Neutrino Magnetic Moment in Left Right Symmetric Model

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December 14, 2022

Outline

- Motivation
- Magnetic Moment in Standard Model
- Left Right Symmetric Model
- Formalism of Transition and Dirac Magnetic Moment
- Results and Discussion
- Summary

ElectroMagnetic properties of Neutrino

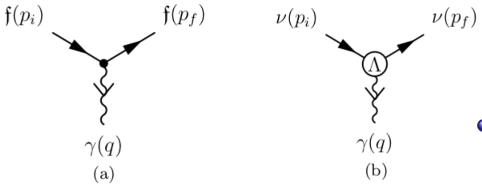


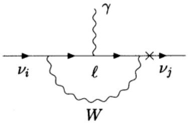
Figure 1: (a)Tree-level coupling of a charged fermion f with a photon , (b) effective one-photon coupling of a neutrino with a photon

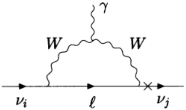
- Hamiltonian and the electromagnetic field are Hermitian, the effective current must be Hermitian
- The current conservation required by gauge invariance
- $\lambda_\mu(q)$ is defined in terms of four form factors:real charge, dipole magnetic and electric, and anapole neutrino form factors.

arxiv.1403.6344

Neutrino electromagnetic properties can be used to distinguish Dirac and Majorana neutrinos.

$$\mu_{ij}^{D} = \frac{eG_F}{8\sqrt{2}\pi^2}(m_i + m_j)\sum_{l} U_{li}U_{lj}^{*} f(r_l)$$





$$i\epsilon_{ij}^{D} = \frac{eG_F}{8\sqrt{2}\pi^2}(m_i - m_j)\sum_{l} U_{li}U_{lj}^{*} f(r_l)$$

with, $f(r_l) \approx \frac{3}{2} - \frac{3r_l}{4} + \ldots.$,
$r_l = (\frac{m_l}{m_W})^2$

For $i = j$

$$\mu_{kk}^{D} \simeq \frac{3eG_F m_k}{8\sqrt{2}\pi^2}$$

Shrock et.al (1982), Nucl. Phys.B206,359.

Dirac Magnetic Moment in Standard Model

For $i \neq j$

$$\mu_{ij}^D = \frac{-3eG_F}{32\sqrt{2}\pi^2}(m_i + m_j)\sum_l U_{li}U_{lj}^* \frac{m_l^2}{m_W^2}$$

$$i\epsilon_{ij}^D = \frac{-3eG_F}{8\sqrt{2}\pi^2}(m_i - m_j)\sum_l U_{li}U_{lj}^* \frac{m_l^2}{m_W^2}$$

- No diagonal electric dipole moment.
- Diagonal magnetic moment is proportional to neutrino mass.
- The transition magnetic moment is suppressed with respect to the largest of the diagonal magnetic moments. This suppression is called "GIM mechanism".
- Dirac neutrinos can have both diagonal and off-diagonal magnetic and electric dipole moments
- Only the off-diagonal ones are allowed for Majorana neutrinos.

Experimental Searches for Neutrino Magnetic Moment

Experiments	Limits
Reactor based Experiments	
KRASNOYARSK (1992):	$\mu_\nu < 2.7 \times 10^{-10} \mu_B$
ROVNO (1993):	$\mu_\nu < 1.9 \times 10^{-10} \mu_B$
MUNU (2005):	$\mu_\nu < 1.2 \times 10^{-10} \mu_B$
TEXONO (2010):	$\mu_\nu < 2.0 \times 10^{-10} \mu_B$
GEMMA (2012):	$\mu_\nu < 2.9 \times 10^{-11} \mu_B$
Accelerator based Experiments	
LAPMF (1993):	$\mu_\nu < 7.4 \times 10^{-10} \mu_B$
LSND (2002):	$\mu_\nu < 6.4 \times 10^{-10} \mu_B$
Solar Neutrino Experiments	
Borexino (2017):	$\mu_\nu < 2.8 \times 10^{-10} \mu_B$
XENON1T (2020):	$\mu_\nu \epsilon (1.4, 2.9) \times 10^{-11} \mu_B$

LRSM particles and their transformation

- Matter Particles: Quarks, Leptons

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\mathbf{2}, \mathbf{1}, -\mathbf{1}, \mathbf{1}), \quad \ell_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, -\mathbf{1}, \mathbf{1})$$

$$q_L = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (\mathbf{2}, \mathbf{1}, \tfrac{\mathbf{1}}{\mathbf{3}}, \mathbf{3}), \quad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, \tfrac{\mathbf{1}}{\mathbf{3}}, \mathbf{3})$$

- Gauge Bosons: $W_{\mu L}, W_{\mu R}, G^a_\mu, B_\mu$
- Higgs: Scalar bidoublet Φ which accommodates SM Higgs and either scalar triplets $\Delta_{L,R}$ or scalar doublets $H_{L,R}$.
  A bi-doublet Higgs in our notation reads as:

$$\Phi \equiv \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (\mathbf{2}, \mathbf{2}, \mathbf{0}, \mathbf{1}),$$

Spontaneous symmetric breaking of LRSM gauge group

Spontaneous symmetry breaking of LRSM:

$$\underbrace{\begin{matrix} SU(2)_L & \times & SU(2)_R & \times & U(1)_{B-L} \\ \{T_L, T_{3L}\} & & \{T_R, T_{3R}\} & & \mathrm{B-L} \\ g_L & & g_R & & g_{BL} \end{matrix}}$$

$$\downarrow \langle H_R(1,2,1)\rangle$$

$$\underbrace{\begin{matrix} SU(2)_L & \times & U(1)_Y \\ \{T_L, T_{3L}\} & & Y \\ g \equiv g_L & & g' \end{matrix}}$$

$$\downarrow \langle \phi(1_L, 1/2_Y)\rangle \subset \Phi(2_L, 2_R, 0_{B-L})$$

$$U(1)_{\mathrm{em}}$$

$$(\mathrm{Q},\ \mathrm{e}) \qquad Q = T_{3L} + Y$$

LRSM with scalar triplets: Manifest LRSM

- The situation gets better when scalar triplet Δ_R carrying a $B-L$ charge of 2 causes SSB from LRSM to SM because it allows lepton flavor violation (allows neutrinoless double beta decay).
- The representations of Δ_R and Δ_L under the LRSM gauge group is given as follows:

$$\Delta_L \equiv \begin{pmatrix} \Delta_L^{++} \\ \Delta_L^{+} \\ \Delta_L^{0} \end{pmatrix} \sim (\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{2})\,, \quad \Delta_R \equiv \begin{pmatrix} \Delta_R^{++} \\ \Delta_R^{+} \\ \Delta_R^{0} \end{pmatrix} \sim (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{2})\,.$$

- The inclusion of Δ_R and Δ_L in the model allows for Majorana mass terms along with the Dirac masses created by the Higgs bidoublet.

Neutrino Magnetic Moment in LRSM

The interaction Lagrangian is defined as:

$$\begin{aligned} \mathrm{L_{int}} &= \frac{g_L}{\sqrt{2}} W^{\dagger}_{L\mu}(\overline{\nu}_{eL}\gamma_\mu e_L + \overline{\nu}_{\mu_L}\gamma_\mu \mu_L + \overline{\nu}_{\tau_L}\gamma_\mu \tau_L) + h.c \\ &+ \frac{g_R}{\sqrt{2}} W^{\dagger}_{R\mu}(\overline{\nu}_{eR}\gamma_\mu e_R + \overline{\nu}_{\mu_R}\gamma_\mu \mu_R + \overline{\nu}_{\tau_R}\gamma_\mu \tau_R) + h.c \end{aligned}$$

The mass term is given by

$$\begin{aligned} \mathrm{L_{mass}} &= \left(\overline{e}_R, \overline{\mu}_R, \overline{\tau}_R\right) \cdot \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \cdot \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \\ &+ \left(\overline{\nu}_{1R}, \overline{\nu}_{2R}, \overline{\nu}_{3R}\right) \cdot \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \cdot \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} \end{aligned}$$

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The unitary transformations $\nu_{iL} = U^{\dagger}_{i\alpha}\nu_{\alpha L}$ and $\nu_{iR} = V^{\dagger}_{i\alpha}\nu_{\alpha R}$ are applied to diagonalise the neutrino mass term. Thus, the interaction term can be rewritten as:

$$\mathrm{L_{int}} = \frac{\mathrm{g_L}}{\sqrt{2}}\mathrm{W^{\dagger}_{L\mu}}\overline{\nu}_{\mathrm{iL}}\mathrm{U^{\dagger}_{i\alpha}}\gamma_{\mu}\mathrm{l_{\alpha L}} + \mathrm{h.c} + \frac{\mathrm{g_R}}{\sqrt{2}}\mathrm{W^{\dagger}_{R\mu}}\overline{\nu}_{\mathrm{iR}}\mathrm{V^{\dagger}_{i\alpha}}\gamma_{\mu}\mathrm{l_{\alpha R}} + \mathrm{h.c}$$

After symmetry breaking of $SU(2)_L$ ansd $SU(2)_R$, we can express the mass term of gauge bosons as:

$$\begin{aligned}
\mathrm{L_{mass}} &= \frac{1}{2}\left(W^{\dagger}_L W^{\dagger}_R\right)\cdot\begin{pmatrix} m^2_{WL} & \delta \\ \delta^* & m^2_{WR}\end{pmatrix}\cdot\begin{pmatrix} W^-_L \\ W^-_R\end{pmatrix} \\
&= \frac{1}{2}\left(W^{\dagger}_1 W^{\dagger}_2\right)\cdot\begin{pmatrix} m^2_{W1} & 0 \\ 0 & m^2_{W2}\end{pmatrix}\cdot\begin{pmatrix} W^-_1 \\ W^-_2\end{pmatrix}
\end{aligned}$$

where $W_1 = W_L \cos\zeta - W_R \sin\zeta e^{i\omega}$,
$W_2 = W_L \sin\zeta e^{-i\omega} + W_R \cos\zeta$

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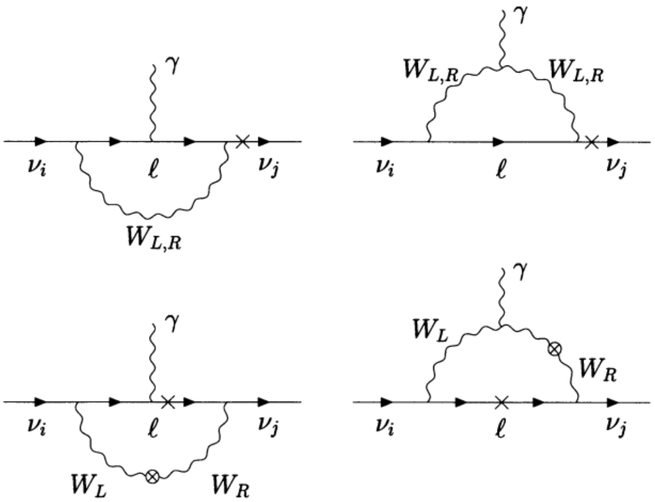
Parameterization

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \cdot \mathrm{P}$$

Here, we have denoted $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and diagonal phase matrix $\mathrm{P} = \mathrm{diag}\left(1, e^{i\alpha}, e^{i\beta}\right)$, where δ is the Dirac CP phase and α, β are Majorana phases varied from $0 \rightarrow 2\pi$.

Loop Level Contributions



Principles of Neutrinos, Application to Astrophysics:Fukugita,Yanagida

Magnetic Moment

$$\mu^{\mathrm{M}}_{\beta\alpha} \simeq i\mu_{\mathrm{B}} \times c \sum_{i=e,\mu,\tau} m_i Im \left[e^{i\omega} \left(V^{\dagger}_{\alpha i} U_{i\beta} - V^{\dagger}_{\beta i} U_{i\alpha} \right) \right]$$

$$\mu^{\mathrm{D}}_{\beta\alpha} \simeq \mu_{\mathrm{B}} \times c \sum_{i=e,\mu,\tau} m_i \left[e^{i\omega} V^{\dagger}_{\beta i} U_{i\alpha} + e^{-i\omega} U^{\dagger}_{\beta i} V_{i\alpha} \right]$$

where

$$\mu_B = \frac{e}{2m_e}$$

$$c = \frac{g_{\mathrm{L}} g_{\mathrm{R}}}{2\,(4\pi)^2} \sin\zeta \cos\zeta\, m_e \frac{m_2^2 - m_1^2}{m_2^2 m_1^2}$$

and

$$\tan 2\zeta = \frac{2}{m_2^2 - m_1^2}$$

Majorana Magnetic Moment Expressions

$$\mu_{32}^{\mathrm{M}} = i\mu_{\mathrm{B}} \times c\Big[m_e Im\Big(e^{i\omega}(V_{21}^{\dagger}U_{13} - V_{31}^{\dagger}U_{12})\Big) + m_{\mu} Im\Big(e^{i\omega}(V_{21}^{\dagger}U_{13} - V_{31}^{\dagger}U_{12})\Big) + m_{\tau} Im\Big(e^{i\omega}(V_{23}^{\dagger}U_{33} - V_{33}^{\dagger}U_{32})\Big)\Big]$$

$$\mu_{31}^{\mathrm{M}} = i\mu_{\mathrm{B}} \times c\Big[m_e Im\Big(e^{i\omega}(V_{11}^{\dagger}U_{13} - V_{31}^{\dagger}U_{11})\Big) + m_{\mu} Im\Big(e^{i\omega}(V_{12}^{\dagger}U_{23} - V_{32}^{\dagger}U_{21})\Big) + m_{\tau} Im\Big(e^{i\omega}(V_{13}^{\dagger}U_{33} - V_{33}^{\dagger}U_{31})\Big)\Big]$$

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$$
\begin{aligned}
\mu_{21}^{\mathrm{M}} &= i\mu_{\mathrm{B}} \times c\Big[m_e Im\Big(e^{i\omega}(V_{11}^{\dagger}U_{12} - V_{21}^{\dagger}U_{11})\Big) \\
&\quad + m_{\mu} Im\Big(e^{i\omega}(V_{12}^{\dagger}U_{22} - V_{22}^{\dagger}U_{21})\Big) \\
&\quad + m_{\tau} Im\Big(e^{i\omega}(V_{13}^{\dagger}U_{32} - V_{23}^{\dagger}U_{31})\Big)\Big]
\end{aligned}
$$

Dirac Magnetic Moment Expressions

$$\begin{aligned}\mu_{32}^{\mathrm{D}} &= \mu_{\mathrm{B}} \times c\Big[m_e\Big(e^{i\omega} V_{31}^{\dagger} U_{12} + e^{-i\omega} U_{31}^{\dagger} V_{12}\Big) \\ &+m_{\mu}\Big(e^{i\omega} V_{32}^{\dagger} U_{22} + e^{-i\omega} U_{32}^{\dagger} V_{22}\Big) \\ &+m_{\tau}\Big(e^{i\omega} V_{33}^{\dagger} U_{32} + e^{-i\omega} U_{33}^{\dagger} V_{32}\Big)\Big]\end{aligned}$$

Continue.....

$$\begin{aligned}
\mu_{31}^{\mathrm{D}} &= \mu_{\mathrm{B}} \times c\Big[m_e\Big(e^{i\omega}V_{31}^{\dagger}U_{11} + e^{-i\omega}U_{31}^{\dagger}V_{11}\Big) \\
&\quad + m_\mu\Big(e^{i\omega}V_{32}^{\dagger}U_{21} + e^{-i\omega}U_{32}^{\dagger}V_{21}\Big) \\
&\quad + m_\tau\Big(e^{i\omega}V_{33}^{\dagger}U_{31} + e^{-i\omega}U_{33}^{\dagger}V_{31}\Big)\Big]
\end{aligned}$$

Continue.....

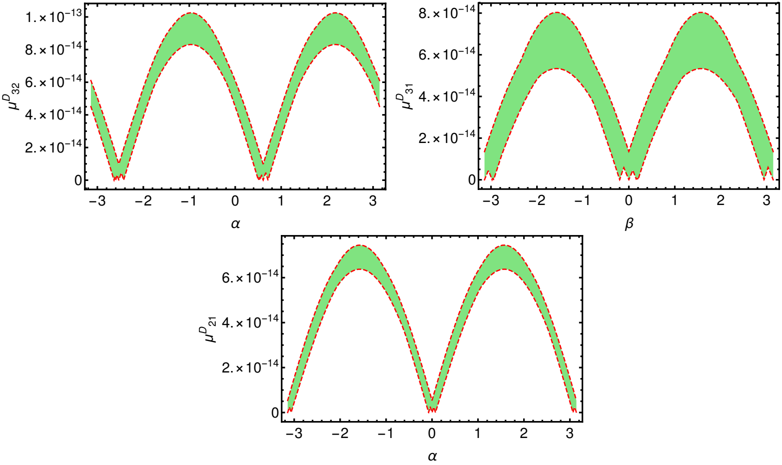
$$
\begin{aligned}
\mu_{21}^{\mathrm{D}} &= \mu_{\mathrm{B}} \times c\Big[m_e\Big(e^{i\omega} V_{21}^{\dagger} U_{11} + e^{-i\omega} U_{21}^{\dagger} V_{11}\Big) \\
&\quad + m_{\mu}\Big(e^{i\omega} V_{22}^{\dagger} U_{21} + e^{-i\omega} U_{22}^{\dagger} V_{21}\Big) \\
&\quad + m_{\tau}\Big(e^{i\omega} V_{23}^{\dagger} U_{31} + e^{-i\omega} U_{23}^{\dagger} V_{31}\Big)\Big]
\end{aligned}
$$

Results and Discussion

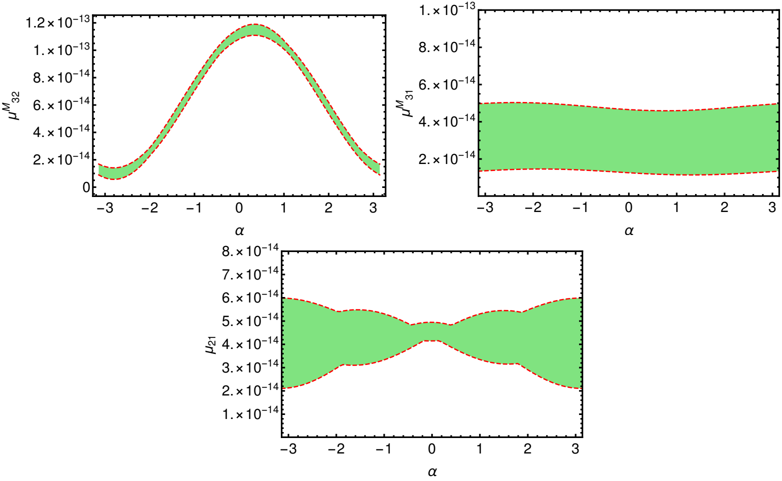
- Transition magnetic moment comes out to be zero for $V = U$ and $V = U^*$ provided $\omega = \pi/2$
- While Dirac magnetic moment is zero only for $V = U$ $(\omega = \pi/2)$ and non-zero for $V = U^*$, a clear cut distinction from transition magnetic moment.
- Both Transition and Dirac magnetic moment are found to be non-zero for $V = U^T$

Parameters	Numerical Value
m_e	0.5198×10^6 (eV)
m_μ	105.6×10^6 (eV)
m_τ	$1776.86 \times 10^6 (eV)$
m_1	80.385×10^9 (eV)
m_2	3×10^{12} (eV)
θ_{12}	34.3 ± 1.0
θ_{13}	49.26 ± 0.79
θ_{23}	8.53 ± 0.13
ω	90°
g_L, g_R	0.63

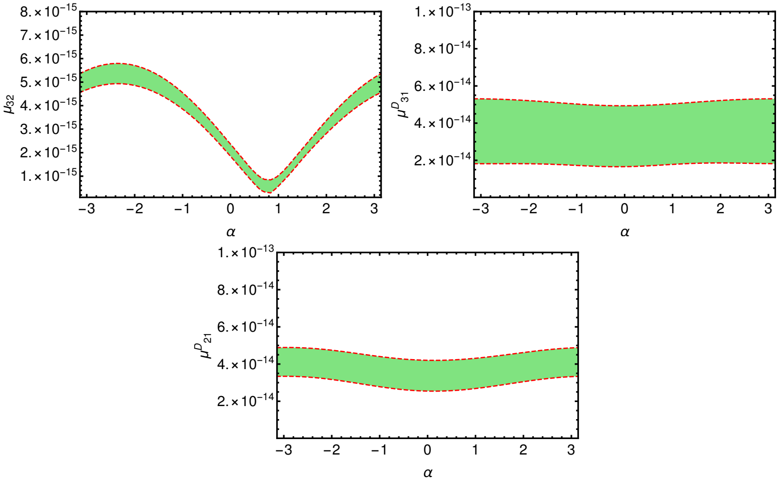
Dirac Magnetic Moment $V = U^*$



Majorana Magnetic Moment $V = U^T$



Dirac Magnetic Moment $V = U^T$



Summary

- If neutrinos have new interactions beyond the SM, then their Dirac or Majorana nature could have observable differences that are not suppressed by neutrino masses.
- We have performed an exhaustive study on all possible criteria that could be used to distinguish between Dirac and Majorana neutrinos in this context.
- The possible criteria includes variou choices of left-handed (U) and right-handed (V) neutrino mixing matrices within the Left-Right Symmetric Model (LRSM).
- In the case of right-handed neutrino mixing equal to the conjugate of the PMNS matrix in the left-handed neutrino sector, this leads to the disappearance of the contribution of Majorana magnetic moments while providing a sizeable contribution to Dirac magnetic moments.

THANK YOU