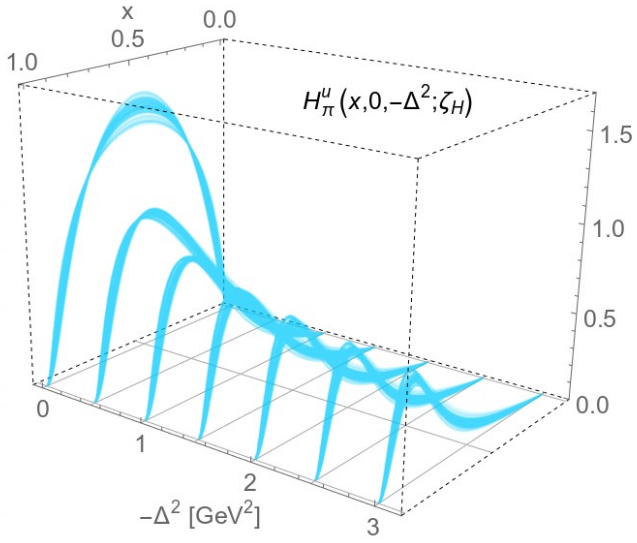
A data-driven approach to construct the pion GPD

Khépani Raya Montaño

José Rodríguez-Quintero, *et al*.



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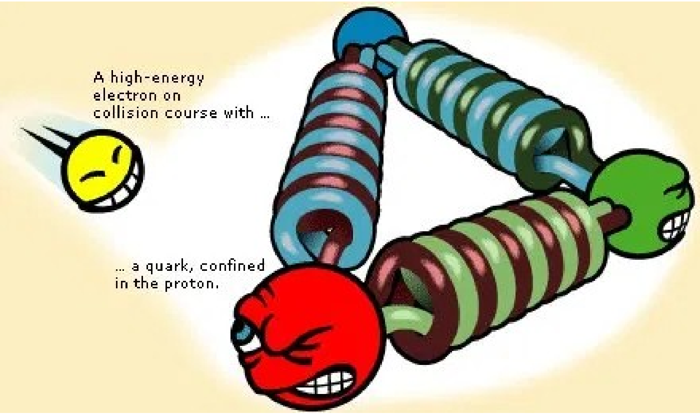
QCD: Basic Facts

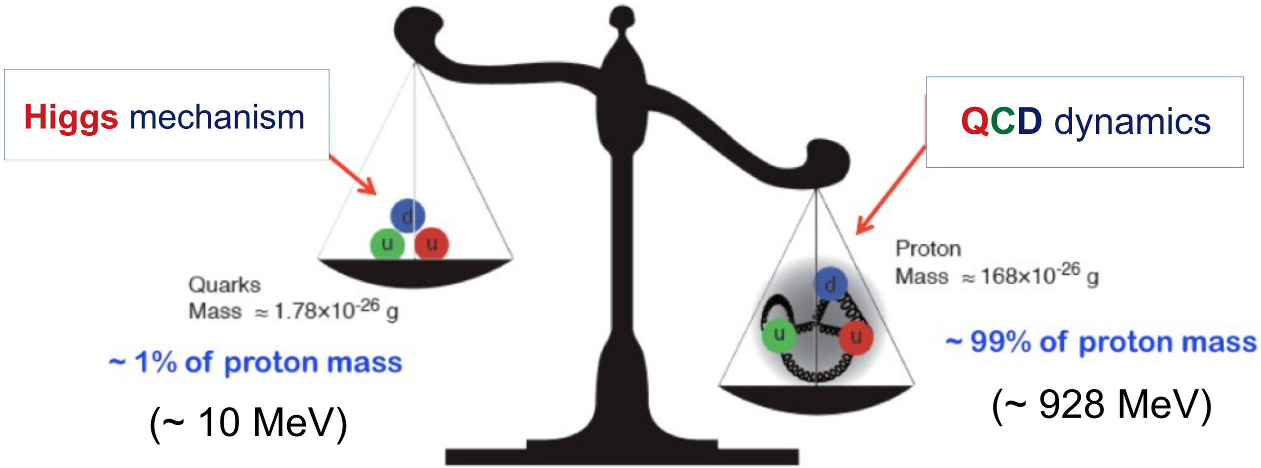
- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?

- Emergence of hadron masses (EHM) from QCD **dynamics**





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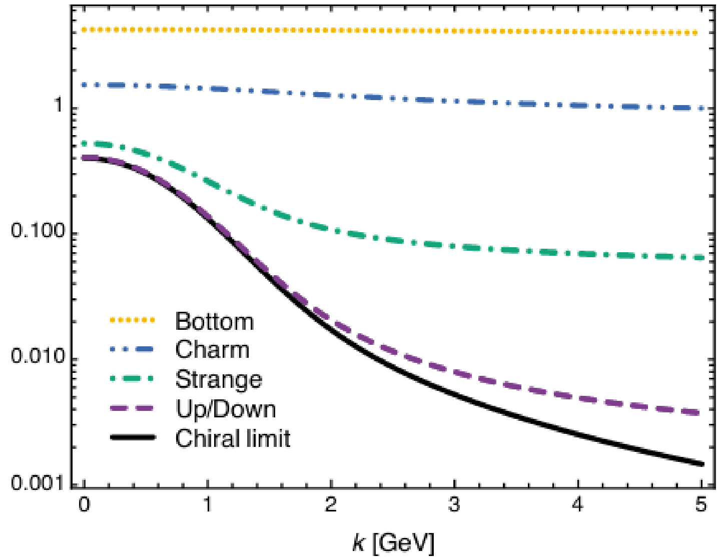
Can we trace them down to fundamental d.o.f?

Dynamical masses

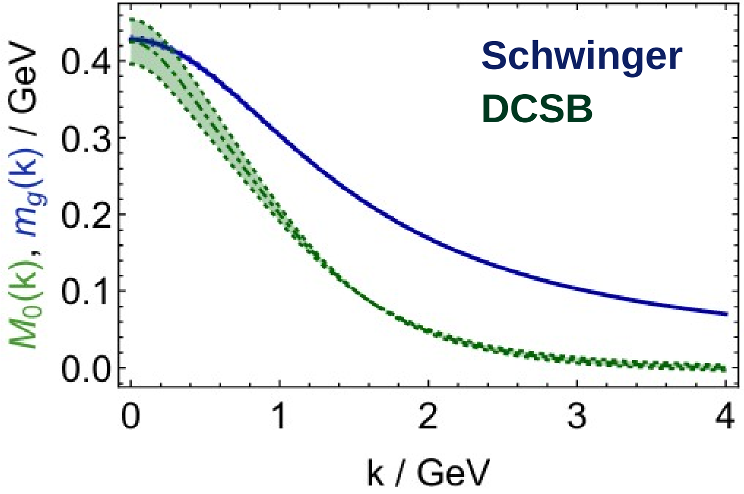
(Dynamical Chiral Symmetry Breaking)



"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$

◆ Emergence of hadron masses (EHM) from QCD **dynamics**



Gluon and quark *running masses*

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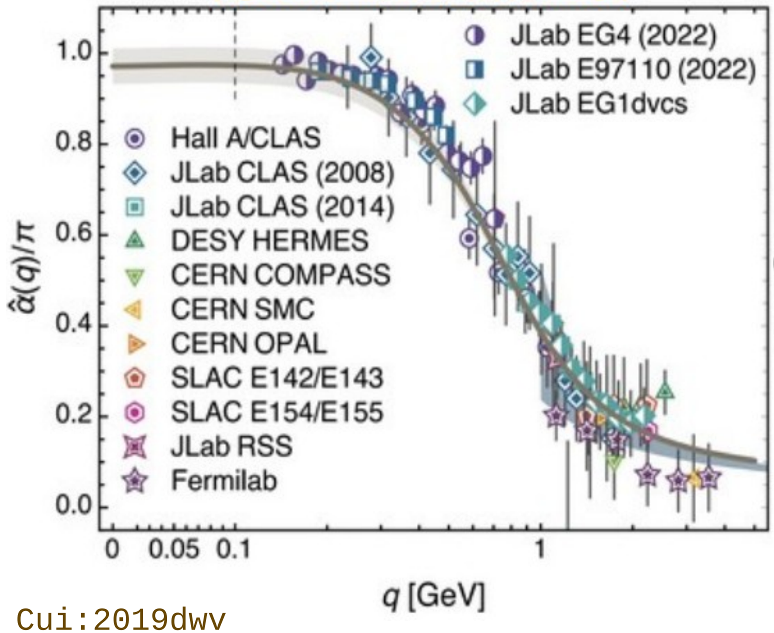
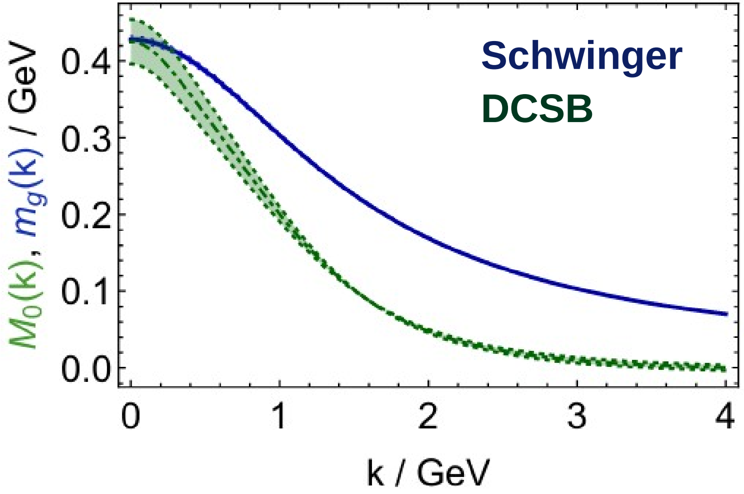
Can we trace them down to fundamental d.o.f?

- Emergence of hadron masses (EHM) from QCD **dynamics**

(figure: D. Binosi's courtesy!)

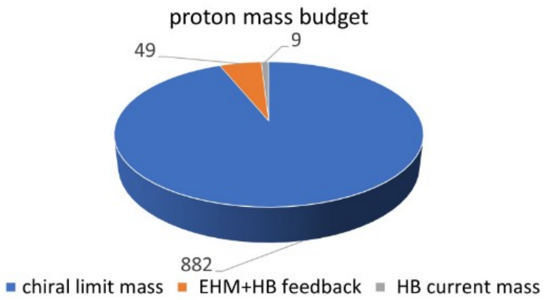


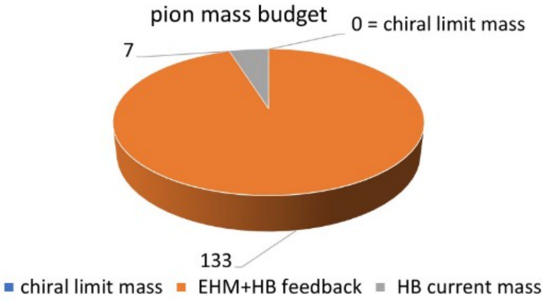
Cui:2019dwv

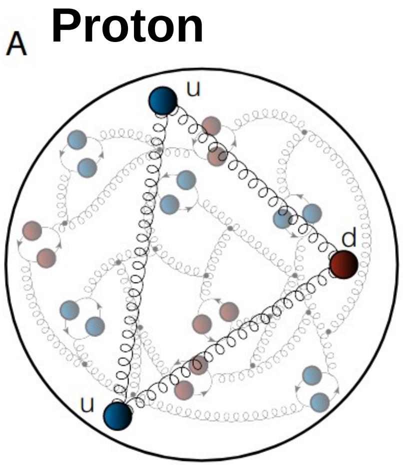


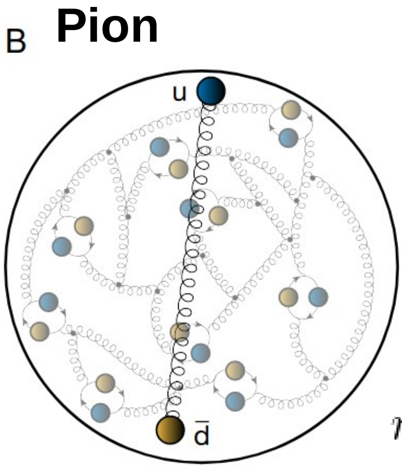
Gluon and quark *running masses*

QCD: A modern goal









$m_p \approx 0.940\ \mathrm{GeV}$

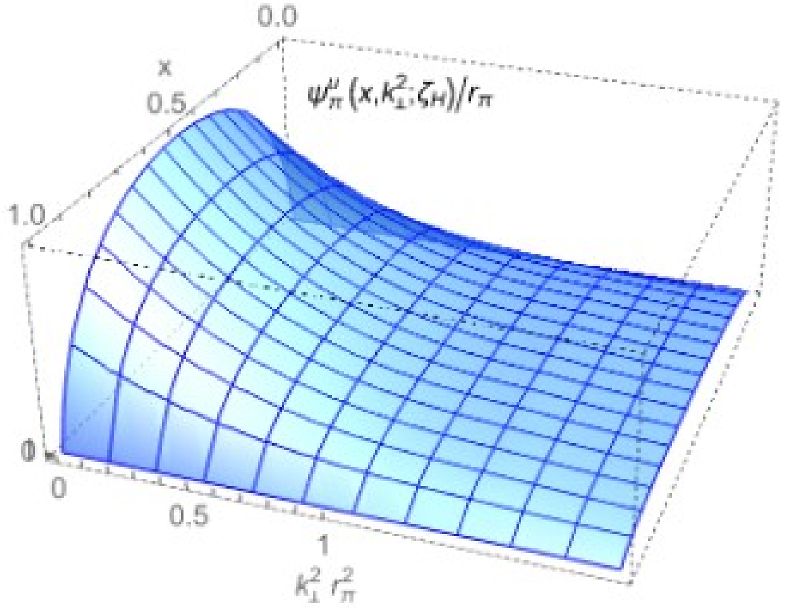
$m_\pi \approx 0.140\ \mathrm{GeV}$

Massive, regardless of **Higgs** mass generation

Massless in the absense of **Higgs** mass generation

- **QCD** should explain both the *massiveness* of the proton and the *masslessness* of the pion

Light-front wave functions



$$\psi^u_{\mathsf{P}}(x, k_\perp^2; \zeta)$$

“One ring to rule them all”

Raya:2021zrz
Raya:2022eqa

Light-front wave functions

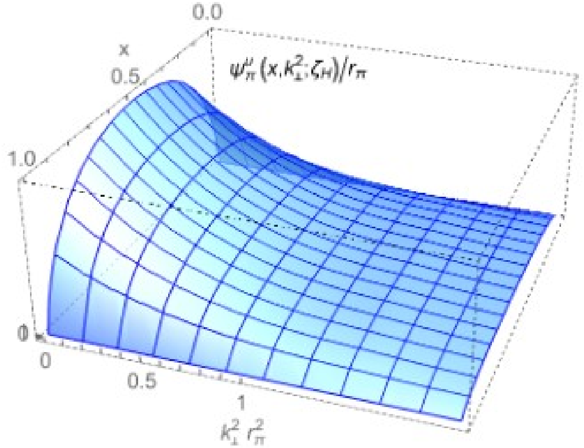
- Many **distributions** are related via the leading-twist light-front wave function (**LFWF**), e.g.:

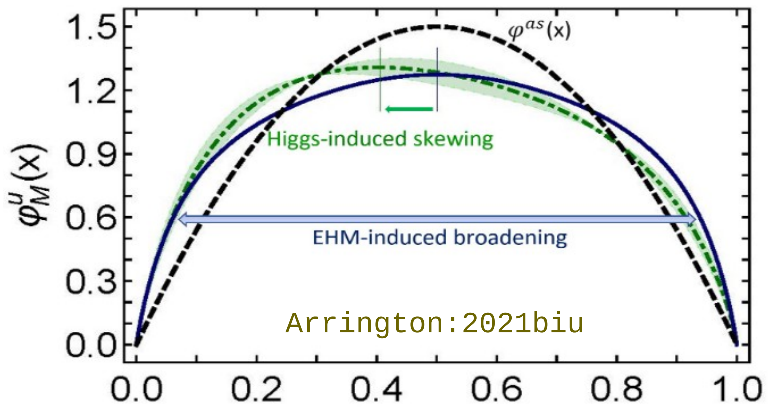
Distribution amplitudes

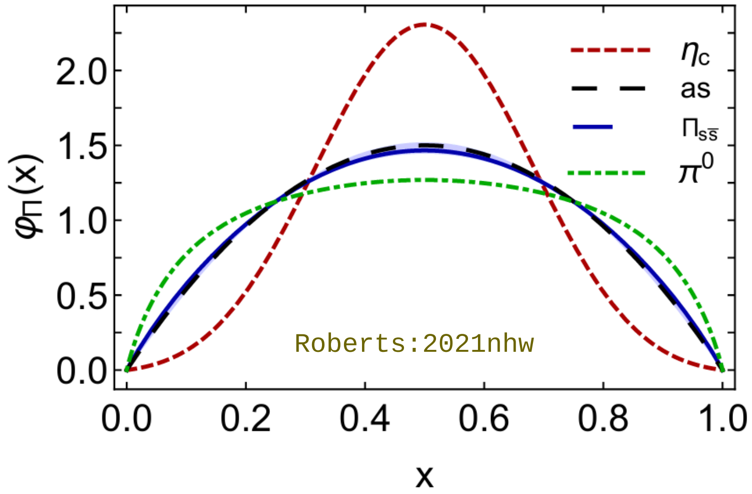
$$f_{\mathrm{P}}\varphi^u_{\mathrm{P}}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi^u_{\mathrm{P}}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

Distribution functions

$$u^{\mathrm{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi^u_{\mathrm{P}}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$







Light-front **wave functions**

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$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

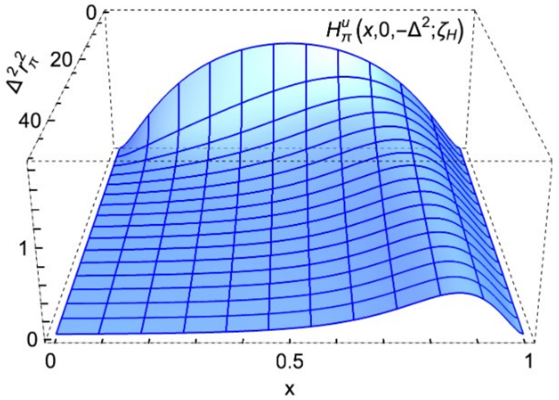
Distribution functions

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$

- In the **DGLAP** kinematic domain, this is also the case of the valence-quark **GPD**:

$$H_{\mathsf{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi_{\mathsf{P}}^{u*}\left(x_-,k_{\perp -}^2;\zeta_{\mathcal{H}}\right)\psi_{\mathsf{P}}^{u}\left(x_+,k_{\perp +}^2;\zeta_{\mathcal{H}}\right)$$

$$x_\mp = (x\mp\xi)/(1\mp\xi)\,,\; t=-\Delta^2$$
$$k_{\perp\mp} = k_\perp \pm (\Delta_\perp/2)(1-x)/(1\mp\xi)$$



- **PDF**: forward limit of the **GPD** (t=0, ξ=0).

- Furthermore, form factors appear as **Mellin** moments of the **GPD**:

$$F_{\mathbf{P}}^{u}(\Delta^2) = \int_{-1}^{1} dx\, H_{\mathbf{P}}^{u}(x,\xi,-\Delta^2;\zeta_H)$$

$$F_{\mathbf{P}} = e_u F_{\mathbf{P}}^{u}(\Delta^2) + e_{\bar h} F_{\mathbf{P}}^{h}(\Delta^2)$$

(e.g. electromagnetic FF)

Light-front wave functions

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Distribution functions

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$

- This connection already **suggests** that:

$$u^{\mathbf{P}}(x;\zeta_H) \sim [\varphi_{\mathbf{P}}^{u}(x;\zeta_H)]^2$$

is a fair approximation, implying:

$$\psi_{\mathbf{P}u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H) = \tilde{\psi}_{\mathbf{P}u}^{\mathbf{u}}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$$

- In fact, we have learned that *x*-*k* crossed terms are weighted by: $M_{\mathbf{P}}^2,\;\; M_h^2 - M_q^2$ (factorised LFWF)

➔So a **factorised Ansatz** should be sensible for the **pion,** implying:

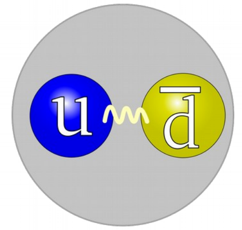
$$H_{\mathsf{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\,\Phi_{\mathbf{P}}(z;\zeta_H)$$

Raya:2021zrz
Raya:2022eqa

PARTON **DISTRIBUTIONS**

- **Fully-dressed valence quarks**
  (quasiparticles)
- **Unveiling of glue and sea d.o.f**
  (partons)

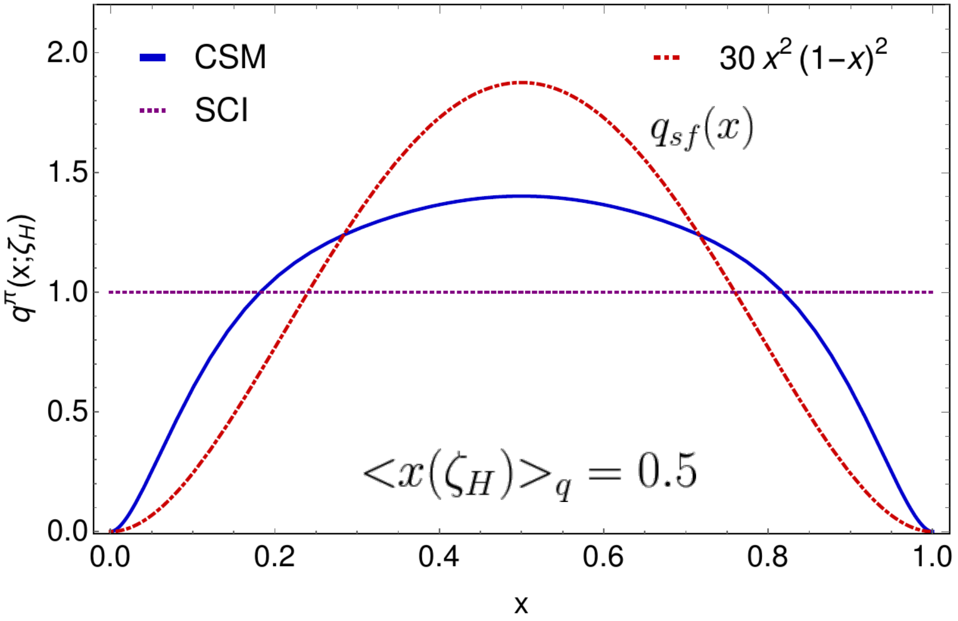
Pion PDF: hadronic scale



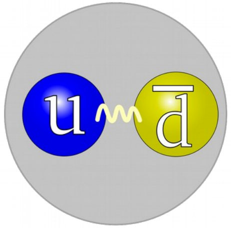
- **Fully-dressed valence quarks** (quasiparticles)

ζ_H : hadronic scale

- At this scale, **all properties** of the hadron are contained within their valence quarks.



Pion PDF: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

ζ_H : hadronic scale

➢ At this scale, **all properties** of the hadron are contained within their valence quarks.

"**Physical**" boundaries:

$$\frac{1}{2^n} \overset{(i)}{\leq} \langle x^n \rangle_{u_\pi}^{\zeta_H} \overset{(ii)}{\leq} \frac{1}{1+n}$$

Produced by

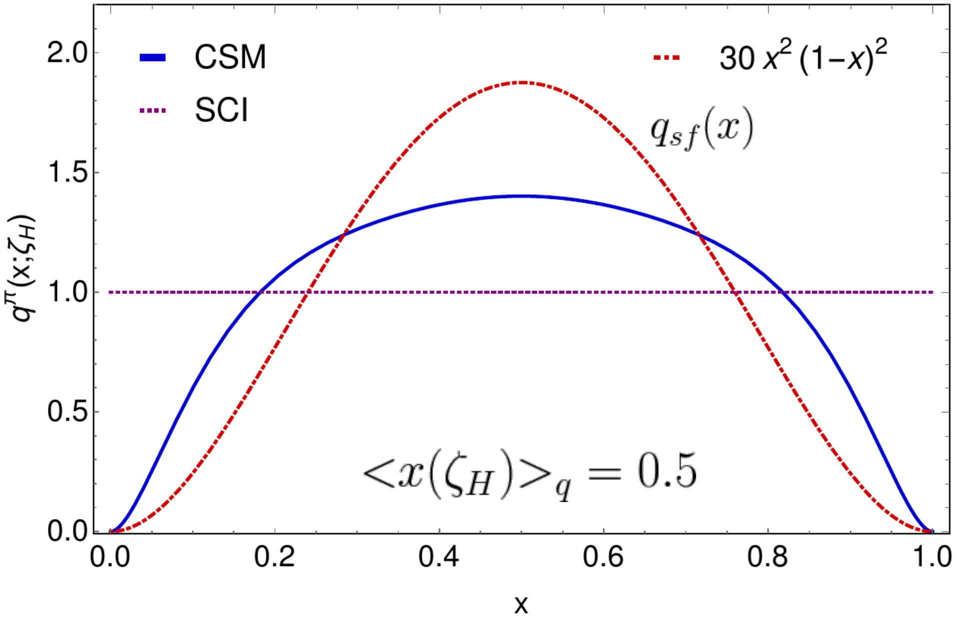
$$q(x;\zeta_H) = \delta(x - 1/2)$$

(infinitely heavy valence quarks)

Produced by

$$q(x;\zeta_H) = 1$$

(massless SCI case)



➢ Besides, **QCD** constrains the **large-x** behavior:

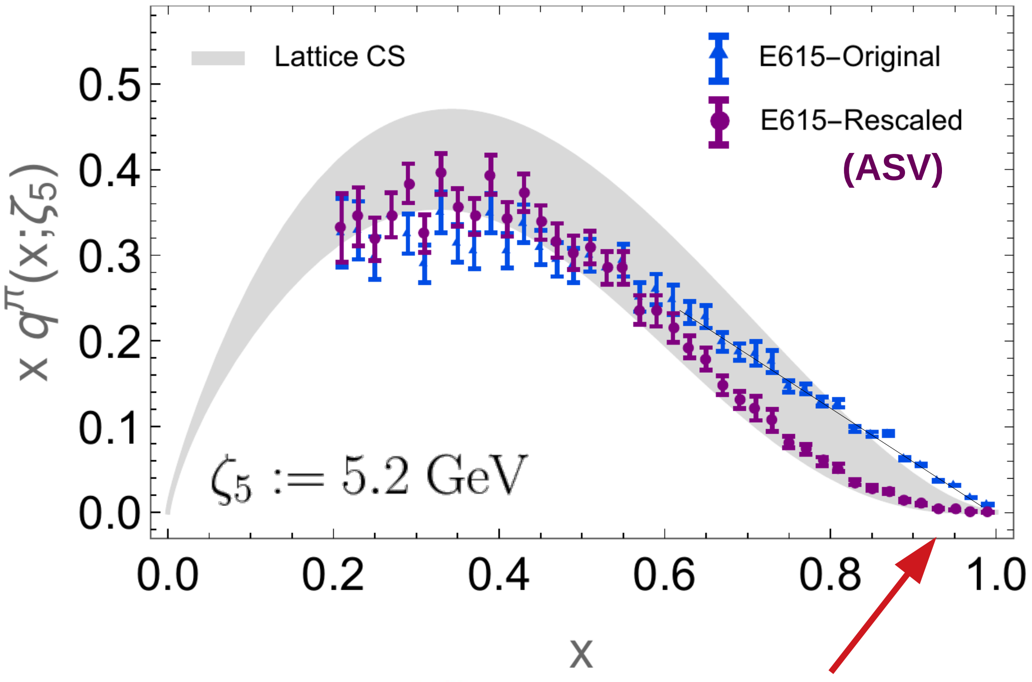
$$u^\pi(x;\zeta) \overset{x\simeq 1}{\approx} (1-x)^{\beta = 2+\gamma(\zeta)}$$

➢ **CSM** results are in agreement with it.

Cui:2020tdf

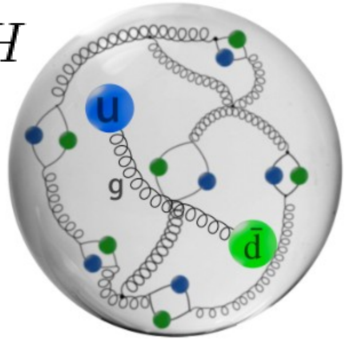
Pion PDF: experimental scale

Conway:1989fs Aicher:2010cb Sufian:2020vzb



$$u^{\pi}(x;\zeta) \overset{x\simeq 1}{\approx} (1-x)^{\beta = 2+\gamma(\zeta)}$$

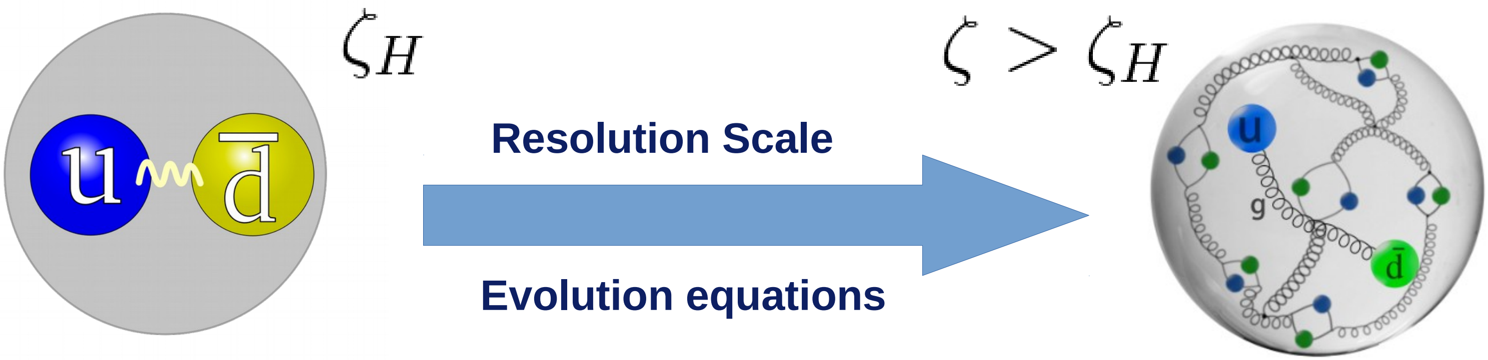
$\zeta > \zeta_H$



- **Unveiling of glue and sea d.o.f** (partons)

- **Experimental** data is given **here**.
- **Lattice QCD** results are also quoted beyond the **hadronic scale**.
  - The interpretation of parton distributions from cross sections demands **special care**.

Pion PDF: energy scales



- **Fully-dressed valence quarks**

  (quasiparticles)

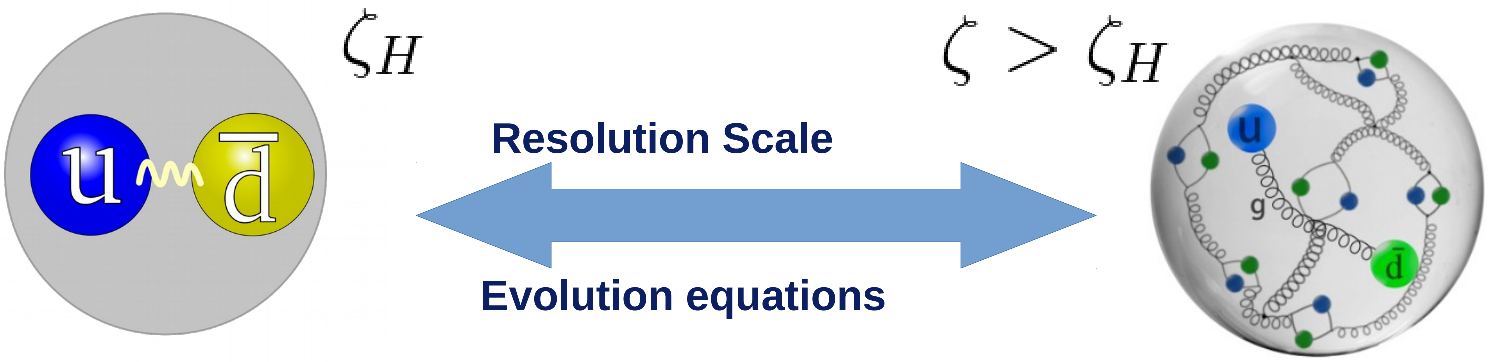
  - Theoretical calculations are perfomed at *some* low energy scale.

- **Unveiling of glue and sea d.o.f**

  (partons)

  - Then evolved via **DGLAP** equations to compare with experiment and lattice.

Pion PDF: energy scales



- **Fully-dressed valence quarks**

  (quasiparticles)

  - Theoretical calculations are perfomed at *some* low energy scale.

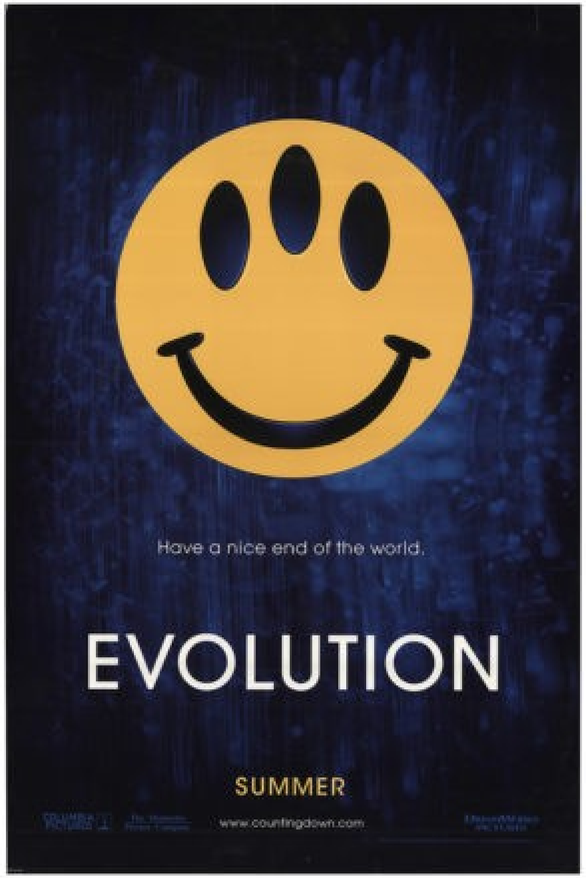
- **Unveiling of glue and sea d.o.f**

  (partons)

  - Then evolved via **DGLAP** equations to compare with experiment and lattice.

- Following our **all orders** evolution, we can go **either way**.
- Besides, the **hadronic scale** becomes unambigously **determined**.

JRQ's talk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DGLAP: All orders evolution

Idea. Define an **effective** coupling such that:

"All orders evolution"

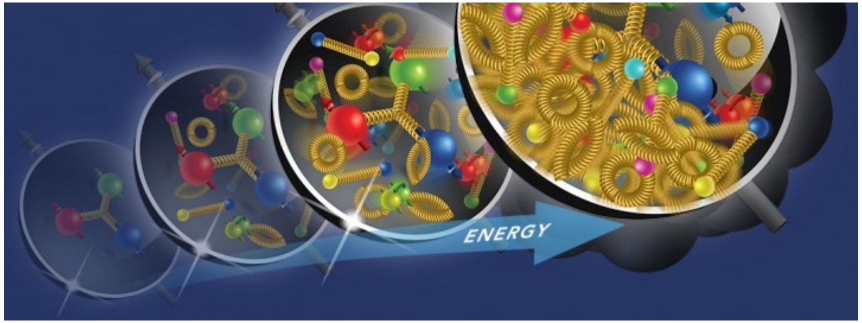
Raya:2021zrz

Cui:2020tdf

Starting from fully-dressed **quasiparticles**, at ζ_H

→ **Sea** and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2\frac{d}{d\zeta^2}\int_0^1 dy\,\delta(y-x) - \frac{\alpha(\zeta^2)}{4\pi}\int_x^1\frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{\rm NS,+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$



- Not the **LO** QCD coupling but an **effective** one.
- Making this equation exact.
- Connecting with the hadron scale, at which the **fully-dressed** valence-**quarks** express **all** of the hadron's properties.

(thus carrying all the momentum)

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q$$

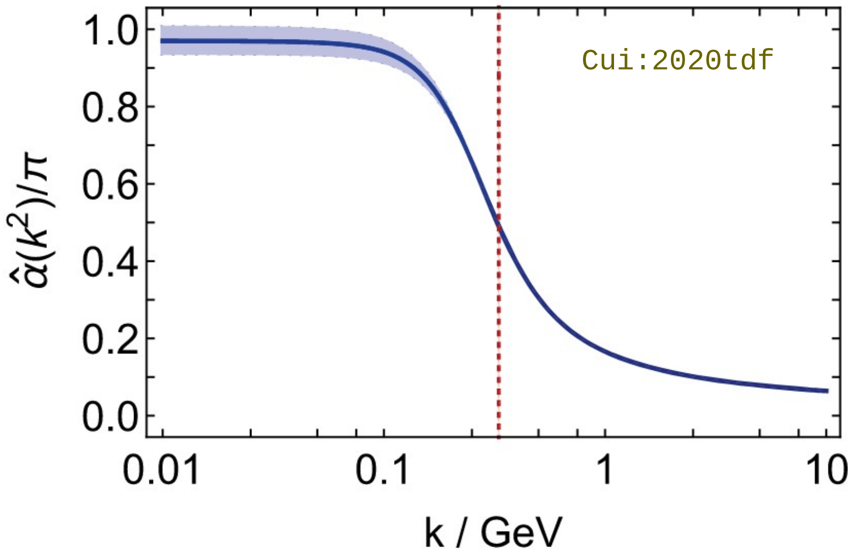
$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\,\alpha(t)$$

Explicitly depending on the **effective charge**

$$\langle x^n(t;\zeta)\rangle_F = \int_0^1 dx\, x^n\, F(x,t;\zeta)$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

- The **QCD PI effective charge** is our best candidate to accommodate our **all orders scheme**.



$$\hat{\alpha}(k^2) = \frac{\gamma_m\pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda_{\rm QCD}^2}\right]} \Rightarrow \zeta_H = 0.331\ \text{GeV}$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{QCD})}^{2\ln(\zeta_f/\Lambda_{QCD})} dt\,\alpha(t)$$

This contains, ***implicitly***, the information of the **effective charge**

- No actual ***need*** to know it. Assuming its existence is **sufficient**.
- ***Unambiguous*** definition of the **hadron scale**:

$$\langle x(\zeta_H)\rangle_q = 0.5 \;\Rightarrow\; \langle x^n(\zeta_f)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\langle 2x(\zeta_f)\rangle_q\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

(pion case)

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

Information on the charge is here

- Details of the effective charge are **encoded** in the ratio of first moments.
- Natural connection with the **hadron scale**.

Implication 2:

$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d}; \\
\langle x(\zeta_f)\rangle_{\text{sea}} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}), \\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right);
\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}}$$

Information on the charge is here

- Can **jump** from one scale to the other. (even downwards)
- Natural connection with the **hadron scale**.

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\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

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\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.
- And, of course, the momentum **sum rule**:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\text{sea}} + \langle x(\zeta_f)\rangle_g = 1$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

Information on the charge is here

- Can **jump** from one scale to the another (even downwards)
- Natural connection with the **hadron scale**.

Implication 3: Recurrence relation

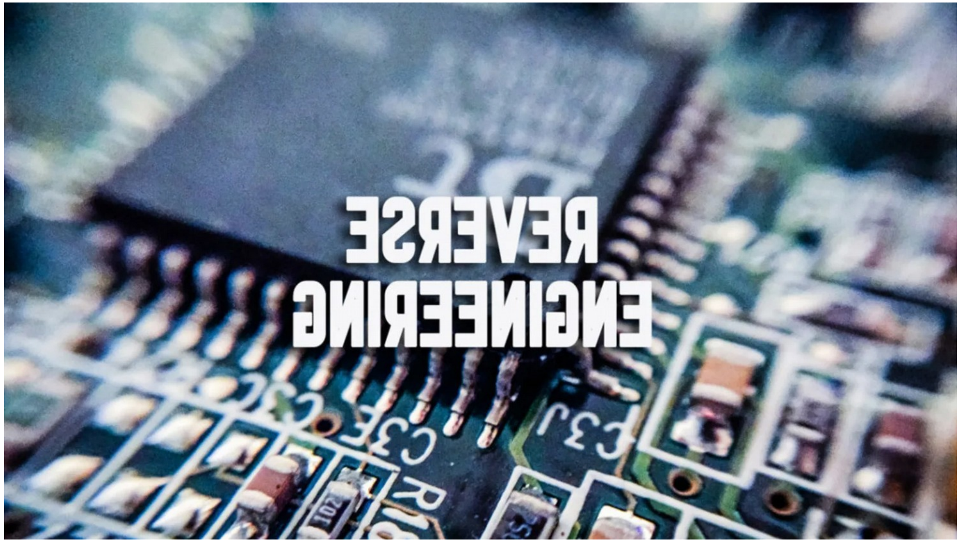
$$\langle x^{2n+1}\rangle_{u_\pi}^{\zeta} = \frac{(\langle 2x\rangle_{u_\pi}^{\zeta})^{\gamma_0^{2n+1}/\gamma_0^1}}{2(n+1)} \times \sum_{j=0,1,\ldots}^{2n} (-)^j \binom{2(n+1)}{j} \langle x^j\rangle_{u_\pi}^{\zeta} (\langle 2x\rangle_{u_\pi}^{\zeta})^{-\gamma_0^j/\gamma_0^1} .$$

- Since **isospin symmetry** limit implies:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$

- **Odd** moments can be expressed in terms of previous **even** moments.
- Thus arriving at the recurrence **relation** on the left.

Reverse engineering the PDF data



Pion **PDF**

D. Binosi's talk

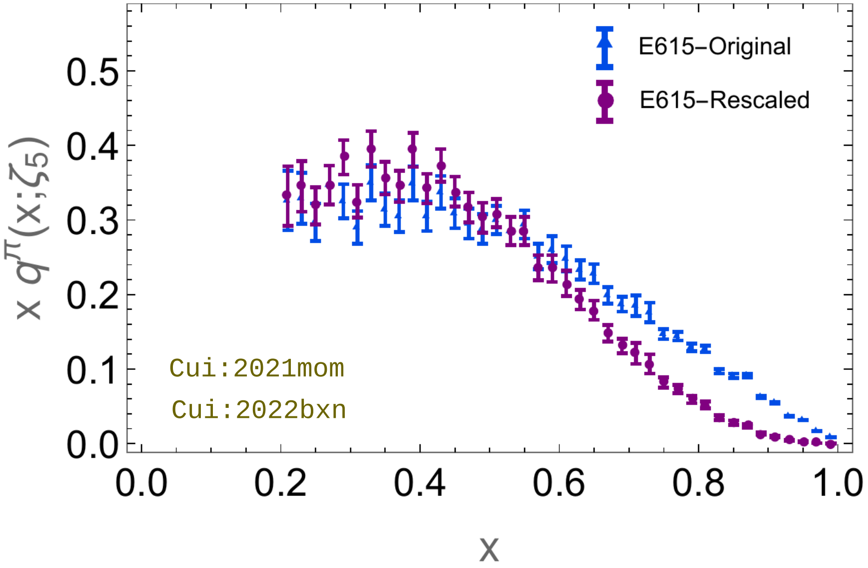
- Let us assume the data can be parameterized with a certain functional form, i.e.:

$$u^\pi(x;[\alpha_i];\zeta) = n_u^\zeta x^{\alpha_1^\zeta}(1-x)^{\alpha_2^\zeta}(1+\alpha_3^\zeta x^2)$$

$$\{\alpha_i^\zeta | i = 1,2,3\}$$

Normalization

Free parameters



- Then, we proceed as follows:

1) Determine the **best values α_i** via least-squares fit to the data.

2) Generate new **values α_i,** distributed randomly around the best fit.

3) Using the latter set, evaluate:

$$\chi^2 = \sum_{l=1}^{N} \frac{(u^\pi(x_l;[\alpha_i];\zeta_5) - u_j)^2}{\delta_l^2}$$

Data point with error

4) Accept a replica with probability:

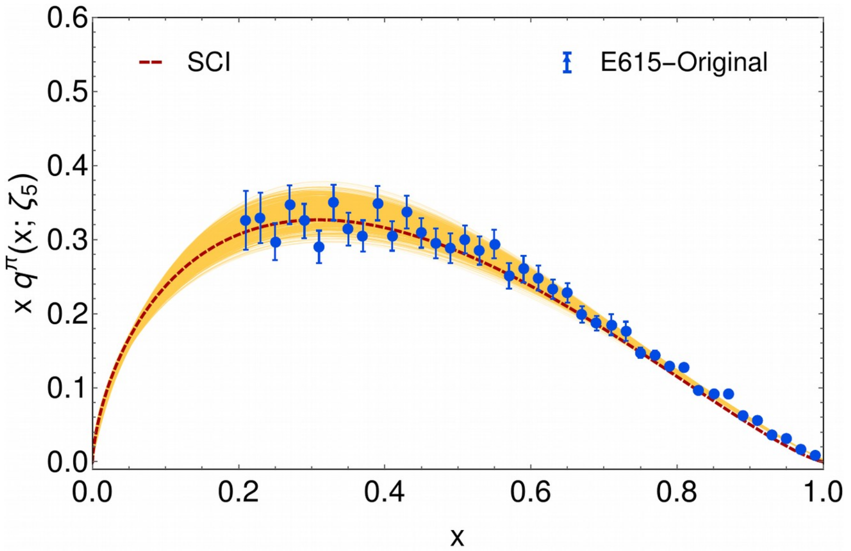
$$\mathcal{P} = \frac{P(\chi^2;d)}{P(\chi_0^2;d)}\,,\quad P(y;d) = \frac{(1/2)^{d/2}}{\Gamma(d/2)} y^{d/2-1} e^{-y/2}$$

5) Evolve back to ζ_H

Repeat (2-5).

Pion PDF: Original Data

➢ Applying this algorithm to the original data yields:



(average)

Mean values (of moments) and errors, ζ_H

{{0.5, 2.52187 × 10^-17}, {0.331527, 0.00803273}, {0.247615, 0.0110893}, {0.19784, 0.0121977}, {0.165066, 0.0124911}, {0.141928, 0.0124198}, {0.124755, 0.0121811}, {0.111521, 0.0118683}, {0.101021, 0.0115275}, {0.0924926, 0.0111824}, {0.085431, 0.010845}, {0.0794897, 0.0105214}, {0.0744232, 0.0102142}, {0.0700521, 0.00992435}, {0.0662432, 0.00965182}}

(SCI)

Moments from SCI, ζ_H

{0.5, 0.332885, 0.249327, 0.199231, 0.165865, 0.142056, 0.124215, 0.11035, 0.0992657, 0.090203, 0.0826552, 0.0762721, 0.0708035, 0.0660661, 0.0619225}

✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✗ But also exhibit agreement with the **SCI results**.

$$q_{\mathrm{SCI}}(x;\zeta_H) \approx 1$$

Thus, given the **QCD prescription**,

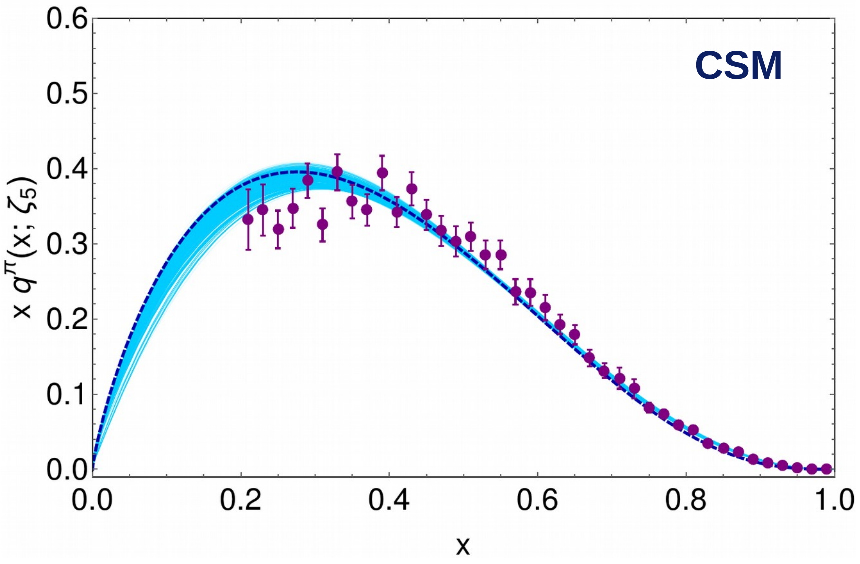
$$u^{\pi}(x;\zeta) \overset{x\simeq 1}{\approx} (1-x)^{\beta=2+\gamma(\zeta)}$$

We shall **discard** this for the upcoming construction of the valence quark GPD

Pion PDF: ASV Data

➢ Applying this algorithm to the **ASV** **data** yields:

(average)



Mean values (of moments) and errors

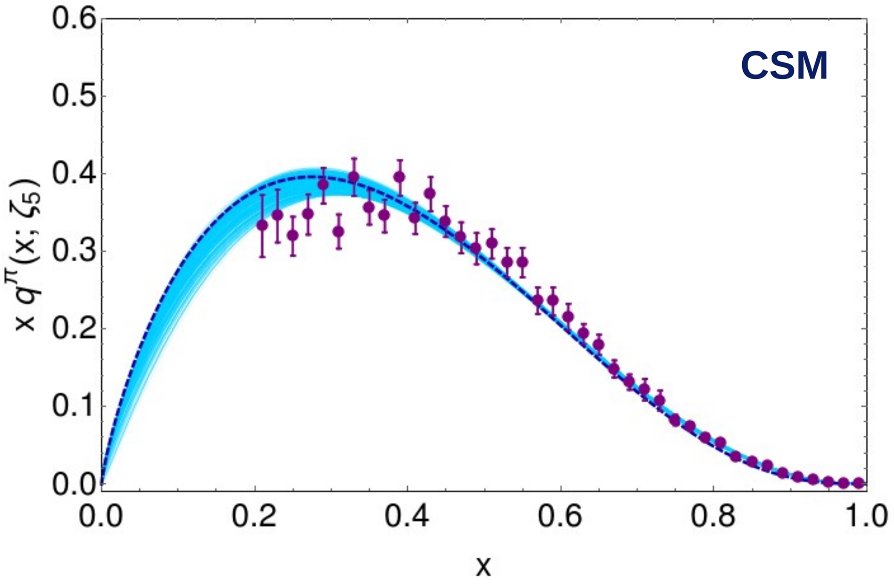
$\{\{0.5, 2.75144\times10^{-17}\}, \{0.299833, 0.00647045\}, \{0.199907, 0.00735448\}, \{0.142895, 0.0068623\}, \{0.107274, 0.00608759\}, \{0.0835168, 0.00532834\}, \{0.0668711, 0.0046596\}, \{0.0547511, 0.00409028\}, \{0.0456496, 0.00361041\}, \{0.0386394, 0.00320609\}\}$

✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✔ Not at all similar to those from SCI

Pion PDF: ASV Data

➢ Applying this algorithm to the **ASV data** yields:

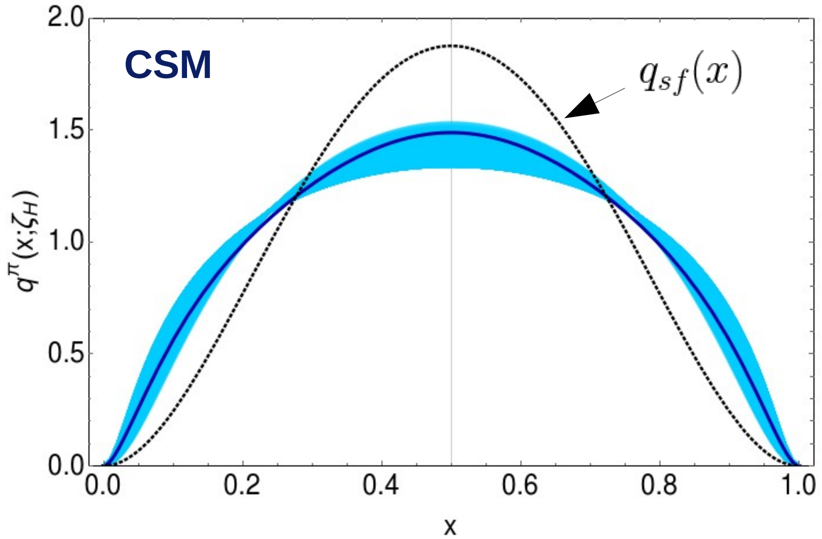


Mean values (of moments) and errors

{{0.5, 2.75144 × 10^{-17}}, {0.299833, 0.00647045}, {0.199907, 0.00735448}, {0.142895, 0.0068623}, {0.107274, 0.00608759}, {0.0835168, 0.00532834}, {0.0668711, 0.0046596}, {0.0547511, 0.00409028}, {0.0456496, 0.00361041}, {0.0386394, 0.00320609}}

✔ Then, we can **reconstruct** the moments produced by each replica, using the single-parameter **Ansatz**:

$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$



✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✔ It exhibits a **soft end-point** behavior…

Pion PDF: Lattice Data

- We can follow an analogous procedure to infer, based upon **lattice data**, how the **hadronic scale PDF** should look like.

- Let us consider the list of **lattice QCD** moments:

n	[61]	[62]	[63]
1	0.254(03)	0.18(3)	0.23(3)(7)
2	0.094(12)	0.064(10)	0.087(05)(08)
3	0.057(04)	0.030(05)	0.041(05)(09)
4	Joo:2019bzr		0.023(05)(06)
5	Sufian:2019bol		0.014(04)(05)
6	Alexandrou:2021mmi		0.009(03)(03)

- Those verify the recurrence relation, thus being compatible with a **symmetric PDF** at ζ_H

- While also falling within the **physical bounds**.

$$\frac{1}{2^n} \leq \langle x^n \rangle^{\zeta}_{u_\pi} \left(\langle 2x \rangle^{\zeta}_{u_\pi} \right)^{-\gamma_0^n/\gamma_0^1} \leq \frac{1}{1+n}$$

Produced by

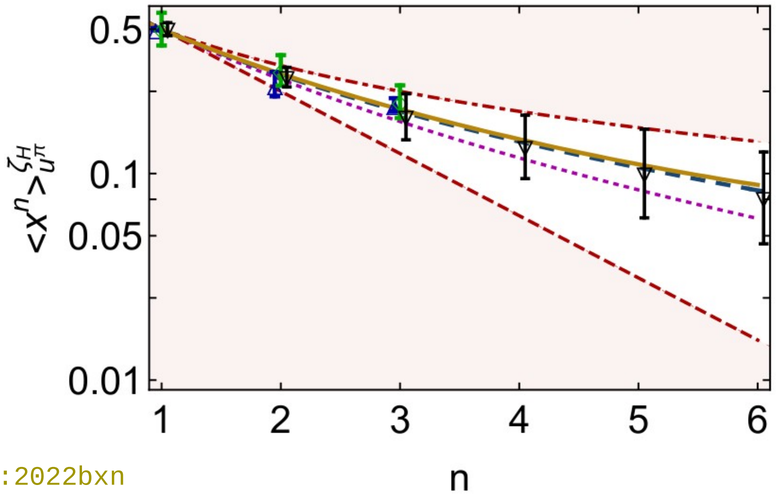
$$q(x;\zeta_H) = \delta(x - 1/2)$$

(infinitely heavy valence quarks)

Produced by

$$q(x;\zeta_H) = 1$$

(massless SCI case)



Cui:2022bxn

Pion PDF: Recap.

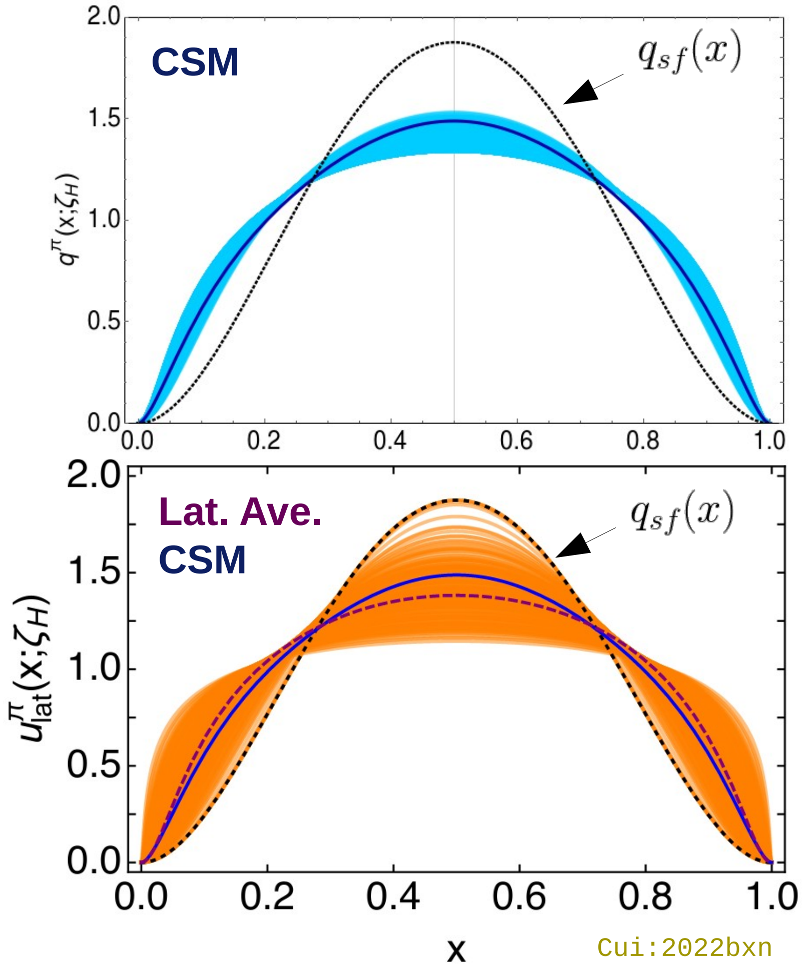
- The (**original**) **experimental** data yield a hadronic scale **PDF** compatible with **SCI results**.

  ➔ Thus should be disfavored since it does not produce the expected large-x behavior.

- Both (**ASV**) **experimental** and **lattice** data yield hadronic scale **PDFs** exhibiting soft end-point behavior and **EHM-induced broadening**.

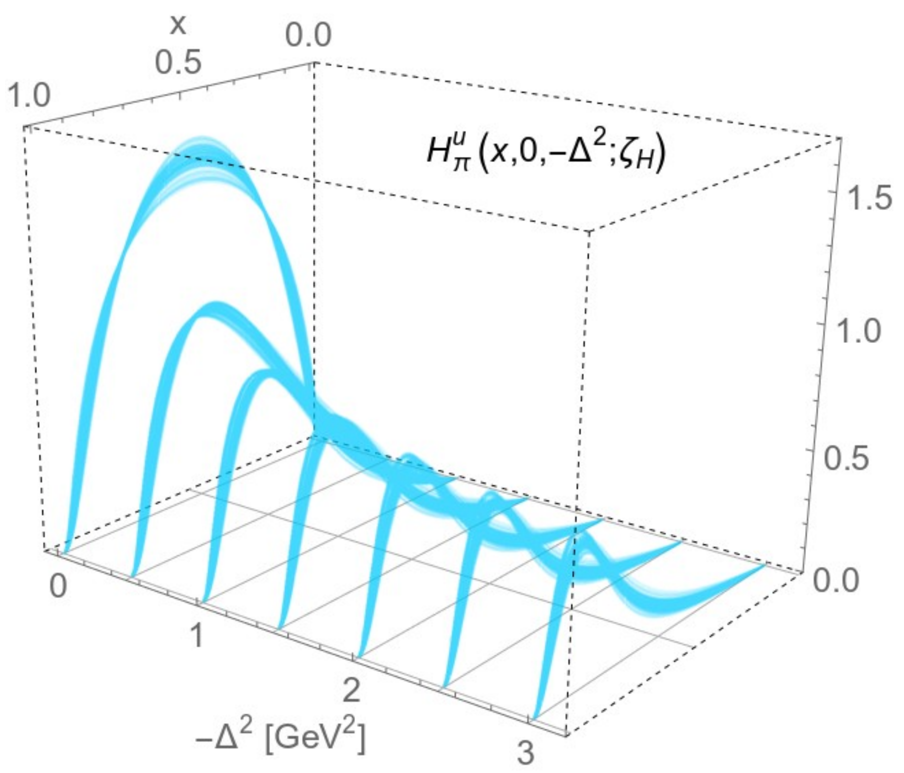
- The results are **compatible**, although current precision of the lattice moments still leaves us with a somewhat **wide band** of **uncertainty**.

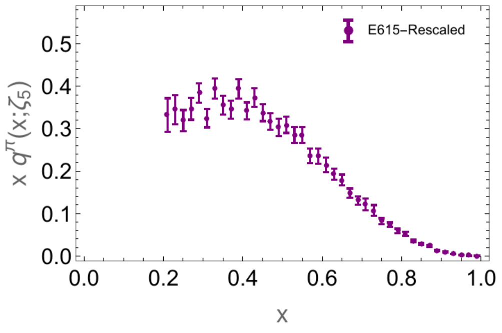
- Thus we focus on the **ASV** data for the rest of the discussion.

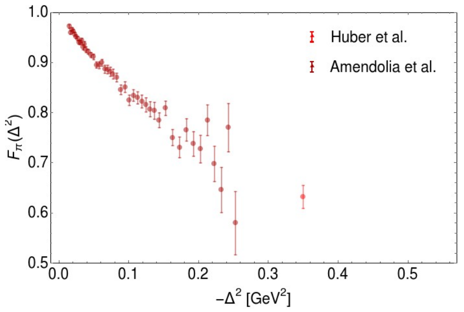


Cui:2022bxn

GPD from PDF and EFF







LFWF: Factorized models

Raya:2021zrz

- Starting with a **factorized LFWF**, $\psi_{\mathbf{P}u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H) = \tilde{\psi}_{\mathbf{P}u}^{\mathbf{u}}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$

This one shall be obtained as in the first part of the talk

- The overlap representation for the **GPD** entails:

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi_{\mathrm{P}}^{u*}\left(x_-,k_{\perp -}^2;\zeta_{\mathcal{H}}\right)\psi_{\mathrm{P}}^{u}\left(x_+,k_{\perp +}^2;\zeta_{\mathcal{H}}\right)$$

$$= \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

Heaviside Theta

This dictates the off-forward behavior of the GPD

- Where $z = s_\perp^2 = -t(1-x)^2/(1-\xi^2)^2$ and:

$$\Phi_{\mathbf{P}}^{u}(z;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\tilde{\psi}_{\mathbf{P}}^{u*}\left(\mathbf{k}_\perp^2;\zeta_H\right)\tilde{\psi}_{\mathbf{P}}^{u}\left((\mathbf{k}_\perp-\mathbf{s}_\perp)^2;\zeta_H\right)$$

… will be driven by the electromagnetic form factor

The GPD **model**

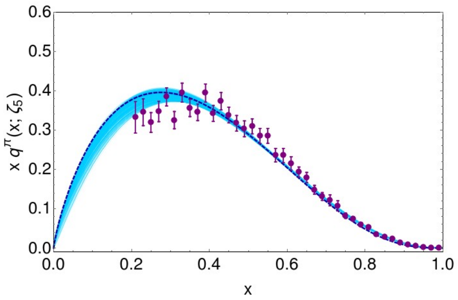
Raya:2021zrz

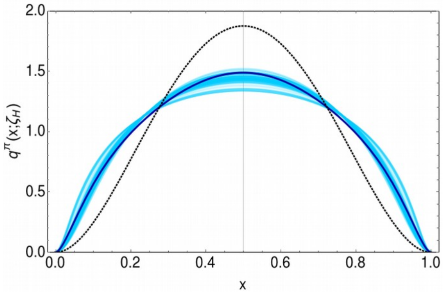
- The factorized **LFWF** thus motivates the following **GPD** model:

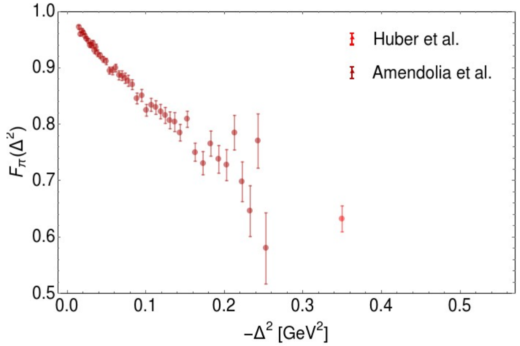
$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathrm{P}}(x_-;\zeta_H)u^{\mathrm{P}}(x_+;\zeta_H)}\Phi_{\mathrm{P}}(z;\zeta_H)$$

- The **PDF** might be inferred from **data**, as described before.
- Thus, **parameterized** by:

$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$







- The **GPD** connects **Φ(z)** with the **EFF** via**:**

$$F_{\pi}(t) = \int_0^1 dx\, u^{\pi}(x;\zeta_H)\Phi_{\pi}(z;\zeta_H)$$

- A useful **parametrization** is:

$$\Phi_{\pi}(z;\zeta_H) = \frac{1+(b_1-1)r_{\pi}^2/(6<x^2>)z}{1+b_1 r_{\pi}^2/(6<x^2>)z + b_2 z^2}$$

- Where $\mathbf{b_{1,2}}$ are parameters to be fitted to the experimental data.

(r_{π} could be treated as free or fixed parameter)

The GPD **model**

$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

- We have a **3-parameter** model for the **GPD**:

$$\{\rho, b_1, b_2\}$$

$$H_{\mathrm{P}}^u(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

$$u^\pi(x;\zeta_{\mathcal{H}}) = n_0 \ln(1+x^2(1-x)^2/\rho^2)$$

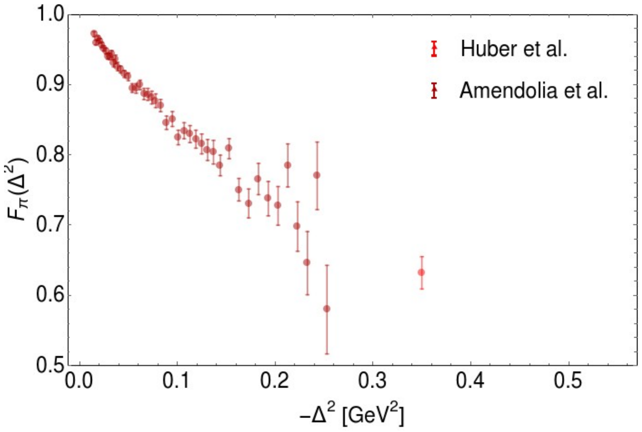
$$\Phi_\pi(z;\zeta_H) = \frac{1+(b_1-1)r_\pi^2/(6<x^2>)z}{1+b_1 r_\pi^2/(6<x^2>)z+b_2 z^2}$$

- The **strategy** is as follows:

  1) Following the described procedure for the **PDF**, generate a replica "*i*", storing the value ρ_i, and its probability of acceptance $P(\rho_i)$.

  2) Using such **replica**, integrate the **GPD** (for ξ=0) using random values for the free parameters.

  3) Compute the χ^2_i by comparing with the **EFF** experimental data [Amendolia:1984nz, JeffersonLab:2008jve].



The GPD model

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$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1+x^2(1-x)^2/\rho^2) \qquad \Phi_{\pi}(z;\zeta_H) = \frac{1+(b_1-1)r_\pi^2/(6<x^2>)z}{1+b_1 r_\pi^2/(6<x^2>)z + b_2 z^2}$$

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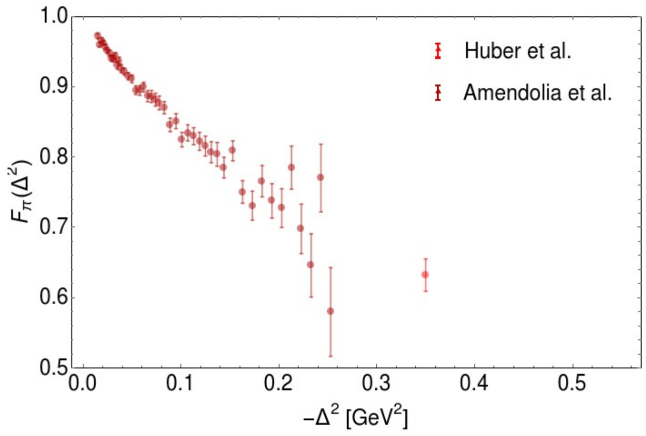


Low Q² data not used to fit the parameters !

The GPD model

- We have a **3-parameter** model for the **GPD**:

$$\{\rho, b_1, b_2\}$$

$$H^u_{\mathrm{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

$$u^\pi(x;\zeta_{\mathcal{H}}) = n_0 \ln(1+x^2(1-x)^2/\rho^2) \qquad \Phi_\pi(z;\zeta_H) = \frac{1+(b_1-1)r_\pi^2/(6<x^2>)z}{1+b_1 r_\pi^2/(6<x^2>)z + b_2 z^2}$$

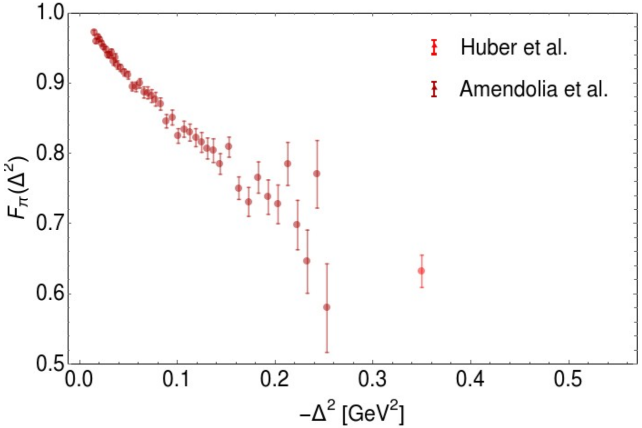
- The **strategy** contiues as follows:

4) Use χ^2_i to **calculate** $P(\{b_1^i, b_2^i\}|\rho_i)$

Accept the set of parameters with probability:

$$P(\{\rho_i, b_1^{(i)}, b_2^{(i)}\}) = P(\{b_1^{(i)}, b_2^{(i)}\}|\rho_i)P(\rho_i)$$

Repeat.



Numerical Results

Numerical **Results**

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

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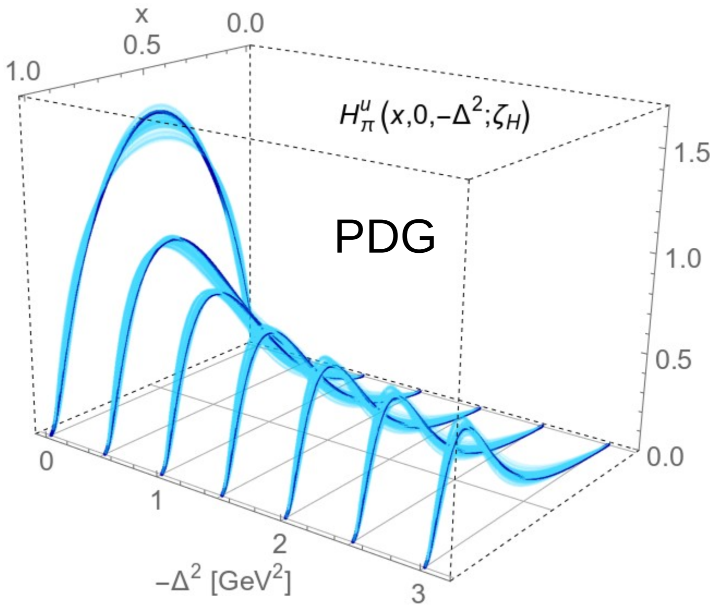
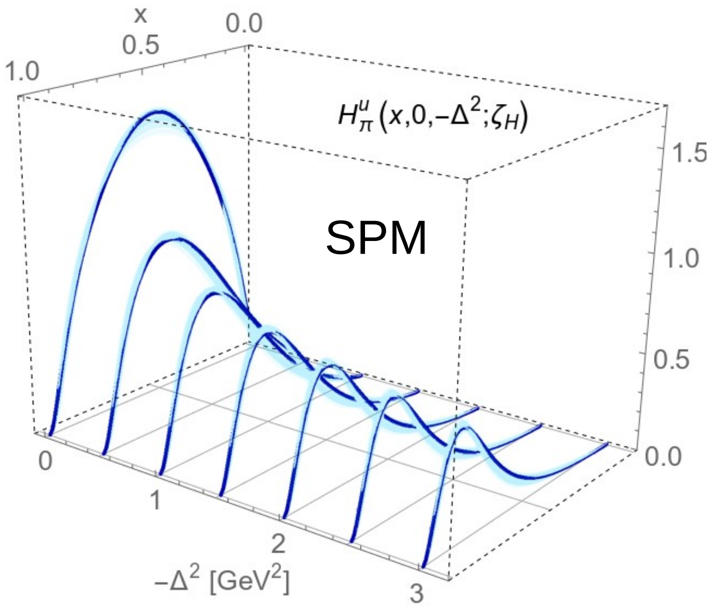
- Combining **pion PDF** (**ASV**) and **pion EFF** data, one arrives at:





CSM:

Raya:2021zrz
Raya:2022eqa

PDG:
$r_{\pi,}$ in (0.63,0.7) fm

SPM:
$r_{\pi,}$ = 0.640(7) fm

Cui:2021aee

Numerical **Results**

$$H_P^u(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

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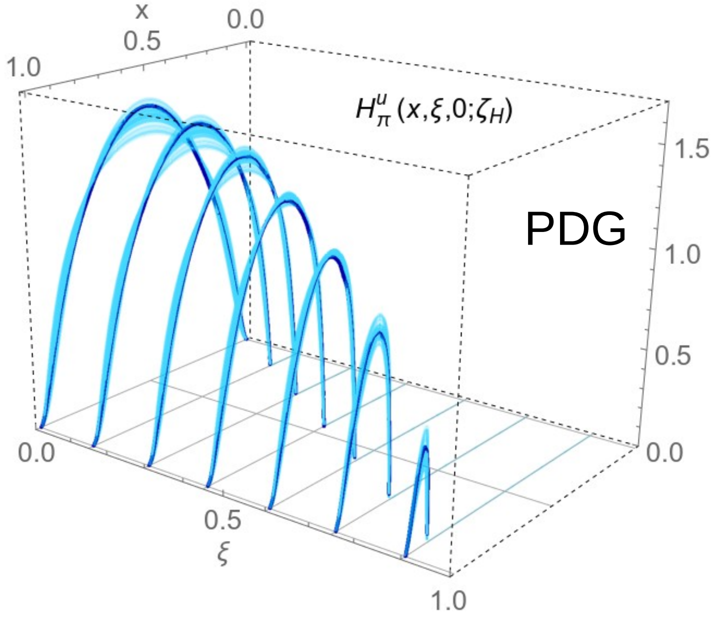

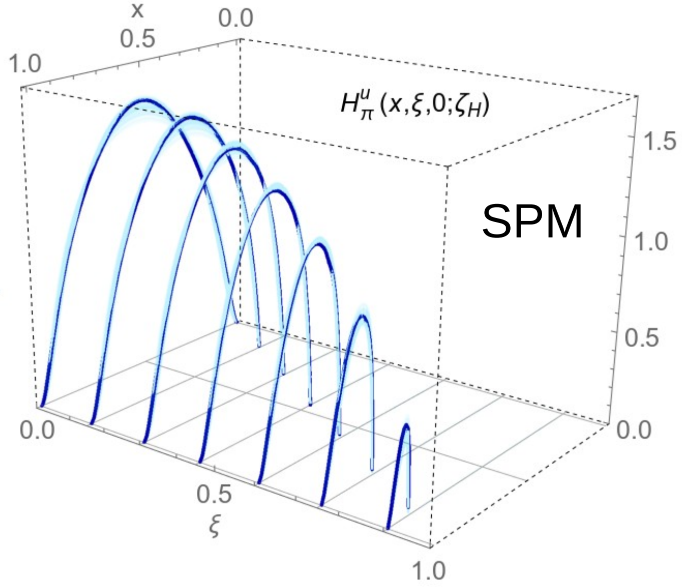

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PDG:
$r_{\pi,}$ in (0.63,0.7) fm

SPM:
$r_{\pi,}$ = 0.640(7) fm

Cui:2021aee

Numerical **Results**

$$q^{\pi}(x) = H_{\pi}^{u}(x,0,0)$$

$$H_{\rm P}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\rm P}(x_-;\zeta_H)u^{\rm P}(x_+;\zeta_H)}\Phi_{\rm P}(z;\zeta_H)$$

$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1+x^2(1-x)^2/\rho^2)$$

$$\Phi_{\pi}(z;\zeta_H) = \frac{1+(b_1-1)r_{\pi}^2/(6<x^2>)z}{1+b_1 r_{\pi}^2/(6<x^2>)z + b_2 z^2}$$

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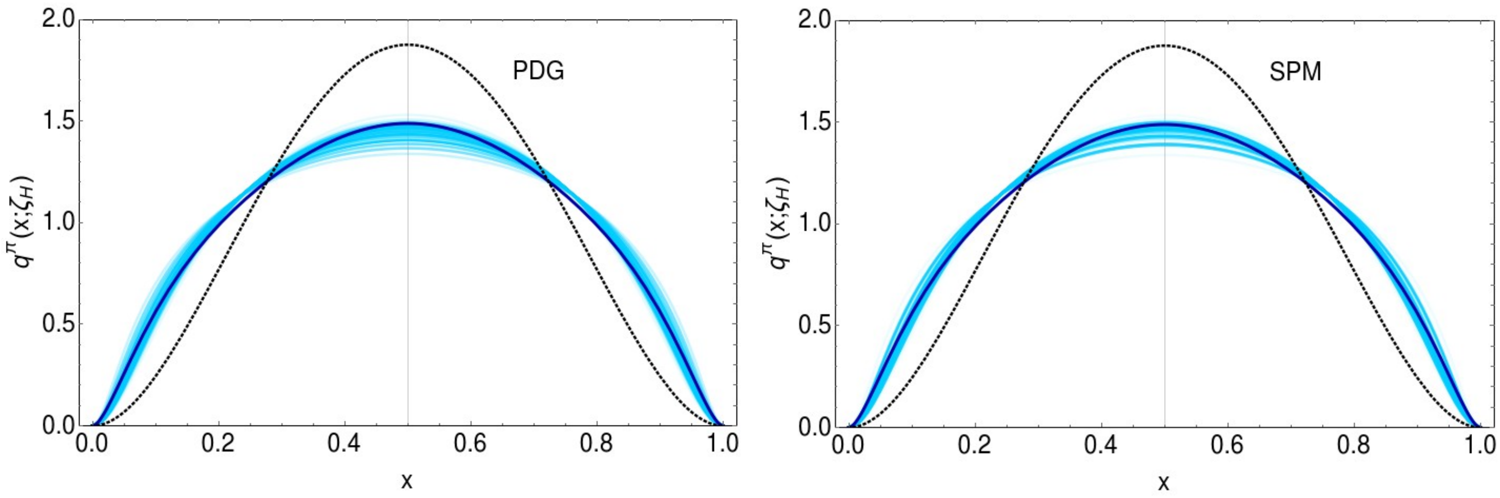

CSM:

Cui:2020tdf

PDG:
$r_{\pi,}$ in (0.63,0.7) fm

SPM:
$r_{\pi,}$ = 0.640(7) fm

Cui:2021aee

Numerical **Results**

$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

$$H^u_{\rm P}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\bf P}(x_-;\zeta_H)u^{\bf P}(x_+;\zeta_H)}\Phi_{\bf P}(z;\zeta_H)$$

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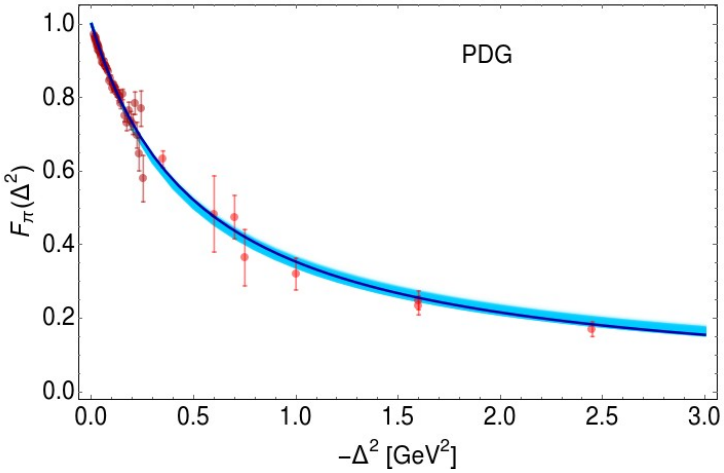

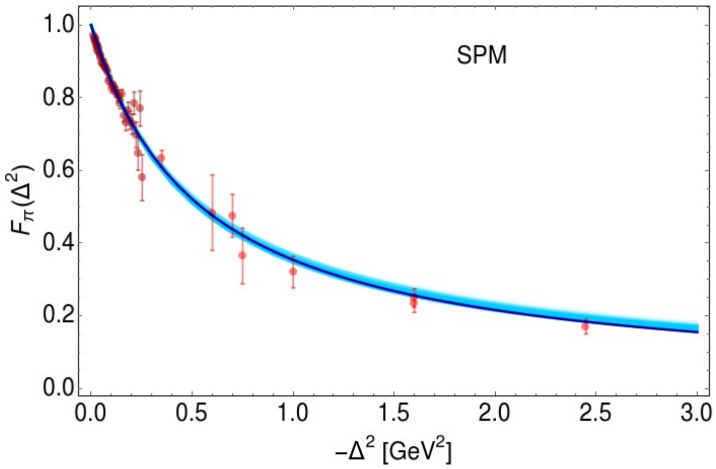

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Cui:2020tdf

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$r_{\pi,}$ in (0.63,0.7) fm

SPM:
$r_{\pi,}$ = 0.640(7) fm

Cui:2021aee

Low Q² data not used to fit the parameters !

Numerical **Results**

$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

$$H_P^u(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

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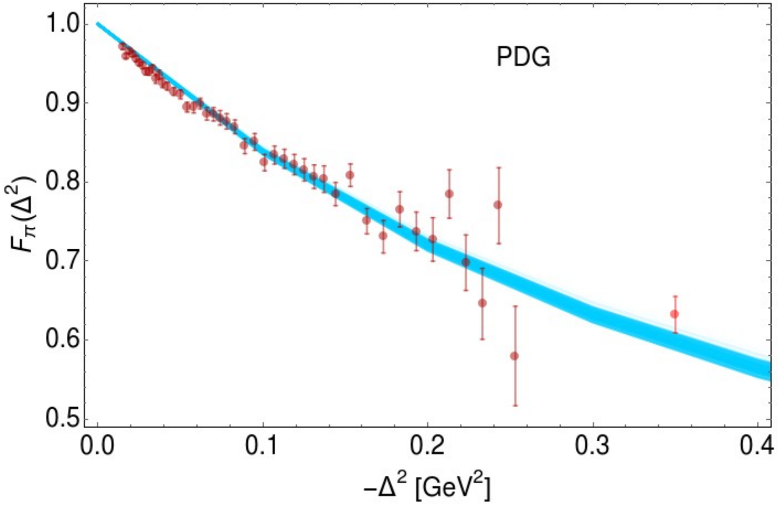
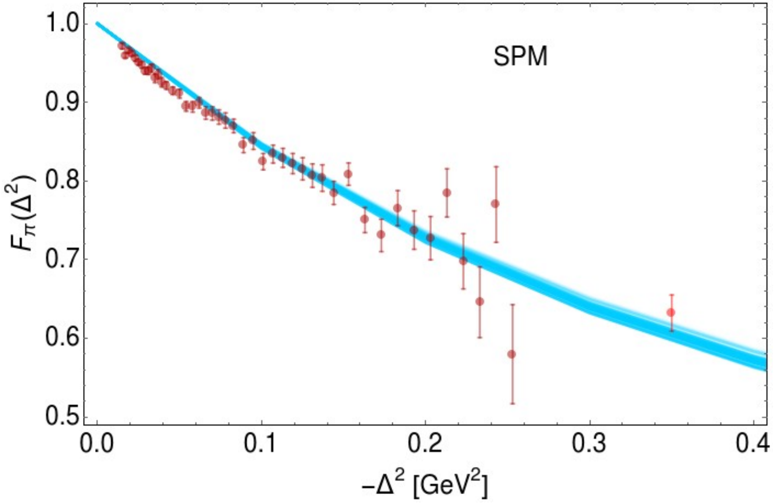




Only Small differences are found, and in a narrow window

… A consequence of the simple Ansatz for Φ

PDG:
$r_{\pi,}$ in (0.63,0.7) fm

SPM:
$r_{\pi,}$ = 0.640(7) fm

Cui:2021aee

Low Q^2 data not used to fit the parameters !

Numerical **Results**

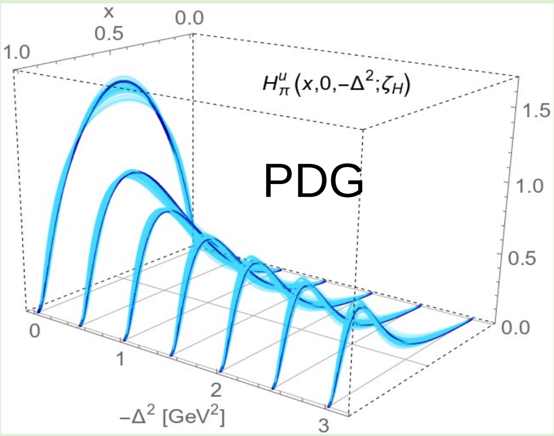
$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

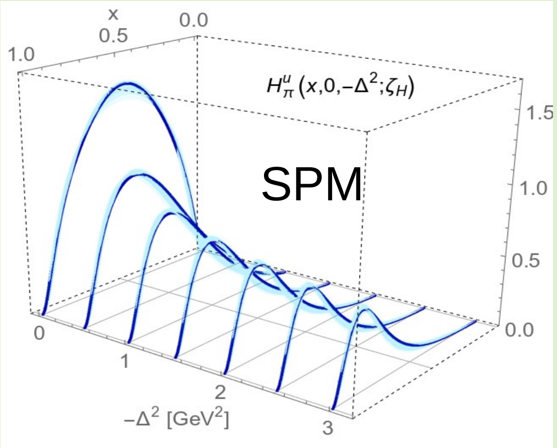
$$H_{\rm P}^u(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\bf P}(x_-;\zeta_H)u^{\bf P}(x_+;\zeta_H)}\,\Phi_{\bf P}(z;\zeta_H)$$

$$u^\pi(x;\zeta_{\mathcal{H}}) = n_0 \ln(1+x^2(1-x)^2/\rho^2) \qquad \Phi_\pi(z;\zeta_H) = \frac{1+(b_1-1)r_\pi^2/(6<x^2>)z}{1+b_1 r_\pi^2/(6<x^2>)z+b_2 z^2}$$

- Combining **pion PDF** (**ASV**) and **pion EFF** data, one arrives at:

Thus, above **$-\Delta^2$=0.1 GeV^2**, the **GPDs** are practically **equivalent**.





Only Small differences are found, and in a narrow window

… A consequence of the simple Ansatz for Φ

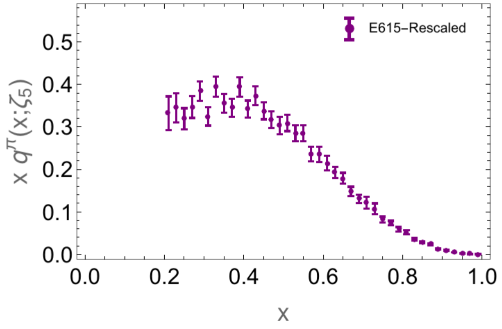
PDG:
$r_{\pi,}$ in (0.63,0.7) fm

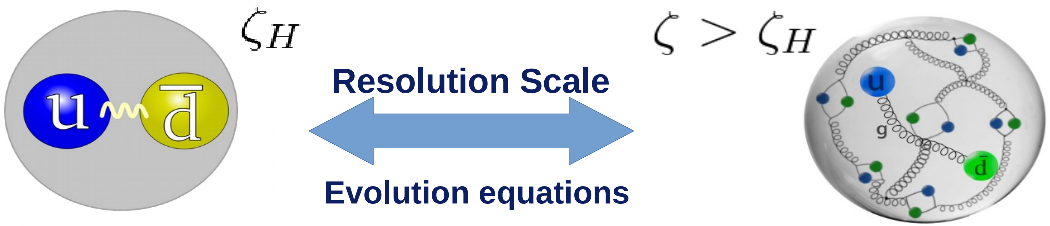
SPM:
$r_{\pi,}$ = 0.640(7) fm

Cui:2021aee

Numerical **Results**

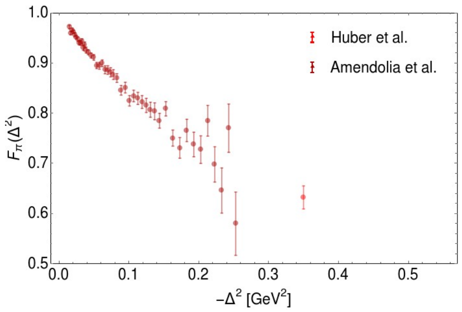
- We can then **evolve** to $\zeta > \zeta_H$ and **compare**!



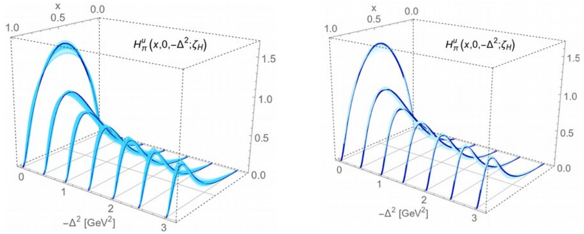


Valence **PDF**, ζ_{exp}

Experimental **EFF**



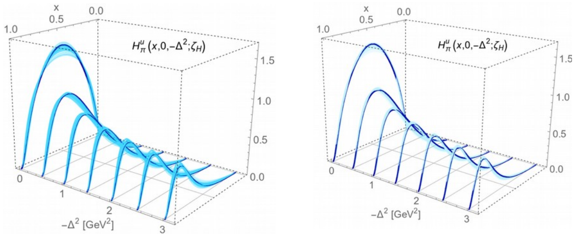
Valence GPD, ζ_H

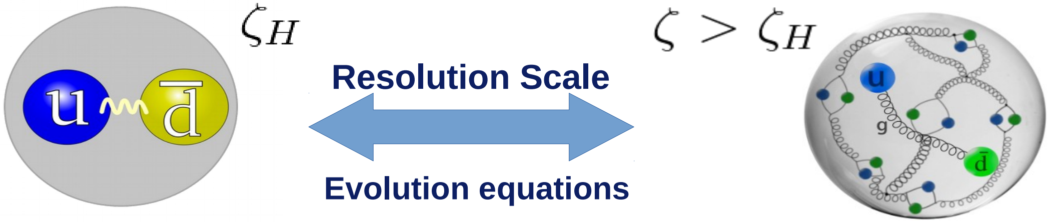


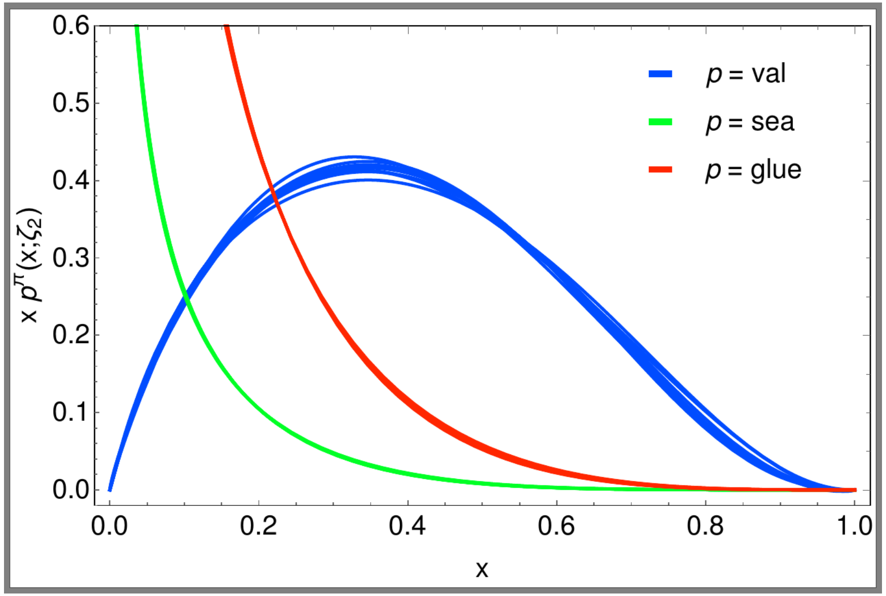
All **GPDs** at some ζ

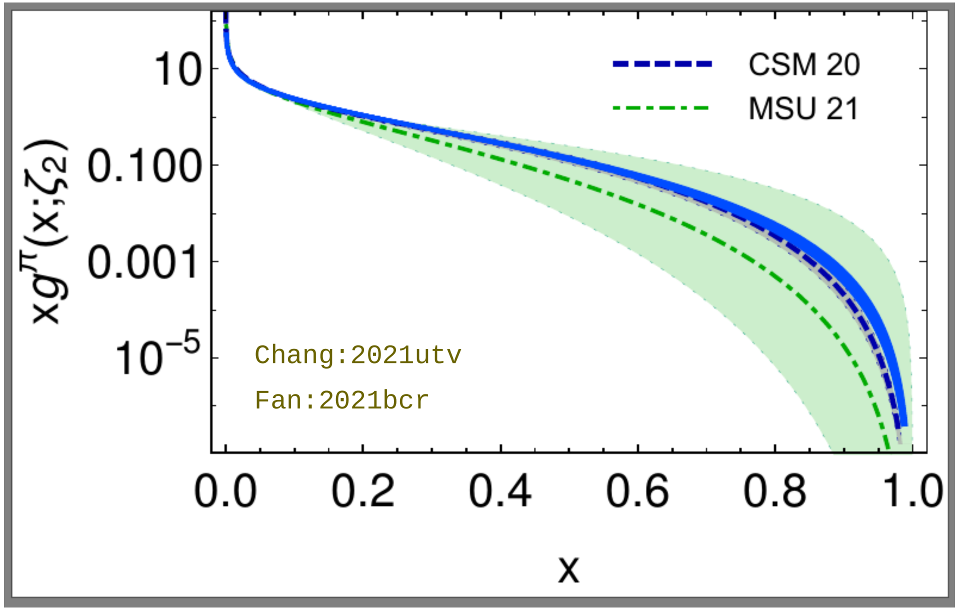
Numerical **Results**

- We can then **evolve** to $\zeta > \zeta_H$ and **compare**!

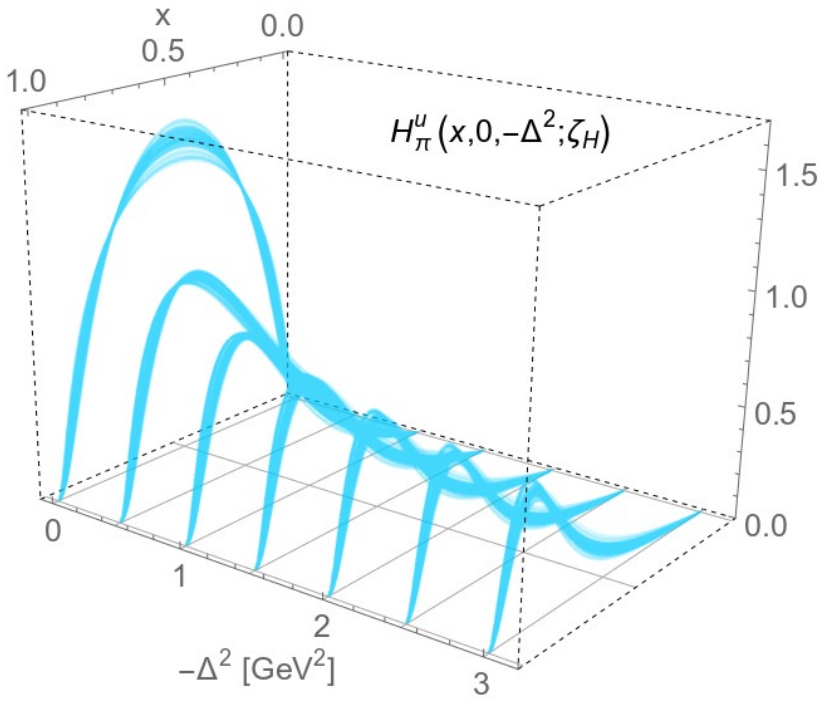








Summary


x
0.0
0.5
1.0
$H_{\pi}^{u}(x,0,-\Delta^2;\zeta_H)$
1.5
1.0
0.5
0.0
0
1
2
3
$-\Delta^2$ [GeV^2]


I just need
the main ideas

Summary

- A compact expression for the hadronic scale pion **GPD** was written on the grounds of the overlap representation of a **factorized** **LFWF**:

$$\psi_{\mathbf{P}u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H) = \tilde{\psi}_{\mathbf{P}u}^{\mathbf{u}}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$$

PDF

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

- Where $z = s_\perp^2 = -t(1-x)^2/(1-\xi^2)^2$ and:

$$\Phi_{\mathbf{P}}^{u}(z;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\tilde{\psi}_{\mathbf{P}}^{u*}\left(\mathbf{k}_\perp^2;\zeta_H\right)\tilde{\psi}_{\mathbf{P}}^{u}\left((\mathbf{k}_\perp-\mathbf{s}_\perp)^2;\zeta_H\right)$$

Summary

- A compact expression for the hadronic scale pion **GPD** was written on the grounds of the overlap representation of a **factorized** **LFWF**:

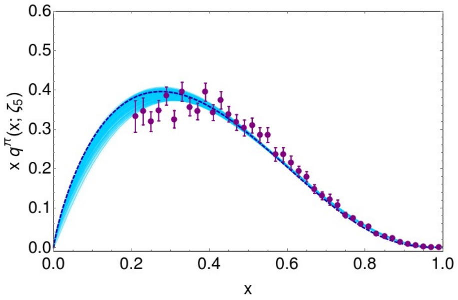
$$\psi^{\uparrow\downarrow}_{\mathbf{P}u}(x,k_\perp^2;\zeta_H) = \tilde{\psi}^{\mathbf{u}}_{\mathbf{P}u}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$$

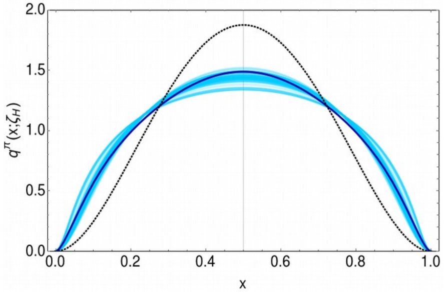
PDF

$$H^u_{\mathrm{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\,\Phi_{\mathbf{P}}(z;\zeta_H)$$

- A hadronic scale **PDF** is obtained from downwards evolution of the **ASV** experimental data, using the **all orders** scheme.

(by chi2 means, we take or discharge a particular replica)





- And parameterized as:

$$u^\pi(x;\zeta_{\mathcal{H}}) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$

(1 parameter)

Summary

- A compact expression for the hadronic scale pion **GPD** was written on the grounds of the overlap representation of a **factorized LFWF**:

$$\psi^{\uparrow\downarrow}_{\mathbf{P}u}(x,k_\perp^2;\zeta_H) = \tilde{\psi}^{\mathbf{u}}_{\mathbf{P}u}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$$

PDF

$$\Rightarrow \quad H^u_{\mathrm{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\,\Phi_{\mathbf{P}}(z;\zeta_H)$$

- A hadronic scale **PDF** is obtained from downwards evolution of the **ASV** experimental data, using the **all orders** scheme.

(by chi2 means, we take or discharge a particular replica)

- The particular **replica** is employed to obtain the **EFF** and compare with **JLab** experimental data.

- Such that we can accept/reject a particular set of parameters for Φ

$$F_\pi(t) = \int_0^1 dx\, u^\pi(x;\zeta_H)\Phi_\pi(z;\zeta_H)$$

$$\Phi_\pi(z;\zeta_H) = \frac{1+(b_1-1)r_\pi^2/(6<x^2>)z}{1+b_1 r_\pi^2/(6<x^2>)z + b_2 z^2}$$

(2-3 parameters)

Summary

- A compact expression for the hadronic scale pion **GPD** was written on the grounds of the overlap representation of a **factorized** **LFWF**:

$$\psi^{\uparrow\downarrow}_{\mathbf{P}u}(x, k_\perp^2; \zeta_H) = \tilde{\psi}^{\mathbf{u}}_{\mathbf{P}u}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$$

PDF

$$\Rightarrow H^u_{\mathrm{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\,\Phi_{\mathbf{P}}(z;\zeta_H)$$

- A hadronic scale **PDF** is obtained from downwards evolution of the **ASV** experimental data, using the **all orders** scheme.
- The particular **replica** is employed to obtain the **EFF** and compare with **JLab** experimental data.
- At the end of the day, the **DGLAP** **GPD** is described by **only 3-4** parameters.
- We can also evolve the **GPD** and produce **sea** and **gluon**.