Orbital angular momentum at small-x

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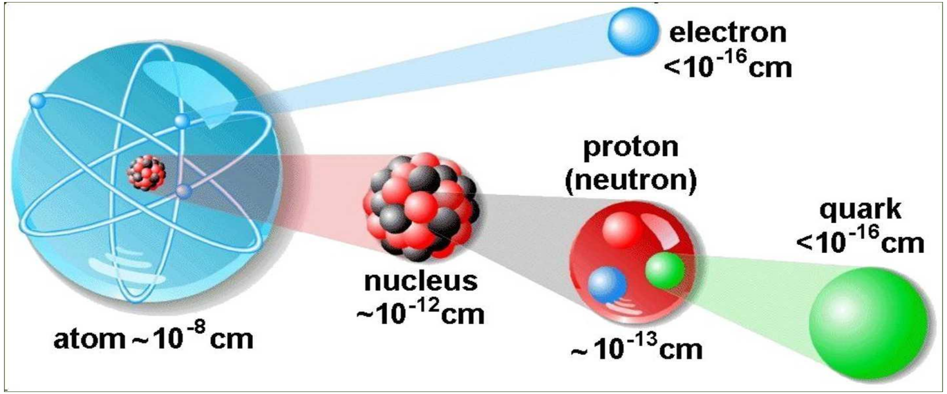
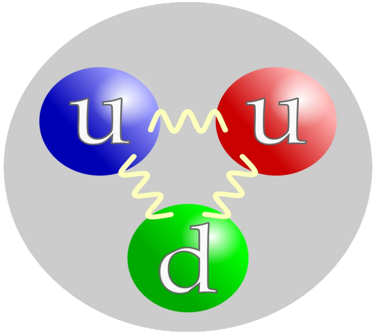
Proton Spin - Quark Model





- Protons are one of the three particles that make up atoms - the building blocks of the universe.
- In the constituent quark model of the proton, the picture is simple: two spin-up quarks and one spin-down quark.
- This predicts that all the proton spin is carried by the quarks, such that

$$S_q = \frac{1}{2}$$

Proton Spin Puzzle

The spin puzzle began when the EMC collaboration measured the proton g1 structure function in 1988. Their data resulted in

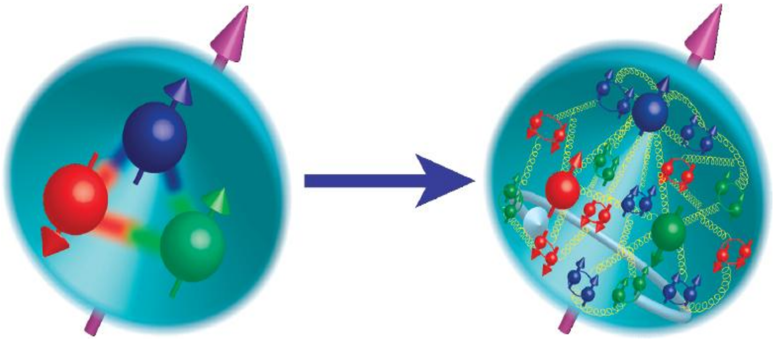
$$S_q \approx 0.05$$

Ashman, J. et al.

It appears (constituent) quarks do not carry all the proton spin (which would have corresponded to $S_q = \frac{1}{2}$).

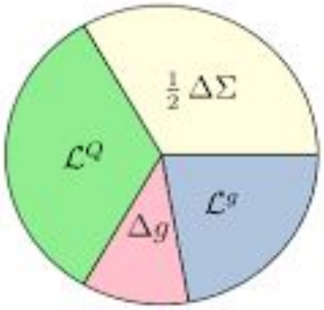
$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

- carried by spin of gluons
- orbital angular momentum of quarks and gluons

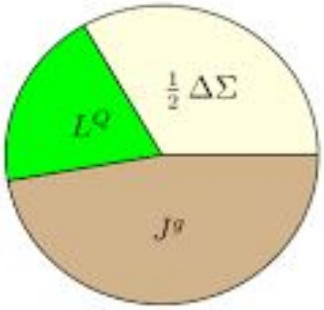


Nucleon Spin decomposition

Jaffe-Manohar



Ji



J-M decomposition

$$\frac{1}{2}\Delta\Sigma = \int \psi^\dagger \frac{1}{2}\Sigma\psi d^3x$$

$$\mathcal{L}^Q = \int \psi^\dagger x \times \frac{1}{i}\nabla\psi d^3x$$

$$\Delta g = \int E^a \times A^a d^3x$$

$$\mathcal{L}^g = \int E^{ai} x \times \nabla A^{ai} d^3x$$

$$\mathcal{L}^Q \neq L^Q$$

$$\Delta g + \mathcal{L}^g \neq J^g$$

Ji decomposition

$$\frac{1}{2}\Delta\Sigma = \int \psi^\dagger \frac{1}{2}\Sigma\psi d^3x$$

$$L^Q = \int \psi^\dagger x \times \frac{1}{i}D\psi d^3x$$

$$J^g = \int x \times (E^a \times B^a) d^3x$$

$$\partial - igA$$

Each term is not separately gauge-invariant

No further decomposition of J_g

Proton spin decompositions

- **Jaffe - Manohar [R. L. Jaffe and A. Manohar,1990]**

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q^{can} + L_g^{can}$$

common and well known

- **Ji [Ji,1997]**

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q^{Ji} + J_g$$

Proton's Spin Status

- Current understanding of spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Quark helicity
Best known

Gluon helicity
Start to know

Orbital angular momentum of quarks and gluons
Not measured yet
not even well defined

$$\frac{1}{2}\int dx(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s})$$

$\sim 30\%$

Spin "puzzle"

$$\Delta G = \int dx \Delta g(x)$$

$\sim 20\%$ (with RHIC data)

from NLO QCD global analysis

Gluon OAM from Wigner distribution

Orbital angular momentum of gluon can be expressed as the phase space average of the classical orbital angular momentum weighted with the Wigner distribution of polarized gluons in a longitudinally polarized nucleon.

$$L_g = \int dx \int d^2 b_\perp \int d^2 k_\perp \, (b_\perp \times k_\perp) \, W(x, b_\perp, k_\perp)$$

The Wigner distribution is the phase space distribution of gluons in transverse momentum ($k_\perp$) - impact parameter ($b_\perp$) space.

$$W_g(x, k_\perp, b_\perp) = \int \frac{d^2\Delta_\perp}{2\pi^2} e^{-i\Delta_\perp . b_\perp} \int \frac{d^2 z_\perp}{2\pi^2} e^{-iz_\perp . k_\perp} \int \frac{dz^-}{2(2\pi) x P^+} e^{-ixP^+ z^-} \times$$

$$\langle P^+, \frac{-\Delta_\perp}{2}, S \, | \mathrm{Tr}\, F^{+i}(0)\, U_{[\eta_1]}(0; z)\, F^{+i}(z)\, U_{[\eta_2]}(z; 0) | P^+, \frac{\Delta_\perp}{2}, S \rangle$$

Gluon OAM

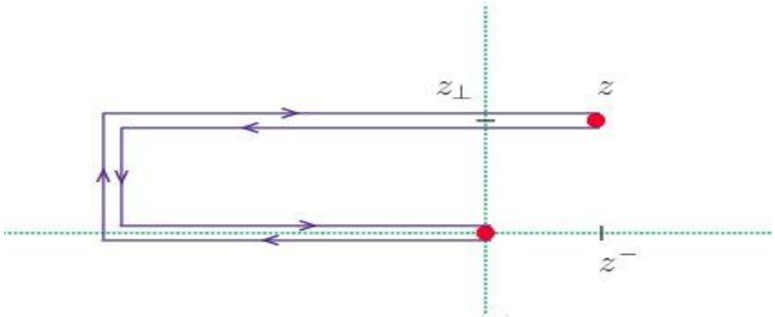
Gluon orbital angular momentum is essentially a two-point correlation function of the field strength tensors at two space-time points.

$$L_g(x) = \int d^2 b_\perp \epsilon^{kj} b_\perp^k \int \frac{d^2\Delta_\perp}{2\pi^2} e^{-i\Delta_\perp . b_\perp} \int \frac{dz^-}{2(2\pi)xP^+} e^{-ixP^+ z^-} \lim_{z_\perp^j \to 0} \frac{1}{i} \frac{\partial}{\partial z_\perp^j}$$
$$\langle P^+, \frac{-\Delta_\perp}{2}, S \,|\mathrm{Tr}\, F^{+i}(0)\, U_{[\eta_1]}(0;z)\, F^{+i}(z) U_{[\eta_2]}(z;0)| P^+, \frac{+\Delta_\perp}{2}, S\,\rangle$$

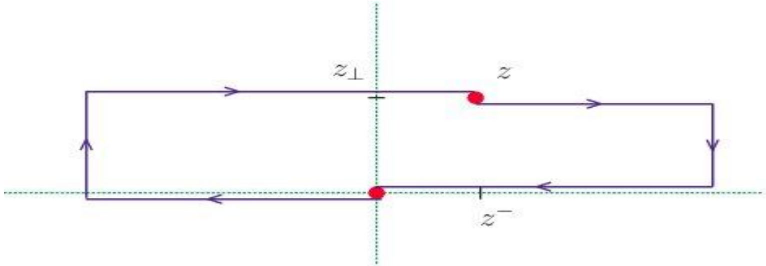
Introducing appropriate **gauge links** between the gauge fields to decide which of type of gluon OAM is this...

Two important choices for gauge links

Staple gauge links (Jaffe-Manohar OAM)

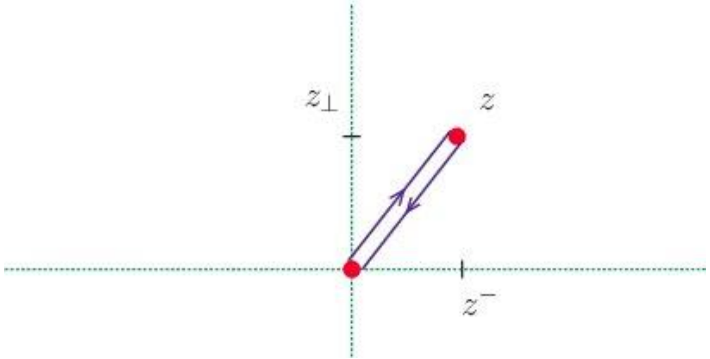


Weizsäcker-William type



Dipole type

Straight gauge links (Ji's OAM)



Gluon OAM at small-x

Taking the transverse derivative of the gluon OAM operator and expanding the exponential, we get,

$$
\begin{aligned}
L_g(x) &= \epsilon^{kj} \lim_{\Delta^k \to 0} \frac{\partial}{\partial \Delta^k} \int \frac{dz^-}{2(2\pi)\, xP^+}\, e^{-ixP^+z^-} \left\langle P^+, -\frac{\Delta_\perp}{2}, S \right| \mathrm{Tr} \sum_{m=1}^{5} \mathcal{O}^j_m \left| P^+, +\frac{\Delta_\perp}{2}, S \right\rangle \\
&\equiv \epsilon^{kj} \left[\frac{1}{x} \mathcal{L}^{jk}_{g,0}(x) + \mathcal{L}^{jk}_{g,1}(x) + x \mathcal{L}^{jk}_{g,2}(x) + x^2 \mathcal{L}^{jk}_{g,3}(x) + \dots \right]
\end{aligned}
$$

eikonal
(leading scattering)

sub-eikonal
(suppressed by one power of x)

sub-sub-eikonal
(suppressed by two power of x)

OAM of gluons at eikonal limit

In the eikonal approximation, $exp(-ixP^+z^-) \simeq 1$ we have derived a generalized form of the gauge-invariant OAM operator at small-x.

$$\mathcal{L}^{jk}_{g,0}(x) = -\lim_{\Delta^k \to 0} \frac{\partial}{\partial \Delta^k} \left\langle P^+, -\frac{\Delta_\perp}{2}, S \middle| \mathrm{Tr}\, F^{+i}(0^-, 0_\perp)\, \mathcal{D}^j_{\mathrm{pure}} A^i_{\mathrm{phys,\,o}}(0^-, 0_\perp)\, \middle| P^+, +\frac{\Delta_\perp}{2}, S \right\rangle - \lim_{\Delta^k \to 0} \frac{\partial}{\partial \Delta^k} \left\langle P^+, -\frac{\Delta_\perp}{2}, S \middle| igF^{+i}(0^-, 0_\perp)\, U(0^-, \eta_1^-; 0_\perp) \left[A^j_{\mathrm{pure}}(\eta_1^-, 0_\perp) U(\eta_1^-, \eta_2^-; 0_\perp) A^i_{\mathrm{phys,o}}(\eta_2^-, 0_\perp) - A^i_{\mathrm{phys,o}}(\eta_1^-, 0_\perp) U(\eta_1^-, \eta_2^-; 0_\perp) A^j_{\mathrm{pure}}(\eta_2^-, 0_\perp) \right] U(\eta_2^-, 0^-; 0_\perp) \middle| P^+, +\frac{\Delta_\perp}{2}, S \right\rangle.$$

Under PT transformation

$$\mathscr{L}^g_0(x, k_\perp, \Delta_\perp, S) = \mathscr{L}^g_0(x, -k_\perp, -\Delta_\perp, -S)$$

No contribution from eikonal term !

The bridge between Jaffe and Ji's OAM

$$\mathcal{L}^{jk}_{g,0}(x) = -\lim_{\Delta^k \to 0} \frac{\partial}{\partial \Delta^k} \left\langle P^+, -\frac{\Delta_\perp}{2}, S \right| \mathrm{Tr}\, F^{+i}\left(0^-, 0_\perp\right) \mathcal{D}^j_{\text{pure}} A^i_{\text{phys, o}}\left(0^-, 0_\perp\right) \left| P^+, +\frac{\Delta_\perp}{2}, S \right\rangle -$$
$$\lim_{\Delta^k \to 0} \frac{\partial}{\partial \Delta^k} \left\langle P^+, -\frac{\Delta_\perp}{2}, S \right| igF^{+i}(0^-, 0_\perp)\, U\left(0^-, \eta_1^-; 0_\perp\right) \Big[A^j_{\text{pure}}\left(\eta_1^-, 0_\perp\right)$$
$$U(\eta_1^-, \eta_2^-; 0_\perp) A^i_{\text{phys,o}}\left(\eta_2^-, 0_\perp\right) - A^i_{\text{phys,o}}\left(\eta_1^-, 0_\perp\right) U(\eta_1^-, \eta_2^-; 0_\perp) A^j_{\text{pure}}\left(\eta_2^-, 0_\perp\right) \Big]$$
$$U(\eta_2^-, 0^-; 0_\perp) \left| P^+, +\frac{\Delta_\perp}{2}, S \right\rangle .$$

$$\mathcal{L}^g_0(x, k_\perp, \Delta_\perp, S)$$

Jaffe-Manohar's OAM

JI's OAM

Gluon OAM at first sub-eikonal order

For the next non-trivial order in the expansion of the exponential, as $exp(-ixP^+z^-) \simeq 1 - ixP^+z^-$

$$\mathcal{L}^{jk}_{g,1}(x) = i\,P^+ \lim_{\Delta^k\to 0} \frac{\partial}{\partial\Delta^k}\left\langle P^+, -\frac{\Delta_\perp}{2}, S\right| \mathrm{Tr}\, F^{+i}(0^-,0_\perp)\, \partial^j \bar{O}^i_{\text{phys},*}(0^-,0_\perp) \left| P^+, +\frac{\Delta_\perp}{2}, S\right\rangle$$

$$+ i\,P^+ \lim_{\Delta^k\to 0} \frac{\partial}{\partial\Delta^k}\left\langle P^+, -\frac{\Delta_\perp}{2}, S\right| \mathrm{Tr}\, F^{+i}(0^-,0_\perp)\, ig\left[\bar{O}^i_{\text{phys},*}, A^j_{\text{res}}\right](0^-,0_\perp)$$

$$\left| P^+, +\frac{\Delta_\perp}{2}, S\right\rangle + i \lim_{\Delta^k\to 0} \frac{\partial}{\partial\Delta^k}\left\langle P^+, -\frac{\Delta_\perp}{2}, S\right| igF^{+i}(0^-,0_\perp)\, U(0^-,\eta_1^-;0_\perp)$$

$$\left[A^j_{\text{pure}}(\eta_1^-,0_\perp)\, U(\eta_1^-,\eta_2^-;0_\perp)\, \bar{A}^i_{\text{phys},*}(\eta_2^-,0_\perp) - \bar{O}^i_{\text{phys},*}(\eta_1^-,0_\perp)\, U(\eta_1^-,\eta_2^-;0_\perp)\, A^j_{\text{pure}}(\eta_2^-,0_\perp)\right]$$

$$U(\eta_2^-,0^-;0_\perp)\left| P^+, +\frac{\Delta_\perp}{2}, S\right\rangle$$

Under PT transformation

$$\mathscr{L}^g_1(x, k_\perp, \Delta_\perp, S) = -\mathscr{L}^g_1(x, -k_\perp, -\Delta_\perp, -S)$$

Sub-eikonal terms contribute to the gluon OAM !

Conclusion

- In this work, we have derived the general operator form of the OAM of gluon in a longitudinally polarized proton, that is valid for all possible geometrics of the gauge links.
- At an appropriate combination of the extent parameters, this correctly reproduces both Jaffe-Manohar and Ji's OAM, and offers a continuous analytical interpolation between the two.
- This also corroborate the fact that in the Taylor expansion of the phase factor $\exp(ixP^+z^-)$ only the odd terms in x can contribute to the gluon OAM for longitudinally polarized proton.

THANK YOU

Chen *et al.* decomposition (2009)

$$J_{QCD} = S'_q + L'_q + S'_g + L'_g$$

Here, gauge field decomposes into pure part and physical part.

$$A^{\mu} = A^{\mu}_{\text{pure}} + A^{\mu}_{\text{phys}}$$

In QED, we have,

$$A_{\text{phys}} = A_{\perp}\,(\text{transverse}) \qquad\qquad A_{\text{pure}} = A_{||}\,(\text{longitudinal})$$

In $A^{+}_{\text{phys}} = 0$

$$A^{\mu}_{\text{phys},\pm}\left(x^{-}, x_{\perp}\right) = \int_{\pm\infty^{-}}^{x^{-}} d\omega^{-}\, U\left(x^{-}, \omega^{-}; x_{\perp}\right) F^{+\mu}\left(\omega^{-}, x_{\perp}\right) U\left(\omega^{-}, x^{-}; x_{\perp}\right)$$

Nucleon Spin decomposition

There are two types of decomposition of the proton spin operator: kinetic and canonical. These two types differ by how the OAM operator is split into the quark and gluon contributions.

- Canonical angular momentum decomposition

$$J^k_{QCD} = \int d^3x \left[\underbrace{\psi^\dagger \sigma^k \psi + \psi^\dagger (\vec{x} \times (-i\overrightarrow{\partial}))^k \psi}_{\text{quark part}} + \underbrace{E^l (\vec{x} \times \overrightarrow{\nabla})^k A^l + (\vec{A} \times \vec{E})^k}_{\text{gluon part}} \right]$$

- Kinetical angular momentum decomposition

$$J^k_{QCD} = \int d^3x \left[\psi^\dagger \sigma^k \psi + \psi^\dagger \underbrace{(\vec{x} \times (-i\overrightarrow{\partial - } gA))^k}_{\text{Kinetic OAM}} \psi + E^l \underbrace{(\vec{x} \times \overrightarrow{\nabla})^k}_{\text{gluon OAM}} A^l + (\vec{A} \times \vec{E})^k \right]$$