Massless bound states in the gauge sector of QCD

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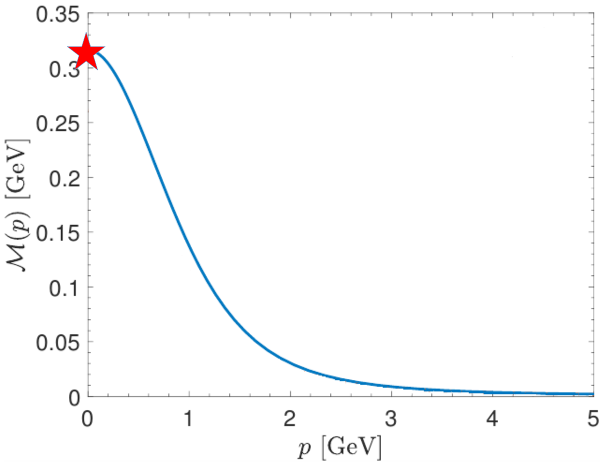
Dynamical mass generation in QFT

In QFT, **particles can acquire masses not present in the lagrangian** defining a particular theory.

In QCD there are two prominent examples:

- **Dynamical Chiral symmetry breaking**: massless lagrangian → massive quarks!
- Origin of **98%** of the observable mass in the universe

H. D. Politzer, Nucl. Phys. B **117**, 397-406 (1976).
V. A. Miransky and P. I. Fomin, Phys. Lett. B **105**, 387-391 (1981).
P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297-365 (2003).
C. S. Fischer and R. Alkofer, Phys. Rev. D **67**, 094020 (2003).
A. C. Aguilar, J. C. Cardona, M. N. F. and J. Papavassiliou, Phys. Rev. D **98**, no.1, 014002 (2018).

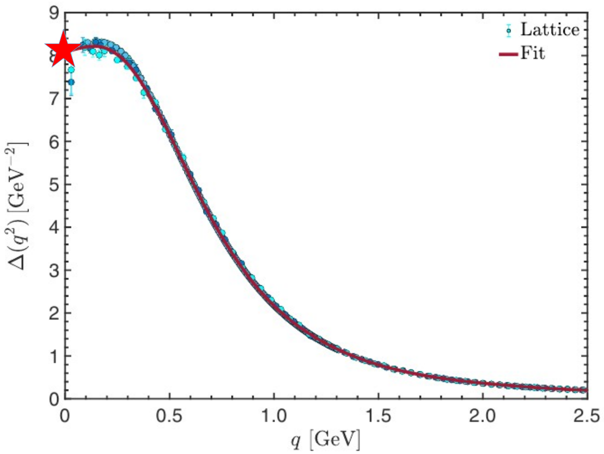


- **Gluon mass generation has also been proposed:**

J. M. Cornwall, Phys. Rev. D 26, 1453 (1982).
A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).

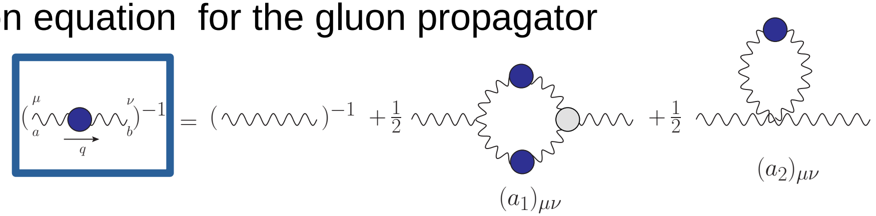
- **On the lattice, gluons also show a mass gap!**

I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
P. Bicudo, et al, Phys. Rev. D **92**, no.11, 114514 (2015).
A. C. Aguilar, et al, Eur. Phys. J. C **80**, no.2, 154 (2020).



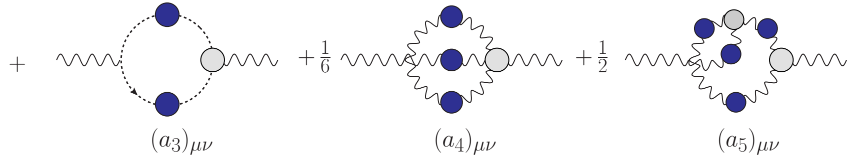
Schwinger mechanism

- The **gluon mass generation must occur without violating gauge symmetry**.
- Recalling the Schwinger-Dyson equation for the gluon propagator



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$



It can be shown that

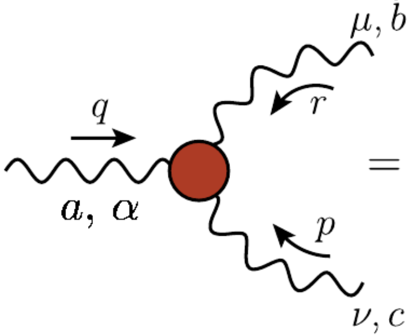
Gauge symmetry + Regular vertices at *q² = 0* → $\Delta^{-1}(0) = 0$

★ **The key to generate gluon mass is to have massless poles, longitudinaly coupled to the gluon momenta, in the vertices of QCD.**

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

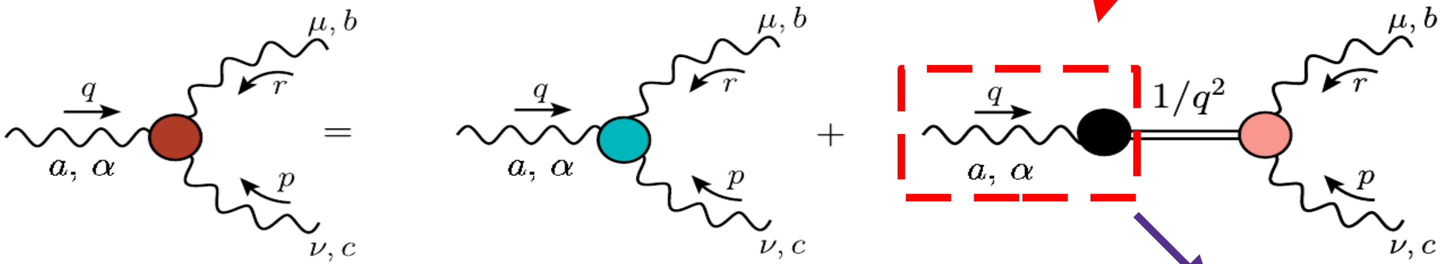
Massless bound state formalism

We assume that the most important pole for gluon mass generation occurs in the **three-gluon vertex**



$$= \quad \mathbb{I}\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \dots$$

The **massless pole** can be interpreted as a **massless bound state** of gluons



$$I_\alpha(q) = q_\alpha I(q^2)$$

- **Logitudinality follows by Lorentz invariance of the transition amplitude**.
- **Longitudinality implies decoupling from spectrum.**

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C **78**, no.3, 181 (2018).

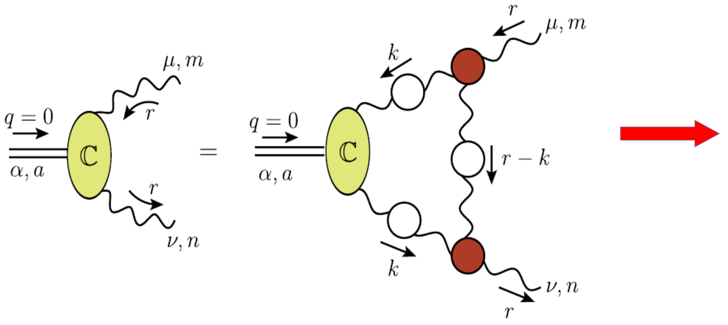
Bethe-Salpeter equation

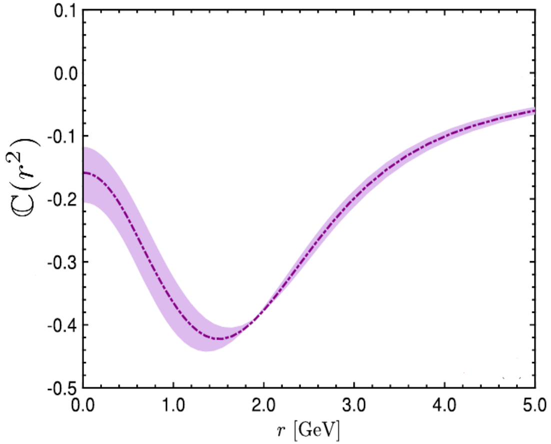
Then, we can derive a **Bethe-Salpeter equation (BSE) to describe the amplitude *$C_1(q,r,p)$* of the pole**.

- From the Bose symmetry of the vertex, $C_1(q,r,p)$ must be antisymmetric in q and r, which implies

$$C_1(0,r,-r) = 0 \;\rightarrow\; C_1(q,r,p) = 2(q\cdot r)\mathbb{C}(r^2) \;\rightarrow\; \mathbb{C}(r^2) := \left[\frac{\partial C_1(q,r,p)}{\partial p^2}\right]_{q=0}$$

- Then, at $q = 0$, the BSE is cast in terms of the amplitude $\mathbb{C}(r^2)$





- Gluon mass gap is determined by $\mathbb{C}(r^2)$

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

$$\Delta^{-1}(0) \sim \int d^4k\, k^2 \Delta^2(k^2)\left[1 - 6\pi\alpha_s C_A Y(k^2)\right]\mathbb{C}(k^2)$$

Bethe-Salpeter equation

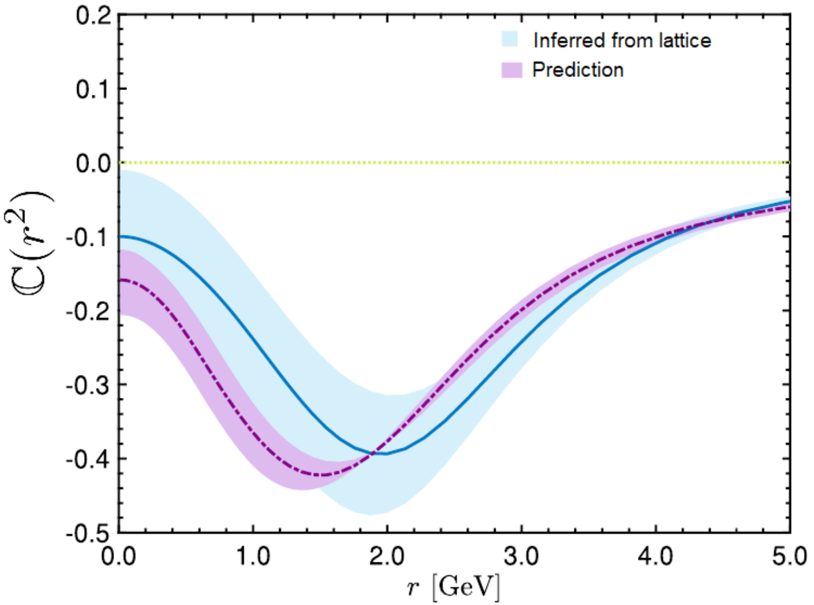
- In previous works it has been shown that the BSE has nontrivial solutions with eigenvalues compatible with the QCD coupling;

- The shape of the amplitude $\mathbb{C}(r^2)$ has been determined;

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C **78**, no.3, 181 (2018).
D. Binosi and J. Papavassiliou, Phys. Rev. D **97**, no.5, 054029 (2018).

- And even been shown to agree with results ***inferred*** from lattice simulations + gauge symmetry;

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).



But BSE is a **homogeneous** equation.

- It determines $\mathbb{C}(r^2)$ only up to a multiplicative constant.

What determines the scale of $\mathbb{C}(r^2)$, and hence of the gluon mass?1

Canonical normalization

The same problem occurs for the BSE amplitudes of any bound states.

Historically, **several normalization conditions** for BSE amplitudes of more conventional bound states have been proposed. In particular, based on

- Charge conservation;
- Energy-momentum tensor conservation;
- **Inhomogeneous BSE.**

All of these were **shown to be equivalent**, where applicable.

See N. Nakanishi, Prog. Theor. Phys. Suppl. **43**, 1-81 (1969) and references therein.

The derivation from the inhomogeneous BSE provides a general method, applicable also to our case.

The inhomogeneous equation for the massless pole

We consider the **three-gluon vertex** contracted with two transverse projectors, to eliminate the poles in the channels *r* and *p*.

Its most general Lorentz structure is comprised of **6 tensors**:

$$P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p)\mathbb{\Gamma}^{\alpha\mu'\nu'}(q,r,p) = \mathbb{L}_1(q,r,p)\ell_1^{\alpha\mu\nu}(q,r,p) + \mathbb{L}_2(q,r,p)\ell_2^{\alpha\mu\nu}(q,r,p) + \sum_{i=1}^{4} N_i(q,r,p)n_i^{\alpha\mu\nu}$$

Non-longitudinal

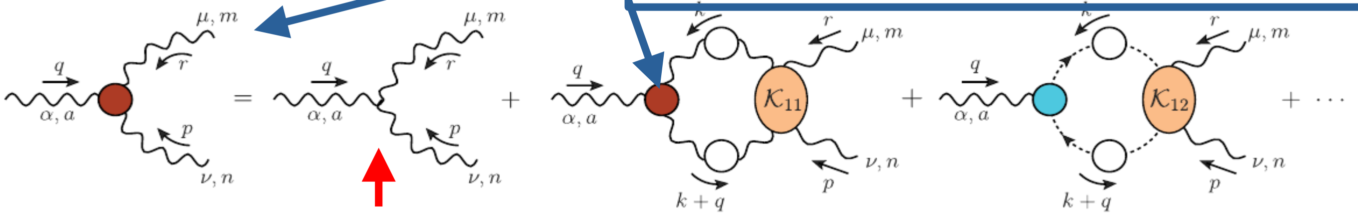
We focus on the form factor $\mathbb{L}_1(q,r,p)$, associated to the tensor structure $\ell_1^{\alpha\mu\nu} = q^{\alpha}P^{\mu}_{\rho}(r)P_{\nu\rho}(p)$

- This form factor has no tree level: $\mathbb{L}_1^{(0)}(q,r,p) = 0$
- It is anti-symmetric in *r* and *p*: $\mathbb{L}_1(q,r,p) = -\mathbb{L}_1(q,p,r)$
- Carries the massless pole: $\mathbb{L}_1(q,r,p) = \dfrac{C_1(r^2, q\cdot r)}{q^2} + \mathcal{R}(r^2, q\cdot r) + \mathcal{O}(q^2)$

The inhomogeneous equation for the massless pole

Now, we consider the Schwinger-Dyson equation for the three-gluon vertex, represented schematically as

$$P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p)\mathrm{I}\Gamma^{\alpha\mu'\nu'}(q,r,p) = \mathbb{L}_1(q,r,p)\ell_1^{\alpha\mu\nu}(q,r,p) + \mathbb{L}_2(q,r,p)\ell_2^{\alpha\mu\nu}(q,r,p) + \sum_{i=1}^{4} N_i(q,r,p)n_i^{\alpha\mu\nu}$$



$$\mathbb{L}_1^{(0)}(q,r,p) = 0$$

Projecting out the form factor $\mathbb{L}_1(q,r,p) = \mathcal{L}_{\alpha\mu\nu}P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p)\mathrm{I}\Gamma^{\alpha\mu'\nu'}(q,r,p)$

$$\mathbb{L}_1(q,r,p) = \mathcal{I}(q,r,p) + \int_k \mathcal{H}(q,r,p,k)\mathbb{L}_1(q,k,-q-k)$$

Inhomogeneous term is purely quantum

$$\mathcal{I}(q,r,p) = \mathcal{L}_{\alpha\mu\nu}\int_k \left[\sum_{i=1}^{4} n_i^{\alpha\rho\sigma}(q,k,-k-q)\right]\Delta(k)\Delta(k+q)\mathcal{K}^{\mu\nu}_{11\rho\sigma}(k,-k-q,r,p)$$

$$\mathcal{H}(q,r,p,k) = \mathcal{L}_{\alpha\mu\nu}\int_k \ell_1^{\alpha\rho\sigma}(q,k,-k-q)\Delta(k)\Delta(k+q)\mathcal{K}^{\mu\nu}_{11\rho\sigma}(k,-k-q,r,p)$$

The inhomogeneous equation for the massless pole

Due the presence of an inhomogeneous term, that **equation fixes the scale** of $\mathbb{C}(r^2)$.

At leading order in q = 0,

$$\mathbb{L}_1(q,r,p) = \mathcal{I}(q,r,p) + \int_k \mathcal{H}(q,r,p,k)\mathbb{L}_1(q,k,-q-k)$$

$$\mathbb{L}_1(q,r,p) = \frac{C_1(r^2, q\cdot r)}{q^2} + \mathcal{R}(r^2, q\cdot r) + \mathcal{O}(q^2)$$

we obtain the BSE, which fixes the shape of the pole amplitude, *i.e.*

$$\mathbb{C}(r^2) = \int_k (r\cdot k)\mathcal{H}(0,r,-r,k)\mathbb{C}(k^2)$$

Taking the expansion to the next order in q = 0, we obtain an equation that fixes its scale

$$\int_k r^2\Delta^2(r)\mathcal{I}'(r^2)\mathbb{C}(r^2) = -\int_r\int_k (r\cdot k)\Delta^2(r^2)\mathcal{H}'(0,r,-r,k)\mathbb{C}(r^2)\mathbb{C}(k^2)$$

Not scale invariant!

Linear in $\mathbb{C}(r^2)$

Quadratic in $\mathbb{C}(r^2)$

Fixing the scale in practice

Let $\mathbb{C}_\star(r^2)$ be the solution of the **homogeneous BSE**, with an arbitrary scale. An let the true $\mathbb{C}(r^2)$ be

$$\mathbb{C}(r^2) = a\mathbb{C}_\star(r^2)$$

$$\int_k r^2\Delta^2(r)\mathcal{I}'(r^2)\mathbb{C}(r^2) = -\int_r\int_k (r\cdot k)\Delta^2(r^2)\mathcal{H}'(0,r,-r,k)\mathbb{C}(r^2)\mathbb{C}(k^2)$$

Substituting into the canonical normalization equation, we obtain

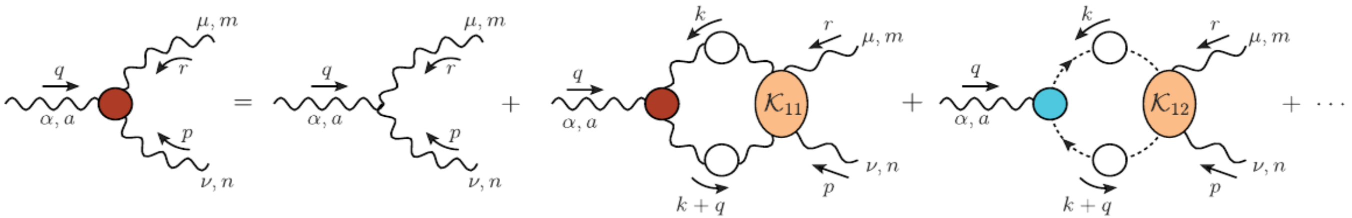
$$a = \frac{\int_k r^2\Delta^2(r)\mathcal{I}'(r^2)\mathbb{C}_\star(r^2)}{\int_r\int_k (r\cdot k)\Delta^2(r^2)\mathbb{C}_\star(r^2)\mathcal{H}'(0,r,-r,k)\mathbb{C}_\star(k^2)}$$

Which fully determines $\mathbb{C}(r^2)$.

Numerical analysis

Objectives: Probe impact of non-longitudinal form factors in the scale of $\mathbb{C}(r^2)$ and compare to lattice lattice inferred result.

Recalling our inhomogeneous equation,



$$\mathcal{I}(q,r,p) = \mathcal{L}_{\alpha\mu\nu} \int_k \left[\sum_{i=1}^{4} n_i^{\alpha\rho\sigma}(q,k,-k-q) \right] \Delta(k)\Delta(k+q)\mathcal{K}^{\mu\nu}_{11\rho\sigma}(k,-k-q,r,p)$$

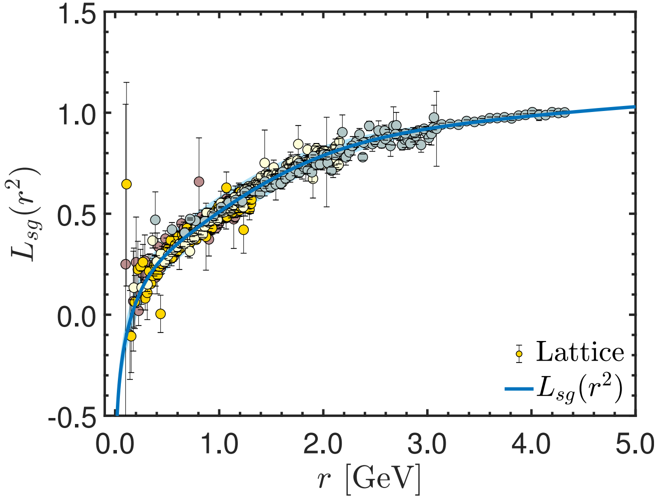
$$\mathcal{H}(q,r,p,k) = \mathcal{L}_{\alpha\mu\nu} \int_k \ell_1^{\alpha\rho\sigma}(q,k,-k-q)\Delta(k)\Delta(k+q)\mathcal{K}^{\mu\nu}_{11\rho\sigma}(k,-k-q,r,p)$$

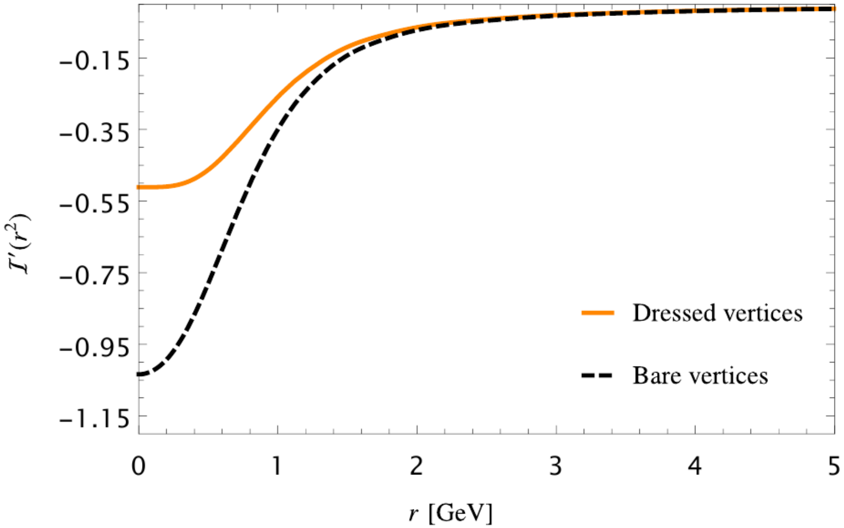
- Although both homogeneous and inhomogeneous terms depend on the same vertices, with different projections, we will keep $\mathcal{H}(q,r,p,k)$ fixed, and only vary the dressings in $\mathcal{I}(q,r,p)$.
- This is done to **keep the shape of the amplitude $\mathbb{C}(r^2)$ fixed, and explore the impact of the dressing on the scale setting in isolation.**

Numerical analysis

One of the **main features of the nonperturbative three-gluon vertex**, established by a variety of lattice simulations and continuum approaches is its **IR suppression**.

P. Boucaud, F . De Soto, J . Rodríguez- Quintero and S . Z afeiropoulos, Phys . Rev . D **95**, 11 , 1 1 4503 (2017).
A . Blum, M . Q . Huber , M . Mitter and L . von Smekal , Phys . Rev . D **89**, 06 1 703 (2014) .
G . Eichmann, R. Williams, R . Alkofer and M . Vujinovic , Phys . Rev . D **89**, 10 , 105014 (2014) .
A . K . Cyrol , L . Fister , M . Mitter , J . M . Pawlowski and N . Strodthoff, Phys . Rev . D **94**, 5, 054005 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).



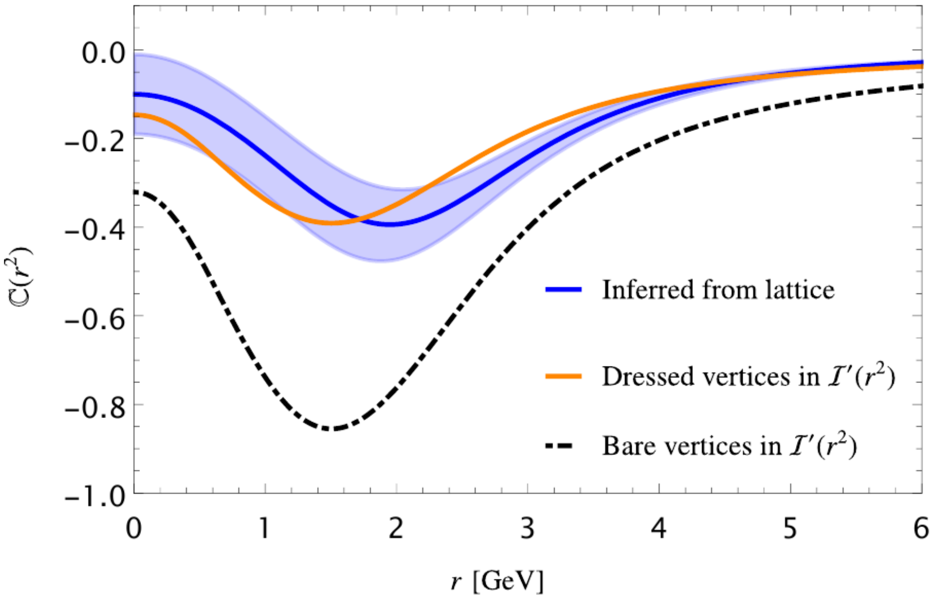


- Infrared suppression of the three-gluon vertex **leads to suppression of** $\mathcal{I}'(r^2)$, in comparison to its result with a bare vertex.

- In turn, this suppression **reduces the scale of** $\mathbb{C}(r^2)$

Numerical analysis

When using the $\mathcal{I}'(r^2)$ computed with **dressed vertices**, we obtain results **in excellent agreement with the $\mathbb{C}(r^2)$ inferred from the lattice.**



The result with bare vertex in the calculation of $\mathcal{I}'(r^2)$ on the other hand, is roughly a factor of 2 too big.

Conclusions

- **Gluon mass can be generated** if the vertices develop nonperturbatively **longitudinally coupled massless poles.**

- The **amplitude $\mathbb{C}(r^2)$ of the pole determines the gluon mass gap**.

- Its shape is determined by a homogeneous Bethe-Salpeter equation.

- Its **scale is determined from the canonical normalization condition**, **derived from the inhomogeneous equation of the vertex.**

- For the longitudinal poles of the Schwinger mechanism, **the inhomogeneous term is purely quantum.**

- Consequently, the **scale of $\mathbb{C}(r^2)$ is sensitive to the details of the dressings.**

- **The infrared suppression of the three-gluon vertex leads to a scale in excellent agreement with lattice inferred results.**