Introduction to Special Relativity

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Speed of light in vacuum

- Waves travel in medium.

- Light can still travel through space.

Medium: **aether**.

- Michelson-Morley experiment tries to measure the velocity of Earth with respect to aether.
- Measuring speed of light in two directions to find the difference.
- No difference found -> failed

Einstein's Postulates

1. The law of physics are the same in all **inertial frames of reference**.
2. The speed of light in vacuum has the same value in all **inertial frames of reference**.

Special theory of relativity

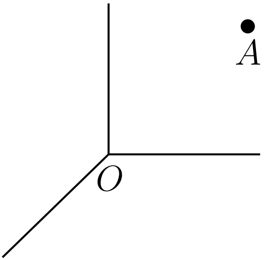
compare quantities measured by two observers in relative motion

only focus on special kind of observers: the ones in inertial frames of reference

Frame of reference

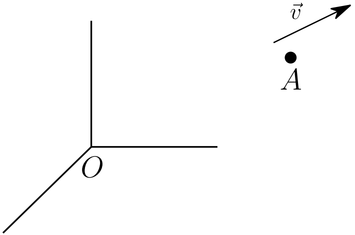
To describe position, velocity, acceleration, etc. of an object, we need to specify "who is measuring".

To quantify this, we set up a coordinate system.



At time t, position, velocity, and acceleration of object A with respect to O are $(x(t), y(t), z(t))$, $\vec{v}(t)$, and $\vec{a}(t)$, respectively.

Inertial Frame of reference



When there is no force acting on an object, if the velocity of that object as measured by a frame of reference is a constant, then that frame of reference is called an inertial frame of reference.

Event

Event: a physical occurence at a certain instant and place.

Examples of event:

- A light bulb going off

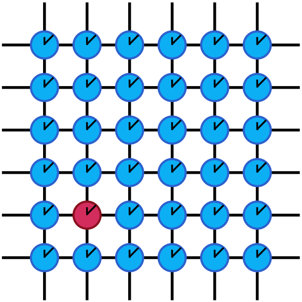
- A tree falling

Each observer describes an event

by specifying time t and spatial coordinates x, y, z

Grid of clocks

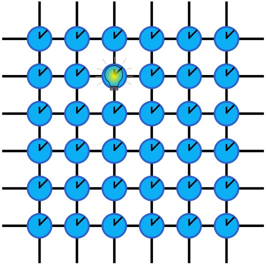
Consider an inertial observer. Let us suppose that there is a 3D grid with a clock at each intersection. These clocks are in sync, and tick at the same rate.



We should specify a point to be the origin (x, y, z) = (0, 0, 0). For example, the position of the red clock.

We should also specify a reference time t=0. For example, today at noon.

Describing event



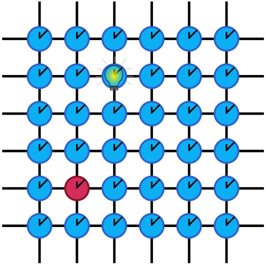
Suppose an event occurs at some instant and place. The clock at that position registers the time that the event occurs.

Imagine e.g. that that clock mysteriously stops when the event occurs, or that some gnome at that position records the time of the event and their position.

The event is then described by this observer O using four numbers which are time t and space x, y, z.

Measurement: The observer records (not see) the event.

Measuring versus seeing



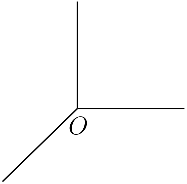
Inertial observer **measures** the event:

- specify the origin (as well as axes)
- specify starting time
- wait until the event happens
- the time and position that the event occurs is recorded (say, when the event occurs, a gnome  at that position records the time on the clock).

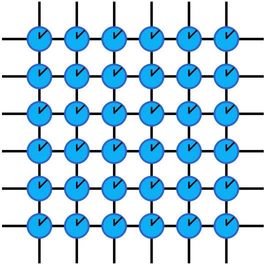
The observer **sees** the event: wait until the signal from the event reaches the origin.

Unless otherwise stated, observer describes event by measuring.

Observer



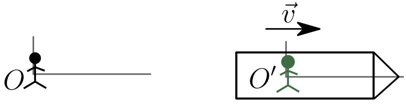
Simple viewpoint: someone staying at the origin of their coordinate system



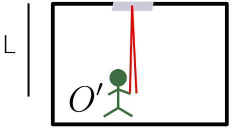
Complicated viewpoint: a 3D grid of clocks (with a gnome at each position recording time and space events)

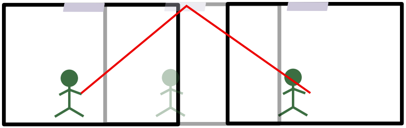
Light and mirror

Consider two inertial observers



Observer O' shines light which reflects off the mirror





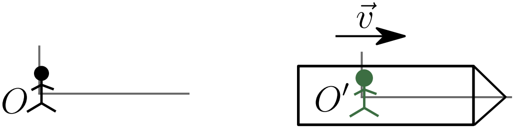
$$\Delta t' = \frac{2L}{c}$$

$$\Delta t = \gamma \Delta t'$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

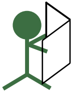
Time dilation

Consider two inertial observers

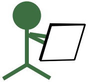


and two events:

A: observer O' starts reading



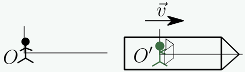
B: observer O' finishes reading



We have to state who is observing the events.

Time dilation (2)

Events as observed by observer O

A: 

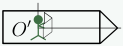
at time t_A

B: 

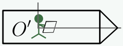
at time t_B

Time interval $\Delta t = t_B - t_A$

Events as observed by observer O'

A: 

at time t'_A

B: 

at time t'_B

Time interval $\Delta t' = t'_B - t'_A$

It turns out that

$$\Delta t \neq \Delta t'$$

- Time is not absolute
- Time interval depends on observers

Q: How long does O' actually takes to finish reading?

A: Depends on observer

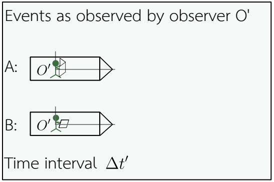
Time dilation (3)

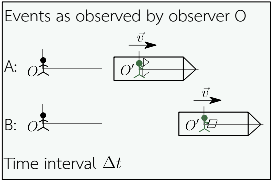
Definition: proper time is a time interval when

two events occur at the same place

Property: For inertial observers

time interval between two events $\geq$ proper time





$\Delta t'$ is the proper time

between the events A and B

$$\Delta t > \Delta t'$$

Time dilation

Time dilation (4)

Let $\Delta\tau$ be a proper time for a(n appropriate) pair of events.

The corresponding frame is called the **rest frame**.

An observer moving with velocity $\vec{v}$ relative to the rest frame will measure the time interval to be

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta\tau$$

Example: Suppose that the proper time is $\Delta\tau = 4\ \mathrm{s}$, and that the relative speed between the two frames is $v = 0.6c$.

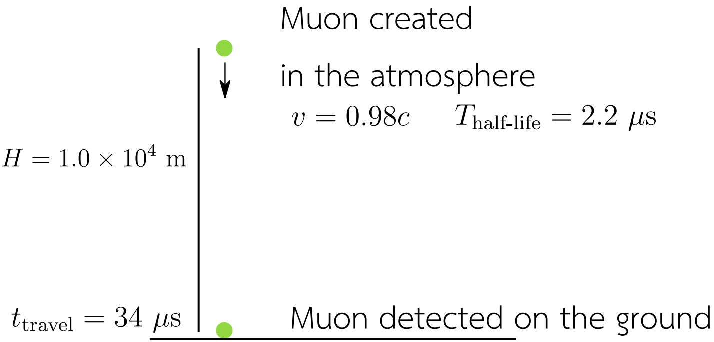
This gives

$$\gamma = \frac{5}{4}$$

So

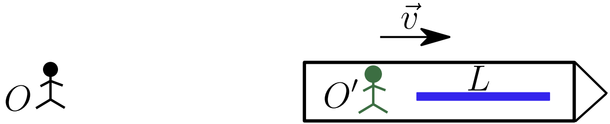
$$\Delta t = 5\ \mathrm{s}$$

Atmospheric muon



Without relativistic effect, a fraction (0.0022%) of muon is expected to survive. But with time dilation, the muon half-life as observed by the Earth observer is $T_{\text{half-life-observed}} = 11\ \mu\text{s}$. So 12% of muon is expected to survive. This agrees with experiment.

Length contraction



A stick of length L is at rest with respect to O'.

The observer O measures the length to be

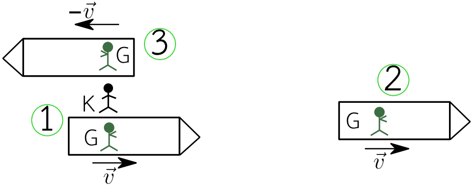
$$\frac{L}{\gamma}$$

NOT see

Length contraction: moving objects are shorter

(by measurement).

Twin paradox

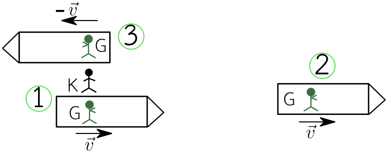


Consider a pair of twin K and G. K stays on Earth. G travels to a distant star with speed v, then immediately returns. When they meet, who are older?

Wrong interpretation 1: From K (G) perspective, G (K) keeps moving. So from "moving clocks are slower", the time G (K) takes for the journey are shorter. So K (G) will become older when they meet. CONFUSION

Wrong interpretation 2: Event (1) and (3) occur at the same position from the perspective of both K and G. So the journey time is proper time. Both of them should be aged the same.

Resolving the twin paradox



When they meet, who is older?

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Wrong interpretation 2: Event (1) and (3) occur at the same position from the perspective of both K and G. So the journey time is proper time. Both of them should be aged the same.

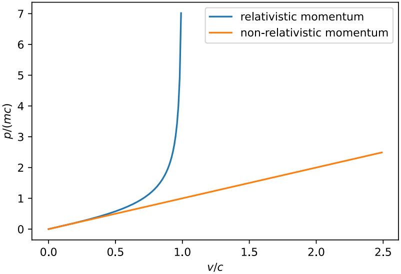
Both of these interpretations are wrong because G is **not** in an inertial frame.

Correct interpretation: Consider events (1) and (2). During these events, both K and G are inertial observers. The time measured by G is proper time, which is shorter than that measured by K. The same conclusion can be obtained when we consider events (2) and (3). So when they meet, K is older.

Relativistic momentum

$$\vec{p} = \gamma m\vec{v} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

This form ensures that conservation of momentum holds in any inertial frame of reference.



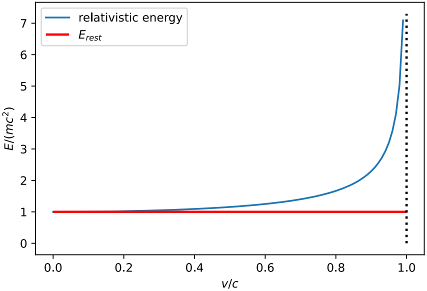
$$p \approx mv \text{ for small } \frac{v}{c}$$

$$p \to \infty \text{ as } v \to c$$

Relativistic energy

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This form ensures that conservation of energy holds in any inertial frame of reference.



$$E \overset{\text{small } v/c}{\approx} mc^2 + \frac{1}{2}mv^2$$

$$E \to \infty \text{ as } v \to c$$

Energy-momentum-mass relation

From $E = \gamma mc^2$ and $\vec{p} = \gamma m\vec{v}$,

it can be shown that

$$E^2 = p^2c^2 + m^2c^4$$

This also applies to massless particle:

$$E = pc \text{ when } m = 0$$

Energy and momentum of photon

A photon of frequency f has wavelength $\lambda = f/c$.

The photon has energy $E = hf$,

and momentum $p = \dfrac{h}{\lambda}$. This agrees with $E = pc$.

Proved by Compton scattering:
wavelength of photon changes
after colliding with an electron.

Conservation laws

Consider collisions of elementary particles.

Energy and momentum are conserved in the process.

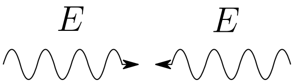
However, **total mass does not need to be conserved.**

Elastic scattering: total mass is conserved.

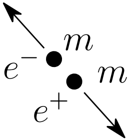
Inelastic scattering: total mass is not conserved.

Pair production

Two photons producing an electron-positron pair



Before



After

Consider two photons, each with energy E colliding with each other to produce an electron-positron pair. The least energy for this process to occur is $E_{\text{threshold}} = mc^2$ $\approx 0.51\ \text{MeV}$ for electron

When $E = E_{\text{threshold}}$, the electron and positron are at rest. But when $E > E_{\text{threshold}}$, they move in the opposite direction with the same speed.