Probing non-unitary neutrino mixing using atmospheric neutrinos at INO-ICAL

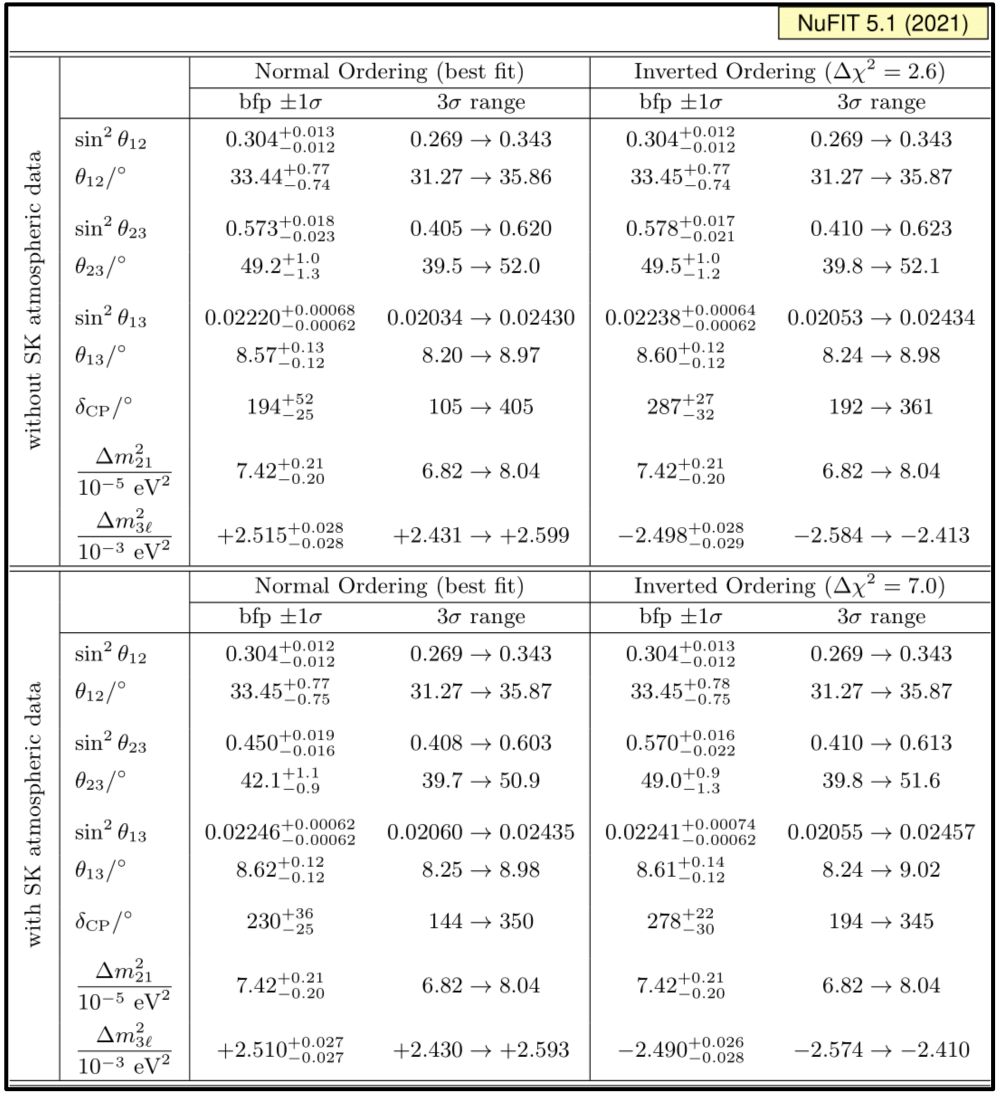
DAE-BRNS HEP Symposium-2022, 15th December 2022
IISER, Mohali

Sadashiv Sahoo†, Sudipta Das
Anil Kumar, and Sanjib Kumar Agarwalla

Plan of Talk:

- Introduction & Motivation
- Formalism
- Constrains by upcoming Long baseline experiments
- Atmospheric Neutrino Oscillation
- The Iron Calorimeter @ INO
- Preliminary Results
- Concluding Remark

Introduction & Motivation



NuFIT 5.1 (2021)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
	$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
	$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{\rm CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
	$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

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	$\delta_{\rm CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
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$$U^{3\times 3}_{\rm PMNS} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - s_{12}s_{13}c_{23} & c_{13}c_{23} \end{bmatrix}$$

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Unitarity

NuFIT 5.1 (2021)

$$|U|^{\text{w/o SK-atm}}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.232 \to 0.507 & 0.459 \to 0.694 & 0.629 \to 0.779 \\ 0.260 \to 0.526 & 0.470 \to 0.702 & 0.609 \to 0.763 \end{pmatrix}$$

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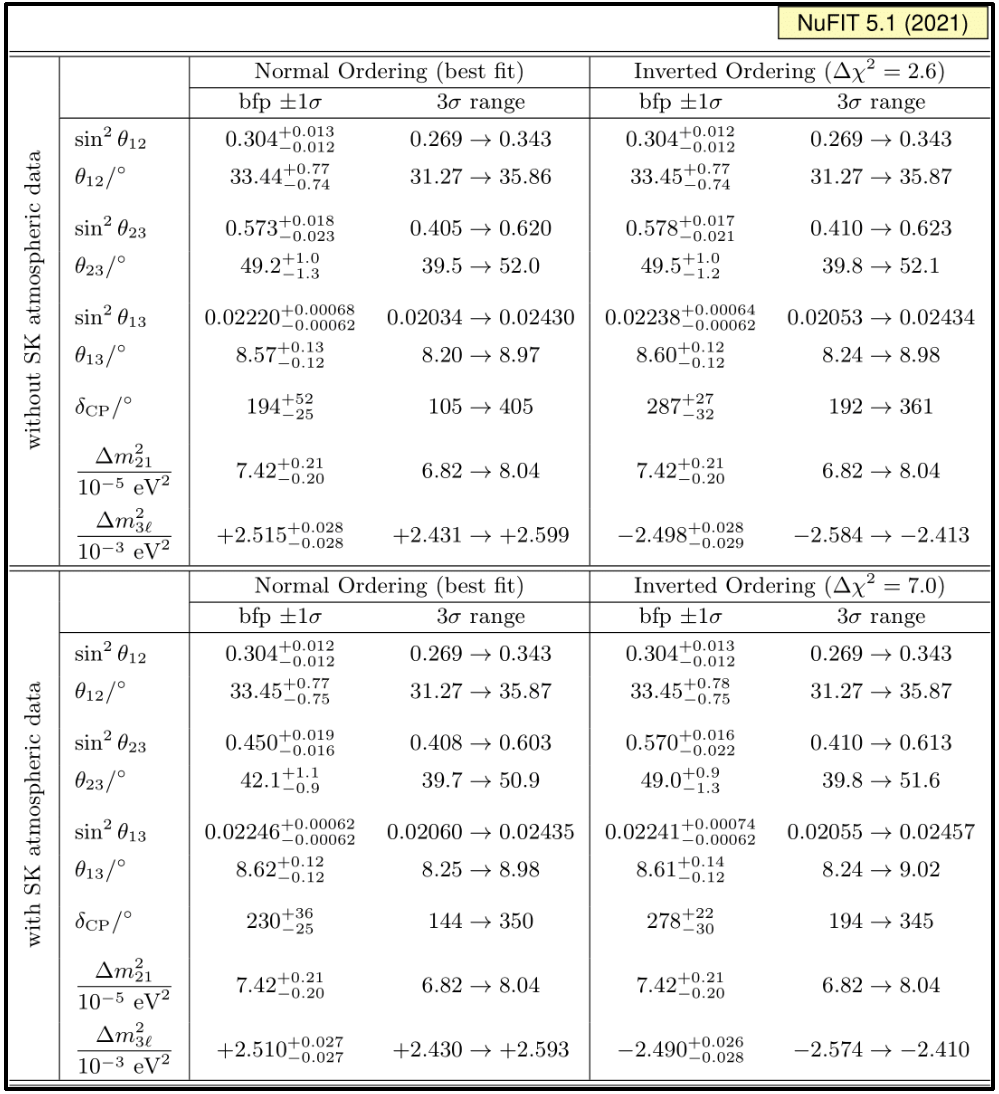
www.nu-fit.org



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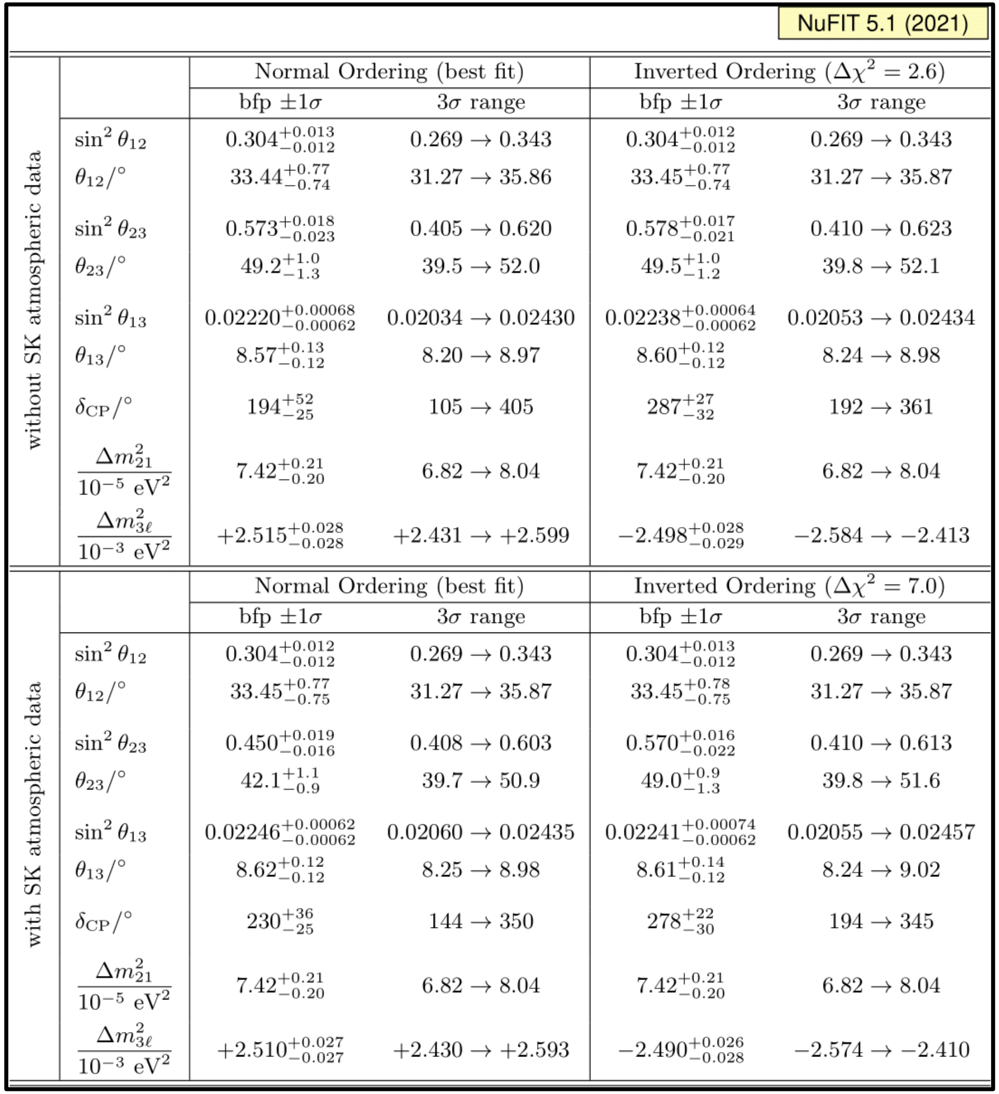
- T2K, NOνA, KamLAND, Daya-Bay, RENO, and Double-Chooz ....
- Super-K, IceCube (DeepCore)



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- Next Gen. ν-detectors are now able to detect all flavours of neutrinos (ν_e, ν_μ, ν_τ). These give opportunities to measure all osc. channels.

Neutrino Oscillations in 'n' neutrino Framework :

$$U_{n\times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix} \qquad H^{m}_{n\times n} = \frac{1}{2E} \cdot \begin{bmatrix} \delta m^2 & 0 \\ 0 & \Delta M^2 \end{bmatrix}_{n\times n}$$

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Neutrino Oscillations in ‘n’ neutrino Framework :

Light-active mixing

Light-sterile mixing

$$U_{n\times n}=\begin{pmatrix} N & S \\ V & T \end{pmatrix}\equiv\left(\begin{array}{c|c} N_{3\times 3} & S_{3\times(n-3)} \\ \hline V_{(n-3)\times 3} & T_{(n-3)\times(n-3)} \end{array}\right)$$

Heavy-active mixing

Heavy-sterile mixing

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ . \\ . \\ . \\ . \end{bmatrix}_{n\times 1}=\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} & \cdots & \cdots & \cdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \cdots & \cdots & \cdots \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}_{n\times n}\cdot\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ . \\ . \\ . \\ . \end{bmatrix}_{n\times 1}$$

Neutrino Oscillations in ‘n’ neutrino Framework :

$$H^m_{n\times n} = \frac{1}{2E}\cdot\begin{bmatrix} \delta m^2 & 0 \\ 0 & \Delta M^2 \end{bmatrix}_{n\times n}$$

$$\delta m^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix}_{3\times 3}$$

$$\Delta M^2 = \begin{bmatrix} \Delta m^2_{41} & 0 & 0 & 0 & 0 \\ 0 & \Delta m^2_{51} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \Delta m^2_{n1} \end{bmatrix}_{(n-3)\times(n-3)}$$

$$\mathrm{m}_i \ll E \;\text{ and }\; \Delta m^2_{ij} \ll E$$

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$$H^{f}_{n\times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix} \cdot \frac{1}{2E} \cdot \begin{pmatrix} \delta m^2 & 0 \\ 0 & \Delta M^2 \end{pmatrix} \cdot \begin{pmatrix} N^\dagger & V^\dagger \\ S^\dagger & T^\dagger \end{pmatrix}$$

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We have only access to neutrinos by detecting neutrinos (ν_e, ν_μ, ν_τ) via their charged cousins.

However, we can feel the presence of Sterile through active-sterile mixing.

Neutrino Oscillations in ‘n’ neutrino Framework :

$$\left[\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \hline \cdot \\ \cdot \\ \cdot \\ \cdot \end{array}\right]_{n\times 1} = \left[\begin{array}{ccc|cccc} U_{e1} & U_{e2} & U_{e3} & \cdots & \cdots & \cdots & \cdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \cdots & \cdots & \cdots & \cdots \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \cdots & \cdots & \cdots & \cdots \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right]_{n\times n} \cdot \left[\begin{array}{c} \nu_1 \\ \nu_2 \\ \nu_3 \\ \hline \cdot \\ \cdot \\ \cdot \\ \cdot \end{array}\right]_{n\times 1}$$

This limitation invokes **non-unitary neutrino mixing** (NUNM) at light three neutrino mixings

Formalism

Formalism

Using Okubo's notation

$$U_{n\times n} = \omega_{n-1\,n}\,\omega_{n-2\,n}\ \ldots\ \omega_{3\,n}\,\omega_{2\,n}\,\omega_{1\,n}\,\omega_{n-2\,n-1}\ \ldots\ \omega_{3\,n-1}\,\omega_{2\,n-1}\,\omega_{1\,n-1}\ \ldots\ \omega_{2\,3}\ \omega_{1\,3}\,\omega_{1\,2}$$

$$U^{n\times n} = U^{n-N}\ \cdot\ U^{N}$$

$$U^{n-N}U^{N} = \left(\begin{array}{cccccccc}
\alpha_{11} & 0 & \cdots & 0 & \vdots & & & \\
\alpha_{21} & \alpha_{22} & \ddots & \vdots & \vdots & & & \\
\vdots & & \ddots & 0 & \vdots & & S & \\
\alpha_{N1} & \cdots & & \alpha_{NN} & \vdots & & & \\
\cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\
 & & & & \vdots & & & \\
 & & V & & \vdots & & T & \\
 & & & & \vdots & & &
\end{array}\right)
\left(\begin{array}{ccccccc}
U^{N}_{11} & U^{N}_{12} & \cdots & U^{N}_{1N} & \vdots & & \\
U^{N}_{21} & U^{N}_{22} & & \vdots & \vdots & & \\
\vdots & & \ddots & & \vdots & 0 & \\
U^{N}_{N1} & \cdots & & U^{N}_{NN} & \vdots & & \\
\cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
 & & & & \vdots & & \\
 & & 0 & & \vdots & I & \\
 & & & & \vdots & &
\end{array}\right)$$

Prog. Theor. Phys. 28, 24 (1962)

Formalism & Conventions :

Using Okubo's notation

$$U_{n\times n} = \omega_{n-1\,n}\,\omega_{n-2\,n}\ \cdots\ \omega_{3\,n}\,\omega_{2\,n}\,\omega_{1\,n}\,\omega_{n-2\,n-1}\ \cdots\ \omega_{3\,n-1}\,\omega_{2\,n-1}\,\omega_{1\,n-1}\ \cdots\ \omega_{2\,3}\ \omega_{1\,3}\,\omega_{1\,2}$$

$$U^{n\times n} = U^{n-N}\cdot U^{N}$$

$$U^{n-N}U^{N} = \begin{pmatrix} \alpha_{11} & 0 & \cdots & 0 & \vdots & & & \\ \alpha_{21} & \alpha_{22} & \ddots & \vdots & \vdots & & & \\ \vdots & & \ddots & 0 & \vdots & & S & \\ \alpha_{N1} & \cdots & & \alpha_{NN} & \vdots & & & \\ \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ & & & & \vdots & & & \\ & & V & & \vdots & & T & \\ & & & & \vdots & & & \end{pmatrix} \begin{pmatrix} U^N_{11} & U^N_{12} & \cdots & U^N_{1N} & \vdots & & \\ U^N_{21} & U^N_{22} & & \vdots & \vdots & & \\ \vdots & & \ddots & & \vdots & 0 & \\ U^N_{N1} & \cdots & & U^N_{NN} & \vdots & & \\ \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ & & & & \vdots & & \\ & & 0 & & \vdots & I & \\ & & & & \vdots & & \end{pmatrix}$$

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Formalism & Conventions :

Using Okubo's notation

$$U_{n\times n} = \omega_{n-1\,n}\,\omega_{n-2\,n}\ \ldots\ \omega_{3\,n}\,\omega_{2\,n}\,\omega_{1\,n}\,\omega_{n-2\,n-1}\ \ldots\ \omega_{3\,n-1}\,\omega_{2\,n-1}\,\omega_{1\,n-1}\ \ldots\ \omega_{2\,3}\ \omega_{1\,3}\,\omega_{1\,2}$$

$$U_{3\times 3} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \omega_{2\,3}\,\omega_{1\,3}\,\omega_{1\,2}$$

$$U_{3\times 3} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \widetilde{U}_{3\times 3}$$

PMNS Matrix

Prog. Theor. Phys. 28, 24 (1962)

Effective Hamiltonian in three neutrino scenario :

In principle α can be complex quantity

$$N = \alpha \cdot \widetilde{U}$$

$$\alpha = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

$$\mathcal{H}_{\nu_{\rm L}} = \frac{1}{2E} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} + N^\dagger \left[\sqrt{2} G_F \begin{pmatrix} N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{G_F}{\sqrt{2}} \begin{pmatrix} N_n & 0 & 0 \\ 0 & N_n & 0 \\ 0 & 0 & N_n \end{pmatrix} \right] N$$

$$N^\dagger \cdot N \neq I$$

arXiv: 1102.3432

arXiv: 1503.08879

arXiv: 2111.00329

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$$\mathcal{H}_{\nu_{\mathrm{L}}} = \frac{1}{2E} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} + N^\dagger \left[\sqrt{2} G_F \begin{pmatrix} N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{G_F}{\sqrt{2}} \begin{pmatrix} N_n & 0 & 0 \\ 0 & N_n & 0 \\ 0 & 0 & N_n \end{pmatrix} \right] N$$

$$N^\dagger \cdot N \neq I$$

$$\text{For } \bar{\nu} \ : \ \sqrt{2}\mathrm{G_F N_e} \rightarrow -\sqrt{2}\mathrm{G_F N_e} \ ; \ -\frac{\mathrm{G_F}}{\sqrt{2}}\mathrm{N_n} \rightarrow +\frac{\mathrm{G_F}}{\sqrt{2}}\mathrm{N_n}$$

Effective survival Probability in three neutrino scenario :

$$\mathcal{H} = \frac{1}{2E}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} + N^\dagger \left[\sqrt{2}G_F \begin{pmatrix} N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{G_F}{\sqrt{2}} \begin{pmatrix} N_n & 0 & 0 \\ 0 & N_n & 0 \\ 0 & 0 & N_n \end{pmatrix}\right] N$$

$$P\left(\nu_\alpha \to \nu_\beta\right) = \left|\left(N \cdot e^{-iHL} \cdot N^\dagger\right)_{\beta\alpha}\right|^2$$

arXiv: 2111.00329

Effective survival Probability in three neutrino scenario :

$$P\left(\nu_\mu \to \nu_\mu\right)(L)\Big|_{\theta_{23}=\pi/4} = \left|\frac{1}{2}\cdot e^{-i\cdot\lambda_1\cdot L}\cdot\left[\left(1+e^{i\cdot\Delta\lambda\cdot L}\right)\right]\right|^2$$

$$\frac{\Delta\lambda}{2} \simeq \frac{\Delta m_{32}^2}{4E} + V_{NC}\cdot\alpha_{32}\cdot\sin 2\theta_{23}$$

λ_1 : is One of the Eigenvalues

$$P\left(\nu_\mu \to \nu_\mu\right)(L)\Big|_{\theta_{23}=\pi/4} = \cos^2\left[\left(\frac{\Delta m_{32}^2}{4E} + V_{NC}\cdot\alpha_{32}\right)\cdot L\right]$$

Constraints by upcoming Long baseline experiments

Future Constraints on α_{32} by LBL

90% C.L.	DUNE	JD	KD	JD+KD	JD+KD+DUNE	T2K+NOνA
$\lvert\alpha_{32}\rvert$	< 0.33	< 1.2	< 0.85	< 0.71	< 0.27	< 1.4

- DUNE : Deep Underground Neutrino Experiment
- JD : Japanese Detector [Tokai to Hyper-Kamiokande (T2HK)]
- KD : Korean Detector [Tokai to Korean Detector]
- NOνA : NuMI Off-axis νe Appearance
- T2K : Tokai to Super-Kamiokande

arXiv: 2111.00329

Here, authors min. Over ϕ_{32} for the following convention $|R| \cdot e^{-i\phi}$

Oscillogram with a benchmarking value of $\alpha_{32} = \pm 0.25$

Benchmark Oscillation Parameters for This Work

$\sin^2 2\theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 2\theta_{13}$	Δm^2_{eff} (eV^2)	Δm^2_{21} (eV^2)	δ_{CP}	Mass Ordering
0.855	0.5	0.0875	2.49×10^{-3}	7.4×10^{-5}	0	Normal (NO)

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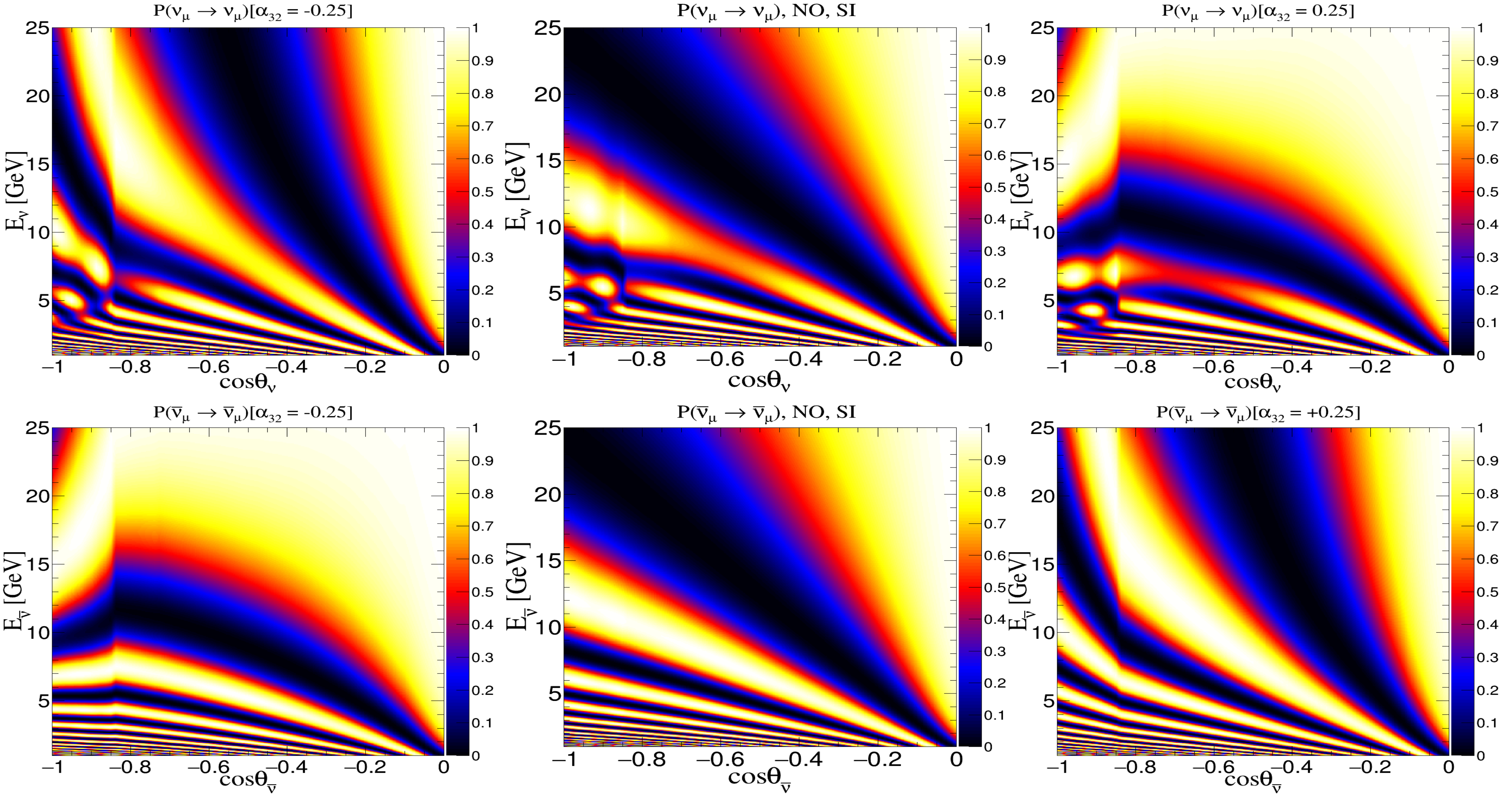
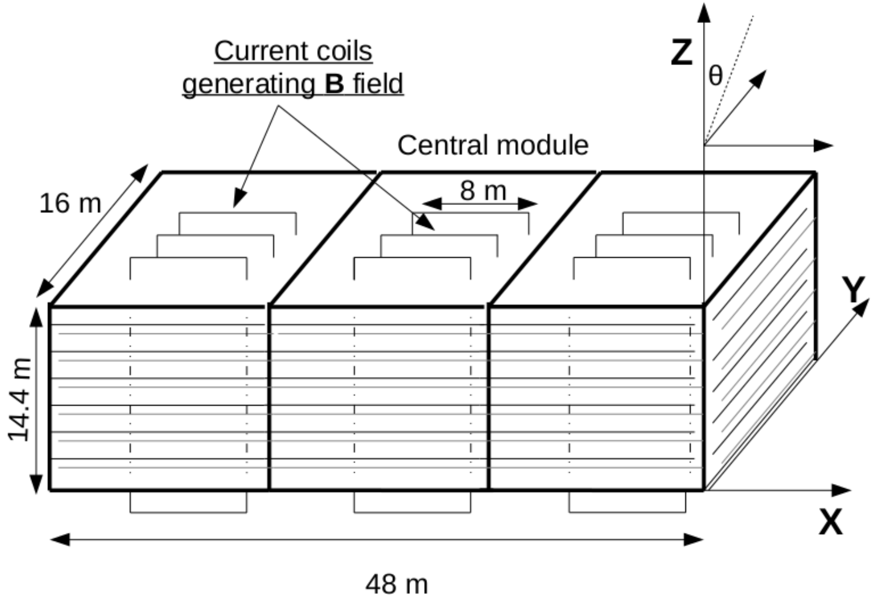
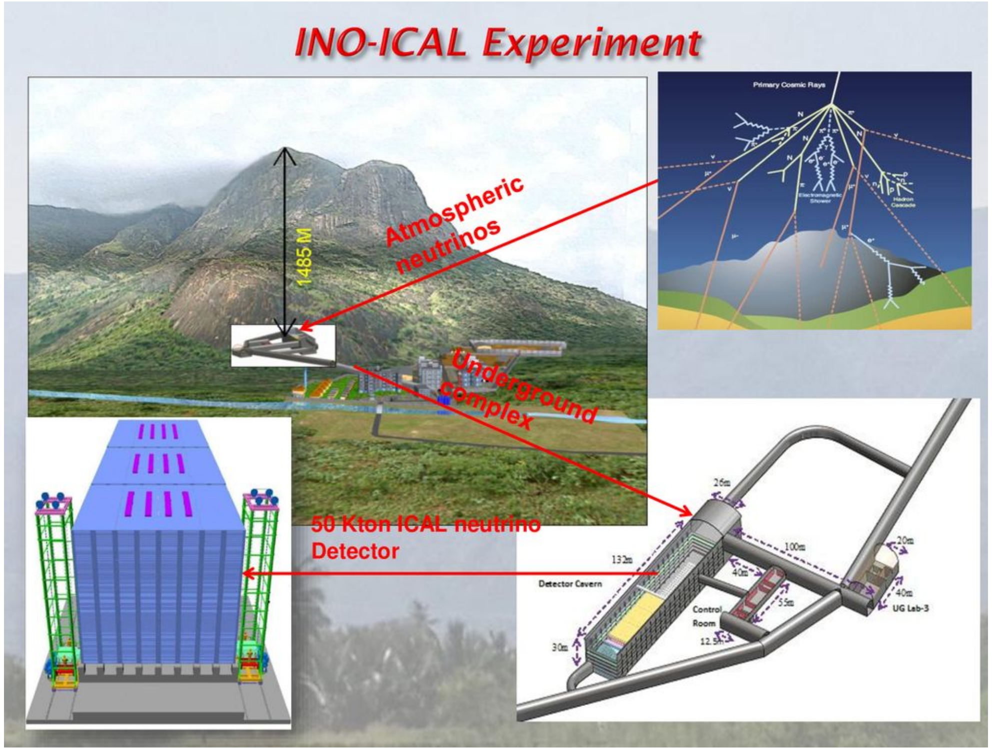
Effect of α_{32} on Muon Survival Channel



Iron-Calorimeter detector at INO





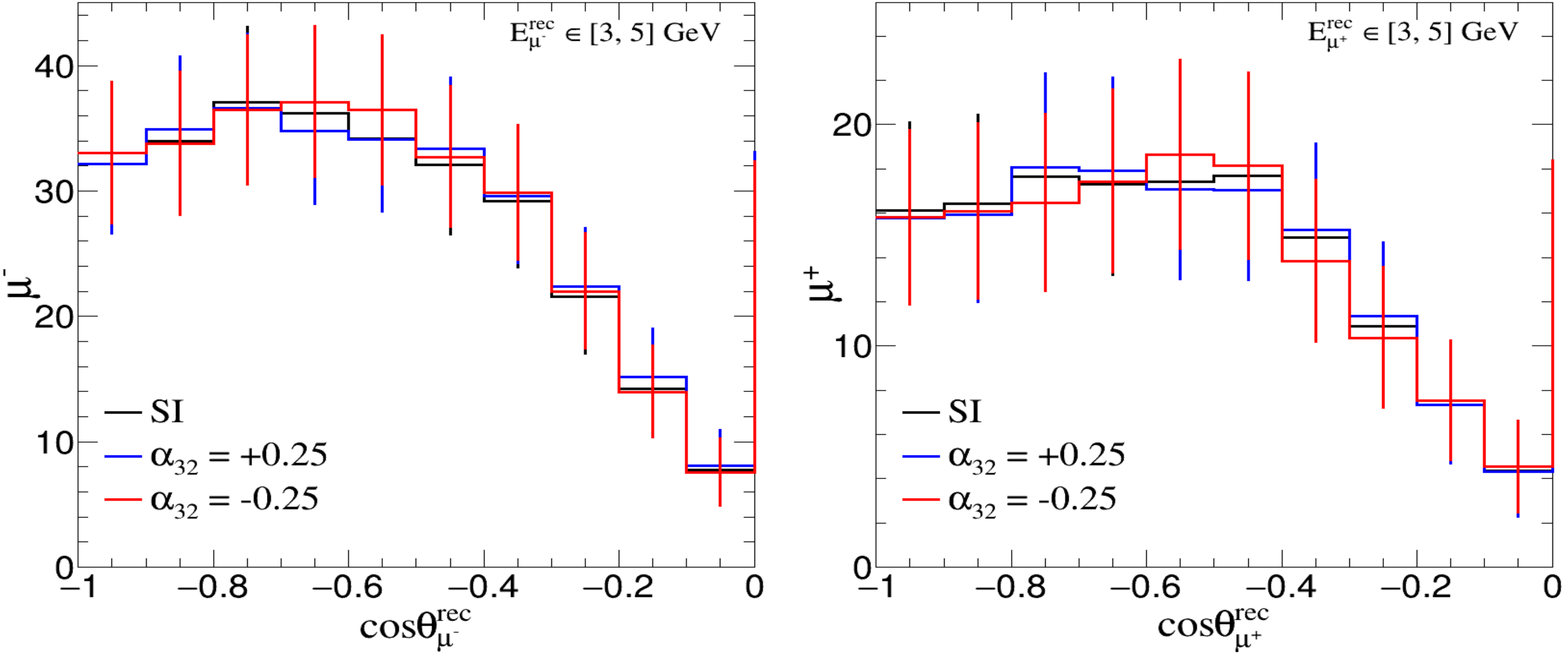
- *50 kt Magnetized Iron Calorimeter (ICAL) of Field strength ~ 1.3 Tesla, enables to distinguish atmospheric neutrino and antineutrino events, separately.*
- *It has ~10% resolution of muon momentum ranging 1-25 GeV and ~1° zenith angle resolution over 15-12800 km range of baselines*

ICAL Event distributions with $\alpha_{32} = \pm 0.25$

- NUANCE MC Generator using Neutrino Flux (Honda) at INO site
- Three-Flavour Oscillation Framework; PREM profile; 500 kt·yr (10 yr)
- Migration matrices from ICAL-Geant4 simulation [arXiv:1304.5115, 1405.7243]
- Binning scheme is adopted from JHEP 03 (2022) 050

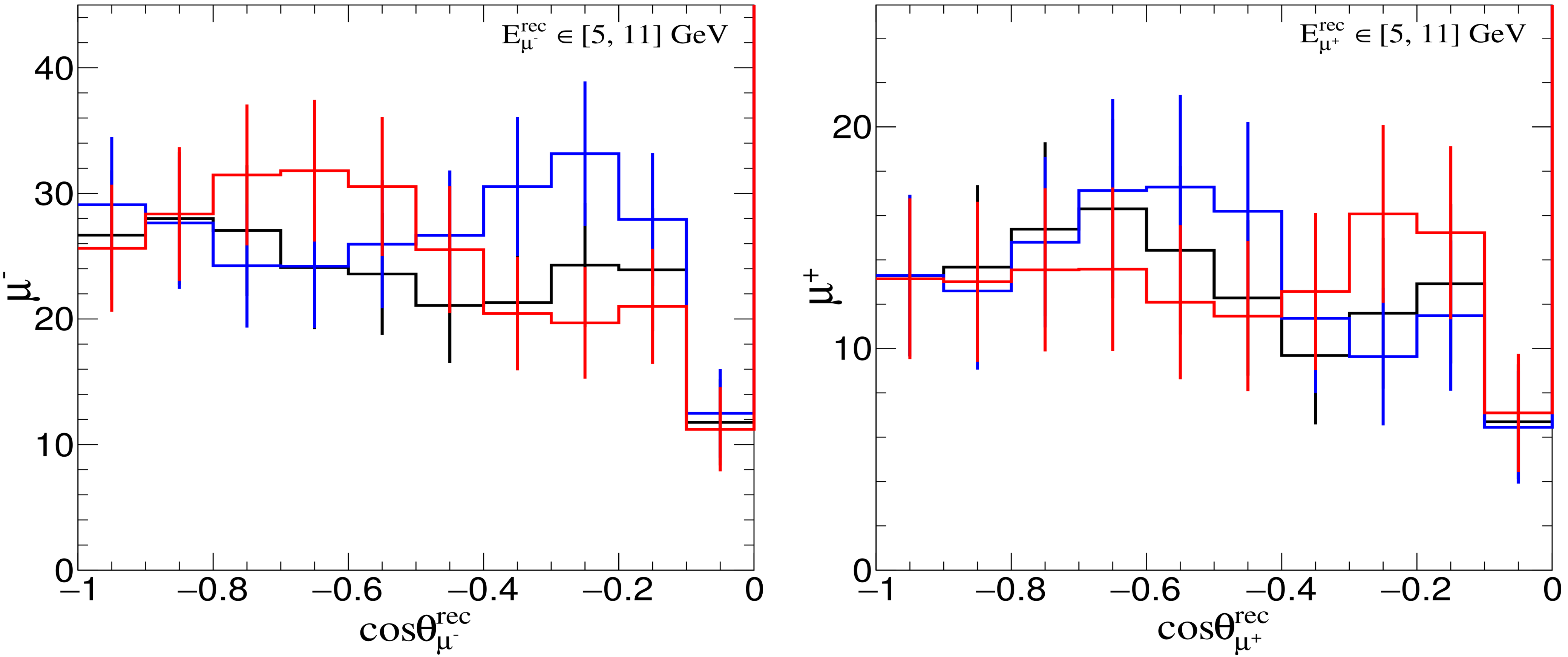
Observable	Range	Bin width	Total bins	
E_μ^{rec} (GeV)	$[1, 11]$	1	10	13
	$[11, 21]$	5	2	
	$[21, 25]$	4	1	
$\cos\theta_\mu^{\text{rec}}$	$[-1.0, 0.0]$	0.1	10	15
	$[0.0, 1.0]$	0.2	5	
$E'^{\text{rec}}_{\text{had}}$ (GeV)	$[0, 2]$	1	2	4
	$[2, 4]$	2	1	
	$[4, 25]$	21	1	

Impact of α_{32} on reconstructed muon events:



Charge Identification Capability of ICAL

Impact of α_{32} on reconstructed muon events:



Charge Identification Capability of ICAL

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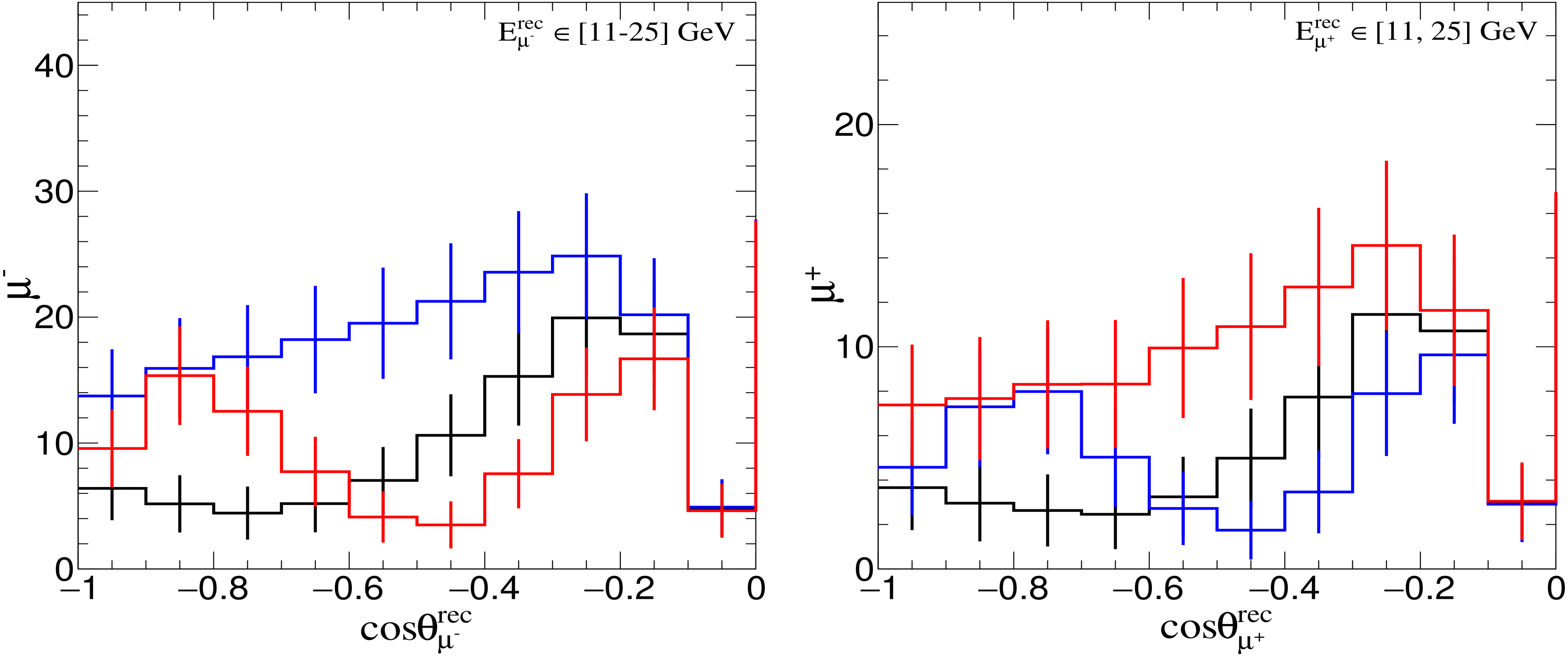


Charge Identification Capability of ICAL

Preliminary results on α_{32}

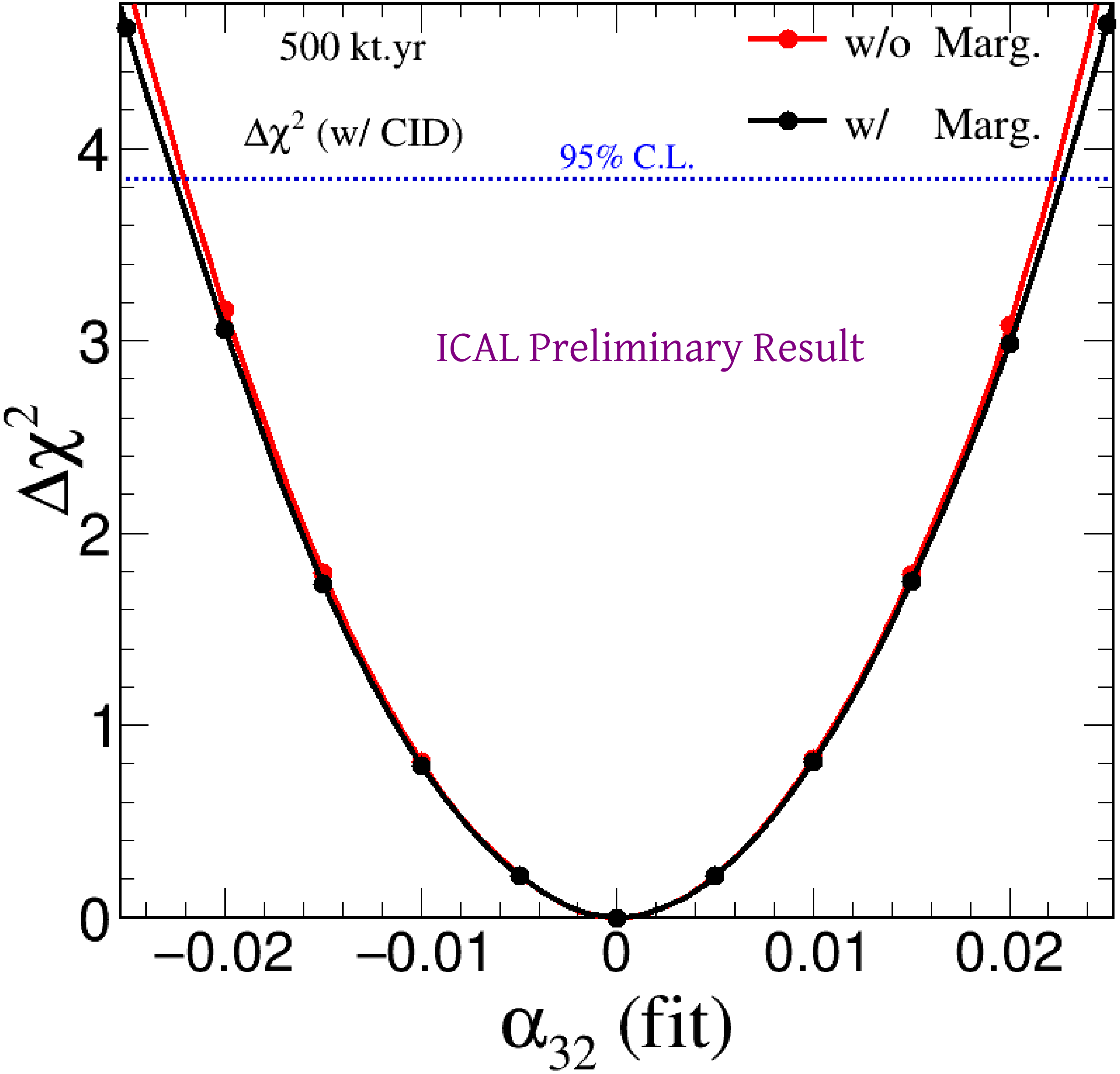
Method of χ^2 Analysis:

$$\chi^2_{\pm} = \min_{\zeta_l} \sum_{i=1}^{N_{E_{\text{had}}}} \sum_{j=1}^{N_{E_{\mu^\pm}}} \sum_{k=1}^{N_{\cos\theta_\mu}} 2\left[N_{ijk}^{\text{test}} - N_{ijk}^{\text{true}} - N_{ijk}^{\text{true}} \ln\left(\frac{N_{ijk}^{\text{test}}}{N_{ijk}^{\text{true}}}\right)\right] + \sum_{l=1}^{5} \zeta_l^2$$

$$N_{ijk}^{\text{test}} = N_{ijk}^{0}\left(1 + \sum_{l=1}^{5} \pi_{ijk}^{l} \zeta_l\right);$$

arXiv:1406.3689v1

- Flux Normalization Error = 20%
- Interaction Cross-section Error = 10%
- Tilt Error = 5%
- Zenith Error = 5%
- Overall Systematic Error = 5%

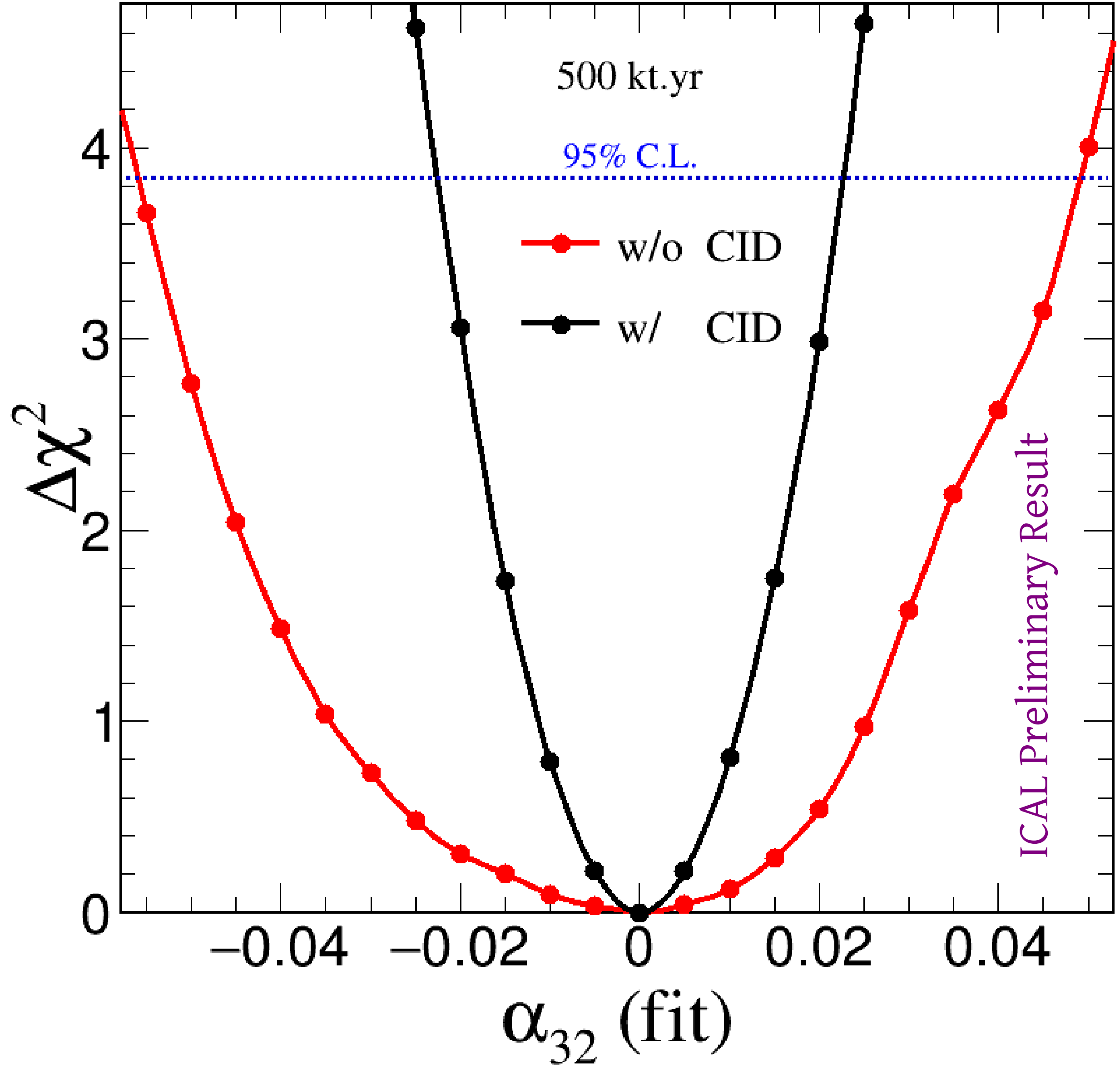


Constraints on "α_{32}" (Impact of Marg.)

w/ Marg.

$\sin^2\theta_{23},\ |\Delta m^2_{eff}|,\ \mathrm{M.O}$

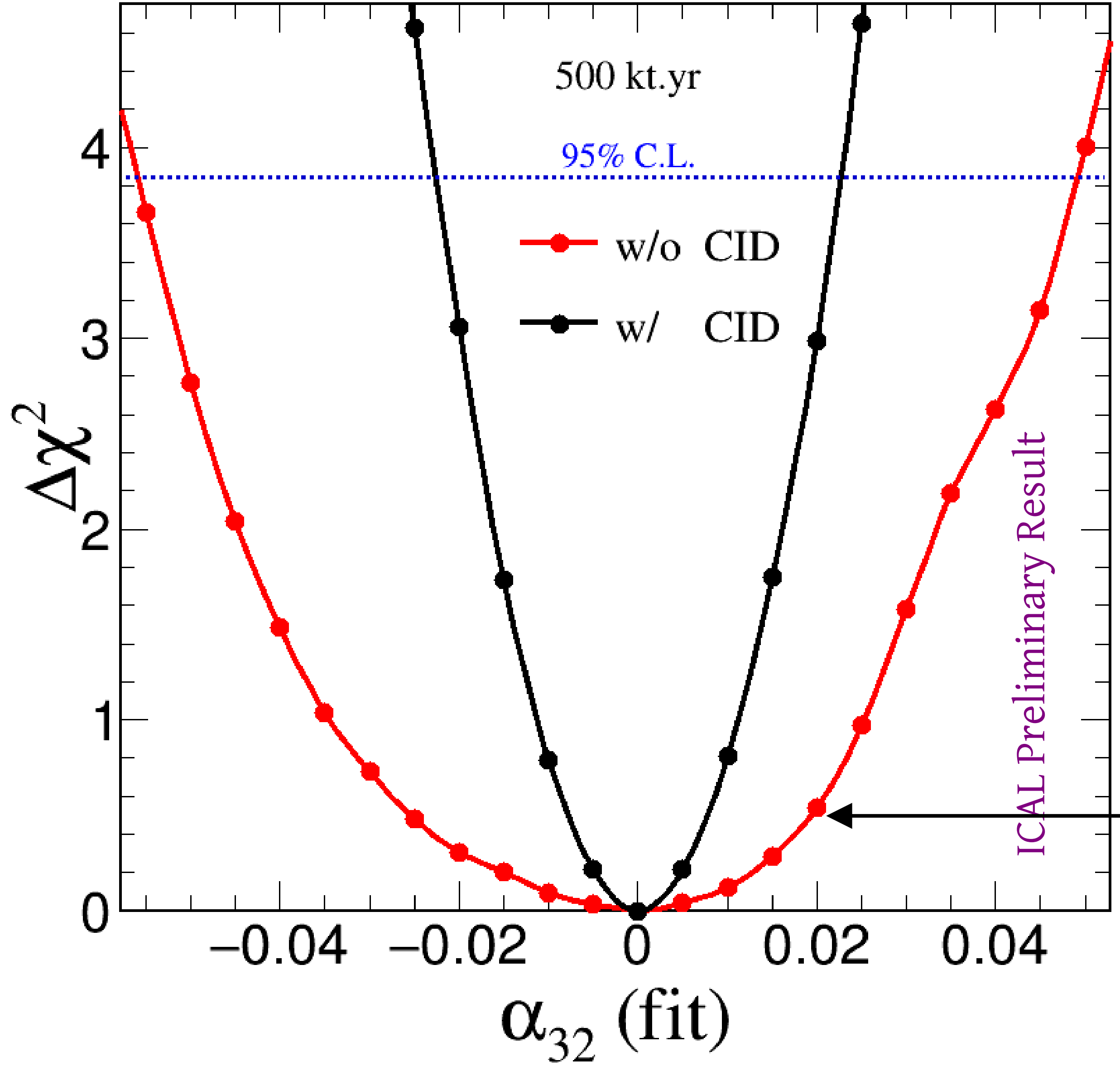
Constraints on "α_{32}" (Impact of CID)



w/ Marg.

$\sin^2\theta_{23},\ |\Delta m^2_{eff}|,$ M.O

Constraints on "α_{32}" (Impact of CID)



w/ Marg.

$\sin^2\theta_{23}$, $|\Delta m^2_{eff}|$, M.O

α_{32} deteriorates by ~128% for w/o CID

90% C.L.	DUNE	JD	KD	JD+KD	JD+KD+DUNE	T2K+NOνA
$\lvert\alpha_{32}\rvert$	< 0.33	< 1.2	< 0.85	< 0.71	< 0.27	< 1.4

arXiv: 2111.00329

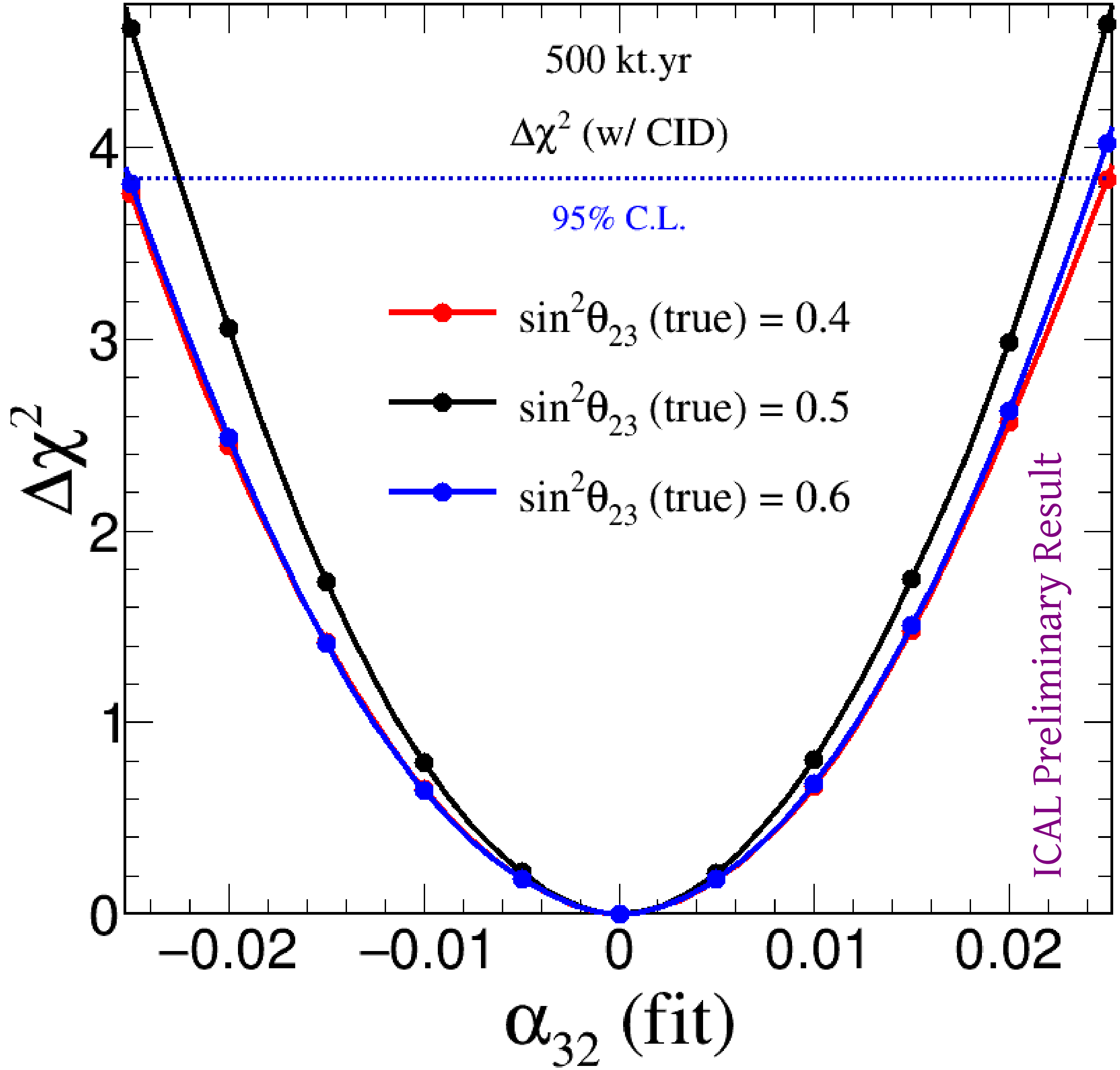
Comparison of limits on α_{32} obtained from ICAL	
Method of Analysis (w/ CID)	α_{32}
$\Delta\chi^2$ (E_μ^{rec}, $\cos\theta_\mu^{\text{rec}}$, E_{had}^{rec}) (90% C.L.)	$[-0.019, +0.019]$
$\Delta\chi^2$ (E_μ^{rec}, $\cos\theta_\mu^{\text{rec}}$, E_{had}^{rec}) (95% C.L.)	$[-0.022, +0.022]$

This Work

Impact of ϕ_{32} on constraining $\lvert\alpha_{32}\rvert$ at 95% C.L.		
Type	DUNE	ICAL
$\phi_{32} = 0$ (fixed)	0.034	0.022
$\phi_{32} \in [-\pi, \pi]$ (free)	0.241	0.119

Keeping all oscillation parameters and min. Condition same for ICAL and DUNE

Impact of θ_{23} (true) on $\Delta\chi^2$ analysis

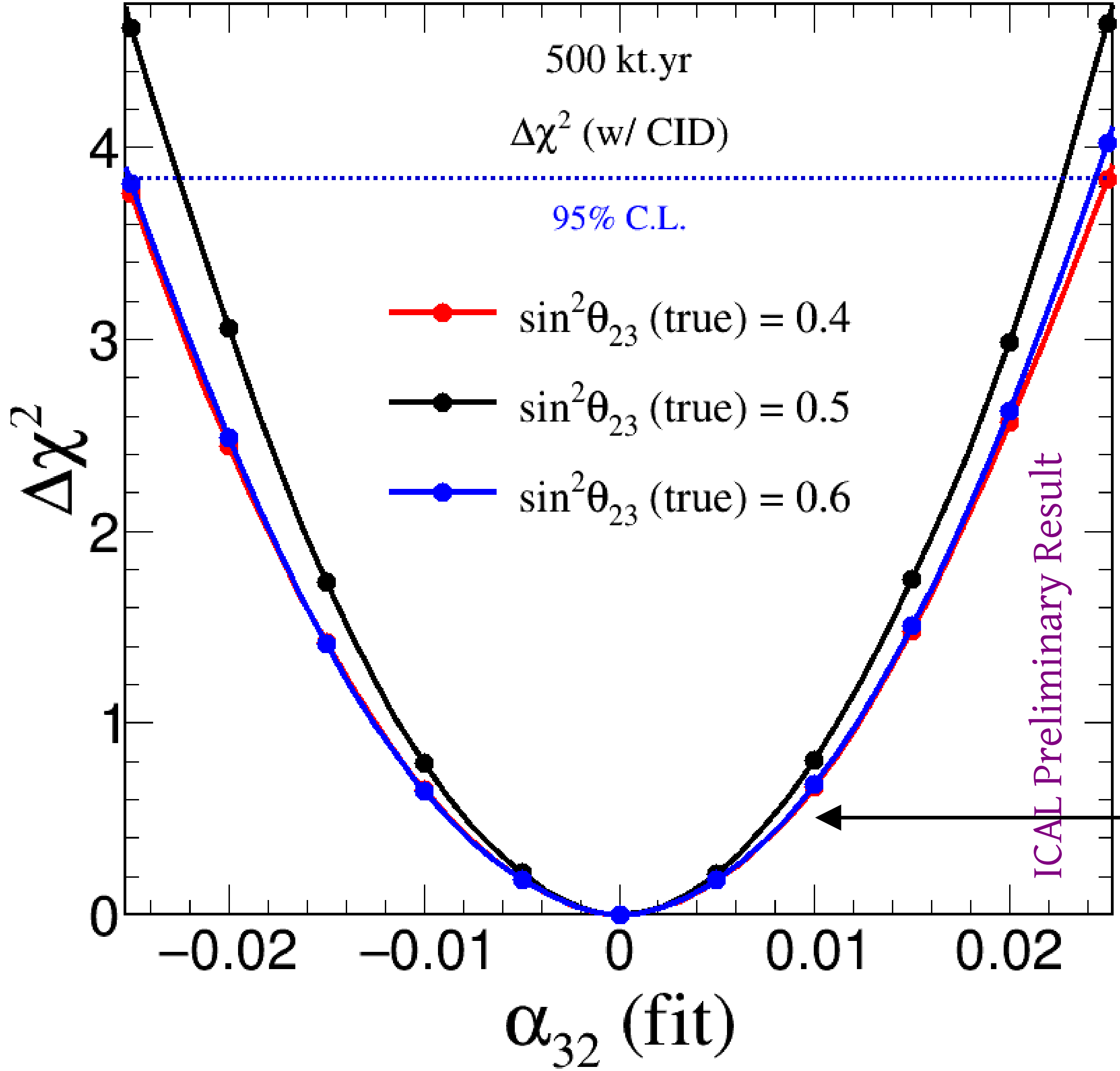


Constraints on "α_{32}" (Impact of θ_{23})

w/ Marg.

$\sin^2\theta_{23},\ |\Delta m^2_{eff}|,\ \text{M.O}$

Constraints on "α_{32}" (Impact of θ_{23})



w/ Marg.
$\sin^2\theta_{23},\ |\Delta m^2_{eff}|,$ M.O

α_{32} deteriorates by ~14% for non-maximal θ_{23}

Concluding Remark :

- For the first time, we have probed the concept of NUNM using atmospheric neutrinos.
- We have explored the impact of neutral-current, playing a crucial to get sensitivity for NUNM parameters.
- We show that how having CID information can improve the constraint of NUNM parameter of the atmospheric detector like INO-ICAL.
- We have explored the sensitivity of NUNM for θ_{23} uncertainties

Concluding Remark :

- We believe, existing high precision data from Super-K, IceCube (DeepCore), KM3NET can certainly improves bound on the NUMM parameters
- ICAL with its CID capability places a competitive bound as compared to next-generation long-baseline experiments.

Thank You

Back Up

Example in 4 x 4 Unitarity Neutrino Mixing

Formalism & Conventions :

Using Okubo's notation

$$U^{4\times4} = R_{34}\cdot R_{24}\cdot R_{14}\cdot R_{23}\cdot R_{13}\cdot R_{12}$$

$$U^{4\times4} = R_{34}\cdot R_{24}\cdot R_{14}\cdot U^{3\times3}$$

Formalism & Conventions :

Using Okubo's notation

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & -s_{34} \\ 0 & 0 & s_{34} & c_{34} \end{pmatrix}, \quad R_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & -s_{24} \\ 0 & 0 & 1 & 0 \\ 0 & s_{24} & 0 & c_{24} \end{pmatrix},$$

$$R_{14} = \begin{pmatrix} c_{14} & 0 & 0 & -s_{24} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_{24} & 0 & 0 & c_{24} \end{pmatrix},$$

Formalism & Conventions :

Using Okubo's notation

$$R_{34} \cdot R_{24} \cdot R_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ -s_{14}s_{24} & c_{24} & 0 & c_{14}s_{24} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & c_{34} & c_{14}c_{24}s_{34} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & -s_{34} & c_{14}c_{24}c_{34} \end{pmatrix}$$

Formalism & Conventions :

Using Okubo’s notation

$$R_{34} \cdot R_{24} \cdot R_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ -s_{14}s_{24} & c_{24} & 0 & c_{14}s_{24} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & c_{34} & c_{14}c_{24}s_{34} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & -s_{34} & c_{14}c_{24}c_{34} \end{pmatrix}$$

Lower Triangular Matrix

Formalism & Conventions :

Using Okubo’s notation

$$U^{4\times 4} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ -s_{14}s_{24} & c_{24} & 0 & c_{14}s_{24} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & c_{34} & c_{14}c_{24}s_{34} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & -s_{34} & c_{14}c_{24}c_{34} \end{pmatrix} \cdot \begin{pmatrix} \tilde{U}_{e1} & \tilde{U}_{e2} & \tilde{U}_{e3} & 0 \\ \tilde{U}_{\mu 1} & \tilde{U}_{\mu 2} & \tilde{U}_{\mu 3} & 0 \\ \tilde{U}_{\tau 1} & \tilde{U}_{\tau 2} & \tilde{U}_{\tau 3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$