RGEs & positivity bounds of the SMEFT dim-8 operators

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In collaboration with M. Chala, Á. Díaz-Carmona, G. Guedes, arxiv:2205.03301

Dimension-8 operators & where do we find them?

❖ **Leading contribution to observables**

- **Neutral triple gauge couplings, anomalous quartic gauge couplings, angular obs. (Drell-Yan)**

arXiv:1308.6323, 2008.04298, 2002.03326, 2003.11615

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- **When EFT cut-off scale is not very high, dim-8 effects are important.**
  - **Higgs measurements** arXiv:1808.00442, 2205.01561, …
  - **Electroweak precision data** arXiv:2102.02819
  - **EFT validity (D6 vs D8 effects)** arXiv:1604.06444, 2003.07862

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- **Dim-8 RGE effects and positivity bounds (restrictions on dim-8 WCs).**

  arXiv:**2205.03301**, 2106.05291, 2110.01624, 1908.09845

SMEFT Dim-8 RGEs

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^4}\sum_j c_j^{(8)} O_j^{(8)} + \cdots$$

$\mathcal{L}_5, \mathcal{L}_7 \to \text{B/LNV}$

$\Lambda = \text{EFT cut-off scale}$

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \gamma_{ij} c_j^{(8)} + \gamma'_{ijk} c_j^{(6)} c_k^{(6)}$$

At $\mathcal{O}\left(\frac{1}{\Lambda^4}\right)$, assuming no B/LNV.

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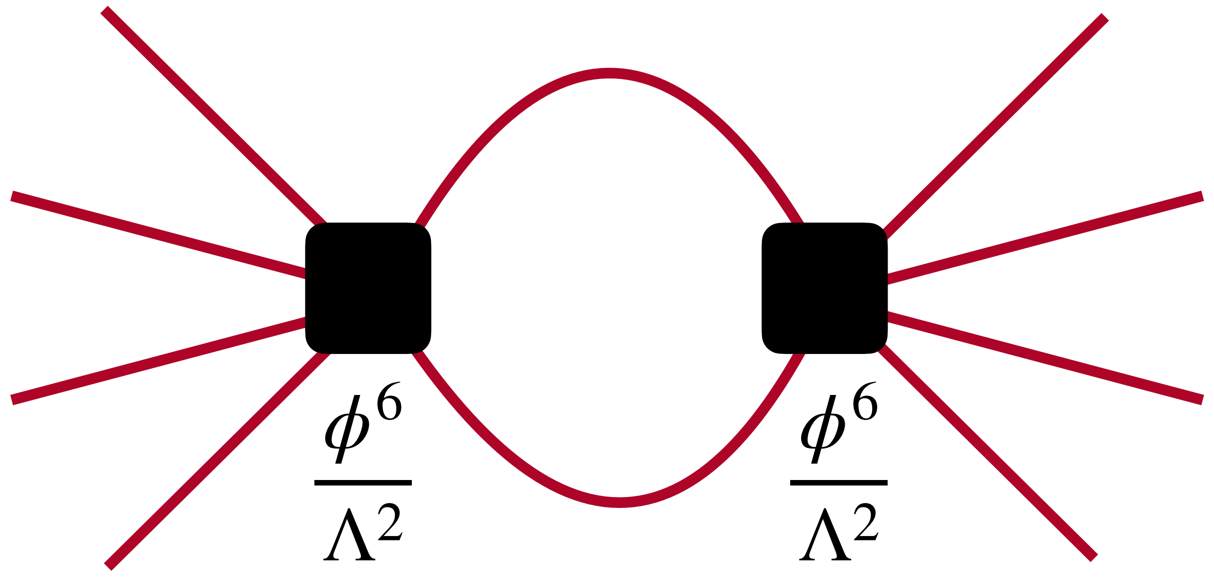
- Two dim-6 operator insertions.

arXiv:2106.05291

Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions I
- M Chala, G Guedes, M Ramos, J Santiago

e.g. :



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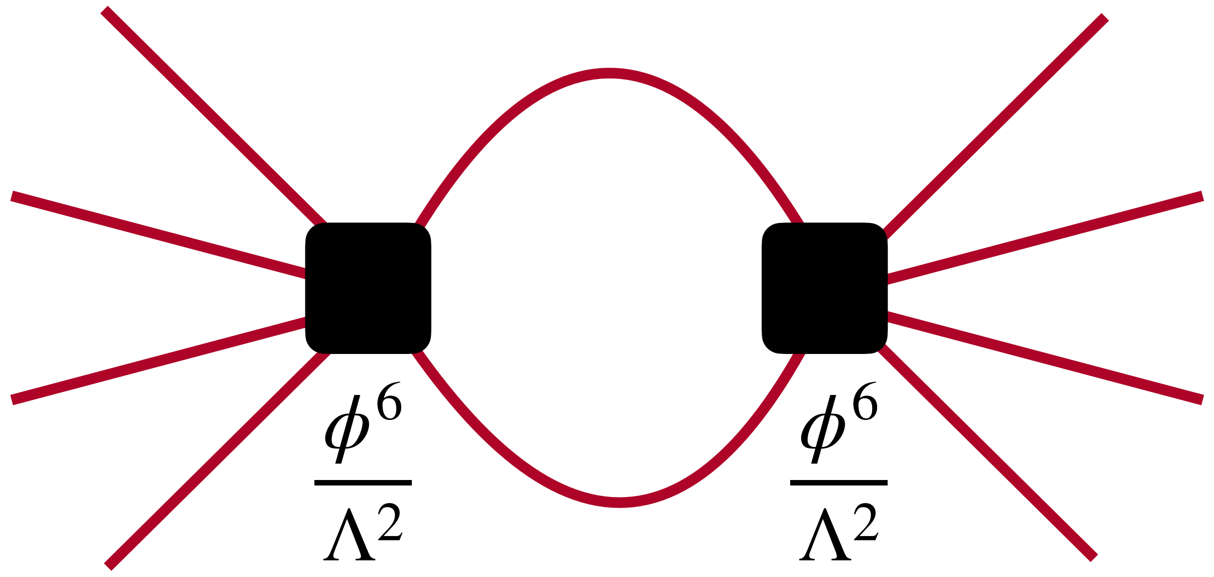

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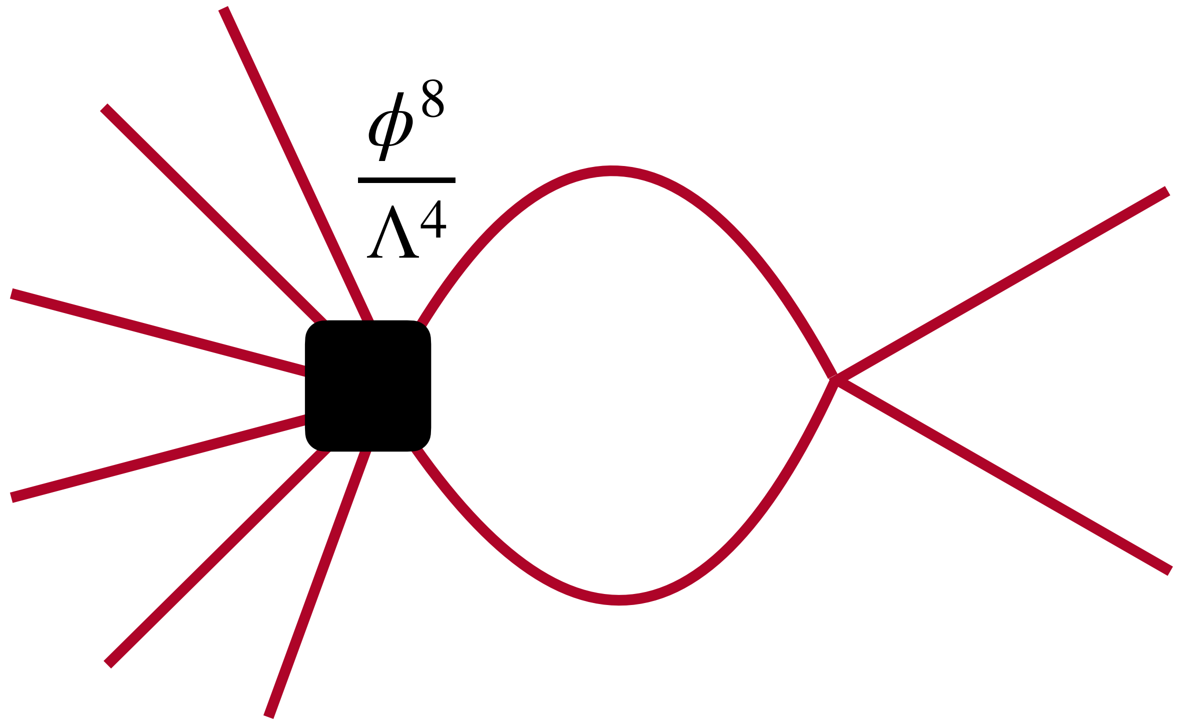

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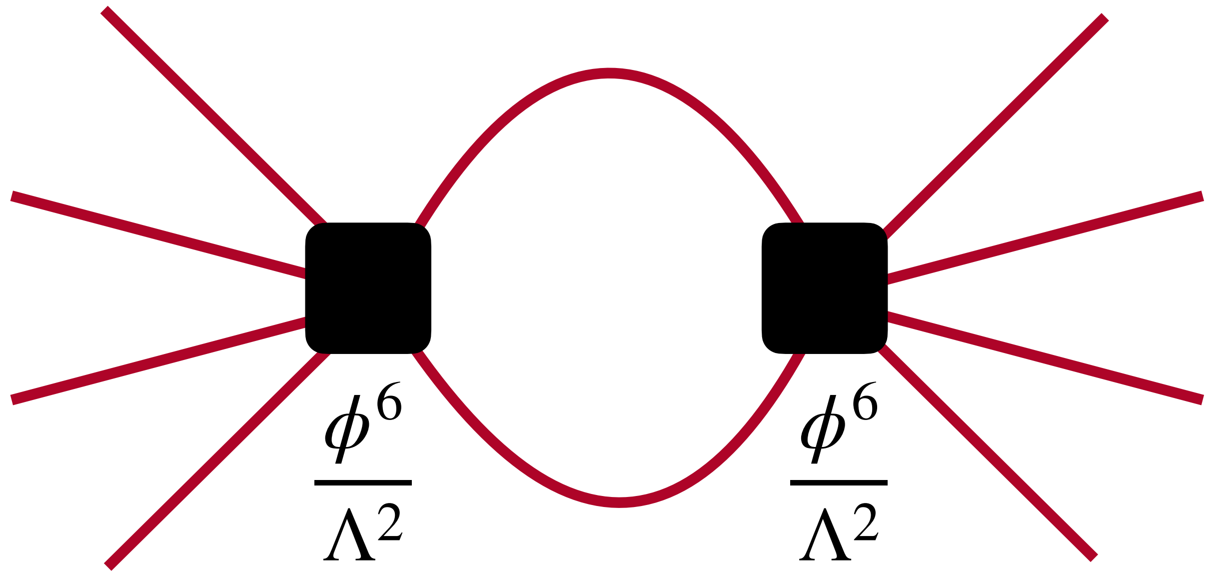

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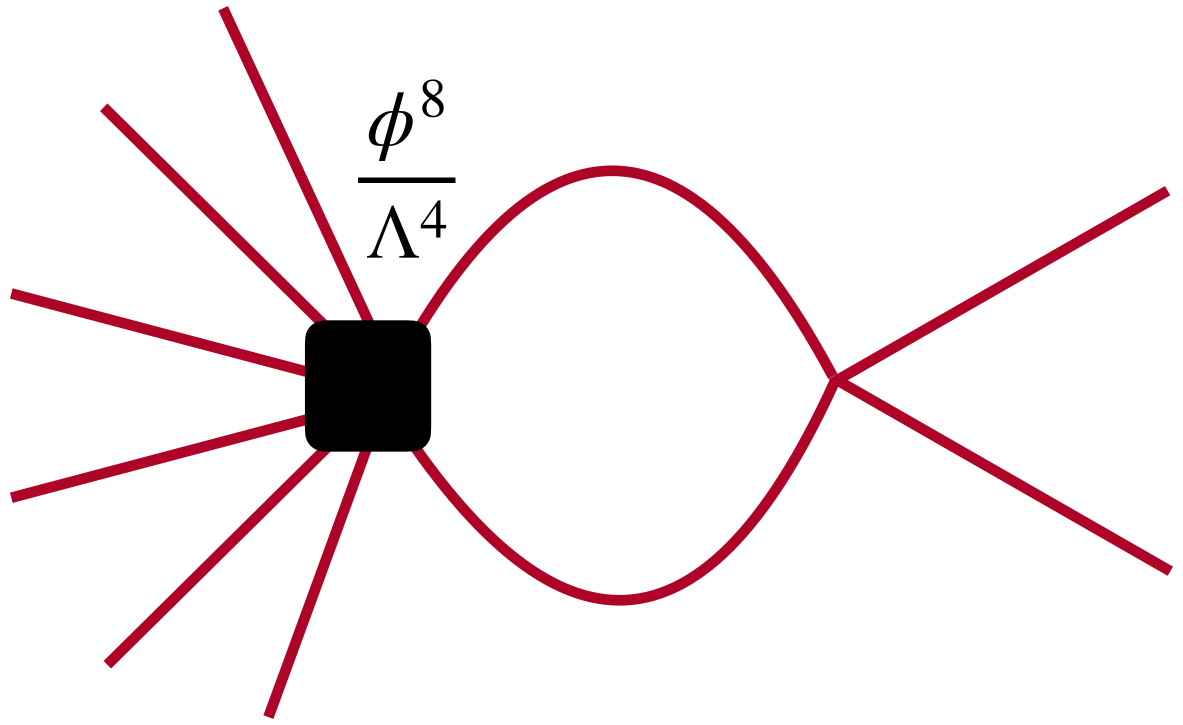

SMEFT Lagrangian

Bosonic operators' RGE: $$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \boxed{\gamma_{ij} c_j^{(8)}} + \gamma'_{ijk} c_j^{(6)} c_k^{(6)}$$

Classes of operator that are **tree-level** generated in weakly coupled UV theories:

arXiv:2001.0001 — Craig, Jiang, Li, Sutherland

Bosonic : $\{\phi^8, \phi^6 D^2, \phi^4 D^4, X^2 \phi^4, X \phi^4 D^2, X^2 H^2 D^2, X^3 H^2, X^4\}$

Fermionic : $\{\psi^2 X \phi^3, \psi^2 \phi^2 D^3, \psi^2 \phi^5, \psi^2 \phi^4 D, \psi^2 X \phi^2 D, \psi^2 \phi^3 D^2, \psi^2 X^2 \phi, \psi^2 X^2 D, \psi^2 X \phi D^2\}$

SMEFT Dim-8 **on-shell basis** : arXiv:2005.00059 — C. W. Murphy

SMEFT Dim-8 **Green's/off-shell basis** : arXiv:2112.12724 — M. Chala, Á. Díaz-Carmona, G. Guedes

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Divergences to RGEs, some details:

- Compute **1-PI loop diagrams**. Use **FeynRules, FeynARTs,** and **FormCalc** packages.

- Divergences are captured by the operators of **off-shell/Green's basis**. arXiv:2112.12724

$$16\pi^2\epsilon\,\mathcal{L}_{\rm DIV} = \tilde{K}_\phi (D_\mu\phi)^\dagger(D^\mu\phi) + \tilde{\mu}^2|\phi|^2 - \tilde{\lambda}|\phi|^4 + \tilde{c}_i^{(6)}\frac{\mathcal{O}_i^{(6)}}{\Lambda^2} + \tilde{c}_j^{(8)}\frac{\mathcal{O}_j^{(8)}}{\Lambda^4}$$

[on RHS we have Green's basis]

- **Removing redundant operators** using on-shell relations. arXiv:2106.05291

- **Cross-checks with MatchMakerEFT**. ✓
  H^8 topologies are computed in MM primarily.

  arXiv:2112.10787
  - A Carmona, A Lazopoulos, P Olgoso, J Santiago

- **Cross-checks** with arXiv:2108.03669 (on-shell amplitude methods). ✓

  arXiv:2108.03669
  - M A Huber, S De Angelis.

Bosonic-bosonic RGE:

Mixing induced by

	ϕ^4D^4	$B\phi^4D^2$	$W\phi^4D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	ϕ^6D^2	ϕ^8
$B^2\phi^2D^2$	g_1^2	0	0	0	0	0	0	0	0
$W^2\phi^2D^2$	g_2^2	0	0	0	0	0	0	0	0
$WB\phi^2D^2$	g_1g_2	0	0	0	0	0	0	0	0
$G^2\phi^2D^2$	0	0	0	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0
$W^2B\phi^2$	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0
ϕ^4D^4	g_2^2	0	0	0	0	0	0	0	0
$B\phi^4D^2$	$g_1g_2^2$	λ	0	0	0	0	0	0	0
$W\phi^4D^2$	g_2^3	0	g_2^2	0	0	0	0	0	0
$B^2\phi^4$	$g_1^2g_2^2$	$g_1\lambda$	$g_1^2g_2$	λ	0	g_1g_2	0	0	0
$W^2\phi^4$	g_2^4	$g_1g_2^2$	g_2^3	0	λ	g_1g_2	0	0	0
$WB\phi^4$	$g_1g_2^3$	$g_2\lambda$	$g_1\lambda$	g_1g_2	g_1g_2	λ	0	0	0
$G^2\phi^4$	0	0	0	0	0	0	g_3^2	0	0
ϕ^6D^2	g_2^4	$g_1\lambda$	$g_2\lambda$	0	0	0	0	λ	0
ϕ^8	λ^3	$g_1\lambda^2$	$g_2\lambda^2$	$g_1^2\lambda$	$g_2^2\lambda$	$g_1g_2\lambda$	0	λ^2	λ

- Largest contribution from each operator class is shown. Zeroes are cross-checked and consistent.
- Loop generated operators that are renormalised by tree-generated operators are grey. (unlike renorm. of dim-6 by dim-6).
- Blue entries contribute larger than naive dimensional analysis expectations.

$$\tilde{\mu}\frac{dc_{\phi^8}}{d\tilde{\mu}} = \frac{1}{16\pi^2}\left(192\lambda - 6(g_1^2+3g_2^2)+\dots\right)c_{\phi^8}$$

Fermionic-bosonic RGE:

Mixing induced by two-fermion operators

	$\psi^2 B\phi^3$	$\psi^2 W\phi^3$	$\psi^2 G\phi^3$	$\psi^2\phi^2 D^3$	$\psi^2\phi^5$	$\psi^2\phi^4 D$	$\psi^2 B\phi^2 D$	$\psi^2 W\phi^2 D$	$\psi^2 G\phi^2 D$	$\psi^2\phi^3 D^2$
$B^2\phi^2 D^2$	0	0	0	g_1^2	0	0	0	0	0	0
$W^2\phi^2 D^2$	0	0	0	g_2^2	0	0	0	0	0	0
$WB\phi^2 D^2$	0	0	0	$g_1 g_2$	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	g_3^2	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	0	0	0	$\|y^t\|^2$	0	0	0	0	0	0
$B\phi^4 D^2$	0	0	0	$g_1\|y^t\|^2$	0	0	$\|y^t\|^2$	0	0	$g_1 y^t$
$W\phi^4 D^2$	0	0	0	$g_2\|y^t\|^2$	0	0	0	$\|y^t\|^2$	0	$g_2 y^t$
$B^2\phi^4$	$g_1 y^t$	0	0	$g_1^2\|y^t\|^2$	0	0	$g_1\|y^t\|^2$	0	0	$g_1^2 y^t$
$W^2\phi^4$	0	$g_2 y^t$	0	$g_2^2\|y^t\|^2$	0	g_2^2	0	$g_2\|y^t\|^2$	0	$g_2^2 y^t$
$WB\phi^4$	$g_2 y^t$	$g_1 y^t$	0	$g_1 g_2\|y^t\|^2$	0	$g_1 g_2$	$g_2\|y^t\|^2$	$g_1\|y^t\|^2$	0	$g_1 g_2 y^t$
$G^2\phi^4$	0	0	$g_3 y^t$	0	0	0	0	0	0	0
$\phi^6 D^2$	0	0	0	$g_2^2\|y^t\|^2$	0	$\|y^t\|^2$	$g_1\|y^t\|^2$	$g_2\|y^t\|^2$	0	$y^t\|y^t\|^2$
ϕ^8	0	0	0	$\lambda\|y^t\|^4$	$y^t\|y^t\|^2$	$\lambda\|y^t\|^2$	$g_1\lambda\|y^t\|^2$	$g_2\lambda\|y^t\|^2$	0	$\lambda y^t\|y^t\|^2$

RGEs of Dim-6,4,2

- Dim-8 operators also induce running of dim-6, dim-4, dim-2 operators.

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
ϕ^2	μ^6	0	0	0	0	0	0	0	0
ϕ^4	$\lambda\mu^4$	$g_1\mu^4$	$g_2\mu^4$	0	0	0	0	μ^4	0
$B^2\phi^2$	$g_1^2\mu^2$	$g_1\mu^2$	0	μ^2	0	0	0	0	0
$W^2\phi^2$	$g_2^2\mu^2$	0	$g_2\mu^2$	0	μ^2	0	0	0	0
$WB\phi^2$	$g_1 g_2\mu^2$	$g_2\mu^2$	$g_1\mu^2$	0	0	μ^2	0	0	0
$G^2\phi^2$	0	0	0	0	0	0	μ^2	0	0
$\phi^4 D^2$	$\lambda\mu^2$	$g_1\mu^2$	$g_2\mu^2$	0	0	0	0	μ^2	0
ϕ^6	$\lambda^2\mu^2$	$\lambda g_1\mu^2$	$\lambda g_2\mu^2$	$g_1^2\mu^2$	$g_2^2\mu^2$	$g_1 g_2\mu^2$	0	$\lambda\mu^2$	μ^2

μ^2 is the squared Higgs mass in the SMEFT.

Lower dim. classes renormalised by bosonic dim-8 operators.
Similar contributions from two-fermion dim-8 operators are computed.

Positivity bounds

arXiv:1908.09845, 2110.01624

- Restrictions on Wilson coefficients of dim-8 operators.

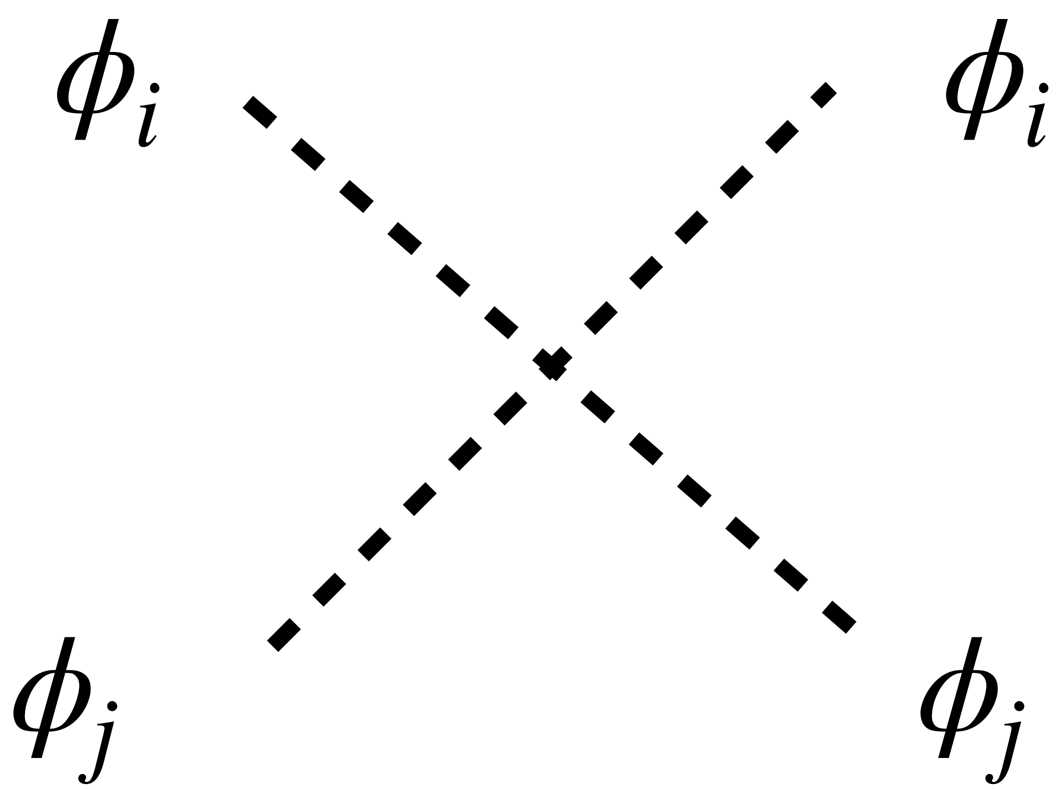
$$\frac{d^2\mathcal{M}(s,t=0)}{ds^2} \geq 0$$

Unitarity, analyticity, crossing symmetry

$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

Tree-level scattering :

$\mathbf{2 \to 2}$



$$c_{\phi^4 D^4}^{(2)} \geq 0$$

$$c_{\phi^4 D^4}^{(1)} + c_{\phi^4 D^4}^{(2)} \geq 0$$

$$c_{\phi^4 D^4}^{(1)} + c_{\phi^4 D^4}^{(2)} + c_{\phi^4 D^4}^{(3)} \geq 0$$

$$\mathcal{M}(s)_{1,2\to 1,2} = -2\lambda + \frac{c_{H^4}^{(2)}}{\Lambda^4} s^2$$

Dim-8 RGEs effects on positivity

$Q^{(1)}_{W^2H^2D^2}$	$(D^\mu H^\dagger D^\nu H) W^I_{\mu\rho} W^{I\rho}_\nu$
$Q^{(2)}_{W^2H^2D^2}$	$(D^\mu H^\dagger D_\mu H) W^I_{\nu\rho} W^{I\nu\rho}$
$Q^{(3)}_{W^2H^2D^2}$	$(D^\mu H^\dagger D_\mu H) W^I_{\nu\rho} \widetilde{W}^{I\nu\rho}$
$Q^{(4)}_{W^2H^2D^2}$	$i\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H) W^J_{\mu\rho} W^{K\rho}_\nu$

- **For $V_1V_2 \to V_1V_2$ process:**

$$g_1^2 c^{(1)}_{B^2\phi^2D^2} + g_2^2 c^{(1)}_{W^2\phi^2D^2} + 2g_1g_2 c^{(4)}_{WB\phi^2D^2} \leq 0\,,$$
$$g_1^2 c^{(1)}_{B^2\phi^2D^2} + g_2^2 c^{(1)}_{W^2\phi^2D^2} - 2g_1g_2 c^{(4)}_{WB\phi^2D^2} \leq 0\,,$$
$$c^{(1)}_{W^2\phi^2D^2} \leq 0\,,$$
$$g_1^2 c^{(1)}_{W^2\phi^2D^2} + 2g_1g_2 c^{(4)}_{WB\phi^2D^2} + g_2^2 c^{(1)}_{B^2\phi^2D^2} \leq 0\,,$$
$$g_1^2 c^{(1)}_{W^2\phi^2D^2} - 2g_1g_2 c^{(4)}_{WB\phi^2D^2} + g_2^2 c^{(1)}_{B^2\phi^2D^2} \leq 0\,.$$

arXiv:1902.08977

$X^2\phi^2D^2$ operators are not generated at tree-level matching of weakly coupled UV completion of the SMEFT.

Dim-8 RGEs effects on positivity

- RGE of $X^2\phi^2D^2$ operators:

$$c^{(1)}_{W^2\phi^2D^2}(\tilde{\mu}) = c^{(1)}_{W^2\phi^2D^2}(\Lambda) - \frac{1}{16\pi^2}\dot{c}^{(1)}_{W^2\phi^2D^2}(\Lambda)\log\frac{\Lambda}{\tilde{\mu}} < 0$$

$$\Rightarrow \frac{1}{6}g_2^2\Big[2c^{(1)}_{\phi^4} + 3c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4D^4} - \frac{16}{3}\left(c^{(1)}_{l^2\phi^2D^3} + c^{(2)}_{l^2\phi^2D^3} + 3c^{(1)}_{q^2\phi^2D^3} + 3c^{(2)}_{q^2\phi^2D^3}\right)_{\alpha_1,\alpha_1}\Big]\log\frac{\Lambda}{\tilde{\mu}} > 0\,,$$

Sufficient conditions :

- Putting the RGEs of the operators.
- Derive relations among operators of same class.
- Wilson coefficients generated from UV theories matched to SMEFT Dim-8 at tree-level are bounded by these positivity (or negativity) constraints.

$$
\begin{aligned}
2c^{(1)}_{\phi^4} + 3c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4} &\geq 0\,,\\
c^{(1)}_{\phi^4} + 2c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4} &\geq 0\,,\\
c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} &\geq 0\,,\\
\left[c^{(1)}_{\psi_R^2\phi^2 D^3} + c^{(2)}_{\psi_R^2\phi^2 D^3}\right]_{\alpha_1,\alpha_1} &\leq 0\,,\\
\left[c^{(1)}_{\psi_L^2\phi^2 D^3} + c^{(2)}_{\psi_L^2\phi^2 D^3} + c^{(3)}_{\psi_L^2\phi^2 D^3} + c^{(4)}_{\psi_L^2\phi^2 D^3}\right]_{\alpha_1,\alpha_1} &\leq 0\,,\\
\left[c^{(1)}_{\psi_L^2\phi^2 D^3} + c^{(2)}_{\psi_L^2\phi^2 D^3} - c^{(3)}_{\psi_L^2\phi^2 D^3} - c^{(4)}_{\psi_L^2\phi^2 D^3}\right]_{\alpha_1,\alpha_1} &\leq 0\,;
\end{aligned}
$$

Summary

- **Renormalization of bosonic SMEFT operators by dim-8 tree-level generated operators are discussed.**

  - **Tree-generated ops. mix with loop-generated ops.**

  - **Mixing induced terms larger than naive dimensional analysis.**

  - **Dim-8 ops. induced running of lower dimensional ops. are computed (loop-generated dim-6 ops. have non-zero mixing).**

  - **Positivity bounds on $X^2\phi^2D^2$ hold at sufficiently small scales at one-loop accuracy.**

Upcoming...

— SDB, A. Díaz-Carmona

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			$\color{green}\checkmark$						$\color{green}\checkmark$		This work
$d_{\leq 4}$ (fermionic)			$\color{green}\checkmark$						$\color{red}\text{X}$		$\color{red}\text{X}$
d_5	$\color{green}\checkmark$				$\color{green}\checkmark$	$\color{orange}\checkmark$					
d_6 (bosonic)		$\color{green}\checkmark$	$\color{green}\checkmark$					$\color{red}\text{X}$	$\color{green}\checkmark$	$\color{red}\text{X}$	This work
d_6 (fermionic)		$\color{green}\checkmark$	$\color{green}\checkmark$					$\color{red}\text{X}$	$\color{red}\text{X}$	$\color{red}\text{X}$	$\color{red}\text{X}$
d_7				$\color{orange}\checkmark$	$\color{orange}\checkmark$	$\color{green}\checkmark$					
d_8 (bosonic)							$\boxed{\color{red}\text{X}}$	$\boxed{\color{red}\text{X}}$	$\color{green}\checkmark$	$\boxed{\color{red}\text{X}}$	This work
d_8 (fermionic)							$\color{red}\text{X}$	$\color{red}\text{X}$	$\color{red}\text{X}$	$\color{red}\text{X}$	$\color{orange}\checkmark$

Blank entries vanish; a tick $\color{green}\checkmark$ represents that the complete contribution is known; the $\color{orange}\checkmark$ implies that only (but substantial) partial results have been already obtained; the $\color{red}\text{X}$ indicates that nothing, or very little, is known. The contribution made in this paper is marked by ▬.

Thanks for your attention !

RGEs of Dim-6,4,2

- Fermionic dim-8 operators also induce running of dim-6, dim-4, dim-2 operators.

	$\psi^2 B\phi^3$	$\psi^2 W\phi^3$	$\psi^2 G\phi^3$	$\psi^2\phi^2 D^3$	$\psi^2\phi^5$	$\psi^2\phi^4 D$	$\psi^2 B\phi^2 D$	$\psi^2 W\phi^2 D$	$\psi^2 G\phi^2 D$	$\psi^2\phi^3 D^2$
ϕ^2	0	0	0	0	0	0	0	0	0	0
ϕ^4	0	0	0	$\mu^4\|y^t\|^2$	0	0	0	0	0	$\mu^4 y^t$
$B^2\phi^2$	0	0	0	0	0	0	0	0	0	0
$W^2\phi^2$	0	0	0	0	0	0	0	0	0	0
$WB\phi^2$	0	0	0	0	0	0	0	0	0	0
$G^2\phi^2$	0	0	0	0	0	0	0	0	0	0
$\phi^4 D^2$	0	0	0	$\mu^2\|y^t\|^2$	0	0	0	0	0	$\mu^2 y^t$
ϕ^6	0	0	0	$\lambda\mu^2\|y^t\|^2$	$\mu^2 y^t$	$\mu^2\|y^t\|^2$	$\mu^2\|y^t\|^2$	$\mu^2\|y^t\|^2$	0	$\mu^2 y^t\|y^t\|^2$

Lower dim. classes renormalised by the fermionic dim-8 operators.