Renormalization Group Evolution of Neutrino Angles and Masses

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1. Background
2. The HSMU hypothesis
3. Wolfenstein Ansatz
4. Conclusion
5. Appendix

Neutrinos and Standard Model

- Neutrino detection: β decay process
  $n \rightarrow p + e^{-} + \bar{\nu}_e$
- Three flavours $(\nu_e, \nu_\mu, \nu_\tau)$
- In SM: Chargeless, massless leptons
- Interact weakly

Neutrino Oscillations

- Flavours mix
- Mathematically, mixing means rotations among flavour eigenstates
- Standard Parameterization
  $U_{PMNS} =$

PMNS matrix

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$U_{PMNS} =$

$$\begin{pmatrix} c_{12}c_{13} & s_{13}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{2}s_{13} \end{pmatrix}$$

- where c_{ij} and s_{ij} are $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively

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- Thus mixing implies $m_\nu \neq 0$

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- where c_{ij} and s_{ij} are $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively
- δ_{CP} is Charge-Parity (CP) violations shown by leptons
- Thus mixing implies $m_\nu \neq 0$
- Contradicts with SM's prediction that neutrinos are massless

Open questions about neutrinos

- Dirac nature or Majorana nature?
- Why are mixing angles so different from that of quarks?

CKM and PMNS matrices

$$U_{CKM} = \begin{pmatrix} 0.97366 - 0.97384 & 0.2237 - 0.2253 & 0.00358 - 0.00406 \\ 0.217 - 0.225 & 0.976 - 0.998 & 0.0396 - 0.0424 \\ 0.0077 - 0.0083 & 0.0377 - 0.399 & 0.983 - 1.043 \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} 0.802 - 0.845 & 0.513 - 0.579 & 0.143 - 0.156 \\ 0.233 - 0.507 & 0.461 - 0.694 & 0.631 - 0.778 \\ 0.261 - 0.526 & 0.471 - 0.701 & 0.611 - 0.761 \end{pmatrix}$$

The idea

- New idea $\implies$ Unification of mixing angles at *some* high energy scale [2]
- θ_q and θ are equal at high scale
- In past, Electromagnetism, EW force
- Present research, Grand Unification Theory (GUT)

RG equations

- How to calculate high scale values?

$$\dot{\theta}_{12} = -\frac{Cy_\tau^2}{32\pi^2}\sin(2\theta_{12})s_{23}^2\frac{|m_1 e^{i\varphi_1} + m_2 e^{i\varphi_2}|^2}{\Delta m_{sol}^2} + O(\theta_{13})$$

RG equations

- How to calculate high scale values?
- Renormalization Group Equations

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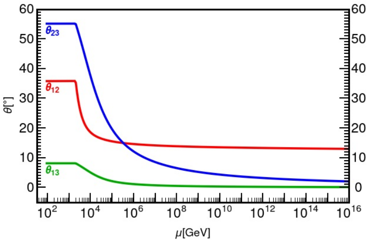
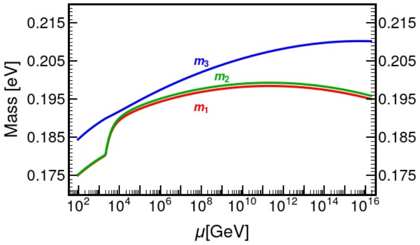
The execution

- Start with low scale quarks parameters (masses, mixing angles)
- Use RG eqns to run them to GUT scale
- HSMU: Equate them to neutrino mixing angles
- Run these neutrino angles down to M_Z scale (include masses at high scale)
- Match the low scale parameters with experimentally measured values(at M_Z scale)

RG running of mixing angles and masses





Experimental values

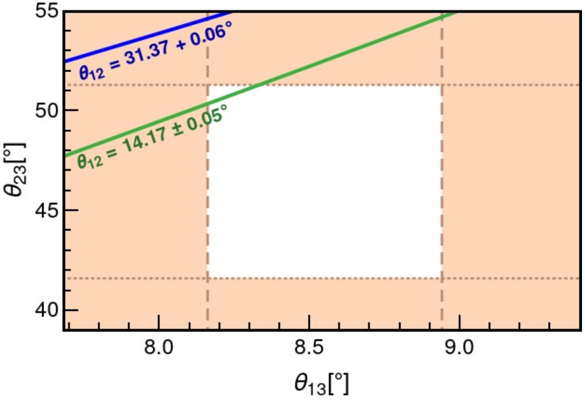
- Exeperimental values, precise upto $3-\sigma$ range [1]

Oscillation parameters	3-σ range	Best fit values
θ_{12}	31.37° - 37.46°	34.33°
θ_{13}	8.16° - 8.94°	8.58°
θ_{23}	41.61° - 51.30°	48.79°
Δm^2_{atm}	2.39×10^{-3}-$2.57 \times 10^{-3} eV^2$	$2.49 \times 10^{-3} eV^2$
Δm^2_{sol}	6.94×10^{-5}-$8.14 \times 10^{-5} eV^2$	$7.50 \times 10^{-5} eV^2$

Dirac Case

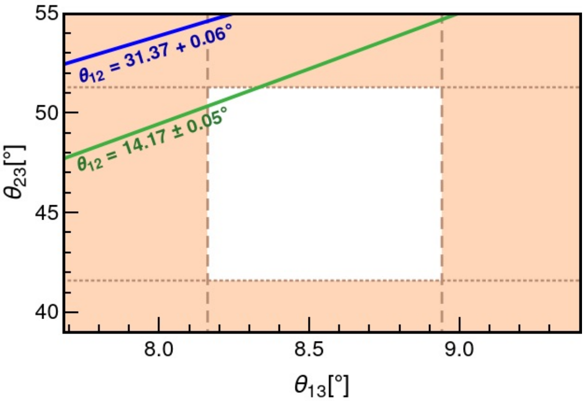
- Aim: To check whether all angles are inside
- Two subcases
- First: $\delta = 0^\circ$
- Second: $\delta \neq 0^\circ \ (= \delta_q)$

Graph and conclusion



- Plot of θ_{23} vs θ_{13} correlation

Graph and conclusion



- Plot of θ_{23} vs θ_{13} correlation
- Conclusion for Dirac case: All angles can't be brought inside

Majorana case

- Two more free parameters: φ_1 and φ_2

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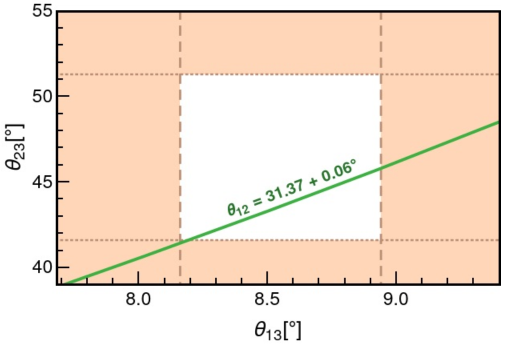
- Extra parameter to check: Effective Majorana Mass ($m_{\beta\beta}$)
- Experimental range: <0.165 eV
- First subcase: $\varphi_1 = \varphi_2 = 0^\circ \implies$ same result as Dirac case

Majorana case: $\varphi_1, \varphi_2 \neq 0°$

- Second subcase: $\varphi_1 = 120°, \varphi_2 = 30°$



Majorana case: $\varphi_1, \varphi_2 \neq 0^\circ$

- Second subcase: $\varphi_1 = 120^\circ, \varphi_2 = 30^\circ$
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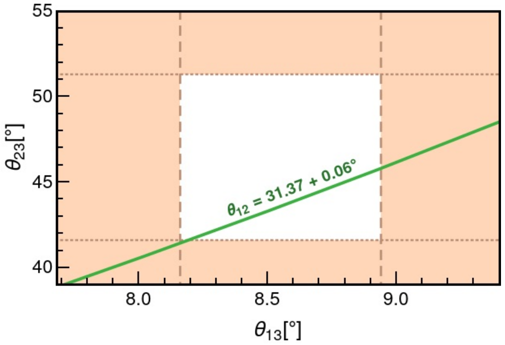

Majorana case: $\varphi_1, \varphi_2 \neq 0^\circ$ Conclusion

- Conclusion: All angles are inside
- Can we bring in Δm^2_{atm}, Δm^2_{sol} and $m_{\beta\beta}$?
- Have to vary φ_1 and φ_2

Majorana case: Vary φ_1, φ_2

$\varphi_1(^\circ)$	$\varphi_2(^\circ)$	θ_{12}	θ_{13}	θ_{23}	Δm^2_{sol}	Δm^2_{atm}
50	0	✓	✓	✓	✓	-
100	0	✓	✓	✓	-	-
200	0	✓	✓	✓	-	-
300	0	✓	-	✓	✓	-
0	50	✓	-	✓	✓	-
50	50	✓	-	✓	✓	-
100	50	✓	✓	✓	-	-
200	50	✓	-	✓	-	-
300	50	✓	✓	✓	✓	-
0	100	✓	-	✓	✓	-
50	100	✓	-	✓	✓	-
100	100	✓	✓	✓	-	-
200	100	✓	-	✓	-	-
300	100	✓	-	✓	✓	-
0	200	✓	-	-	-	-
50	200	✓	-	✓	✓	-
100	200	✓	-	✓	-	-
200	200	✓	-	✓	-	-
300	200	✓	✓	-	-	✓
0	300	✓	-	✓	✓	-
50	300	✓	✓	✓	✓	-
100	300	✓	✓	✓	-	-
200	300	✓	✓	✓	✓	-
300	300	✓	-	✓	✓	-

$\varphi_1(^\circ)$	$\varphi_2(^\circ)$	θ_{12}	θ_{13}	θ_{23}	Δm^2_{sol}	Δm^2_{atm}
50	0	✓	✓	✓	Only one	
100	0	✓	✓	✓	Only one	
200	0	✓	✓	✓	Only one	
300	0	✓	Only one		Only one	
0	50	✓	Only one		Only one	
50	50	✓	Only one		Only one	
100	50	✓	✓	✓	Only one	
200	50	✓	Only one		Only one	
300	50	✓	✓	✓	Only one	
0	100	✓	Only one		Only one	
50	100	✓	Only one		Only one	
100	100	✓	✓	✓	Only one	
200	100	✓	Only one		Only one	
300	100	✓	Only one		Only one	
0	200	✓	Only one		Only one	
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100	200	✓	Only one		Only one	
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300	200	✓	Only one		Only one	
0	300	✓	Only one		Only one	
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- Conclusion: At best, 4 parameters are brought in

Wolfenstein Parameterization

- Introduce Wolfenstein parameter λ

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$$\theta_{12} = \arcsin(\lambda)$$
$$\theta_{23} = \alpha \arcsin(\lambda^2)$$
$$\theta_{13} = \beta \arcsin(\lambda^3)$$

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- $\lambda = \sin\theta_{12}$

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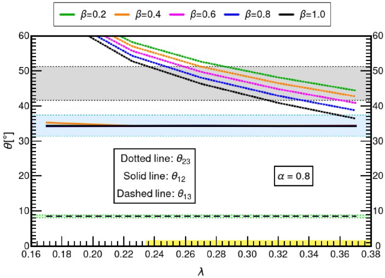
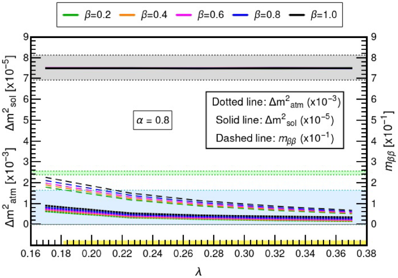
$$\theta_{23} = \alpha \arcsin(\lambda^2)$$
$$\theta_{13} = \beta \arcsin(\lambda^3)$$

- α, β are linear coefficients

Effect of λ





- λ varies in inverse correlation with Δm^2_{atm}, Δm^2_{sol} and $m_{\beta\beta}$

φ_1, φ_2 variations

- Select the best φ_1, φ_2 pairs from HSMU case

  $\varphi_1 = 50^\circ$; $\varphi_2 = 0^\circ$

  $\varphi_1 = 50^\circ$; $\varphi_2 = 300^\circ$

  $\varphi_1 = 200^\circ$; $\varphi_2 = 300^\circ$

  $\varphi_1 = 300^\circ$; $\varphi_2 = 50^\circ$

α, β variations

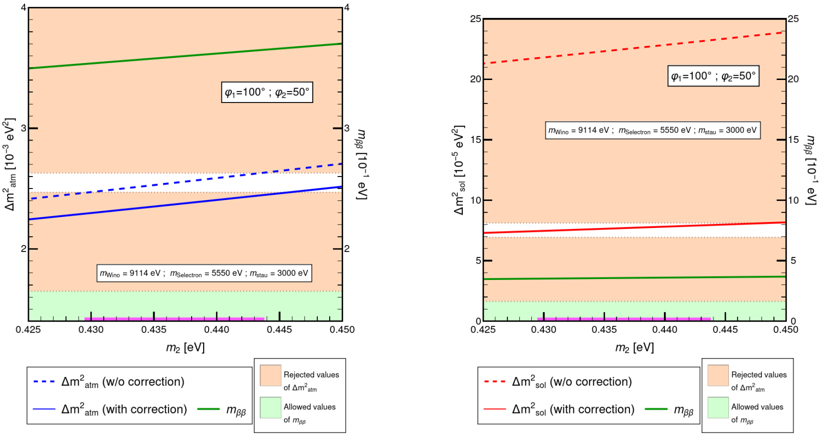
- For already chosen φ_1, φ_2, vary α, β
- Ready for λ variations at the end

$\varphi_1 = 300^\circ$, $\varphi_2 = 50^\circ$ with α, β variations

α	β	θ_{23}	Δm^2_{atm}	$m_{\beta\beta}$
1.0	1.0	⋆	• (red)	•
1.0	0.8	⋆ (green)	• (red)	•
1.0	0.6	⋆ (green)	• (red)	•
1.0	0.4	⋆ (green)	• (red)	•
1.0	0.2	⋆ (green)	• (red)	•
0.8	1.0	⋆	• (red)	•
0.8	0.8	⋆	• (red)	•
0.8	0.6	⋆ (green)	• (red)	•
0.8	0.4	⋆ (green)	• (red)	•
0.8	0.2	⋆ (green)	• (red)	•
0.6	1.0	⋆	• (red)	•
0.6	0.8	⋆	• (red)	•
0.6	0.6	⋆	• (red)	•
0.6	0.4	⋆	• (red)	•
0.6	0.2	⋆ (green)	• (red)	•
0.4	1.0	⋆	• (red)	•
0.4	0.8	⋆	• (red)	•
0.4	0.6	⋆	• (red)	•
0.4	0.4	⋆	• (red)	•
0.4	0.2	⋆	• (red)	•
0.2	1.0	⋆	• (red)	⋆
0.2	0.8	⋆	• (red)	• (green)
0.2	0.6	⋆	• (red)	• (green)
0.2	0.4	⋆	• (red)	• (green)
0.2	0.2	⋆	• (red)	• (green)

- We can rule out invalid set of values from this easily

Threshold corrections



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- Threshold corrections $\implies$ 5 out of 6 low scale parameters are successfully brought in (Only one Δm^2 remains)

Conclusion

- HSMU $\implies$ Leaning towards Majorana nature of neutrinos; but too stringent constraints
- Wolfenstein ansatz $\implies$ Better; but still can't get all parameters in
- Threshold corrections $\implies$ 5 out of 6 low scale parameters are successfully brought in (Only one Δm^2 remains)
- Time to put a full stop on SUSY high scale unification models

RG equations: Angles

$$\dot{\theta}_{12} = -\frac{Cy_\tau^2}{32\pi^2} \sin(2\theta_{12}) s_{23}^2 \frac{|m_1 e^{i\varphi_1} + m_2 e^{i\varphi_2}|^2}{\Delta m_{sol}^2} + O(\theta_{13})$$

$$\dot{\theta}_{13} = \frac{Cy_\tau^2}{32\pi^2} \sin(2\theta_{12}) \sin(2\theta_{23}) \frac{m_3}{\Delta m_{atm}^2 (1+\zeta)}$$

$$\times \left[m_1 \cos(\varphi_1 - \delta) - (1+\zeta) m_2 \cos(\varphi_2 - \delta) - \zeta m_3 \cos(\delta) \right] + O(\theta_{13})$$

$$\dot{\theta}_{23} = -\frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m_{atm}^2}$$

$$\left[c_{12}^2 |m_2 e^{i\varphi_2} + m_3|^2 + s_{12}^2 \frac{m_2 e^{i\varphi_2} + m_3}{1+\zeta} \right] + O(\theta_{13})$$

RG equations: Masses

$$16\pi^2 \dot{m}_1 = \left[\alpha + C y_\tau^2 (2 s_{12}^2 s_{23}^2 + F_1)\right] m_1$$
$$16\pi^2 \dot{m}_2 = \left[\alpha + C y_\tau^2 (2 c_{12}^2 s_{23}^2 + F_2)\right] m_2$$
$$16\pi^2 \dot{m}_3 = \left[\alpha + 2 C y_\tau^2 c_{13}^2 c_{23}^2\right] m_3$$

$$8\pi^2 (\Delta \dot{m}_{sol}^2) = \alpha \Delta m_{sol}^2 + C y_\tau^2 \left[2 s_{23}^2 (m_2^2 c_{12}^2 - m_1^2 s_{12}^2) + F_{sol}\right]$$
$$8\pi^2 (\Delta \dot{m}_{atm}^2) = \alpha \Delta m_{atm}^2 + C y_\tau^2 \left[2 m_3^2 s_{13}^2 c_{23}^2 - 2 m_2^2 c_{12}^2 s_{23}^2 + F_{atm}\right]$$

RG equations: Variables defined

where $\dot{\theta}$, $\dot{m}$ and $\dot{m^2}$ represent $\frac{d\theta}{dt}$, $\frac{dm}{dt}$ and $\frac{dm^2}{dt}$ respectively

$t = \ln\left(\frac{\mu}{\mu_0}\right)$, where μ is the variable energy scale and μ_0 is the initial energy scale from where RG running starts.

$\Delta m^2_{sol} = (m_2^2 - m_1^2)$ & $\Delta m^2_{atm} = (m_3^2 - m_2^2)$

s_{ij} and c_{ij} are $\sin\theta_{ij}$ and $\cos\theta_{ij}$ respectively.

$C = -3/2$ for MSSM and $C = 1$ for SM.

$\zeta = \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$

y_τ is 3^{rd} generation element of Yukawa coupling matrix Y_e.

RG equations: Variables defined

$$F_1 = -s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta + 2s_{13}^2 c_{12}^2 c_{23}^2$$
$$F_2 = s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta + 2s_{13}^2 s_{12}^2 c_{23}^2$$
$$F_{sol} = (m_1^2 + m_2^2) s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta + 2s_{13}^2 c_{23}^2 \left(m_2^2 s_{12}^2 - m_1^2 c_{12}^2 \right)$$
$$F_{atm} = -m_2^2 s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\delta - 2m_2^2 s_{12}^2 s_{13}^2 c_{23}^2$$

Initial values

- M_Z scale = 91.1876 GeV
- GUT scale = 2×10^{16} GeV
- tanβ = 55 (β is the ratio of expectation values of Higgs doublets in 2HDM)
- SUSY cutoff scale = 2000 GeV
- Values of gauge coupling constants
  - Higgs coupling = 0.4615 (at M_Z scale) & 0.7013 (at GUT scale)
  - Weak coupling = 0.6519 (at M_Z scale) & 0.6904 (at GUT scale)
  - Strong coupling = 1.2198 (at M_Z scale) & 0.6928 (at GUT scale)

Majorana case sector

- "Effective Majorana mass" $m_{\beta\beta} \equiv \left[\sum_i U_{ei}^2 \, m_i \right]$
- PMNS matrix

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{13}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{2}s_{13} \end{pmatrix} \times \begin{pmatrix} e^{\frac{-i\varphi_1}{2}} & 0 & 0 \\ 0 & e^{\frac{-i\varphi_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Lagrangians used

Below SUSY breaking scale:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_5$$

Above SUSY breaking scale,

but below seesaw scale:

$$\mathcal{L} = \mathcal{L}_{\mathrm{MSSM}} + \mathcal{L}_5$$

Above seesaw scale:

$$\mathcal{L} = \mathcal{L}_{\mathrm{MSSM}} + \mathcal{L}_{\mathrm{seesaw}}$$

$$= \mathcal{L}_{\mathrm{MSSM}} - \mathrm{Y}_{\nu}^{\mathrm{ij}}\bar{\mathrm{L}}^{\mathrm{i}}\tilde{\mathrm{H}}\nu_{\mathrm{R}}^{\mathrm{i}} - \frac{1}{2}\bar{\nu}^{\mathrm{j}}\mathrm{M}^{\mathrm{ij}}\nu^{\mathrm{j}} + \mathrm{H.c.}$$

$$\mathcal{L}_5 = -\frac{\mathrm{f_{ik}}}{\Lambda_{\mathrm{ss}}}(\epsilon_{\mathrm{ab}}\mathrm{L_a^i H_b})(\epsilon_{\mathrm{cd}}\mathrm{L_c^k H_d}) + \mathrm{H.c.}$$

[1] P. F. de Salas, D. V. Forero, S. Gariazzo, P. Martínez-Miravé, O. Mena, C. A. Ternes, M. Tórtola, and J. W. F. Valle.
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[2] R. N. Mohapatra, M. K. Parida, and G. Rajasekaran.
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