Tensor renormalization group analysis of two-dimensional generalized $O(2)$ model

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IN PURSUIT OF KNOWLEDGE

Outline

1. Tensor Networks
2. Tensor Network formulation
3. 2D generalized O(2) Model
4. HOTRG Method

OUTLINE

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Lagrangian approach

The systems that can be studied using this approach are

CLASSICAL MANY-BODY SYSTEM

PATH INTEGRAL REPRESENTATION OF QUANTUM SYSTEM

Lagrangian approach

We try to study the

PARTITION FUNCTION

$$Z = \int [d\phi] \exp[-S(\{\phi\})]$$

COARSE GRAINING METHODS

TRG,SRG,HOTRG,TNR,HOSRG,...

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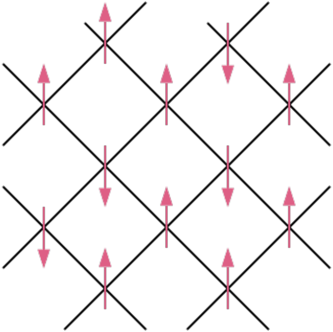
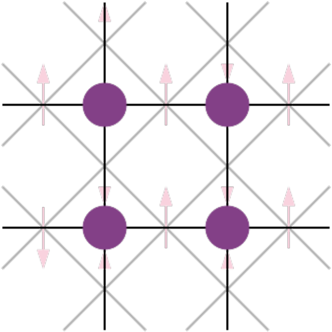
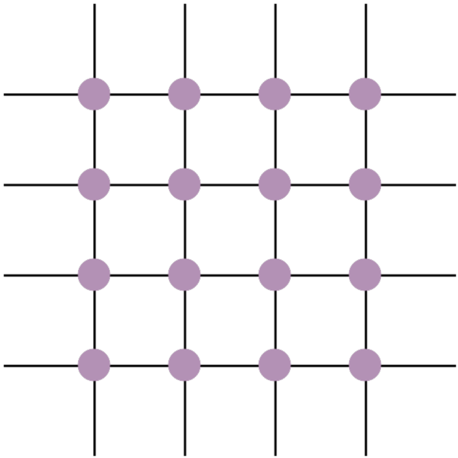
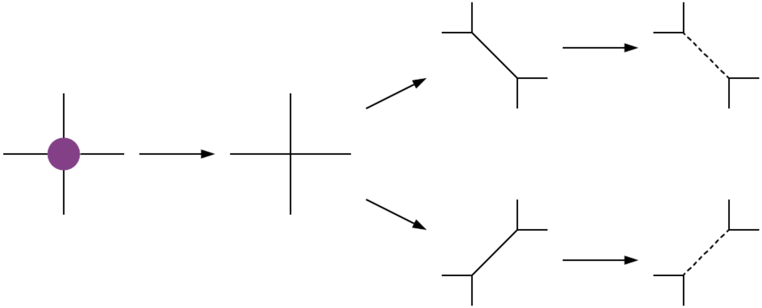
2D Spin Model

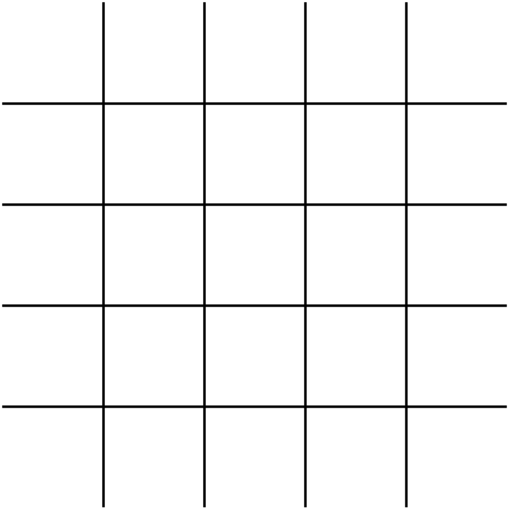


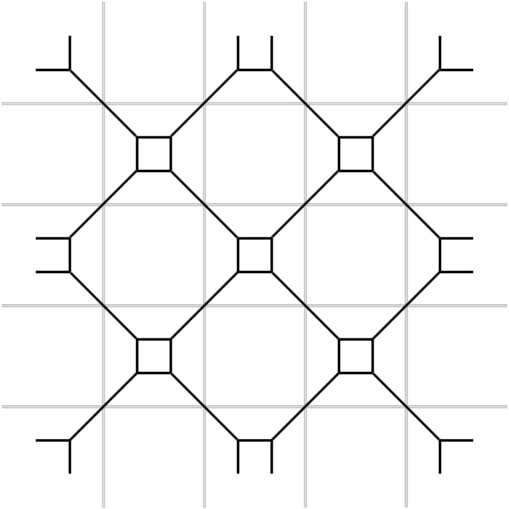
2D Spin Model

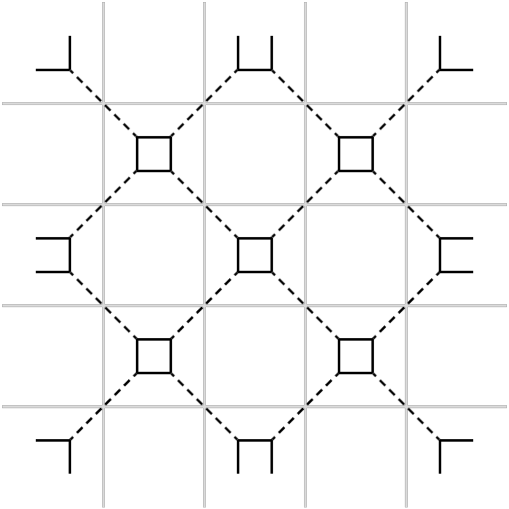


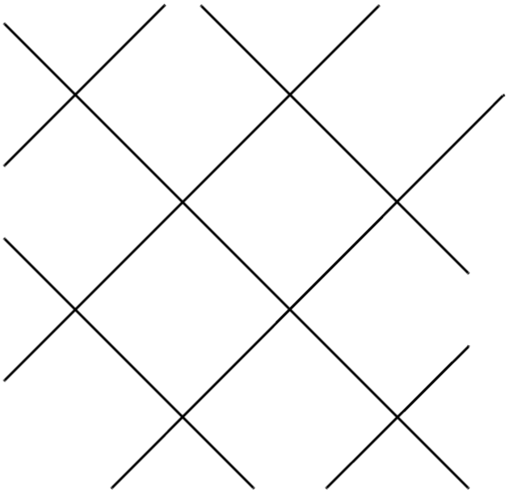


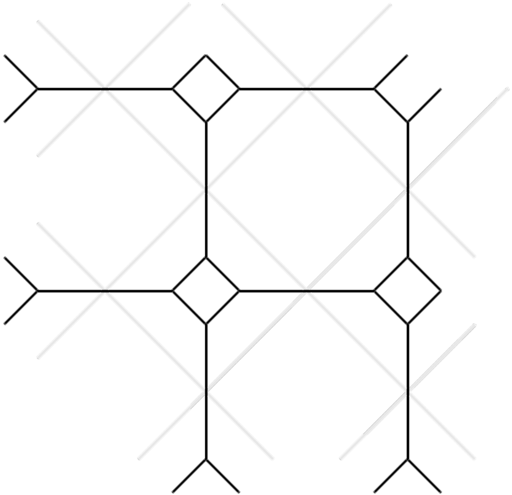


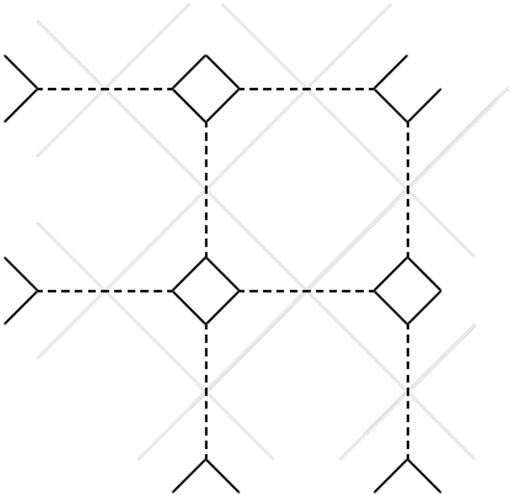


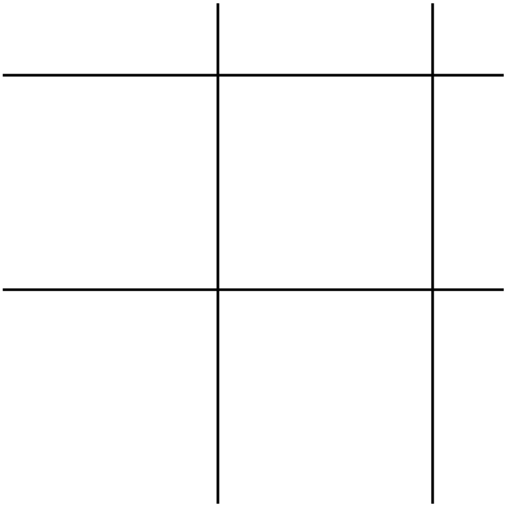


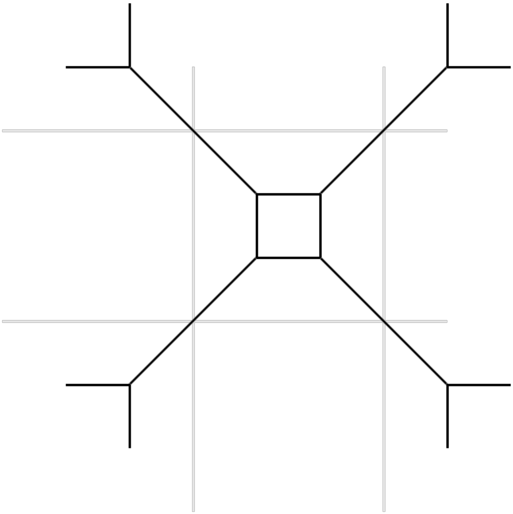


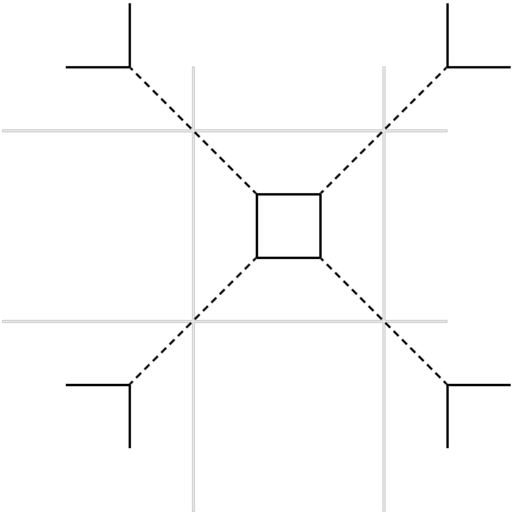


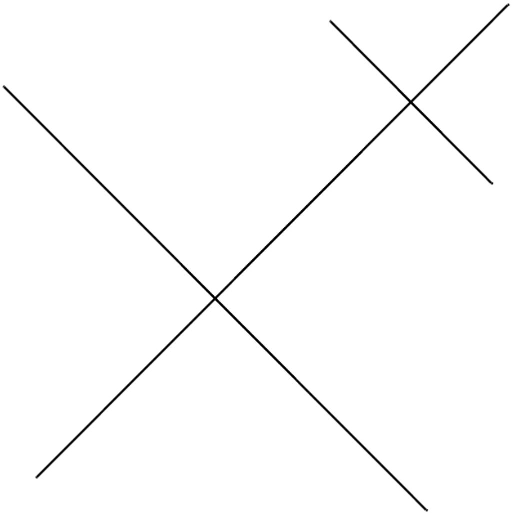


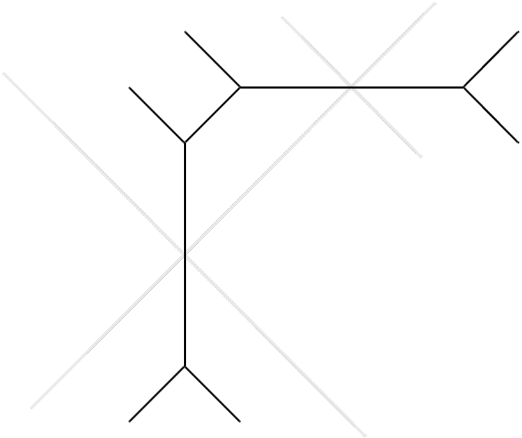


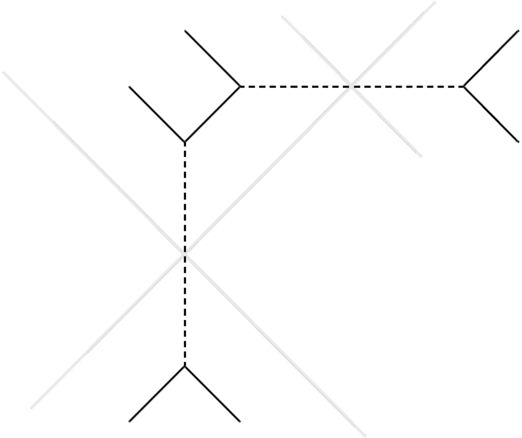


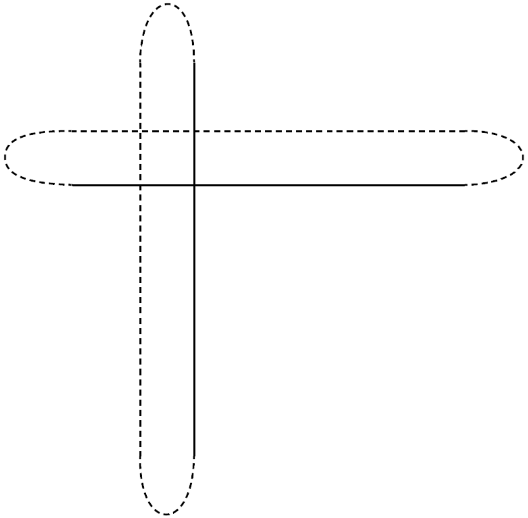


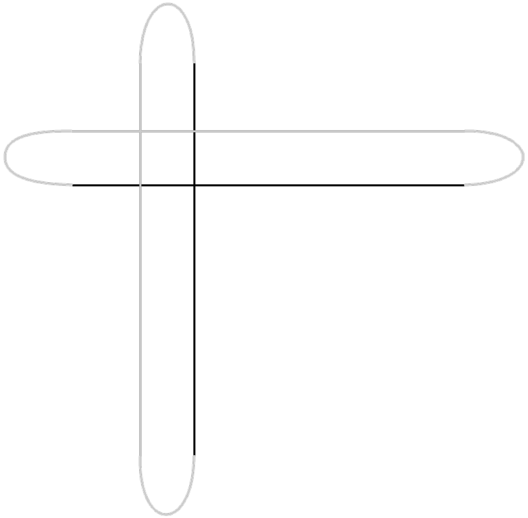












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2D MODEL

The modified XY spin model on a two-dimensional square lattice

$$\mathrm{H} = -\mathrm{J}\sum_{<\mathrm{ij}>}\cos(\theta_\mathrm{i}-\theta_\mathrm{j}) - \mathrm{J}_1\sum_{<\mathrm{ij}>}\cos(\mathrm{q}(\theta_\mathrm{i}-\theta_\mathrm{j})) - \mathrm{h}\sum_\mathrm{i}\cos\theta_\mathrm{i},$$

$$\theta_\mathrm{i} \in [0, 2\pi] \quad \& \quad \mathrm{J}, \mathrm{J}_1 > 0$$

$$\mathrm{For}\quad \mathrm{q}=2,\quad \mathrm{J}=\Delta\quad \mathrm{and}\quad \mathrm{J}_1=1-\Delta,\qquad \mathrm{with}\quad 0\leq\Delta\leq 1$$

$$\mathrm{H}=-\sum_{<\mathrm{ij}>}[\Delta\cos(\theta_\mathrm{i}-\theta_\mathrm{j})+(1-\Delta)\cos(2\theta_\mathrm{i}-2\theta_\mathrm{j})]-\mathrm{h}\sum_\mathrm{i}\cos\theta_\mathrm{i},$$

$$\Delta = 1$$

$$\mathrm{H} = -\sum_{<\mathrm{ij}>} [\cos(\theta_\mathrm{i} - \theta_\mathrm{j})]$$

usual XY model

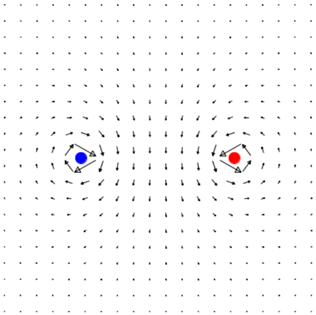
$$\Delta = 0$$

$$\mathrm{H} = -\sum_{<\mathrm{ij}>} [\cos(2\theta_\mathrm{i} - 2\theta_\mathrm{j})]$$

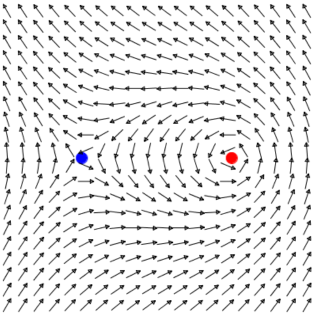
purely spin-nematic model

For $\Delta = 0$ - purely spin-nematic term

integer vortex-antivortex pair



half-integer vortex-antivortex pair



Our goal is to look into the physics for the region

$$0 \leq \Delta \leq 1$$

Our goal is to look into the physics for the region

$$0 \leq \Delta \leq 1$$

and determine the multi-critical point accurately

$$\Delta_{mc}.$$

TENSOR NETWORK CONSTRUCTION

$$Z = \prod_{i} \int \frac{d\theta_i}{2\pi} \prod_{<ij>} e^{\beta[\Delta \cos(\theta_i - \theta_j) + (1-\Delta)\cos(2(\theta_i - \theta_j))]} \prod_{i} e^{\beta[h \cos(\theta_i)]}$$

$$Z = \prod_{s} \int \frac{d\theta_s}{2\pi} \prod_{l \in L} \sum_{n_l} a_{n_l}(\beta, \Delta) e^{\iota n_l(\theta_{s_i} - \theta_{s_j})} \sum_{p_l} I_{p_l/2}(\beta h) e^{\iota p_l \theta_i}$$

Using character expansion for

$$e^{x\cos\theta} = \sum_{n=-\infty}^{\infty} I_n(x) e^{\iota n\theta},$$

where $I_n(x)$ are the modified Bessel functions of the first kind.

$$Z = \prod_{s} \int \frac{d\theta_s}{2\pi} \prod_{l \in L} \sum_{n_l} a_{n_l}(\beta, \Delta) e^{\iota n_l (\theta_{s_i} - \theta_{s_j})} \sum_{p_l} I_{p_l/2}(\beta h) e^{\iota p_l \theta_i},$$

$$a_n(\beta, \Delta) = \sum_{m=-\infty}^{\infty} I_{n-2m}(\beta \Delta) I_m(\beta(1-\Delta))$$

Integrating over the θ degrees of freedom,

$$\mathrm{Z} = \mathrm{tTr}\left(\prod \mathrm{T_{ijkl}}\right),$$

where

$$\mathrm{T_{ijkl}} = \mathrm{T_{i,j,k,l}} = \sqrt{\mathrm{a_{n_i}}(\beta,\Delta)\mathrm{a_{n_j}}(\beta,\Delta)\mathrm{a_{n_k}}(\beta,\Delta)\mathrm{a_{n_l}}(\beta,\Delta)}\ \mathrm{I_{i+k-j-l}}(\beta \mathrm{h})$$

Observable

There are two main observales calculated

$$C_v = \frac{\partial^2 F}{\partial \beta^2},$$

and

$$M = -\frac{\partial F}{\partial h} = \frac{1}{\beta}\frac{\partial \ln Z}{\partial h}$$

where,

$$F = -\frac{\ln Z}{\beta}.$$

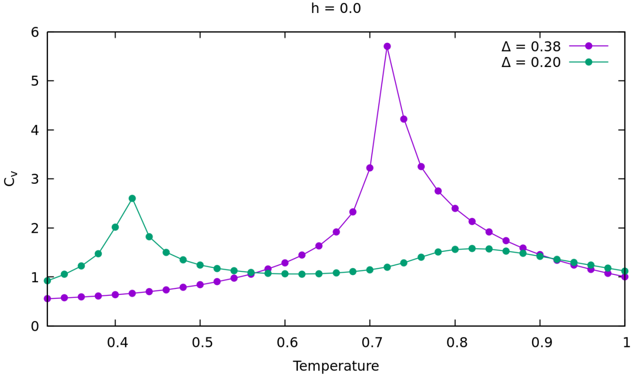
Results

We obtain two different types of transition

- the multi-critical point is obtained at $\Delta = 0.32$
- for $\Delta < 0.32$ we have two phase transition
- for $\Delta > 0.32$ we have BKT phase transition

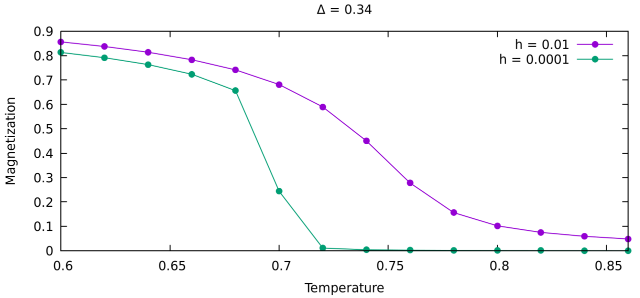
Results

System size $2^{30} \times 2^{30}$ with $\chi = 91$.



Results

$T_{h\to 0}$



Results

Δ	T_{Cv}	$T_{h\to 0}$
0.34	0.72	0.68
0.60	0.90	0.86

THANK YOU

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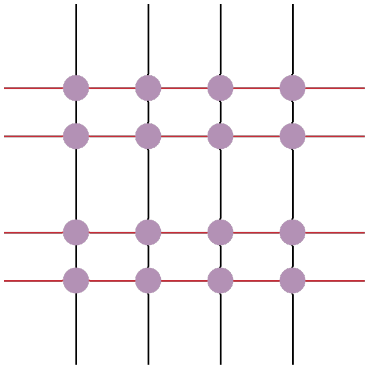
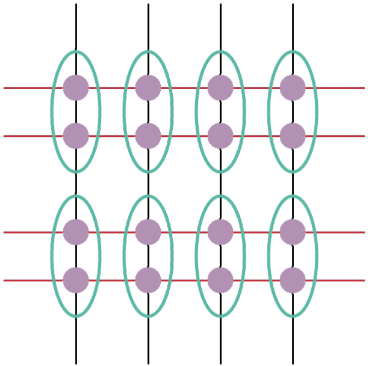
HOTRG



HOTRG



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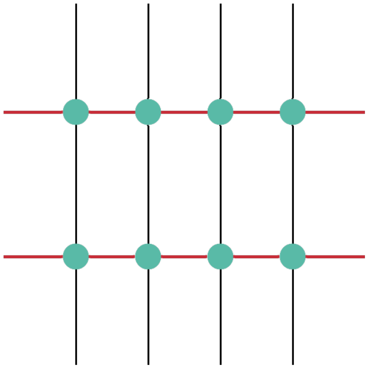
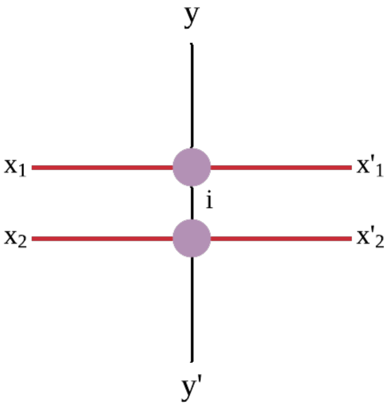


$$M_{xx'yy'} = \sum_i T_{x_1 x_1' y i} T_{x_2 x_2' i y}$$

where

$$x = x_1 \otimes x_2, x' = x_1' \otimes x_2'$$

HOTRG



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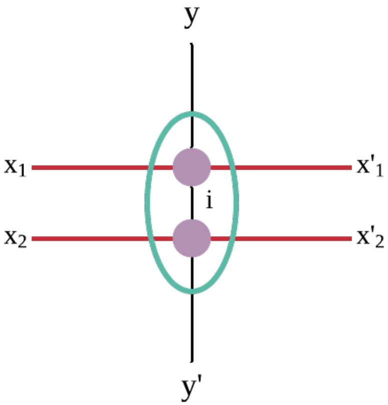

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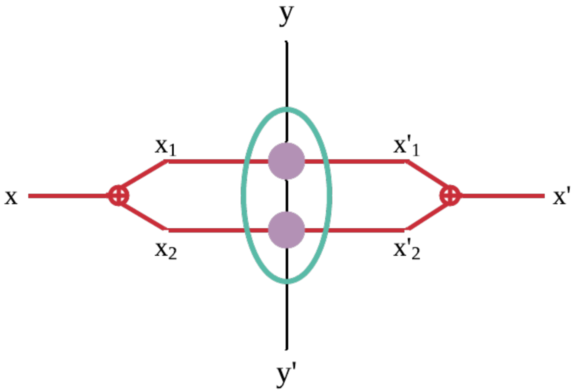

HOTRG