

Perceiving the Emergence
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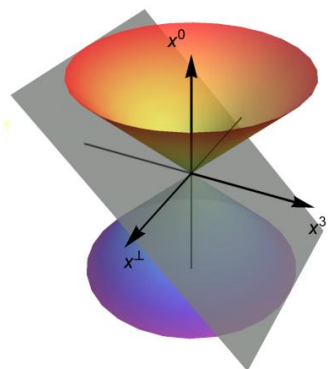
10 - 13 May 2022
CERN, Geneva - Switzerland



GPDs and TMDs of Light Mesons from a Basis Light-front Approach

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CERN, Geneva-Switzerland 13 May 2022

Outline

- Basis Light-front Quantization approach
- Application to light mesons
 - A review of BLFQ-NJL
 - **Light mesons with one dynamical gluon**
- Application to strangeonia and strange mesons
- Conclusion & outlook

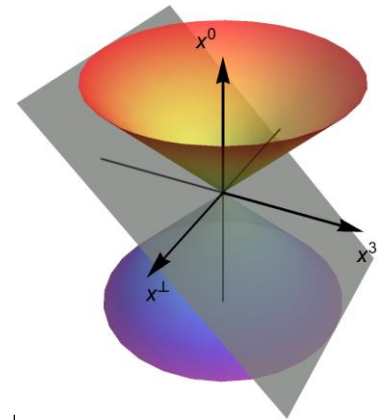
Basis Light-front Quantization

- Nonperturbative eigenvalue problem

$$P^- |\beta\rangle = P_\beta^- |\beta\rangle$$

- P^- : light-front Hamiltonian
- $|\beta\rangle$: mass eigenstate
- P_β^- : eigenvalue for $|\beta\rangle$

[Vary et al, 2008]



- Evaluate observables for eigenstate

$$O \equiv \langle \beta | \hat{O} | \beta \rangle$$

- Fock sector expansion

- Eg. $|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \dots$

- Discretized basis

- Transverse: 2D harmonic oscillator basis: $\Phi_{n,m}^b(\vec{p}_\perp)$.
- Longitudinal: plane-wave basis, labeled by k .
- Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$

$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

Light Meson Structure from NJL Interaction

$$H_{\text{eff}} = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) \vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x) \partial_x) + H_{\text{eff}}^{\text{NJL}}$$

$|\text{meson}\rangle = |q\bar{q}\rangle + \dots$

Diagonalize H_{eff}



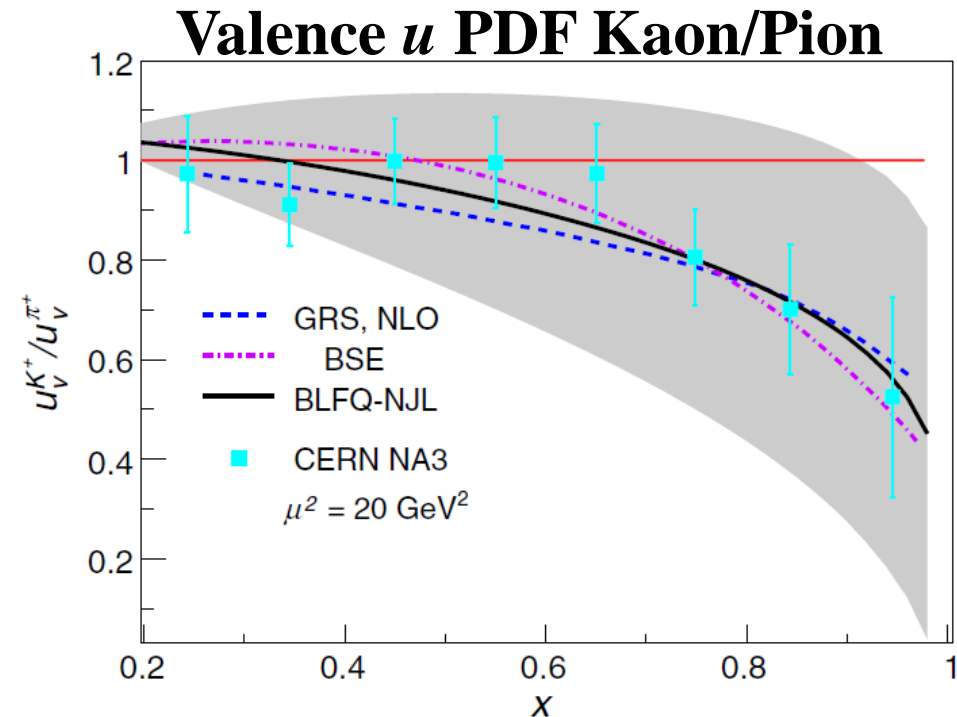
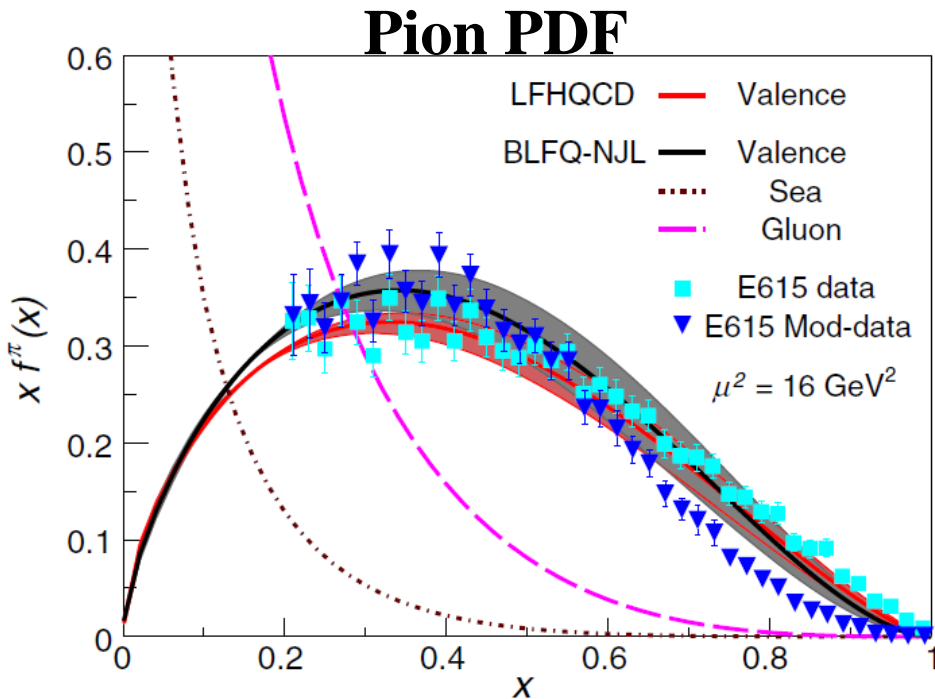
LF wave function



PDF@initial scale



Scale evolution



Dynamical Gluon in Light Mesons

[Lan, et al, PRL 19']

$$|\text{meson}\rangle = |q\bar{q}\rangle + \dots$$

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]



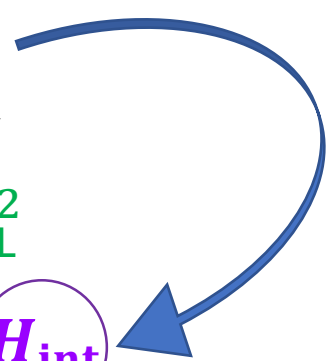
$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

H_{int}	$ q\bar{q}\rangle$	$ q\bar{q}g\rangle$
$\langle q\bar{q} $		
$\langle q\bar{q}g $		0

$$H_{\text{eff}} = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$



$$P^- = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) + H_{\text{int}}$$



Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{LFQCD}^- = \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi - \frac{1}{2} \int d^3x A_a^i (i\partial^\perp)^2 A_a^i$$

$$+g \int d^3x \bar{\psi} \gamma_\mu A^\mu \psi$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+} \gamma_\nu A^\nu \psi$$

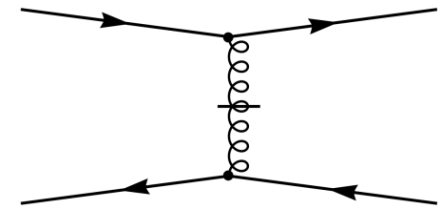
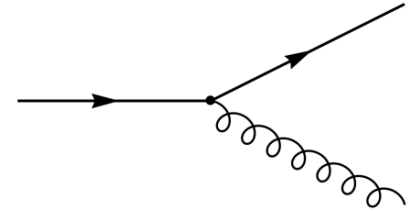
$$-i g^2 \int d^3x f^{abc} \bar{\psi} \gamma^+ T^c \psi \frac{1}{(i\partial^+)^2} (i\partial^+ A_a^\mu A_{\mu b})$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi$$

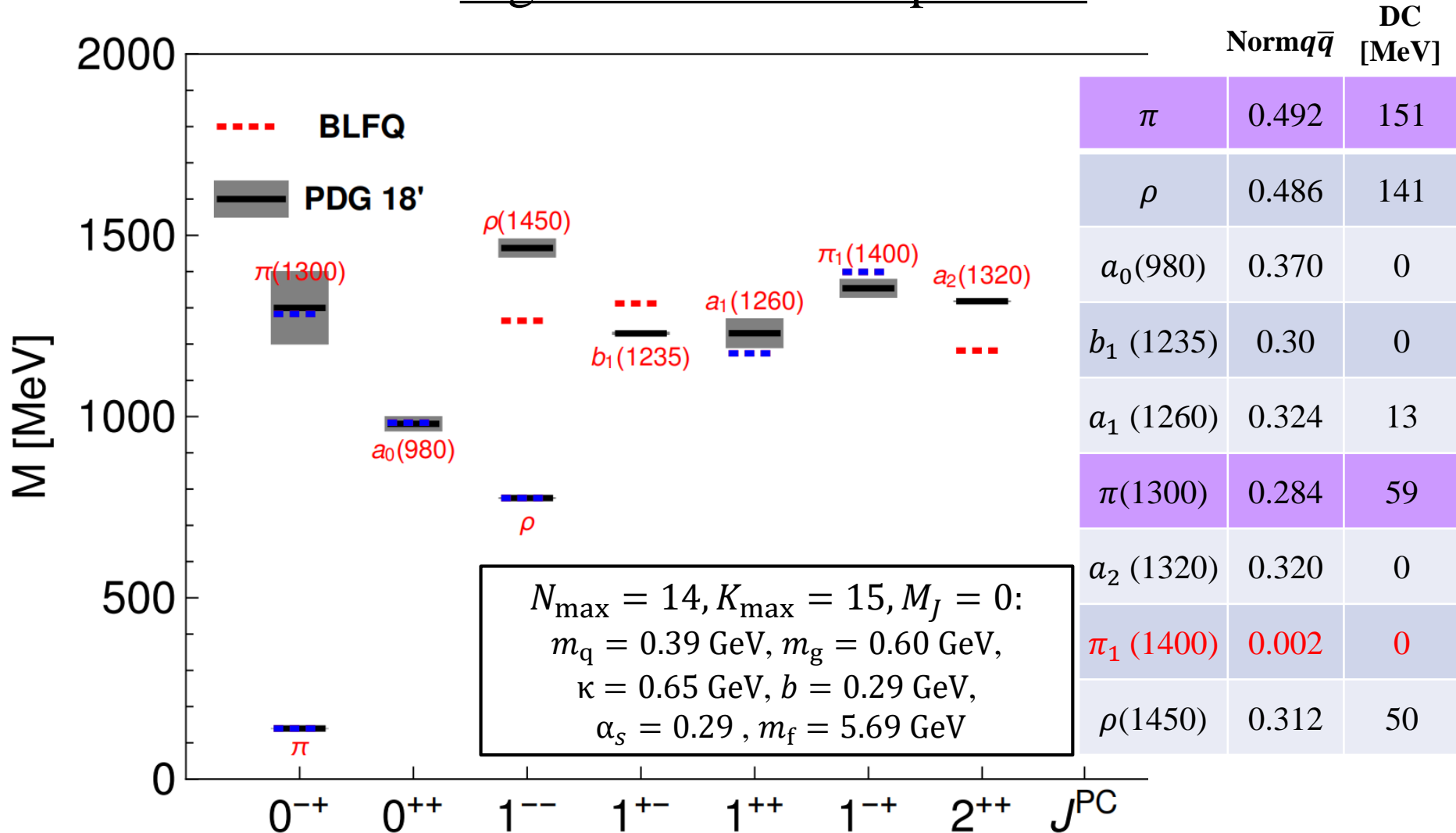
$$+i g \int d^3x f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c$$

$$- \frac{1}{2} g^2 \int d^3x f^{abc} f^{ade} i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2} (i\partial^+ A_d^+ A_{ve})$$

$$+ \frac{1}{4} g^2 \int d^3x f^{abc} f^{ade} A_b^\mu A_c^\nu A_{\mu d} A_{ve}$$



Light Meson Mass Spectrum



$N_{\max} = 14, K_{\max} = 15, M_J = 0:$
 $m_q = 0.39 \text{ GeV}, m_g = 0.60 \text{ GeV},$
 $\kappa = 0.65 \text{ GeV}, b = 0.29 \text{ GeV},$
 $\alpha_s = 0.29, m_f = 5.69 \text{ GeV}$

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

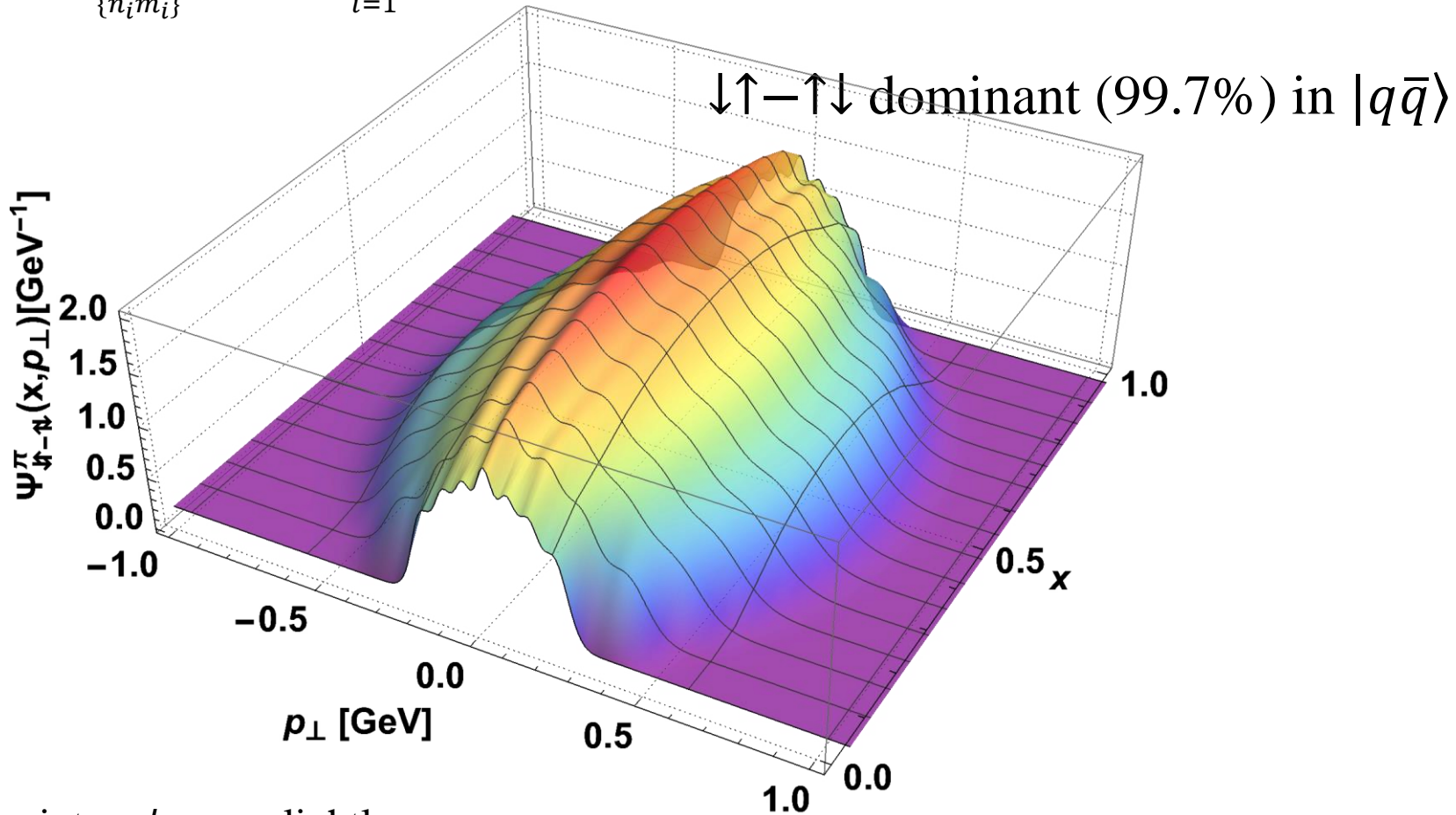
Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

The Wave Function in Leading Fock Sector

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



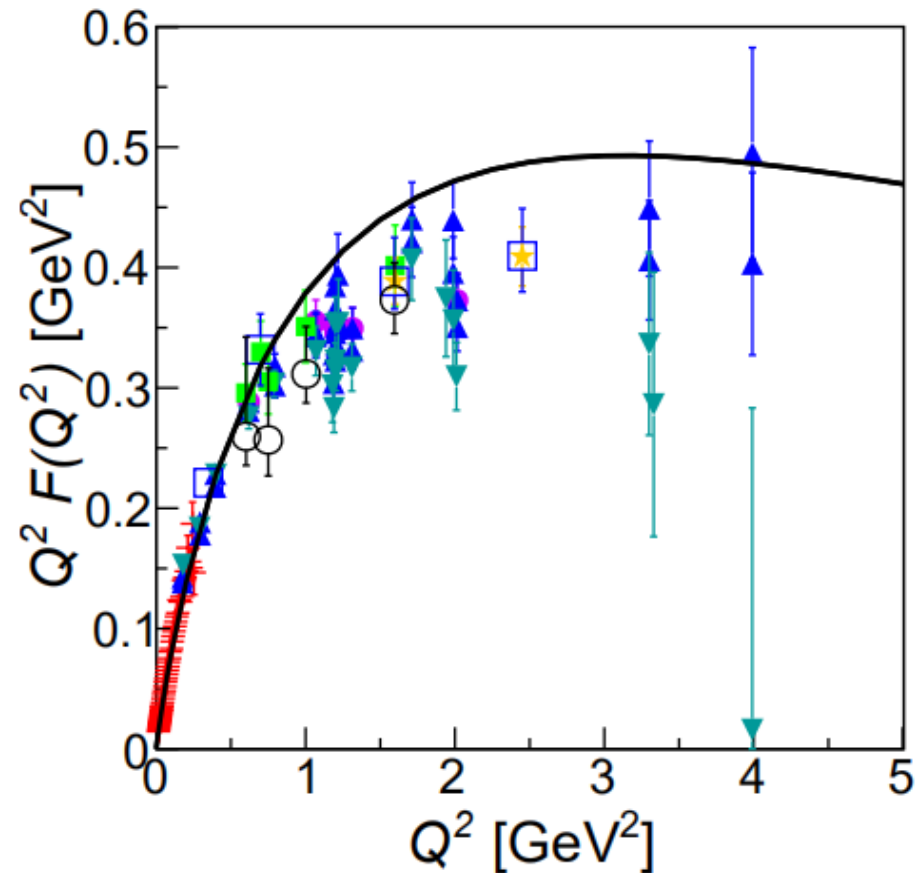
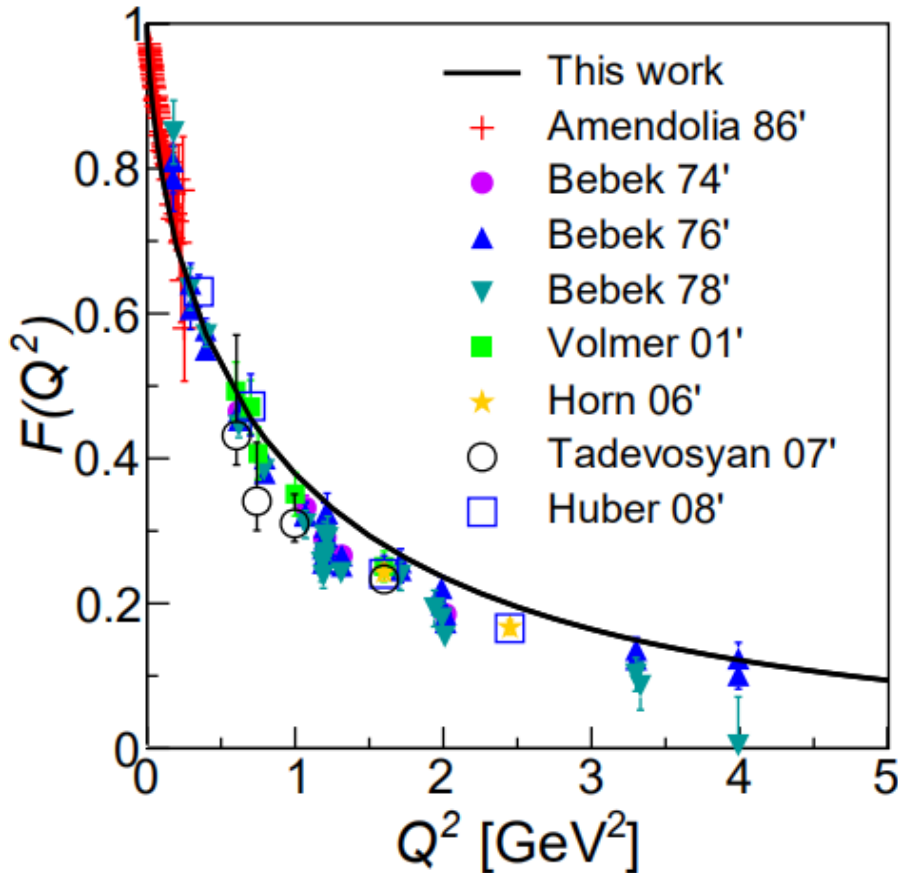
- At endpoint x , $\psi \sim p_{\perp}$: lightly narrow
- At middle x , $\psi \sim p_{\perp}$: a little bit wide

Pion Electromagnetic Form Factor

[Brodsky & de Teramond, PRD 77:056007 (2008)]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$\langle\Psi(p')|J_{EM}^+(0)|\Psi(p)\rangle = (p+p')^+ F(Q^2)$$

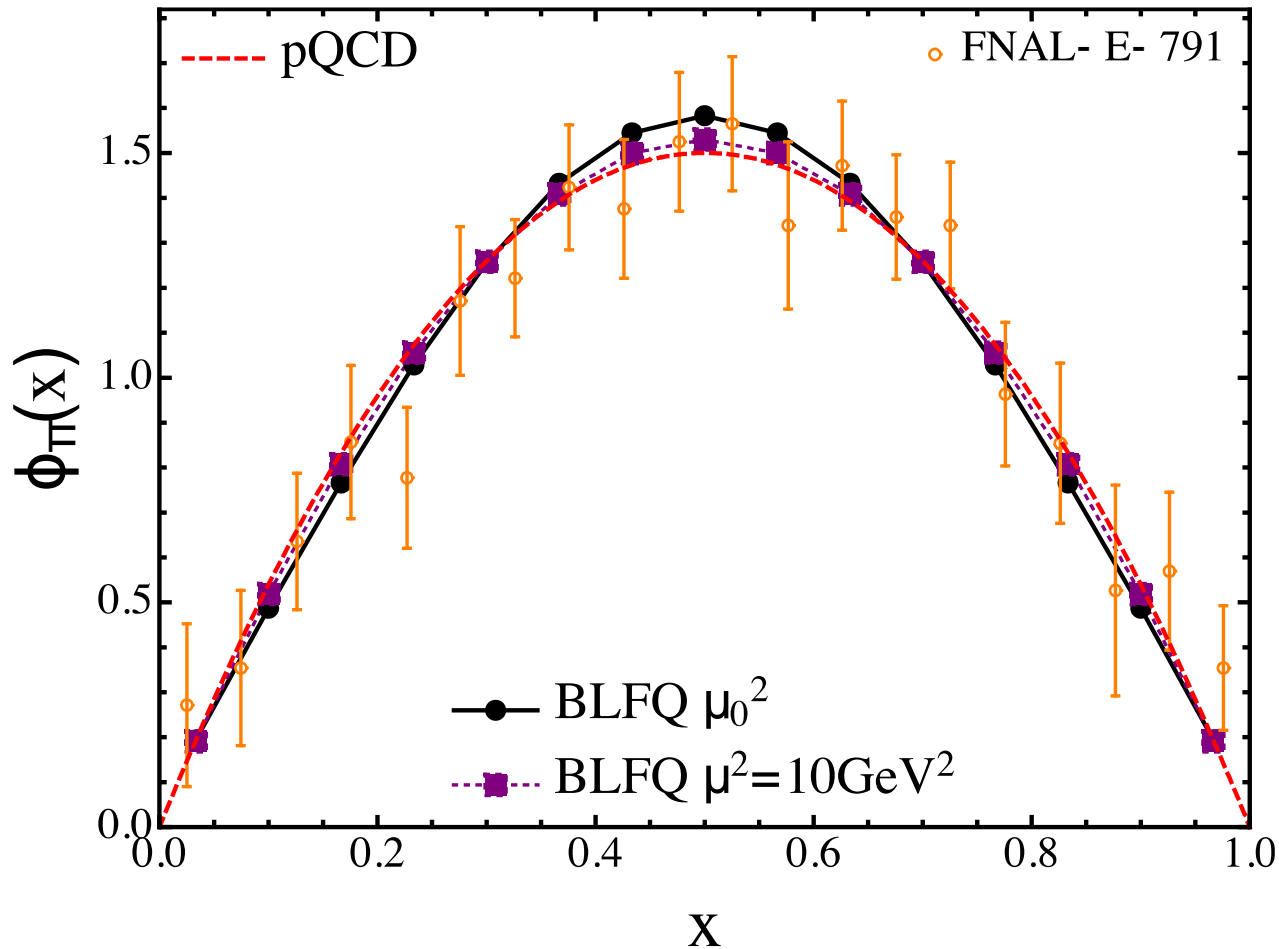


- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/Q^2$ for large Q^2

Pion PDA

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

[Ruiz Arriola & Broniowski, PRD 66:094016 (2002)]

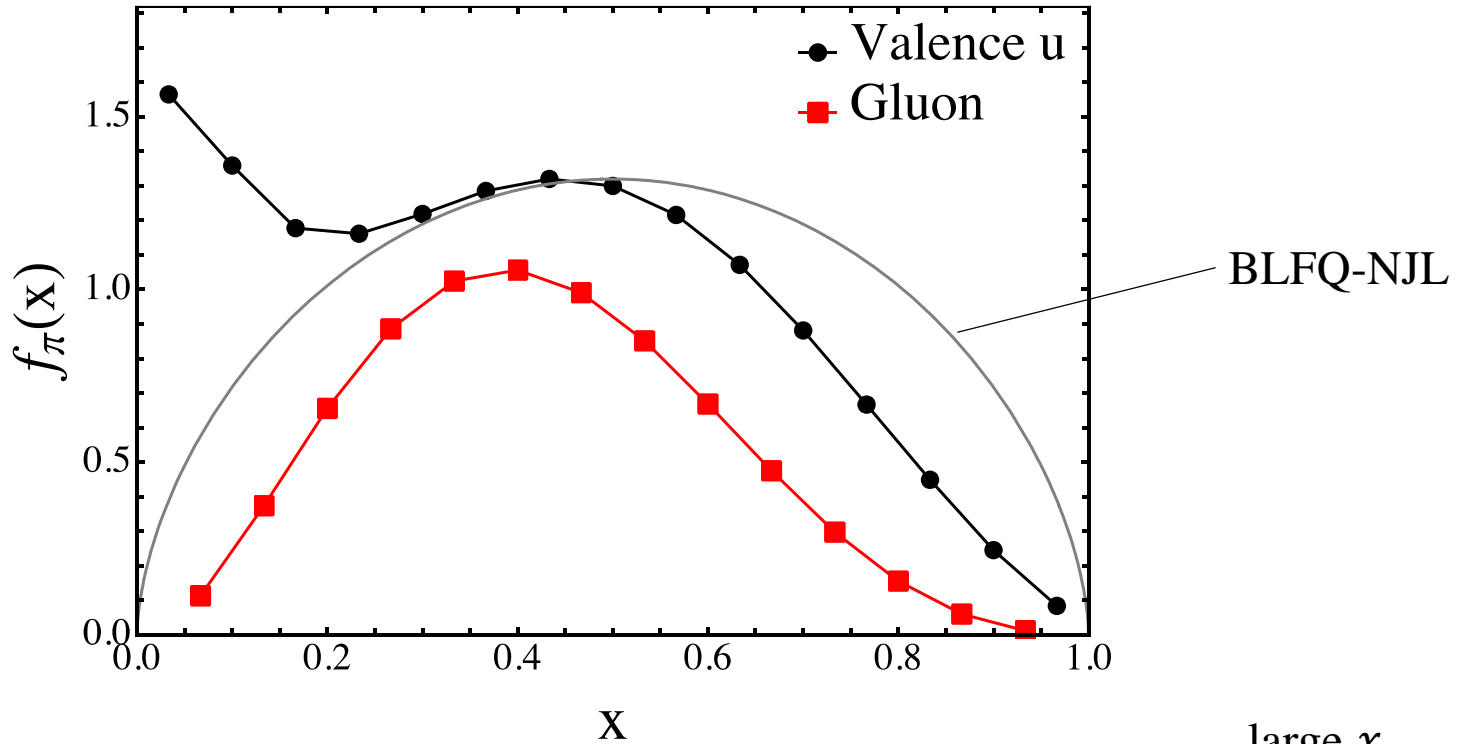


- Endpoint behavior almost agrees with pQCD
- Consistent with FNAL-E-791 experiment

Preliminary

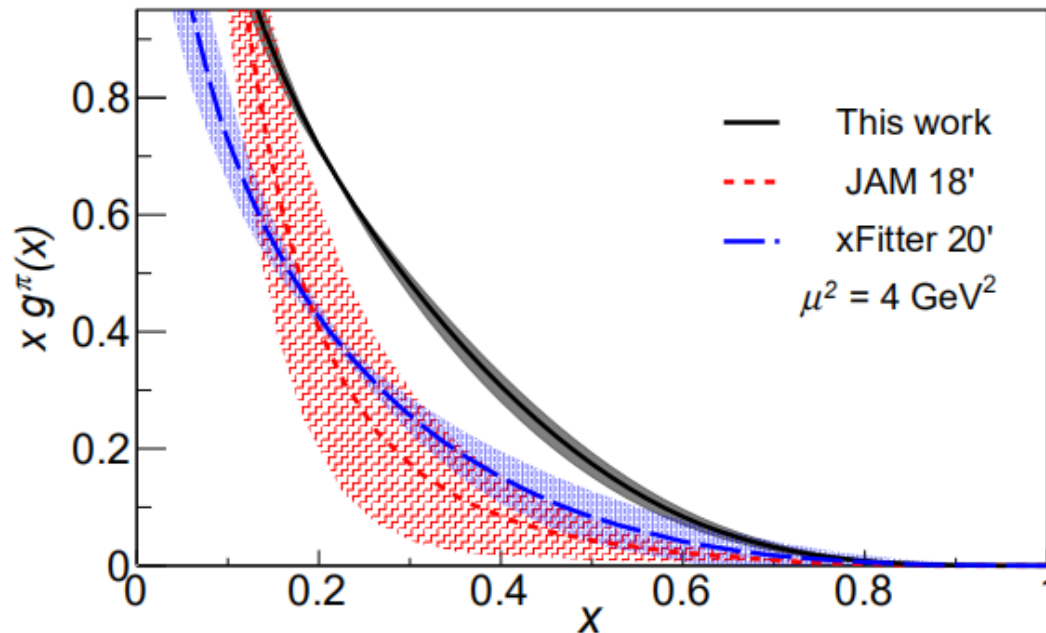
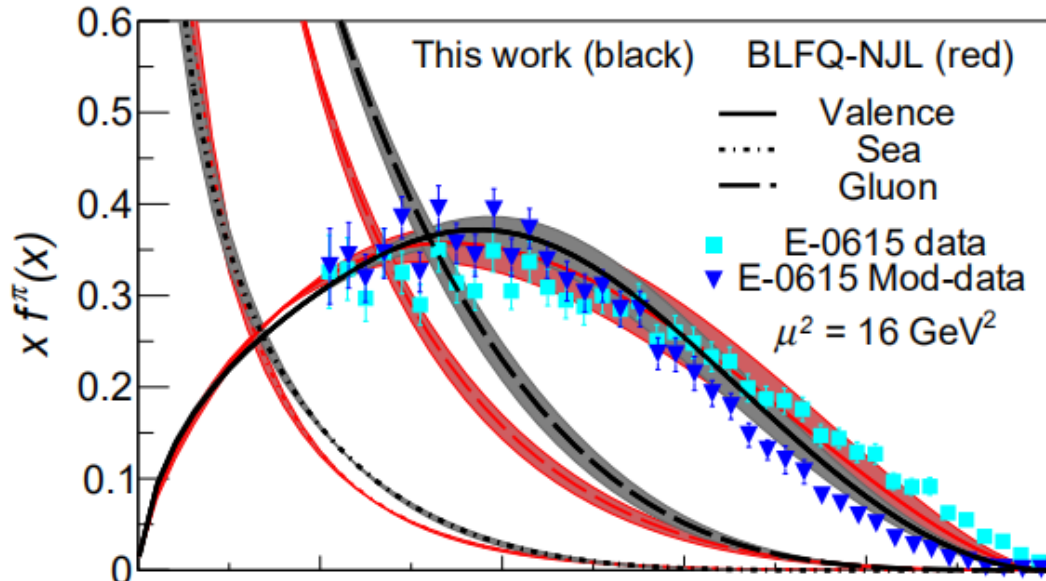
Pion PDF at Model Scale

$$f_i(x) = \sum_{\mathcal{N}, \lambda_i} \int [d\mathcal{X} d\mathcal{P}^\perp]_{\mathcal{N}} \left| \psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_j=0} \right|^2 \delta(x - x_i) \quad |\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



$\mu_{0\text{BLFQ-NJL}}^2 = 0.240 \text{ GeV}^2$	$\langle x \rangle_{\text{gluon}} = 0$;	$\langle x \rangle_{\text{valence } u} = 0.5$	$(1-x)^{0.596}$
$\mu_{0\text{BLFQ}}^2 = 0.34 \text{ GeV}^2$	$\langle x \rangle_{\text{gluon}} = 0.216$;	$\langle x \rangle_{\text{valence } u} = 0.392$	$(1-x)^{1.4}$

Pion PDF with QCD Evolution



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

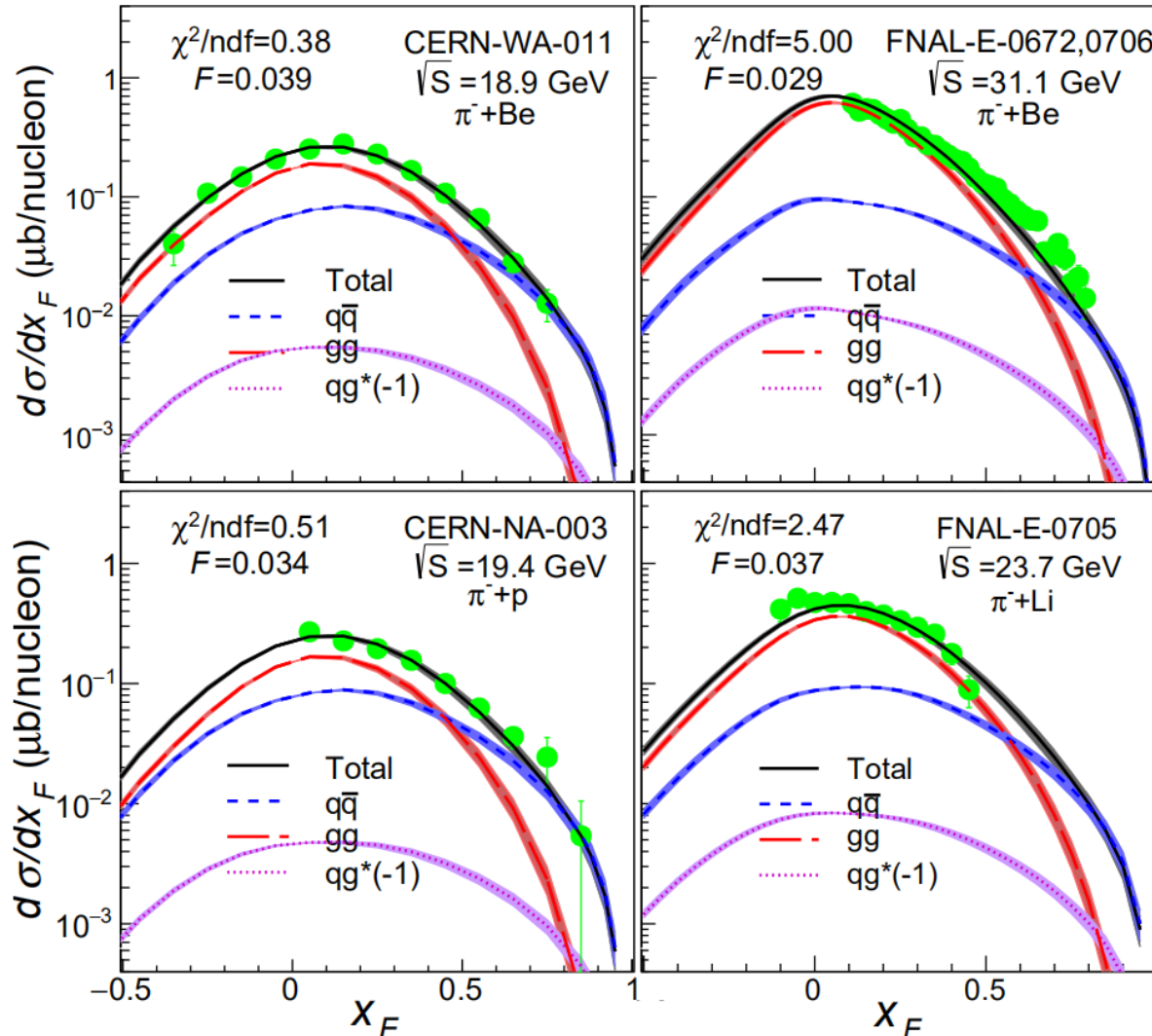
- Large- x behavior $(1 - x)^{1.77}$ closer to pQCD
- The gluon distribution significantly increases

$\langle x \rangle @ 4 \text{ GeV}^2$	Valence	Gluon	Sea
BLFQ	0.483	0.421	0.096
BLFQ-NJL	0.489	0.398	0.113
[BSE 2019']	0.48(3)	0.41(2)	0.11(2)

J/ψ production cross section $\pi^\pm N \rightarrow J/\psi X$

$$\frac{d\sigma}{dx_F} |J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S \sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^\pm}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

[Chang, et al, PRD 102 (2020) 054024];
 [Nason, et al, NPB 303 (1988) 607];
 [Mangano, et al, NPB 405 (1993) 507]

- significantly gg contribution
- various energies of pions
- different target

Agree with experimental data (FNAL E672, E706, E705, CERN NA3, WA11).

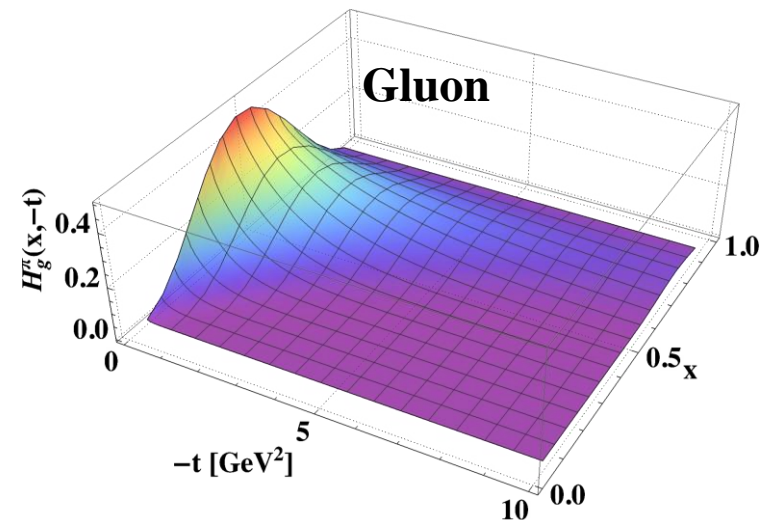
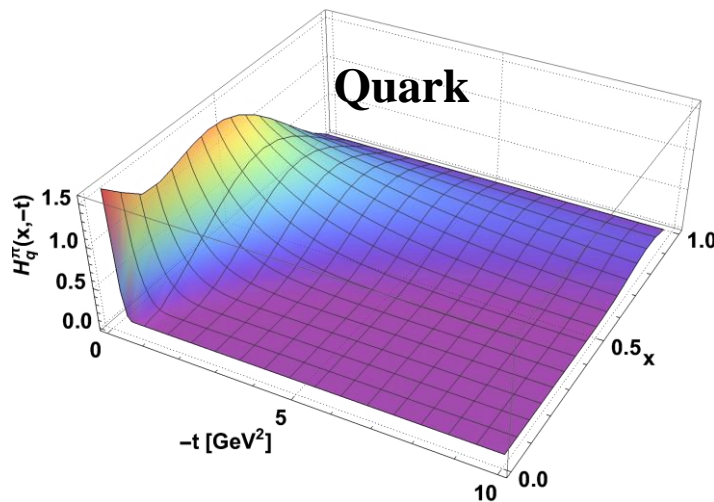
Pion GPD

[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$H_{\pi}^q(x, \xi = 0, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q} \left(-\frac{Z}{2} \right) \gamma^+ q \left(\frac{Z}{2} \right) \right| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_{\perp}=0}}$$

$$H_{\pi}^g(x, \xi = 0, t) = \frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \left| G^{+\mu} \left(-\frac{Z}{2} \right) G_{\mu}^+ \left(\frac{Z}{2} \right) \right| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_{\perp}=0}}$$



- Quark content enhanced at small x with $|q\bar{q}g\rangle$
- Falls slowly at larger x
- Emerge at larger x range for larger $-t$

Pion GPD

[M. Diehl, Phys. Rep. 388 (2003) 41-277]

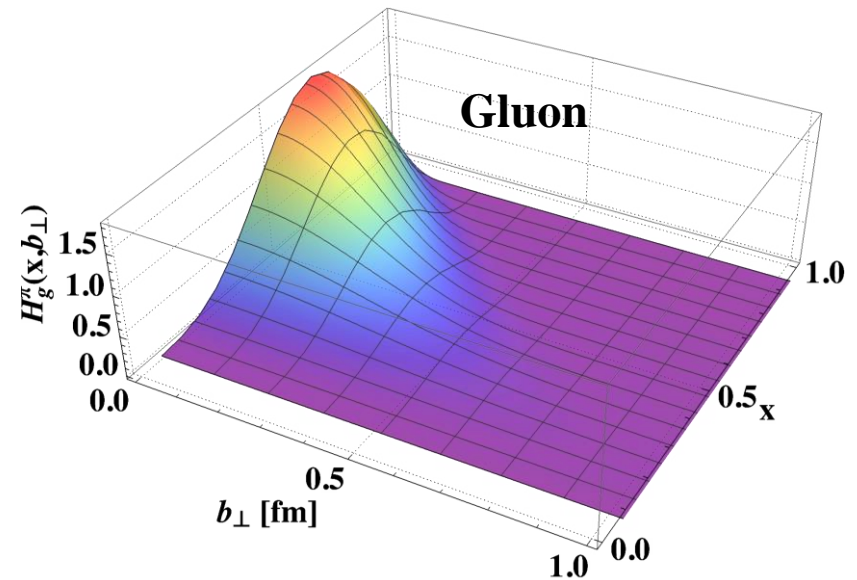
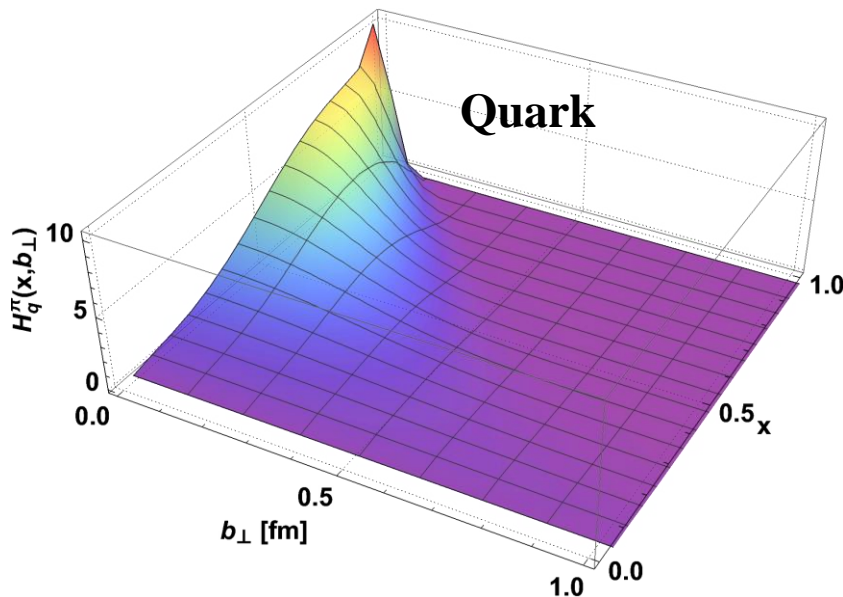
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$H_{\pi}^q(x, \xi = 0, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q} \left(-\frac{Z}{2} \right) \gamma^+ q \left(\frac{Z}{2} \right) \right| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_{\perp}=0}}$$

$$H_{\pi}^g(x, \xi = 0, t) = \frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \left| G^{+\mu} \left(-\frac{Z}{2} \right) G_{\mu}^+ \left(\frac{Z}{2} \right) \right| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_{\perp}=0}}$$



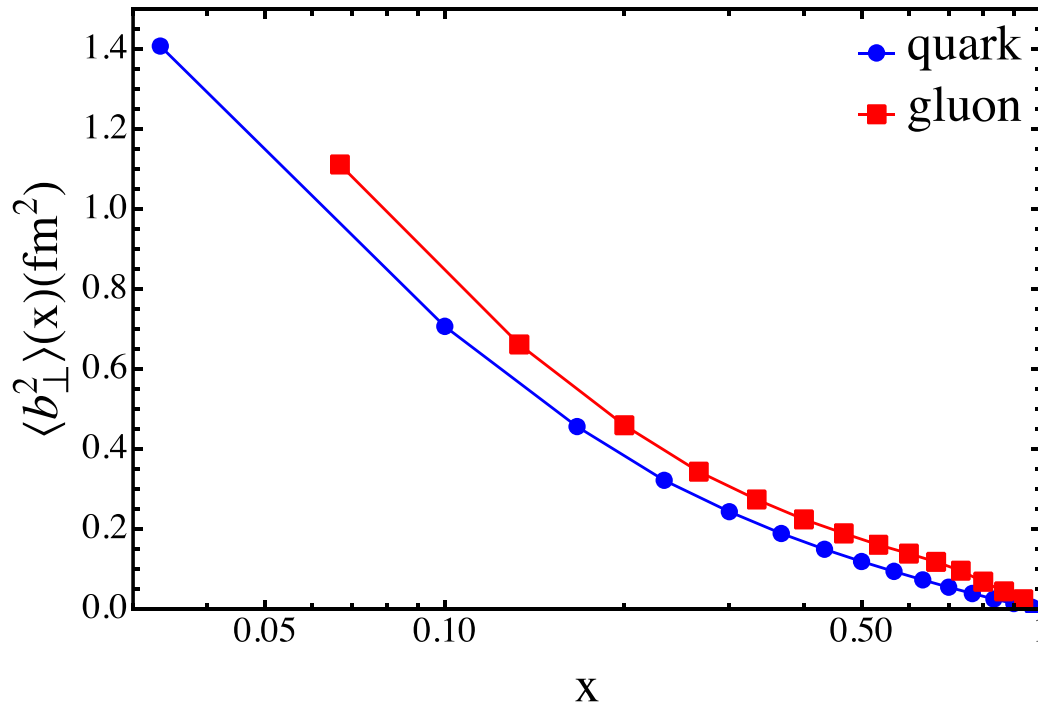
The impact parameter distributions (IPDs)



Pion Impact Parameter Distribution

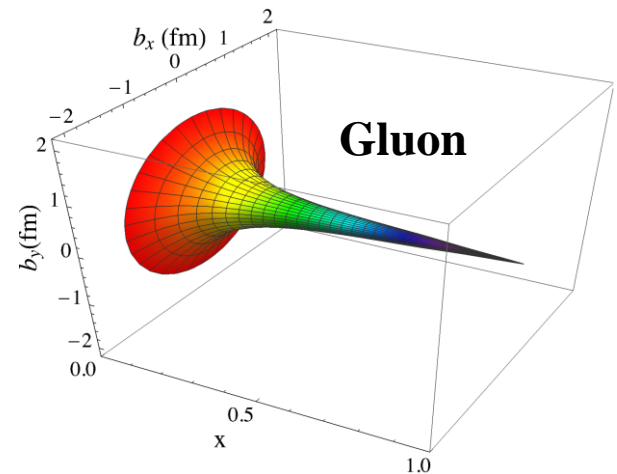
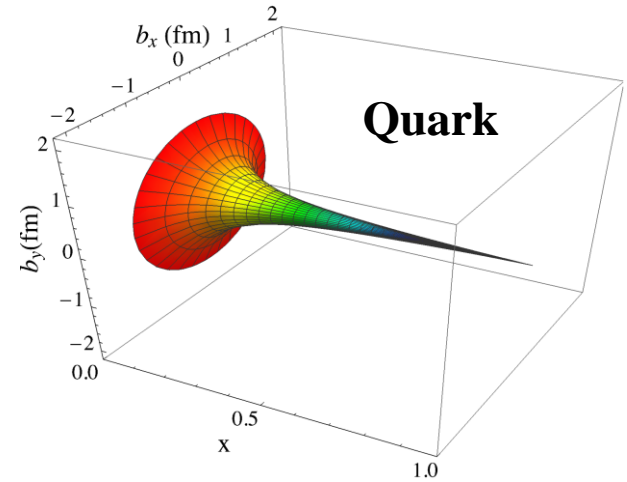
One can define the x -dependent squared radius of the quark and the gluon density in the transverse plane as

$$\langle b_{\perp}^2 \rangle^{q,g}(x) = \frac{\int d^2 \mathbf{b}_{\perp} \mathbf{b}_{\perp}^2 H_{\pi}^{q,g}(x, \mathbf{b}_{\perp})}{\int d^2 \mathbf{b}_{\perp} H_{\pi}^{q,g}(x, \mathbf{b}_{\perp})}$$



➤ Gluon distribution is slightly broader than the quark

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



Preliminary

Pion TMD

Preliminary

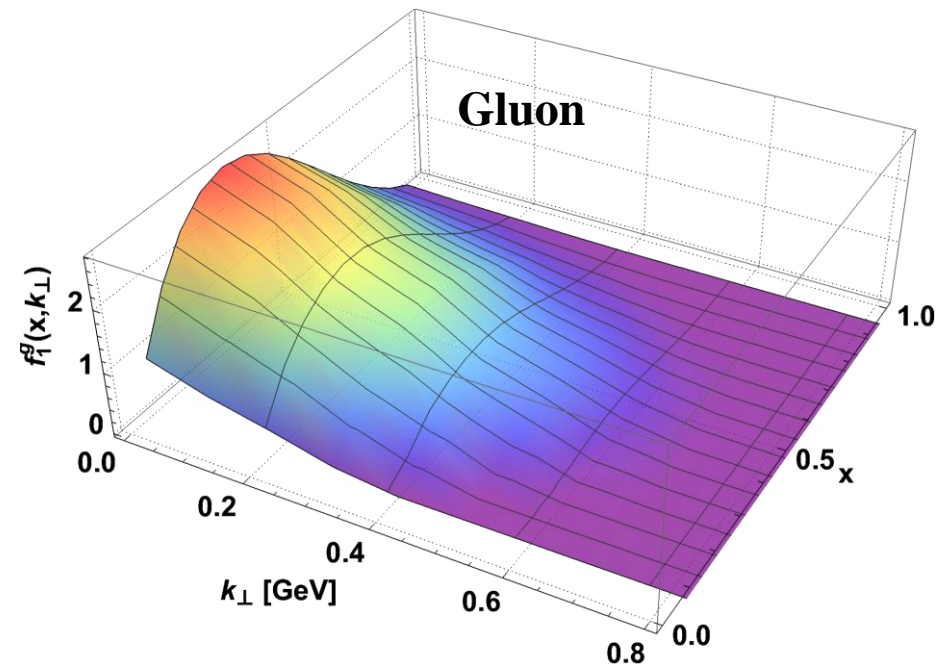
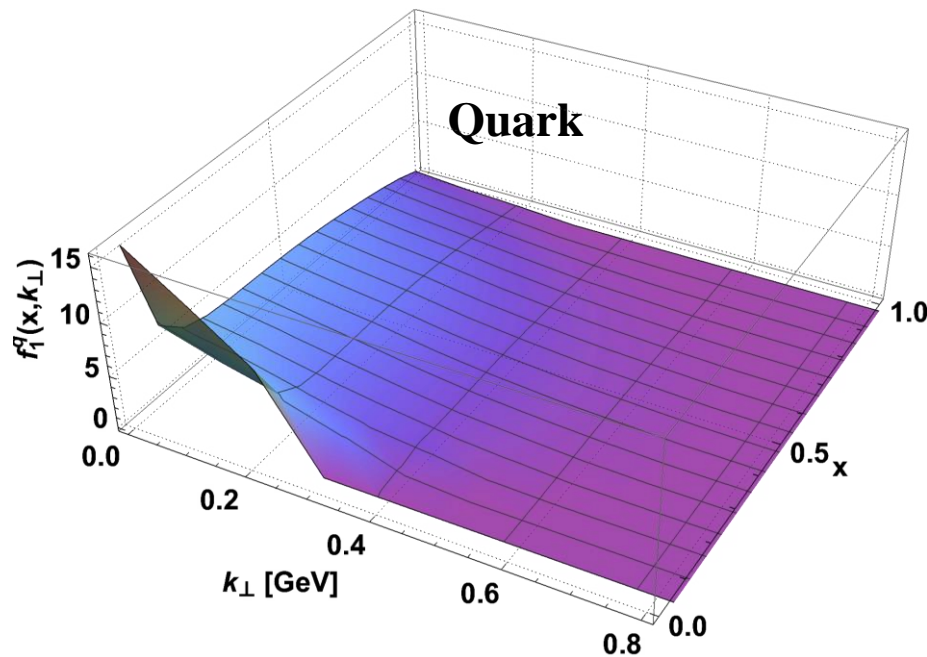
[Boer & Mulders PRD 57 (1998) 5780]

[Pasquini et al, PRD 90 (2014) 014050]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$f_1^q(x, k_\perp) = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(z^- k^+ - z_\perp k_\perp)} \left\langle \pi, P \left| \bar{q} \left(-\frac{Z}{2} \right) \gamma^+ q \left(\frac{Z}{2} \right) \right| \pi, P \right\rangle_{z^+=0}$$

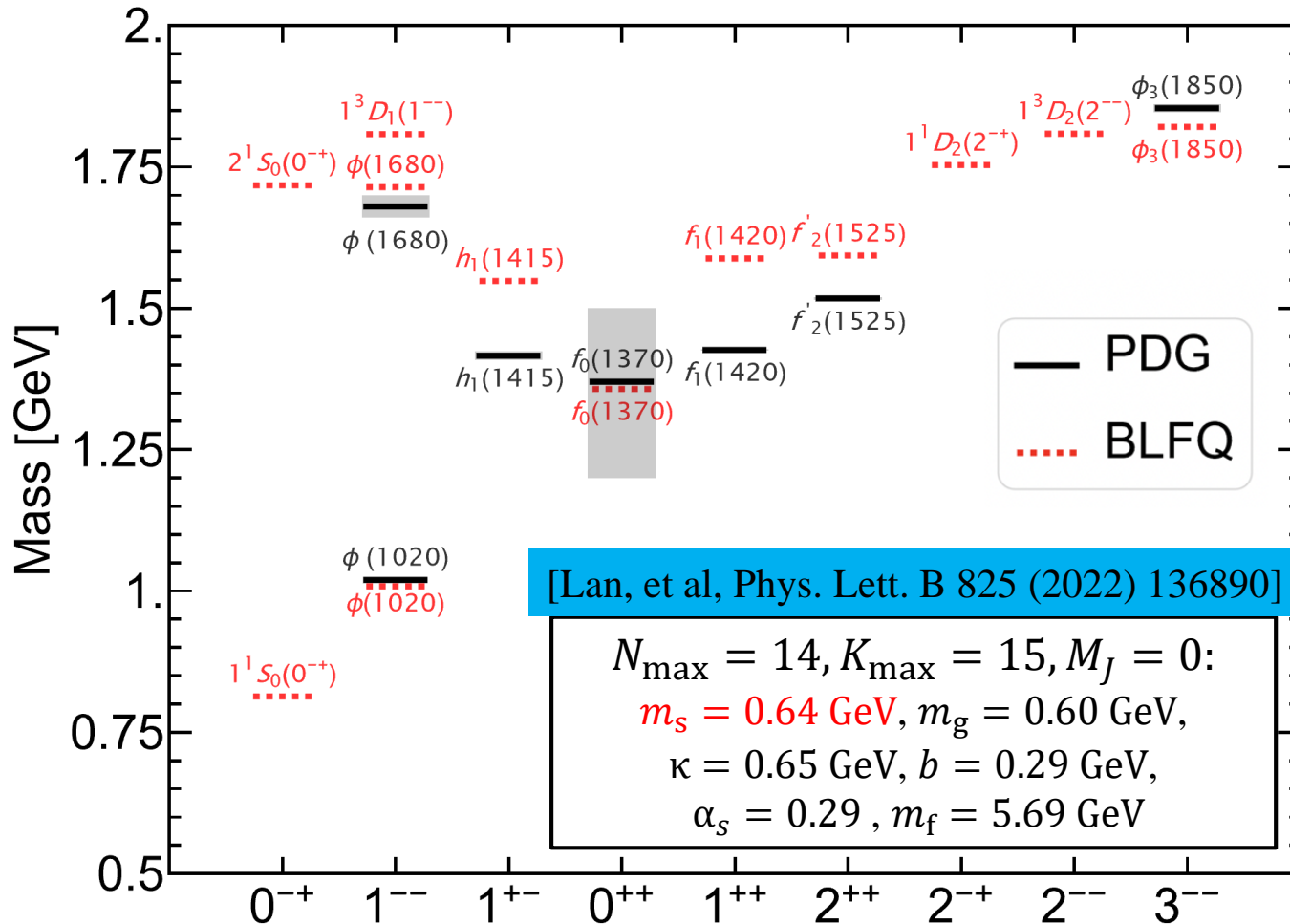
$$f_1^g(x, k_\perp) = \frac{1}{xP^+} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(z^- k^+ - z_\perp k_\perp)} \left\langle \pi, P \left| G^{+\mu} \left(-\frac{Z}{2} \right) G_\mu^+ \left(\frac{Z}{2} \right) \right| \pi, P \right\rangle_{z^+=0}$$



- The TMD decreases with k_\perp
- Vanishes after $k_\perp \sim 0.7$ GeV

[Jiangshan Lan, et al, in preparation]

Strangeonium Mass Spectrum

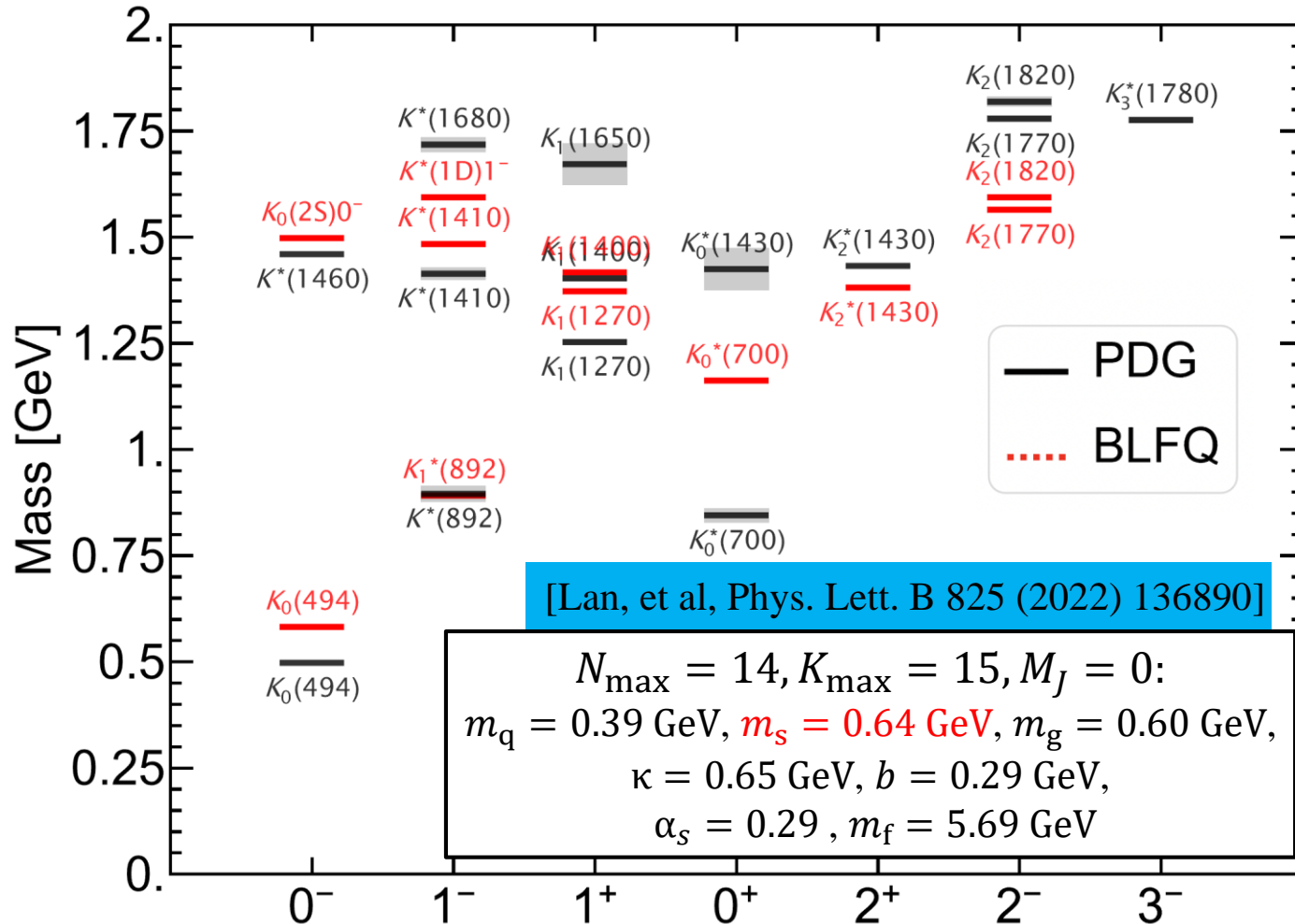


$$|\text{meson}\rangle = a|s\bar{s}\rangle + b|s\bar{s}g\rangle + \dots$$

Fix the only additional parameter m_s by fitting vector states

$$\phi(1020), \phi(1680), \phi_3(1850)$$

Strange Meson Mass Spectrum



$$|\text{meson}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

➤ Zero parameter

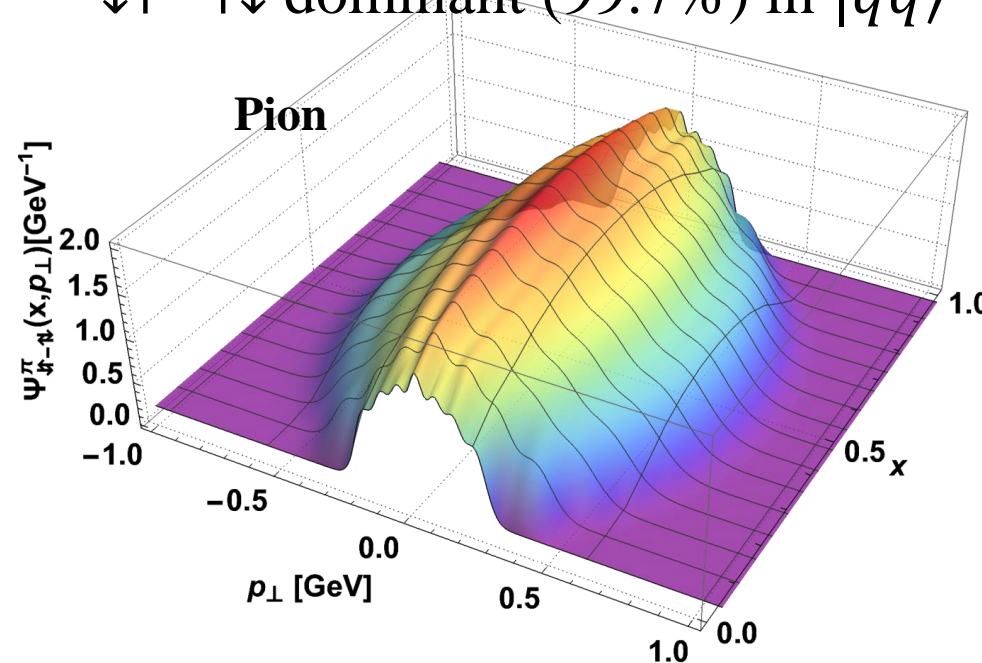
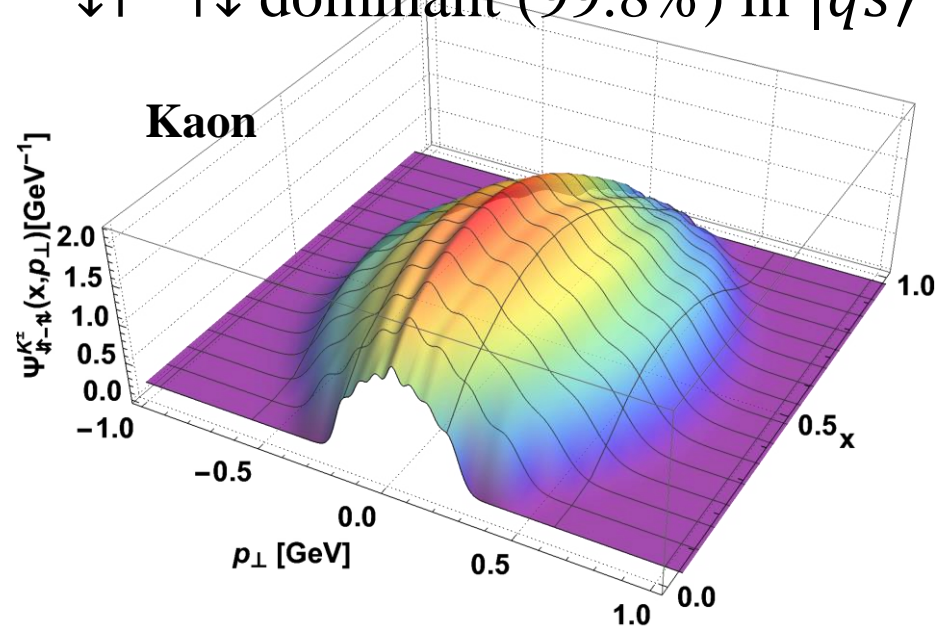
The Wave Function in Leading Fock Sector

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

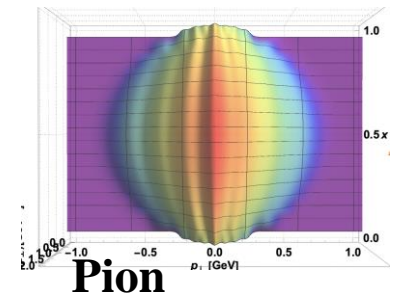
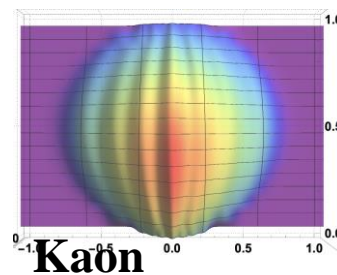
$\downarrow\uparrow - \uparrow\downarrow$ dominant (99.8%) in $|q\bar{s}\rangle$

$\downarrow\uparrow - \uparrow\downarrow$ dominant (99.7%) in $|q\bar{q}\rangle$



Compared with pion

- Narrower in longitudinal direction
- Asymmetric in longitudinal direction
- Comparable in transverse direction

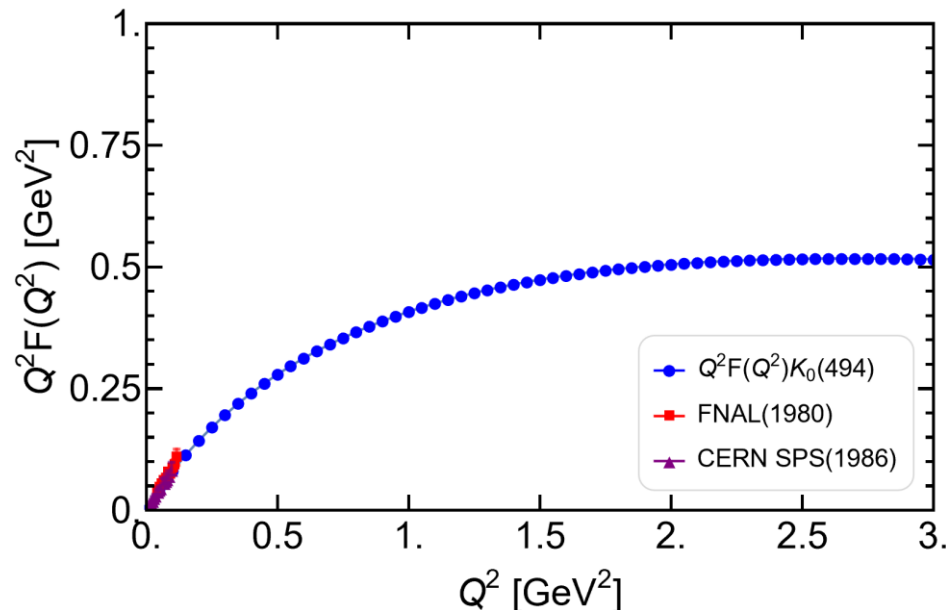
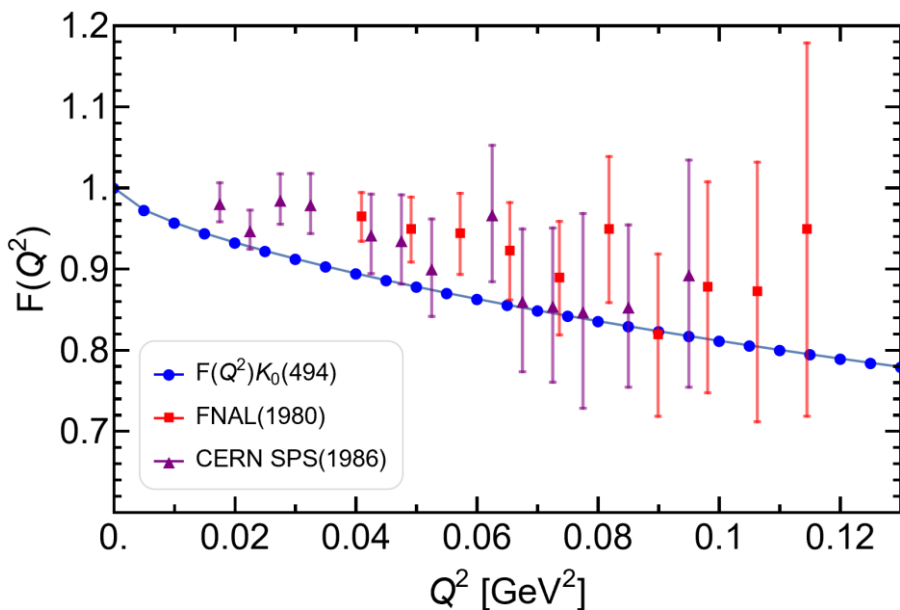


Kaon Electromagnetic Form Factor

[Brodsky & de Teramond, PRD 77:056007 (2008)]

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

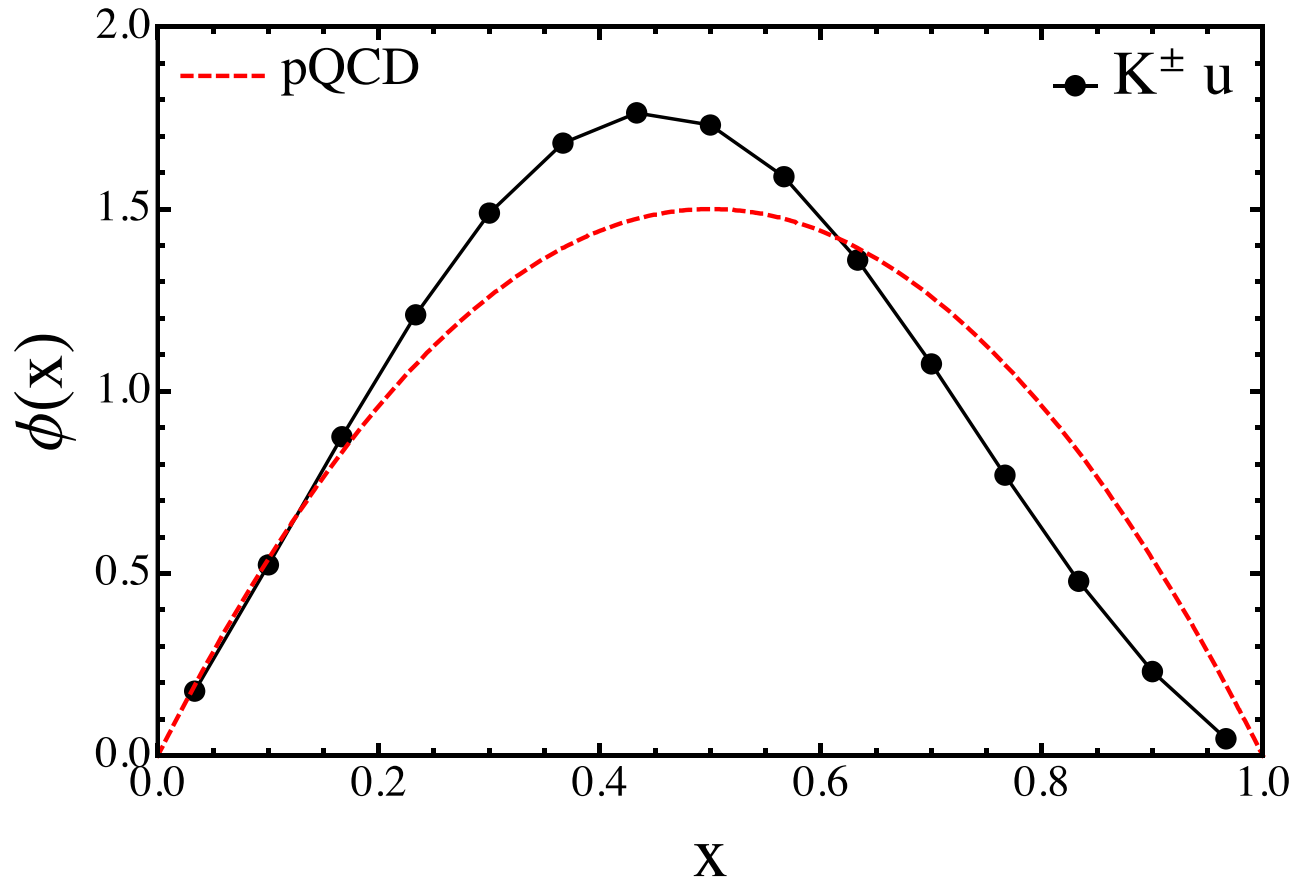
$$\langle\Psi(p')|J_{EM}^+(0)|\Psi(p)\rangle = (p + p')^+ F(Q^2)$$



- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/Q^2$ for large Q^2

Kaon PDA

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

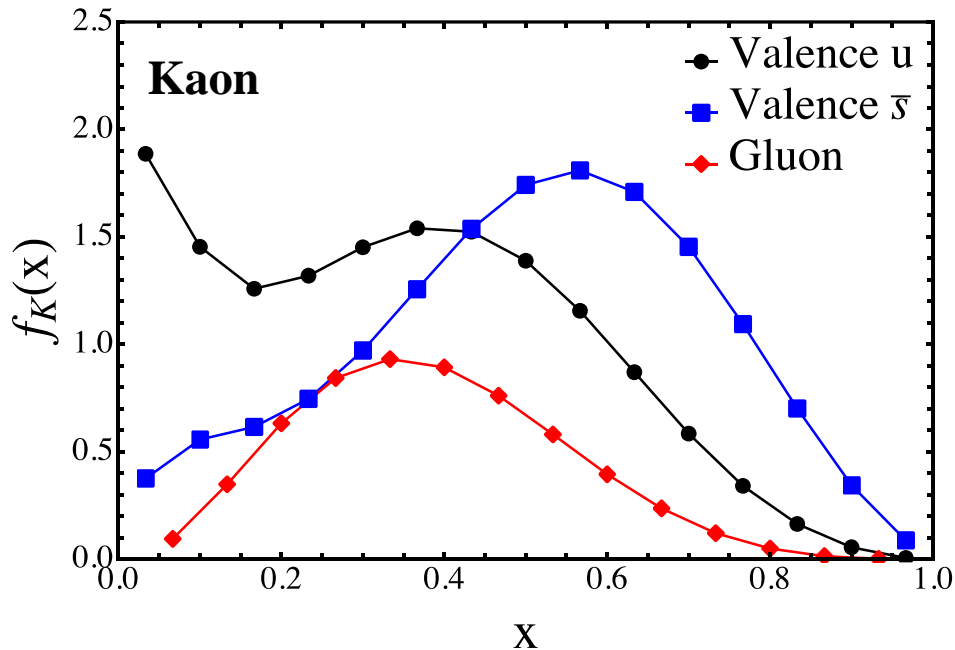


- Endpoint behavior at small x almost agrees with pQCD

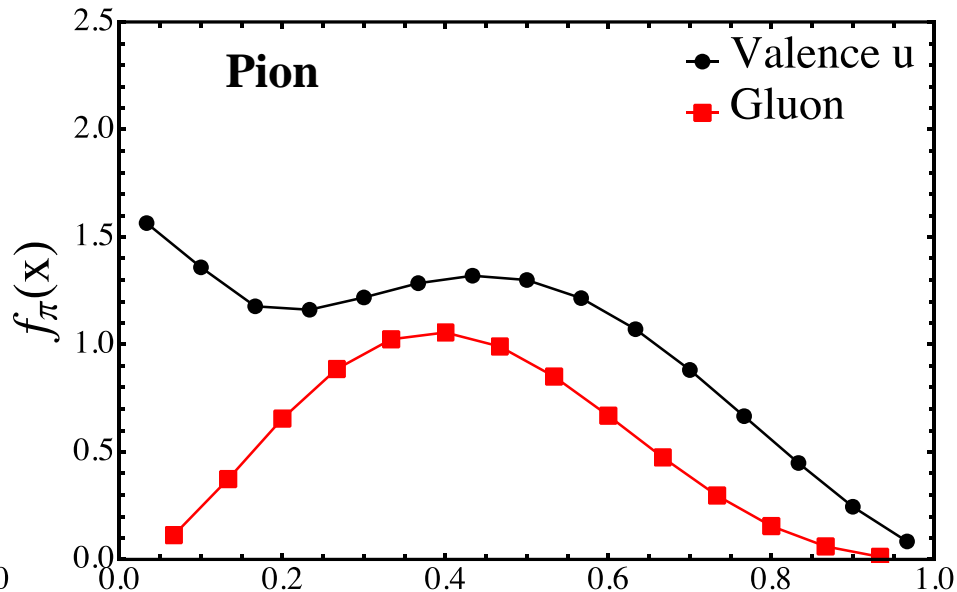
Preliminary

Kaon PDF at Model Scale

$$f_i(x) = \sum_{\mathcal{N}, \lambda_i} \int [d\mathcal{X}d\mathcal{P}^\perp]_{\mathcal{N}} \left| \psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_j=0} \right|^2 \delta(x - x_i) \quad |K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$



$$\mu_{\text{0BLFQ } kaon}^2 = 0.50 \text{ GeV}^2$$



$$\mu_{\text{0BLFQ } pion}^2 = 0.34 \text{ GeV}^2$$

$$\langle x \rangle_{\text{gluon}} = 0.152; \quad \langle x \rangle_{\text{valence } u} = 0.343; \quad \langle x \rangle_{\text{valence } \bar{s}} = 0.505$$

- Smaller gluon content in kaon
- At small x , u dominates over \bar{s}

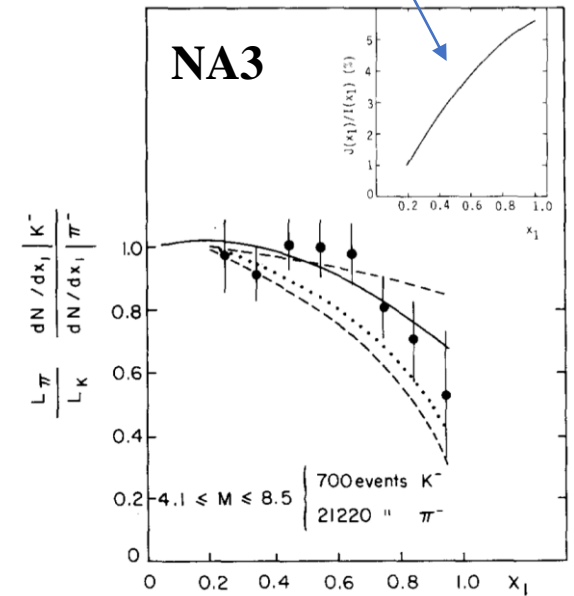
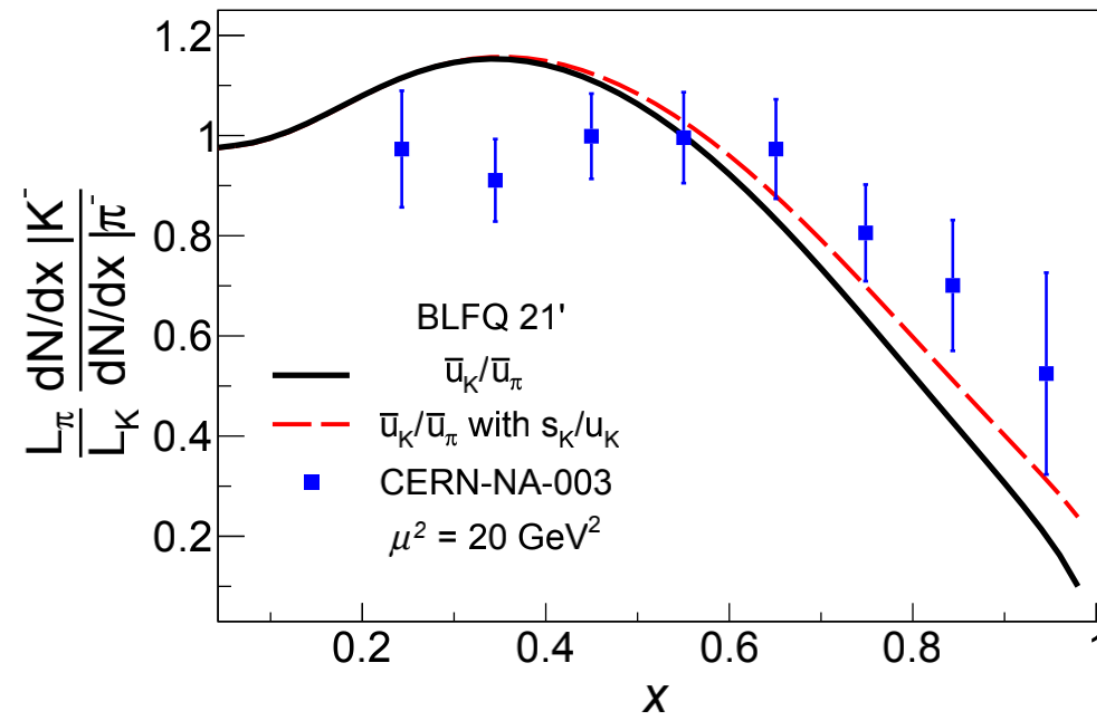
Kaon PDF with QCD Evolution

[Badier et al. PLB 1980 (NA3)];

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

$$\begin{aligned} \frac{L_\pi}{L_K} \frac{dN/dx|K^-}{dN/dx|\pi^-} &= \frac{\bar{u}_K(x)I(x) + s_K(x)J(x)}{\bar{u}_\pi(x)I(x) + d_\pi(x)J(x)} = \frac{\bar{u}_K(x)I(x)(1 + 0.055x \frac{s_K(x)}{\bar{u}_K(x)})}{\bar{u}_\pi(x)I(x)(1 + 0.055x)} \\ &= \frac{\bar{u}_K(x)}{\bar{u}_\pi(x)} \times \frac{(1 + 0.055x \frac{s_K(x)}{\bar{u}_K(x)})}{(1 + 0.055x)} = R(x)C(x), \end{aligned}$$

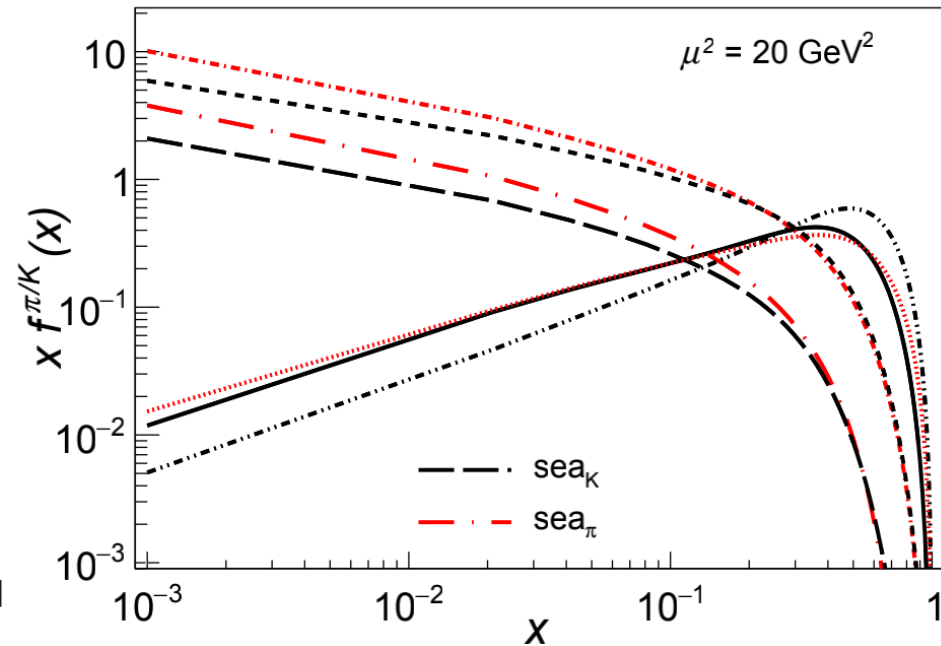
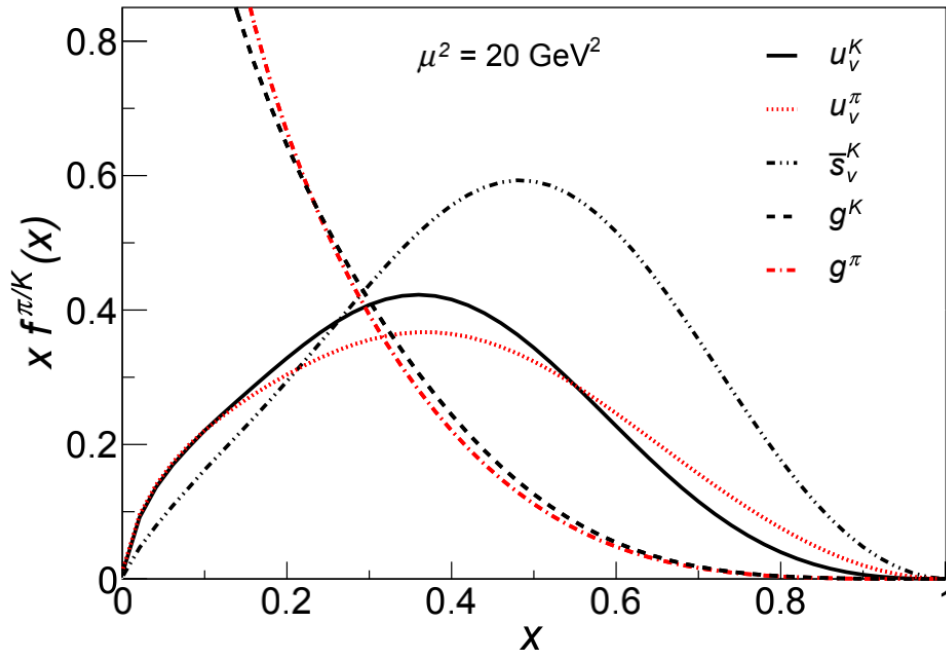
$$J(x)/I(x) \sim 0.055x$$



- Roughly touch the result from NA3

Kaon PDFs VS Pion PDFs

At $\mu^2 = 20 \text{ GeV}^2$



Kaon

$$\langle x \rangle_{\text{gluon}} = 0.381; \quad \langle x \rangle_{\text{valence } u} = 0.214; \quad \langle x \rangle_{\text{valence } \bar{s}} = 0.316; \quad \langle x \rangle_{\text{sea}} = 0.089$$

Pion

$$\langle x \rangle_{\text{gluon}} = 0.449; \quad \langle x \rangle_{\text{valence } u} = 0.211; \quad \langle x \rangle_{\text{sea}} = 0.129$$

- The gluon in the kaon carries less longitudinal moment is slightly less than the gluon in the pion

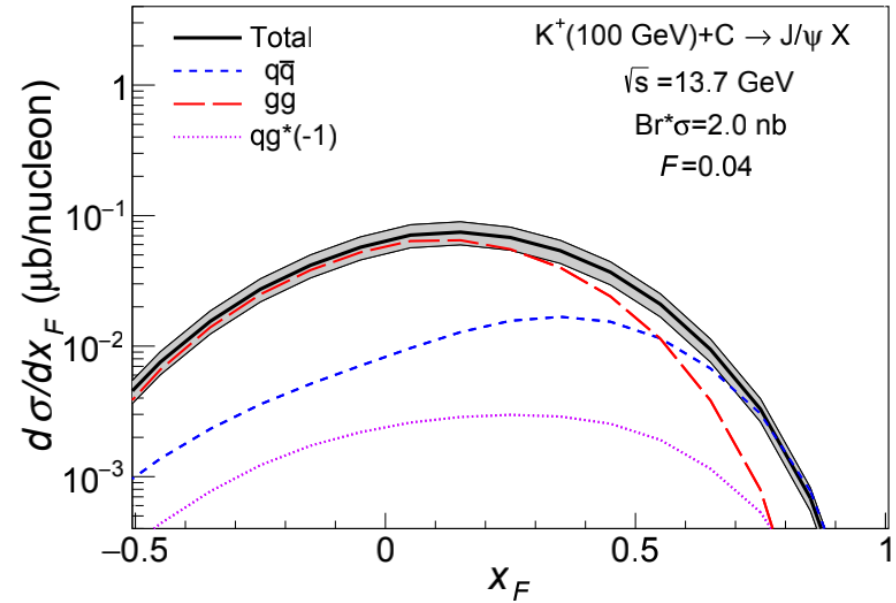
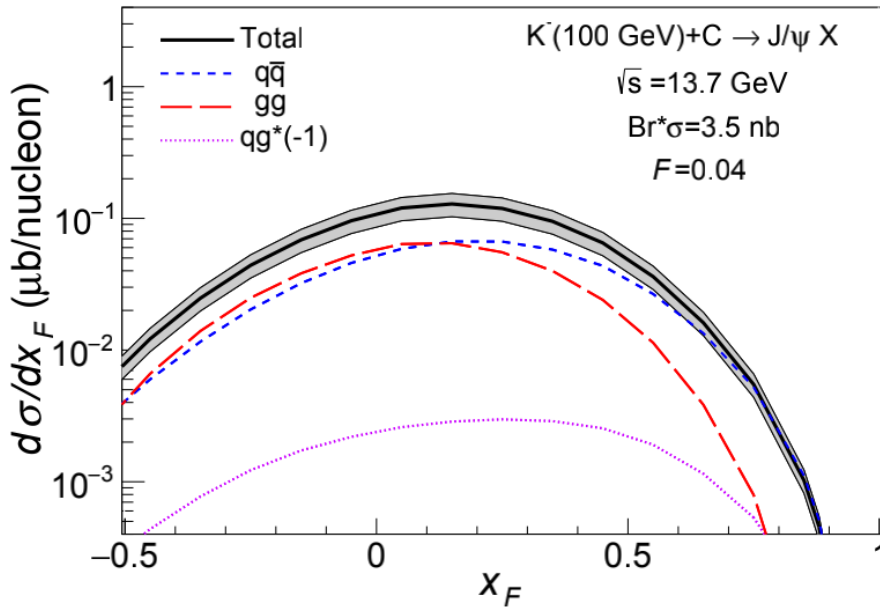
J/ψ production cross section

$$K^\pm N \rightarrow J/\psi X$$

[Chang, et al, PRD 102 (2020) 054024]; [Nason, et al, NPB 303 (1988) 607]; [Mangano, et al, NPB 405 (1993) 507]

$$\frac{d\sigma}{dx_F} |J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S \sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^\pm}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

CEM [nCTEQ 2015]



assuming the cross section of $K^- + C$ is 3.7 nb, then the cross section of 2.0 nb

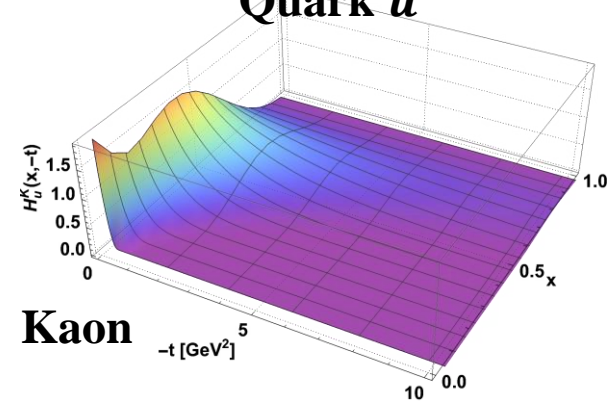
➤ $q\bar{q}$ contribution different between $K^- + C$ and $K^+ + C$

Kaon GPD

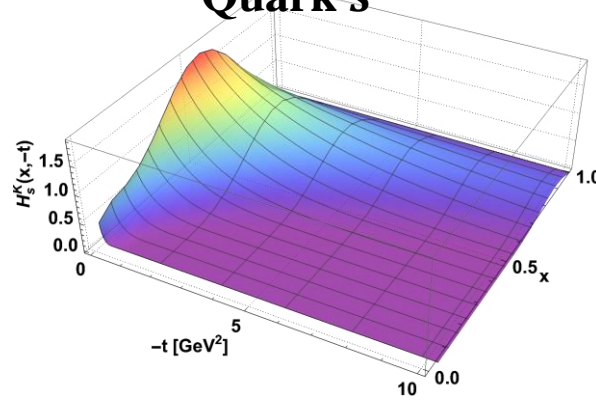
Preliminary

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

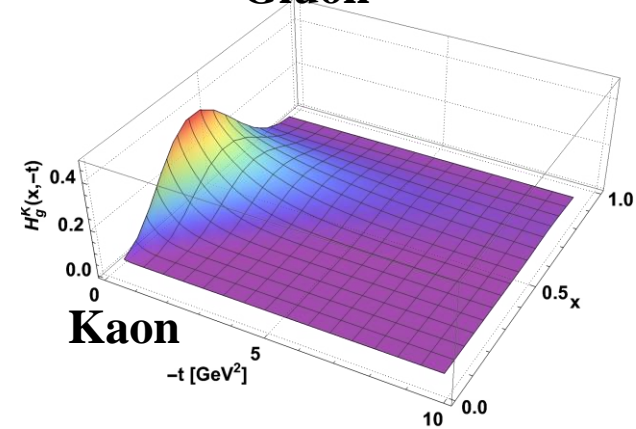
Quark u



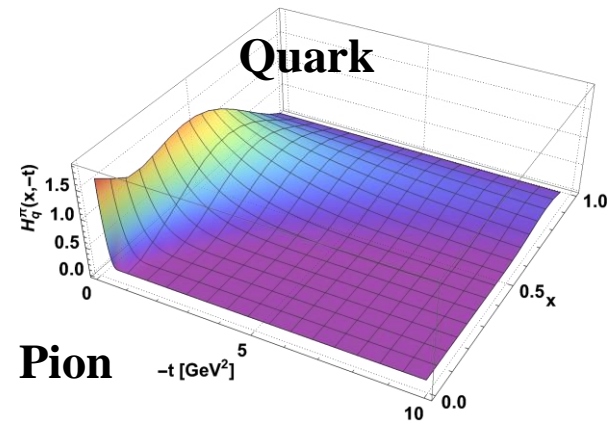
Quark \bar{s}



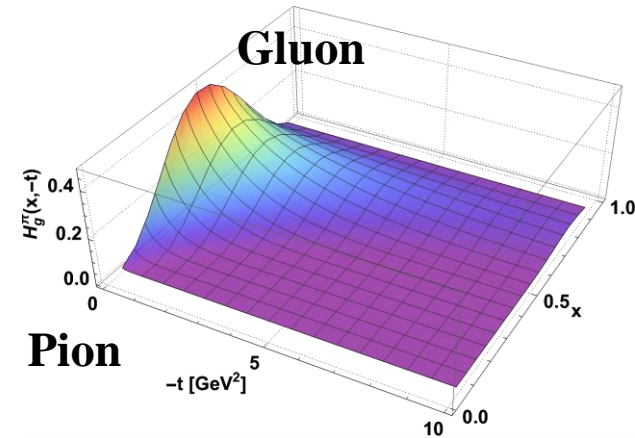
Glueon



Quark



Glueon



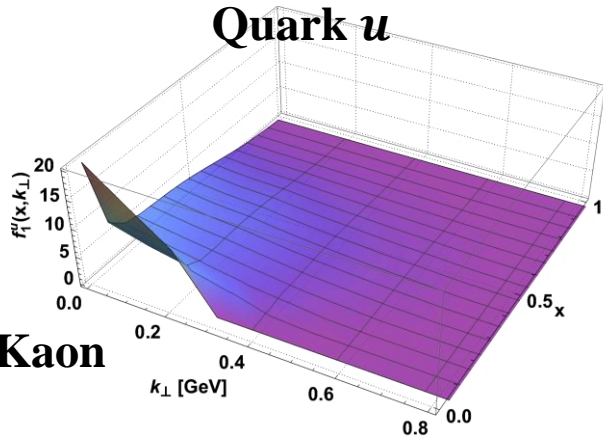
- Quark u content enhanced at small x with $|q\bar{s}g\rangle$
- Falls slowly at larger x
- Emerge at larger x range for larger $-t$

Kaon TMD

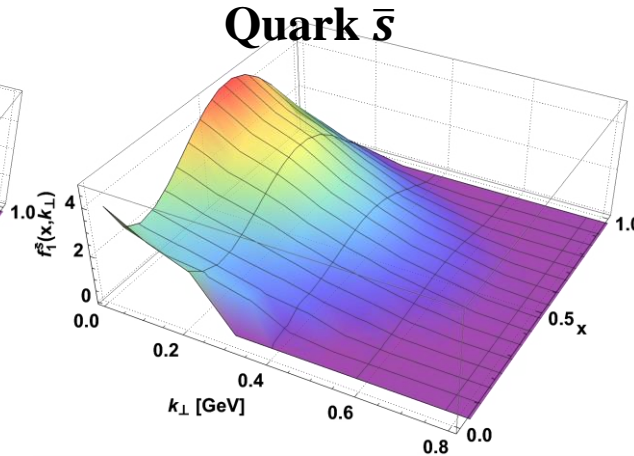
Preliminary

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

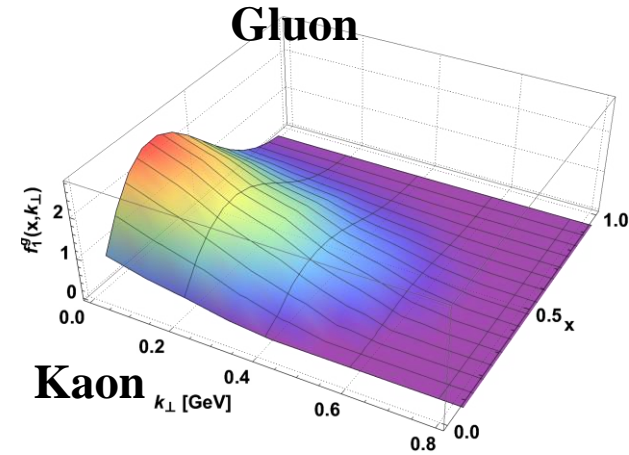
Quark u



Quark \bar{s}

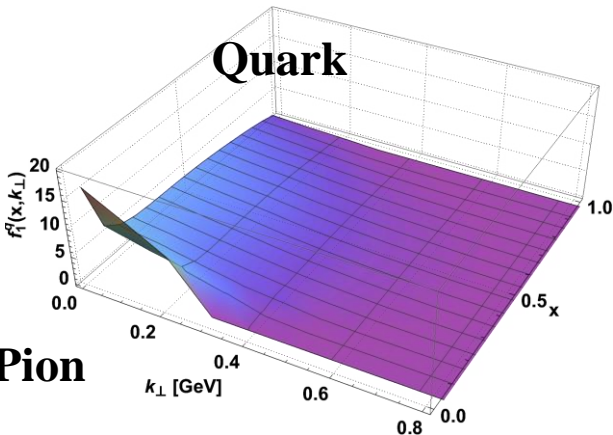


Gluon

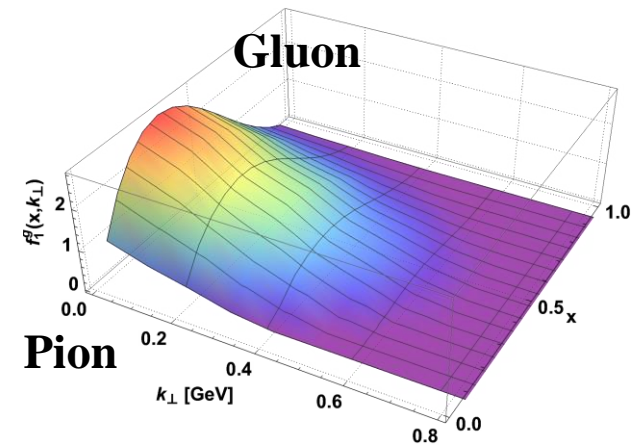


Kaon

Quark



Gluon



Pion

- The TMD decreases with k_{\perp}
- Quark u and \bar{s} contents enhanced at small x with $|q\bar{s}g\rangle$
- A peak for \bar{s} quark

Conclusion & Outlook

- Light-front Hamiltonian approach: **Mass Spectrum** \leftrightarrow **Structure**
 - Compared to NJL interaction, dynamical gluon in light meson:
 - ✓ Explains the properties of excited/exotic states such as $\pi(1300)$, $\pi_1(1400)$
 - ✓ Describes EMFF, $F(Q^2) \propto 1/Q^2$ for large Q^2
 - ✓ Improves endpoint behavior in PDF/PDA
 - ✓ Generates more gluon at moderate x /less gluon at small x
 - ✓ Improves agreement on J/ψ production cross section with experimental data
 - Preliminary results on **gluon GPDs** and **TMDs** of light mesons
 - It works on strangeonia and strange mesons
-
- Systematically expandable by including higher Fock sectors

$$|\text{Meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|gg\rangle + e|q\bar{q}gg\rangle + \dots$$

Thank you for your attention!