

# EHM, pion DFs

# and $J/\psi$ production

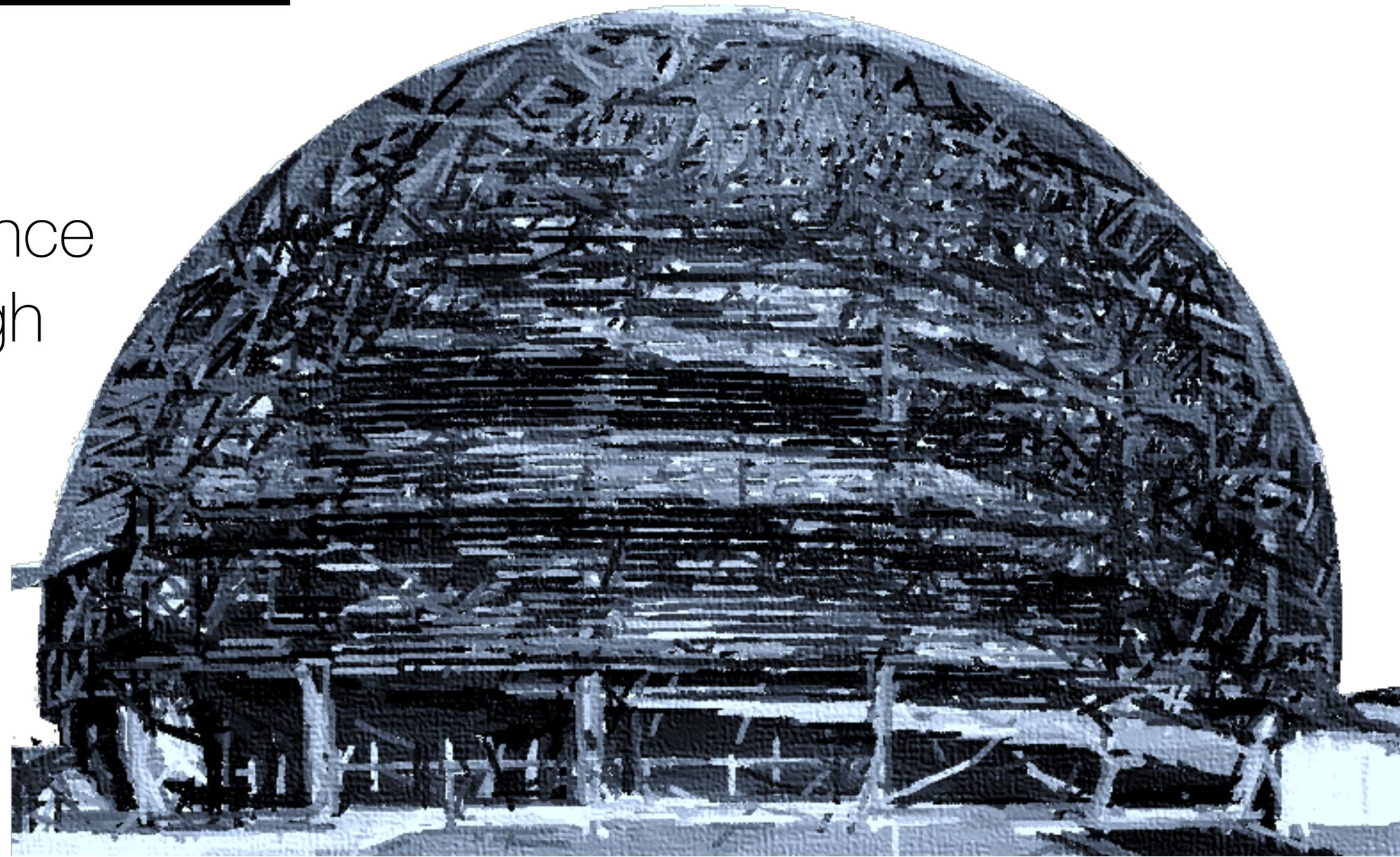
DANIELE BINOSI  
ECT\* - FONDAZIONE BRUNO KESSLER

Perceiving the Emergence  
of Hadron Mass through  
AMBER@CERN **7**

MAY 10 - MAY 13



**ECT\***  
EUROPEAN CENTRE  
FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS  
FONDAZIONE BRUNO KESSLER



# QCD LAGRANGIAN

$$\mathcal{L}_{\text{QCD}} = \sum_{j=u,s,d,\dots} \bar{q}_j [\gamma_\mu D_\mu + m_j] q_j + \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 + \partial_\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial_\mu \bar{c}^a) A_\mu^b c^c$$

(linear) gauge fixing      Faddeev-Popov ghost term

$$D_\mu = \partial_\mu + ig \frac{1}{2} \lambda^a A_\mu^a$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

## GLUON SELF-INTERACTION

pure-gluon QCD displays a **mass gap**

$$m_g \sim 0.5 \text{ GeV}$$

Cornwall, PRD 26 (1982)

## GAUGE SYMMETRY IS FINE

2-point STI can be still satisfied with

$$\Delta_{\mu\nu}(q) = \frac{P_{\mu\nu}(q)}{q^2 [1 + \Pi(q^2)]}, \quad q^\mu P_{\mu\nu}(q) = 0$$

$$\lim_{q^2 \rightarrow 0} q^2 \Pi(q^2) = m_g$$

(“only” requires the presence of longitudinally coupled massless poles)

Schwinger, PR 125 and 128 (1962)

## STRESS-ENERGY TENSOR IS ANOMALOUS

$$T_{\mu\mu} = \frac{\beta}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$

but no size prescribed...

## 1 RGI MASSES

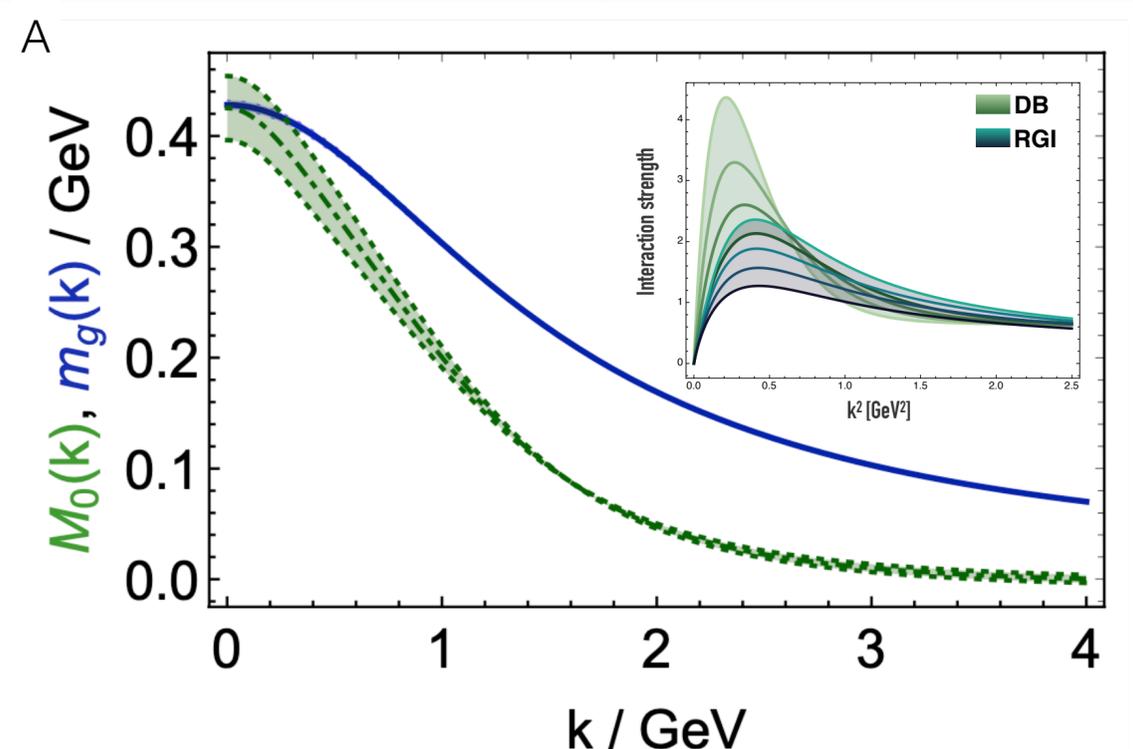
40 years+ non-perturbative methods uncover the size of the gluon mass

$$m_g = 0.43(1) \text{ GeV}$$

Aguilar et al., EPJC 80 (2020)

and reveal the associated RGI running masses, unifies matter-based and gauge-focused understanding of QCD interactions,...

DB et al., PLB 742 (2015)



# A QCD EHM PRIMER

## 2 PI EFFECTIVE CHARGE

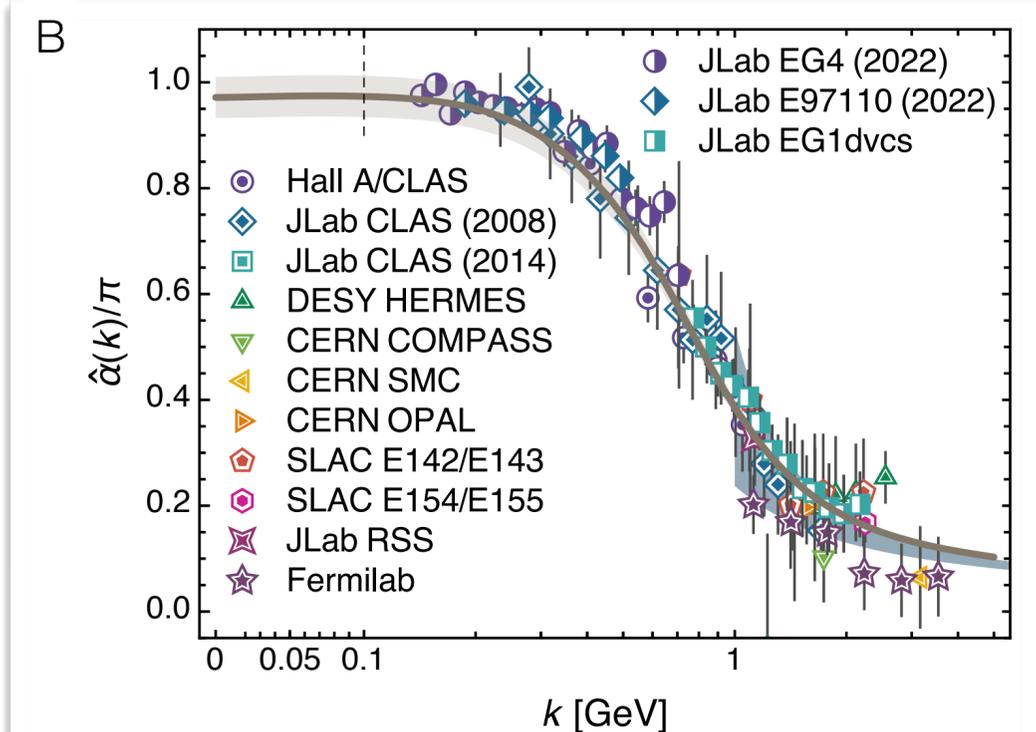
owing to the emergence of a non-zero gluon mass scale a process independent effective charge emerges

$$\hat{\alpha}(s) = \frac{4\pi}{(11 - 2n_f/3) \log[\mathcal{K}^2(s)/\Lambda^2]}, \quad \mathcal{K}^2(s) = \frac{a_0^2 + a_1 s + s^2}{b_0 + s}$$

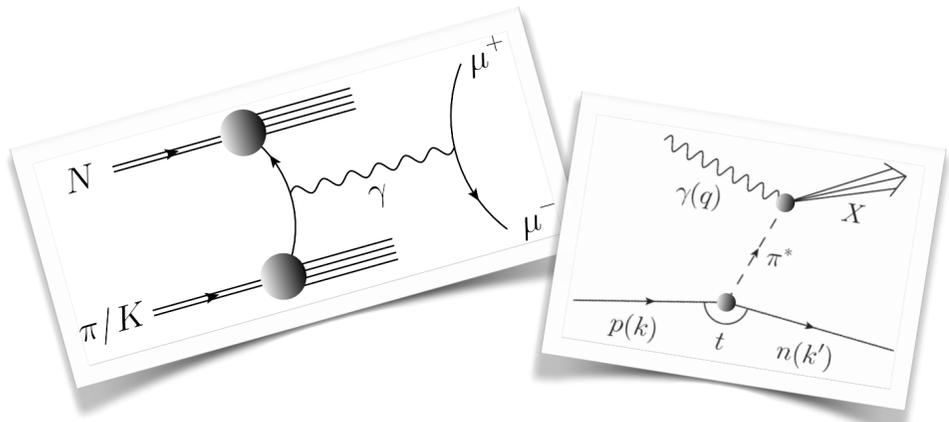
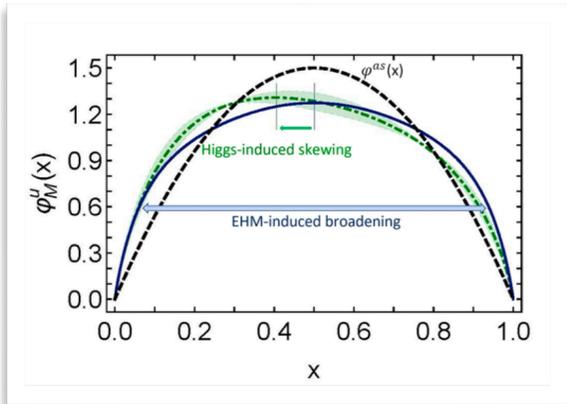
parameter free prediction

defines a screening mass of  $\zeta_H \approx 1.4\Lambda = 0.331(2) \text{ GeV}$

practically identical to Bjorken sum rule coupling measured in DIS candidate for QCD interaction strength @ all moment

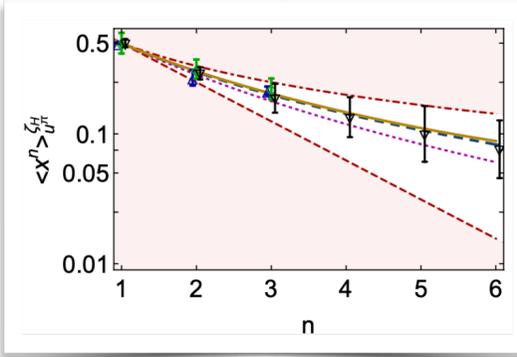


Cui et al., CPC 44 (2020)



## 2 alpha + DGLAP

$$\langle x^n \rangle_{u_\pi}^\zeta = \langle x^n \rangle_{u_\pi}^{\zeta_H} (\langle 2x \rangle_{u_\pi}^\zeta)^{\gamma_0^n / \gamma_0^1} \quad \frac{1}{2^n} \leq \langle x^n \rangle_{u_\pi}^{\zeta_H} \leq \frac{1}{1+n}$$



### DISTRIBUTION AMPLITUDES

$$f_M \varphi_M^u(x; \zeta_H) = \frac{1}{16\pi^3} \int d^2 k_\perp \psi_{M_u}^{\uparrow\downarrow}(x, k_\perp^2; \zeta_H)$$

### DISTRIBUTION FUNCTIONS

$$u^M(x; \zeta_H) = \int d^2 k_\perp |\psi_{M_u}^{\uparrow\downarrow}(x, k_\perp^2; \zeta_H)|^2$$

Brodsky, and Lepage, ASDHEP 5 (1989)

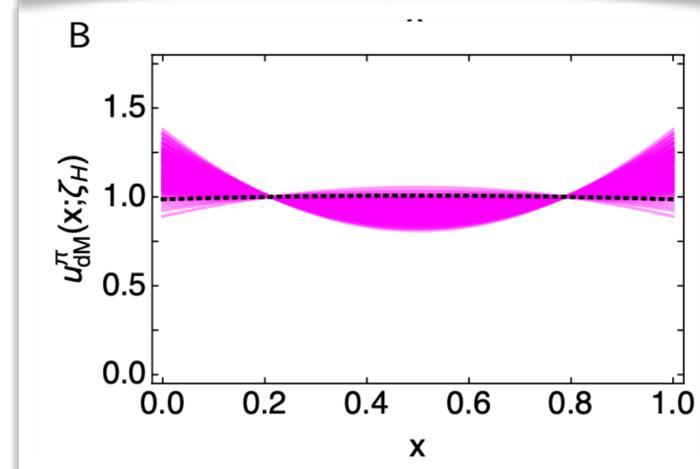
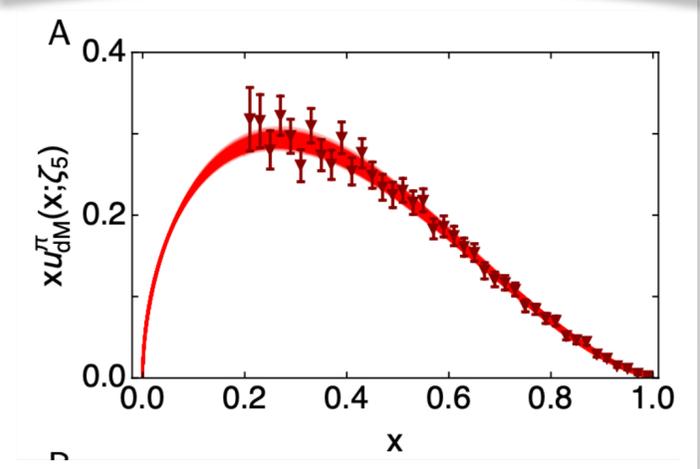
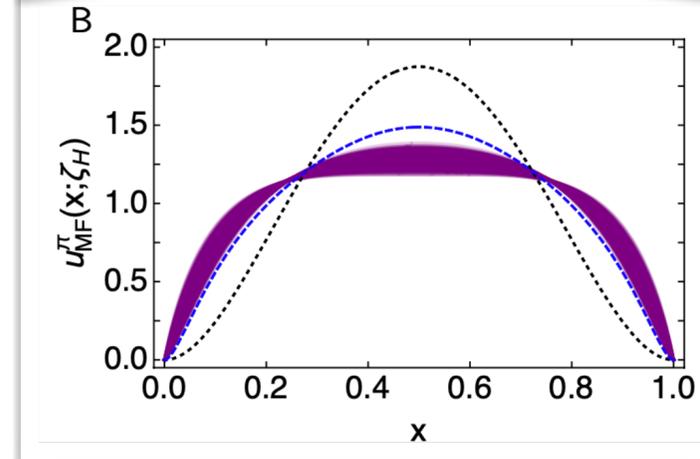
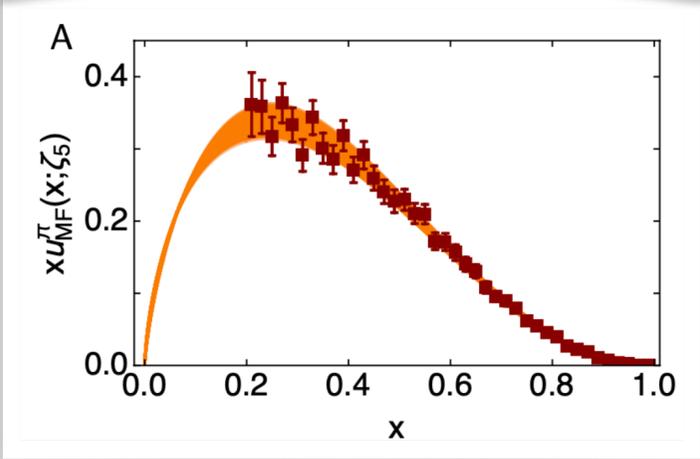
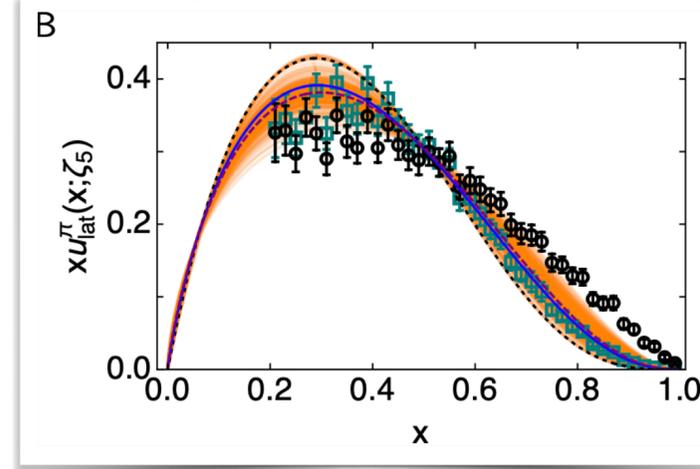
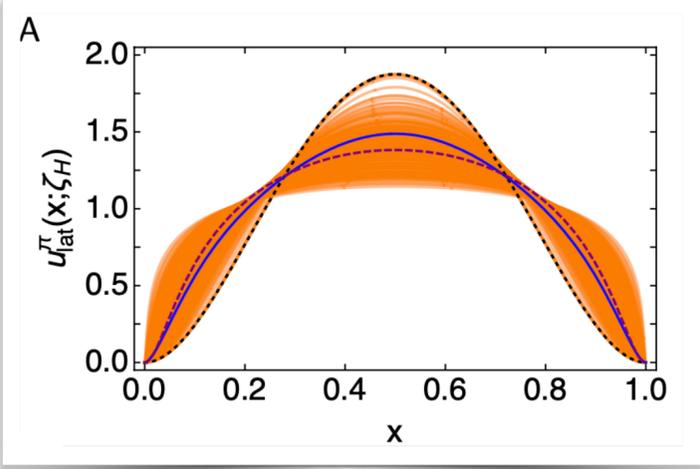
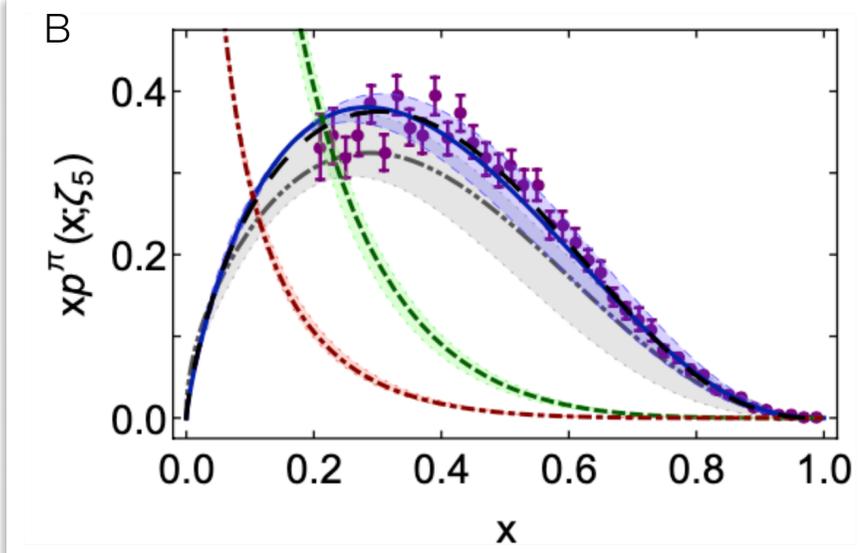
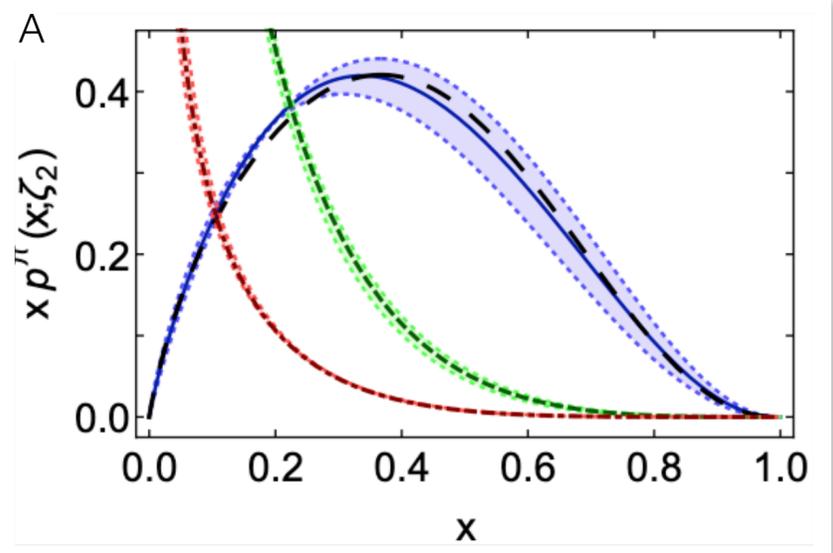
## FACTORIZED APPROXIMATION

$$\psi_{M_u}^{\uparrow\downarrow}(x, k_\perp^2; \zeta_H) = \varphi_M^u(x; \zeta_H) \psi_{M_u}^{\uparrow\downarrow}(k_\perp^2; \zeta_H) \implies u^M(x; \zeta_H) \propto |\varphi_M^u(x; \zeta_H)|^2$$

Xu et al PRD 97 (2018)  
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## 1 Pion

|           | $\langle x \rangle_u^\pi$ | $\langle x^2 \rangle_u^\pi$ | $\langle x^3 \rangle_u^\pi$ |                                     |  |
|-----------|---------------------------|-----------------------------|-----------------------------|-------------------------------------|--|
| $\zeta_2$ | 0.24(2)                   | 0.094(13)                   | 0.047(08)                   | $\langle x \rangle_g^\pi = 0.41(2)$ | $\langle x \rangle_{\text{sea}}^\pi = 0.11(2)$ |
| $\zeta_5$ | 0.20(2)                   | 0.074(10)                   | 0.035(6)                    | $\langle x \rangle_g^\pi = 0.45(2)$ | $\langle x \rangle_{\text{sea}}^\pi = 0.14(2)$ |



# J/ψ PRODUCTION

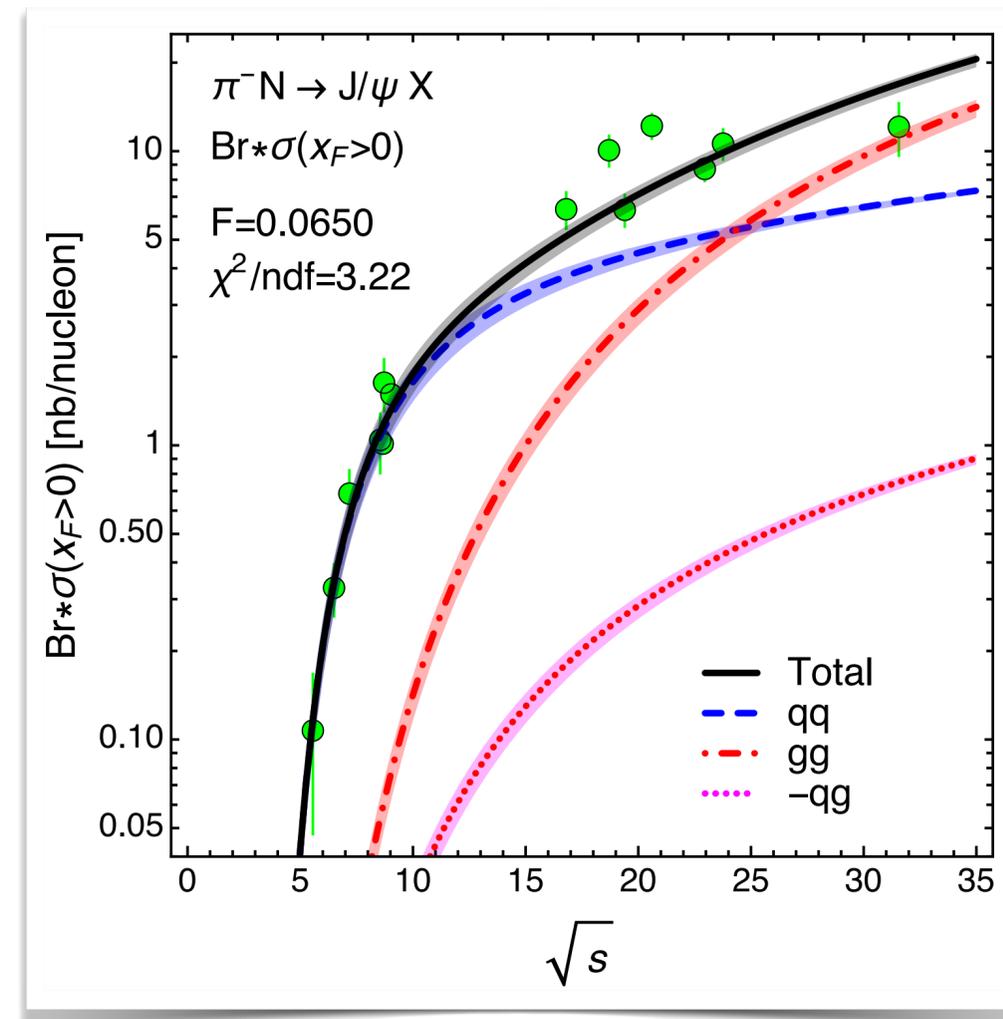
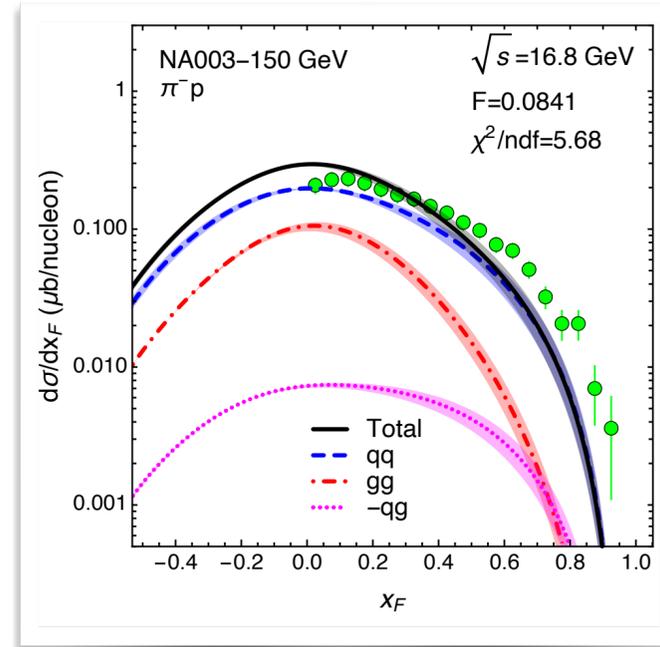
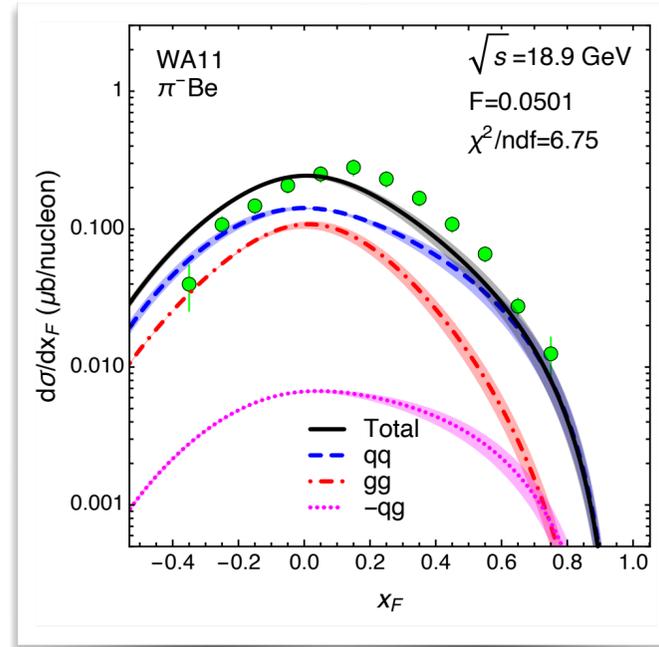
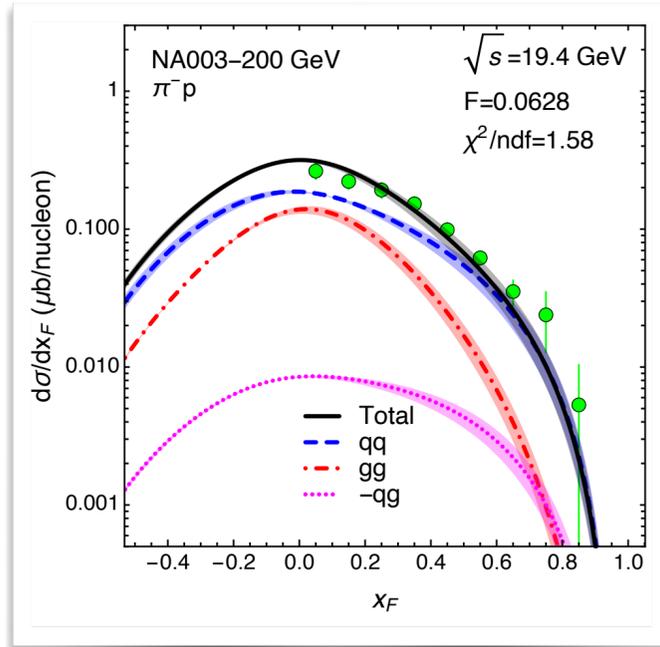
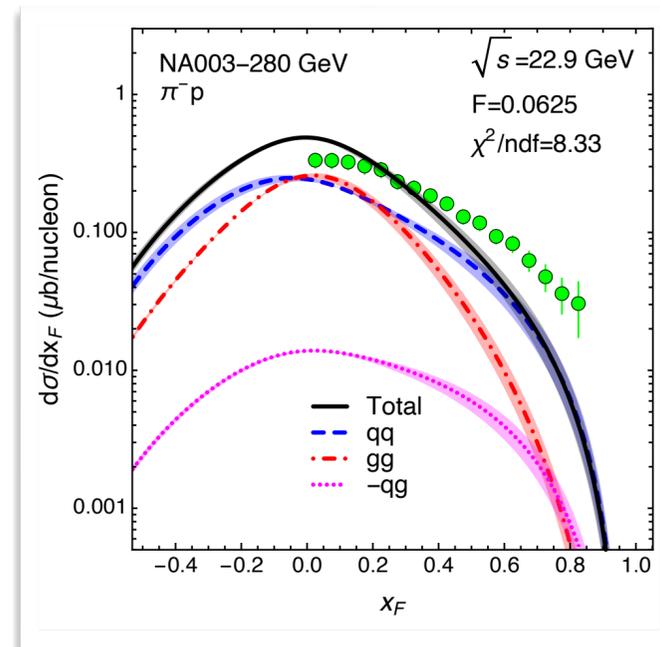
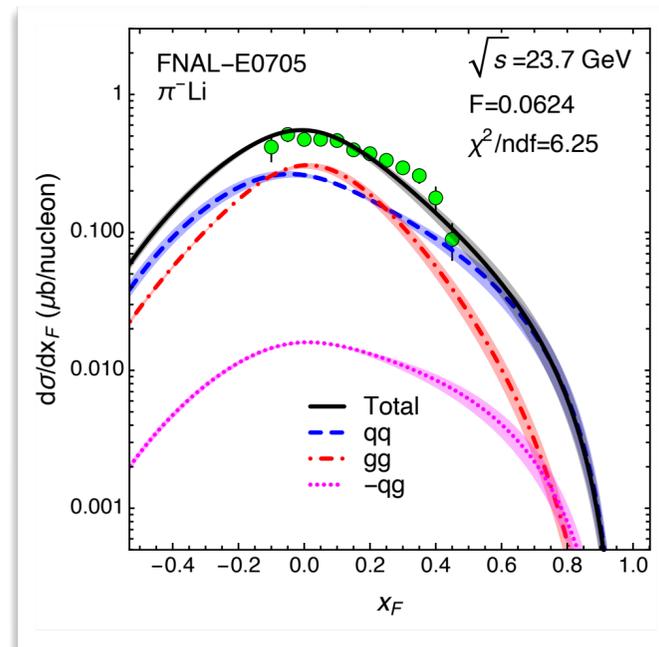
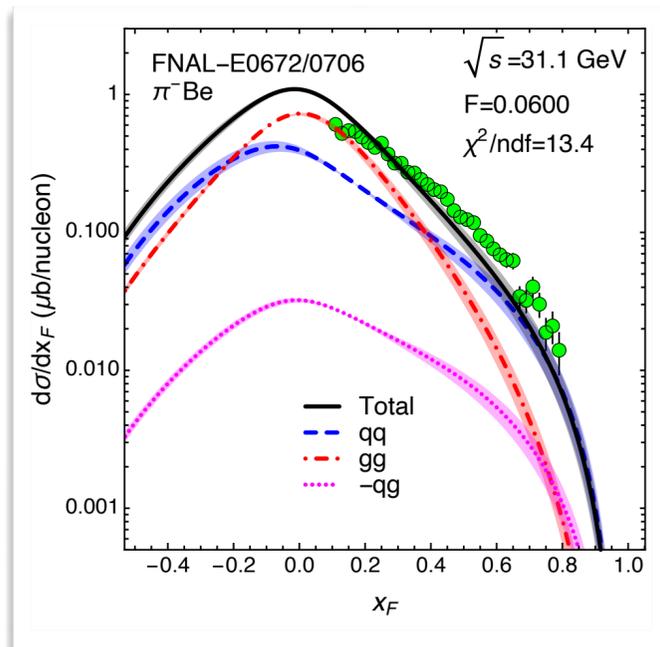
$$\left. \frac{d\sigma}{dx_F} \right|_{J/\psi} = F \sum_{i,j=q,\bar{q},g} \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 \frac{1}{s \sqrt{x_F^2 + 4 \frac{M_{c\bar{c}}^2}{s}}} \hat{\sigma}_{ij} \left( 4m_c^2/M_{c\bar{c}}^2, \mu_R^2/m_c^2 \right) f_i^{\pi^\pm}(x_1, \mu_F) f_j^N(x_2, \mu_F)$$

fit to data

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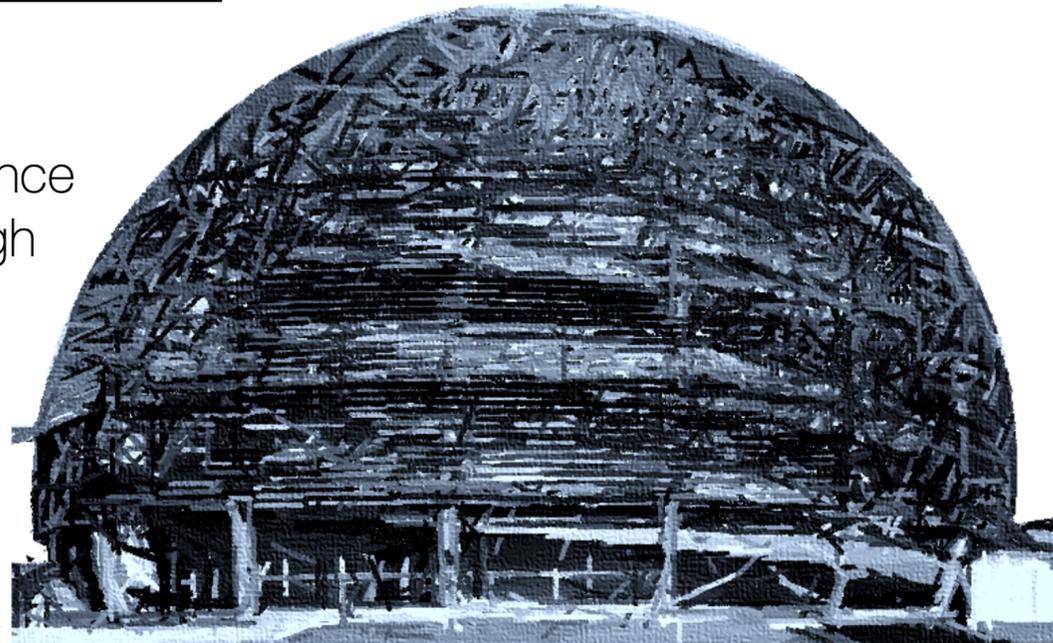
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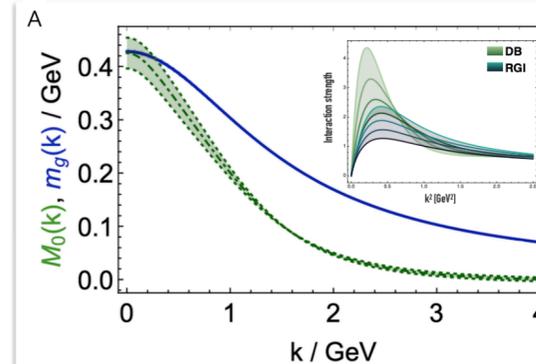
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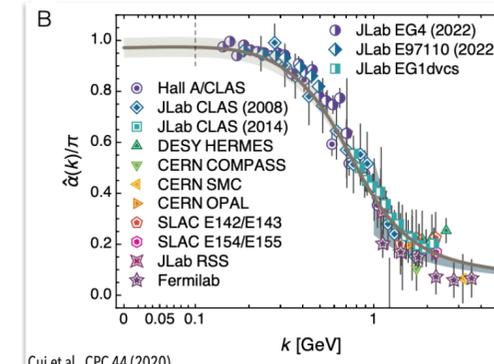
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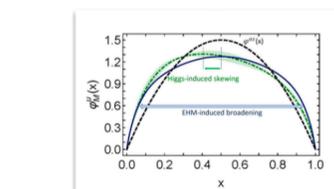
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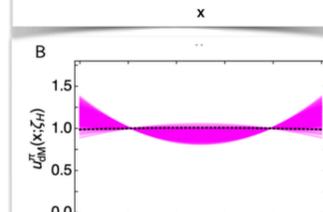
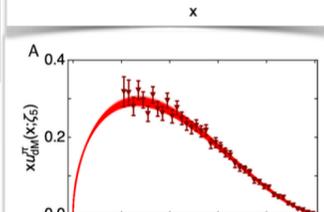
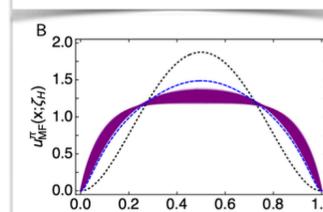
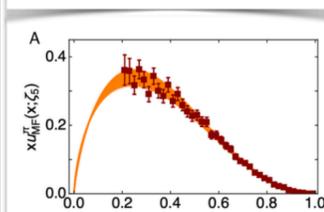
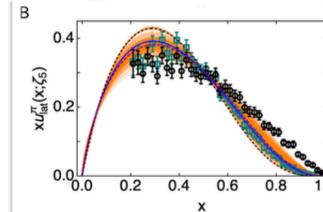
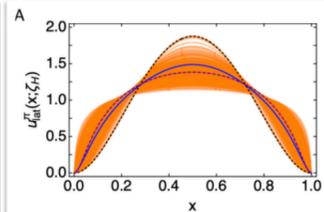
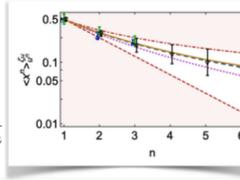
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### J/psi PRODUCTION

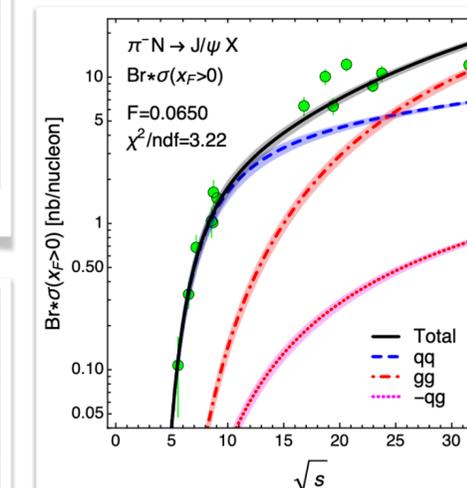
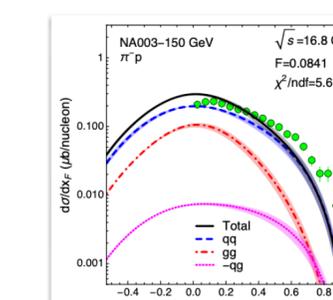
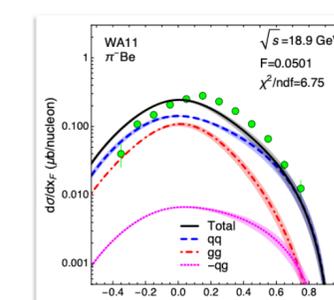
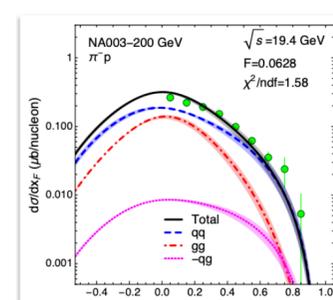
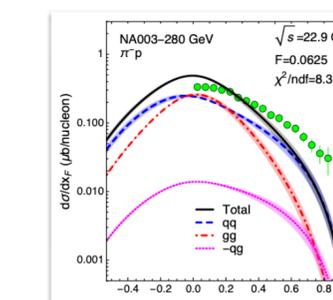
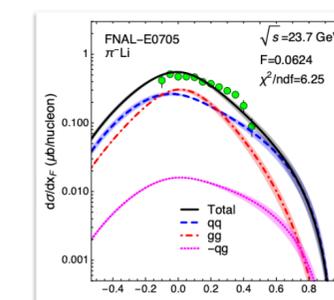
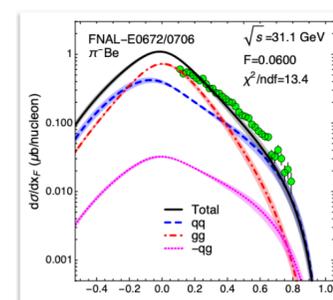
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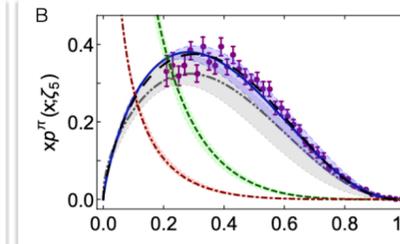
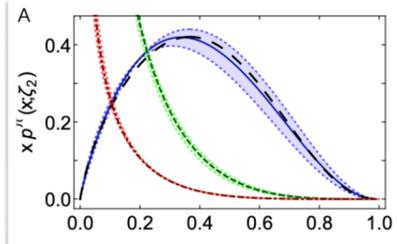
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