Radiatively generated quark and lepton masses in extended gauge theories

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Gurucharan Mohanta

Physical Research Laboratory
Ahmedabad

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[1] G.Mohanta and Dr. K.M Patel

Outline

- Introduction
- General framework for radiative mechanism in the U(1) extended SM
- A Model
- Phenomenological aspects of the model
- Summary and results

Intoduction to Fermion mass problem

- In SM, masses are the incalculable parameters.
- The masses of the charged fermions, $m_f \propto \frac{y\ v}{\sqrt{2}}$.

  i.e., hierarchical couplings: 10^{-6} to 1.

  But, the gauge couplings are $O(1)$ numbers.
- The masses of the fermions follow the pattern

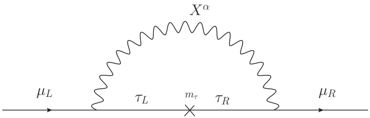
$$\frac{\mathbf{m_{f_1}}}{\mathbf{m_{f_2}}} \sim 10^{-2} \qquad \frac{\mathbf{m_{f_2}}}{\mathbf{m_{f_3}}} \sim 10^{-2}$$

- Flavour $[U(3)]^5$ symmetry is broken very badly for third generations.
- Yukawa couplings in SM are not Dirac natural parameters.

Possible explanation

- In QFTs, sometimes the small values are protected by symmetry, like of proton decay, $0\nu\beta\beta$.
- Lighter generation masses can be protected by some new symmetry, and arises radiatively(quantum corrections). S. Weinberg,PRL(1972)

1-loop supression $\sim \frac{1}{16\pi^2}$



- Natural guess: At tree level third generation, at one loop level second generation and at two loop level first generations get mass. Balakrishna,PRL(1988), Balakrishna and Mahapatra,Phy.Let.B(1988), etc,
- Can this be possible in SM?............Answer: No.

$\Downarrow$

New symmetry (and/or new fields)

General framework for U(1) extended SM

- Gauge interaction for the new $U(1)$ symmetry

$$-\mathcal{L}_{\rm gauge} = g_X X_\mu \left(q_{L\alpha}\, \overline{f}'_{L\alpha} \gamma^\mu f'_{L\alpha} + q_{R\alpha}\, \overline{f}'_{R\alpha} \gamma^\mu f'_{R\alpha} \right)$$

- The mass matrix is defined by

$$-\mathcal{L}_m = \overline{f}'_{L\alpha}\, \mathcal{M}^{(0)}_{\alpha\beta}\, f'_{R\beta} + \text{h.c.}$$

$$\mathcal{M}^{(0)} = \begin{pmatrix} 0_{3\times 3} & (\mu)_{3\times 1} \\ (\mu')_{1\times 3} & m_F \end{pmatrix} \implies M^{(0)}_{ij} = -\frac{1}{m_F} \mu_i \mu'_j.$$

- Physical basis can be obtained by transformation

$$f'_{L,R} = \mathcal{U}_{L,R}\, f_{L,R}\,, \qquad \mathcal{U}_L^\dagger\, \mathcal{M}^{(0)}\, \mathcal{U}_R = \mathcal{D} \equiv \text{Diag.}(0, 0, m_3, m_4)\,.$$

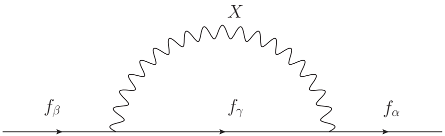
- The guage interactions in physical basis will be

$$-\mathcal{L}_{\rm gauge} = g_X\, X_\mu \overline{f}_\alpha \gamma^\mu \mathcal{C}_{\alpha\beta}\, f_\beta\,,$$

where $\mathcal{C}_{\alpha\beta} = (\mathcal{Q}_L)_{\alpha\beta}\, P_L + (\mathcal{Q}_R)_{\alpha\beta} P_R$, $\mathcal{Q}_{L,R} = \mathcal{U}^\dagger_{L,R}\, q_{L,R}\, \mathcal{U}_{L,R}$.

General framework...2

- The 1-loop fermion self energy correction induced by the gauge boson will be



$$-i\Sigma_{\alpha\beta}(p) = \sum_\gamma \int \frac{d^4k}{(2\pi)^4} (-ig_X \gamma^\mu \mathcal{C}^\dagger_{\alpha\gamma}) \frac{i(\not k + \not p + m_\gamma)}{[(k+p)^2 - m_\gamma^2 + i\epsilon]} (-ig_X \gamma^\nu \mathcal{C}_{\gamma\beta}) \Delta_{\mu\nu}(k)$$

- The final result can be written as

$$\Sigma_{\alpha\beta}(p=0) = \sigma^L_{\alpha\beta}\, P_L + \sigma^R_{\alpha\beta}\, P_R \,,$$

$$\sigma^R_{\alpha\beta} = \frac{g_X^2}{4\pi^2} \quad \sum_\gamma (\mathcal{Q}_L)_{\alpha\gamma} \, (\mathcal{Q}_R)_{\gamma\beta} \times m_\gamma \, B_0[M_X^2, m_\gamma^2]$$

PV Function

$$B_0[M^2, m^2] \equiv \Delta_\epsilon - \frac{M^2 \ln M^2 - m^2 \ln m^2}{M^2 - m^2} \,,$$
$$\Delta_\epsilon = \frac{2}{\epsilon} + 1 - \gamma + \ln 4\pi$$

- The 1-loop corrected fermion mass matrix can be written as

$$\mathcal{M} = \mathcal{M}^{(0)} + \delta\mathcal{M} \,, \qquad \delta\mathcal{M} = \mathcal{U}_L\, \sigma^R\, \mathcal{U}_R^\dagger \,.$$

General framework..3

- The divergent part of $\delta\mathcal{M}$ ($\delta\mathcal{M}_{\rm div}$) will be

$$\delta\mathcal{M}_{\rm div} \propto \mathcal{U}_L\, \mathcal{Q}_L\, \mathcal{D}\, \mathcal{Q}_R\, \mathcal{U}_R^\dagger = q_L\, \mathcal{M}^{(0)}\, q_R\,, \implies (\delta\mathcal{M}_{\rm div})_{ij} = 0\,.$$

- The finite part of $\delta\mathcal{M}$ is of our main interest.

$$(\delta\mathcal{M})_{\alpha\beta} = \frac{g_X^2}{4\pi^2} q_{L\alpha} q_{R\beta} \sum_\gamma (\mathcal{U}_L)_{\alpha\gamma}\, (\mathcal{U}_R^*)_{\beta\gamma}\; m_\gamma\, b_0[M_X^2, m_\gamma^2]\,,$$

- Further simplification is possible in the seesaw approximation.

$$\mathcal{U}_{L,R} \simeq \begin{pmatrix} U_{L,R} & -\rho_{L,R} \\ \rho_{L,R}^\dagger U_{L,R} & 1 \end{pmatrix}\,,$$

$$\rho_L = -m_F^{-1}\mu\,, \rho_R^\dagger = -m_F^{-1}\mu'$$
$$U_L^\dagger\, M^{(0)}\, U_R = {\rm Diag.}(0, 0, m_3)\,.$$

$\Downarrow$

$$(\delta\mathcal{M})_{ij} \simeq \frac{g_X^2}{4\pi^2} q_{Li} q_{Rj}\, M_{ij}^{(0)}\, \left(b_0[M_X^2, m_3^2] - b_0[M_X^2, m_F^2]\right)$$

$$(\delta\mathcal{M})_{i4} \propto q_{Li} q_{R4}\, \mu_i$$
$$(\delta\mathcal{M})_{4i} \propto q_{L4} q_{Ri}\, \mu'_i$$
$$(\delta\mathcal{M})_{44} \propto q_{L4} q_{R4}\, m_F$$

For large M_X,$(b_0[M_X^2, m_3^2] - b_0[M_X^2, m_F^2]) \simeq -\frac{m_F^2}{M_X^2}\, \ln\, \frac{m_F^2}{M_X^2}$

General framework: Choice of new U(1)

1-loop corrected mass matrix

$$\mathcal{M} = \begin{pmatrix} (\delta M)_{3\times 3} & (\tilde{\mu})_{3\times 1} \\ (\tilde{\mu}')_{1\times 3} & \tilde{m}_F \end{pmatrix}$$

$$\Downarrow$$

$$M = \delta M - \frac{1}{\tilde{m}_F}\tilde{\mu}\tilde{\mu}'$$

$$\begin{aligned} (\delta M)_{ij} &= (\delta \mathcal{M})_{ij} \\ \tilde{\mu}_i &= \mu_i + (\delta \mathcal{M})_{i4} \\ \tilde{\mu}'_i &= \mu'_i + (\delta \mathcal{M})_{4i} \\ \tilde{m}_F &= m_F + (\delta \mathcal{M})_{44} \ . \end{aligned}$$

- For $q_{L1} = q_{L2} = q_{L3}$ and $q_{R1} = q_{R2} = q_{R3}$ (flavour universal)

  $M \propto M^{(0)}$ $\quad$ $(rank\ 1)$ M. Lindner, etal, PRD(2022)

- **Flavour non-universal** $U(1)$ is required to generate lighter generation fermion masses.
- For a generic $U(1)$ charges, δM could lead to masses of a similar magnitude for the first and second generation fermions at 1-loop.

Choice 1

With one U(1) and upto two loops

Choice 2(Our choice)

$G_F = U(1)_1 \times U(1)_2$

A MODEL

- We choose new symmetry as $G_F = U(1)_{2-3} \times U(1)_{1-2}$ (Generalisation of $U(1)_{L_\mu - L_\tau} \times U(1)_{L_e - L_\mu}$)

The SM charges

Particles	$\mathcal{G}_{SM}$
H_{u_i}	$(1, 2, -\frac{1}{2})$
H_{d_i}	$(1, 2, \frac{1}{2})$
η_i	$(1, 1, 0)$
$T_{L,R}$	$(3, 1, \frac{2}{3})$
$B_{L,R}$	$(3, 1, \frac{-1}{3})$
$E_{L,R}$	$(1, 1, -1)$

Charges under G_F

Particles	G_F charges
$\mathcal{F}_1,\ H_{u_1}, H_{d_1}, \eta_1$	$(0, 1)$
$\mathcal{F}_2,\ H_{u_2}, H_{d_2}, \eta_2$	$(1, -1)$
$\mathcal{F}_3,\ H_{u_3}, H_{d_3}, \eta_3$	$(-1, 0)$
VL fermions	*Neutral*

$\mathcal{F} = Q, u_R, d_R, L, e_R$

- The allowed Yukawa lagrangian with mass terms is given as

$$
\begin{aligned}
-\mathcal{L}_Y &= y_{u\,i}\, \overline{Q_L}_i\, H_{u\,i}\, T_R + y'_{u\,i}\, \overline{T_L}\, \eta_i^*\, u_{Ri} + y_{d\,i}\, \overline{Q_L}_i\, H_{d\,i}\, B_R + y'_{d\,i}\, \overline{B_L}\, \eta_i^*\, d_{Ri} \\
&+ y_{e\,i}\, \overline{L_L}_i\, H_{d\,i}\, E_R + y'_{e\,i}\, \overline{E_L}\, \eta_i^*\, e_{Ri} + m_T\, \overline{T_L}\, T_R + m_B\, \overline{B_L}\, B_R \\
&+ m_E\, \overline{E_L}\, E_R + \text{h.c.}\,.
\end{aligned}
$$

EXAMPLE SOLUTIONS

Parameters	Solution 1 (S1)	Solution 2 (S2)	Solution 3 (S3)
M_{Z_1}	10^4	10^6	10^8
M_{Z_2}	2.8708×10^5	1.4470×10^7	1.9110×10^9
m_T	1.1000×10^4	1.1000×10^6	1.1003×10^8
m_B	3.0754×10^5	2.4839×10^7	1.4015×10^9
m_E	2.8128×10^5	4.9462×10^7	6.7820×10^8
μ_{u1}	1.8023×10^1	-1.6702×10^1	1.5410×10^1
μ_{u2}	3.0901×10^2	3.0969×10^2	-0.6860
μ_{u3}	1.4763	-0.5133	2.8599×10^2
μ'_{u1}	-3.9573×10^3	3.3923×10^5	-3.8518×10^7
μ'_{u2}	3.5446×10^3	-3.9343×10^5	-3.7762×10^7
μ'_{u3}	-2.8507×10^3	29653×10^5	-3.7718×10^7
μ_{d1}	$2.3809 \times 10^1 + i\,6.7460$	$-1.0402 \times 10^1 + i\,6.7204$	$4.6117 + i\,4.3534$
μ_{d2}	$1.4422 \times 10^2 + i\,2.6938 \times 10^1$	$2.7279 \times 10^1 - i\,1.2285 \times 10^2$	$-0.1868 - i\,0.8509$
μ_{d3}	5.6345	-2.1306	1.0439×10^2
μ'_{d1}	-6.1152×10^2	8.8600×10^4	-2.2724×10^7
μ'_{d2}	3.7385×10^3	3.4743×10^5	2.4618×10^7
μ'_{d3}	-7.5116×10^2	1.2636×10^5	-1.9748×10^7
μ_{e1}	8.1306×10^1	-7.6362×10^1	-0.9180
μ_{e2}	2.7874×10^1	-1.5295×10^2	-9.6383
μ_{e3}	-1.1628	3.2793	-2.2318×10^1
μ'_{e1}	-1.2295×10^3	-1.9688×10^5	2.7449×10^7
μ'_{e2}	-3.9625×10^3	-3.8735×10^5	-3.5794×10^7
μ'_{e3}	3.9792×10^3	2.9732×10^4	2.0714×10^7

Phenomenological aspects: New physics effects

- In the physical basis, the couplings of $Z_{1,2}$ can be written as:

$$-\mathcal{L}_{Z_{1,2}} = \sum_{k=1,2} g_k \left[\left(X_{f_L}^{(k)} \right)_{ij} \overline{f_{Li}} \gamma^\mu f_{Lj} + \left(X_{f_R}^{(k)} \right)_{ij} \overline{f_{Ri}} \gamma^\mu f_{Rj} \right] Z_{k\mu}$$

  where

$$X_{f_L}^{(k)} = U_{f_L}^\dagger \, q_{fL}^{(k)} \, U_{f_L} \, ,$$

- U_{f_L}, U_{f_R} will be determined from diagonalization equation

$$U_{f_L}^\dagger M_f U_{f_R} = \mathrm{Diag.}(m_{f_1}, m_{f_2}, m_{f_3})$$

- $X^{(k)}$'s are non-diagonal. So, there is FCNCs.
- Z_1 is the lightest among all the new particles. So, we consider the FCNC effects of Z_1.

Phenomenological aspects: Quark Flavour Violation

Wilson coefficient	Allowed range	S1	S2	S3
$\mathrm{Re}C_K^1$	$[-9.6, 9.6] \times 10^{-13}$	-9.5×10^{-10}	-5.4×10^{-14}	6.2×10^{-18}
$\mathrm{Re}\tilde{C}_K^1$	$[-9.6, 9.6] \times 10^{-13}$	-1.6×10^{-9}	-1.6×10^{-13}	2.8×10^{-17}
$\mathrm{Re}C_K^4$	$[-3.6, 3.6] \times 10^{-15}$	6.2×10^{-9}	5.0×10^{-13}	-7.5×10^{-17}
$\mathrm{Re}C_K^5$	$[-1.0, 1.0] \times 10^{-14}$	5.4×10^{-9}	4.2×10^{-13}	-5.9×10^{-17}
$\mathrm{Im}C_K^1$	$[-9.6, 9.6] \times 10^{-13}$	5.9×10^{-25}	9.5×10^{-30}	1.7×10^{-33}
$\mathrm{Im}\tilde{C}_K^1$	$[-9.6, 9.6] \times 10^{-13}$	-1.0×10^{-24}	-3.8×10^{-29}	3.9×10^{-31}
$\mathrm{Im}C_K^4$	$[-1.8, 0.9] \times 10^{-17}$	9.5×10^{-26}	1.5×10^{-29}	-5.3×10^{-31}
$\mathrm{Im}C_K^5$	$[-1.0, 1.0] \times 10^{-14}$	8.3×10^{-26}	1.3×10^{-29}	-4.2×10^{-31}
$\lvert C_{B_d}^1 \rvert$	$< 2.3 \times 10^{-11}$	1.6×10^{-12}	9.9×10^{-18}	5.8×10^{-22}
$\lvert \tilde{C}_{B_d}^1 \rvert$	$< 2.3 \times 10^{-11}$	2.9×10^{-12}	3.8×10^{-18}	1.0×10^{-18}
$\lvert C_{B_d}^4 \rvert$	$< 2.1 \times 10^{-13}$	5.1×10^{-12}	1.6×10^{-17}	6.7×10^{-20}
$\lvert C_{B_d}^5 \rvert$	$< 6.0 \times 10^{-13}$	9.1×10^{-12}	2.6×10^{-17}	1.0×10^{-19}
$\lvert C_{B_s}^1 \rvert$	$< 1.1 \times 10^{-9}$	8.3×10^{-11}	2.8×10^{-15}	3.0×10^{-19}
$\lvert \tilde{C}_{B_s}^1 \rvert$	$< 1.1 \times 10^{-9}$	2.0×10^{-10}	5.8×10^{-14}	4.2×10^{-17}
$\lvert C_{B_s}^4 \rvert$	$< 1.6 \times 10^{-11}$	3.1×10^{-10}	3.3×10^{-14}	9.8×10^{-18}
$\lvert C_{B_s}^5 \rvert$	$< 4.5 \times 10^{-11}$	5.5×10^{-10}	5.4×10^{-14}	1.5×10^{-17}
$\lvert C_D^1 \rvert$	$< 7.2 \times 10^{-13}$	2.0×10^{-10}	2.9×10^{-15}	6.5×10^{-19}
$\lvert \tilde{C}_D^1 \rvert$	$< 7.2 \times 10^{-13}$	3.5×10^{-9}	2.7×10^{-13}	2.8×10^{-17}
$\lvert C_D^4 \rvert$	$< 4.8 \times 10^{-14}$	3.2×10^{-9}	1.1×10^{-13}	1.7×10^{-17}
$\lvert C_D^5 \rvert$	$< 4.8 \times 10^{-13}$	3.7×10^{-9}	1.2×10^{-13}	1.9×10^{-17}

Phenomenological aspects: Lepton Flavour Violation

LFV observable	Limit	S1	S2	S3
$\mathrm{BR}[\mu \to e]$	$< 7.0 \times 10^{-13}$	7.2×10^{-7}	4.0×10^{-15}	3.7×10^{-22}
$\mathrm{BR}[\mu \to 3e]$	$< 1.0 \times 10^{-12}$	7.9×10^{-9}	6.0×10^{-17}	2.5×10^{-25}
$\mathrm{BR}[\tau \to 3\mu]$	$< 2.1 \times 10^{-8}$	2.3×10^{-8}	1.7×10^{-18}	1.1×10^{-24}
$\mathrm{BR}[\tau \to 3e]$	$< 2.7 \times 10^{-8}$	9.2×10^{-11}	6.5×10^{-19}	4.2×10^{-28}
$\mathrm{BR}[\mu \to e\gamma]$	$< 4.2 \times 10^{-13}$	2.0×10^{-11}	1.3×10^{-19}	3.5×10^{-27}
$\mathrm{BR}[\tau \to \mu\gamma]$	$< 4.4 \times 10^{-8}$	9.3×10^{-13}	7.1×10^{-20}	3.1×10^{-27}
$\mathrm{BR}[\tau \to e\gamma]$	$< 3.3 \times 10^{-8}$	3.9×10^{-13}	2.1×10^{-21}	4.0×10^{-29}

- QFV and LFV process imply that $M_{Z_1} > 10^3$ TeV for phenomenologically consistent solutions.

Summary and results

- Extended abelian guage symmetry prevents tree level mass terms for all the SM fermions and allows to generate lighter generation fermion masses radiatively.
- The SM fermions should have flavour non-universal charges under the new symmetry to generate radiative masses.
- Two U(1) symmetries can used to induce hierarchical lighter generation radiative masses at 1-loop level.
- Our explicit model $G_F = U(1)_1 \times U(1)_2$ is able to generate hierarchical radiative masses for first and second generation.
- The Yukawa couplings of the model can be choosen $O(1)$.
- The model contains large number of parameters.

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THANK YOU

Backup Slides

Problems with Previous works

- Difficulties with earlier models
  - Lack of precise measured values.
  - Have not used values from detailed calculation.
- Problems with S. Weinberg's $SO(3)_L \times SO(3)_R$ model S.Weinberg, PRD(2020)
  - Predicts following equal mass ratios

$$\frac{m_c}{m_t} = \frac{m_s}{m_b} = \frac{m_\mu}{m_\tau} .$$

  - Doesn't have CKM mixings.

"............These models are not realistic, for reasons that will be spelled out later, but it is hoped that they may help to revive interest in this program, and to lay out some of the methods and problems that it confronts. ...." by S. Weinberg in PRD(2020)

Rank-1 Structure of mass matrix

To obtain rank-1 structure:

- Set 3^{rd} gen massive at tree level and add scalars with appropriate charges to generate radiative masses for lighter generations. B A Dobrescu and P. J Fox,JHEP(2008)
- Enhance guage sector and increase fermion contents by adding VL fermions to obtain mass matrix of the type Balakrishna,PRL(1988), Balakrishna and Mahapatra,PRL(1988), etc,

$$\mathcal{M}^{(0)} = \begin{pmatrix} 0 & 0 & 0 & \mu_1 \\ 0 & 0 & 0 & \mu_2 \\ 0 & 0 & 0 & \mu_3 \\ \mu'_1 & \mu'_2 & \mu'_3 & m_F \end{pmatrix}.$$

In seesaw approximation $m_F >> \mu_i, \mu'_i$, the effective 3×3 mass matrix can be written as

$$M^{(0)}_{ij} = -\frac{1}{m_F} \mu_i \mu'_j$$

Elements of the correction matrix

- The components of $\delta\mathcal{M}$ can be simplified as

$$
\begin{aligned}
(\delta\mathcal{M})_{ij} &\simeq \frac{g_X^2}{4\pi^2} q_{Li} q_{Rj}\, M_{ij}^{(0)} \left(b_0[M_X^2, m_3^2] - b_0[M_X^2, m_F^2]\right) \\
(\delta\mathcal{M})_{i4} &\simeq \frac{g_X^2}{4\pi^2} q_{Li} q_{R4}\, \mu_i \left(b_0[M_X^2, m_F^2] + \sum_j \frac{|\mu'_j|^2}{m_F^2}\, b_0[M_X^2, m_3^2] \right) , \\
(\delta\mathcal{M})_{4i} &\simeq \frac{g_X^2}{4\pi^2} q_{L4} q_{Ri}\, \mu'_i \left(b_0[M_X^2, m_F^2] + \sum_j \frac{|\mu_j|^2}{m_F^2}\, b_0[M_X^2, m_3^2] \right) , \\
(\delta\mathcal{M})_{44} &\simeq \frac{g_X^2}{4\pi^2} q_{L4} q_{R4}\, m_F \left(b_0[M_X^2, m_F^2] - \frac{m_3^2}{m_F^2}\, b_0[M_X^2, m_3^2] \right) .
\end{aligned}
$$

General framework: Choice of U(1)

- For a generic $U(1)$ charges, δM could lead to masses of a similar magnitude for the first and second generation fermions at 1-loop.
- Choosing $q_{L1} = q_{R1} = 0$, it will generate only the second generation fermion masses at 1-loop.
- Adding another flavoured $U(1)$ with $q_{L1}, q_{R1} \neq 0$, will generate first generation fermion masses.
- The mass hierarchy between the first and second generations can be arranged by choosing hierarchical masses for the gauge bosons of two $U(1)$s.

$$b_0[M_X^2, m_3^2] - b_0[M_X^2, m_F^2] \simeq -\frac{m_F^2}{M_X^2} \ln \frac{m_F^2}{M_X^2} .$$

- Choice of new symmetry is $G_F = U(1)_1 \times U(1)_2$

A MODEL...2

- The effective 3×3 mass matrix in each sector

$$M^{(0)}_{u,d,e} \equiv -\frac{1}{m_{T,B,E}}\,\mu_{u,d,e}\,\mu'_{u,d,e}\,.$$

- The 1-loop corrected effective 3×3 mass matrices will be

$$M_f = \delta M_f - \frac{1}{\tilde{m}_F}\tilde{\mu}\tilde{\mu}' = \delta M_f + M_f^{(0)}$$

$$\delta M_f = \delta M_{f1}(Z_1) + \delta M_{f2}(Z_2)$$

.

$$\delta M_f = \frac{N_f g_1^2}{4\pi^2}\left(b_0[M^2_{Z_1}, m^2_{f3}] - b_0[M^2_{Z_1}, m^2_F]\right)\begin{pmatrix} 0 & 0 & 0 \\ 0 & \left(M_f^{(0)}\right)_{22} & -\left(M_f^{(0)}\right)_{23} \\ 0 & -\left(M_f^{(0)}\right)_{32} & \left(M_f^{(0)}\right)_{33} \end{pmatrix}$$

$$+\frac{N_f g_2^2}{4\pi^2}\left(b_0[M^2_{Z_2}, m^2_{f3}] - b_0[M^2_{Z_2}, m^2_F]\right)\begin{pmatrix} \left(M_f^{(0)}\right)_{11} & -\left(M_f^{(0)}\right)_{12} & 0 \\ -\left(M_f^{(0)}\right)_{21} & \left(M_f^{(0)}\right)_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Neutrino masses in the model

- Neutrino masses can be accommodated by introducing three RH neutrinos, singlet under both the SM gauge symmetry and G_F,

$$-\mathcal{L}_\nu = y_{D\,ij}\,\overline{L_L}_i\,H_{u\,i}\,\nu_{R\,j} + \frac{1}{2} M_{R\,ij}\,\nu_{Ri}^T C^{-1} \nu_{Rj} + \mathrm{h.c.}\,.$$

- For $M_R \gg M_D$, the usual type I seesaw mechanism can be realized and the light neutrino mass matrix can be written as

$$M_\nu = -M_D\, M_R^{-1}\, M_D^T\,.$$

- All the three neutrino masses can arise at the tree level.

EXAMPLE SOLUTIONS

- The mass lagrangian has 25 real parameters which can be used to determine 13 observables.
- 25 real parameters are obtained through χ^2 function minimization technique. The χ^2 function consists of 13 observables whose mean and SD are given below

Observable	Value	Observable	Value
m_u	1.27 ± 0.50 MeV	m_e	0.487 ± 0.049 MeV
m_c	0.619 ± 0.084 GeV	m_μ	1.027 ± 0.103 MeV
m_t	171.7 ± 3.0 GeV	m_τ	1.746 ± 0.174 GeV
m_d	2.90 ± 1.24 MeV	$\lvert V_{us}\rvert$	0.22500 ± 0.00067
m_s	0.055 ± 0.016 GeV	$\lvert V_{cb}\rvert$	0.04182 ± 0.00085
m_b	2.89 ± 0.09 GeV	$\lvert V_{ub}\rvert$	0.00369 ± 0.00011
		J_{CP}	$(3.08 \pm 0.15) \times 10^{-5}$

- We impose $M_{Z_1} \leq m_T << m_B, m_E$; $|y_{fi}| < \sqrt{4\pi}$ and $v_{fi} < 174$ GeV.