Thermal photon production rate from Transverse-Longitudinal (T-L) mesonic correlator on the lattice

Dibyendu Bala

Bielefeld University, Germany

Sajid Ali, Olaf Kaczmarek, Anthony Francis, Greg Jackson, Tristan Ueding
HotQCD Collaboration

Outline

- Photons and dileptons produced from QGP are important probes to study Quark-Gluon-Plasma.
- Photons and dileptons will directly come out of the plasma without further interaction with the plasma.
- The photon production rate (R_γ) and di-lepton production rate ($R_{l^+l^-}$) from a thermalized QGP can be calculated in terms of the spectral function L.D. McLerran and T. Toimela, Phys. Rev. D 31 (1985) 545.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)}{2\pi^2 k} g^{\mu\nu}\rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_{l^+l^-}}{d\omega d^3\vec{k}} = \frac{\alpha_{em}^2 n_b(\omega)}{3\pi^2(\omega^2 - k^2)} g^{\mu\nu}\rho_{\mu\nu}(\omega, \vec{k})$$

- Need to estimate $\rho_{\mu\nu}$ from lattice.
- We need correlation function of the current operator $J_\mu(\vec{x},\tau) = \bar{\psi}(\vec{x},\tau)\gamma_\mu\psi(\vec{x},\tau)$ on the lattice.

$$G^E_{\mu\nu}(\tau,\vec{k}) = \int d^3\vec{x}\exp\left(i\vec{k}.\vec{x}\right)\langle J_\mu(\vec{x},\tau)J_\nu(\vec{0},0)\rangle$$

- Relation with spectral function,

$$G^E_{\mu\nu}(\tau,\vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_{\mu\nu}(\omega,\vec{k})\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically ill-posed problem.
  1) Difference in the number of degrees of freedom.
  2) Small error in G^E become very large error in ρ.

- $\rho_{\mu\nu}$ can be decomposed,

$$\rho_{\mu\nu}(\omega,\vec{k}) = P^T_{\mu\nu}\rho_T(\omega,\vec{k}) + P^L_{\mu\nu}\rho_L(\omega,\vec{k})$$

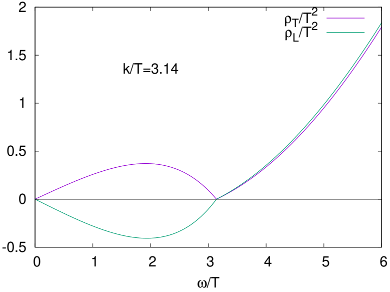
$$\rho_V(\omega,\vec{k}) = \rho^{\mu}_{\mu}(\omega,\vec{k}) = 2\rho_T(\omega,\vec{k}) + \rho_L(\omega,\vec{k})$$

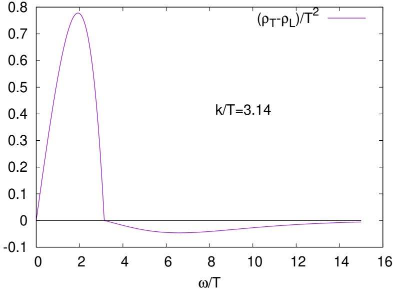
- At the photon point $\rho_L(|\vec{k}|,\vec{k}) = 0$.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\rho_T(|\vec{k}|,\vec{k})$$

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2(\rho_T(|\vec{k}|,\vec{k}) - \rho_L(|\vec{k}|,\vec{k}))$$

- Free result for ρ_T and ρ_L, G. Aarts and J.M. Resco,Nucl. Phys. B 726 (2005) 93





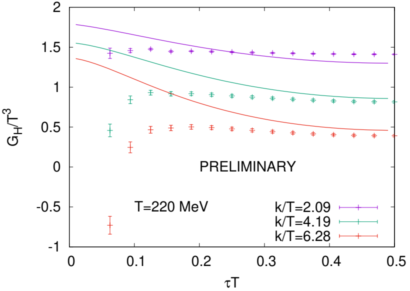
- $\rho_V = 2\rho_T + \rho_L$ has large UV part. G_V^E has large UV contribution.
- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.
  M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato , Phys. Rev. D 102, 091501(R)
- Sum rule $\int_0^\infty d\omega\omega\rho_H(\omega) = 0$ and from OPE at large ω, $\rho_H(\omega) \sim \frac{1}{\omega^4}$
- Photon production rate can be estimated from ρ_H.

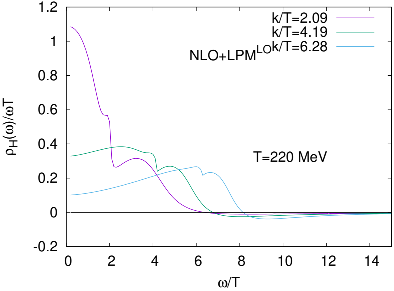
- We use $N_f = 2+1$ flavor HISQ configurations with $m_l = \frac{m_s}{5}$.
- Lattice spacing $a \sim 0.028$ fm and pion mass $m_\pi \sim 313$MeV.
- Lattice size: $96^3 \times 32$ (spatial extent ~ 2.67 fm).
- Temperatures: $1.15\,T_{pc}$(220 MeV).
- On these configurations, we calculate the correlation function of clover Wilson fermion at finite momentum $\frac{k}{T} = \frac{2\pi n}{3}$.
- $\kappa = 0.13515$ and $C_{sw} = 1.34108$ (Tadpole improvement)
- This corresponds to a quark mass $<<$ T.

$$G_H^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} 2(\rho_T(\omega, \vec{k}) - \rho_L(\omega, \vec{k})) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- G_H has multiplicative renormalization.
- $G_H/2\chi_q T$ is does not need renormalization.
- $\chi_q = 0.872\,T^2$ in $g^6 \log(g)$ perturbation theory.

  A. Vuorinen, Phys. Rev. D 67, (2003) 074032





G. Jackson & M. Laine, J. High Energy. Phys. 2019, 144

- Non-perturbative modeling of the spectral function is required.

- The polynomial fit ansatz for spectral function,

$$G_H^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho_H(\omega, \vec{k}) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- For $\omega \leq \omega_0$

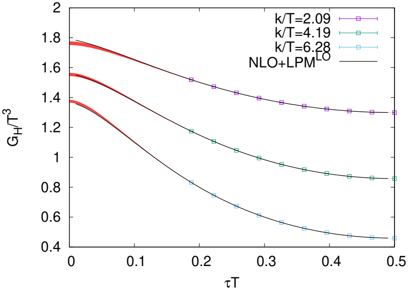
$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \delta_0\left(\frac{\omega}{\omega_0}\right)\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

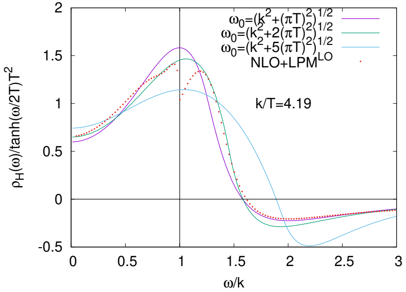
J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, Phys. Rev. D 94, 016005. and $\omega \geq \omega_0$

$$\rho_H(\omega) = \frac{a}{\omega^4} + \frac{b}{\omega^6} + \frac{c}{\omega^8}$$

- $\beta = \rho_H(\omega_0)$ and $\gamma = \rho'_H(\omega_0)$
- a, b, c are determined from the smoothness condition at $\omega = \omega_0$ along with the sum rule $\int_0^\infty d\omega\omega\rho_H(\omega) = 0$.
- Constrained fit with $\delta_0 \geq 0$,$\rho_H(k, \vec{k}) \geq 0$ and $\frac{\partial G_H}{\partial \tau} \leq 0$

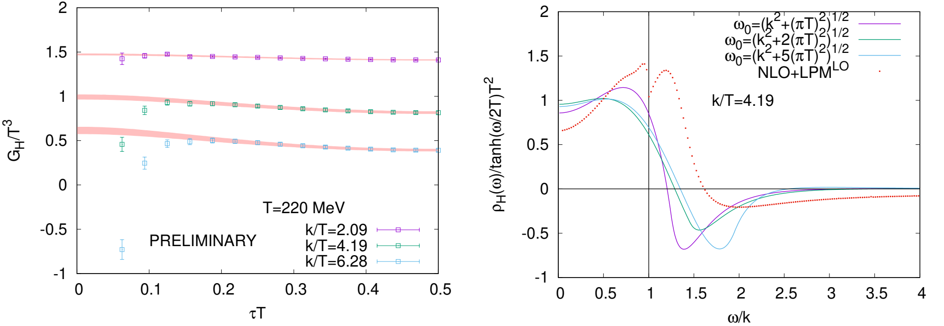
- Ten perturbative data points between 0.1875 to 0.5 in τT.
- An artificial error was introduced to the order $\delta G/G = 0.001$ and tried to reconstruct the spectral function.





- The exact spectral function can be captured within a systematic uncertainty between $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$ and $\omega_0 = \sqrt{k^2 + 5\pi^2 T^2}$

- Reconstruction with non-perturbative data.



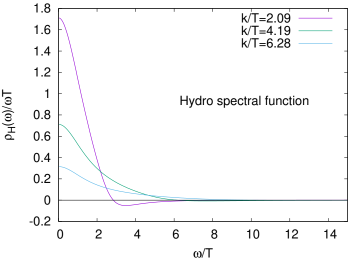
- The non-perturbative estimate is smaller at the photon point than the perturbative estimate.

- Hydro-motivated spectral function, arxiv.2112.12497

$$\rho_H^{hydro}(\omega,\vec{k}) = A\frac{\tanh(\omega/T)\,k^2\,(1+a^2\,k^2-2\,a\,b\,\omega^2+b^2\omega^2)}{(1+b^2\,\omega^2)(a^2\,k^4+\omega^2-2a\,k^2\,b\,\omega^2+b^2\,\omega^4)}$$

- The sum rule will relate to a and b.
- OPE prediction $\frac{1}{\omega^4}$.

  M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato , Phys. Rev. D 102, 091501(R)



- Backus Gilbert's estimate of the spectral function,

  G. Backus, F. Gilbert, Geophysical Journal of the Royal Astronomical Society 16, 169 (1968)

- $G_H(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho_H(\omega)}{f(\omega)} f(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$

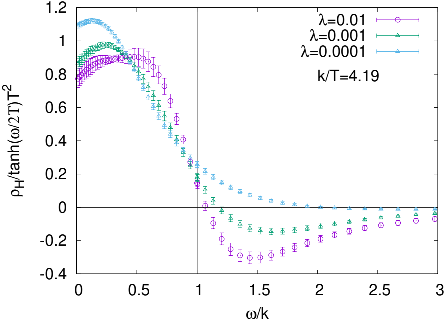
- $$\frac{\rho^{BG}(\omega)}{f(\omega)} = \sum_i q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \frac{\rho(\bar{\omega})}{f(\bar{\omega})}.$$

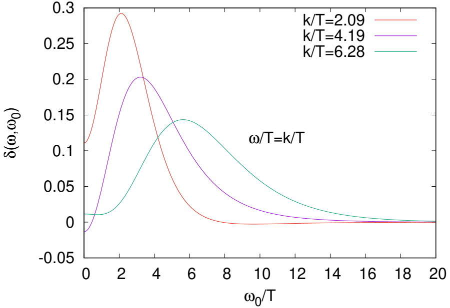
  $\delta(\omega, \bar{\omega}) = \sum_i q_i(\omega) K(\bar{\omega}, \tau_i) f(\bar{\omega})$.

- Minimize $F(\omega) = \lambda\, Width[\delta(\omega, \bar{\omega})] + (1 - \lambda)\, var[\rho_{BG}(\omega)]$

$$f(\omega) = \frac{\tanh(\omega/T)}{(1 + (\omega/\omega_0)^2 + (\omega/\omega_0)^4)}$$

where, $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$.

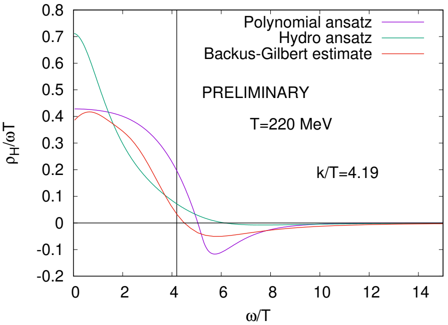

λ=0.01
λ=0.001
λ=0.0001
k/T=4.19
$\rho_H/\tanh(\omega/2T)T^2$
ω/k

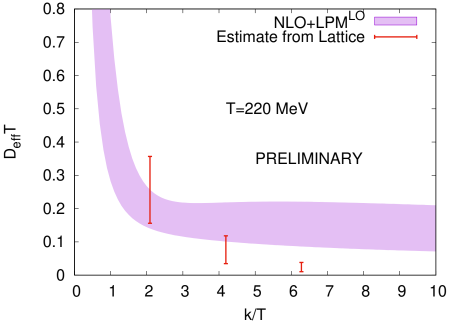

k/T=2.09
k/T=4.19
k/T=6.28
ω/T=k/T
$\delta(\omega,\omega_0)$
ω_0/T

- Photon production rate,

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)\chi_q}{\pi^2} Q_i^2 D_{eff}(k)$$

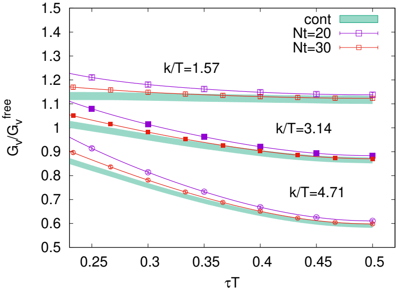
- The effective diffusion coefficient,
$D_{eff}(k) = \frac{\rho_H(|\vec{k}|,\vec{k})}{2\chi_q|\vec{k}|}$
$\lim_{k\to 0} D_{eff}(k) = D$

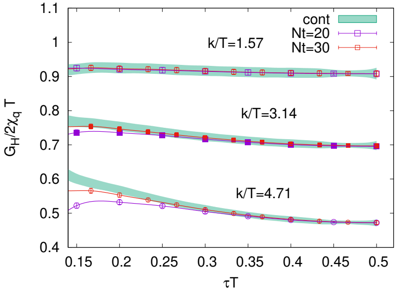




- We calculated the T-L correlator in Full QCD.
- Polynomial spectral function with smoothly connected OPE expected expansion at large ω fit the lattice correlator.
- We have used spectral function motivated from Hydro ansatz.
- The Backus-Gilbert method has also been used.
- Photon production rate estimated from all these methods has been compared.

- Continuum extrapolation of Clover Wilson fermion correlation function in Quenched QCD.





- Motivated by the quenched data, we assume the cut-off effects on the T-L correlator are also small in 2+1 flavor QCD shows the result from a single lattice spacing.

- Backus Gilbert estimate of the spectral function, $G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) K(\omega, \tau)$.
-

$$\rho^{BG}(\omega) = q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \rho(\bar{\omega}).$$

$\delta(\omega, \bar{\omega}) = \sum_i q^i(\omega)\, K(\bar{\omega}, \tau_i) = \boldsymbol{q^t(\omega).K(\bar{\omega})}$.

- $A(\omega) = \int d\bar{\omega}(\omega - \bar{\omega})^2 \delta(\omega, \bar{\omega})^2 = \boldsymbol{q^t(\omega).W(\omega).q(\omega)}$.

$$W_{ij}(\omega) = \int_0^\infty (\omega - \bar{\omega})^2 K(\omega, \tau_i) K(\bar{\omega}, \tau_j)$$

$\boldsymbol{W}$ has very small eigenvalue.

- Regulate the problem with, $B(\omega) = var(\rho^{BG}(\omega)) = \boldsymbol{q^t(\omega).S.q(\omega)}$. $S_{ij} = COV(\tau_i, \tau_j)$.
- Minimize, $F(\omega) = \lambda\, A(\omega) + (1 - \lambda)\, B(\omega)$.

$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \sum_{i=0}^{n}\delta_i\left(\frac{\omega}{\omega_0}\right)^i\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

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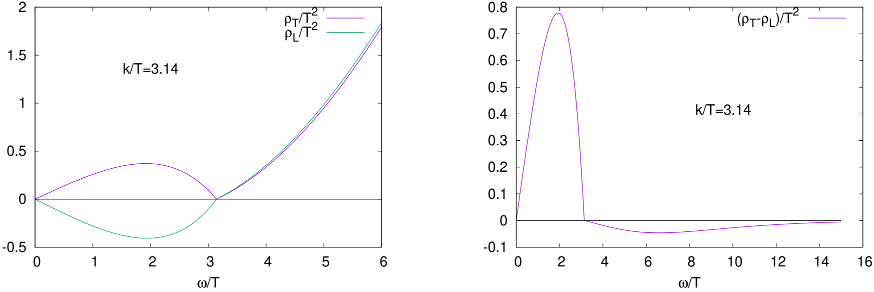

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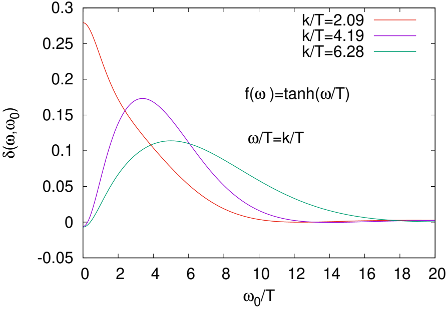
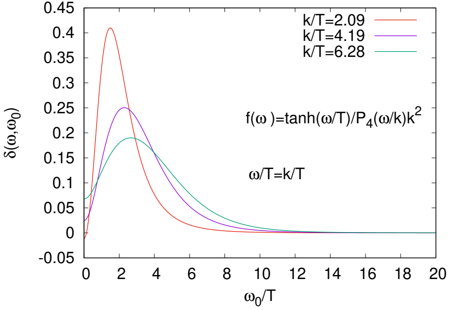

k/T=2.09
k/T=4.19
k/T=6.28
f(ω)=tanh(ω/T)
ω/T=k/T
δ(ω,ω0)
ω0/T


k/T=2.09
k/T=4.19
k/T=6.28
f(ω)=tanh(ω/T)/P4(ω/k)k2
ω/T=k/T
δ(ω,ω0)
ω0/T