Proton momentum distributions from light-front dynamics

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Introduction

- In hadron physics, one of the most important remaining challenges is to describe the dynamics and structure of the proton in terms of its basic constituents (quarks and gluons).
- The proton light-front wave function, defined on the null plane $x^+ = t + z = 0$, gives through the parton probability densities access to various observables.
- For example:
  - Electromagnetic form factors
  - The parton distribution function
  - Generalized parton distribution functions
- Additionally, the double parton scattering cross section depends on the double parton distribution function (DPDF) [1]:

$$D(x_1, x_2, \vec{\eta}_\perp) = \sum_{n=3}^{\infty} D_n(x_1, x_2, \vec{q}_\perp) = \sum_{n=3}^{\infty} \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{d^2k_{2\perp}}{(2\pi)^2} \left\{ \prod_{i\neq 1,2} \int \frac{d^2k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\}$$
$$\times \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta\left(\sum_{i=1}^{n} \vec{k}_{i\perp}\right) \Psi_n^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp, x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp, \ldots) \Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, \ldots)\,, \tag{1}$$

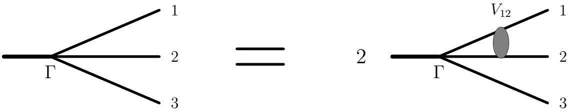
- The first of Mellin moments of DPDF has recently been calculated within lattice QCD [2].

[1] B. Blok et al, PRD 83 (2011) 071501 (R).

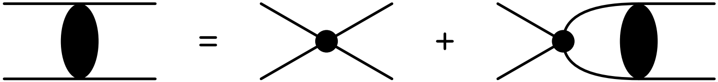
[2] G. S. Bali, JHEP09 (2021) 106.

General goals

- In the long-term perspective, to create a fully dynamical model for the proton in Minkowski space including the infinite number of Fock components in the state vector.
- It will then give direct access to observables defined on the light-front hyperplane.
- In that sense complementary to the BLFQ (talk by Xingbo Zhao) and the quark-diquark model by C. Roberts et al.
- As a first step, Fock basis truncated to valence order and spin degree-of-freedom not included.
- Quark-diquark model were the the quark-quark transition amplitude has a pole representing the s-wave diquark introduced through the zero-range interaction between two of the quarks. In that sense it is an effective low-energy model.
- The proton structure will be explored through the LF wave function and its Ioffe-time representation. Results for the momentum distributions will also be presented.



- Three spinless particles of mass m. Spectator + pair of interacting particles. Factor of two due to symmetry of wave function with respect to exchange of the particles.



- In the present work a zero-range interaction with four-leg-vertex $i\lambda$ used. Then, for the two-body amplitude (see figure)

$$i\mathcal{F}(M_{12}^2)) = i\lambda + (i\lambda)^2\mathcal{B} + (i\lambda)^3\mathcal{B}^2 + ... = \frac{1}{(i\lambda)^{-1} - \mathcal{B}(M_{12}^2)} \tag{2}$$

with

$$\mathcal{B}(M_{12}^2) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k^2 - m^2 + i\epsilon)} \frac{i}{[(k-P)^2 - m^2 + i\epsilon]} \tag{3}$$

- Regularized and renormalized by fixing a diquark pole in the scattering amplitude.

- Faddeev-Bethe-Salpeter (FBS) equation with zero interaction [1]:

$$v(q,p) = 2\mathrm{i}\mathcal{F}(M_{12}^2) \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon} \frac{\mathrm{i}}{(p-q-k)^2 - m^2 + i\epsilon} v(k,p) \tag{4}$$

- Currently, bare propagators for the quarks.
- $v(q,p)$ is one of the Faddeev components of the total vertex function.
- Di-quark concept introduced via assuming a pole in $\mathcal{F}(M_{12}^2)$, corresponding either to a two-body bound ($a > 0$) or virtual ($a < 0$) state, where a denotes the scattering length
- $\mathcal{F}(M_{12}^2)$, where $M_{12}^2 = (p-q)^2$, given by

$$\mathcal{F}(M_{12}^2) = \frac{\Theta(-M_{12}^2)}{\frac{1}{16\pi^2 y} \log \frac{1+y}{1-y} - \frac{1}{16\pi m a}} + \frac{\Theta(M_{12}^2)\,\Theta(4m^2 - M_{12}^2)}{\frac{1}{8\pi^2 y'} \arctan y' - \frac{1}{16\pi m a}} + \frac{\Theta(M_{12}^2 - 4m^2)}{\frac{y''}{16\pi^2} \log \frac{1+y''}{1-y''} - \frac{1}{16\pi m a} - \frac{iy''}{16\pi}}, \tag{5}$$

- The FBS equation was recently solved including the infinite number of Fock components in Euclidean [2] and Minkowski [3] space.

[1] T. Frederico, PLB 282 (1992) 409

[2] E. Ydrefors et al, PLB 770 (2017) 131

[3] E. Ydrefors et al, PLB 791 (2019) 276

- After the LF projection, i.e. introducing $k_\pm = k_0 \pm k_z$ and integrating over k_-, one obtains the three-body LF equation [1, 2]:

$$\Gamma(x,k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{x'(1-x-x')}\int_0^\infty \frac{d^2k'_\perp}{M_0^2 - M_N^2}\Lambda(M_0^2)\Gamma(x',k'_\perp) \tag{6}$$

with the squared free three-body mass

$$M_0^2 = (k_\perp'^2 + m^2)/x' + (k_\perp^2 + m^2)/x + ((k'_\perp + k_\perp)^2 + m^2)/(1-x-x') \tag{7}$$

- Form factor introduced via substraction, i.e.

$$[M_0^2 - M_N^2]^{-1} - [M_0^2 + \mu^2]^{-1} = \Lambda(M_0^2)[M_0^2 - M_N^2]^{-1} \to \Lambda(M_0^2) = [M_0^2 + \mu^2]^{-1}[M_N + \mu^2], \tag{8}$$

where μ is a cut-off mass.

- The form factor eliminates the unphysical ground state, with $M_N^2 < 0$, and also lead to an infrared enhancement.
- The three-body valence LF wave function is given by

$$\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}) = \frac{\Gamma(x_1,\vec{k}_{1\perp}) + \Gamma(x_2,\vec{k}_{2\perp}) + \Gamma(x_3,\vec{k}_{3\perp})}{\sqrt{x_1x_2x_3}(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))}, \tag{9}$$

where due to momentum conservation: $x_3 = 1 - x_2 - x_3$ and $\vec{k}_{3\perp} = -\vec{k}_{1\perp} - \vec{k}_{2\perp}$.

[1] J. Carbonell and V.A. Karmanov, PRC 67 (2003) 037001

[2] T. Frederico, PLB 282 (1992) 409

Electromagnetic form factor

- The valence contribution to the Dirac form factor is given by

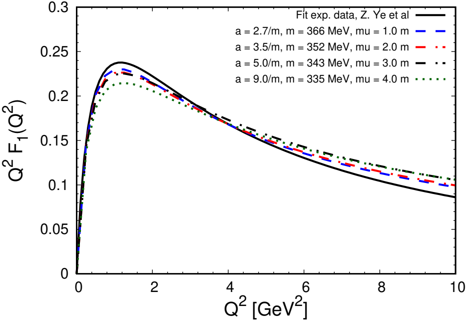
$$F_1(Q^2) = \left\{ \prod_{i=1}^{3} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta\left(1 - \sum_{i=1}^{3} x_i\right) \delta\left(\sum_{i=1}^{3} \vec{k}^{\mathrm{f}}_{i\perp}\right) \Psi_3^\dagger(x_1, \vec{k}^{\mathrm{f}}_{1\perp}, \ldots) \Psi_3(x_1, \vec{k}^{\mathrm{i}}_{1\perp}, \ldots), \tag{10}$$

where $Q^2 = \vec{q}_\perp \cdot \vec{q}_\perp$ and the magnitudes of the momenta read

$$\left|\vec{k}^{\mathrm{f(i)}}_{i\perp}\right|^2 = \left|\vec{k}_{i\perp} \pm \frac{\vec{q}_\perp}{2} x_i\right|^2 = \vec{k}^2_{i\perp} + \frac{Q^2}{4} x_i^2 \pm \vec{k}_{i\perp} \cdot \vec{q}_\perp x_i \quad (i = 1, 2), \tag{11}$$

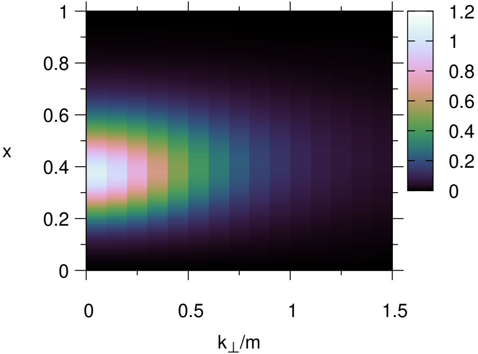
and

$$\begin{aligned} \left|\vec{k}^{\mathrm{f(i)}}_{3\perp}\right|^2 &= \left|\pm \frac{\vec{q}_\perp}{2}(x_3 - 1) - \vec{k}_{1\perp} - \vec{k}_{2\perp}\right|^2 = \\ &(1 - x_3)^2 \frac{Q^2}{4} \pm (1 - x_3)\vec{q}_\perp \cdot (\vec{k}_{1\perp} + \vec{k}_{2\perp}) + (\vec{k}_{1\perp} + \vec{k}_{2\perp})^2. \end{aligned} \tag{12}$$



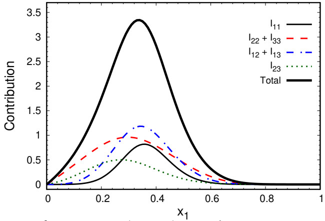
- In figure $Q^2F(Q^2)$ for different values of a and μ compared with fit to exp. data by Z. Ye et al.
- Best agreement obtained for $a = 2.7/m$ and $\mu = m$ with a constituent quark mass $m = 366$ MeV, and this parameters will be used in the following.
- Fair agreement with exp. data for $Q^2 < 5\,\text{GeV}^2$ but for larger values of Q^2 they deviate, presumably due to lack of a finite-range interaction.

Results for the vertex function



- The proton structure contained in the vertex function $\Gamma(x, k_\perp)$. Concentrated at small $k_\perp$ and $x \sim 1/3$.

Parton distribution function at model scale



- The single parton distribution function (PDF), is the integrand of the form factor at $Q^2 = 0$, i.e.

$$f_1(x_1) = \frac{1}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} |\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp})|^2 = I_{11} + I_{22} + I_{33} + I_{12} + I_{13} + I_{23}. \tag{13}$$

with the Faddeev contributions

$$\begin{aligned} I_{ii} &= \frac{1}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma^2(x_i,\vec{k}_{i\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))^2} \\ I_{ij} &= \frac{2}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma(x_i,\vec{k}_{i\perp})\Gamma(x_j,\vec{k}_{j\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))^2}; \quad i \neq j. \end{aligned} \tag{14}$$

- The PDF at model scale is peaked around $x = 1/3$ and quite narrow. None of the Faddeev contributions are negligible.

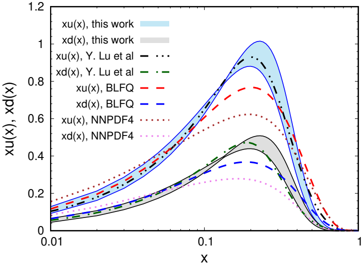
PDF evolution

- For the comparison with other frameworks and/or experimental data the PDF should be evolved from the model scale to a higher scale.
- In this work we use the all-order DGLAP evolution (see e.g. talk by Craig) and the process-independent charge (EPJC 80 (2020) 1064):

$$\alpha(k^2) = \frac{\gamma_m \pi}{\log[\mathcal{K}^2(k^2)/\Lambda^2_{QCD}]}, \quad \mathcal{K}^2(y) = (a_0^2 + a_1 y + y^2)/(b_0 + y) \tag{15}$$

- The initial scale is given by the hadron scale $Q_0 = 0.330 \pm 0.03$ GeV.
- The same evolution framework was used successfully for the pion (see previous talk by W. de Paula).

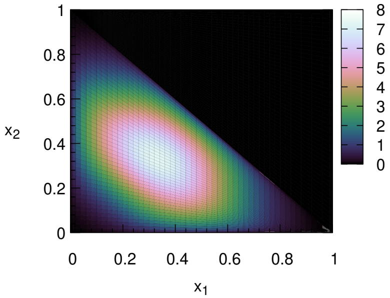
Proton PDF at $Q = 3.097$ GeV



- Colored areas: Computed u and d-quark xpdfs at $Q = 3.097$ GeV with the areas corresponding to the uncertainty in initial scale $Q_0 = 0.330 \pm 0.03$ GeV.
- Dash-dotted lines: Results from Y. Lu et al (see talk by Craig). Good agreement at least for $x \lesssim 0.5$. Disagreement at large x probably due to the use of contact interaction in our model.
- Dashed-lines: BLFQ [1] but evolved using same framework as in this work. Only good agreement for small x.
- Dotted lines: Results from the NNPDF 4.0 global fit. None of the models agree well with these results.
- A few remarks:
  - Model of this work and the one by Y. Lu et al, are both quark-diquark models, but the latter one has also axial-vector diquark and a more realistic quark-quark interaction.
  - The BLFQ which is a Hamiltonian approach include (at least effectively) confinement, which is lacking in the two other models.

[1] PRD 104, 094036 (2021)

Distribution amplitude



- The distribution amplitude is defined as

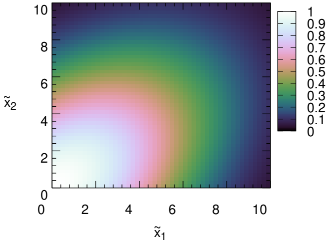
$$\phi(x_1, x_2) = \int d^2k_{1\perp} d^2k_{2\perp} \Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}). \tag{16}$$

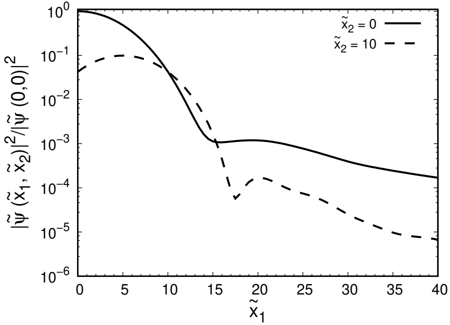
- It shows the dependence of the wave function on the momentum fractions for the case when the quarks share the same position.
- Triangular shape due to $x_1 + x_2 \leq 1$. Distribution centered around $x_1 = x_2 = 1/3$ but quite wide.

Ioffe-time image of the valence state

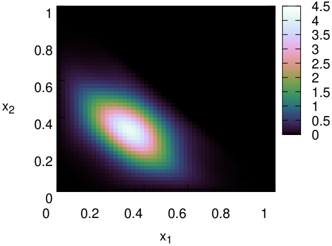
- Alternatively, the proton can be studied in the on the null-plane, in terms of the transverse position ($\vec{b}_{i\perp}$) and the Ioffe-time $\tilde{x}_i = b_i^- p^+$. The image of the proton is then obtained through the Fourier transform of the proton LF wave function.
- For simplicity, we consider here the case $\vec{b}_{1\perp} = \vec{b}_{2\perp} = \vec{0}_\perp$, and then one has

$$\Phi(\tilde{x}_1, \tilde{x}_2) \equiv \tilde{\Psi}_3(\tilde{x}_1, \vec{0}_\perp, \tilde{x}_2, \vec{0}_\perp) = \int_0^1 dx_1 \, e^{i\tilde{x}_1 x_1} \int_0^{1-x_1} dx_2 \, e^{i\tilde{x}_2 x_2} \, \phi(x_1, x_2) \,, \tag{17}$$





- For $\tilde{x}_1 >= 10$ a rather dramatic decrease of the amplitude is seen.
- An exponential damping is seen with respect to the relative distance in Ioffe-time between the two quarks. We expect this damping to be even more significant if confinement is incorporated, as its more effective at large distances.

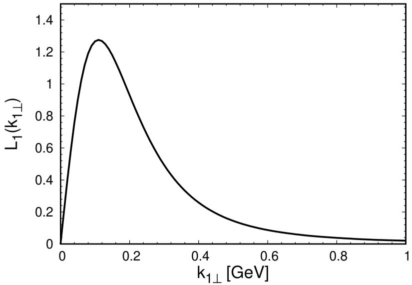


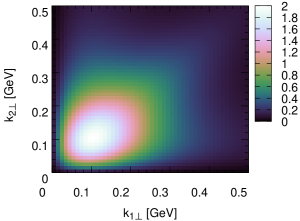
- The valence double parton distribution function (DPDF) is given by

$$D_3(x_1, x_2; \vec{\eta}_\perp) = \frac{1}{(2\pi)^6} \int d^2k_{1\perp} d^2k_{2\perp} \times \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp; x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}). \tag{18}$$

- Fourier transform of $D_3(x_1, x_2, \vec{\eta}_\perp)$ in $\vec{\eta}_\perp$ gives the probability of finding the quarks 1 and 2 with momentum fractions x_1 and x_2 at a relative distance $\vec{y}_\perp$ within the proton.
- In the figure is shown results for $\eta_\perp = 0$, showing a distribution centered around $x_1 = x_2 = 1/3$.

Transverse momentum densities





- The single quark transverse momentum density in the forward limit and integrated in the longitudinal momentum is associated with the probability density to find a quark with momentum $k_\perp$.
- It can be computed as:

$$L_1(k_{1\perp}) = \frac{k_{1\perp}}{(2\pi)^6} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{2\pi} d\theta_1 \int d^2k_{2\perp} |\psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2. \qquad (19)$$

- Two-quark one:

$$L_2(k_{1\perp}, k_{2\perp}) = \frac{k_{1\perp} k_{2\perp}}{(2\pi)^6} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 |\psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2. \qquad (20)$$

Work in progress: Going beyond the valence order

- The three-body FBS equation with zero-range interaction, including the infinite number of Fock components, was solved by direct integration in Minkowski space in Ref. [1]. However, the solution was quite difficult from numerical point of view.
- However, like in the two-body case, the Nakanishi integral representation be used for vertex function:

$$v(q;p) = \int_{-4/3}^{2/3} dz \int_0^\infty \frac{d\gamma g(\gamma,z)}{\gamma - k^2 - (p\cdot q)z - i\epsilon} \tag{21}$$

- For the two-body scattering amplitude

$$\mathcal{F}(M_{12}^2) = \int_{4m^2}^\infty d\gamma \frac{\rho(\gamma)}{M_{12}^2 - \gamma + i\epsilon} \tag{22}$$

with the spectral function

$$\rho(\gamma) = -\frac{\theta(s-4m^2)}{16\pi^2} \frac{y''}{\left(\frac{y''}{16\pi^2}\log\frac{1+y''}{1-y''} - \frac{1}{16\pi m a}\right)^2 + \left(\frac{y''}{16\pi}\right)^2} \tag{23}$$

- Construction of the integral equation for $g(\gamma,z)$ and its solution is under development.
- Observables could then be computed including all the infinite number of Fock components.

[1] E. Ydrefors et al, PLB 791 (2019) 276

Conclusions

- We have, in this work, studied the proton in a simple but fully dynamical valence LF model based on a zero-range interaction.
- The model is based on the concept of a strongly interacting scalar diquark.
- We have studied the structure of the proton by computing the LF wave function in its Ioffe-time representation and also momentum distributions.
- However, the model is rather crude since e.g. the spin degree of freedom hasn't been included yet. But is a first step towards studying the proton directly in Minkowski space.
- Future plans:
  - Generalization to the infinite set of Fock components (The Faddeev-Bethe-Salpeter equation solved in PLB 791 (2019) 276)
  - Implementation of a more realistic interaction (gluon exchange)
  - Inclusion of spin degree of freedom