Investigating spontaneous SO(10) symmetry breaking in type IIB matrix model

Arpith Kumar†, Anosh Joseph and Piyush Kumar

Department of Physical Sciences,
Indian Institute of Science Education and Research (IISER) Mohali,
Knowledge City, Sector 81, SAS Nagar, Punjab 140306, India

Overview

Non-perturbative formulations are essential to understand the dynamical compactification of extra dimensions in superstring theories. The type IIB (IKKT) matrix model in the large-N limit is one such conjectured formulation for a ten-dimensional type IIB superstring. In this model, a smooth spacetime manifold is expected to emerge from the eigenvalues of the ten bosonic matrices. When this happens, the SO(10) symmetry in the Euclidean signature must be spontaneously broken. The Euclidean version has a severe sign-problem since the Pfaffian obtained after integrating out the fermions is inherently complex. In recent years, the complex Langevin method (CLM) has successfully tackled the sign problem. We apply the CLM method to study the Euclidean version of the IKKT matrix model and investigate the possibility of spontaneous symmetry breaking (SSB) of SO(10) symmetry. In doing so, we encounter a singular-drift problem. To counter this, we introduce supersymmetry-preserving deformations with a Myers term. We study the spontaneous symmetry breaking in the original model at the vanishing deformation parameter limit. Our analysis indicates that the phase of the Pfaffian induces the spontaneous SO(10) symmetry breaking in the Euclidean IKKT model.

1. Review of Euclidean IKKT matrix model

The IKKT matrix model was proposed in 1996 as a constructive definition of the ten-dimensional type IIB superstring theory [1]. The Euclidean IKKT matrix model, obtained by a Wick rotation of the Lorentzian version, has a finite well-defined partition function [2],

$$Z = \int dX d\psi e^{-S_{\rm IKKT}}, \text{ where } S_{\rm IKKT} = S_{\rm b} + S_{\rm f},$$

$$S_{\rm b} = -\frac{1}{4} N \,{\rm tr}\left([X_\mu, X_\nu]^2\right) \quad \text{and} \quad S_{\rm f} = -\frac{1}{2} N \,{\rm tr}\left(\psi_\alpha (\mathscr{C}\Gamma^\mu)_{\alpha\beta} [X_\mu, \psi_\beta]\right)$$

The $N \times N$ traceless Hermitian matrices, $X_\mu (\mu = 1,2,3,\cdots, 10)$ and $\psi_\alpha (\alpha = 1,2,3,\cdots, 16)$ transform respectively as vectors and Majorana-Weyl spinors under SO(10) transformations.

- **Symmetries of theory**: The action manifests SU(N) gauge symmetry, extended $\mathcal{N} = 2$ supersymmetry (SUSY), and **SO(10) rotational symmetry**.
- **Fermionic operator** $\mathscr{M}$: The partition function, after integrating out the fermions reads, $Z = \int dX \ {\rm Pf}\mathscr{M} \ e^{-S_{\rm b}} = \int dX \ e^{-S_{\rm eff}}; \quad S_{\rm eff} = S_{\rm b} - \ln {\rm Pf}\mathscr{M}$, where fermionic operator, $\mathscr{M}$ is a complex $16(N^2-1) \times 16(N^2-1)$ anti-symmetric matrix. It has been conjectured that the phase of Pfaffian is responsible for spontaneous SO(10) symmetry breaking.
- **SSB order parameter**: Eigenvalues of X_μ interpreted as spacetime points.The *radial extent* of spacetime in each direction as follows $\langle \lambda_\mu \rangle = \left\langle \frac{1}{N} {\rm tr}\left(X_\mu^2\right) \right\rangle$. In the large-$N$ limit, if these extents are not equivalent, the SO(10) symmetry spontaneously breaks down to SO(d).

2. Applying complex Langevin to the IKKT model

We apply the complex Langevin method to the Euclidean IKKT model. The update of bosonic matrices X_μ at Langevin time τ reads $\frac{d(X_\mu)_{ij}}{d\tau} = -\frac{\partial S_{\rm eff}}{\partial (X_\mu)_{ji}} + (\eta_\mu)_{ij}(\tau)$, where $\eta_\mu(\tau)$ is a Hermitian Gaussian noise.

- **Reliability criteria for simulations**: Distribution of magnitude of the drift, $u_\tau = \left| \partial S_{\rm eff} / \partial (X_\mu)_{ji} \right|$, should be suppressed exponentially or faster.
- **Bosonic IKKT matrix model**: Simulations indicate absence of SSB (Fig. 1).

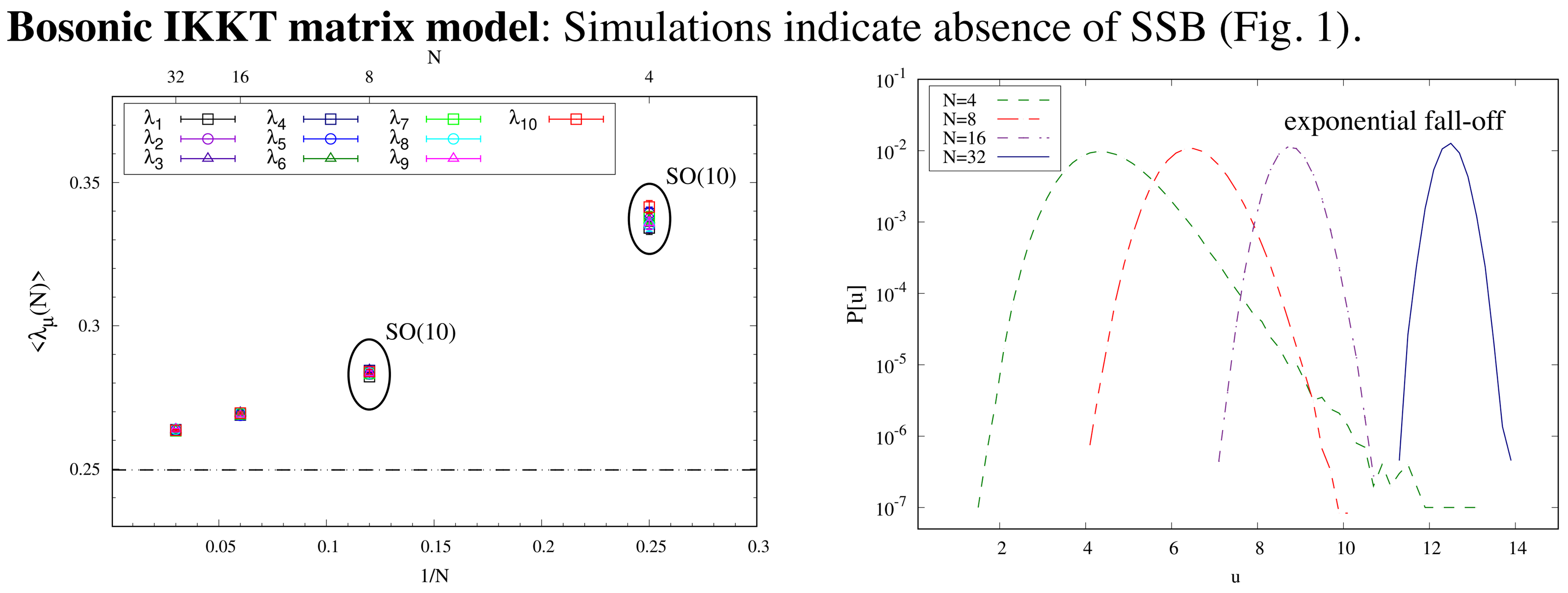

Figure 1: Bosonic IKKT model. (Left) The expectation value of order parameter λ_μ and (Right) the corresponding probability of the magnitude of the drift for various N.

- **Problems in complex Langevin simulations of the Euclidean IKKT model**: *Excursion problem* and the *singular-drift problem* violate reliability criteria. We resolved excursion problem using the *gauge cooling method* [3].
- **Singular-drift problem**: Arises when the eigenvalues of $\mathscr{M}$ accumulate densely near zero. Eigenvalues can be shifted away from the origin by adding fermion bilinear mass deformation terms to action [4], in general, $\Delta S = \frac{N}{2} \epsilon m_\mu {\rm tr}\left(X_\mu^2\right) + \frac{N}{2} {\rm tr}\left(\psi_\alpha (\mathscr{C}\mathscr{A})_{\alpha\beta} \psi_\beta\right)$, where m_μ is the mass vector and $\mathscr{A}$ is a complex 16×16 anti-symmetric matrix. Using the above deformations, the authors of a recent prominent study [5] concluded spontaneous SO(10) symmetry breaking down to SO(3). Apart from explicitly breaking the SO(10) symmetry, such deformations also induce SUSY breaking.

3. SUSY-preserving mass deformations

We introduce SUSY-preserving deformations [6], which includes a Myers term, to the original IKKT model ($S_{\rm IKKT}$). The deformed model has action $S = S_{\rm IKKT} + S_\Omega$,

$$S_\Omega = N \,{\rm tr}\left(M^{\mu\nu} X_\mu X_\nu + i N^{\mu\nu\sigma} X_\mu [X_\nu, X_\sigma] + \frac{i}{8} \overline{\psi} N_3 \psi\right),$$

$$N_3 = -\Omega \Gamma^8 \Gamma^{9\dagger} \Gamma^{10}, \; N^{\mu\nu\sigma} = \frac{\Omega}{3!} \sum_{\mu,\nu,\sigma=8}^{10} \epsilon^{\mu\nu\sigma} \quad \text{and} \quad M = \frac{\Omega^2}{4^3}\left(\mathbb{I}_7 \oplus 3\mathbb{I}_3\right).$$

- **Eigenvalues of** $\mathscr{M}$: $\mathscr{M}_{\alpha a \beta b} \to \tilde{\mathscr{M}}_{\alpha a \beta b} = \frac{N}{2} \Gamma^\mu_{\alpha\beta} {\rm tr}\left(X_\mu [t^a, t^b]\right) - \frac{i\Omega N}{8} \left(\Gamma^8 \Gamma^{9\dagger} \Gamma^{10}\right)_{\alpha\beta} \delta_{ab}$

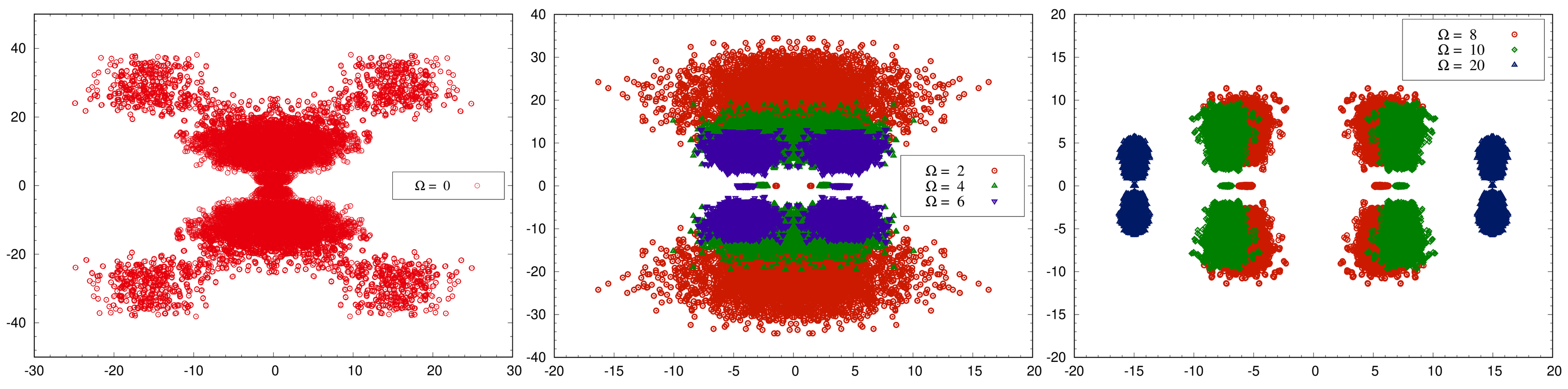

Figure 2: IKKT model with SUSY-preserving mass deformations. Scatter plot of real versus imaginary part of the eigenvalues of the fermion operator $\mathscr{M}$. The plots are for various mass deformation parameters Ω and fixed $N = 6$.

- **Deformed bosonic IKKT model**: We append the bosonic Gaussian mass deformation terms and a Myers term to the bosonic IKKT matrix model. The action of the deformed model reads $S_{\rm b} = S_{\rm bIKKT} + S_{\rm G} + S_{\rm Myers}$, where

$$S_{\rm G} = \frac{\Omega^2 N}{4^3} {\rm tr}\left(\sum_{i=1}^{7} X_i^2 + 3 \sum_{a=8}^{10} X_a^2\right) \text{ and } S_{\rm Myers} = \frac{i\Omega N}{3!} {\rm tr}\left(\sum_{a,b,c=8}^{10} X_a [X_b, X_c]\right).$$

We investigate whether SO(10) symmetry is intact in the $\Omega \to 0$ limit using normalized extent values $\langle \rho_\mu(\Omega) \rangle \equiv \left\langle \frac{\lambda_\mu(\Omega)}{\sum_\mu \lambda_\mu(\Omega)} \right\rangle$. We find that SO(10) symmetry is restored (Fig. 3).

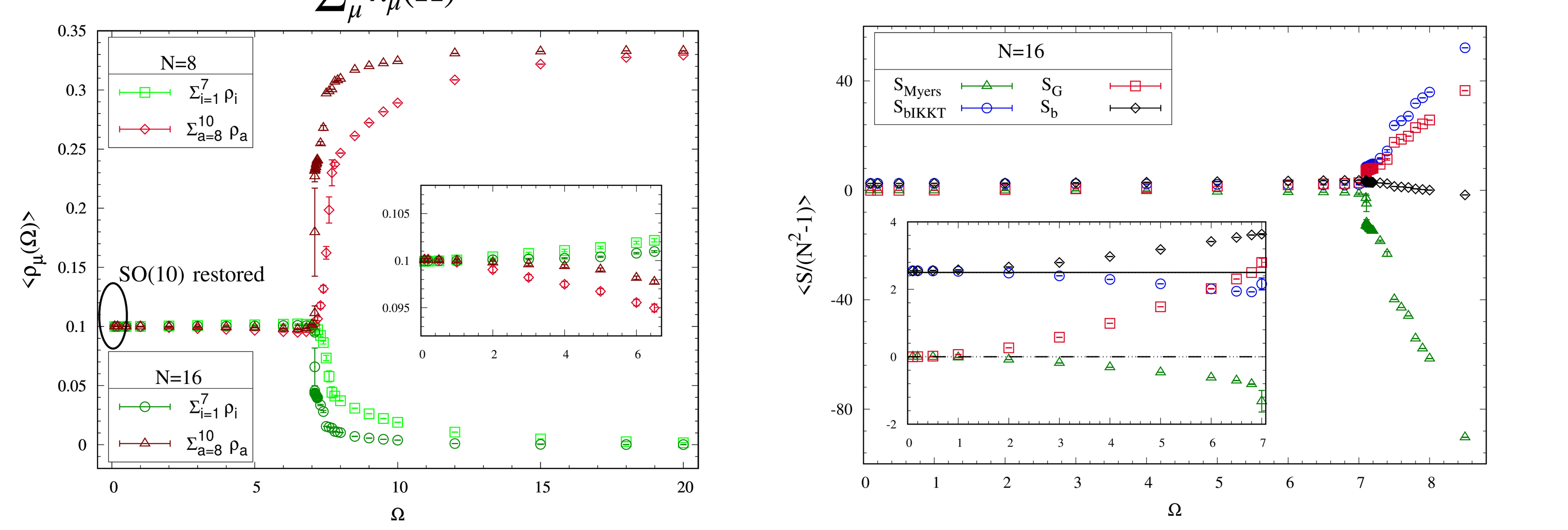

Figure 3: Deformed bosonic IKKT model with a Myers term. (Left) The averaged extents, $\rho_i(\Omega)/7$ and $\rho_a(\Omega)/7$ versus mass-deformation parameter Ω for $N = 8,16$. (Right) The bosonic action terms versus mass-deformation parameter Ω for $N = 16$.

- **IKKT model with SUSY-preserving mass deformations**: We perform complex Langevin simulations for various mass deformation and finite-N values. For a large enough Ω value, we observe an explicit SO(7) × SO(3) symmetry breaking. Finite-N results suggest extents ρ_μ independent of N, but need large-N computations to find the exact behaviour.

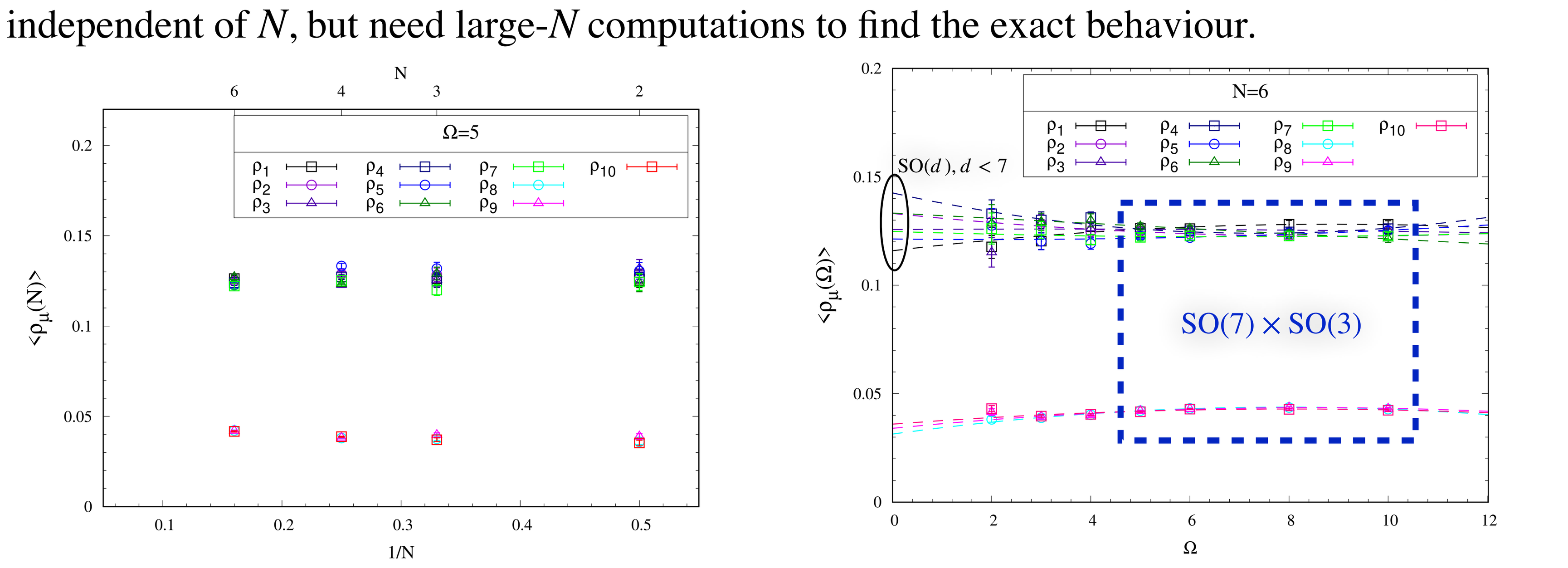

Figure 4: IKKT model with SUSY-preserving mass deformations. (Left) The normalized extents (order parameter) ρ_μ versus N for fixed $\Omega = 5$. (Right) The normalized extents ρ_μ versus Ω for fixed $N = 6$.

In the limit $\Omega \to 0$, we recover the original IKKT matrix model, and even for $N = 6$, the spontaneous breaking of SO(10) → SO(7) × SO(3) is apparent (Fig. 4).

4. Take Home Message

Our complex Langevin analysis of IKKT matrix model indicates:

- SUSY-preserving deformations can evade singular-drift problem.
- Phase of the Pfaffian does indeed trigger the SSB of ten-dimensional rotational symmetry.

5. References

[1]. Ishibashi et. al., *Nucl. Phys. B* **498** *(1997) 467* [hep-th/9612115]

[2]. Krauth et. al., *Phys. Lett. B* **431** *(1998) 31* [hep-th/9803117]

[3]. Seiler et. al., *Phys. Lett. B* **723** *(2013) 213* [1211.3709], Nagata et. al., Phys. Rev. D **94** (2016) [1606.07627]

[4]. Ito et. al., *JHEP* **12** (2016) 009 [1609.04501]

[5]. Anagnostopoulos et. al., *JHEP* **06** (2020) 069 [2002.07410]

[6]. Bonelli, *JHEP* **08** (2002) 022 [hep-th/0205213]

6. Contact

† speaker

Email ID: *arpithk.phy@gmail.com*

POSTER PDF

PARTIALLY FUNDED BY