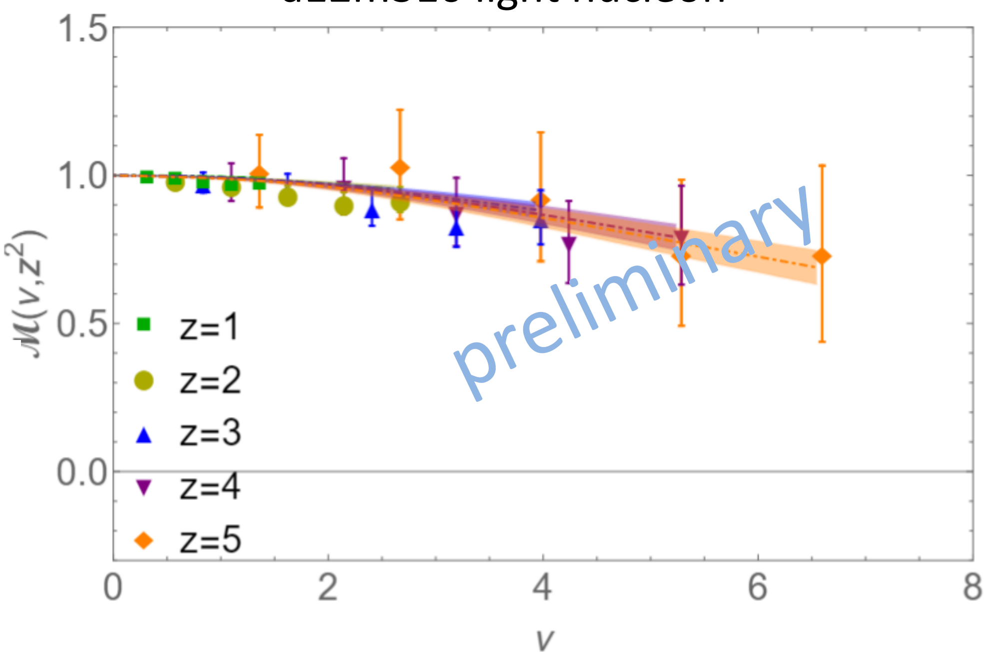
$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R(x\nu, z^2\mu^2)$$

Functional form light-cone PDF,

$$f_g(x,\mu) = \frac{xg(x,\mu)}{\langle x \rangle_g(\mu)} = \frac{x^A(1-x)^C}{B(A+1,C+1)},$$

$$\chi^2(\mu, a, M_\pi) = \sum_{\nu,z} \frac{(\mathscr{M}^{\rm fit}(\nu,\mu,z^2,a,M_\pi) - \mathscr{M}^{\rm lat}(\nu,z^2,a,M_\pi))^2}{\sigma^2_{\mathscr{M}}(\nu,z^2,a,M_\pi)}.$$

a12m310 light nucleon



Matrix element → RpITD → $xg(x)/\langle x \rangle$ → $xg(x)$

Reconstructed $xg(x)/\langle x \rangle_g$

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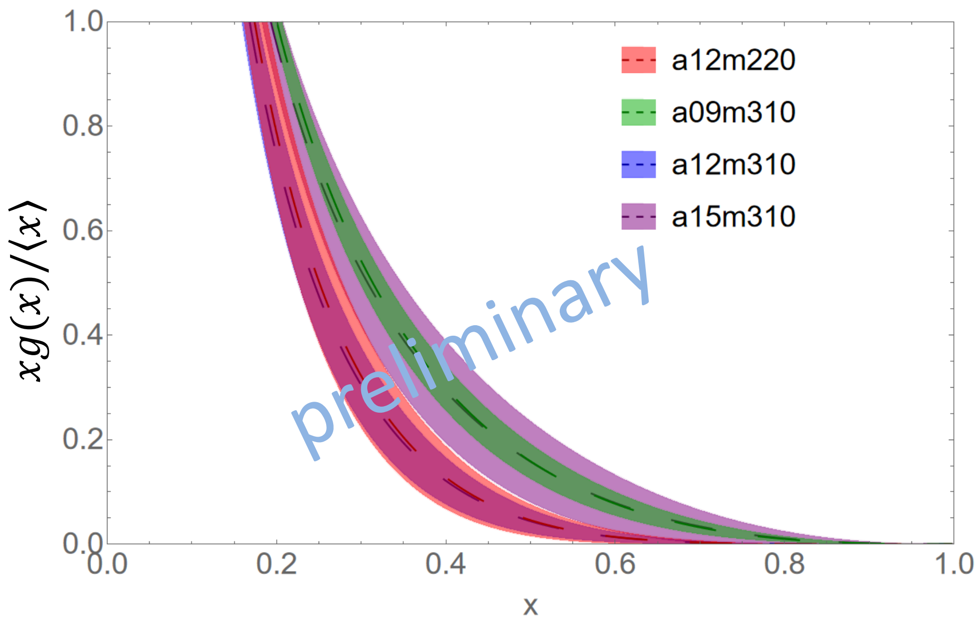
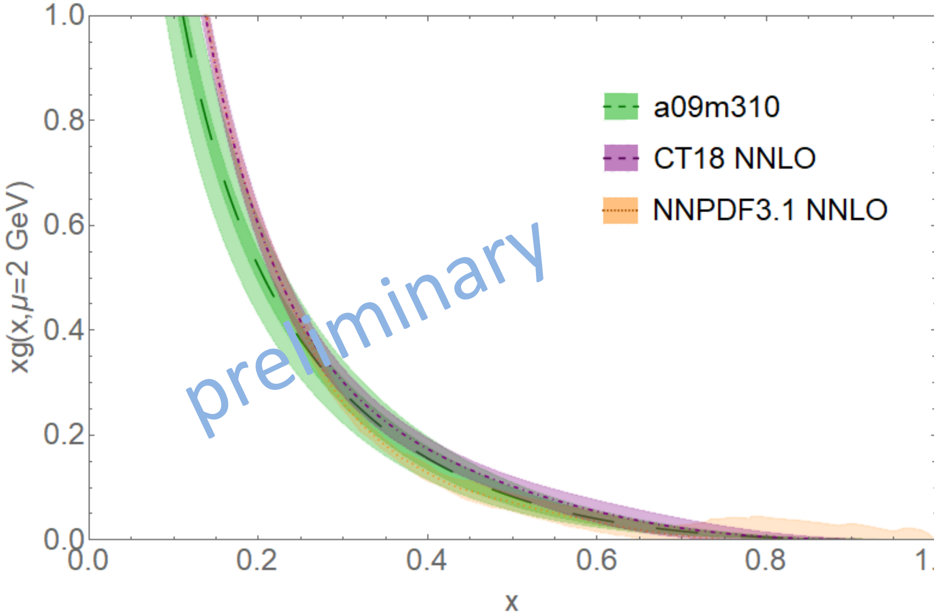
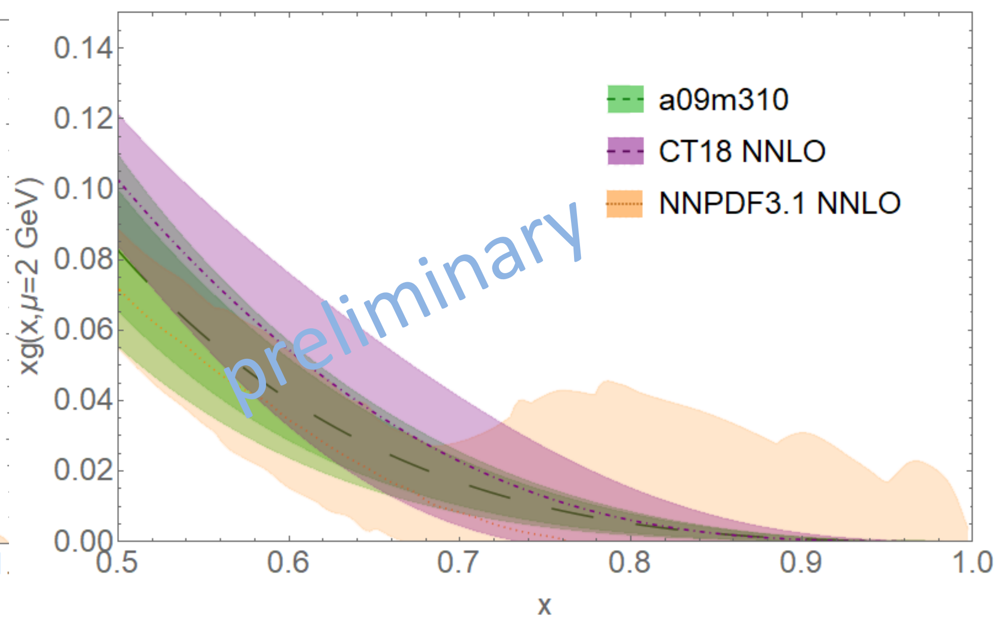


Matrix element → RpITD → $xg(x)/\langle x \rangle$ → $xg(x)$

Nucleon gluon PDF $xg(x)$

With the renormalized $\langle x\rangle_g^{\overline{MS}}$ calculated, we obtain the nucleon gluon PDF $xg(x)$ for the first time.





- In the small-x region, $xg(x)$ results are in a better agreement with CT18 or NNPDF3.1 NNLO $xg(x)$, comparing to the previous $xg(x)/\langle x\rangle$ results

Matrix element → RITD → $xg(x)/\langle x\rangle$ → $xg(x)$

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$L^3 \times T$	$32^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
N_{meas}^{2pt}	731,200	143,680	21,600

[Fan and Lin, Phys.Lett.B 2021]

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First x-dependence pion gluon PDF (2020)

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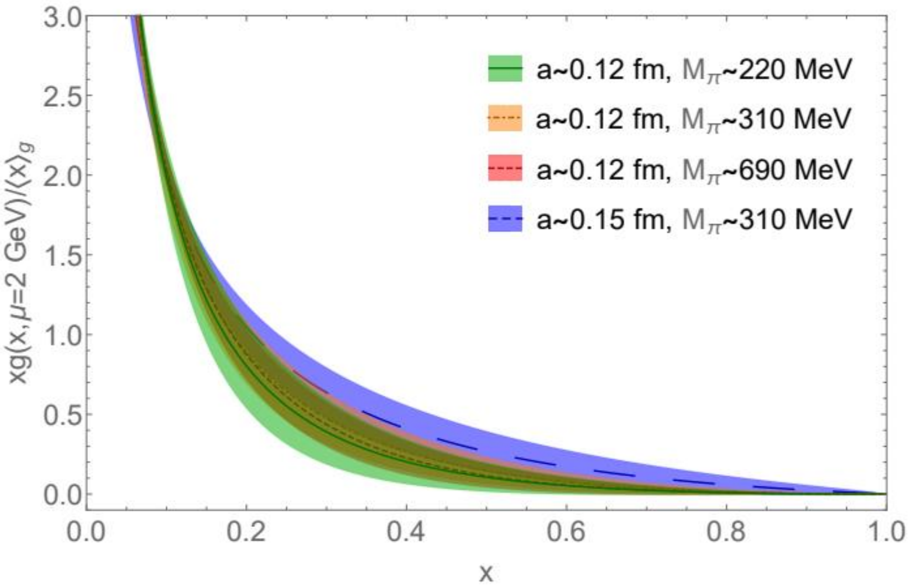
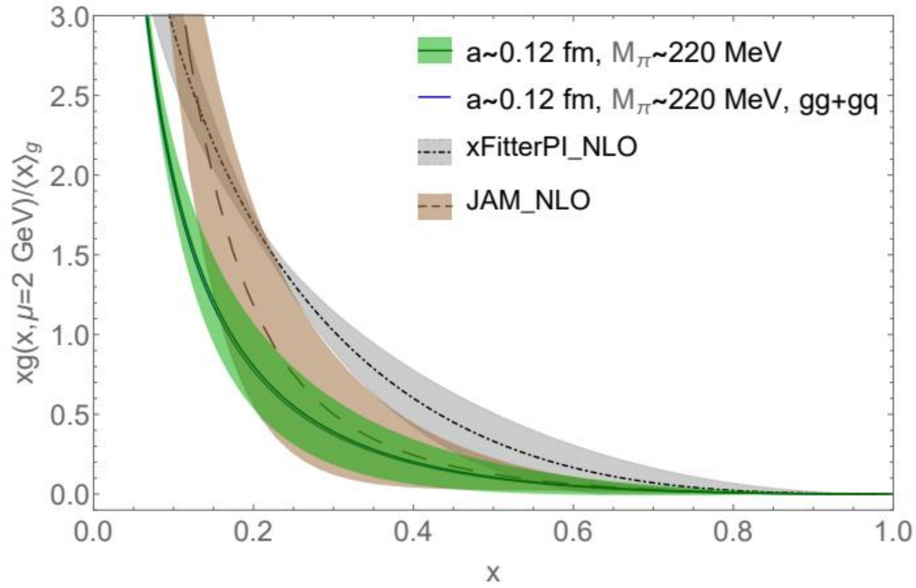
We used the same procedure to calculate pion gluon PDF as we used in 2020 nucleon PDF calculation. In our pion calculation [Fan and Lin, Phys.Lett.B 2021], we used three lattice ensembles with 3 pion masses and two lattice spacings.

We found:

- Our pion gluon PDF for the lightest pion mass is consistent with JAM for $x > 0.2$ and with xFitter for $x > 0.5$ within uncertainty
- The asymptotic behavior of the pion gluon PDF in the large-x region in terms of $(1-x)^C$. $C > 3$ is implied from our study



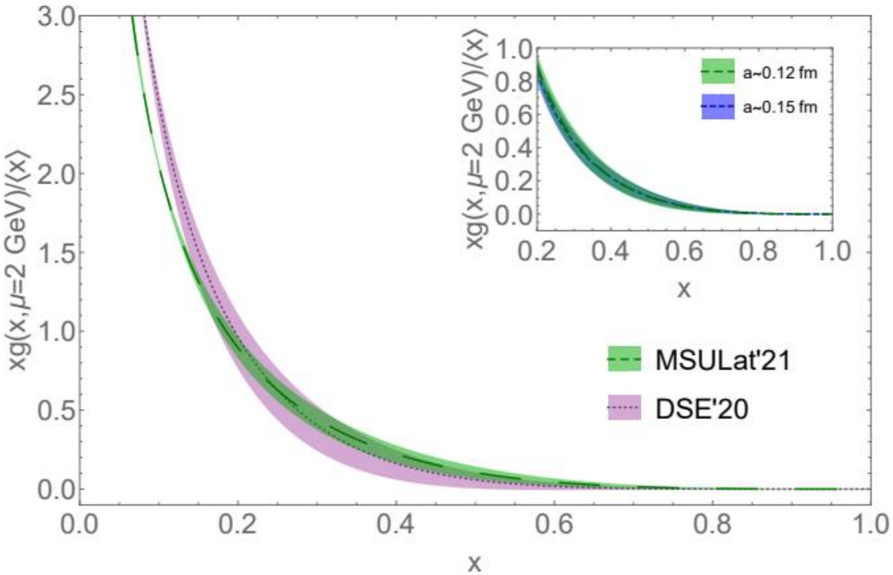


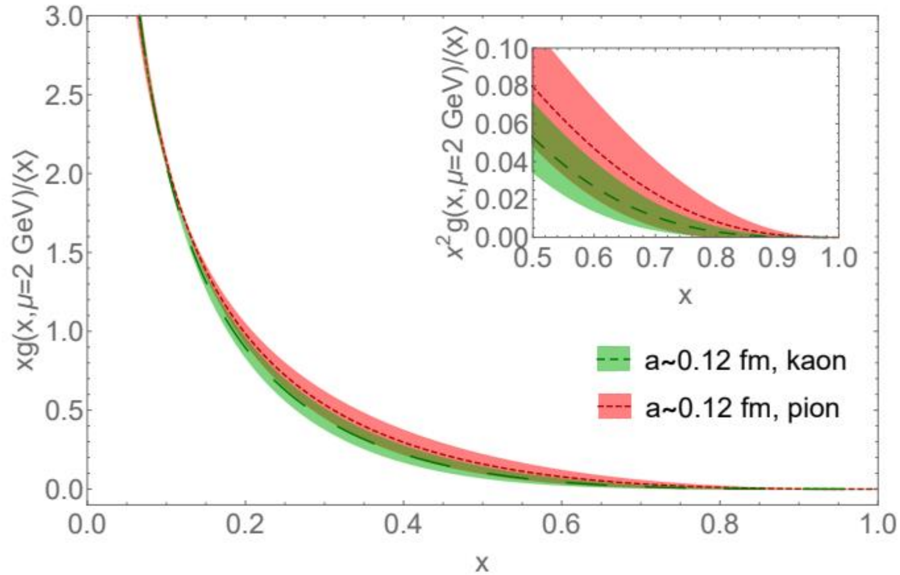
First x-dependence kaon gluon PDF (2021)

Following the same pseudo-PDF matching kernel, we also studied the kaon gluon PDF [Alejandro Salas-Chavira et al, 2112.03124]. We improved our fit procedure and statistics in this work.

We found:

- The kaon PDF to be slightly smaller in central value for most of the x > 0.2 region comparing with the pion PDF result.
- The Kaon gluon PDF at the finer lattice spacing is consistent with the DSE result [Cui et al, Eur.Phys.J.C 2020] within statistical uncertainties.





[Alejandro Salas-Chavira et al, 2112.03124]

Conclusion and outlook

- The x-dependence nucleon, pion, and kaon gluon PDFs are calculated using lattice QCD via the pseudo-PDF method

- The nucleon gluon PDF we calculated are consistent with the global fit PDFs within the one-sigma error, though the error is still quite large

- The extrapolation of nucleon gluon PDF to physical pion mass and continuum limit will be the next to-do