Light front chiral sum rule and the implication to the pion structure

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AMBER

Outline

- The two faces of the pion
- Chiral sum rule on the light front
- Holographic QCD
- 3D image of the pion
- Summary

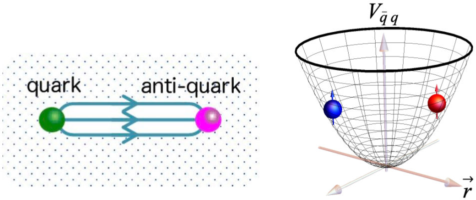
Based on: YL, Maris, Vary, arXiv:2203.14447 [hep-th]


Janus, the mythological two-faced Roman god, is the god of beginnings, gates, transitions, time, duality, doorways, passages, frames, and endings.

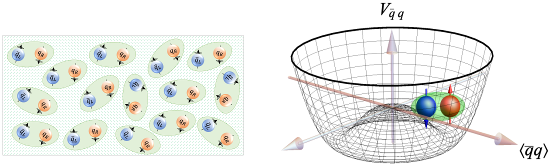
Introduction

Two distinguished non-perturbative properties of QCD:

Confinement



Chiral symmetry breaking

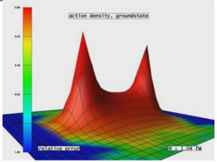


Implication to hadron physics:

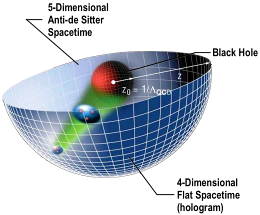
- Quark confinement
- Nambu-Goldstone boson
  - Gell-Mann-Oakes-Renner relation
  - Low-energy theorems

Pion, the Janus meson, holds the key

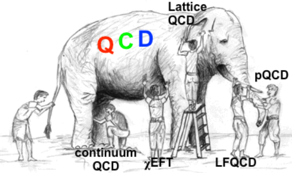
Confinement & chiral symmetry breaking in QCD



- Nambu-Jona-Lasinio model, chiral effective field theory: low energy effective theories
  cf. QCD sum rule, instanton vacuum, chiral soliton model, ...
- Lattice gauge theory
  - *Emergence of linear confining potential between heavy quarks*
  - *Light mesons: approaching the physical pion mass* [FLAG, '21]
    Nielsen-Ninomiya theorem, ``Berlin wall'' $C = K\left(\frac{M_\rho}{M_\pi}\right)^4 a^{-7}L^5, \quad L \gtrsim 4M_\pi^{-1}$ [Urbach, '05]
  - *It is fair to say, no simple picture has emerged so far*



- Holographic QCD [e.g., Erlich, '05]
  - *Hard/soft-wall AdS/QCD, confinement, chiral symmetry breaking, ...*
  - *No direct access to the hadron structure (except light-front holography)*
- Dyson-Schwinger equations/Bethe-Salpeter equations
  - *Exact relations between pion BSA and quark self-energy from AvWTI*
    $f_\pi E_\pi(k,0) = B_q(k^2), \cdots$: *implication to EHM* [Maris, '98; cf. Gross, '83]
  - *Maris-Tandy model, rainbow-ladder, relativized confinement, UV*
- Get Minkowskian: moments, LaMET, Nakanishi rep'n, CST, un-Wick rotation, ...
  [Nakanishi '63; Ji '13; Chang '13; Biernat '14; Frederico, '19; Maris '20; Eichmann '21

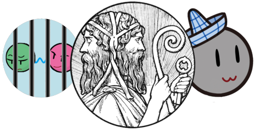


Talks by Lin (W), Liu (W), Stadler (R), Zhao (F), de Paula(F), Ydrefors (F), Shi (F)

Parton structure of the pion

The structure of the pion has to accommodate two seemingly opposing faces:

- Pion as a elementary particle (Goldstone boson): chiral effective field theory
- Pion as a quark-antiquark bound state: quark model, perturbative QCD

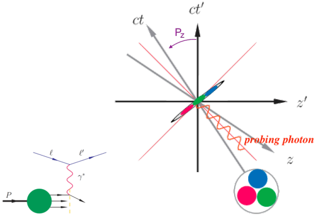


How to describe the structure of hadrons? $(r_h \sim \lambda_c)$

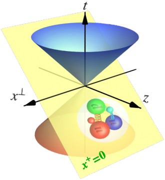
- Light front approach is the only (known) consistent way to describe hadron structure as measured in the experiments [Burkardt, '00; Miller '02&'08]
- Examples: hard exclusive process: $z^2 \sim 1/Q^2 \to 0$ (LC dominance) [Lepage, '80]

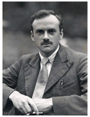
$$i\mathcal{M}^{\mu\nu} = \int \mathrm{d}^4 z\, e^{iq\cdot z} \langle 0 | J^\mu(z) J^\nu(0) | P \rangle \sim \int \mathrm{d}x\, T_H(x, Q^2)\, \overbrace{\phi_P(x;\mu)}^{\text{light-cone distribution amplitude}} \qquad x = \frac{k^+}{P^+}$$

infinite momentum frame ($P_z \to \infty$)



light front quantization ($x^+ = 0$)





$$x^\pm = x^0 \pm x^3$$

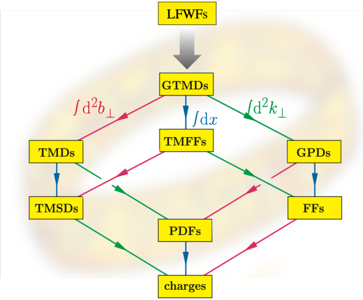
Light front wave functions

[Brodsky, Phys. Rep. '98]

$$|P(p)\rangle = \sum_{s,\bar{s}} \int \frac{\mathrm{d}x}{2x(1-x)} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \psi_{s\bar{s}/P}(x,\vec{k}_\perp) \frac{1}{\sqrt{N_C}} \sum_i b^\dagger_{si}(p_1) d^\dagger_{\bar{s}i}(p_2)|0\rangle + \cdots$$

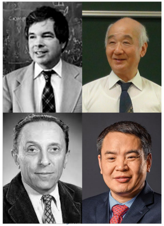
where $x = p_1^+/p^+$, and $\vec{k}_\perp = \vec{p}_{1\perp} - x\vec{p}_\perp$.

- LFWFs are relativistic (Minkowskian) & frame independent
- Direct access to hadron structures, e.g. parton distributions
  - *local matrix elements (e.g. form factors):* $\langle h'|O(x)|h\rangle$
  - *light-like correlators (e.g. GPDs, LCDAs):* $\langle h'|O(x)O(y)|h\rangle_{x^+=y^+}$
  - *scattering amplitudes: time-ordered perturbation theory (TOPT)*
  - *external fields, time-dependent problems*
  - *intrinsic densities, spectral densities, entropy & entanglement, ...*

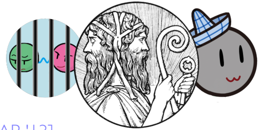


Access to the light front amplitudes:

- OPE + moments reconstruction [Talk by Shi (F)]
- Nakanishi representation & light-front projection of BSA [Talks by de Paula & Ydrefors (F)]
- un-Wick rotation [Talk by Stadler (R)]
- IMF and large momentum effective theory [Talk by Lin (W)]
- Schrödinger-Einstein equation [Talk by Zhao (F)]



Chiral symmetry breaking on the light front



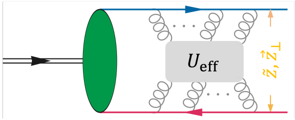
Common myths & myth busters:

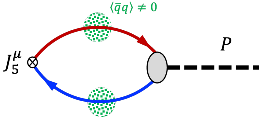
- Light front vacuum is trivial -- no χ condensate/NGB
  Answer: LF vacuum is not trivial -- there is chiral condensate on the LF [Wu, JHEP '04; Beane, AP '13]
- Light front spinors are chiral spinors -- no χSB
  Answer: only true in free theory [Burkardt PRD '97]
- χSB is a collective pheno./property of vacuum/zero modes -- no effect on hadrons (FB phys.)
  Answer: $Q_5|0\rangle_{LF} = 0$ *-- in-vacuum condensate* $\to$ *in-hadron condensate* [Maris '97; Brodsky '13; Casher '74]

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]





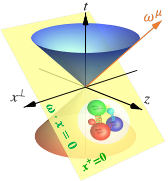
Covariant light-front dynamics

The most general covariant structure of the pion valence LFWF: [Carbonell, PR '98]

$$\psi_{s\bar{s}/P}(x,\vec{k}_\perp) = \bar{u}_s(p_1)\Big[\gamma_5\,\phi_1(x,k_\perp) + \hat{f}_\chi \frac{\gamma_5 \not{\omega}}{\omega\cdot p}\phi_2(x,k_\perp)\Big] v_{\bar{s}}(p_2),$$

where ω is the null vector ($\omega^2=0$) indicating the orientation of the quantization surface, $\not{\omega} = \gamma^+ \sim \not{p}$.

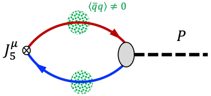
- ω dependent terms are needed to maintain rotational invariance: $L^{\mu\nu}_{\text{int}} = i\omega^{[\mu}\partial/\partial\omega_{\nu]}$
- Conformal symmetry: $\omega^\mu \to \xi\omega^\mu \Rightarrow$ Lorentz structure $\gamma_5\not{\omega}/\omega\cdot p$
- Need a $\hat{f}_\chi$ is in mass dimension
- In QCD, χSB $\Rightarrow\ f_\pi \neq 0\ \Rightarrow\ \hat{f}_\chi \neq 0$ (previous works chose $\hat{f}_\chi = 0, m_q, M_\pi$, wrong!)



$$\psi_{\uparrow\uparrow/P} = \psi^*_{\downarrow\downarrow/P} = -\frac{k_\perp e^{-i\arg\vec{k}_\perp}}{\sqrt{x(1-x)}}\phi_1;\quad \psi_{\uparrow\downarrow-\downarrow\uparrow/P} = \frac{\sqrt{2}m_q}{\sqrt{x(1-x)}}\phi_1 - \hat{f}_\chi\sqrt{8x(1-x)}\phi_2 \to -\hat{f}_\chi\sqrt{8x(1-x)}\phi_2;$$

Decay constant: $\langle 0|J_5^+(0)|P(p)\rangle = ip^+ f_P$

$$\frac{f_P}{2\sqrt{2N_C}} = \int\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp).$$



$J_5^\mu = i\partial^\mu\phi_\pi + J^\mu_{5,N}$

Chiral sum rule

Partially conserved axial-vector current (PCAC):

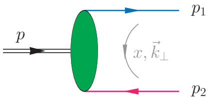
$$\partial_\mu J_5^\mu = 2im_q\bar{q}\gamma_5 q \quad \Rightarrow \quad \langle 0|\partial_\mu J_5^\mu - 2im_q\bar{q}\gamma_5 q|P(p)\rangle = 0.$$

where, $J_5^\mu(x) = \bar{q}(x)\gamma^\mu\gamma_5 q(x)$ is the axial-vector current; $q(x)$ is the quark field operator.

$$q(x) = \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2p^+}\Big\{b_s(p)u_s(p)e^{ip\cdot x} + d_s^\dagger(p)v_s(p)e^{-ip\cdot x}\Big\}\Big|_{x^+=0},$$

$$\overset{m_q\to 0}{\Rightarrow} \quad \boxed{\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2 k_\perp}{(2\pi)^3}\frac{k_\perp^2}{x(1-x)}\psi^{(0)}_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp) = 0}$$

- Gell-Mann-Oakes-Renner relation: $f_P^{(0)2}M_P^2 = 2m_q g_P^{(0)} + O(m_q^2)$, where $g_P = \langle 0|j_5|P(p)\rangle$
- Exact relation -- no Fock sector truncation
- Wave functions contain self-energy -- no assumption of quarks as physical eigenstates
- Uncertainty principle: $\frac{1}{2}\Delta x^+ \, \Delta P^- \gtrsim 1$
- Additional sum rules from further light-front current algebra [Beane, '13; Hobbs, '16]

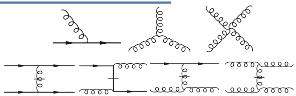


First approximation to QCD

[Review: Brodsky, Phys. Rep. '98]

Light-front QCD in light cone gauge $A^+ = 0$

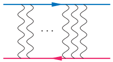
$$H_{\mathrm{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + U + \sum_{ij} V_{ij}^{(\mathrm{QCD})} - \delta_{ij} U_i$$

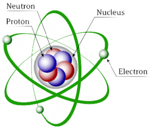
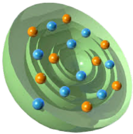
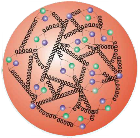


$$\Downarrow$$

Light-front Schrödinger wave equation (LFSWE)

$$\Big(\frac{\vec{k}_\perp^{\,2} + m_q^2}{x} + \frac{\vec{k}_\perp^{\,2} + m_{\bar{q}}^2}{1-x} + U\Big)\psi(x, \vec{k}_\perp) = M^2 \psi(x, \vec{k}_\perp)$$

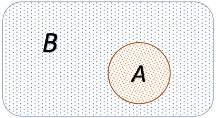


Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
$\downarrow$	$\downarrow$	$\downarrow$
Bohr Model 	Shell Model 	?
$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$	

Separation of variables

In LFD, there is a natural separation of the transverse and longitudinal d.o.f.'s:

$$\Big\{\underbrace{\frac{\vec{k}_\perp^2}{x(1-x)}}_{\text{chiral limit },\perp} + \underbrace{\frac{(1-x)m_q^2 + x m_{\bar{q}}^2}{x(1-x)}}_{\text{mass term },\parallel} + U\Big\}\psi(x,\vec{k}_\perp) = M^2\psi(x,\vec{k}_\perp)$$



$$S_A = -\mathrm{tr}\,\rho_A \ln \rho_A$$

- Separation ansatz:

$$U = U_\perp(\zeta_\perp) + U_\parallel(\tilde{z}) \Rightarrow M^2 = M_\perp^2 + M_\parallel^2,\ \psi(x,\vec{\zeta}_\perp) = \varphi(\vec{\zeta}_\perp)\chi(x)$$

Here, $\vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$, $\tilde{z} = \frac{1}{2}P^+x^- = i\partial/\partial x|_{\vec{\zeta}_\perp}$. [Miller & Brodsky, PRC 2020]

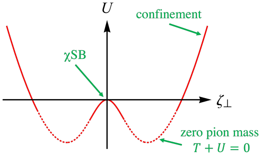
- The LFSWE can be split into two equations: [Chabysheva, AP 2012]

$$\Big[-\nabla_\zeta^2 + U_\perp(\vec{\zeta}_\perp)\Big]\varphi(\vec{\zeta}_\perp) = M_\perp^2\varphi(\vec{\zeta}_\perp), \quad \Big[\frac{\cancel{m_q^2}}{x} + \frac{\cancel{m_{\bar{q}}^2}}{1-x} + U_\parallel(\tilde{z})\Big]\chi(x) = M_\parallel^2\chi(x)$$

Brodsky et al. took $\chi_\pi(x) = 1 \Rightarrow \phi_\pi(x) = (8f_\pi/\pi)\sqrt{x(1-x)}$. ['t Hooft, NPB '76]

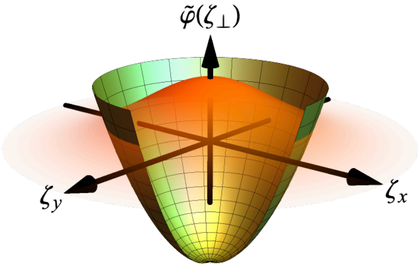
- Chiral sum rule becomes:

$$f_P\nabla_{\zeta_\perp}^2\varphi_P(\vec{\zeta}_\perp = 0) = 0 \quad \Rightarrow \quad U_\perp(\vec{\zeta}_\perp = 0) = 0.$$

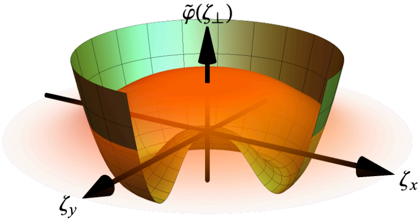


Example

$$\Big[-\nabla_\perp^2 + U(\vec{\zeta}_\perp)\Big]\widetilde{\varphi}_P(\vec{\zeta}_\perp) = M_P^2 \widetilde{\varphi}_P(\vec{\zeta}_\perp), \qquad f_P \nabla_\perp^2 \widetilde{\varphi}_P(\vec{\zeta}_\perp = 0) = 0.$$



Without χSB



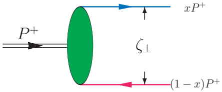
With χSB

* For the excited pions, the chiral sum rule is fulfilled by $f_P = 0$.

Light-front holography: equation of motion

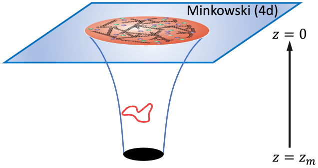
[Review: Brodsky, Phys. Rep. '15]

LFH is a unique mapping between LFQCD_{3+1} and string motion in AdS/QCD



- Based on LFQCD
- $\zeta_\perp = \sqrt{x(1-x)}r_\perp$,
- Conjugate to off-shell energy μ

LFH	$\leftrightarrow$	soft-wall AdS/QCD
$\zeta_\perp$	$\leftrightarrow$	z,
ϕ_J	$\leftrightarrow$	$\widetilde{\phi}_{Jm}$
U_J	$\leftrightarrow$	$\frac{1}{4}\Phi'^2 + \frac{1}{2}\Phi'' + \frac{2J-3}{2z}\Phi'$
$(\mu R)^2$	$\leftrightarrow$	$m^2 - (J-2)^2$

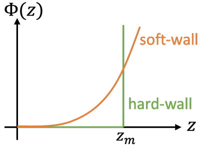


- Based on gravity/gauge duality
- $z \to 0$: CFT
- $z \to \infty$: low-energy QCD pheno.
- $z^{-1} \sim \mu_R$, RG scale

$$\Big[-\frac{1}{\zeta_\perp}\frac{\mathrm{d}}{\mathrm{d}\zeta_\perp}\Big(\zeta_\perp\frac{\mathrm{d}}{\mathrm{d}\zeta_\perp}\Big) + \frac{m^2}{\zeta_\perp^2} + U_J(\zeta_\perp)\Big]\widetilde{\varphi}_{Jm}(\zeta_\perp) = M^2\widetilde{\varphi}_{Jm}(\zeta_\perp)$$

$$\Big[-\frac{z^{3-2J}}{e^{\Phi(z)}}\frac{\mathrm{d}}{\mathrm{d}z}\Big(\frac{e^{\Phi(z)}}{z^{3-2J}}\frac{\mathrm{d}}{\mathrm{d}z}\Big) + \frac{\mu^2R^2}{z^2}\Big]\varphi_J(z) = M^2\varphi_J(z)$$

where $\varphi_J(z) = \left(\frac{R}{z}\right)^{J-\frac{3}{2}} e^{-\frac{1}{2}\Phi(z)}\phi_J(z)$, $\widetilde{\varphi} = \widetilde{\phi}/\sqrt{\zeta_\perp}$, and $(\mu_{\text{eff}}R)^2 = \big(\mu(z)R\big)^2 - Jz\Phi'(z) + J(5-J)$.



Light-front holography: form factor

[Review: Brodsky, Phys. Rep. '15]

- On the light front, the form factor is obtained from the Drell-Yan-West formula

$$
\begin{aligned}
F_\pi(Q^2) &= \int_0^1 \mathrm{d}x \int \mathrm{d}^2\zeta_\perp \rho_\pi(\vec\zeta_\perp) e^{i\sqrt{\frac{1-x}{x}}\vec\zeta_\perp\cdot\vec q_\perp} + \cdots \\
&= \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) \zeta_\perp Q K_1(\zeta_\perp Q) + \cdots
\end{aligned}
$$

  where $\rho_\pi(x, \vec\zeta_\perp) = N^2 \big|\widetilde\varphi_\pi(\zeta_\perp)\big|^2$.

- In AdS/QCD, the form factor is obtained from the 5D current $A_\mu(x^\mu, z) = e^{iq\cdot x} V(q^2, z)\epsilon_\mu(q)$

  [Hong, '06; Grigoryan, '07ab&'08; Kwee '08]

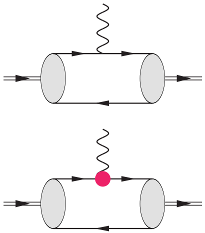
$$
F(Q^2) = \int \frac{\mathrm{d}z}{z^3} V(Q^2, z)\phi^2(z)
$$

  In hard-wall model,

$$
V_{\mathrm{HW}}(Q^2, z) = zQK_1(zQ) + zQI_1(zQ)\frac{K_0(Qz_m)}{I_0(Qz_m)}.
$$

  It consists of a point-like part, and a second part due to the hard-wall confinement.

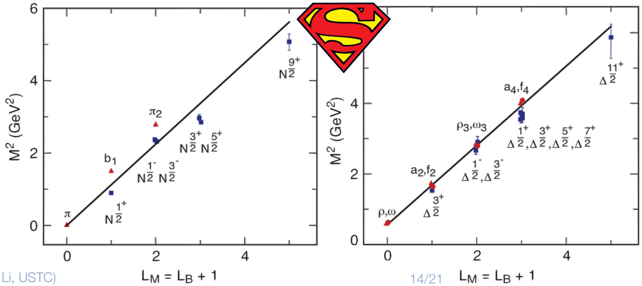
- Pole representation & QCD sum rule [Grigoryan '07; Afonin, '22]

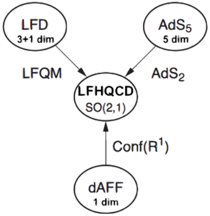


Light-front holography: soft-wall confinement

[Review: Brodsky, Phys. Rep. '15]

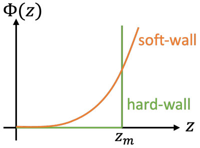
- Confinement entails a dilaton field $\Phi(z)$ breaking the conformal symmetry at large z
  - $\Phi(z) \to 0$ *at* $z \to 0$
- Soft-wall model: $\Phi(z) \sim z^2$ at $z \to \infty$ agrees with the Regge trajectories in meson spectrum [Karch '06]
  - *Effective light-front confining potential:* $U(z) \sim z^2$ *as* $z \to \infty$
  - *Karch et al adopted* $\Phi(z) = \lambda z^2 \ \Rightarrow U(z) = \lambda^2 z^2 + 2(J-2)\lambda \ \Rightarrow$ *massless pion*
- Brodsky and de Téramond further show this choice is consistent with the superconformal symmetry
  - *Pion appears as a massless susy singlet* [Fubini, '84; de Alfaro, '76; Miyazawa '66&'68]
  - *Emergent hadron mass: (dynamical) superconformal symmetry*
- What about chiral symmetry breaking?





Chiral symmetry breaking in soft-wall AdS/QCD

[Erlich '05; Karch '06; Gherghetta, '09]



PRL **95**, 261602 (2005)

QCD and a Holographic Model of Hadrons

Joshua Erlich,[1] Emanuel Katz,[2] Dam T. Son,[3] and Mikhail A. Stephanov[4]

1,069 citations

Hard-wall model: with chiral symmetry breaking, but no Regge trajectory

PHYSICAL REVIEW D **74**, 015005 (2006)

Linear confinement and AdS/QCD

Andreas Karch,[1,*] Emanuel Katz,[2,†] Dam T. Son,[3,‡] and Mikhail A. Stephanov[4,§]

1,017 citations

Soft-wall model: with Regge trajectory, but no chiral symmetry breaking

tion between bulk and boundary theories [12,19]. The action at quadratic order in the fields and derivatives reads

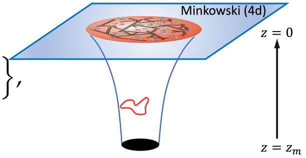
$$I = \int d^5x e^{-\Phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

asymptotics $e^{z^2} \to \infty$ and $\exp\{-(3/4)z^{-2}\} \to 1$. Since the equation is linear, selecting one of the solutions in the IR (the $X < \infty$ one, of course) gives Σ simply proportional to M. This is not what one wants in a theory with spontaneous symmetry breaking such as QCD. It is clear that one has to consider higher order terms in the potential $U(X, \ldots)$ for X and all other scalar condensates. Such a potential would

Chiral symmetry breaking in soft-wall AdS/QCD

[Gherghetta, '09; Kapusta, '10]

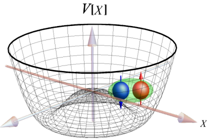
- Scalar field X dual to $\bar{q}q$, with a non-vanishing VEV: $\langle X\rangle = \frac{1}{2}\chi(z)$



$$S = -\int \mathrm{d}^5x\sqrt{-g}e^{-\Phi(z)}\mathrm{Tr}\Big\{\big|DX\big|^2 - V[X] + \frac{1}{4g_5^2}F^2\Big\},$$

where $V[X] = -m_X^2|X|^2 + \kappa|X|^4$ is the Higgs potential, $m_X^2 = -3$

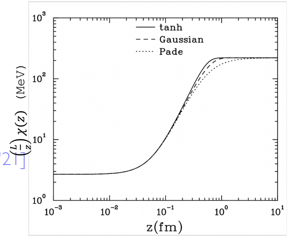
- Eq. of motion: $\frac{\mathrm{d}}{\mathrm{d}z}\Big(\frac{e^{-\Phi}}{z^3}\frac{\mathrm{d}}{\mathrm{d}z}\chi\Big) - \frac{e^{-\Phi}}{z^5}\Big(m_X^2\chi - \frac{\kappa}{2}\chi^3\Big) = 0$

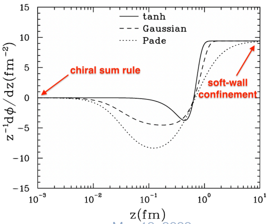


- At the CFT boundary: $\chi(z) \sim \xi m_q z + \xi^{-1}\Sigma z^3$ at $z \to 0$ [Klebanov & Witten, '99]
- $\Phi(z) \sim z^6 \Rightarrow U \sim -z^4$ at $z \to 0$, consistent with the chiral sum rule
- Recall confinement requires $\Phi(z) \sim z^2$ at $z \to \infty$
- The Mexican-hat potential is the relic of the Higgs potential in 5D
- Similar models with χSB:

  [Babingto '04; Casero '07&'10; Jarvinen '12; Li, '13; Sui '10;]

  [Cui '16; Chelabi '16; Braga '19; Capossoli, '20; Ballon-Bayona '20&'21]





Analytic model and 3D image of the pion

$$\Big[- \nabla_\perp^2 + U(\zeta_\perp) \Big] \widetilde{\varphi}_P(\zeta_\perp) = M_P^2 \widetilde{\varphi}_P(\zeta_\perp), \qquad f_P \nabla_\perp^2 \widetilde{\varphi}(\zeta_\perp = 0) = 0.$$

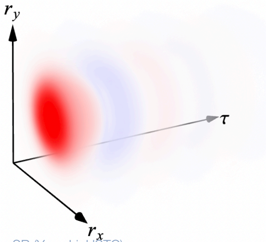
Ansatz pion wave function based on LFH: $\widetilde{\varphi}_\pi(\zeta_\perp) = \Big(1 + \frac{1}{2}\zeta_\perp^2 + \frac{1}{8}\zeta_\perp^4\Big) e^{-\frac{\zeta_\perp^2}{2}}$.

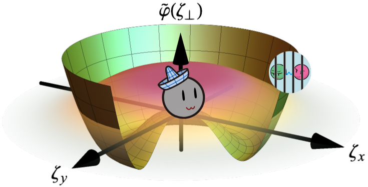
Given the pion wave function, the potential is,

$$U(\zeta_\perp) = \frac{\widetilde{\varphi}''_\pi(\zeta_\perp) + \zeta_\perp^{-1} \widetilde{\varphi}'_\pi(\zeta_\perp)}{\widetilde{\varphi}_\pi(\zeta_\perp)} = \frac{\zeta_\perp^4(\zeta_\perp^2 - 6)}{\zeta_\perp^4 + 4\zeta_\perp^2 + 8} \to \begin{cases} \zeta_\perp^2 & \zeta_\perp \to \infty \\ -\zeta_\perp^4 & \zeta_\perp \to 0 \end{cases}$$

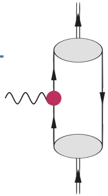
Coordinate space wave function: (τ is the Ioffe time [Miller, '20])

$$\widetilde{\psi}_\pi(\tau, \vec{r}_\perp) = \int_0^1 \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} e^{ix\tau - i\vec{k}_\perp \cdot \vec{r}_\perp} \psi_\pi(x, \vec{k}_\perp)$$





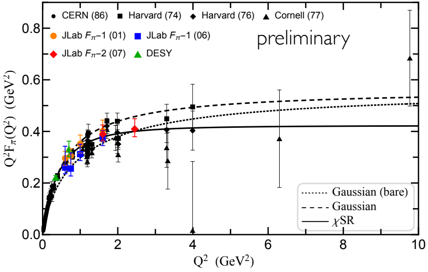
Pion form factors

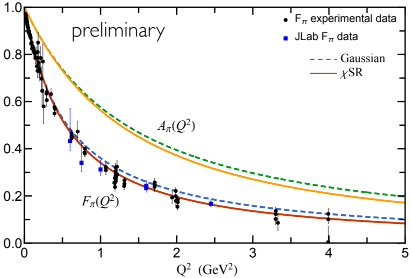


- Electromagnetic form factor: $\langle p+q|J^+(0)|p\rangle = 2p^+F_\pi(-q^2)$, gravitational form factor: $\langle p+q|T^{++}(0)|p\rangle = 2p^+p^+A_\pi(-q^2)$ with $q^+=0, Q^2=-q^2=q_\perp^2$

$$F_\pi(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) V(Q^2,\zeta_\perp), \quad A_\pi(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) H(Q^2,\zeta_\perp)$$

- One single parameter, $\kappa = 0.43\,\mathrm{GeV}$, sets the scale [Brodsky, '08]
- Pion electromagnetic radius $r_\pi|_{\mathrm{em}}$ = 0.64 fm (PDG value: 0.67 fm), mass radius $r_\pi|_{\mathrm{mass}}$ = 0.40 fm (Belle II: 0.32--0.39 fm, extracted from the GDA analysis of the $\gamma^*\gamma^* \to \pi^0\pi^0$ process) [Kumano, '18]
- χSB modify the large-Q^2 behavior: incorporating high-twist contributions automatically



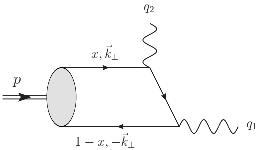


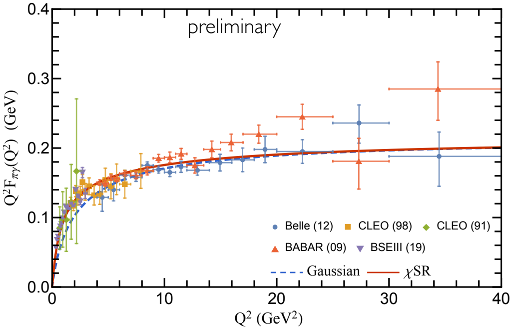
Transition form factor

Singly-virtual two-photon transition form factor:

$$F_{\pi\gamma}(Q^2) = e_f^2 2\sqrt{2N_C} \int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \frac{\psi_\pi(x,\vec{k}_\perp)}{k_\perp^2 + x(1-x)Q^2}$$

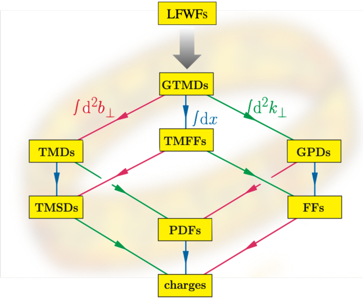
- VMD: $Q^2 \to Q'^2 = Q^2 + M_\rho^2$
- pQCD normalization: $Q^2 F_{\pi\gamma}(Q^2) \to 2f_\pi$, $Q^2 \to \infty$





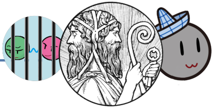
Further observables

- Electromagnetic form factor: Brodsky, PRD '08
- Gravitational form factors: Brodsky, PRD '08
- Transition form factor: Brodsky, PRD '11
- Nucleon form factors: Sufian, PRD '17
- Generalized Parton distributions (GPDs): de Teramond, PRL '18
- PDFs: Liu, PRL '20
- Gluon distribution: de Teramond, PRD '21
- ......



See T. Liu's talk (W)

Summary



- Pion is the key to understand confinement and chiral symmetry breaking in QCD
- Light-front wave functions provide the direct access to the parton structure of the pion
- Obtained an exact sum rule for the valence sector wave function based on the most general covariant structure and PCAC
- This chiral sum rule is consistent with chiral symmetry breaking in soft-wall AdS/QCD and leads to a remarkable feature of the pion structure