Light Front Wave Functions of Vector Mesons From CSMs

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Outline

- From Bethe-Salpeter WFs to Light-Front WFs: Formalism
- LF-LFWFs of light and heavy pseudoscalar mesons.
- LF-LFWFs of light and heavy vector mesons.
- Diffractive vector meson electroproduction.
- Vector meson TMDs.

BSE approach

- An alternative way to calculate the LFWFs.

"...'t Hooft did not use the light–cone formalism and which nowadays might be called standard. Instead, he started from covariant equations... The light–cone Schrodinger equation was then obtained by projecting the Bethe–Salpeter equation onto hyper-surfaces of equal light–cone time."

(T. Heinzl arXiv:hep-th/0008096)

$$\int \frac{dk^-}{2\pi} \Psi_{BS}(k;p) = \frac{u^{(1)}(x_1, k_\perp)}{\sqrt{x_1}} \frac{u^{(2)}(x_2, -k_\perp)}{\sqrt{x_2}} \psi(x_i, k_\perp)$$

(G. Lepage and S. Brodsky, PRD 1980)

$$\psi(x, \mathbf{p}; s_1, s_2) = \frac{1}{2P^+} \int \frac{dp^-}{2\pi} \bar{u}(xP^+, \mathbf{p}; s_1)\gamma^+\Phi(p)\gamma^+ v((1-x)P^+, -\mathbf{p}; s_2)\,.$$

(H. Liu and D. Soper, PRD1993)

(W. de Paula, E. Ydrefors, J.H. Alvarenga Nogueira, T. Frederico, and G. Salme, PRD2021)

$$\langle 0|\bar{d}_+(0)\gamma^+\gamma_5 u_+(\xi^-, \xi_\perp)|\pi^+(P)\rangle = i\sqrt{6}P^+\psi_0(\xi^-, \xi_\perp),$$

$$\langle 0|\bar{d}_+(0)\sigma^{+i}\gamma_5 u_+(\xi^-, \xi_\perp)|\pi^+(P)\rangle = -i\sqrt{6}P^+\partial^i\psi_1(\xi^-, \xi_\perp).$$

(M. Burkardt, X. Ji, F. Yuan, PLB 2002)

$$2P^+\Psi_{\uparrow\downarrow}(k^+, \mathbf{k}_\perp) = \int \frac{dk^-}{2\pi} \mathrm{Tr}\left[\gamma^+\gamma_5\chi(k, P)\right],$$

$$ik^i 2P^+\Psi_{\uparrow\uparrow}(k^+, \mathbf{k}_\perp) = \int \frac{dk^-}{2\pi} \mathrm{Tr}\left[\sigma^{+i}\gamma_5\chi(k, P)\right].$$

(C. Mezrag, H. Moutarde and J. Rodríguez-Quintero, Few-Body Syst 2016)

All equivalent.

BSE approach

- From BS WF to LF WF

Covariant Bethe-Salpeter wave function in the instant form

$$\phi_i(x,\vec{k}_T) \sim \int dk^- dk^+ \delta(xP^+ - k^+) \mathrm{Tr}[\Gamma_i \chi(k,P)]$$

LFWF

spin configurations

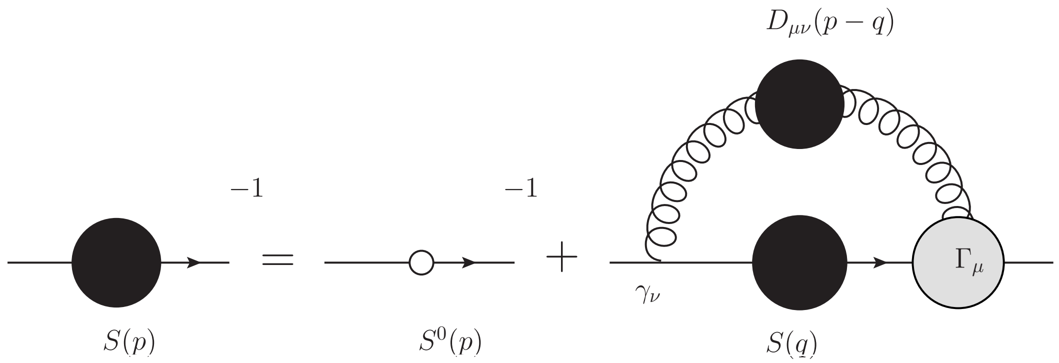
set $k^+ = xP^+$

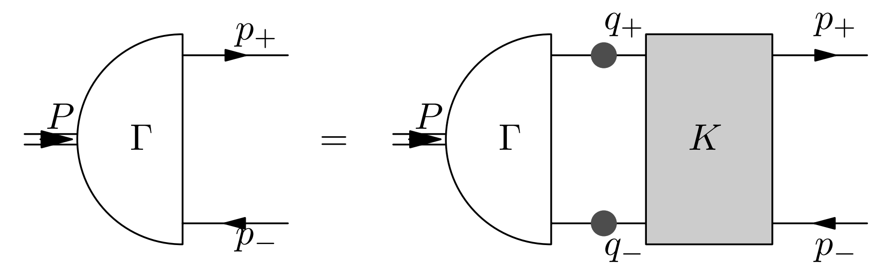
Project onto the light front null plane $\xi^+ = 0$

(C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, Few Body Syst. 2016)

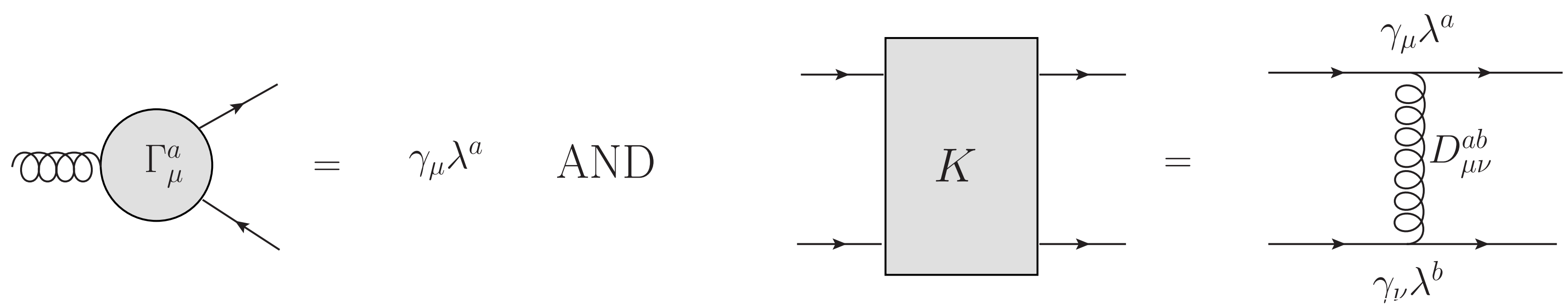
Bethe-Salpeter WFs

- The BS wave function can be solved by aligning the quark DSE and meson BSE.

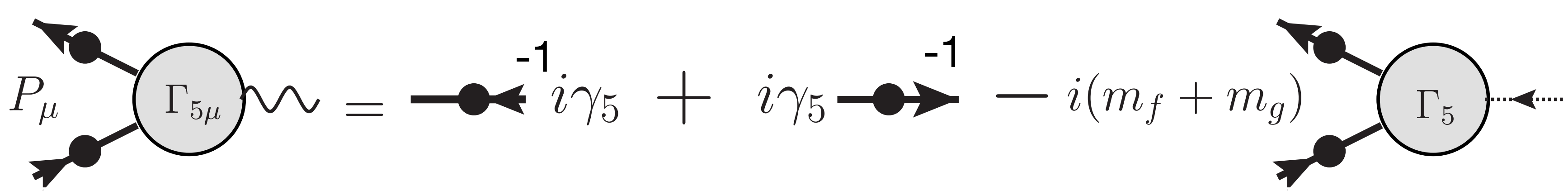




- The simplest (and widely used) truncation is the Rainbow-Ladder (RL) truncation



- The RL truncation preserves QCD's explicit chiral symmetry by respecting the axial vector WTI, and thereby place a firm ground for its dynamical breaking.



Pion and kaon: (P. Maris and C. D. Roberts, PRC1997)

ρ and ϕ: (P. Maris and P. C. Tandy, PRC1999)

Heavy mesons (M. Black, A Krassnigg, et al)

Pseudo-scalar LF-LFWFs

BS WFs

$$\chi_M(k;P) = \gamma_5\Big[iE(k;P) + \not{P}F(k;P) + (k\cdot P)\not{k}\,G(k;P) + i[\not{k},\not{P}]\,H(k;P)\Big].$$

$$\mathcal{F}(k;P) \equiv \mathcal{F}(k^2, P^2, k\cdot P)$$

$$\mathcal{F} = E, F, G, H$$

LF-LFWFs

$$|M\rangle = \sum_{\lambda_1,\lambda_2}\int \frac{d^2\boldsymbol{k}_T}{(2\pi)^3}\frac{dx}{2\sqrt{x\bar{x}}}\frac{\delta_{ij}}{\sqrt{3}}\,\Phi_{\lambda_1,\lambda_2}(x,\boldsymbol{k}_T)\,b^\dagger_{f,\lambda_1,i}(x,\boldsymbol{k}_T)\,d^\dagger_{h,\lambda_2,j}(\bar{x},\bar{\boldsymbol{k}}_T)|0\rangle.$$

$$\Phi_{\uparrow,\downarrow}(x,\boldsymbol{k}_T) = \psi_0(x,\boldsymbol{k}_T^2),\qquad \Phi_{\downarrow,\uparrow}(x,\boldsymbol{k}_T) = -\psi_0(x,\boldsymbol{k}_T^2),$$

$$\Phi_{\uparrow,\uparrow}(x,\boldsymbol{k}_T) = k_T^-\psi_1(x,\boldsymbol{k}_T^2),\qquad \Phi_{\downarrow,\downarrow}(x,\boldsymbol{k}_T) = k_T^+\psi_1(x,\boldsymbol{k}_T^2),$$

(X. Ji, J.-P. Ma, F. Yuan, EPJC2004)

4 Lorentz scalar functions

2 independent scalar functions

$$\phi_i(x,\vec{k}_T) \sim \int dk^- dk^+ \delta(xP^+ - k^+)\mathrm{Tr}[\Gamma_i\chi(k,P)]$$

$$\Gamma_i \sim \gamma^+\gamma_5, \sigma^{+i}\gamma_5$$

Pion (at different masses)

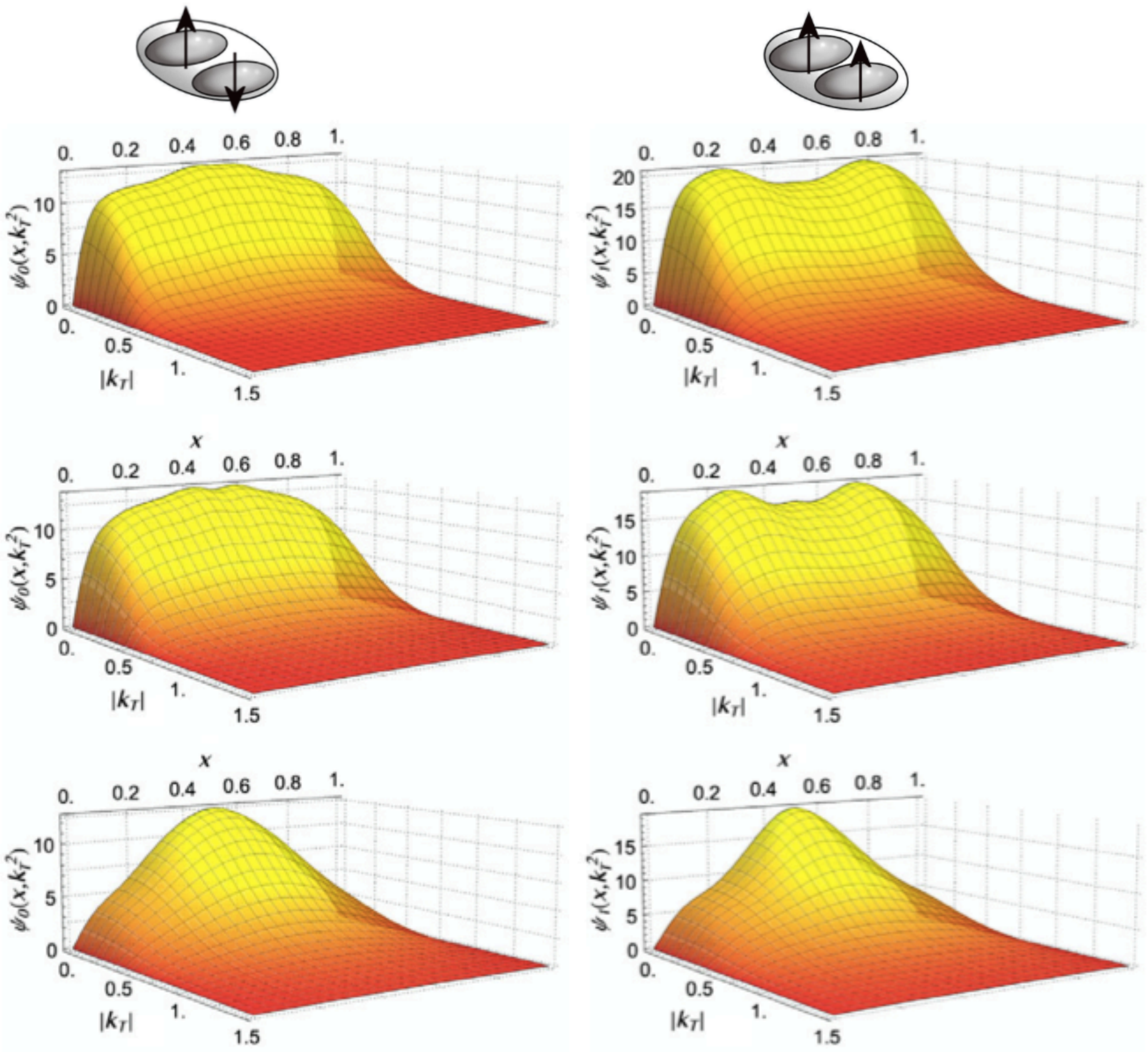


FIG. 1. The $\psi_0(x, \boldsymbol{k}_T^2)$ and $\psi_1(x, \boldsymbol{k}_T^2)$ of pion at $m_\pi = 130$ (top row), 310 (middle row), and 690 MeV (bottom row).

(C. S., M. Li, X. Chen, W. Jia, PRD2021)

Lattice&LaMET formalism: X. Ji and Y. Liu, PRD2022

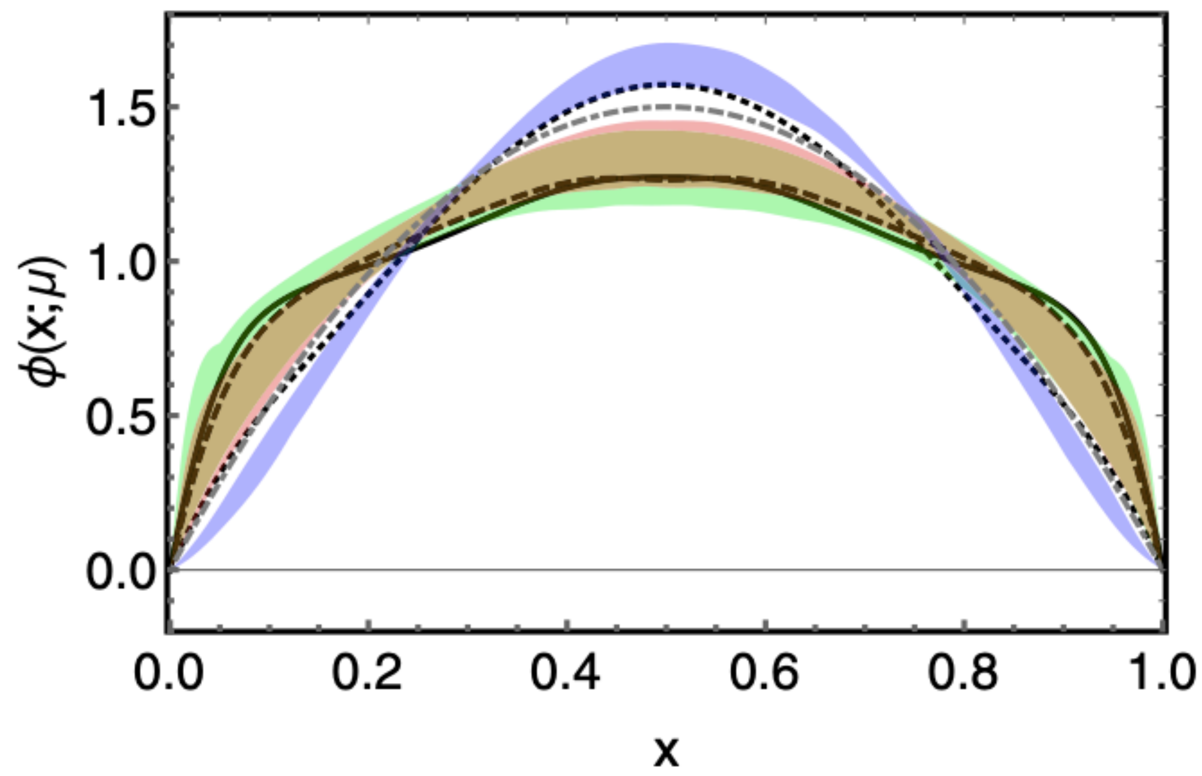


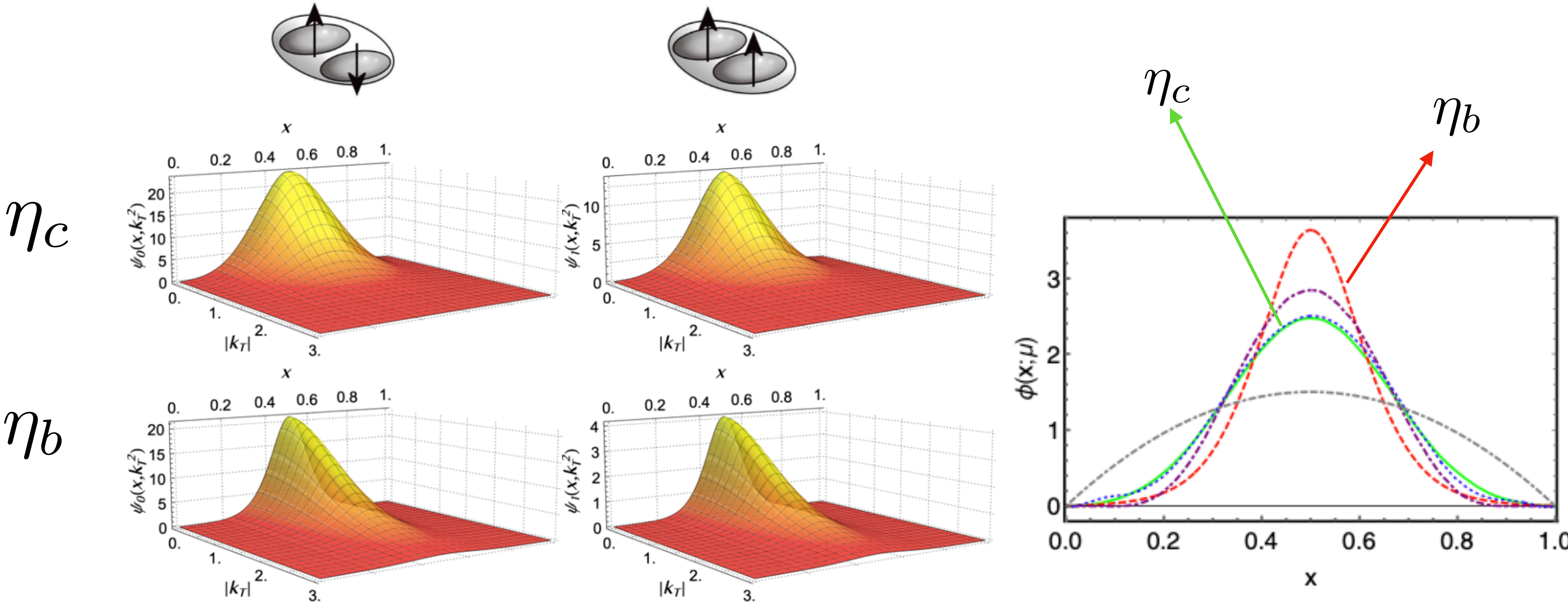
FIG. 3. The PDA of the pion at masses of $m_\pi = 130$ (solid), 310 (dashed), and 690 MeV (dotted). The colored bands are results from lattice QCD [22] at the same masses of $m_\pi = 130$ (red), 310 (green), and 690 MeV (blue).

(M. Ding, et al, PLB2016)

(R. Zhang, et al, PRD2020)

- Prdiction: LF-LFWFs evolve slowly from pion mass of 130 MeV to 310 MeV, but significantly different at 690 MeV̄ (ss pion)

eta_c and eta_b LF-LFWFs



(C. S., M. Li, X. Chen, W. Jia, PRD2021)

- Narrowly distributed in x in heavy pseudoscalar mesons.
- Evolution of LF-LFWF with current quark mass (EHM & Higgs mechanism).

Normalization

Normalization of LFWFs.

In general, all LFWFs (including higher Fock states) should normalize to 1.

Condition 1: $$\sum_{n,\lambda_i} \int \overline{\prod_i} dx_i \frac{d^2 k_{\perp i}}{16\pi^3} |\psi_{n/\pi}(x_i, \mathbf{k}_{\perp i}, \lambda_i)|^2 = 1 \,.$$ ✓

In practice, two normalization conditions were usually employed.

Condition 1': $$1 = ||\psi_2||^2 = ||\psi_{2\uparrow\downarrow}||^2 + ||\psi_{2\uparrow\uparrow}||^2 + ||\psi_{2\downarrow\uparrow}||^2 + ||\psi_{2\downarrow\downarrow}||^2$$?

Condition 2: $$\int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \psi_{2\uparrow\downarrow}(x, \mathbf{k}_\perp) = \frac{f_\pi}{2\sqrt{3}}$$ ✓

"While Condition 1' enforces a constituent picture, Condition 2 is exact and holds beyond a constituent picture."

(T. Heinzl arXiv:hep-th/0008096)

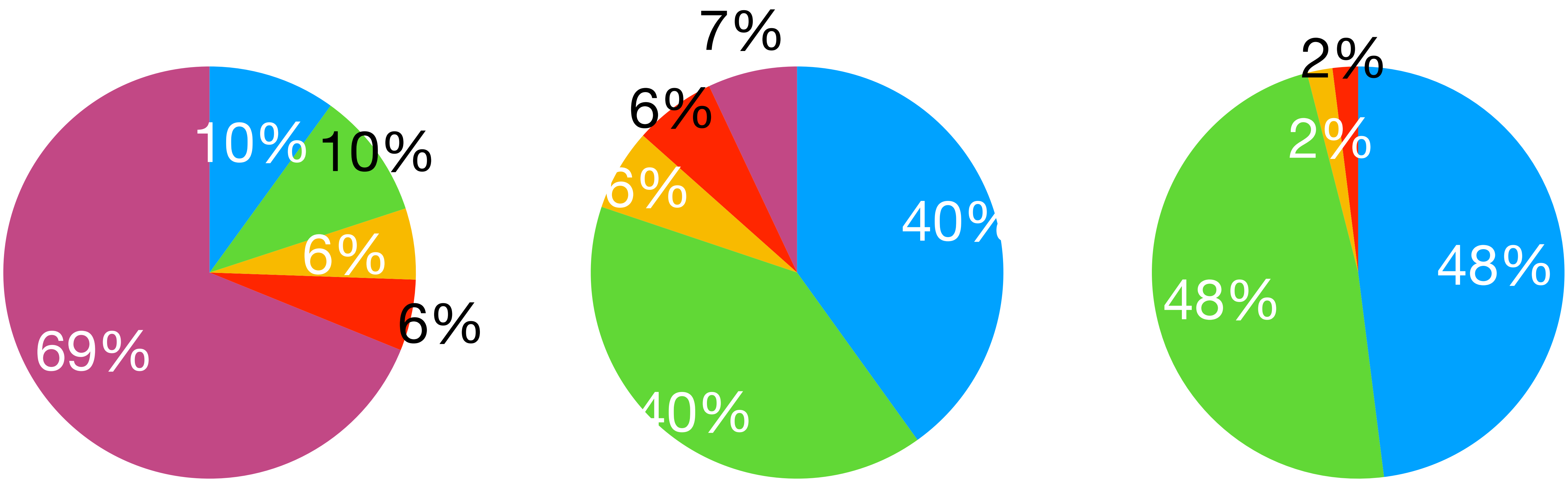
Canonical normalization of BS wave functions

$$G_4(p,q;P) = \Sigma_i \frac{\chi_i(p,P_i)\bar{\chi}_i(q,-P_i)}{P^2 - P_i^2} + R(p,q;P)$$ ⟷ **Condition 1**

$$f_\pi P_\mu = \int_q^\Lambda \mathrm{Tr}[\gamma_5 \gamma_\mu \chi(q;P)]$$ ⟷ **Condition 2**

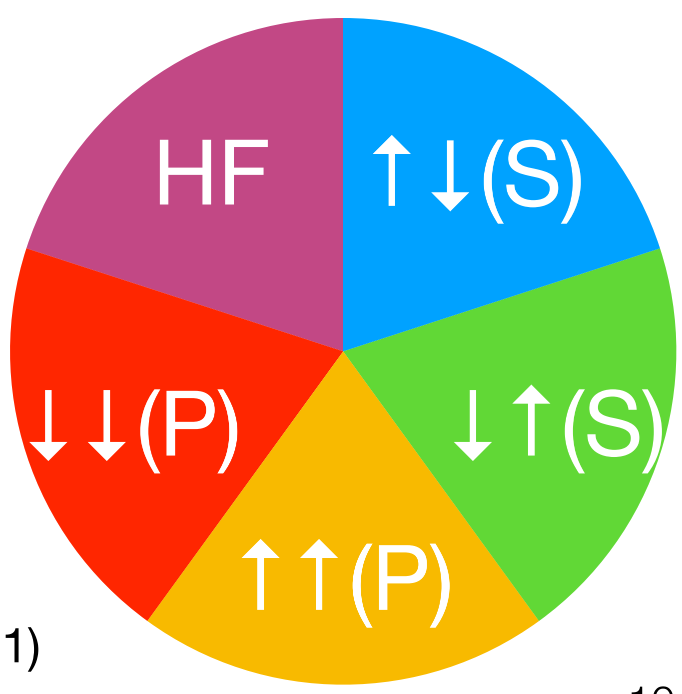
Leading Fock state contribution to total normalization

pion η_c η_b



(C. S., M. Li, X. Chen, W. Jia, PRD2021)

HF=Higher Fock states



- Indication of higher Fock-states contribution in pion.
- Higher Fock-states are significantly suppressed in heavy mesons.

(W. de Paula, E. Ydrefors, J. H. Alvarenga Nogueira, T. Frederico, and G. Salme, PRD2021)

Vector meson LF-LFWFs

BS WFs

$$\chi_\mu^M(k,P) = \sum_{i=1}^{8} T_\mu^i(k,P) F^i(k^2, k\cdot P, P^2)$$

$$A_1 = k_\mu - \frac{P_\mu P\cdot k}{P^2},\ A_2 = \gamma_\mu - \frac{P_\mu \not{P}}{P^2},$$
$$B_1 = I_4,\ B_2 = \not{P},\ B_3 = \not{k},\ B_4 = [\not{k}, \not{P}],$$

$$T_\mu^1 = iA_1.B_1, \quad T_\mu^2 = A_1.B_2(k\cdot P),$$
$$T_\mu^3 = A_1.B_3, \quad T_\mu^4 = -iA_1.B_4,$$
$$T_\mu^5 = A_2.B_1, \quad T_\mu^6 = -iA_2.B_2,$$
$$T_\mu^7 = -i[A_2, B_3](k\cdot P),\ T_\mu^8 = \{A_2, B_4\}\,.$$

LF-LFWFs

$$|M\rangle^\Lambda = \sum_{\lambda,\lambda'} \int \frac{d^2\boldsymbol{k}_T}{(2\pi)^3} \frac{dx}{2\sqrt{x\bar{x}}} \frac{\delta_{ij}}{\sqrt{3}}$$
$$\Phi^\Lambda_{\lambda,\lambda'}(x,\boldsymbol{k}_T)\, b^\dagger_{f,\lambda,i}(x,\boldsymbol{k}_T)\, d^\dagger_{f,\lambda',j}(\bar{x},\bar{\boldsymbol{k}}_T)|0\rangle.$$

$$\Phi^0_{\pm,\mp} = \psi^0_{(1)}, \quad \Phi^0_{\pm,\pm} = \pm k_T^{(\mp)} \psi^0_{(2)};$$
$$\Phi^{\pm1}_{\pm,\pm} = \psi^1_{(1)}, \quad \Phi^{\pm1}_{\pm,\mp} = \pm k_T^{(\pm)} \psi^1_{(2)};$$
$$\Phi^{\pm1}_{\mp,\pm} = \pm k_T^{(\pm)} \psi^1_{(3)},\ \Phi^{\pm1}_{\mp,\mp} = (k_T^{(\pm)})^2 \psi^1_{(4)}$$

8 Lorentz scalar functions

6 independent scalar functions, reduce to 5 for Charge parity eigenstate

$$\Phi^\Lambda_{\lambda,\lambda'}(x,\vec{k}_T) = -\frac{1}{2\sqrt{3}}\int \frac{dk^- dk^+}{2\pi}\delta(xP^+ - k^+)\mathrm{Tr}\left[\Gamma_{\lambda,\lambda'}\gamma^+\chi^M(k,P)\cdot\epsilon_\Lambda(P)\right].$$
$$\Gamma = I \pm \gamma_5, \mp(\gamma^1 \pm i\gamma^2)$$

Vector meson LF-LFWFs

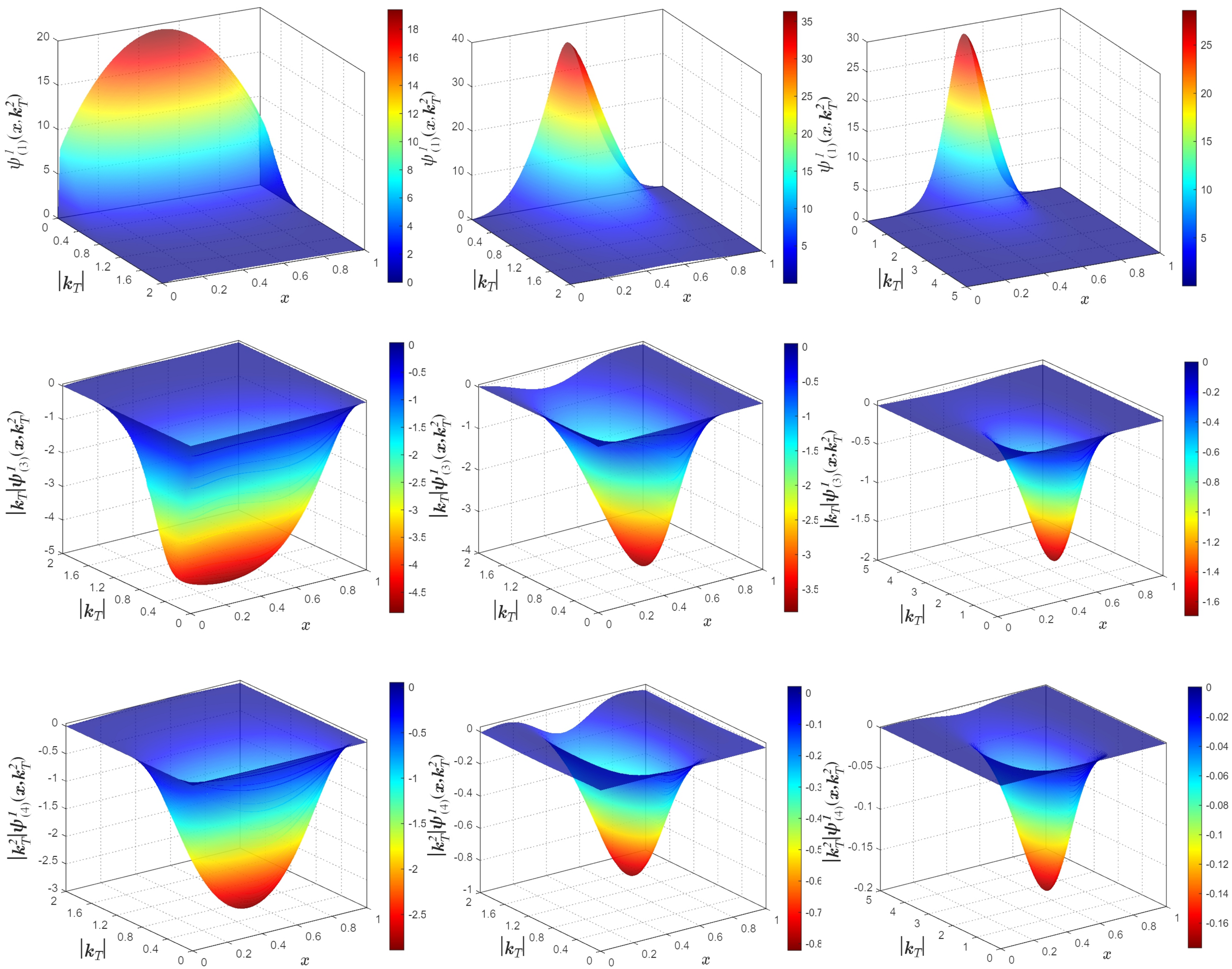
$$\Lambda = \lambda + \lambda' + L_z$$

|Λ|=1 | **Rho** | **J/psi** | **Upsilon**

|Lz|=0

|Lz|=1

|Lz|=2

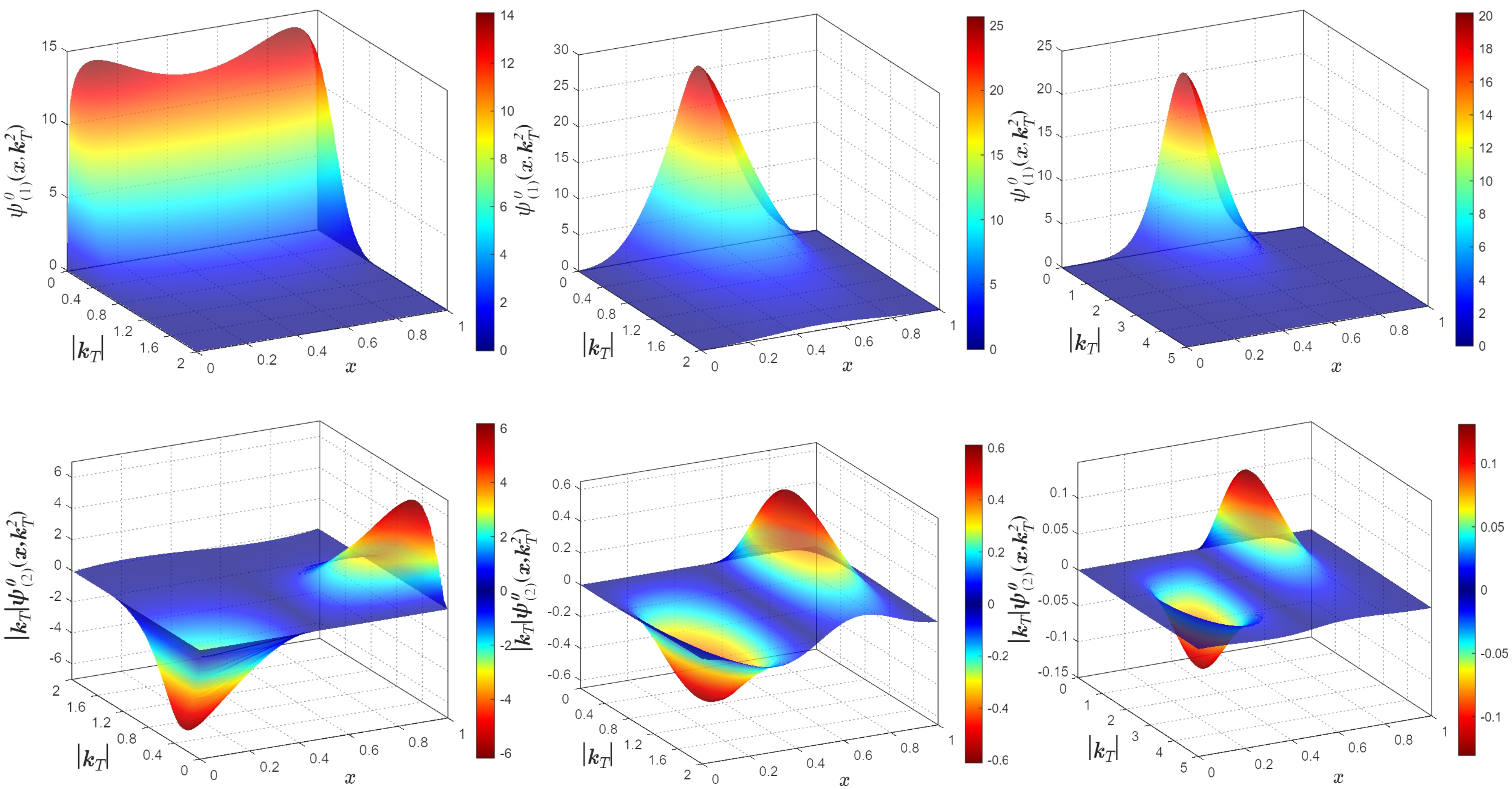


Vector meson LF-LFWFs

Λ=0 **Rho** **J/psi** **Upsilon**

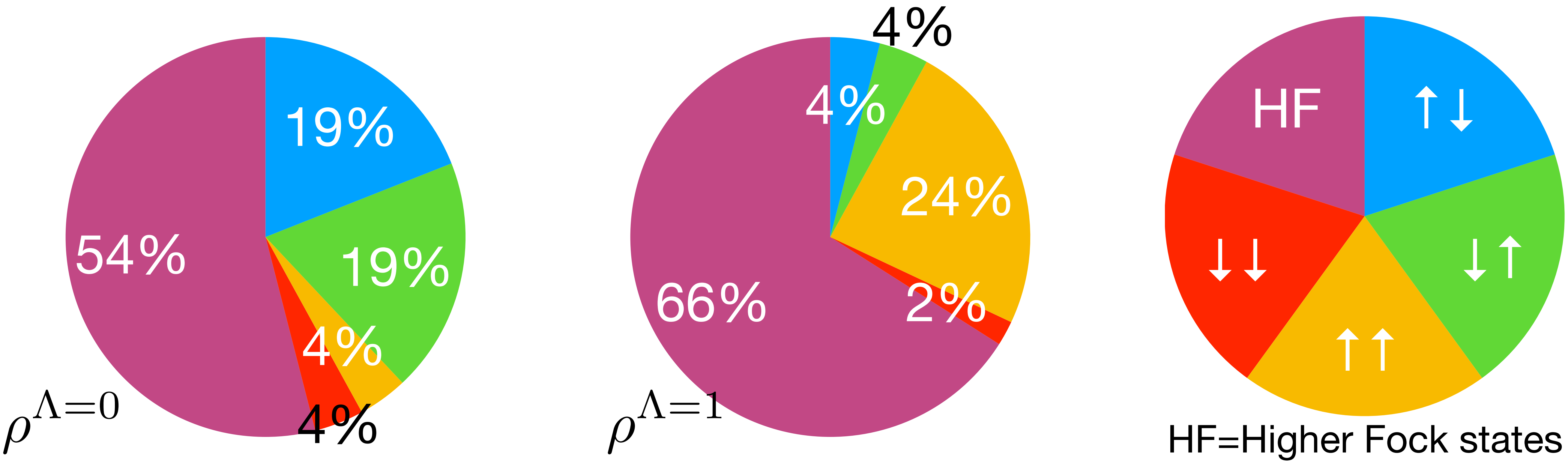
|Lz|=0

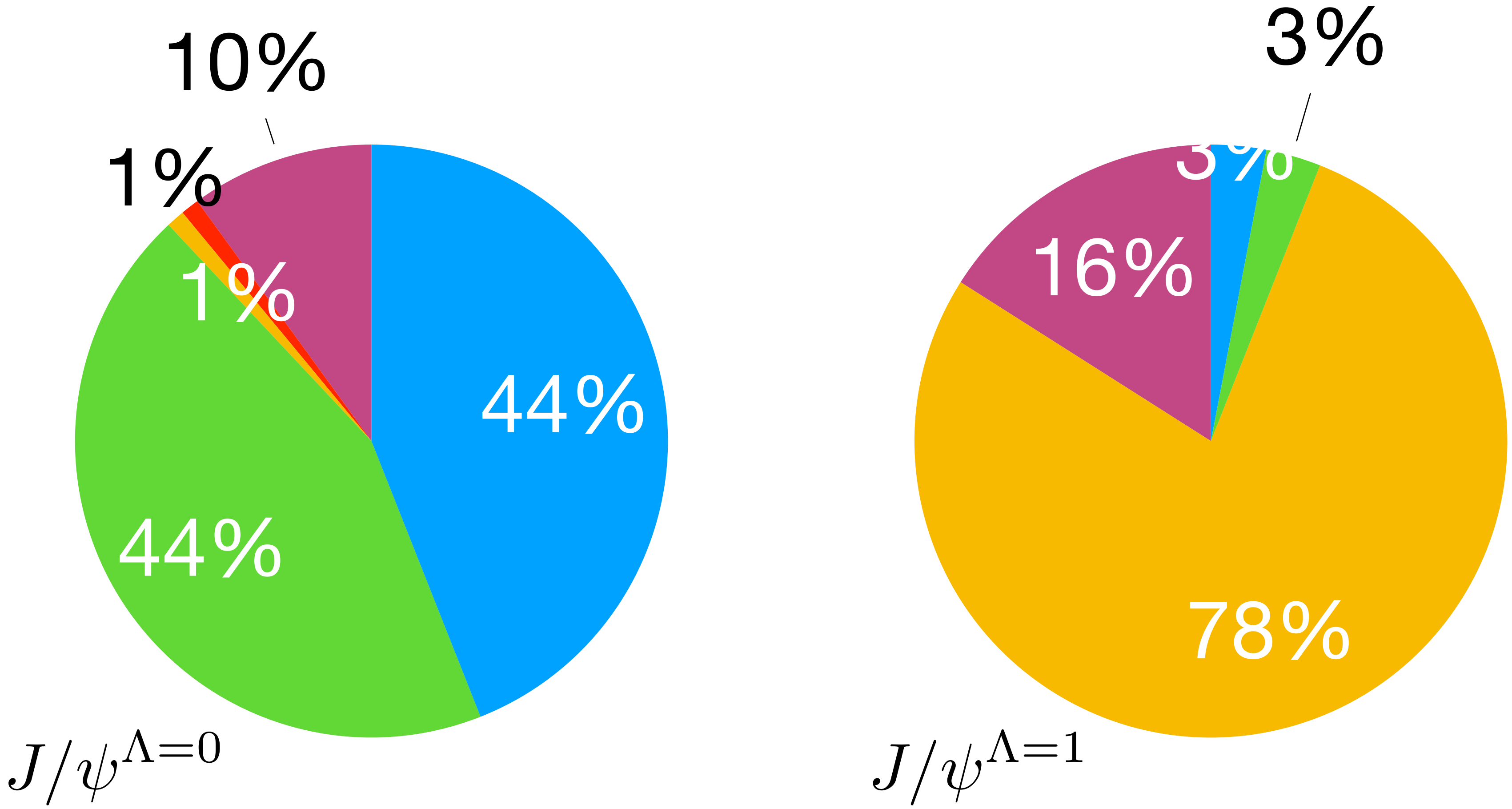
|Lz|=1



- All LF-LFWFs are nonvanishing.
- Evolution of LF-LFWF with current quark mass (EHM & Higgs).

Leading Fock state contribution to total normalization

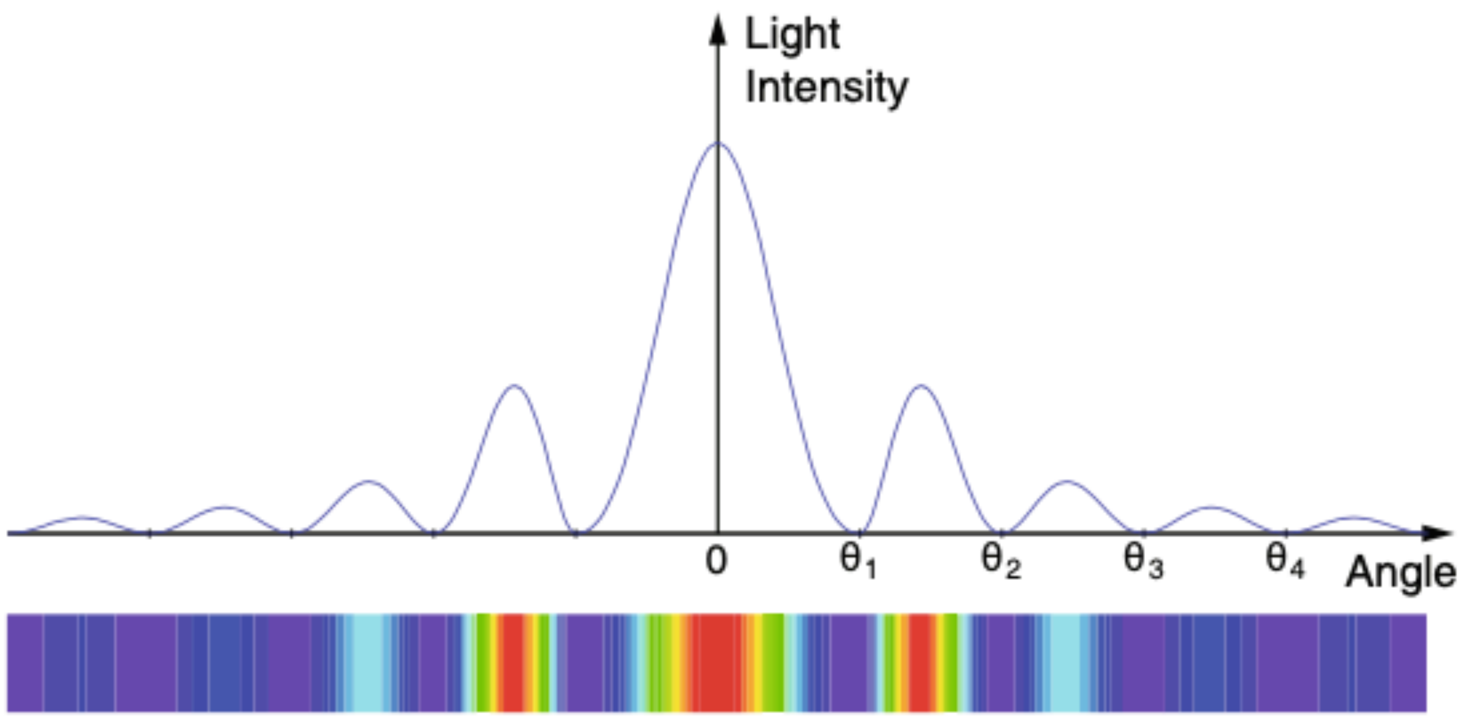


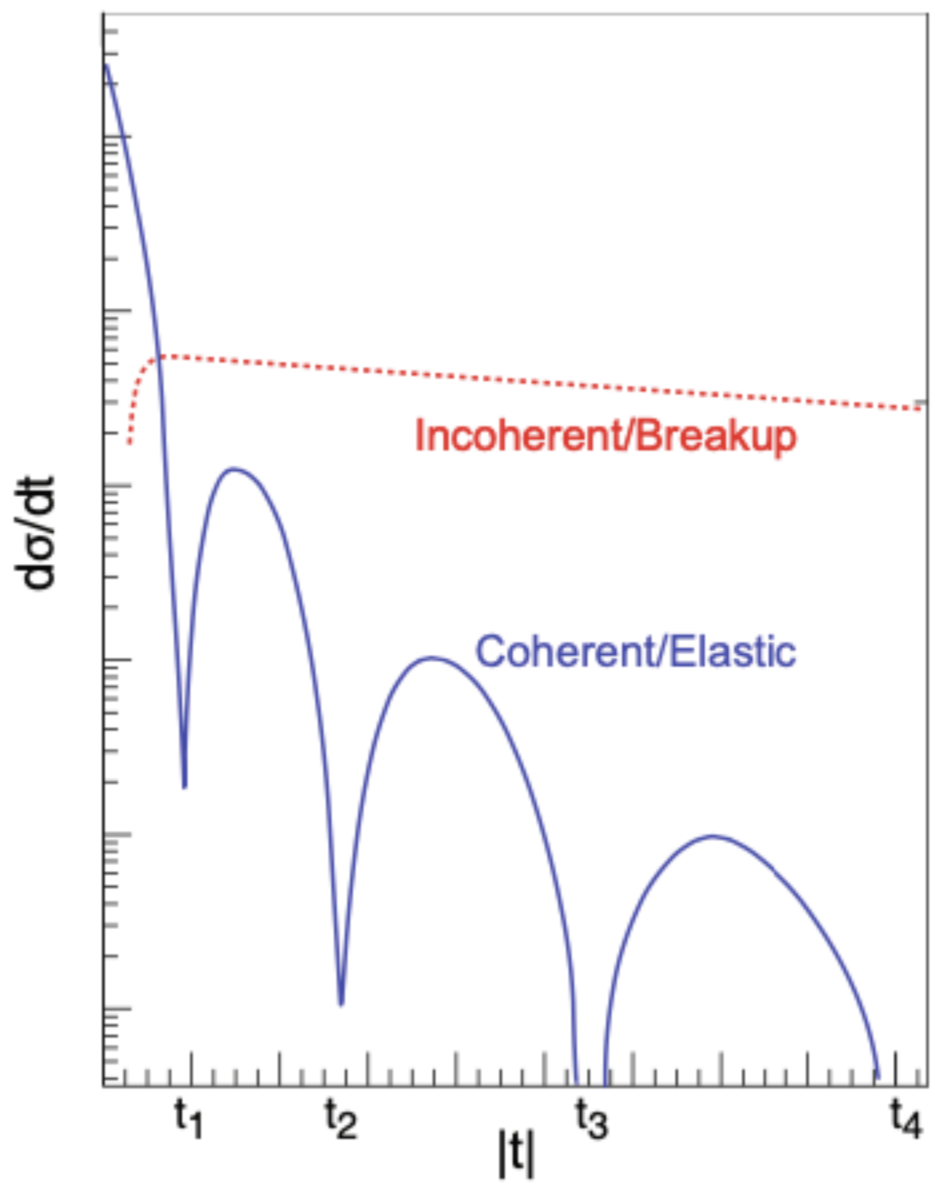


- s-wave component generally dominates.
- Heavy mesons dominated by LF-LFWFs.

Application I: Diffractive vector meson production

Diffraction





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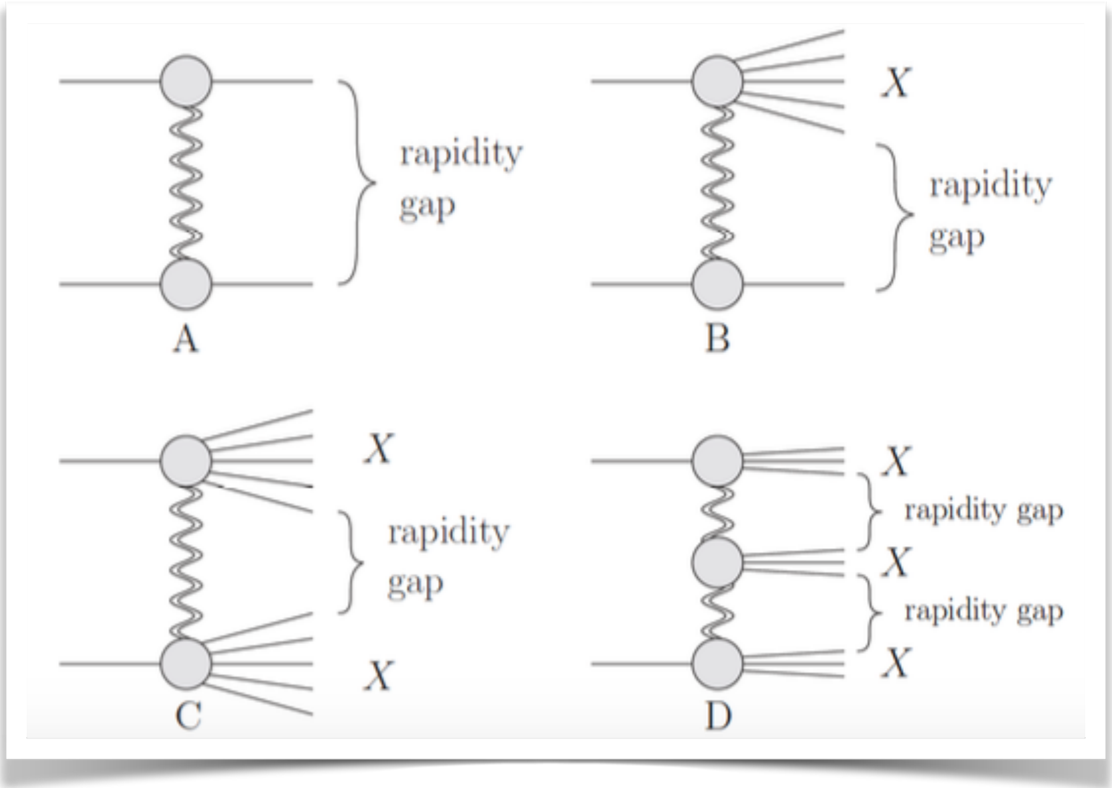
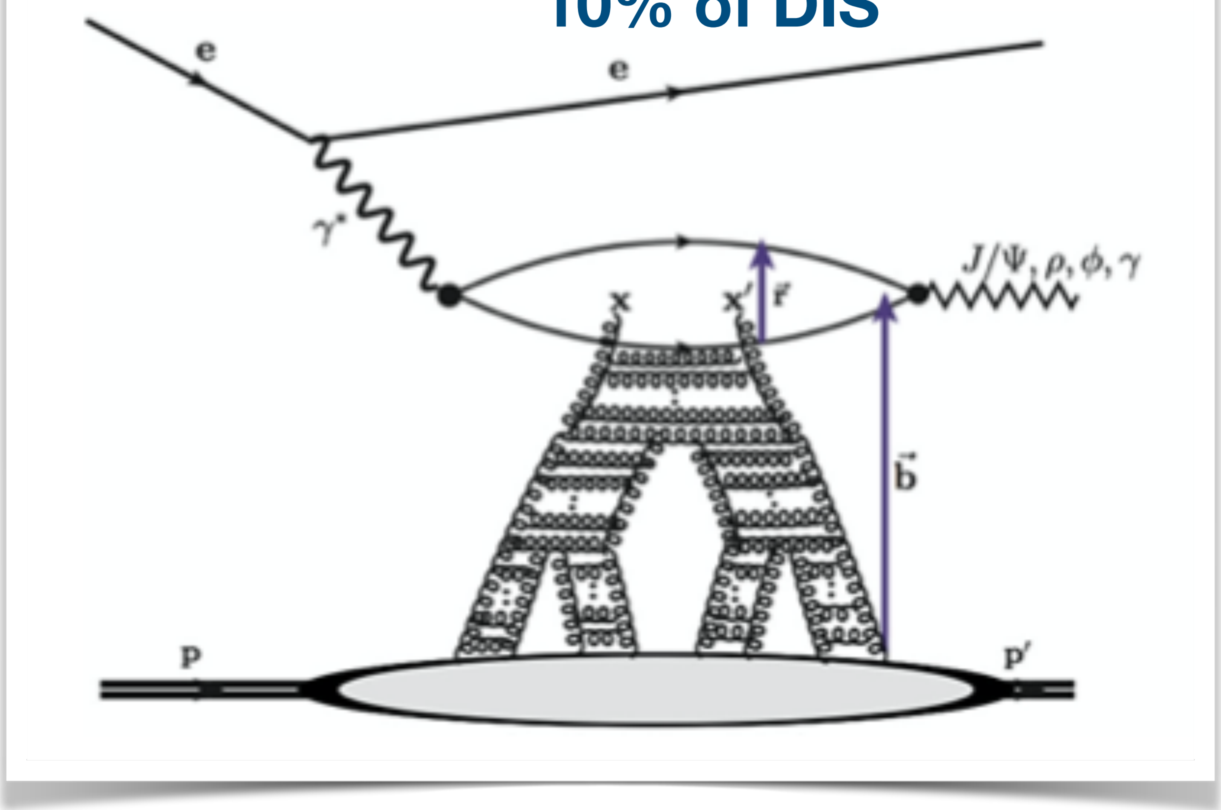
Diffractive vector mesons production



10% of DIS



Colorless exchange

- Color Dipole Picture of diffractive VM production

$$\sigma \sim \phi^{q\bar{q}}_{\gamma^*} \otimes \sigma_{q\bar{q},N} \otimes \phi^{q\bar{q}}_{V}$$

$$\sigma_{q\bar{q},N} \sim g^2(x)$$

Sensitive to small x gluons!

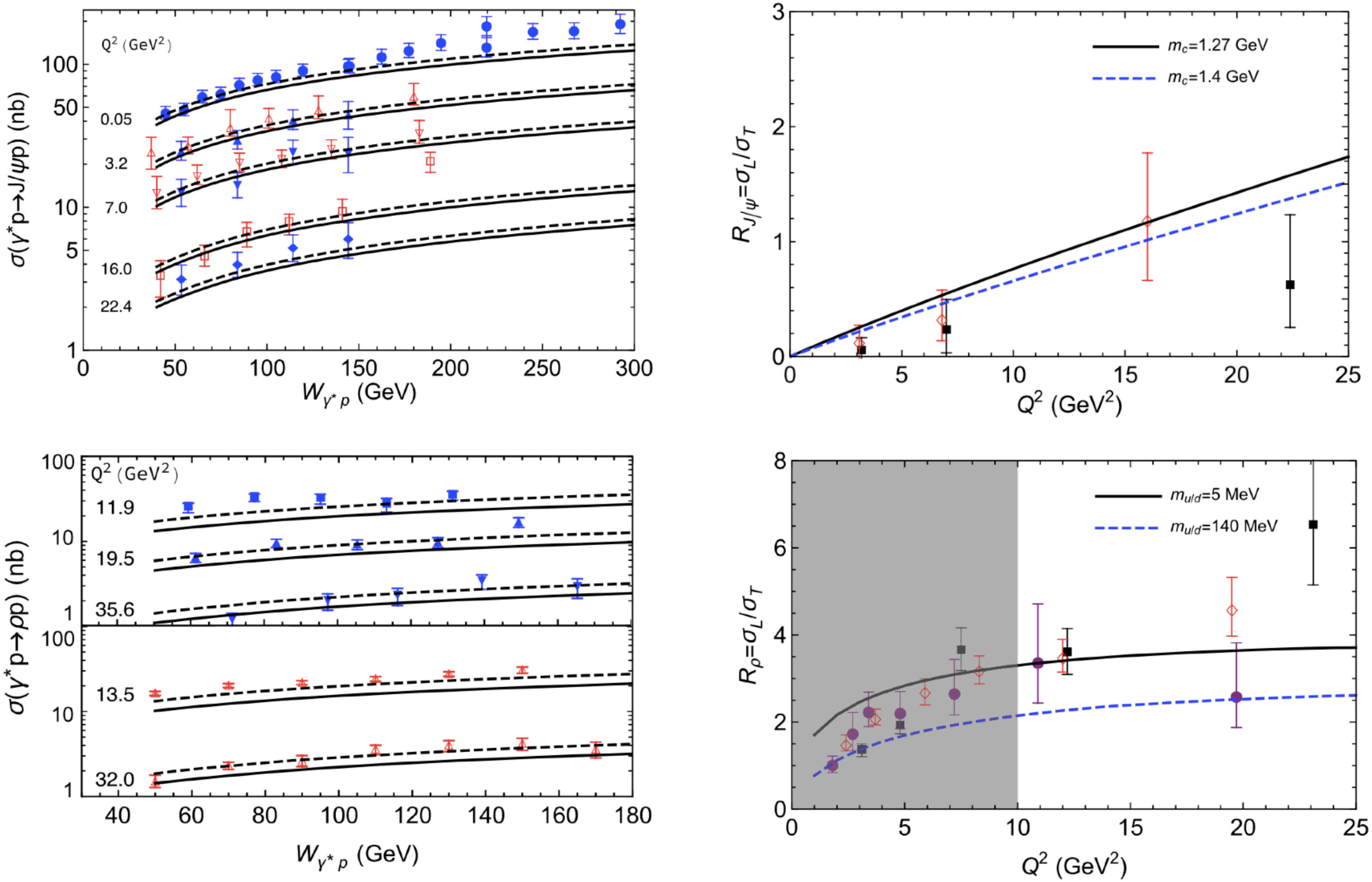
$\phi^{q\bar{q}}_{\gamma*}$ light-cone perturbation

$\sigma_{q\bar{q},N}$ b-dependent CGC model (saturation).

(A. H. Rezaeian and I. Schmidt, PRD2013)

$\phi^{q\bar{q}}_{V}$ DS-BSEs LF-LFWFs

Calculation & HERA data



(C.S., Ya-Ping Xie, Ming Li, Xurong Chen and Hong-Shi Zong, PRD(L)2021)

- No new parameters introduced.
- Rho and J/psi production agrees well with HERA data in a reasonable kinematical range.

Application II: vector meson TMDs

$$\Theta^{(\Lambda)\vec{S}}_{\beta\alpha}(x,\vec{k}_T) = \int \frac{dz^- \, d^2\vec{z}_T}{(2\pi)^3} \, e^{i(xP^+ z^- - \vec{k}_T\cdot\vec{z}_T)} \, {}_{\vec{S}}\langle P,\Lambda|\overline{\psi}_\alpha(0)\psi_\beta(z^-,\vec{z}_T)|P,\Lambda\rangle_{\vec{S}}.$$

$$\frac{1}{2}\mathrm{Tr}_D\left[\gamma^+\,\Theta^{(\Lambda)s}(x,\boldsymbol{k}_T)\right] = f_1(x,\boldsymbol{k}_T^2) + S_{LL} f_{1LL}(x,\boldsymbol{k}_T^2) + \frac{\boldsymbol{S}_{LT}\cdot\boldsymbol{k}_T}{m_V} f_{1LT}(x,\boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_{TT}\cdot\boldsymbol{k}_T}{m_V^2} f_{1TT}(x,\boldsymbol{k}_T^2),$$

$$\frac{1}{2}\mathrm{Tr}_D\left[\gamma^+\gamma_5\,\Theta^{(\Lambda)s}(x,\boldsymbol{k}_T)\right] = \Lambda\left[S_L\, g_{1L}(x,\boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{m_V} g_{1T}(x,\boldsymbol{k}_T^2)\right],$$

$$\frac{1}{2}\mathrm{Tr}_D\left[-i\sigma^{+i}\gamma_5\,\Theta^{(\Lambda)s}(x,\boldsymbol{k}_T)\right] = \Lambda\left[S_T^i\, h_1(x,\boldsymbol{k}_T^2) + S_L\,\frac{k_T^i}{m_V}\, h_{1L}^{\perp}(x,\boldsymbol{k}_T^2) + \frac{1}{2\,m_V^2}\left(2\,k_T^i\,\boldsymbol{k}_T\cdot\boldsymbol{S}_T - S_T^i\,\boldsymbol{k}_T^2\right) h_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\right].$$

TMD overlap representation (Helicity basis)

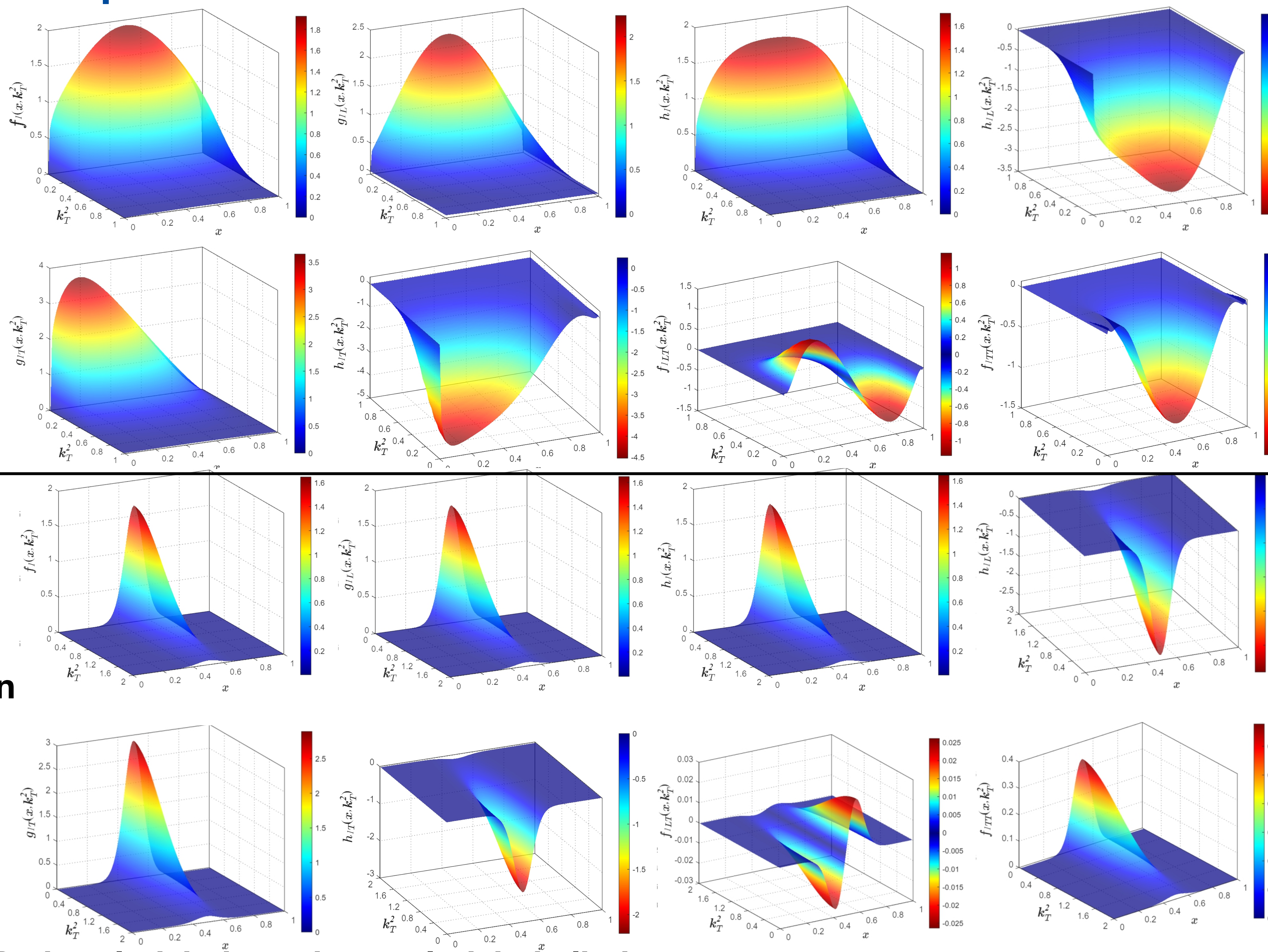
$$M^{(\Lambda)\vec{S}}(x,\vec{k}_T) = [\Theta^{(\Lambda)\vec{S}}_{\beta\alpha}(x,\vec{k}_T)\gamma^+]^T$$

(S. Kaur, et al, JHEP2021)

Rho & Upsilon TMDs

Rho

Upsilon



- **TMDs in relativistic and nonrelativistic limits.**

OAM & Rho TMDs

(C.S. J. Li, M. Li, X. Chen and W. Jia, arXiv:2205.02757)

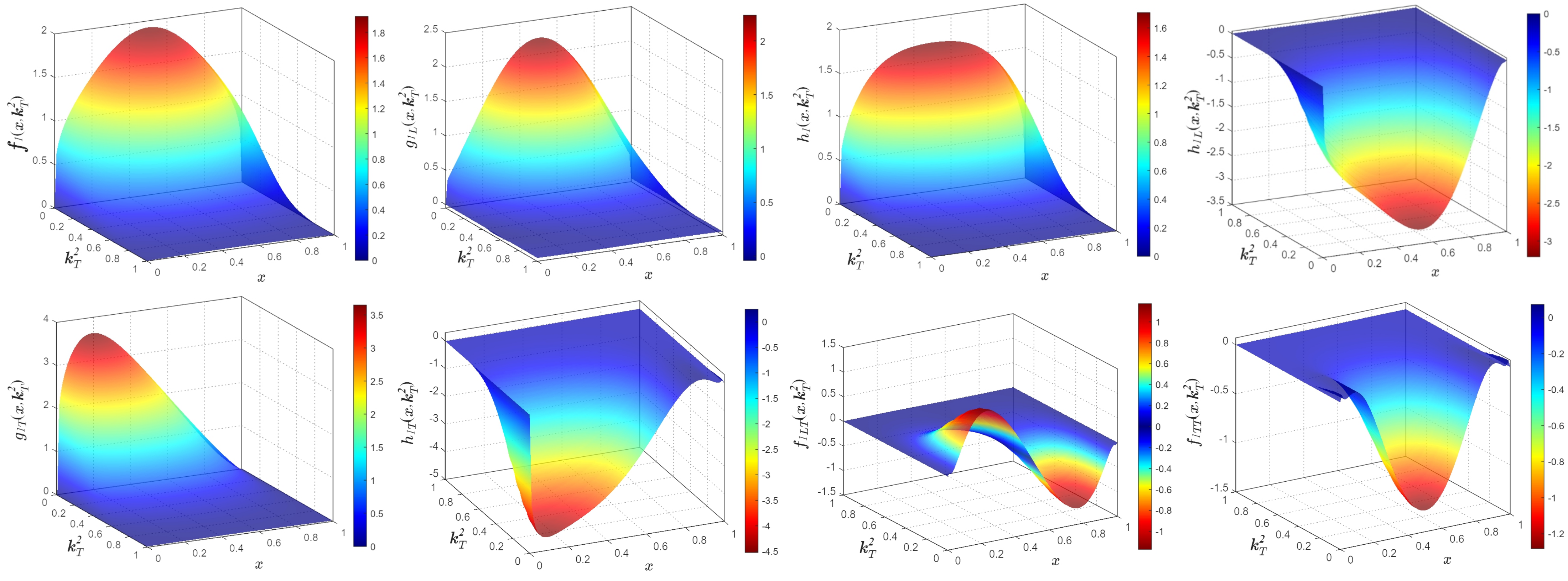


FIG. 5. The ρ TMDs from the full BSE-based LF-LFWFs.

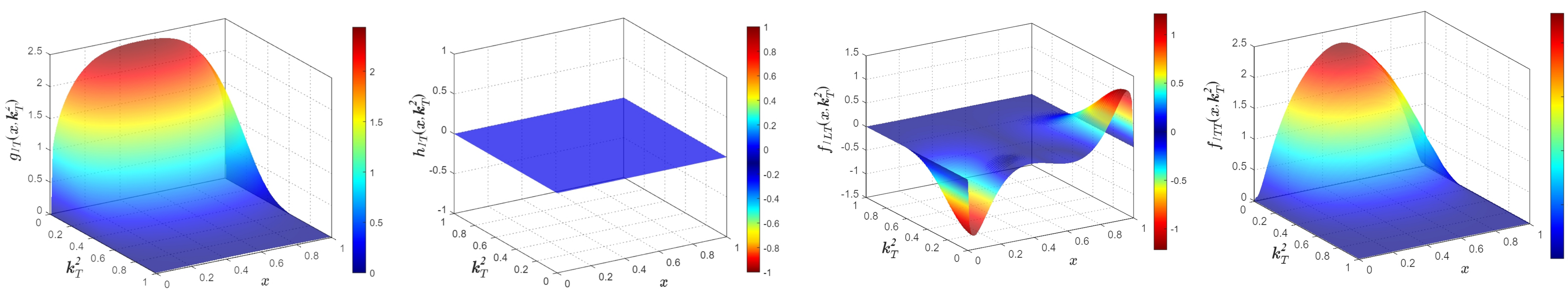


FIG. 6. The ρ TMDs obtained by setting $\phi^{\Lambda=0}_{|l_z|=1} = \phi^{\Lambda=1}_{|l_z|=2} = 0$ in the full BSE-based LF-LFWFs.

(Y. Ninomiya, et al, PRC2017) (S. Kaur, et al, JHEP2021)

- BSE-based TMDs agree with NJL and/or holographic model if higher OAM are set to zero.
- High OAM LF-LFWFs have sizable effect in determining certain polarized TMDs.

PDFs

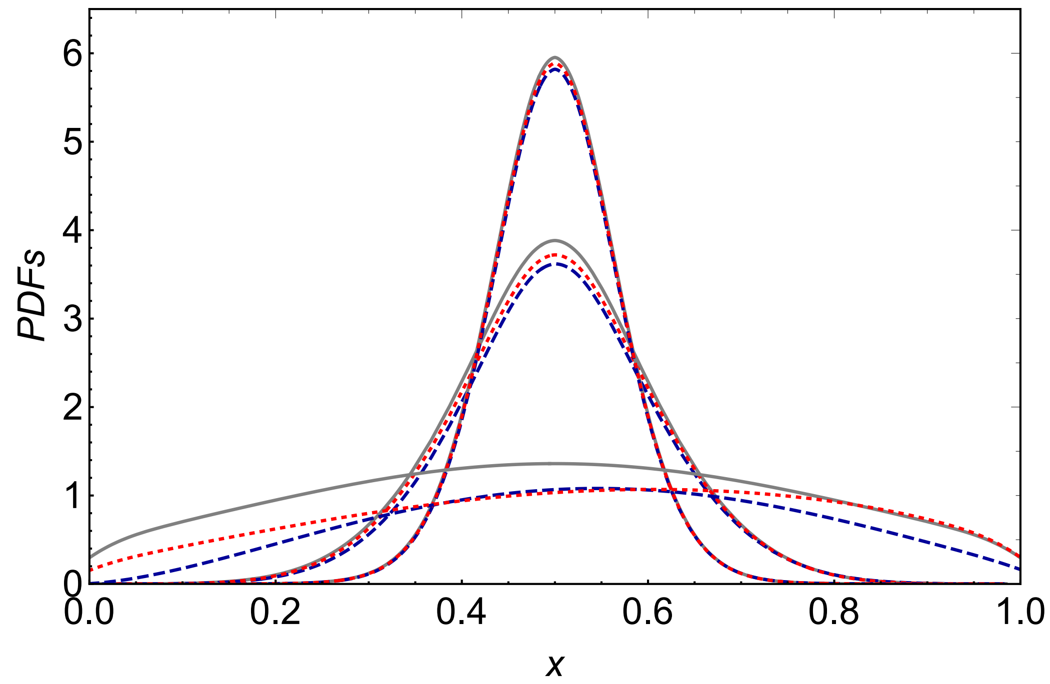


FIG. 7. The $f_1(x)$ (gray solid), $g_{1L}(x)$ (red dotted) and $h_1(x)$ (blue dashed) of vector mesons at hadron scale. At $x = 0.5$, from top to bottom, the three sets of curves correspond to Υ, J/ψ and ρ respectively.

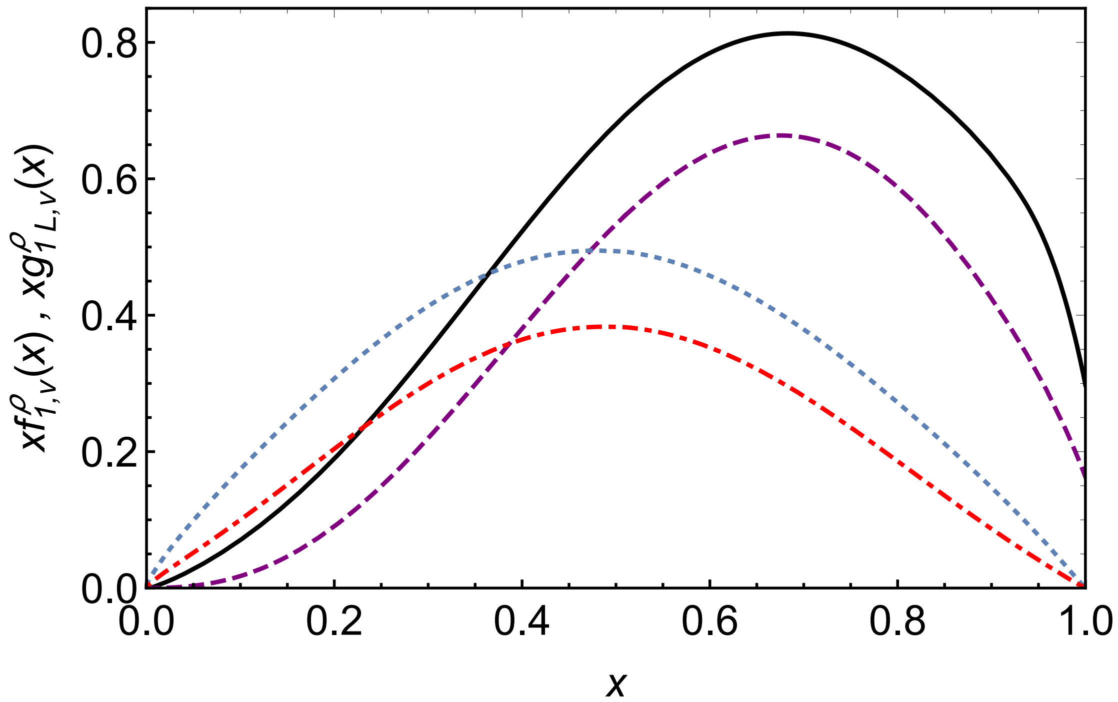


FIG. 8. The $xf^{\rho}_{1,v}(x)$ at hadronic scale (black solid) and evolved scale of 2.4 GeV (blue dotted), and $xg^{\rho}_{1L,v}(x)$ at hadronic scale (purple dashed) and evolved scale of 2.4 GeV (red dot-dashed).

Moments: $a_n = \langle x^{n-1} \rangle_{f_{1,v}}$ $\qquad r_n = \langle x^{n-1} \rangle_{g_{1L,v}}$

lattice (C. Best et al, PRD1997):

$$a_2 = 0.334(21), a_3 = 0.174(47), \; a_4 = 0.066(39)$$
$$r_1 = 0.57(32), \quad r_2 = 0.212(17), \; r_3 = 0.077(34)$$

Herein (C.S. et al, arXiv:2205.02757):

$$a_2 = 0.316, \; a_3 = 0.155, \; a_4 = 0.091$$
$$r_1 = 0.66, \quad r_2 = 0.227, \; r_3 = 0.111.$$

Summary

- LF-LFWFs are extracted from BS wave functions of light and heavy, pseudoscalar and vector mesons, based on rainbow-ladder & Maris-Roberts-Tandy model.
- The LF-LFWFs contribution decrease from heavy to light mesons, indicating the presence of higher Fock-states and thereby revealing a complex parton picture.
- The LF-LFWFs in light mesons are generally broad in x, in connection with EHM.
- Diffractive vector meson production data are well reproduced, in a combined study of DSEs, color dipole model and color glass condensate.
- LF-LFWFs with higher OAM have sizable effect in determining polarized distributions.

Thank you!