Precision jet event shapes for DIS

in Soft-Collinear Effective Field theory (SCET)

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IISER Mohali

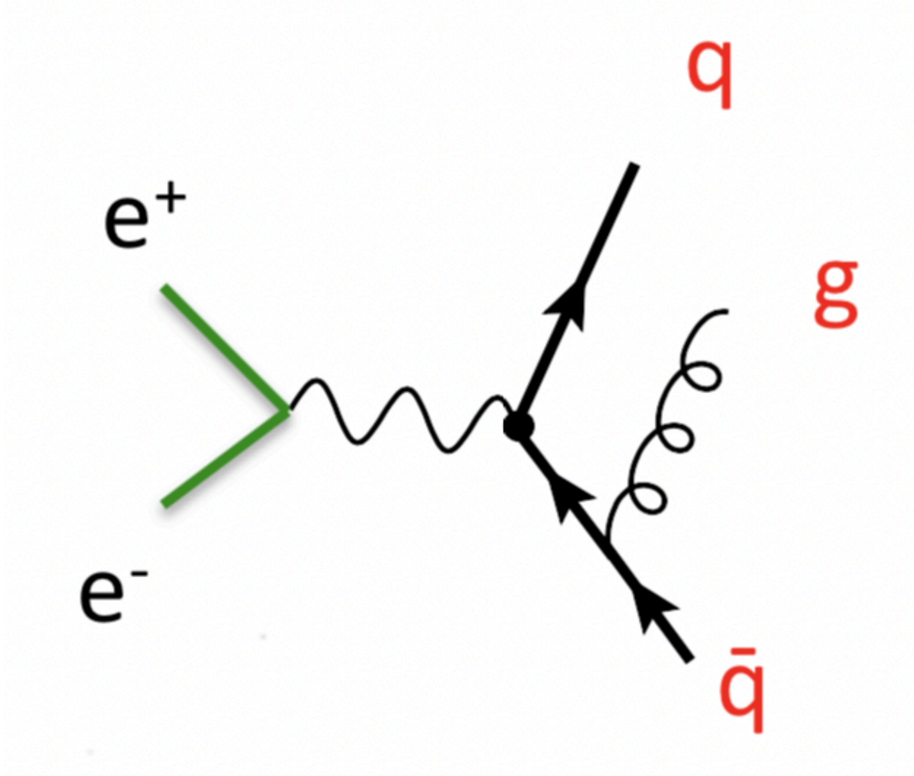
Coauthors: Jiawei Zhu, Daekyoung Kang, Fudan University, Shanghai

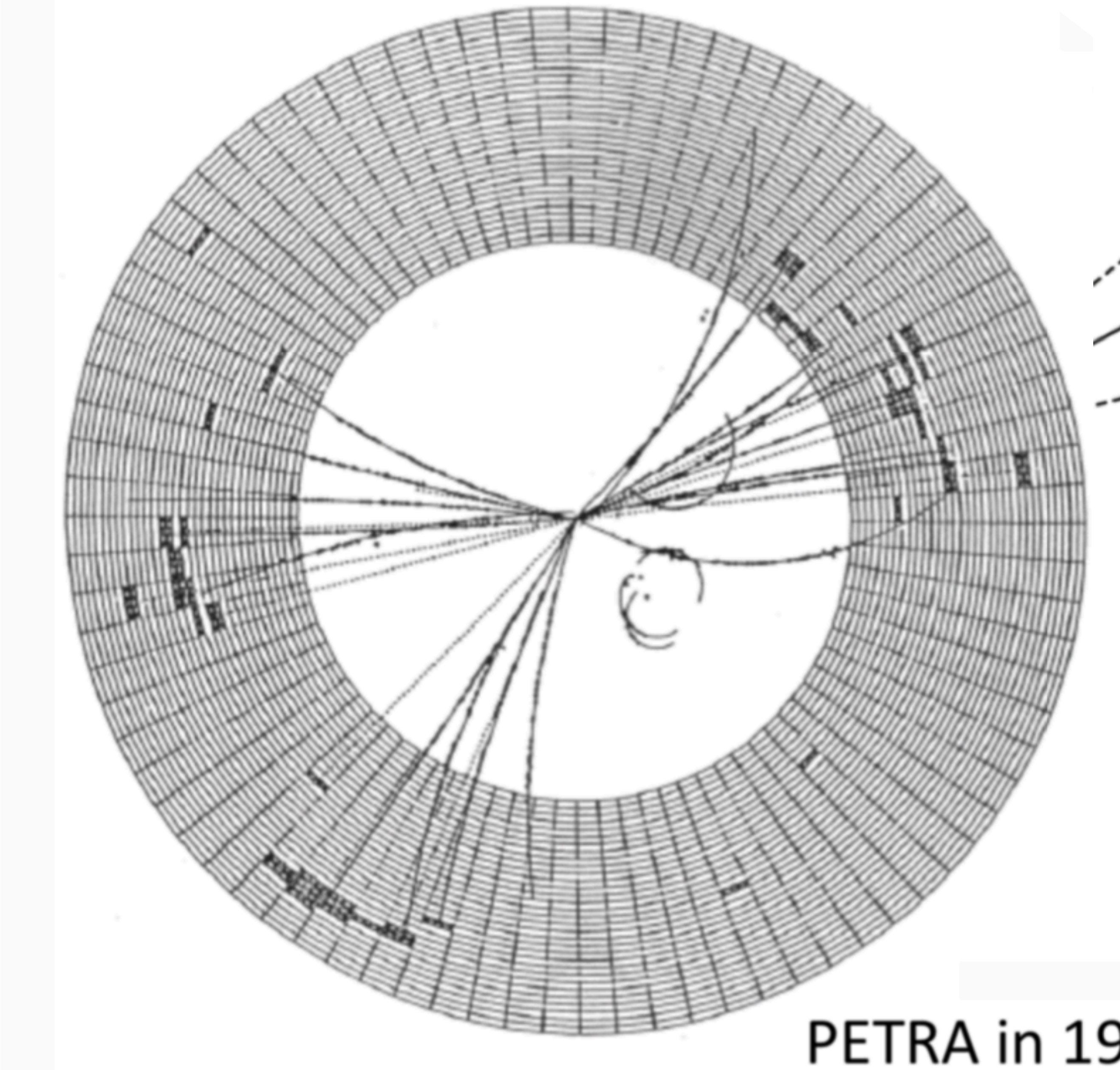
Talk organized as…

- Motivation
- Jet Event Shapes and angularity for deep inelastic scattering (DIS)
- Angularity beam functions at next-to-next-to-leading log (NNLL)
- Angularity differential cross-section at NNLL
- Prediction & Remarks

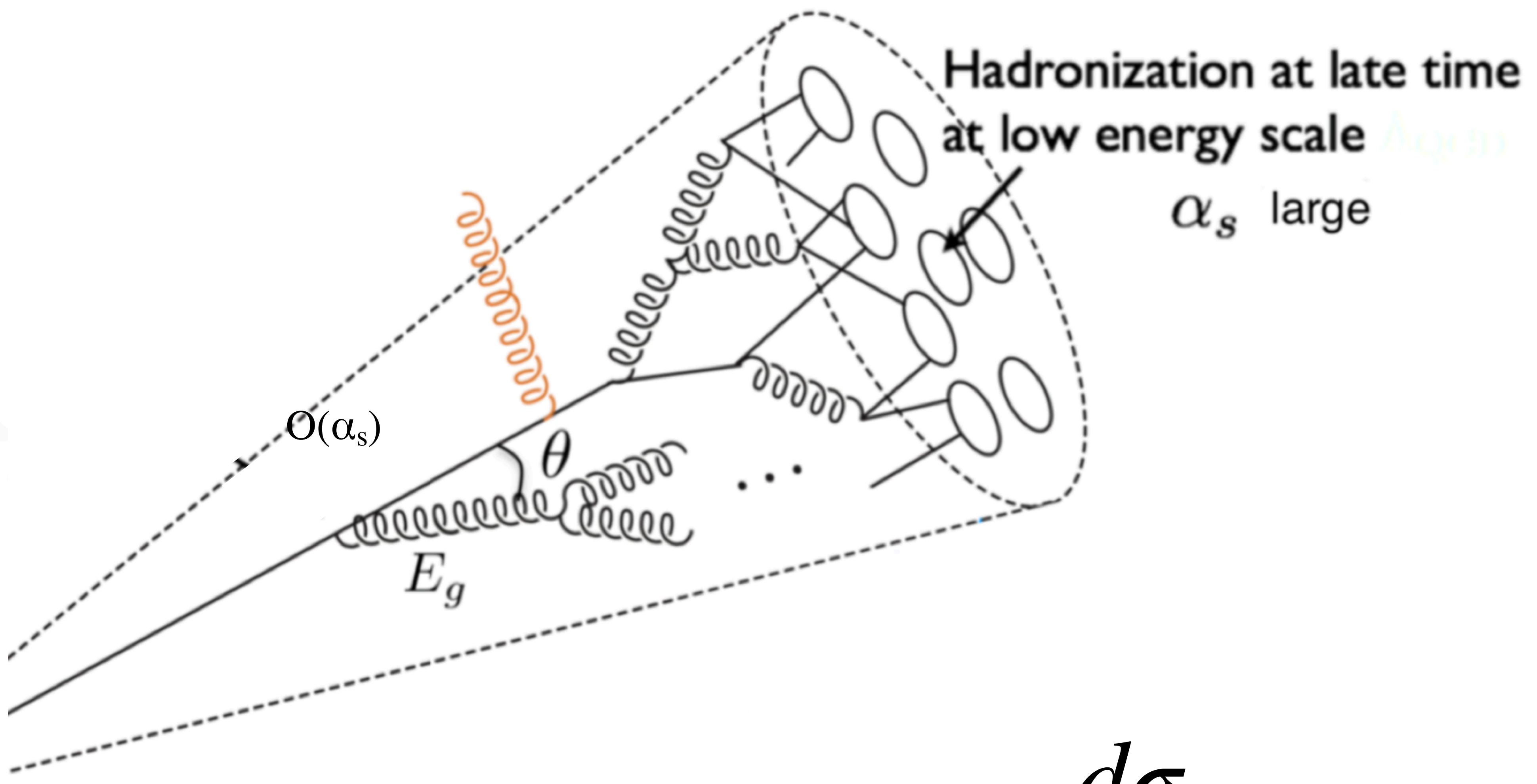
Jets and jet event shapes

The most common final states are collimated branches of strongly interacting particles, called jet.





PETRA in 1979



$$\frac{d\sigma}{dx dQ^2 d\tau_a} = ??$$

Jet Observables are called

Event Shapes

e.g., Thrust, Jet Broadening, Angularity…

Thrust event shape

Most extensively studied event shape

$$\tau = \frac{2}{Q} \sum_{i \in \chi} |p_\perp^i| e^{-|\eta_i|}$$

Rapidity: $\eta = \frac{1}{2} \ln \left(\frac{p^-}{p^+} \right)$

Thrust event shape

Most extensively studied event shape

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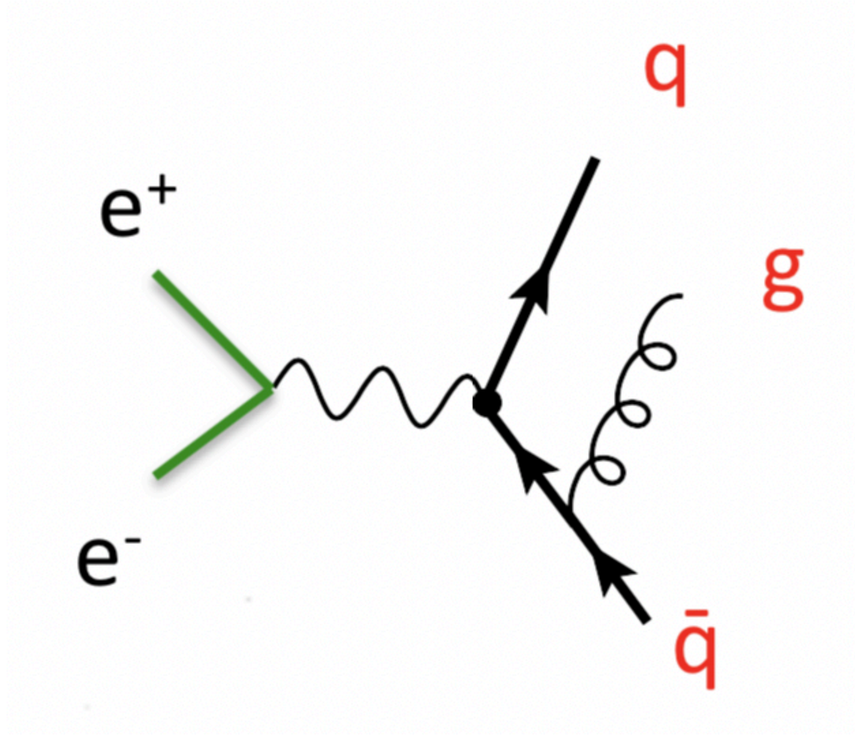
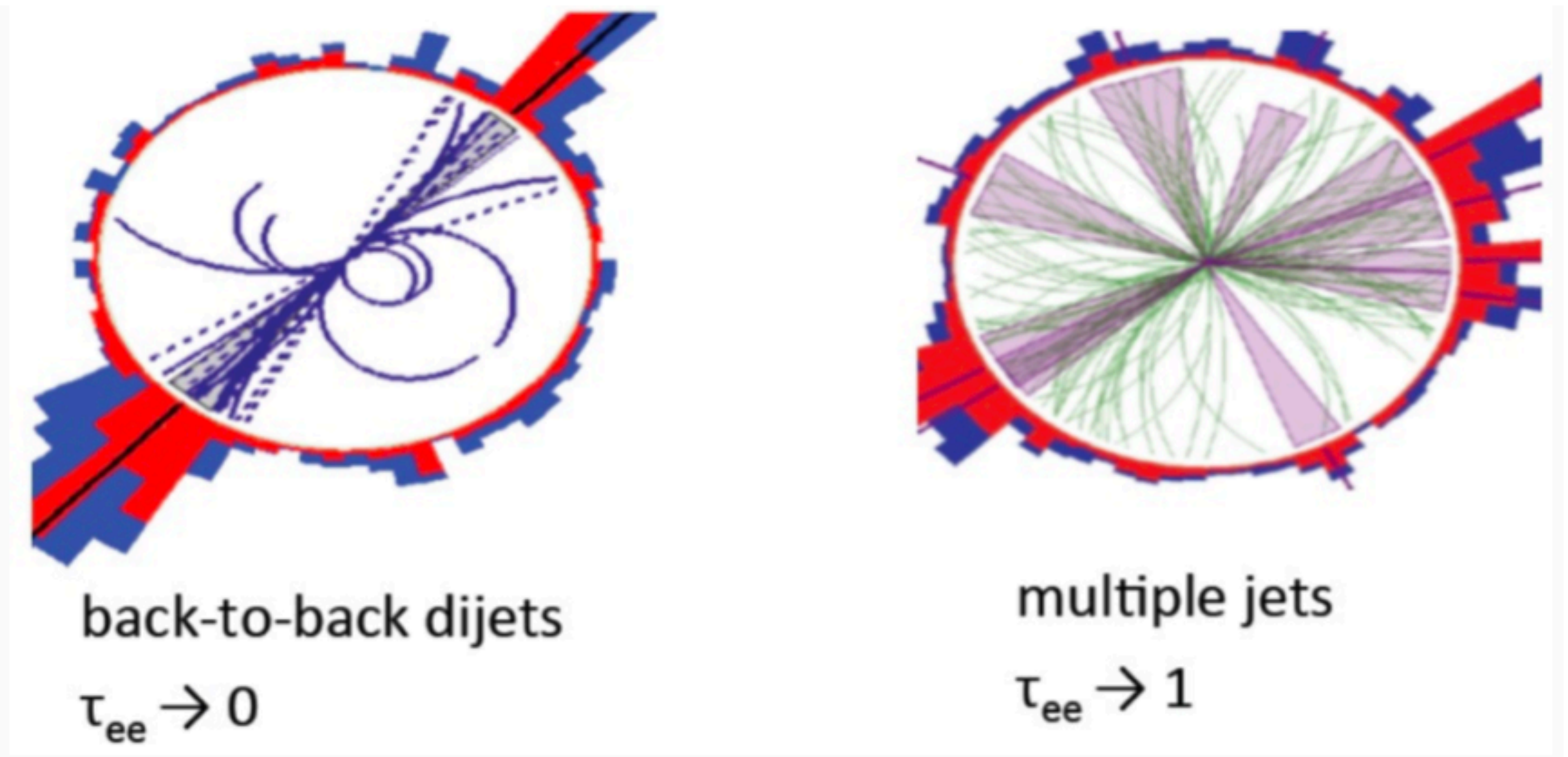
An example:



$$n = (1, 0, 0, 1)$$
$$\bar{n} = (1, 0, 0, -1)$$

$$\tau_{ee} = \frac{2}{Q^2} \sum_{i \in \chi} min\{p_i . n, p_i . \bar{n}\}$$



Thrust characterizes the geometry of collision!

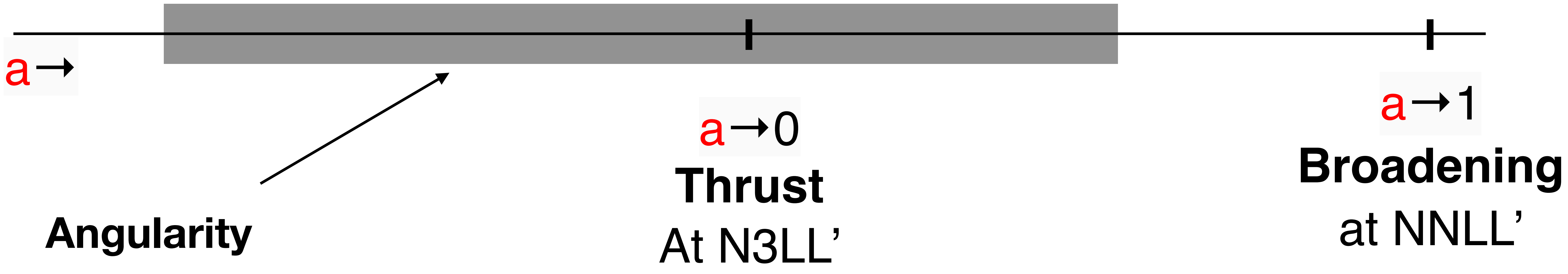
Angularity event shapes

—C. F. Berger, T. Kucs and G. F. Sterman' 2003

$$\tau^a = \frac{2}{Q}\sum_{i\in\chi} |p_\perp^i|\, e^{-|\eta_i|(1-a)}$$

Depends on continuous parameter

A more general event shape!
provides access from thrust to jet broadening in continuous manner



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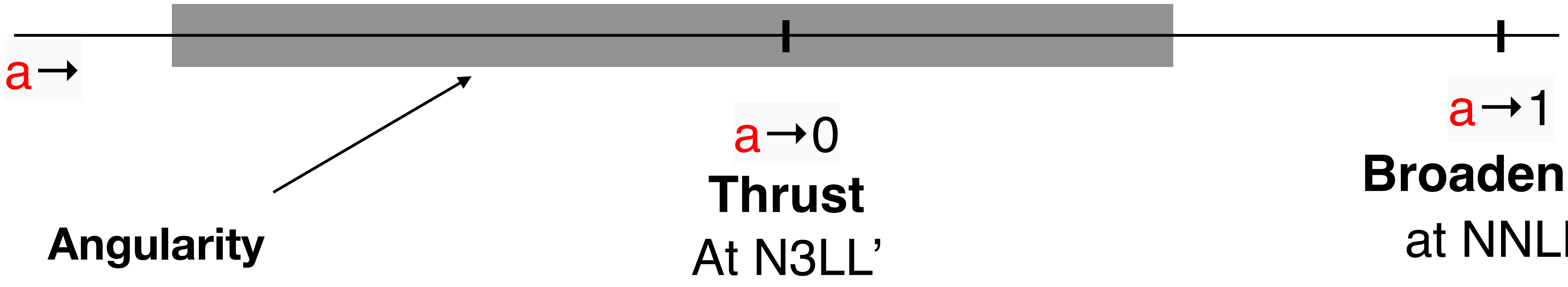
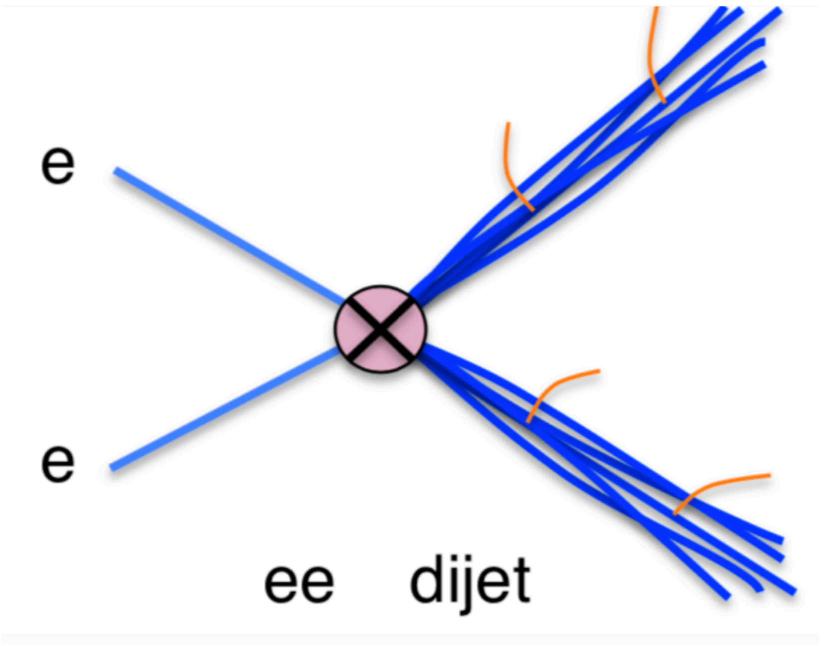


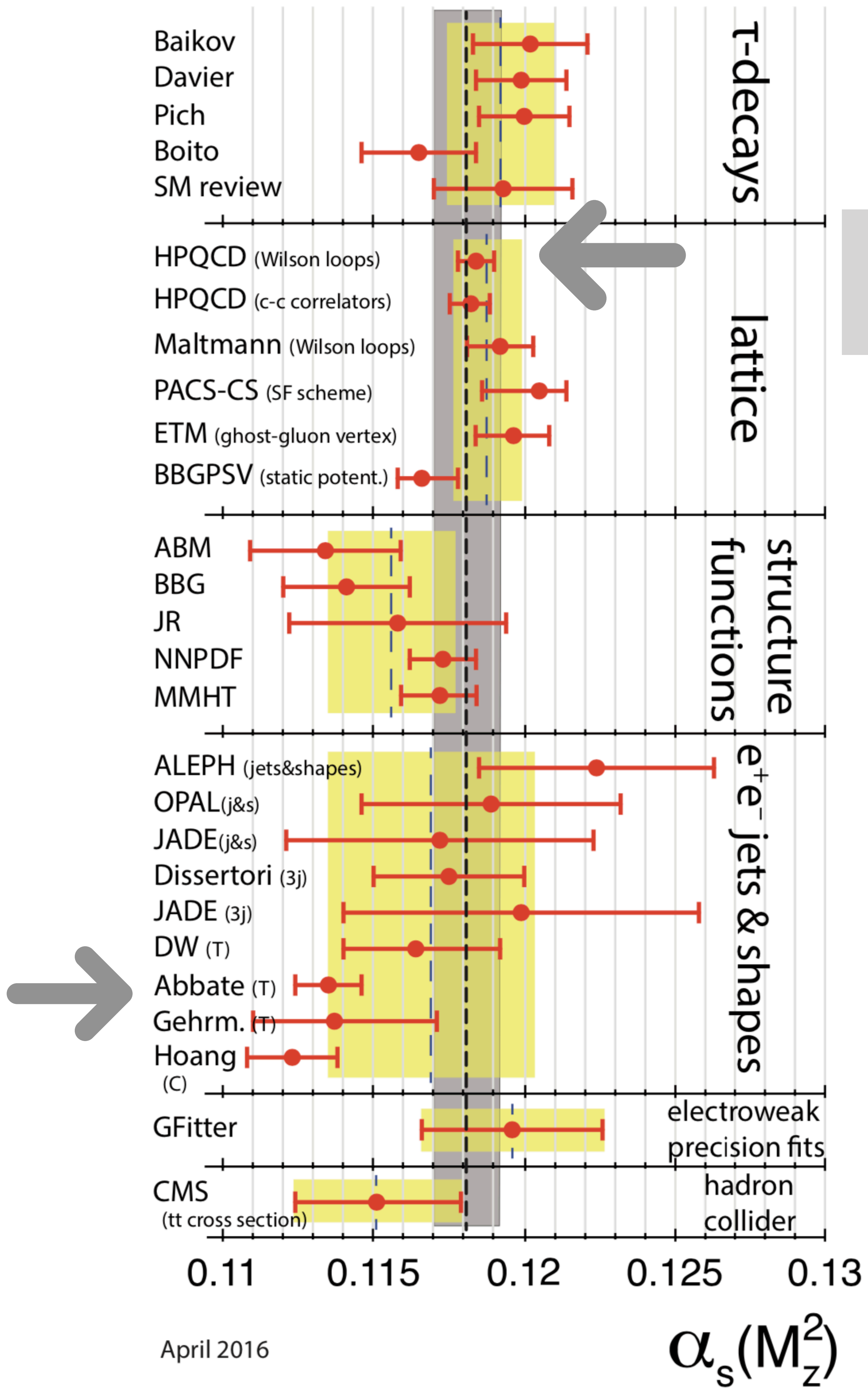
For e^+e^- = dijet

$$\tau^a_{ee} = \frac{2}{Q^2}\sum_{i\in\chi} min\left\{(p_i.n)^{a/2}(p_i.\bar{n})^{(1-a/2)}\right\}$$



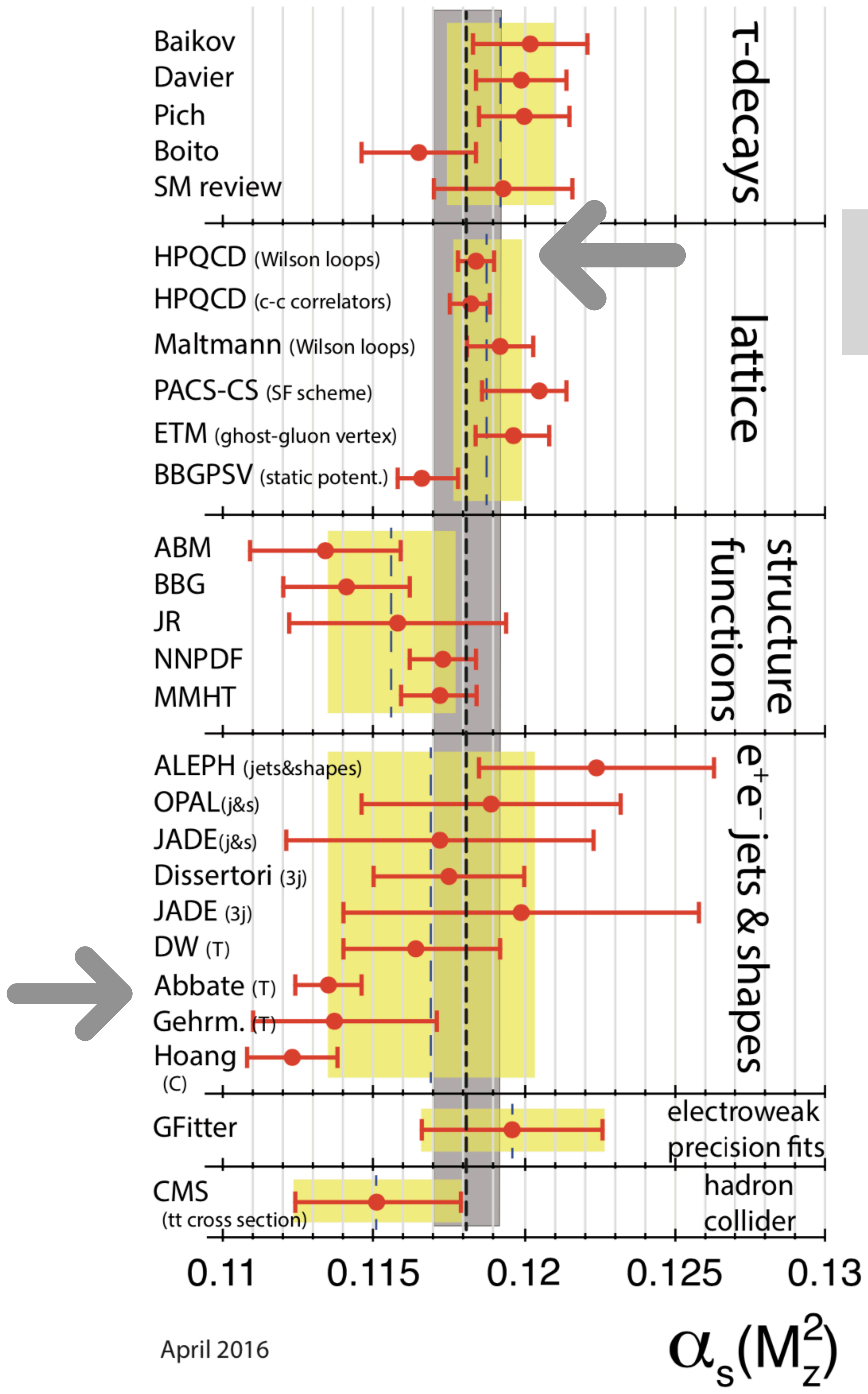
$$n.\bar{n} = 2$$

Why DIS angularity?



Discrepancy> 3-Sigma from Lattice

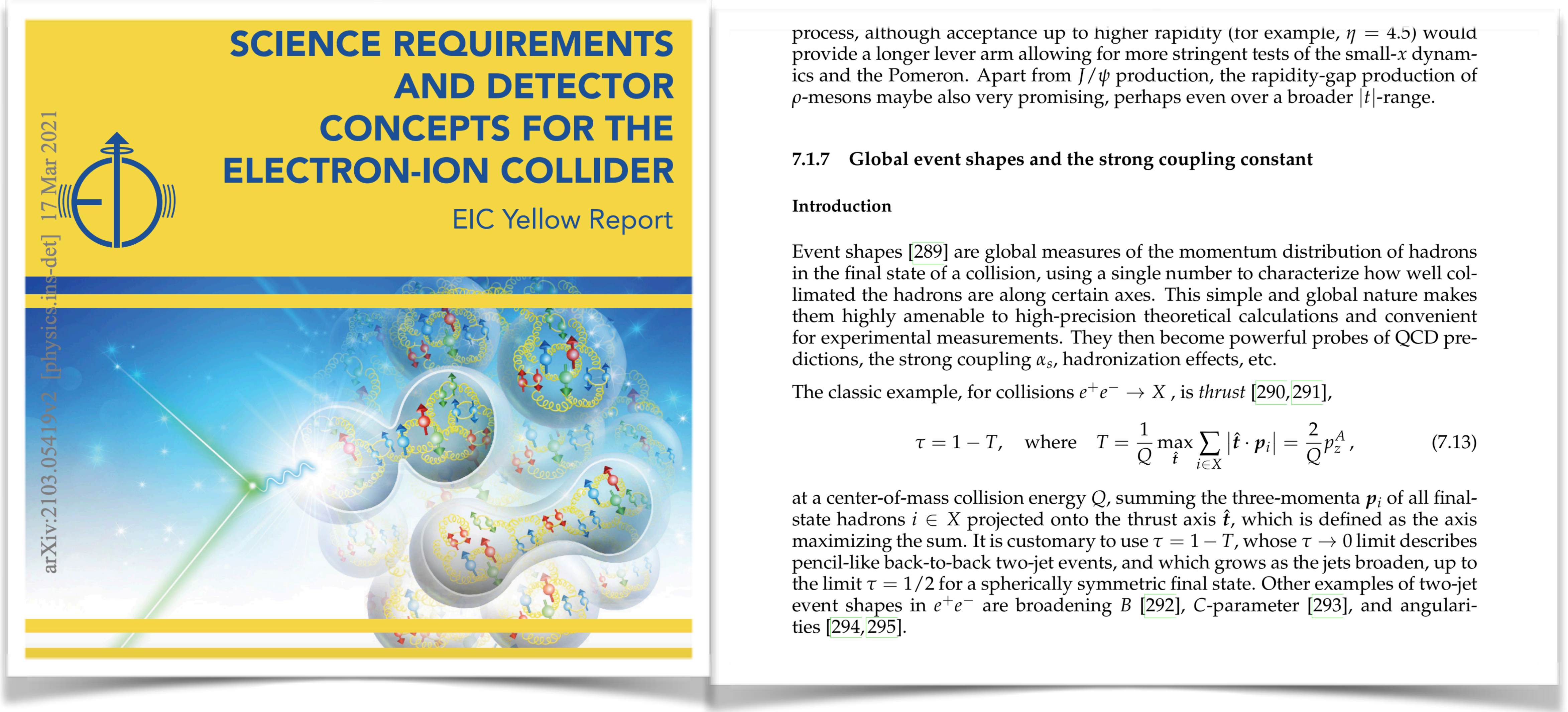
Why DIS angularity?



Discrepancy> 3-Sigma from Lattice

Need a new test from an independent experiment and new event shapes!

DIS event shapes for **future Electron-Ion-Collider (EIC)** at BNL!!



process, although acceptance up to higher rapidity (for example, $\eta = 4.5$) would provide a longer lever arm allowing for more stringent tests of the small-x dynamics and the Pomeron. Apart from J/ψ production, the rapidity-gap production of ρ-mesons maybe also very promising, perhaps even over a broader $|t|$-range.

7.1.7 Global event shapes and the strong coupling constant

Introduction

Event shapes [289] are global measures of the momentum distribution of hadrons in the final state of a collision, using a single number to characterize how well collimated the hadrons are along certain axes. This simple and global nature makes them highly amenable to high-precision theoretical calculations and convenient for experimental measurements. They then become powerful probes of QCD predictions, the strong coupling α_s, hadronization effects, etc.

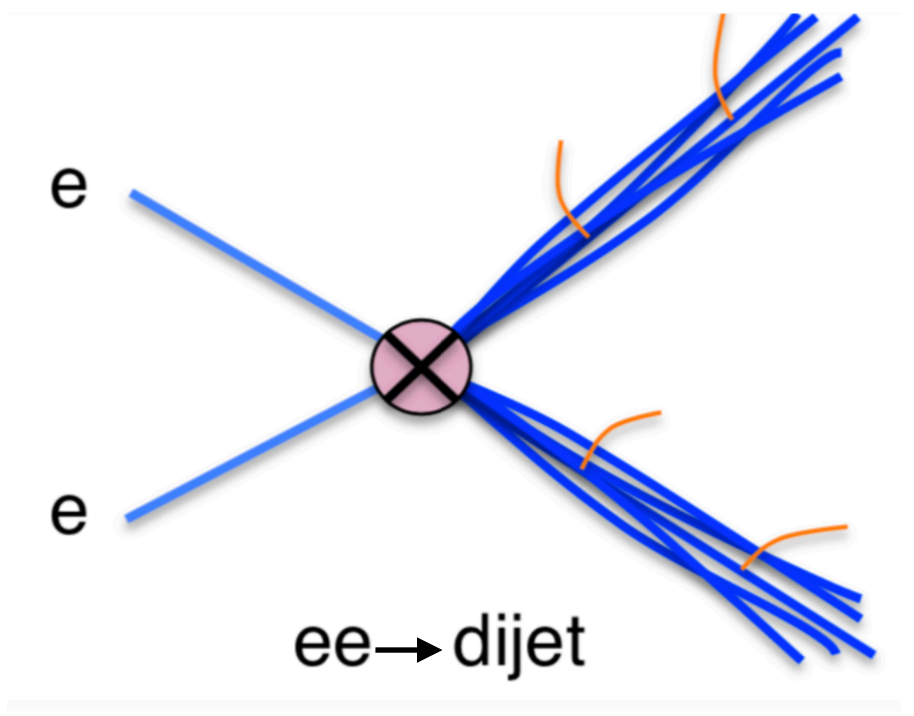
The classic example, for collisions $e^+e^- \to X$, is *thrust* [290,291],

$$\tau = 1 - T, \quad \text{where} \quad T = \frac{1}{Q}\max_{\hat{t}} \sum_{i\in X} |\hat{t}\cdot \boldsymbol{p}_i| = \frac{2}{Q} p_z^A, \qquad (7.13)$$

at a center-of-mass collision energy Q, summing the three-momenta $\boldsymbol{p}_i$ of all final-state hadrons $i \in X$ projected onto the thrust axis $\hat{t}$, which is defined as the axis maximizing the sum. It is customary to use $\tau = 1 - T$, whose $\tau \to 0$ limit describes pencil-like back-to-back two-jet events, and which grows as the jets broaden, up to the limit $\tau = 1/2$ for a spherically symmetric final state. Other examples of two-jet event shapes in e^+e^- are broadening B [292], C-parameter [293], and angularities [294,295].

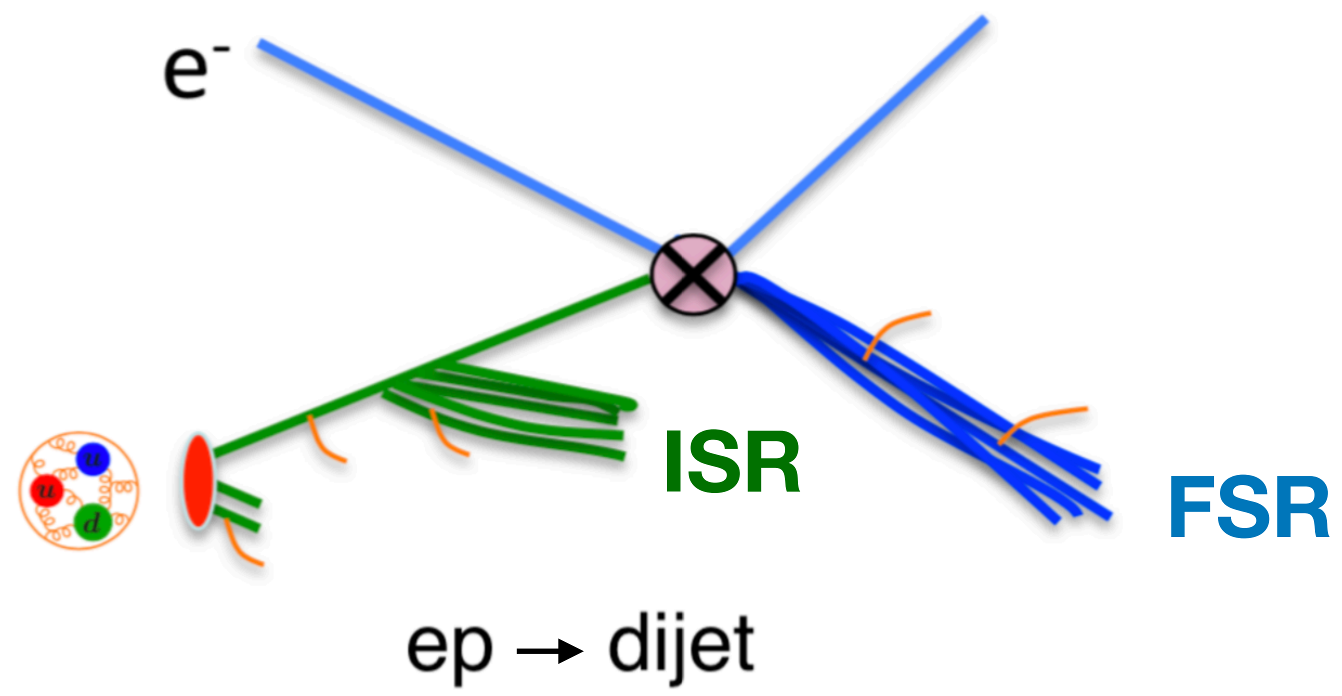
Could be an early milestone!

Angularity for DIS



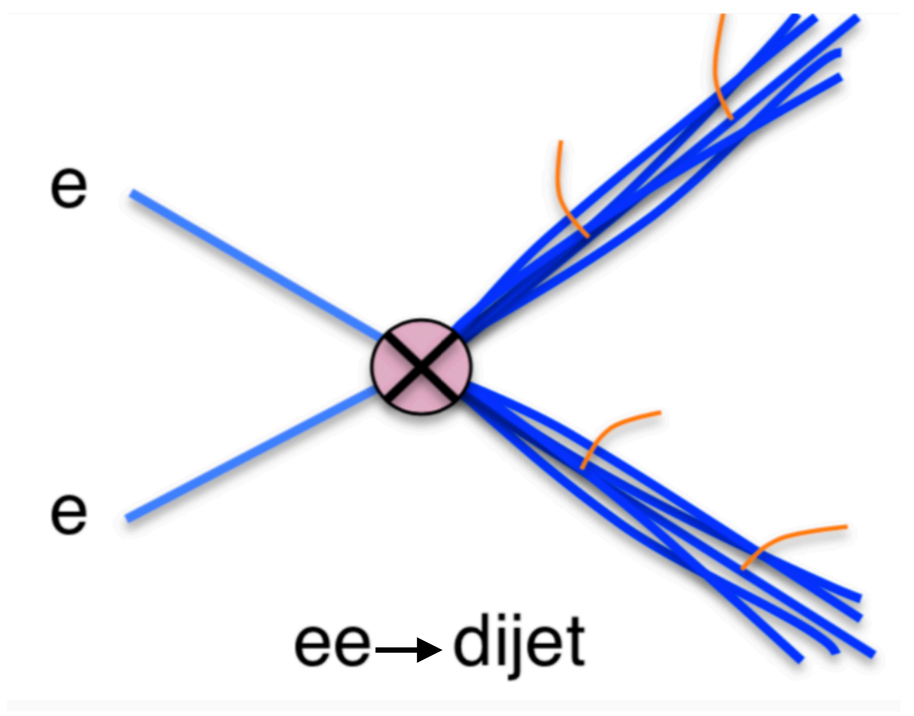
Back to back in CM frame

$$n.\bar{n} = 2$$



Not back to back even in CM !!

Angularity for DIS



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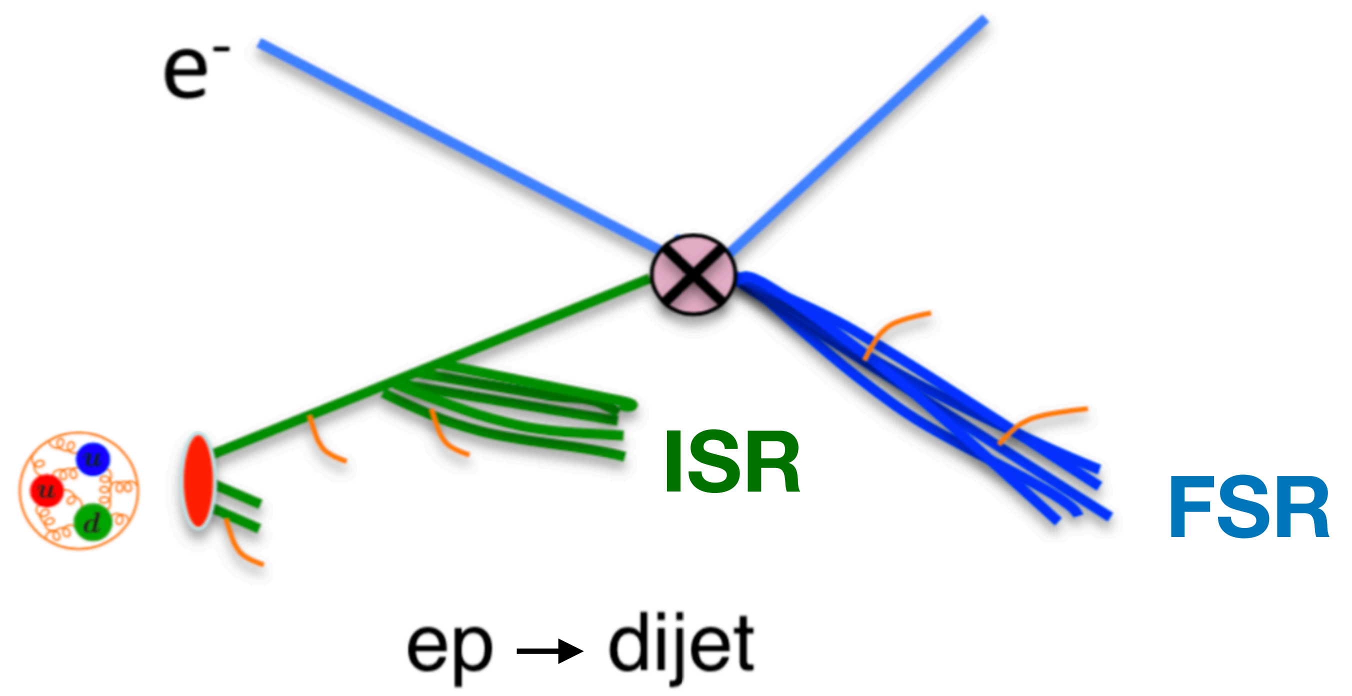


Not back to back even in CM !!

Axis Choice: qB = xP, qJ= jet axis

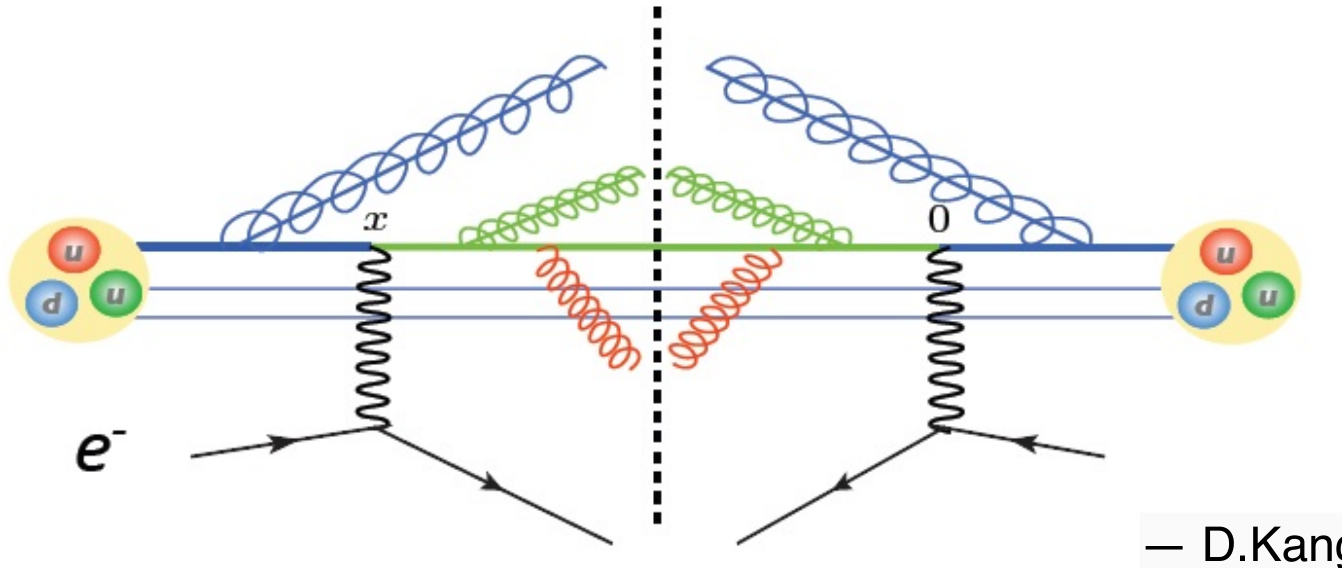
$q_B^\mu = \omega_B \frac{n_B^\mu}{2}$ and $q_J^\mu = \omega_J \frac{n_J^\mu}{2}$ with $n_i \cdot \bar{n}_i = 2$

we obtain $\omega_B = \bar{n}_B \cdot q_B$ and $\omega_J = \bar{n}_J \cdot q_J$

$$\tau_a = \frac{2}{Q^2} \sum_{i \varepsilon \mathscr{X}} min\left\{ (q_B.p_i)\left(\frac{q_B.p_i}{q_J.p_i}\right)^{-a/2}, (q_J.p_i)\left(\frac{q_J.p_i}{q_B.p_i}\right)^{-a/2} \right\}$$

SCET factorization: Angularity diff. cross-section for DIS

$$\frac{d\sigma}{dx\,dQ^2\,d\tau_a} = L_{\mu\nu}(x, Q^2)\; W^{\mu\nu}(x, Q^2, \tau_a)$$



— D.Kang,Lee,Stewart'2013

Z.Kang,Mantry,Qiu'2012

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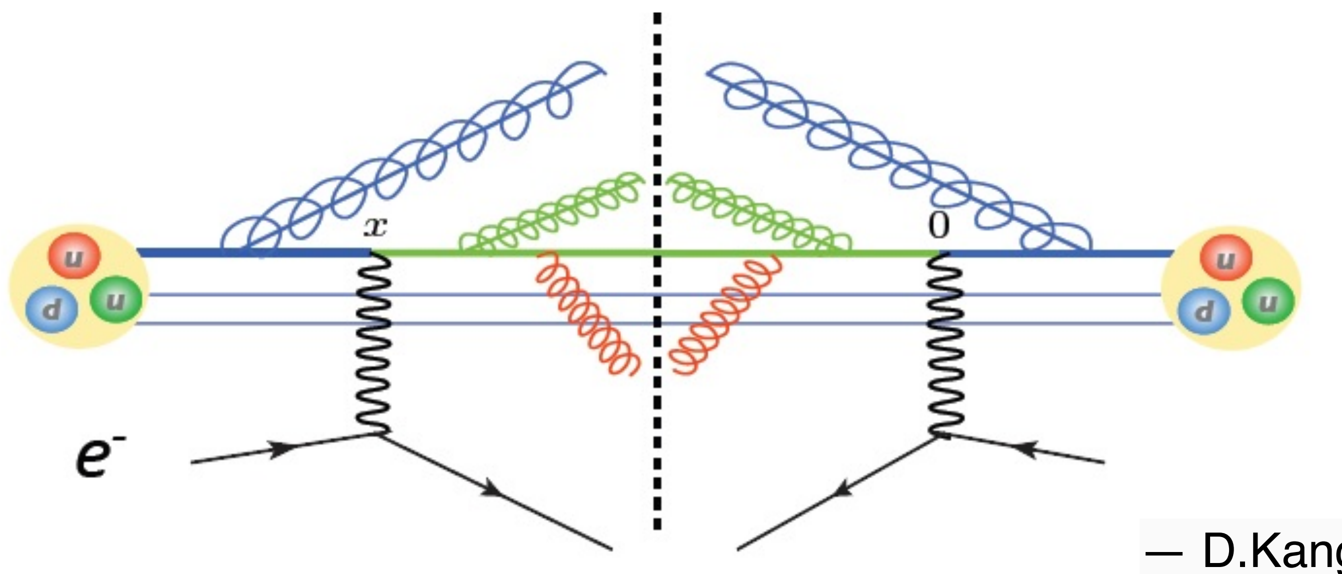
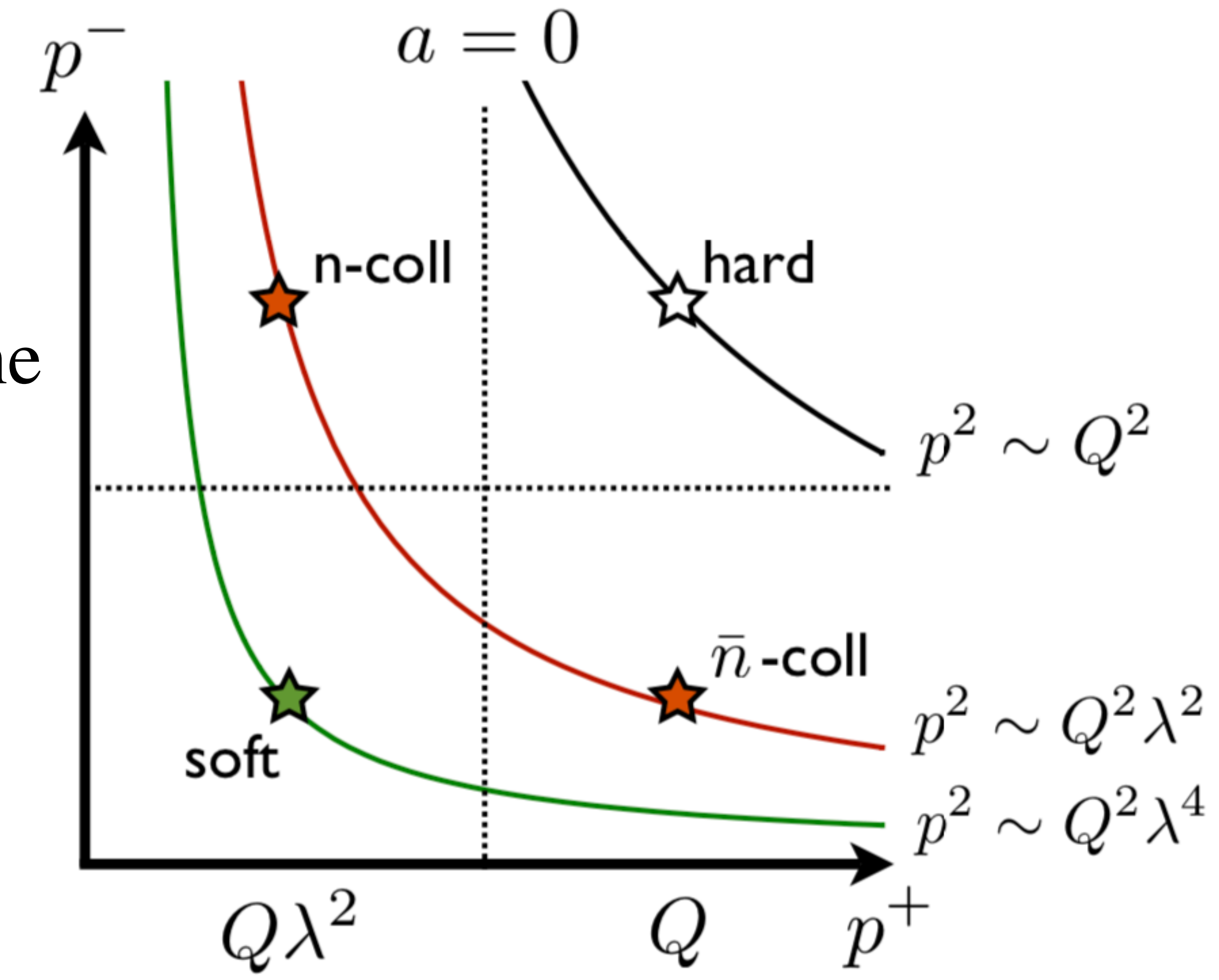


— D.Kang,Lee,Stewart'2013
Z.Kang,Mantry,Qiu'2012

$p_c^2 \sim Q^2\lambda^{2-a}$ $\quad p_s^2 \sim Q^2\lambda^{2(2-a)}$

$p_\perp = 0$ plane

$p^2 = p^+ p^-$



$$\frac{d\sigma}{dx\,dQ^2\,d\tau_a} = \frac{d\sigma_0}{dx\,dQ^2}\int d\tau_a^J\, d\tau_a^B\, d\tau_a^S\, \delta\left(\tau_a - \tau_a^J - \tau_a^B - \tau_a^S\right) \times \sum_{i=q,\bar{q}} H_i(Q^2,\mu)\,\mathcal{B}_i(\tau_a^B,x,\mu)\,J(\tau_a^J,\mu)\,S(\tau_a^S,\mu)$$

$a \leq 0.5$

Angularity Beam function

$$\frac{d\sigma}{dx dQ^2 d\tau_a} = \frac{d\sigma_0}{dx dQ^2} \int d\tau_a^J \, d\tau_a^B \, d\tau_a^S \, \delta\left(\tau_a - \tau_a^J - \tau_a^B - \tau_a^S\right)$$
$$\times \sum_{i=q,\bar{q}} H_i(Q^2, \mu) \mathcal{B}_i(\tau_a^B, x, \mu) J(\tau_a^J, \mu) S(\tau_a^S, \mu)$$

—Hornig, Lee, Ovanesyan'09;

—Bell, Hornig, Lee, Talbert'18,

Beam func.: $B(\tau_a, x, \mu) = \text{pdf} \otimes \left[\delta_{qj}\delta(\tau_a) + \tilde{\mathcal{I}}_{qj}^{(1)} + \mathcal{O}(\alpha_s^2) + ...\right]$

$NP \quad LO \quad NLO \quad NNLO$

—Tanmay Maji, D. Kang, J. Zhu, JHEP11(2021) 026

Angularity Beam function

$$\frac{d\sigma}{dx dQ^2 d\tau_a} = \frac{d\sigma_0}{dx dQ^2} \int d\tau_a^J \, d\tau_a^B \, d\tau_a^S \, \delta\Big(\tau_a - \tau_a^J - \tau_a^B - \tau_a^S\Big)$$
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$NP \quad LO \quad NLO \quad NNLO$

$$G^{\text{fixed}}(L_G,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left[-j_G \kappa_G \frac{\Gamma_0}{2} L_G^2 - \gamma_0^G L_G + c_1^G\right], \qquad G = \{H, \widetilde{S}, \widetilde{J}\}$$

$$L_B(\tau_a) = \log\Big[\frac{Q}{\mu_B}(\tau_a e^{-\gamma_E})^{1/j_B}\Big]$$

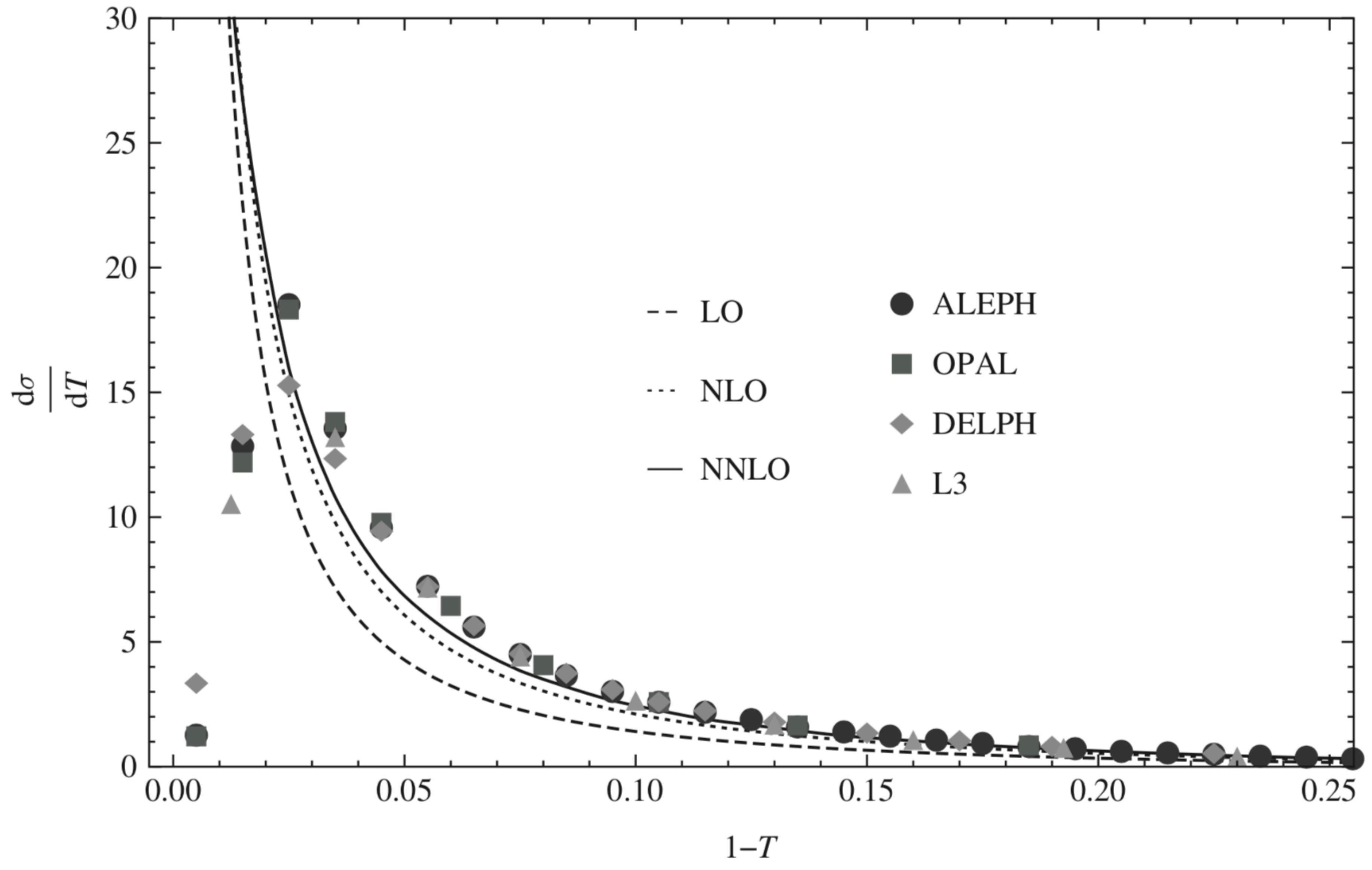
With jB = 2-a

Large logs at threshold limit demands Resummation!

—Tanmay Maji, D. Kang, J. Zhu, JHEP11(2021) 026

Resummation in Laplace space

$$G^{\text{fixed}}(L_G,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left[-j_G\kappa_G\frac{\Gamma_0}{2}L_G^2 - \gamma_0^G L_G + c_1^G\right], \qquad G = \{H, \widetilde{S}, \widetilde{J}\}$$



$$\frac{d\sigma}{dx\,dQ^2\,d\tau_a}$$

Resummation in Laplace space

$$\tilde{\sigma}(\nu)$$

$\sigma = \sigma^{(0)}$ leading order (**LO**)

$+\alpha_s\sigma^{(1)}$ next-to-leading order (**NLO**)

$+\alpha_s^2\sigma^{(2)}$ next-next-to-leading order (**NNLO**)

$+\cdots$

Resummation in Laplace space

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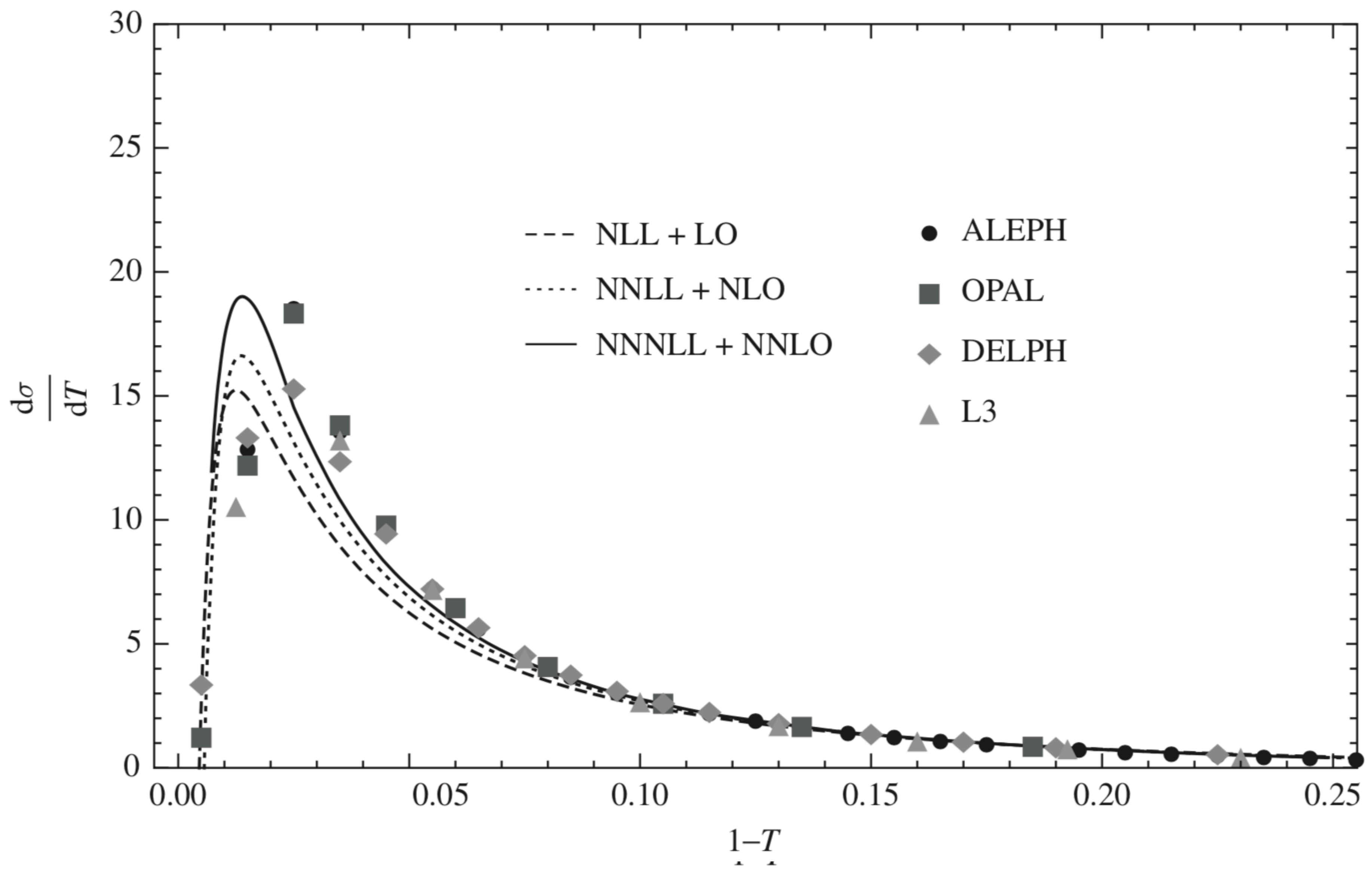


$$\frac{d\sigma}{dx dQ^2 d\tau_a}$$

Resummation in Laplace space

$$\tilde{\sigma}(\nu)$$

$$\begin{aligned} \sigma = \sigma^{(0)} & \quad \text{leading order (LO)} \\ + \alpha_s \sigma^{(1)} & \quad \text{next-to-leading order (NLO)} \\ + \alpha_s^2 \sigma^{(2)} & \quad \text{next-next-to-leading order (NNLO)} \\ + \cdots & \end{aligned}$$

$$K_B(\mu_B,\mu) \quad = \quad L_B \sum_{k=1}^{\infty} (\alpha_s L_B)^k$$ Leading Log (**LL**)

$$+ \sum_{k=1}^{\infty} (\alpha_s L_B)^k$$ Next-to-leading Log (**NLL**)

$$+ \ldots$$ Next-to-next-leading Log (**NNLL**)

Results

Angularity Differential Cross-section

$$\frac{d\sigma^{DIS}}{dx dQ^2 d\tau_a} = ?$$

Angularity diff. cross-section

$$\frac{d\sigma}{dx\,dQ^2 d\,\tau_a} = \frac{d\sigma_0}{dxdQ^2}\sum_\nu H_\nu(Q^2,\mu) \int d\tau_a^J\, d\tau_a^B\, dk_S\, J_q(\tau_a^J,\mu) B_{\nu/q}(\tau_a^B,x,\mu) \times S(k_S,\mu)\, \delta\left(\tau_a - \tau_a^J - \tau_a^B - \frac{k_S}{Q_R}\right),$$

Angularity diff. cross-section

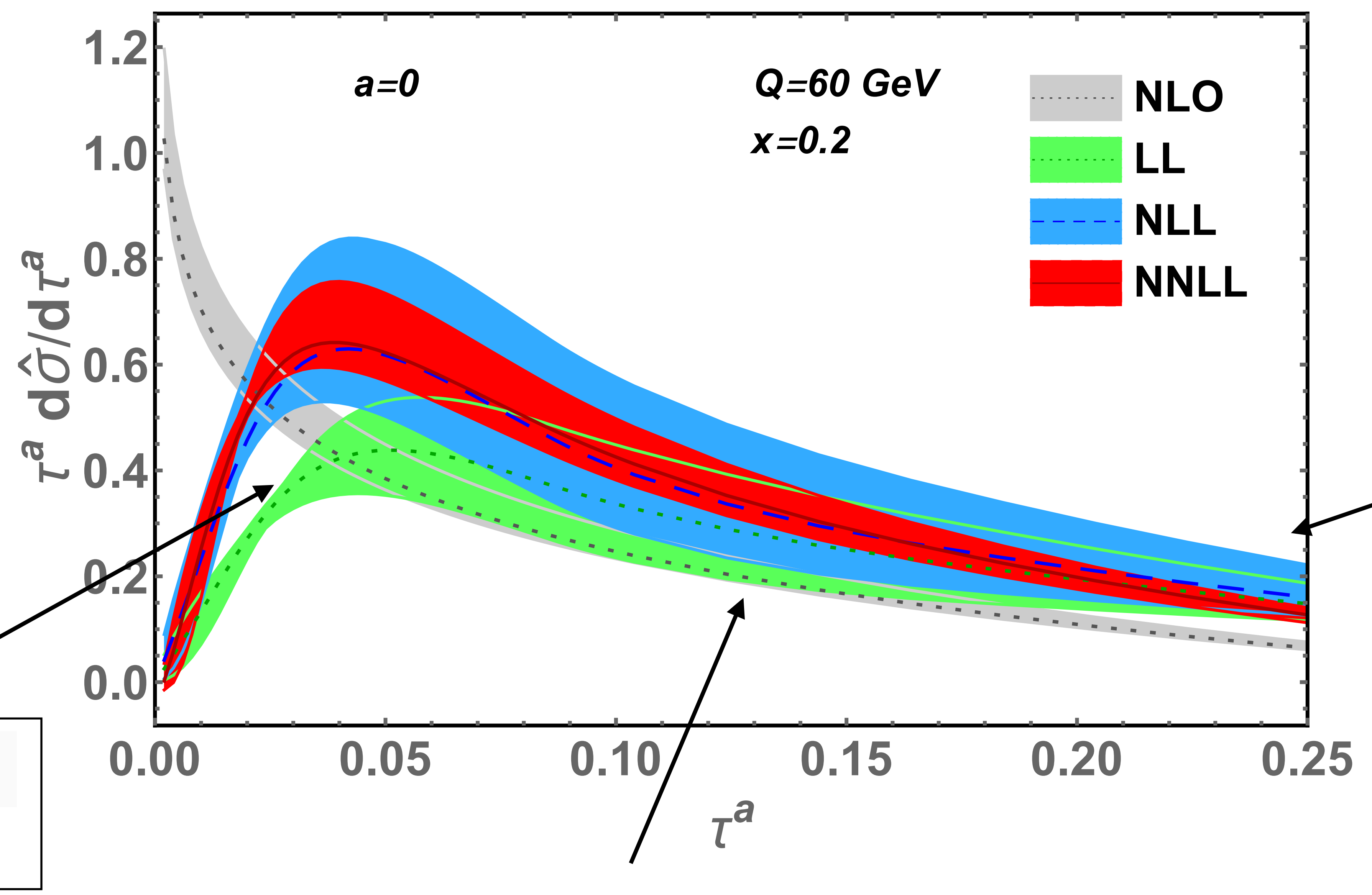
$$\frac{d\sigma}{dx\,dQ^2 d\,\tau_a} = \frac{d\sigma_0}{dxdQ^2} \sum_\nu H_\nu(Q^2,\mu) \int d\tau_a^J\, d\tau_a^B\, dk_S\, J_q(\tau_a^J,\mu) B_{\nu/q}(\tau_a^B,x,\mu) \times S(k_S,\mu)\,\delta\left(\tau_a - \tau_a^J - \tau_a^B - \frac{k_S}{Q_R}\right),$$

Resummed result

$$\begin{aligned}
\sigma(x,Q^2,\tau_a,\mu) \;=\;& \sigma_0(x,Q^2)\left(\frac{Q}{\mu_H}\right)^{\eta_H(\mu,\mu_H)} e^{\kappa(\mu_H,\mu_J,\mu_B,\mu_S,\mu)} \\
&\times\left(\left(\frac{Q}{\mu_J}\right)^{2-a}\tau_a e^{-\gamma_E}\right)^{\eta_J(\mu,\mu_J)}\left(\left(\frac{Q}{\mu_B}\right)^{2-a}\tau_a e^{-\gamma_E}\right)^{\eta_B(\mu,\mu_B)}\left(\frac{Q^2}{\mu_S}\tau_a e^{-\gamma_E}\right)^{2\eta_S(\mu,\mu_S)} \\
&\times\tilde{j}_q\left(\partial_\Omega + \log\left(\frac{Q^{2-a}}{\mu_J^{2-a}}\tau_a e^{-\gamma_E}\right),\mu_J\right)\tilde{s}\left(\frac{1}{Q_R}\left(\partial_\Omega + \log(\frac{Q}{\mu_S}\tau_a e^{-\gamma_E})\right),\mu_S\right) \\
&\times\Bigg[H_q(y,Q^2,\mu_H)\tilde{b}_q\left(\partial_\Omega + \log\left(\frac{Q^{2-a}}{\mu_B^{2-a}}\tau_a e^{-\gamma_E}\right),x,\mu_B\right) \\
&\quad + H_{\bar{q}}(y,Q^2,\mu_H)\tilde{b}_{\bar{q}}\left(\partial_\Omega + \log\left(\frac{Q^{2-a}}{\mu_B^{2-a}}\tau_a e^{-\gamma_E}\right),x,\mu_B\right)\Bigg]\frac{1}{\tau_a\Gamma(\Omega)}
\end{aligned}$$

Angularity Cross-section at NNLL

$$\frac{d\sigma}{dxdQ^2d\tau_a}$$

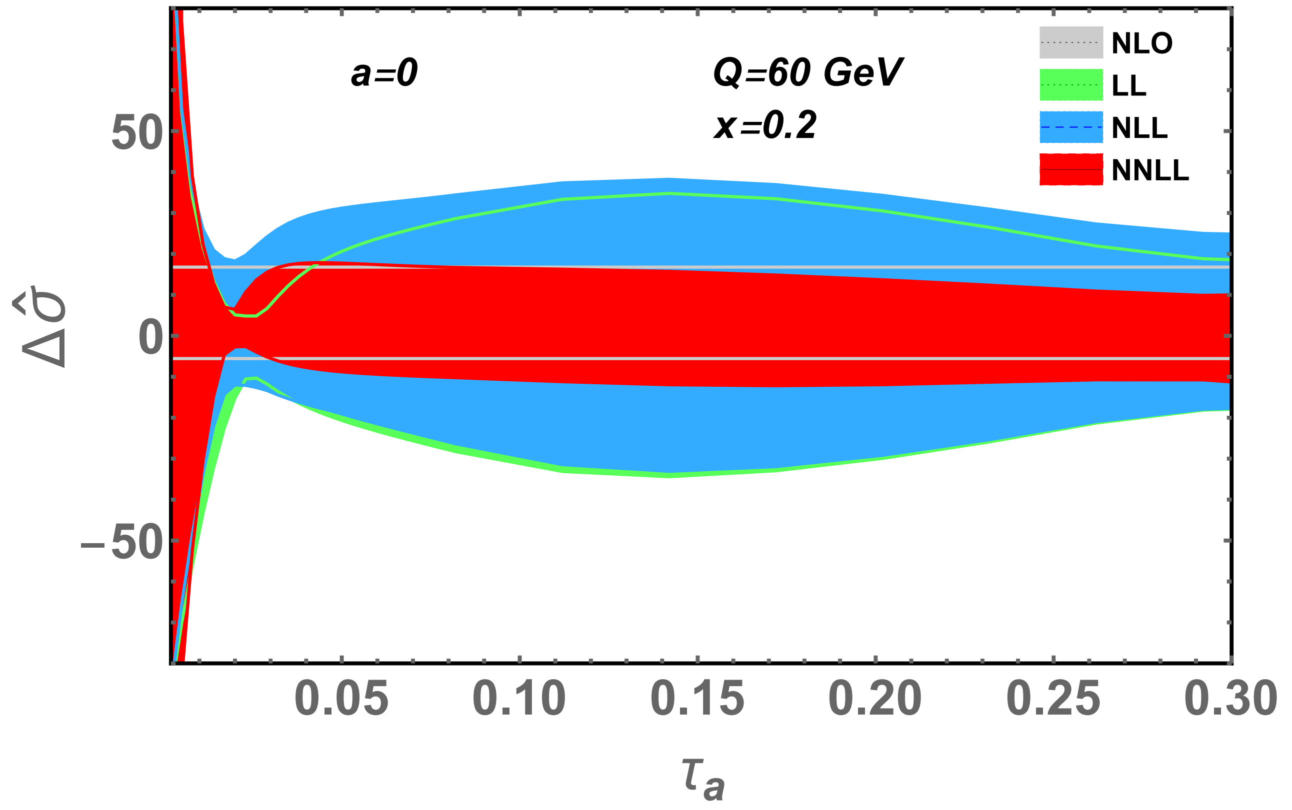


Far-tail region: Need full QCD

Low angularity region: Log singularity at NLO

Tail region: Resummation gives convergence from LL to NNLL

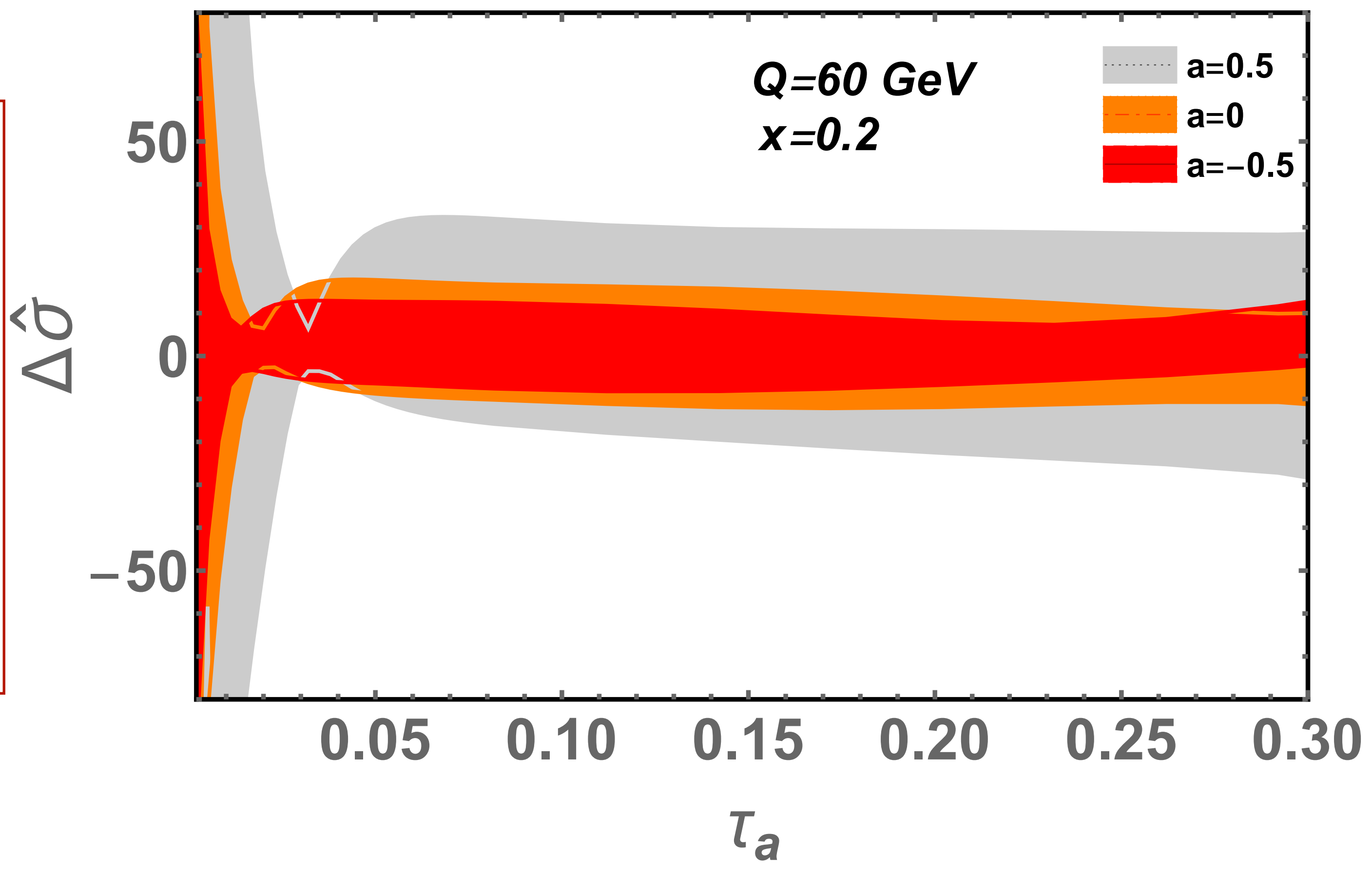
Relative Uncertainty



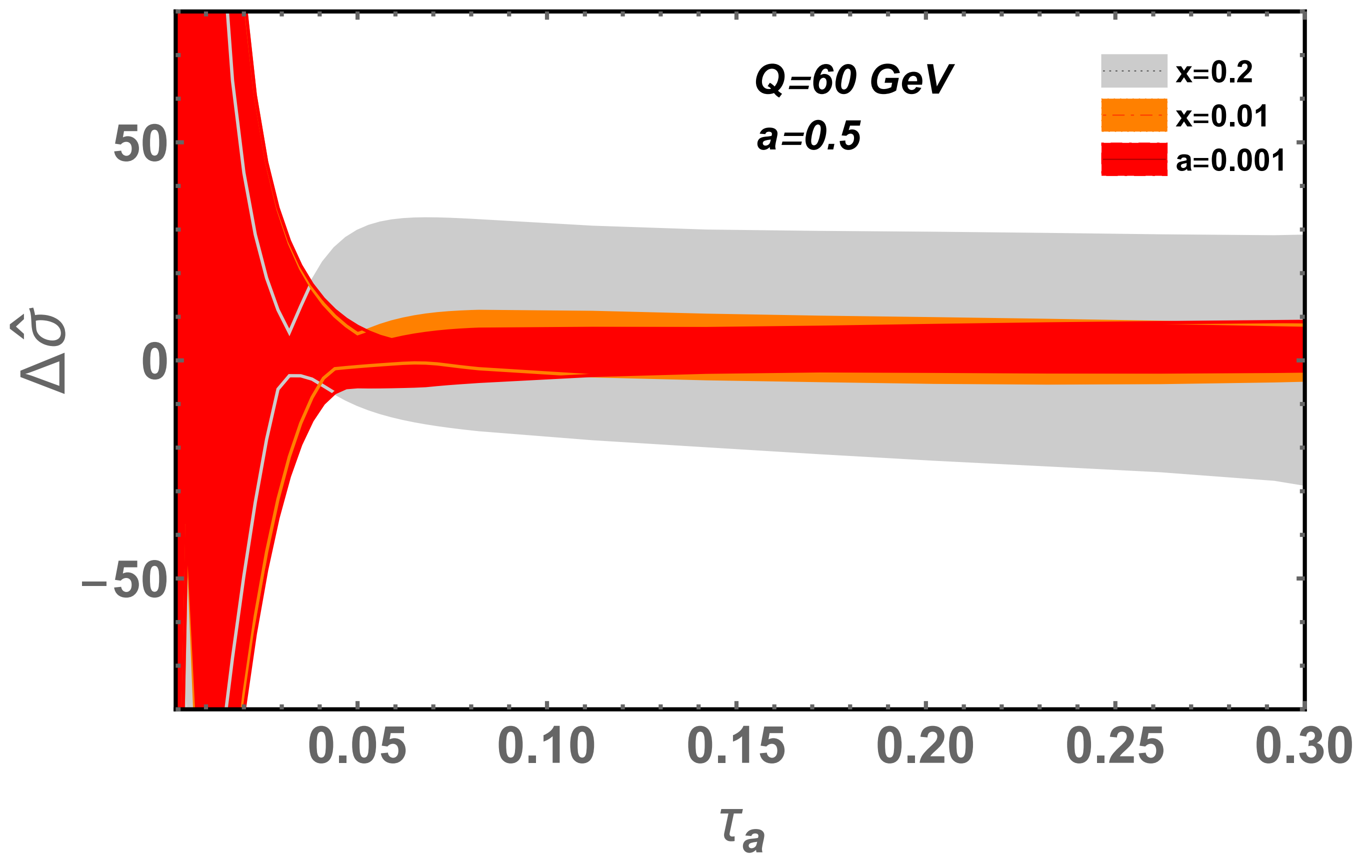
Angularity Cross-section for EIC

Soft-Collinear Effective Theory (SCET)

Sensitivity to ‘a’



Sensitivity to ‘x’



Angularity Cross-section for EIC

Soft-Collinear Effective Theory (SCET)

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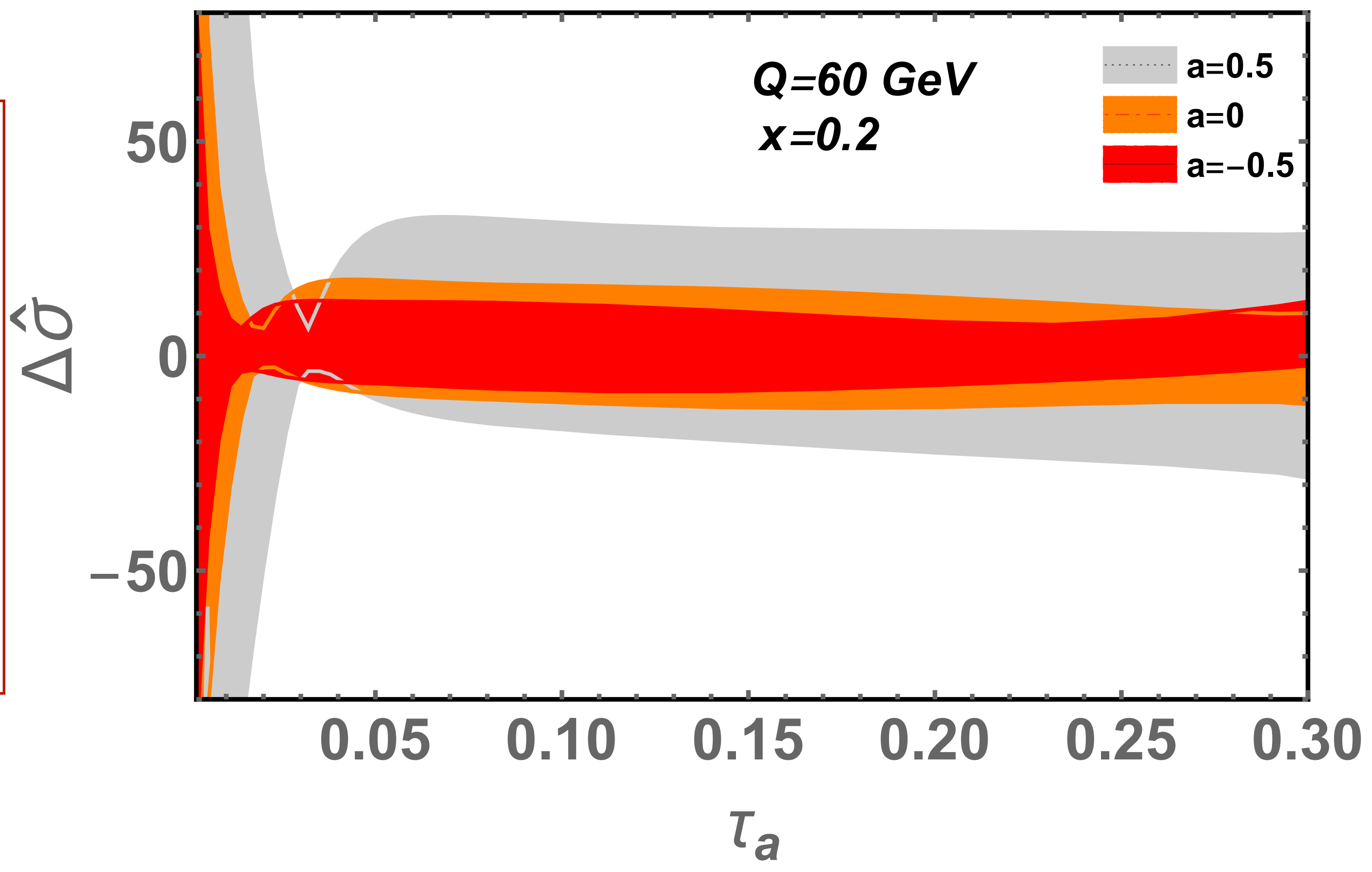
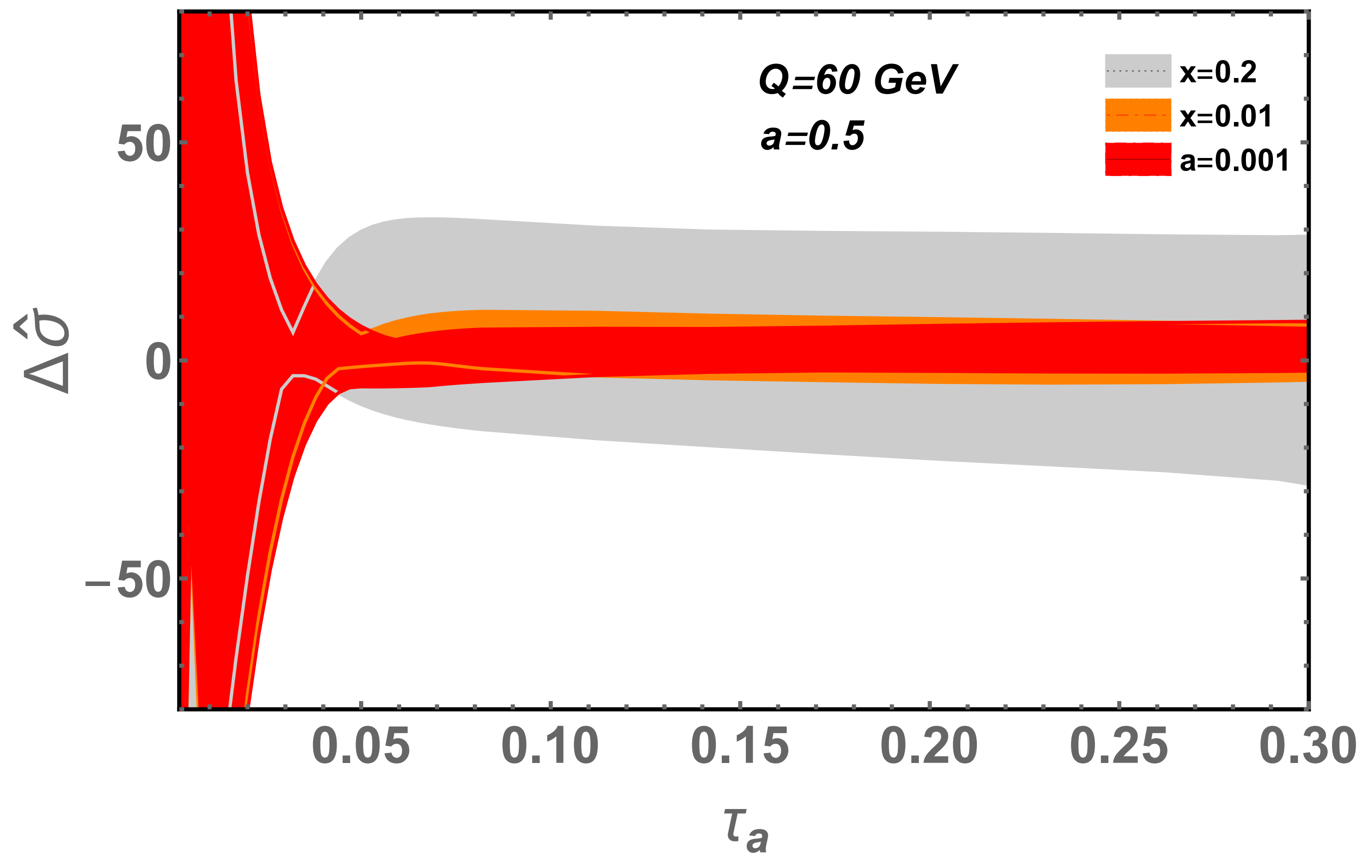
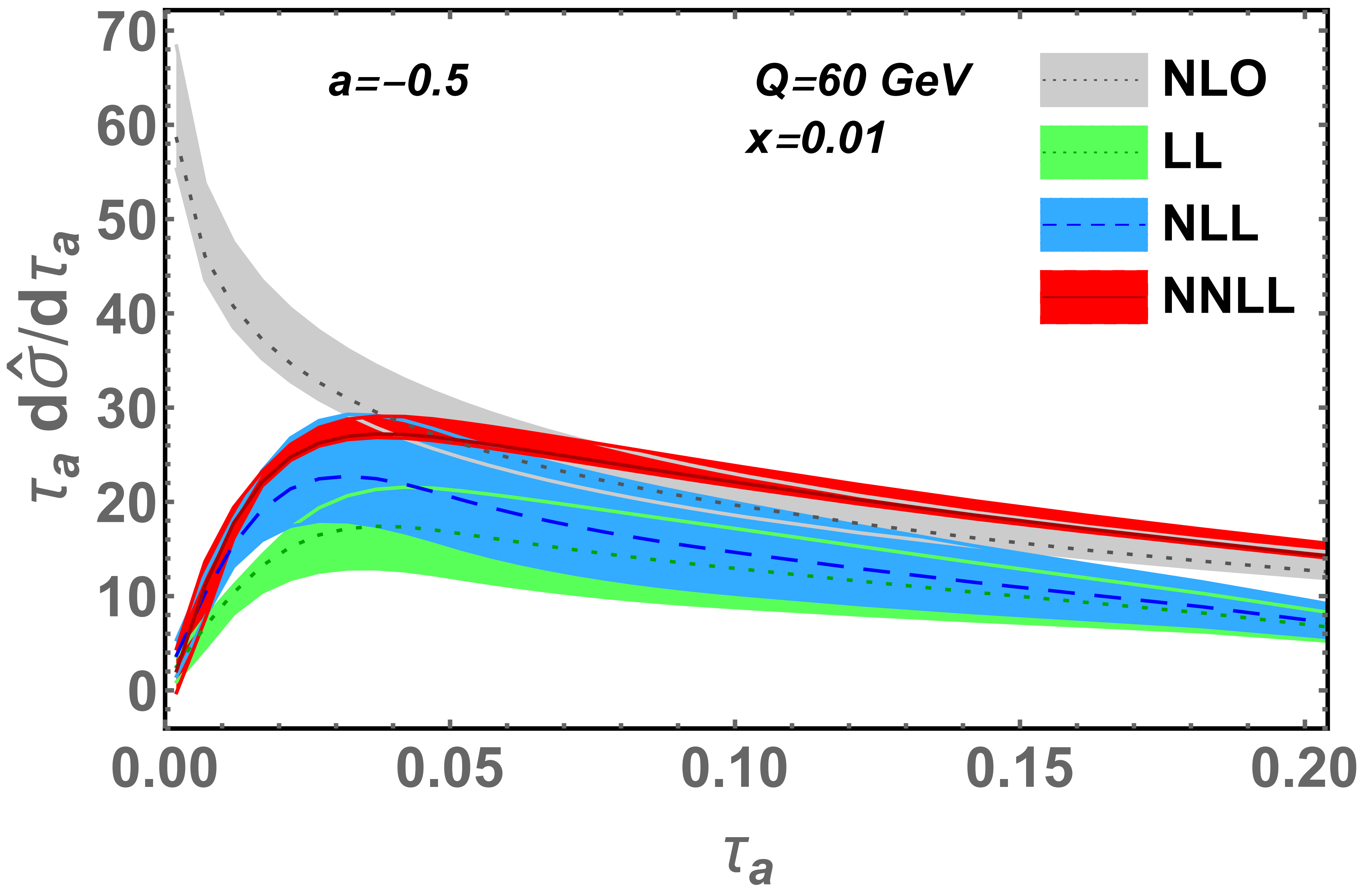


Sensitivity to ‘x’





Angularity measurement would be more precise for a < 0 & small-x

Summary and conclusions

☑ The angularity event shape is defined for deep inelastic scattering process and the angularity beam function is presented at one-loop for the first time.

☑ We present angularity differential cross-section at the NNLL accuracy and give prediction to the future Electron-Ion-Collider kinematics.

Summary and conclusions

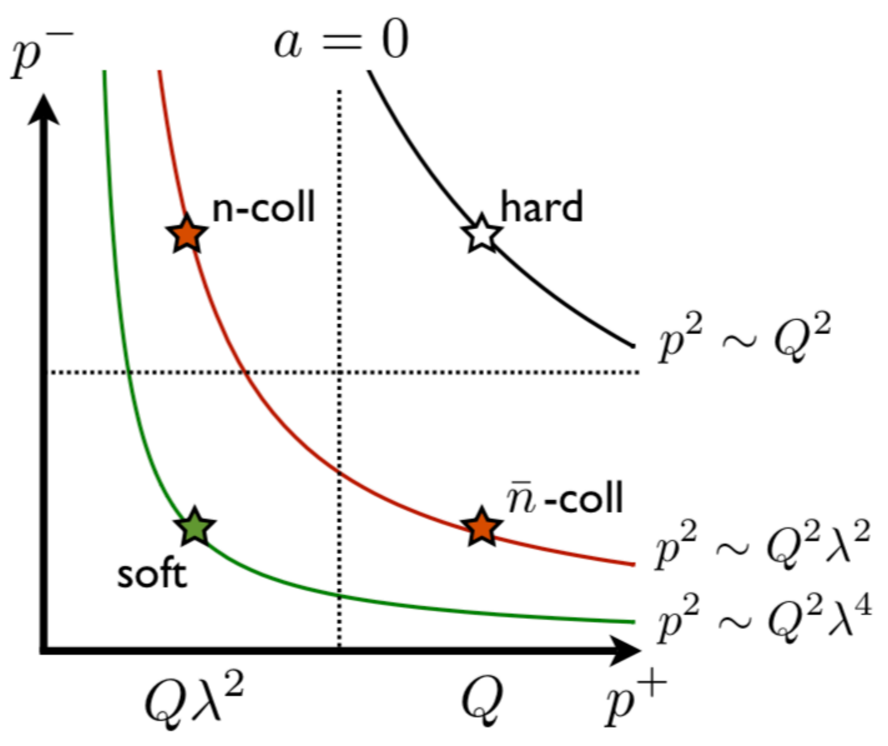
- ☑ The angularity event shape is defined for deep inelastic scattering process and the angularity beam function is presented at one-loop for the first time.

- ☑ We present angularity differential cross-section at the NNLL accuracy and give prediction to the future Electron-Ion-Collider kinematics.

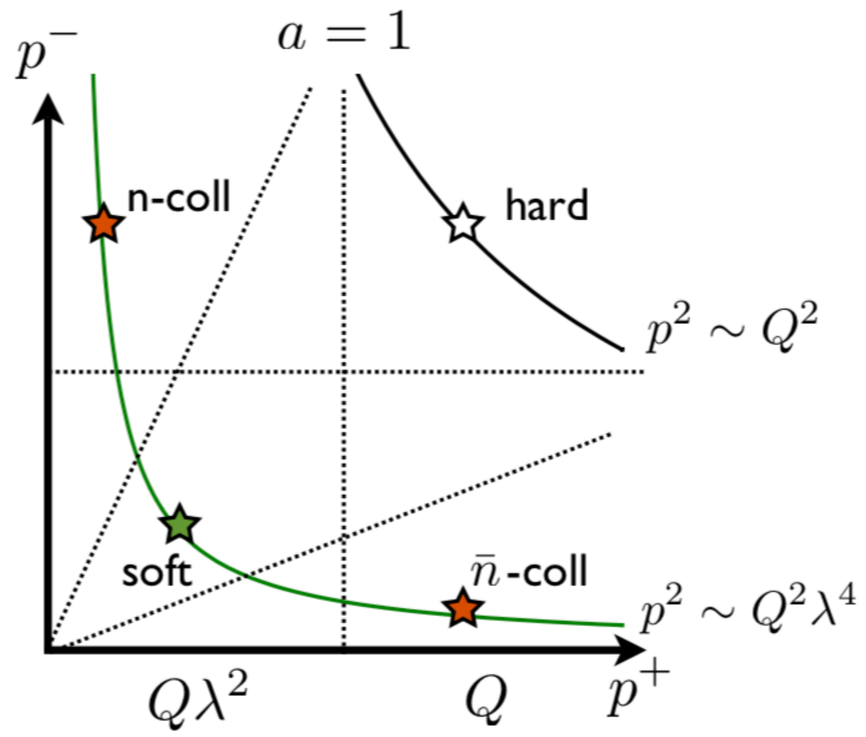
Future direction

- ☑ An extension of this work to access the entire **a** space, specially **a**~1 region, by incorporating the recoil effect.

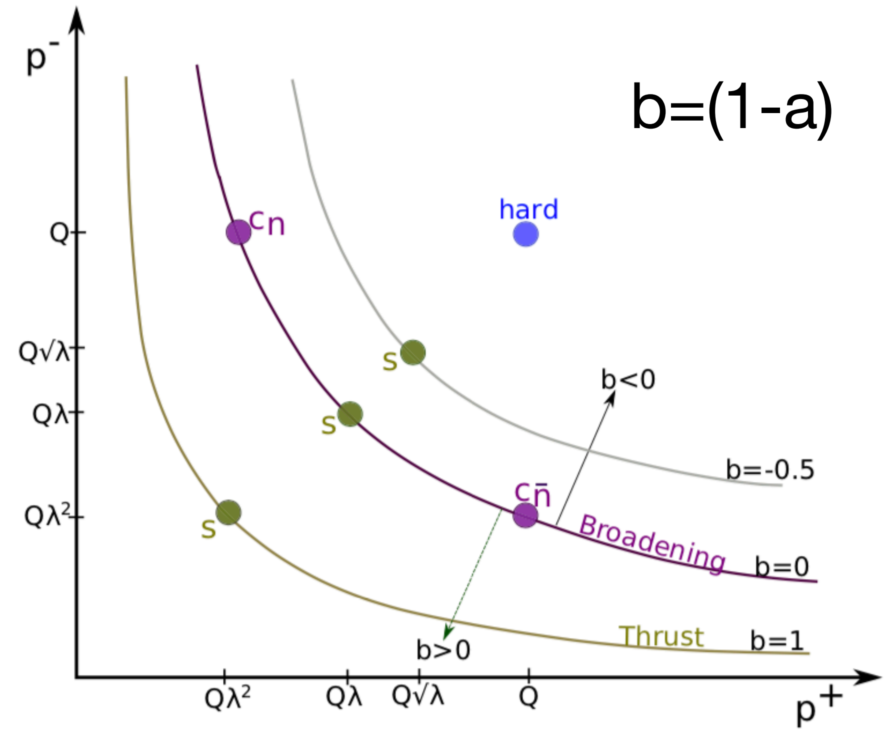
- ☑ Uncertainty in the cross-section is sensitive to Q, '**a**' and 'x' and we need to find out a reasonable profile function for DIS angularity.



SCET-I



SCET-II



Recoil effect

—Andrew Hornig, Christopher Lee, and Grigory Ovanesyan, JHEP 05 (2009) 122

—A. Budhraja, Ambar Jain and Massimiliano Procura, JHEP08(2019)144

Thank you!

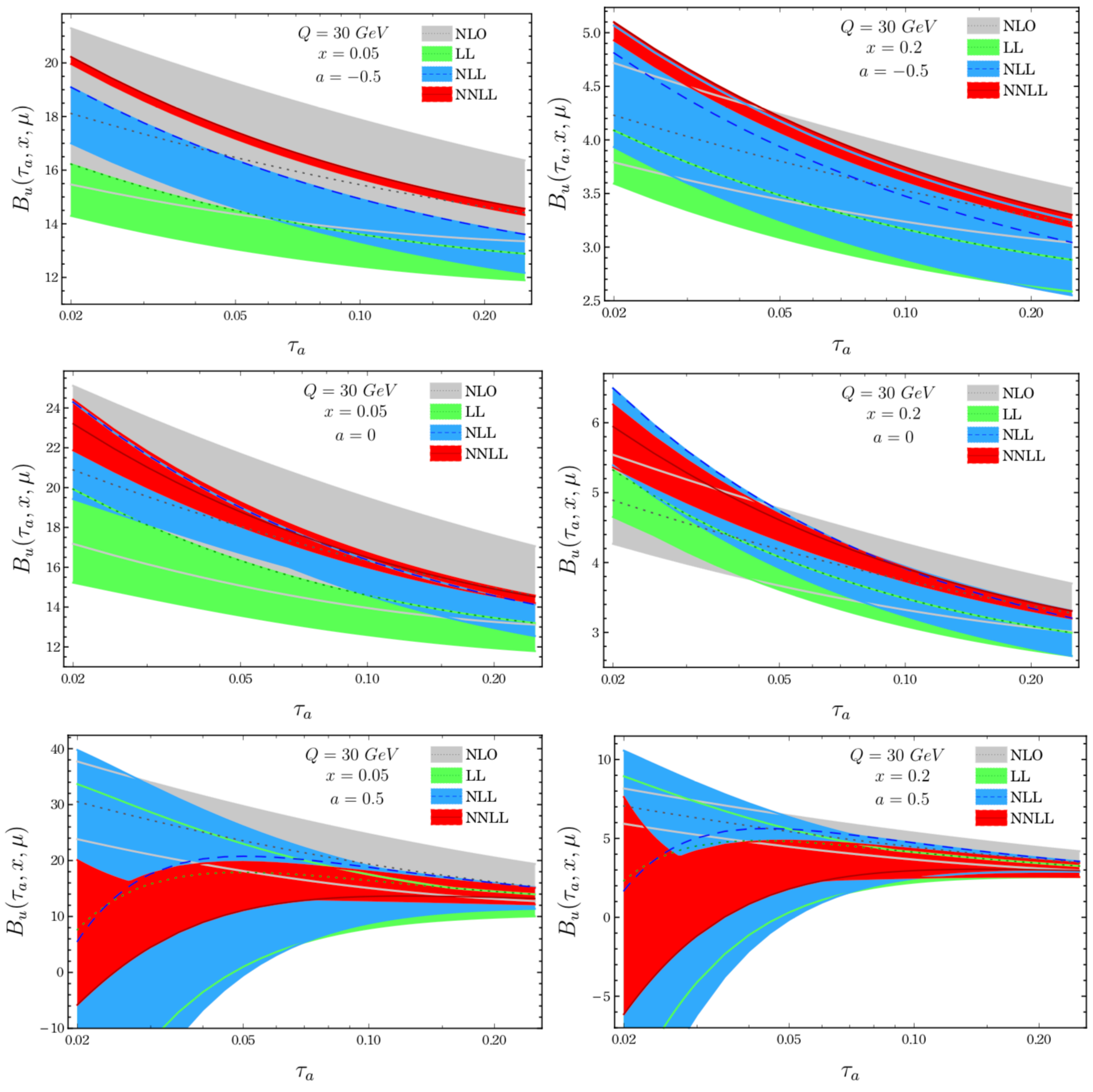
Back up

Results:

Angularity Beam function

$$B(\tau_a^B, x, \mu)$$

Angularity Beam function at NNLL



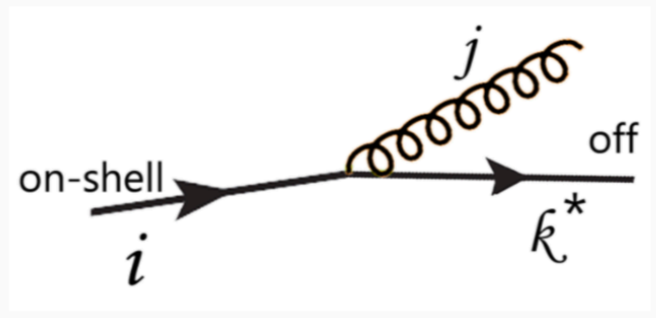
<- a= - 0.5

<- Thrust limit

<- a= 0.5

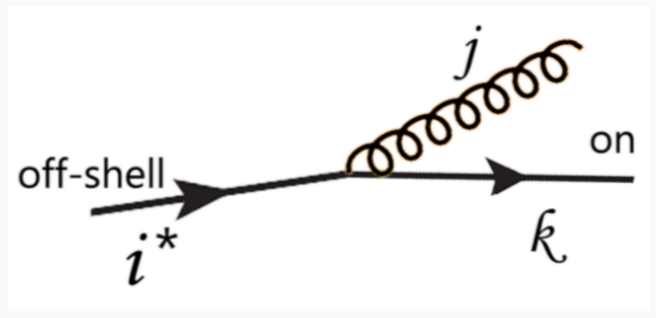
Beam Func. & Fragmentation func.

Beam at NLO: $i \to k^* j$



Crossing Symmetry!

Fragmentation at NLO: $i^* \to kj$



Splitting Function:

[M.Ritzmann,W.J.Waalewijn,PRD90(2014)]

$$P_{i\to k^* j}(2pi.pj, x) \equiv (-1)^{\Delta_f} P_{k*\to ij}(-2pi.pj, 1/x)$$

$$I^{(1)} \sim \ldots \frac{\alpha_s C_F}{2\pi} \frac{2(1-a)}{2-a} \frac{1+x^2}{1-x} \log(x)$$

Resummation in Laplace space

$$\frac{d\sigma}{dx\,dQ^2\,d\tau_a} = \frac{d\sigma_0}{dx\,dQ^2} \int d\tau_a^J\, d\tau_a^B\, d\tau_a^S\, \delta\Big(\tau_a - \tau_a^J - \tau_a^B - \tau_a^S\Big) \times \sum_{i=q,\bar{q}} H_i(Q^2,\mu) \mathcal{B}_i(\tau_a^B, x, \mu) J(\tau_a^J,\mu) S(\tau_a^S,\mu)$$

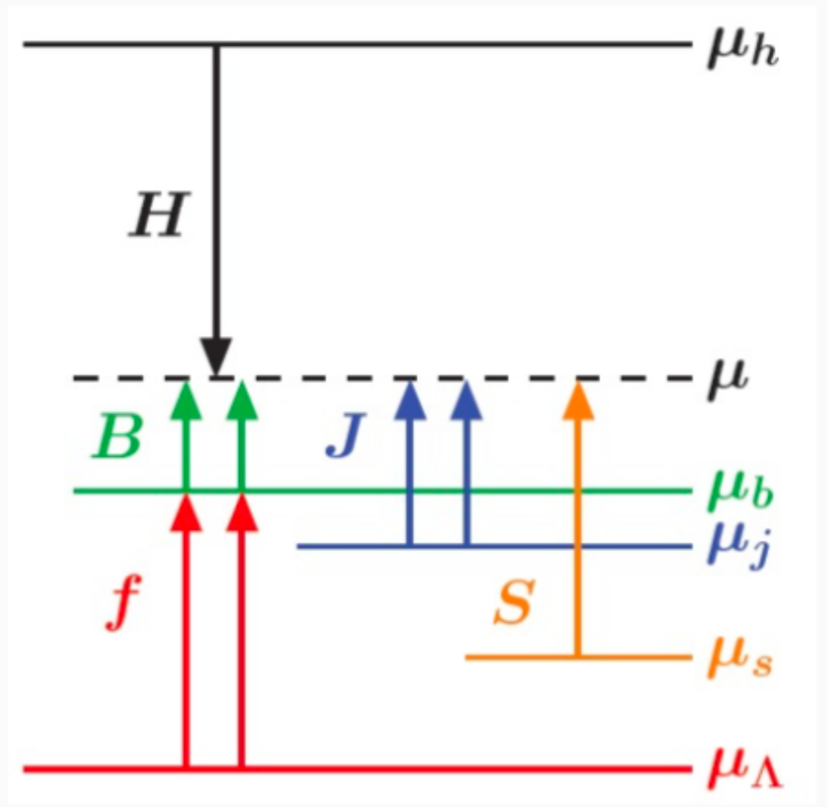
$$G(\nu,\mu) = \int_0^\infty d\tau_a e^{-\nu\tau_a} G(\tau_a,\mu)$$

$$\tilde{\sigma}_q(\nu) = H_q(Q^2,\mu)\tilde{\mathcal{B}}_q(\nu,\mu)\tilde{J}(\nu,\mu)\tilde{S}(\nu,\mu)$$

$$p_c \sim Q(\lambda_c^2, 1, \lambda_c)\,, \qquad \tau_a^B(p_c) \sim \lambda_c^{2-a}$$

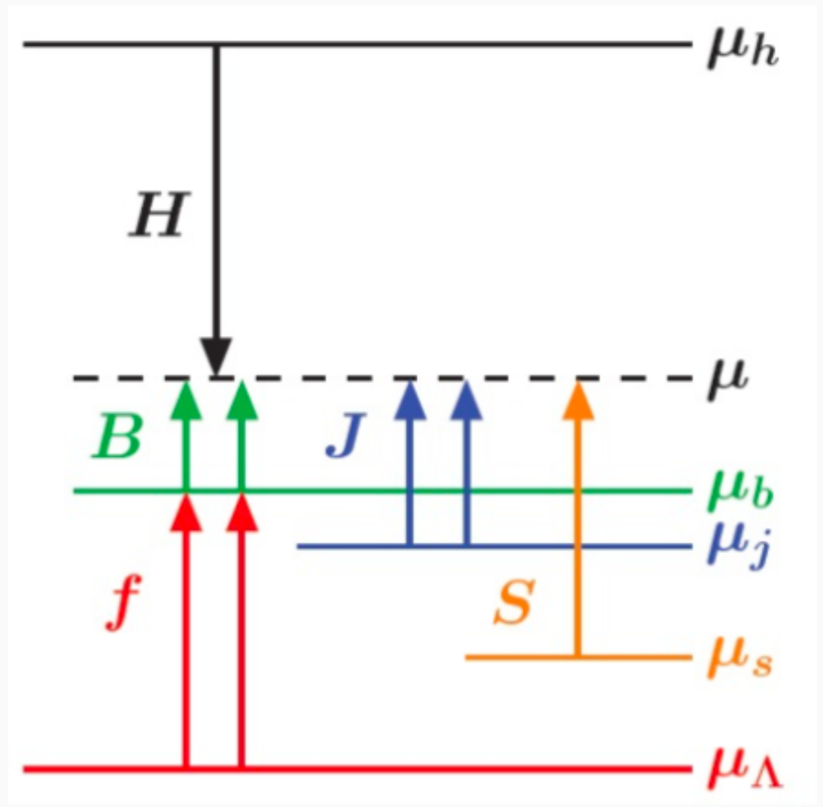
$$p_s \sim Q(\lambda_s, \lambda_s, \lambda_s)\,, \qquad \tau_a^B(p_s) \sim \lambda_s$$

Resummation of large logs



$$\mu_H = Q, \;\; \mu_{J,B} = Q\tau_a^{1/(2-a)}, \;\; \mu_S = Q\tau_a$$

Resummation of large logs



$$\mu_H = Q, \;\; \mu_{J,B} = Q\tau_a^{1/(2-a)}, \;\; \mu_S = Q\tau_a$$

Evolution Equation for beam function

$$\mu \frac{d}{d\mu} B(\nu, \mu) = \gamma_G(\mu)\, B(\nu, \mu) \;\; ; \qquad \text{similar to} J, S, H$$

$$\text{Solution}: \; B(\nu, \mu) = B(\nu, \mu_B)\, e^{K_B(\mu_B,\mu) + j_B \eta_B(\mu_B,\mu) L_B},$$

- Jet and beam functions are defined by same collinear operator: $\gamma_J(\mu) = \gamma_B(\mu)$

$$K_B(\mu_B, \mu) = L_B \sum_{k=1}^{\infty} (\alpha_s L_B)^k + \sum_{k=1}^{\infty} (\alpha_s L_B)^k + \ldots$$

LL $\qquad NLL$

$\alpha_s L_B \sim 1$

$L_B = \ln(\mu/\mu_B)$

LL: Leading Log;
NLL: Next-to-Leading Log

Anomalous dimension

The universal cusp anomalous dimension $\Gamma_{\rm cusp}(\alpha_s)$ and non-cusp anomalous dimension $\gamma_G(\alpha_s)$ are expressed in powers of α_s as

$$\Gamma_{\rm cusp}(\alpha_s) = \sum_{n=0} \Gamma_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} , \qquad \gamma_G(\alpha_s) = \sum_{n=0} \gamma_n^G \left(\frac{\alpha_s}{4\pi}\right)^{n+1} , \tag{4.15}$$

where Γ_n are given in appendix D and one-loop result for γ_n^G are given in [36]

$$\gamma_0^G = \{-12C_F \,, 0 \,, 6C_F\} \qquad G = \{H, S, J\} \,, \tag{4.16}$$

which again satisfies the consistency in eq. (4.13) at the order α_s. The two-loop hard anomalous dimension is well known [61, 63] and available up to three-loops [64]

$$\gamma_1^H = -2C_F\Big[\Big(\frac{82}{9} - 52\zeta_3\Big)C_A + (3 - 4\pi^2 + 48\zeta_3)C_F + \Big(\frac{65}{9} + \pi^2\Big)\beta_0\Big] \,. \tag{4.17}$$

$$\gamma_G(\mu) = j_G\,\kappa_G \Gamma_{\rm cusp}(\alpha_s) L_G + \gamma_G(\alpha_s)\,, \tag{4.11}$$

where $\Gamma_{\rm cusp}(\alpha_s)$ and $\gamma_G(\alpha_s)$ are the cusp and non-cusp anomalous dimensions. The characteristic logarithm L_G is defined as

$$L_G = \begin{cases} \ln\left(\frac{Q}{\mu}\right) & G = H\,, \\ \ln\left[\frac{Q}{\mu}(\nu e^{\gamma_E})^{-1/j_G}\right] & G = \{\widetilde{S}, \widetilde{J}, \widetilde{\mathcal{B}}\}\,, \end{cases} \tag{4.12}$$

The consistency relation followed by scale independence of cross section $d\sigma(\mu)/d\mu = 0$ is given by $\gamma_H(\mu) + \gamma_{\widetilde{S}}(\mu) + 2\gamma_{\widetilde{J}}(\mu) = 0$, which is valid for any values of Q, μ, ν in eq. (4.11) and it turns into three consistency relations

$$\begin{aligned} j_H\,\kappa_H + j_S\,\kappa_S + 2 j_J\,\kappa_J &= 0\,, \\ \kappa_S + 2\kappa_J &= 0\,, \\ \gamma_H(\alpha_s) + \gamma_S(\alpha_s) + 2\gamma_J(\alpha_s) &= 0\,. \end{aligned} \tag{4.13}$$

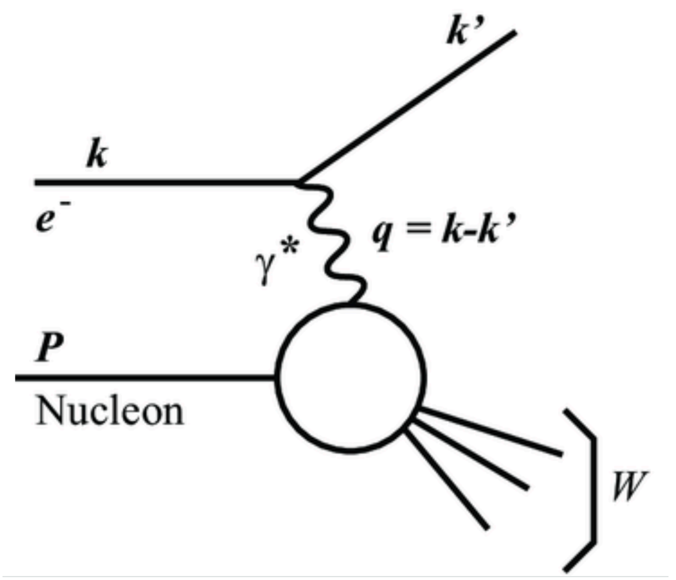
The constants j_G and κ_G are given by

$$\begin{aligned} j_G &= \{1, 1, 2-a\}\,, \\ \kappa_G &= \left\{4, \frac{4}{1-a}, -\frac{2}{1-a}\right\}\,, \qquad G = \{H, S, J\} \end{aligned} \tag{4.14}$$

where $C_{qj} = C_F, T_F$ for $j = q, g$. One of the logarithmic terms L_B is associated with PDF with the splitting functions P_{qj}

$$\begin{aligned} P_{qq}(z) &= \left[\frac{\theta(1-z)}{1-z}\right]_+ (1+z^2) + \frac{3}{2}\delta(1-z) = \left[\theta(1-z)\frac{1+z^2}{1-z}\right]_+\,, \\ P_{qg}(z) &= \theta(1-z)[(1-z)^2 + z^2]\,. \end{aligned} \tag{5.5}$$

DIS factorization in SCET



$$\frac{d\sigma}{dxdQ^2d\tau_a} = L_{\mu\nu}(x, Q^2)\, W^{\mu\nu}(x, Q^2, \tau_a)$$

The hadronic tensor defined by QCD current $J^\mu(x) = \bar{\psi}\gamma^\mu\psi(x)$

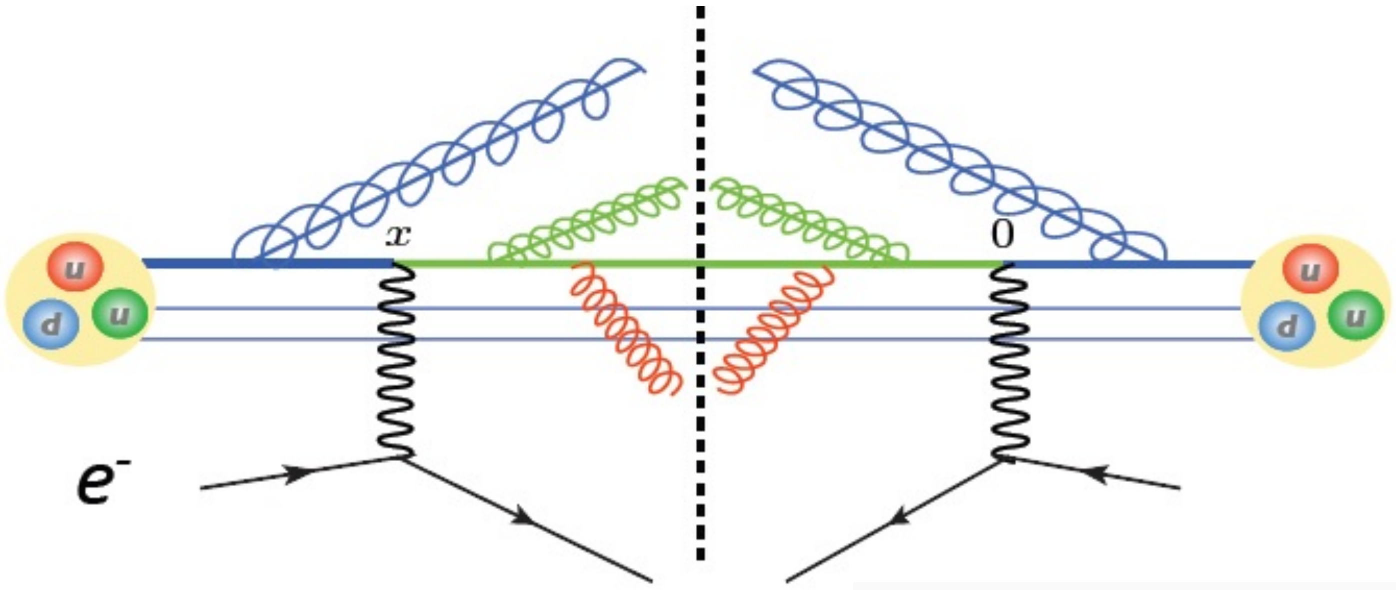
$$W^{\mu\nu}(x, Q^2, \tau_a) = \sum_X \langle P|J^{\mu\dagger}|X\rangle\langle X|J^\nu|P\rangle(2\pi)^{(4)}\delta^4(P+q-p_X)\delta(\tau_a - \tau_a(X))$$
$$= \int d^4x\, e^{iq\cdot x}\langle P|J^{\mu\dagger}(x)\delta(\tau_a - \hat{\tau}_a)J^\nu(0)|P\rangle\,.$$

Neglecting the power correction O(λ^2), we match the current $J^\mu(x) = \bar{\psi}\gamma^\mu\psi(x)$ onto the operators in SCET and perform the field redefinition to have factorized form of the hadronic tensor as

$$W_{\mu\nu}(x, Q^2, \tau_a) = \left(\frac{8\pi}{n_J\cdot n_B}\right)\int d\tau_a^J\, d\tau_a^B\, d\tau_a^S\, \delta\Big(\tau_a - \tau_a^J - \tau_a^B - \tau_a^S\Big)$$
$$\times H_{\mu\nu}(q^2,\mu)\mathcal{B}_i(\tau_a^B, x, \mu)J(\tau_a^J,\mu)S(\tau_a^S,\mu)$$

Measurement operator: $\hat{\tau}_a = \hat{\tau}_a^{cB} + \hat{\tau}_a^{cJ} + \hat{\tau}_a^S$

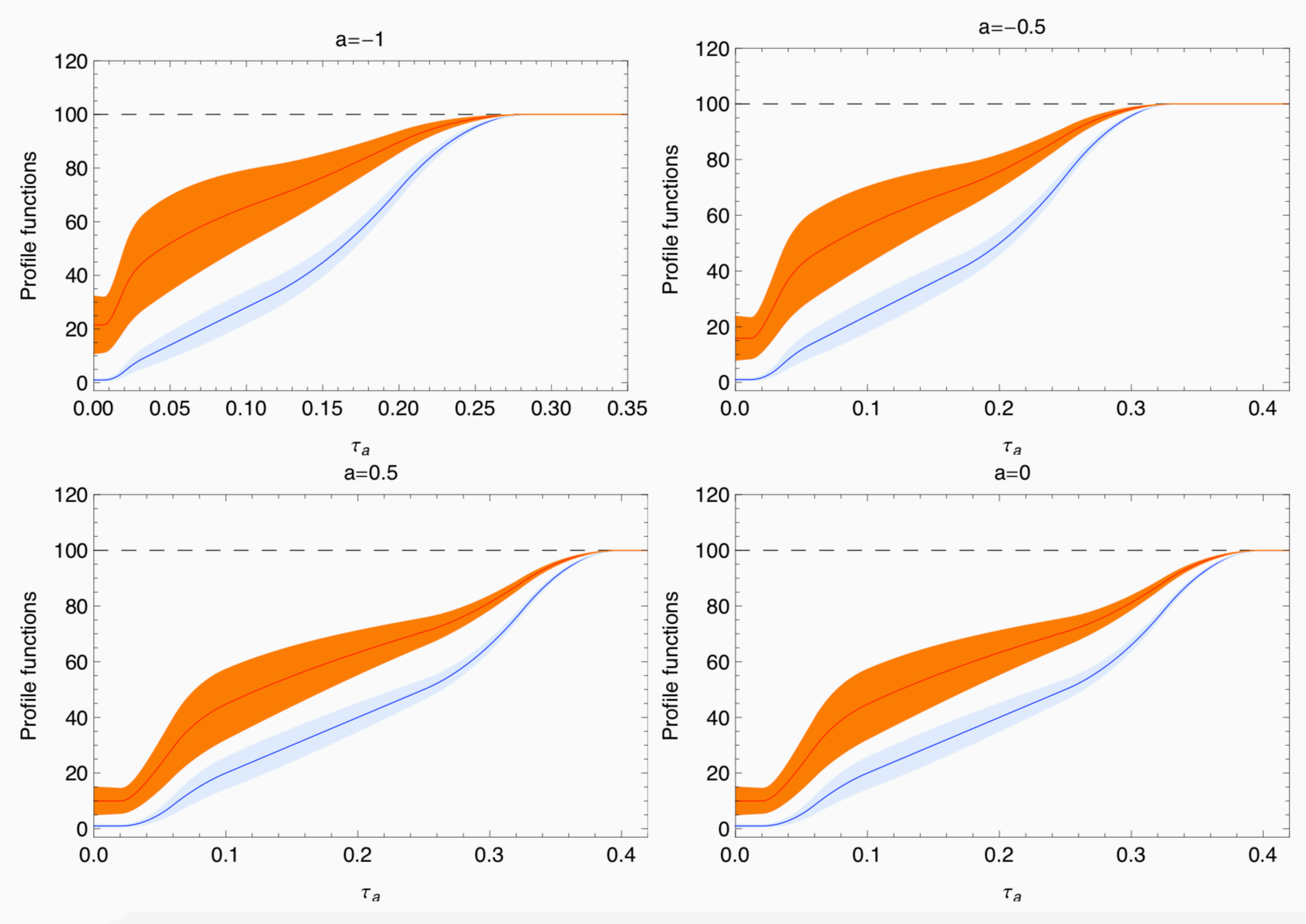
$$\frac{d\sigma}{dxdQ^2d\tau_a} = \frac{d\sigma_0}{dxdQ^2}\int d\tau_a^J\, d\tau_a^B\, d\tau_a^S\, \delta\Big(\tau_a - \tau_a^J - \tau_a^B - \tau_a^S\Big)$$
$$\times \sum_{i=q,\bar{q}} H_i(Q^2,\mu)\mathcal{B}_i(\tau_a^B, x, \mu)J(\tau_a^J,\mu)S(\tau_a^S,\mu)$$



— D.Kang,Lee,Stewart'2013
Z.Kang,Mantry,Qiu'2012

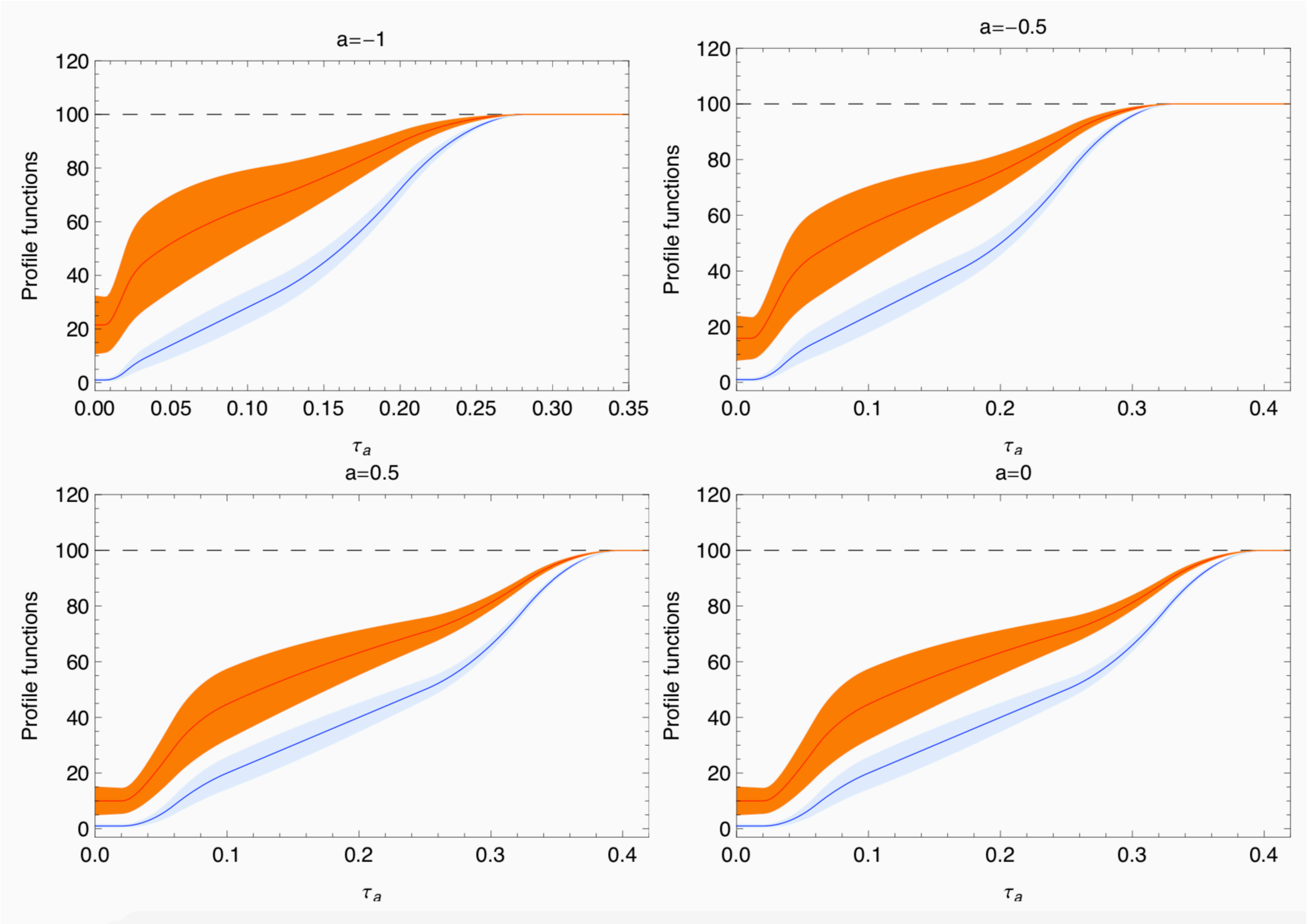
SCET facto.: dσ = Hard × Beam ⊗ Jet ⊗ Soft

Profile function

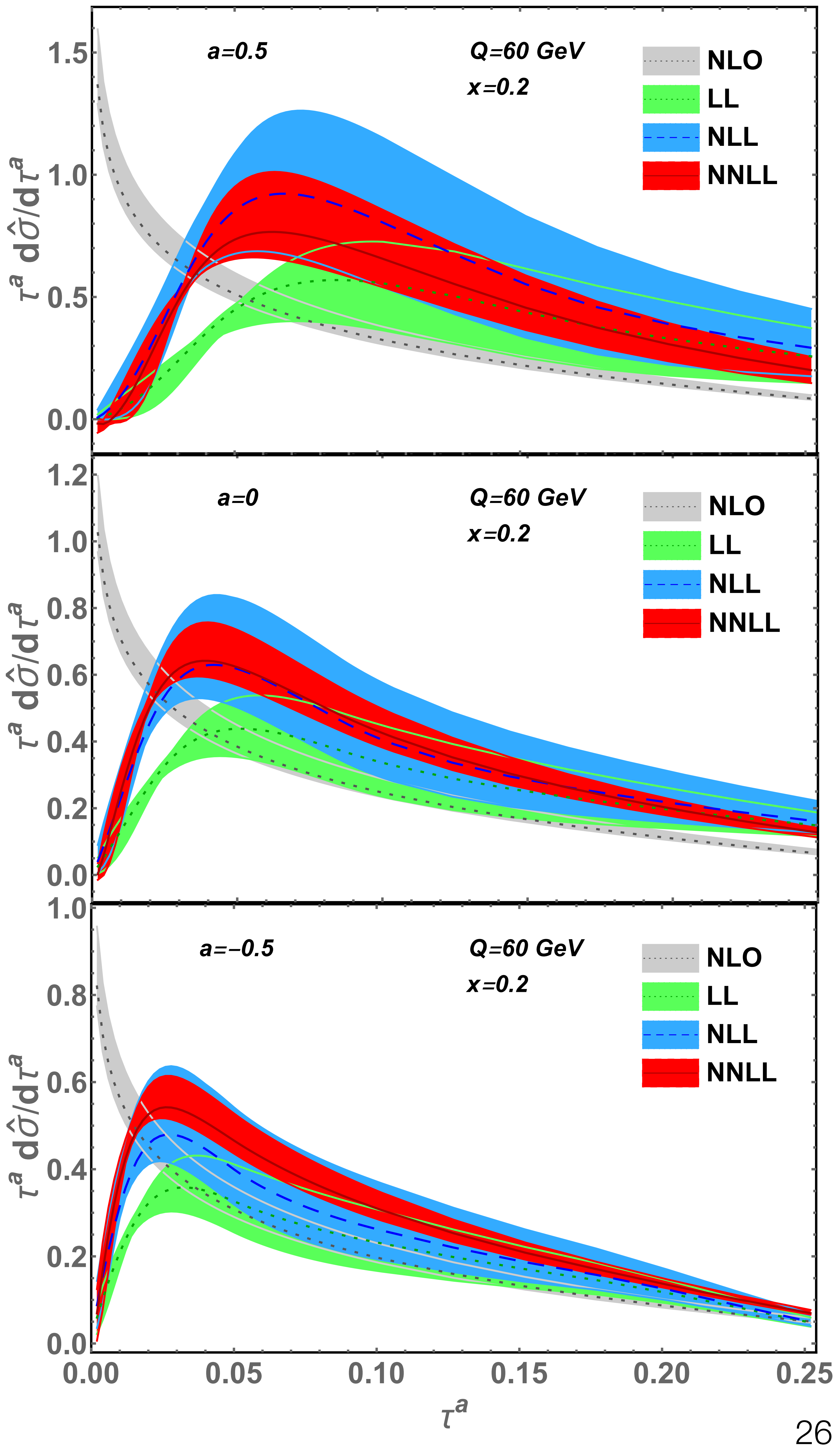


Profile function

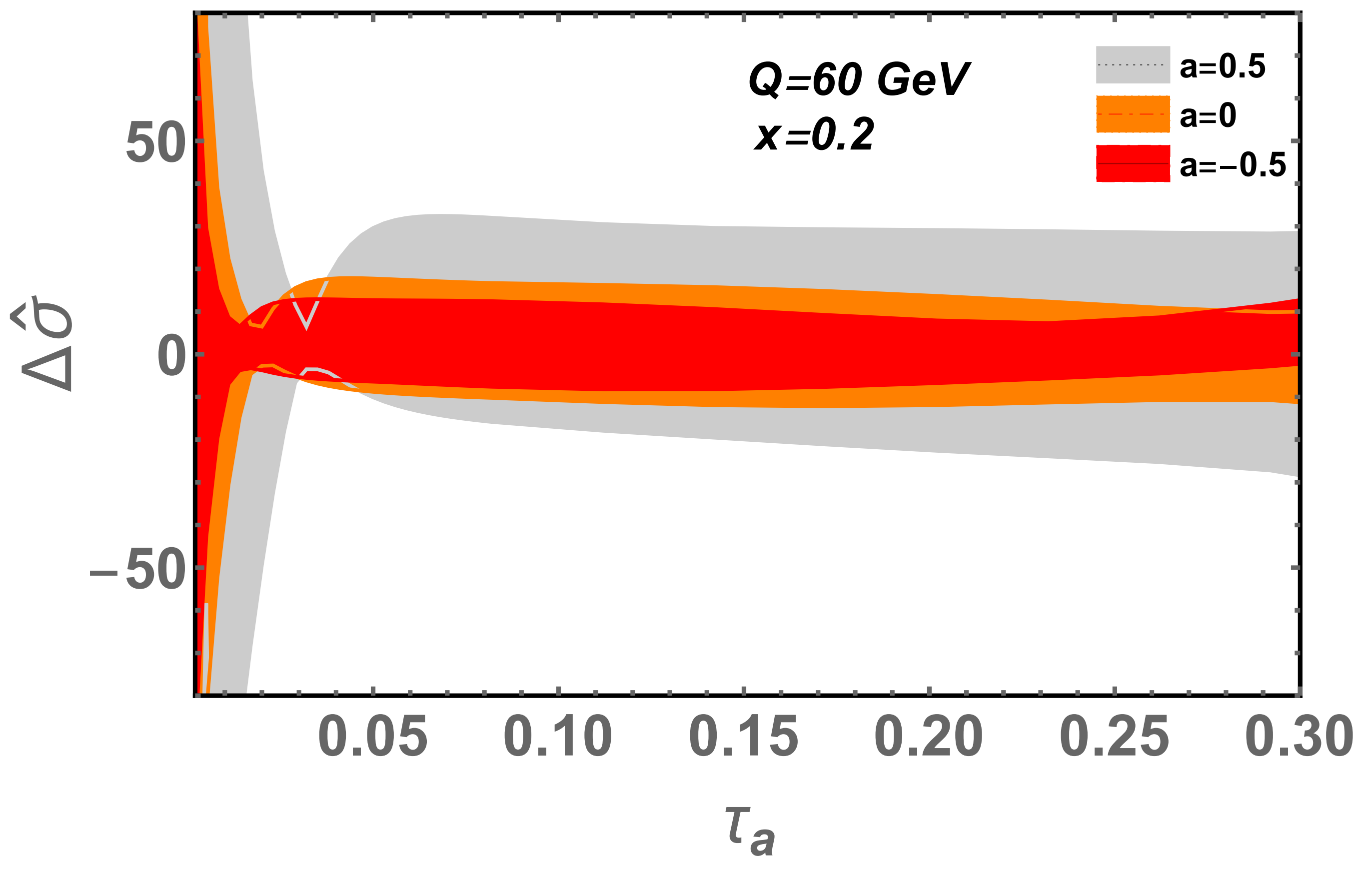
- We adopt electron-positron angularity profile function from Bell, Hornig, Lee, Talbert, 18



'a' dependency

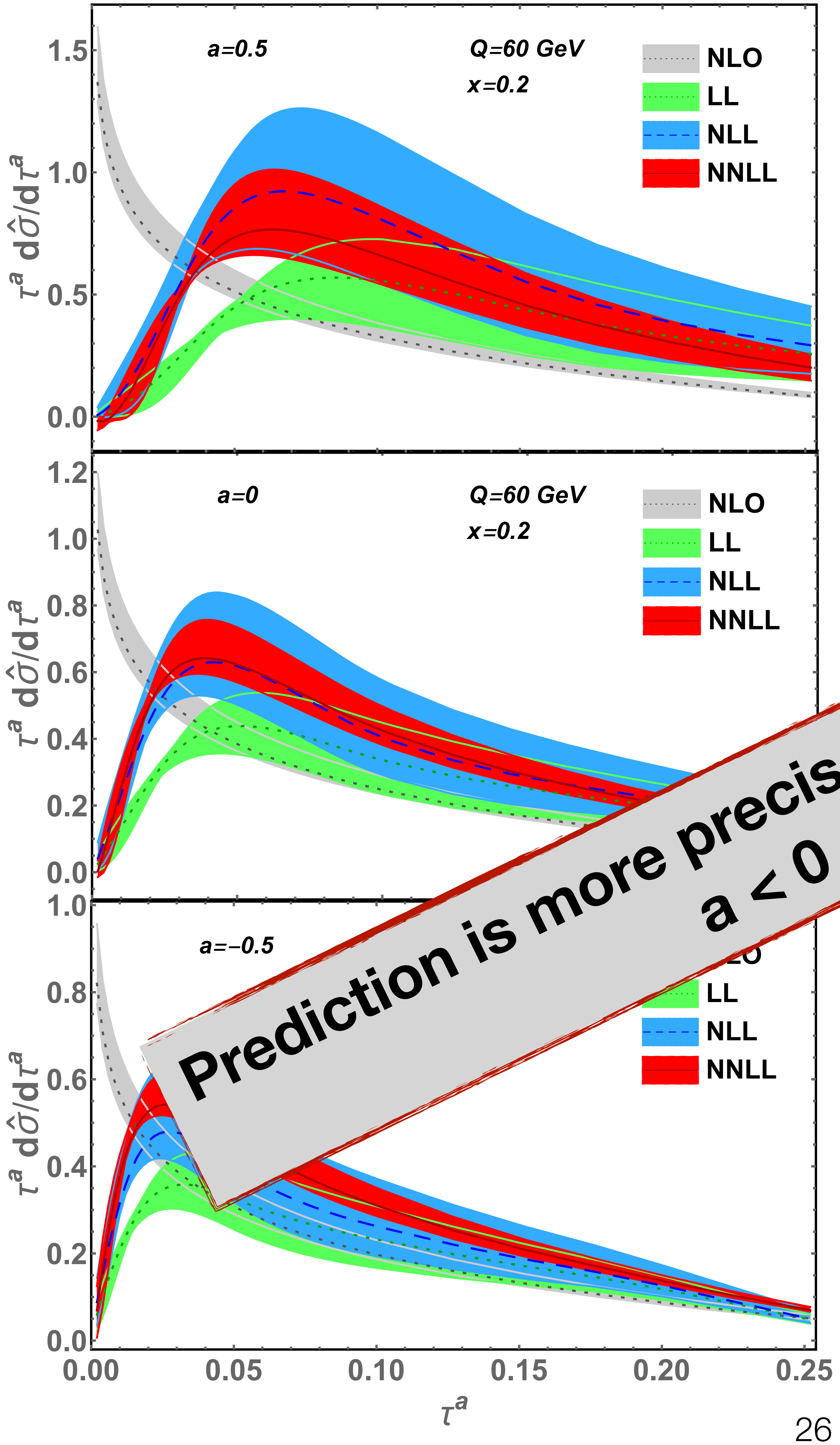


Relative uncertainty at NNLL

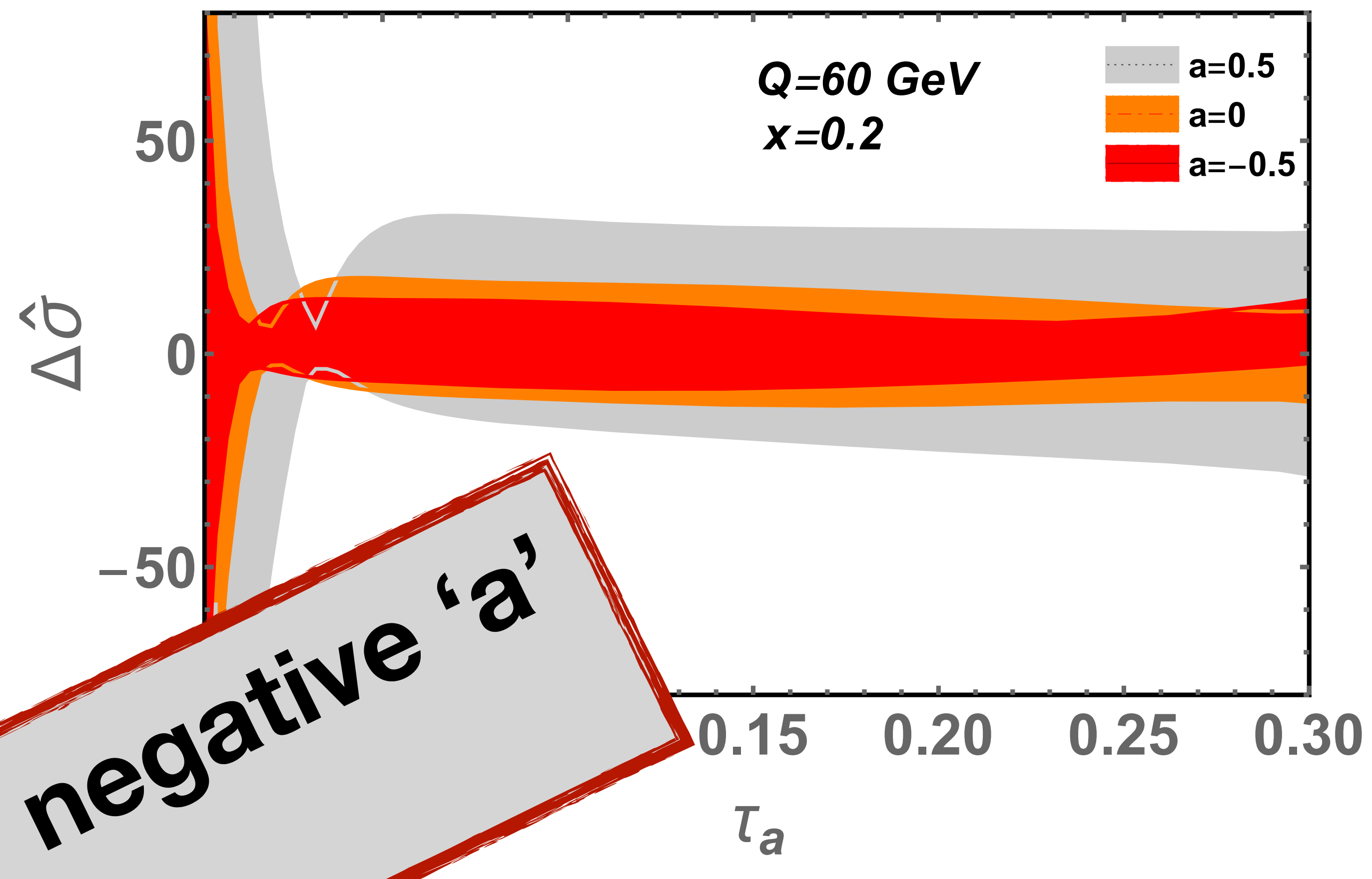


uncertainty in the DIS angularity cross-section depends on the angularity parameter 'a'

‘a’ dependency



Relative uncertainty at NNLL



Prediction is more precise for negative ‘a’
a < 0

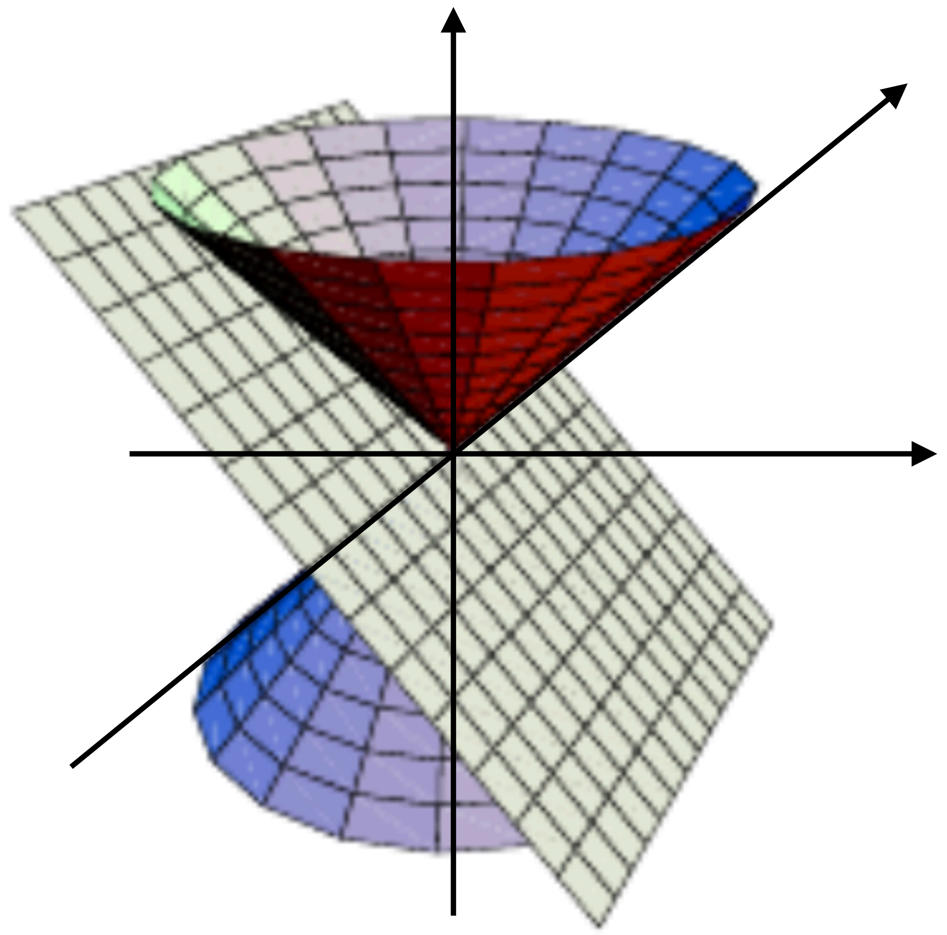
- uncertainty in the DIS angularity cross-section depends on the angularity parameter ‘a’

Tool: Light-front dynamics

$$(x^+, x^-, x_\perp)$$

$$x^+ = x^0 + x^3$$

$$x^- = x^0 - x^3$$



Scalar product $$x \, . \, p = \frac{1}{2} x^+ p^- + \frac{1}{2} x^- p^+ - \mathbf{x}_\perp \, . \, \mathbf{p}_\perp$$

Dispersion relation $$p^2 = m^2 \; ; \qquad p^- = \frac{p_\perp^2 + m^2}{p^+}$$

- **Vacuum is trivial in the Light-Front theory:** $p_i^+ \geq 0$
- **Time order not Needed: backward going diagram vanishes**