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Mechanical Properties and Gravitational Form Factors of a Dressed Quark in Light-Front Hamiltonian QCD

Sudeep Saha[1‡], Jai More[1⋆], Asmita Mukherjee[1†], Sreeraj Nair[2,3,4‡]

[1]Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India
[2]Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
[3]School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China
[4]CAS Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

‡sudeepsaha@iitb.ac.in, ⋆jai.more@iitb.ac.in, †asmita@phy.iitb.ac.in, ‡sreeraj@impcas.ac.cn

1. Introduction

A major interest in hadron physics and QCD is, understanding the mechanical properties like mass, angular momentum, and pressure distribution inside the nucleon in terms of quarks and gluons.

- These mechanical properties are encoded in gravitaional form factors (GFFs). They are functions of the square of the momentum transfer (q^2) in the process.
- GFFs are related to generalized parton distributions (GPDs), and can be accessed in exclusive electron-proton scattering process, e.g. deeply virtual compton scattering (DVCS).
- $A(q^2)$, $B(q^2)$, $C(q^2)$ and $\bar{C}(q^2)$ are four GFFs of a spin$-\frac{1}{2}$ system.

2. Gravitational Form Factors

- The electromagnetic interaction of a nucleon with an external EM field is described by $\langle p'|J^\mu|p\rangle A_\mu$,

$$\langle P'|J^0_q|P\rangle A_0 = 2e_q\, M\phi\,|_{(\text{Rest frame})}$$

- Interaction of a nucleon with the weak classical gravitational field is

$$\frac{1}{2}\sum_{q,G}\langle P'|\theta^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu} = 2M\, M\phi\,|_{(\text{Rest frame})}$$

- $\langle P'|\theta^{\mu\nu}_{q,G}|P\rangle$ is the current that couples to gravity.
- The standard parametrization for spin$-\frac{1}{2}$ system in QCD:

$$\langle P',S'|\theta^{\mu\nu}_i(0)|P,S\rangle = \overline{U}(P',S')\Big[-B_i(q^2)\frac{\overline{P}^\mu\,\overline{P}^\nu}{M} + \big(A_i(q^2)+B_i(q^2)\big)\frac{1}{2}(\gamma^\mu\overline{P}^\nu+\gamma^\nu\overline{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \overline{C}_i(q^2)M\, g^{\mu\nu}\Big]U(P,S),$$

where $\overline{P}^\mu = \frac{1}{2}(P'+P)^\mu$, $\overline{U}(P',S')$, $U(P,S)$ are the Dirac spinors and M is the mass of the target state, $i\equiv(Q,G)$.

- Sum Rules:
  $\sum_{(i=Q,G)}A_i(0)=1$, $\sum_{(i=Q,G)}B_i(0)=0$.
  Ji's sum rule: $A(x)+B(x)=\frac{1}{2}$, $B(0)=0$, $J(0)=\frac{1}{2}$.
  $\partial_\mu\theta^{\mu\nu}=0 \rightarrow \sum_{(i=Q,G)}\overline{C}_i(q^2)=0$.

3. D-term and Mechanical Properties

- The $C_i(q^2)$ form factor also known as D-term is unconstrained at zero momentum transfer.
- $D_i(q^2)=4C_i(q^2)$ is related to the pressure $p(\boldsymbol{b}^\perp)$ and shear $s(\boldsymbol{b}^\perp)$ distributions inside the nucleon as

$$p(\boldsymbol{b}^\perp) = \frac{1}{2M\boldsymbol{b}}\frac{d}{d\boldsymbol{b}^\perp}\left[\boldsymbol{b}^\perp\frac{d}{d\boldsymbol{b}^\perp}D_i(\boldsymbol{b}^\perp)\right] - M\,\overline{C}_i(\boldsymbol{b}^\perp),$$

$$s(\boldsymbol{b}^\perp) = -\frac{\boldsymbol{b}^\perp}{M}\frac{d}{d\boldsymbol{b}^\perp}\left[\frac{1}{\boldsymbol{b}^\perp}\frac{d}{d\boldsymbol{b}^\perp}D_i(\boldsymbol{b}^\perp)\right],$$

where

$$F(\boldsymbol{b}^\perp) = \frac{1}{(2\pi)^2}\int d^2\boldsymbol{q}^\perp\, e^{-iq^\perp b^\perp}\mathscr{F}(q^2) = \frac{1}{2\pi}\int_0^\infty d\boldsymbol{q}^{\perp 2}J_0\left(\boldsymbol{q}^\perp\boldsymbol{b}^\perp\right)\mathscr{F}(q^2),$$

where $\mathscr{F}=(A,B,C,\overline{C})$, J_0 : Bessel function of zeroth order, $\boldsymbol{b}^\perp$: impact parameter, M : mass of the dressed quark state.

- D-term has been extracted from the Jlab data and it is found to be negative.
- $p(\boldsymbol{b}^\perp)$ calculated from the data is found to be repulsive at the core and attractive towards the periphery.
- Theoretical models on $p(\boldsymbol{b}^\perp)$ and $s(\boldsymbol{b}^\perp)$ distributions: Bag model, chiral quark soliton model, AdS/QCD motivated quark-diquark model, multipole model. But these are phenomenological models and do not incorporate any gluonic degree of freedom. Lattice results are also there.
- Total quark + gluon EMT:

$$\theta^{ij}(x_\perp) = \left(\frac{x^i_\perp x^j_\perp}{x^2_\perp} - \frac{1}{3}\delta^{ij}\right)s(x^2_\perp) + \delta^{ij}p(x^2_\perp)$$

$s(x^2_\perp)$: Shear stress, $p(x^2_\perp)$: Pressure.

4. Dressed Quark Model (DQM)

- A simple relativistic spin-1/2 state, like a quark dressed with a gluon at one loop in QCD.
- This model employs a gluonic degree of freedom.
- The dressed quark state can be expanded in Fock space in terms of multiparton light-front wavefunctions (LFWFs), which can be calculated using the light-front Hamiltonian.
- LFWFs can be written in terms of relative momenta that are frame-independent. Thus, LFWFs are boost invariant.

$$|P,\lambda\rangle = \psi_1(P,\lambda)b^\dagger_\lambda(P)|0\rangle + \sum_{\lambda_1,\lambda_2}\int[k_1][k_2]\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2) \times \psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)b^\dagger_{\lambda_1}(k_1)a^\dagger_{\lambda_2}(k_2)|0\rangle,$$

where $[k] = \frac{dk^+d^2\boldsymbol{k}^\perp}{\sqrt{2(2\pi)^3k^+}}$, ψ_1 :Normalization.

Under Jacobi transformation,

$$k^+_i = x_iP^+,\ \boldsymbol{k}^\perp_i = \boldsymbol{\kappa}^\perp_i + x_i\boldsymbol{P}^\perp,\ x_1+x_2=1,\ \boldsymbol{\kappa}^\perp_1+\boldsymbol{\kappa}^\perp_2=0,$$

$$\phi^{\lambda a}_{\lambda_1,\lambda_2}(x_i,\boldsymbol{\kappa}^\perp_i) = \left[\frac{x(1-x)}{\boldsymbol{\kappa}^{\perp 2}+m^2(1-x)^2}\right]\frac{g}{\sqrt{2(2\pi)^3}} \times\frac{T^a}{\sqrt{1-x}}\chi^\dagger_{\lambda_1}\Big[\frac{-2(\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\varepsilon}^{\perp *}_{\lambda_2})}{1-x} - \frac{1}{x}(\tilde{\sigma}^\perp\cdot\boldsymbol{\kappa}^\perp)(\tilde{\sigma}^\perp\cdot\boldsymbol{\varepsilon}^{\perp *}_{\lambda_2}) + im(\tilde{\sigma}^\perp\cdot\boldsymbol{\varepsilon}^{\perp *}_{\lambda_2})\frac{1-x}{x}\Big]\chi_\lambda\psi^\lambda_1,$$

where $\phi^{\lambda a}_{\lambda_1,\lambda_2}(x_i,\kappa^\perp_i)=\sqrt{P^+}\psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)$, g : quark-gluon coupling, T^a : colour SU(3) matrices, $\varepsilon^\perp_{\lambda_2}$: polarization vector of gluon, m : quark mass, χ_λ : two-component spinor for the quark respectively, $\lambda=1,2$: helicity up/down, $\tilde{\sigma}_1=\sigma_2$, $\tilde{\sigma}_2=-\sigma_1$.

5. Matrix Elements of EMT & Extraction of GFFs

- Two component formulation of light-front QCD, with $A^+=0$.
  Drell-Yan frame:

$$P^\mu = (P^+,\boldsymbol{P}^\perp,P^-) = \left(P^+,\boldsymbol{0}^\perp,\frac{M^2}{P^+}\right),$$

$$P'^\mu = \left(P^+,\ \boldsymbol{q}^\perp,\frac{\boldsymbol{q}^{\perp 2}+M^2}{P^+}\right),$$

$$q^\mu = (P'-P)^\mu = \left(0,\boldsymbol{q}^\perp,\frac{\boldsymbol{q}^{\perp 2}}{P^+}\right).$$

- Fermionic part of QCD EMT:

$$\theta^{\mu\nu}_Q = \frac{1}{2}\overline{\psi}i\left[\gamma^\mu D^\nu + \gamma^\nu D^\mu\right]\psi.$$

6. Results

$$A_Q(q^2) = 1+\frac{g^2\,C_F}{2\pi^2}\left[\frac{11}{10}-\frac{4}{5}\left(1+\frac{2m^2}{q^2}\right)\frac{f_2}{f_1} - \frac{1}{3}\log\left(\frac{\Lambda^2}{m^2}\right)\right],$$

$$B_Q(q^2) = \frac{g^2C_F}{12\pi^2}\frac{m^2}{q^2}\frac{f_2}{f_1},$$

$$D_Q(q^2) = \frac{5g^2C_F}{6\pi^2}\frac{m^2}{q^2}\left(1-f_1f_2\right) = 4\ C_Q(q^2),$$

$$\overline{C}_Q(q^2) = \frac{g^2C_F}{72\pi^2}\left(29-30\ f_1\ f_2+3\ \log\left(\frac{\Lambda^2}{m^2}\right)\right),$$

where $f_1=\frac{1}{2}\sqrt{1+\frac{4m^2}{q^2}}$, $f_2=\log\left(1+\frac{q^2(1+2f_1)}{2m^2}\right)$.

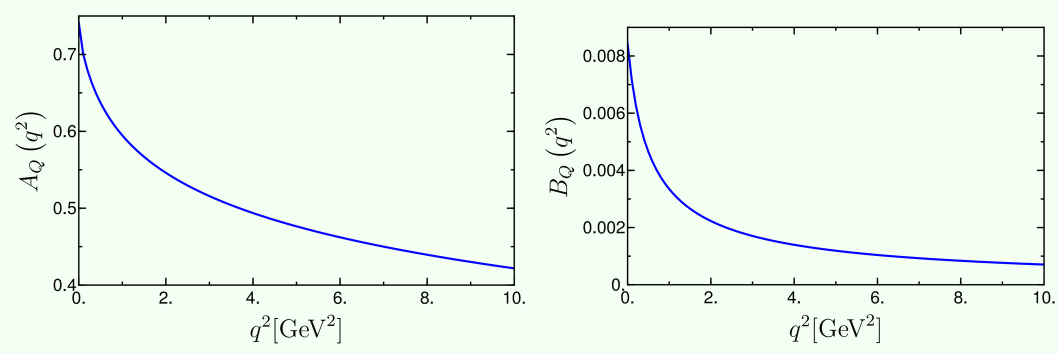


Figure: GFFs $A_Q(q^2)$ & $B_Q(q^2)$ as a function of q^2.

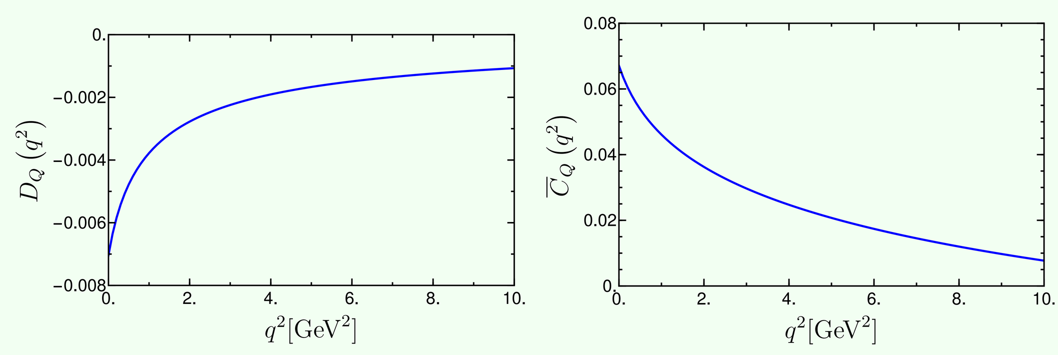
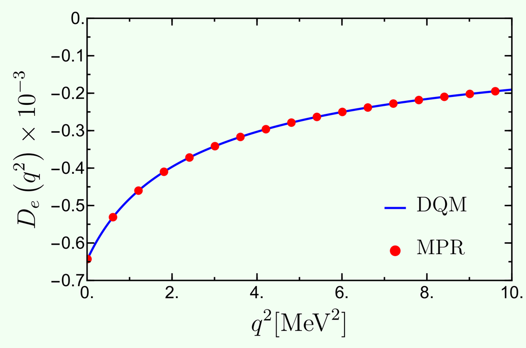


Figure: GFFs $D_Q(q^2)$ & $\overline{C}_Q(q^2)$ as a function of q^2.



MPR: Metz *et al.* Phys.Lett.B 820 (2021), 136501

Figure: Electron D-term as a function of q^2.

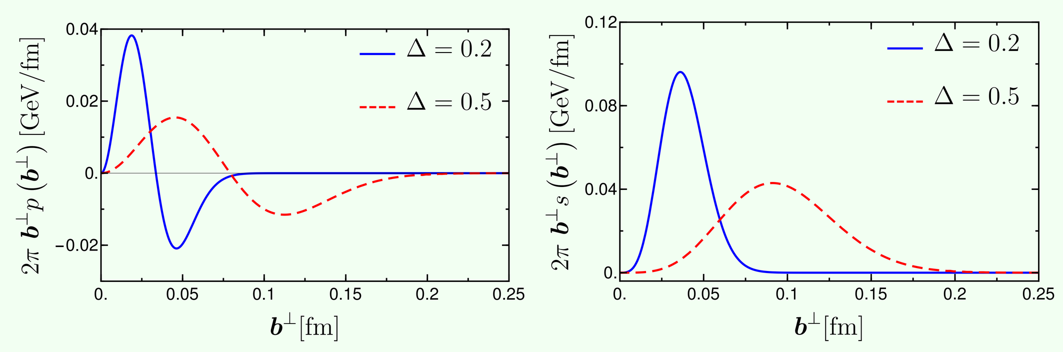


Figure: $2\pi\boldsymbol{b}^\perp p(\boldsymbol{b}^\perp)$ & $2\pi\boldsymbol{b}^\perp s(\boldsymbol{b}^\perp)$ as a function of $\boldsymbol{b}^\perp$.

Gaussian Wave Packet state:

$$\frac{1}{16\pi^3}\int\frac{d^2\boldsymbol{p}^\perp dp^+}{p^+}\phi(p)\,|\,p^+,\boldsymbol{p}^\perp,\lambda\rangle,$$

with $\phi(p)=p^+\ \delta(p^+-p^+_0)\ e^{-\frac{p^{\perp 2}}{2\Delta^2}}$.

7. Conclusions

- We have studied the four gravitational form factors in a composite spin-1/2 system, a quark dressed with a gluon at one loop level in QCD.
- We have also analysed the pressure and shear distributions in this model.

8. Acknowledgements

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9. Reference

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