Exploring millicharged dark matter components from the shadows

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Outline

- Black hole shadow
- Shadow for millicharged dark matter plasma around a Schwarzschild black hole
  1) Constant mass density
  2) Radially in-falling
- Possible future bounds on millicharged dark matter based on black hole shadow measurements
- Summary

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Exploring millicharged dark matter components from the shadows

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Black hole shadow

Image of M87* black hole

EHT Collaboration (2019)

- Black holes (BHs) are **astrophysical compact object** having such **high gravity** that below some radial distance (event horizon), no matter or radiation can escape it.
- The "**shadow**" is defined as the region of the observer's sky that is left dark when there are light sources distributed everywhere at distances behind the black hole.
- For a static and spherically symmetric spacetime, the metric can be defined as,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) .$$

- **Photon trajectories** (null geodesics) is determined by the Hamiltonian, $H_\gamma = \frac{1}{2}g^{\mu\nu}p_\mu p_\nu = 0$.
- Using above equation, the **photon sphere radius** is determined by the condition,

$$r_{\text{ph.}} f'(r_{\text{ph.}}) = 2f(r_{\text{ph.}}) .$$

- Using the equation for photon trajectory, **shadow radius** can be obtained as,

$$r_{\text{sh.}} = r_{\text{ph.}}\sqrt{\frac{f(r_{\text{obs.}})}{f(r_{\text{ph.}})}} .$$

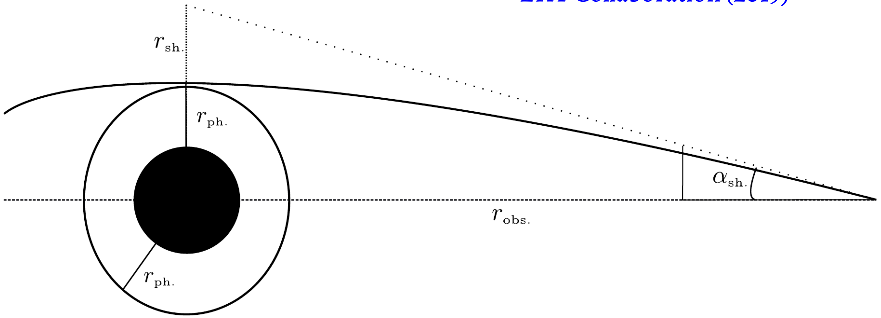


Fig 1 : Black hole Shadow Schematic

- For Schwarzschild black hole,

$$f(r) = 1 - \frac{2M_{\text{BH}}}{r} ,$$

$$\boxed{r_{\text{ph.}}^{\text{Sch.}} = 3M_{\text{BH}}} . \qquad \boxed{r_{\text{sh.}}^{\text{Sch.}} = 3\sqrt{3}M_{\text{BH}}} .$$

S. Chandershekhar (1983) Synge (1966)

Millicharged dark matter plasma around black hole

- Various models of dark matter proposes an effective (but very weak) interaction of dark matter with light, which leads them to have a fractional charge and are called **millicharged (mCP) dark matter**. B. Holdom (1985)
- We consider a presence of two mCP species : $\tilde{e}$ and $\tilde{p}$, with charges $-\epsilon q_e$, and $+\epsilon q_e$, and mass $m_{\tilde{e}}$ and $m_{\tilde{p}}$ respectively.
- The DM plasma frequency due to $\tilde{e}$ and $\tilde{p}$ is given by

$$\tilde{\omega}_p^2(r) = \frac{\epsilon^2 q_e^2}{\epsilon_o m_{\tilde{e}} m_{\tilde{p}}} \tilde{\rho}(r) \, .$$

$\tilde{\rho}(r)$: Mass density of mCP DM

- The modified Hamiltonian for photon trajectories is given by,

$$H_\gamma = \frac{1}{2}\left(g^{\mu\nu} p_\mu p_\nu + \tilde{\omega}_p^2(r)\right) = 0 \, .$$

Breuer and Ehlers (1981)

- Using above equation, we can obtain the condition for determining photon sphere radius ,

$$\frac{d}{dr}\left[\frac{r^2}{f(r)} - \frac{\tilde{\omega}_p^2(r) r^2}{E^2}\right]\Bigg|_{r=r_{\rm ph.}} = 0 \, .$$

- The expression of shadow radius is obtained using the photon trajectory,

$$r_{\rm sh.} = r_{\rm ph.} \sqrt{\frac{f(r_{\rm obs.})}{f(r_{\rm ph.})} \frac{\left(1 - \frac{\tilde{\omega}_p^2(r_{\rm ph.}) f(r_{\rm ph.})}{E^2}\right)}{\left(1 - \frac{\tilde{\omega}_p^2(r_{\rm obs.}) f(r_{\rm obs.})}{E^2}\right)}} \, .$$

Case I : Constant mass density dark matter plasma

- We take the mcP DM distribution to have constant mass density,

$$\tilde{\rho}(r) = \begin{cases} f_{\rm mCP}\,\rho_0 & R_{\rm Sch.} < r < \kappa R_{\rm Sch.}\,, \\ 0 & \kappa R_{\rm Sch.} \leq r\,. \end{cases}$$

$f_{\rm mCP}$: Mass fraction of mCP DM to total DM

ρ_0 : Total DM density

- Dimensionless dispersion parameter,

$$\chi_d \equiv \frac{\epsilon^2 q_e^2 f_{\rm mCP}\rho_0}{\eta\epsilon_o m_{\tilde{p}}^2 \left(\frac{2\pi c}{\lambda_{\rm EM}}\right)^2}$$

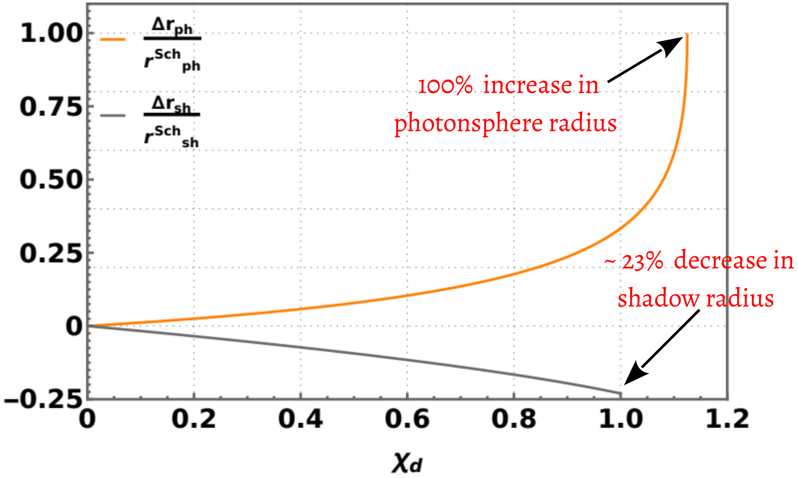
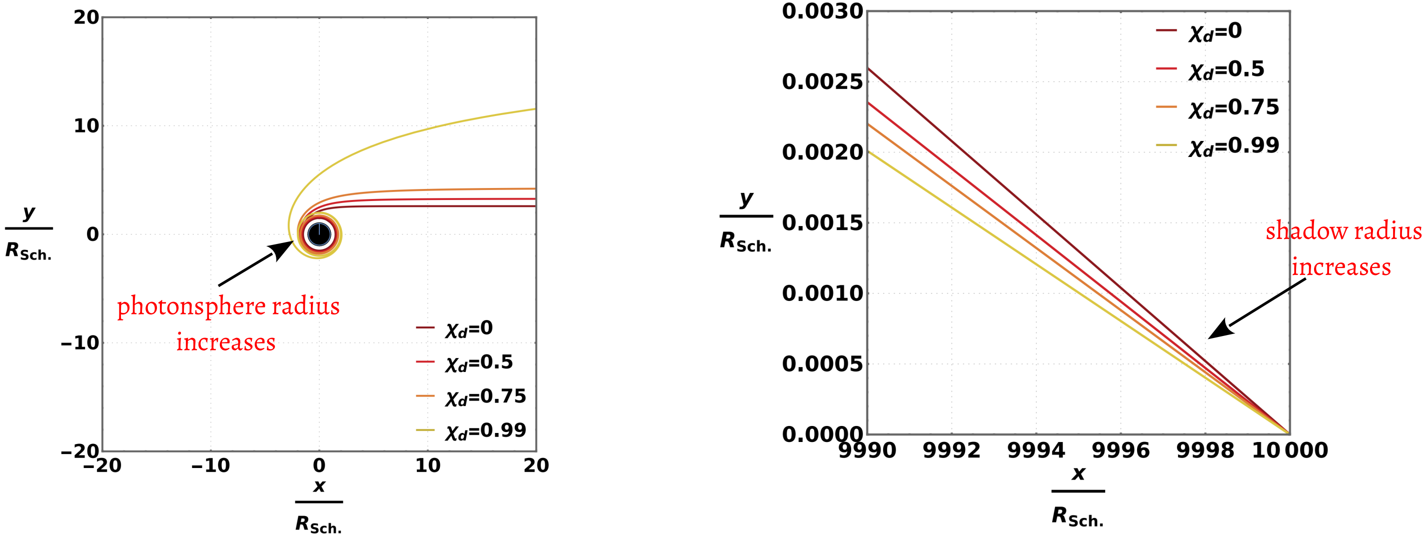


Fig. 1: Variation in the shadow/photon sphere radii with χ_d

- We found that the **photon sphere radius increases** and the **shadow radius decreases** as the dark matter plasma effect becomes more pronounced.
- It is found that the photon sphere radius can be as large as $6M_{\rm BH}$ (100% increase).
- The shadow radius can become $4M_{\rm BH}$ (~25% decrease).

Case I : Constant mass density dark matter plasma (cont.)



(a) (b)

Fig. 2: Trajectory of the outgoing light rays near vivinity of (a) BH (b) observer

$f_{\rm mCP}\rho_0 = 6.9\times10^{13}{\rm M}_\odot/(Kpc)^3,\ M_{\rm BH} = 6.5\times10^9{\rm M}_\odot,\ m_{\tilde e}/m_{\tilde p} = 1$ and $\lambda = 1.3\,{\rm mm},\ \kappa = 10^3$

ϵ	$m_{\rm mCP.}\ (eV)$	χ_d	$r_{\rm ph.}\ (R_{\rm Sch.})$	$\frac{\Delta r_{\rm ph.}}{r^{\rm Sch.}_{\rm ph.}}$	$r_{\rm sh.}\ (R_{\rm Sch.})$	$\frac{\Delta r_{\rm sh.}}{r^{\rm Sch.}_{\rm sh.}}$
10^{-17}	10^{-11}	~ 0	1.500	$\sim 0\%$	2.598	$\sim 0\%$
	10^{-12}	~ 0	1.500	$\sim 0\%$	2.598	$\sim 0\%$
	10^{-13}	0.02	1.503	0.23%	2.589	−0.34%
	5×10^{-14}	0.08	1.514	0.94%	2.562	−1.4%
	1.5×10^{-14}	0.90	1.856	24%	2.089	−20%
	1×10^{-14}	2.03	(No photon sphere)	—	(No shadow)	—

Table. 1: Photon sphere and shadow radii for few representative points in the viable mCP parameter space

Case II : Radially in-falling dark matter plasma

- We take the steady radial accretion of mcP DM. It will have a distribution ,

$$\tilde{\rho}(r) = \begin{cases} \frac{f_{\rm mCP}\dot{M}_{ac}}{4\pi c\sqrt{R_{\rm Sch.}\,r^3}} & R_{\rm Sch.} < r < \kappa R_{\rm Sch.}\ , \\ 0 & \kappa R_{\rm Sch.} \leq r\ . \end{cases}$$

$f_{\rm mCP}$: Mass fraction of mCP DM to total DM
$\dot{M}_{\rm ac}$: Acceration rate of the DM

- Dimensionless dispersion parameter,

$$\chi_d \equiv \frac{\epsilon^2 q_e^2 f_{\rm mCP}\dot{M}_{ac}}{4\pi\epsilon_o c m_{\tilde{e}} m_{\tilde{p}} R^2_{\rm Sch.}\left(\frac{2\pi c}{\lambda_{\rm EM}}\right)^2}\ .$$

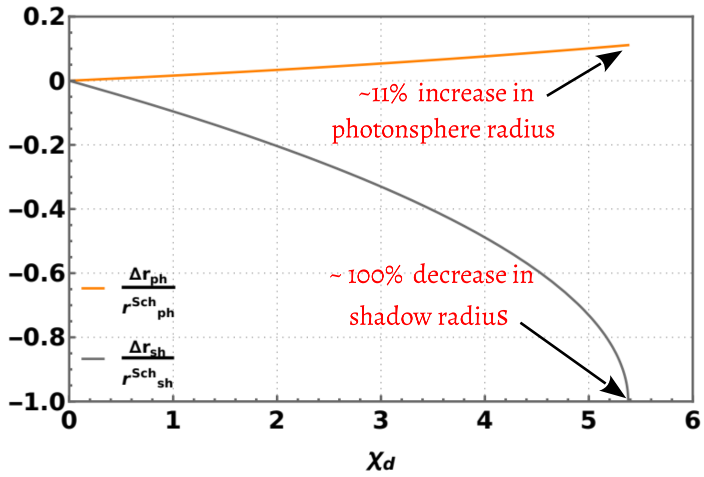


Fig. 1: Variation in the shadow/photon sphere radius with χ_d

- We found that the **photon sphere radius increases** and the **shadow radius decreases** as the dark matter plasma effect becomes more pronounced.
- It is found that the photon sphere radius can be as large as $10/3\ M_{\rm BH}$(~11% increase).
- The shadow radius can vanish to 0 (~100% decrease).

Case II : Radially in-falling dark matter plasma (cont.)

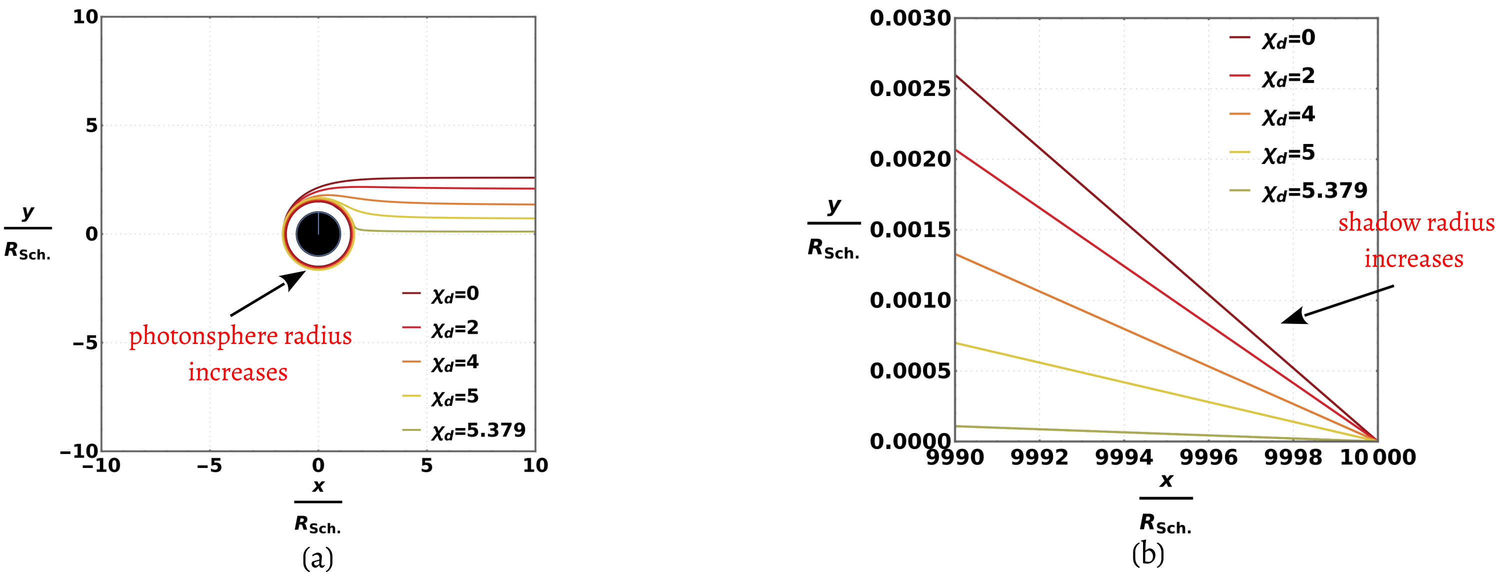


Fig. 2: Trajectory of the outgoing light rays near vivinity of (a) BH (b) observer

$f_{\rm mCP}\dot{M}_{ac} = 0.1{\rm M}_{\odot}/{\rm Year},\ M_{\rm BH} = 6.5\times10^{9}{\rm M}_{\odot},\ m_{\tilde{e}}/m_{\tilde{p}} = 1$ and $\lambda = 1.3\,{\rm mm},\ \kappa = 10^{3}$

ϵ	$m_{\rm mCP.}\ (eV)$	χ_d	$r_{\rm ph.}\ (R_{\rm Sch.})$	$\frac{\Delta r_{\rm ph.}}{r^{\rm Sch.}_{\rm ph.}}$	$r_{\rm sh.}\ (R_{\rm Sch.})$	$\frac{\Delta r_{\rm sh.}}{r^{\rm Sch.}_{\rm sh.}}$
2×10^{-18}	10^{-14}	0.08	1.502	0.12%	2.579	−0.72%
	5×10^{-15}	0.32	1.507	0.49%	2.523	−2.9%
	1.5×10^{-15}	3.51	1.597	6.5%	1.545	−41%
	1.25×10^{-15}	5.05	1.653	10%	0.649	−75%
	1×10^{-15}	7.88	(No photon sphere)	—	(No shadow)	—

Table. 1: Photon sphere and shadow radii for few representative points in the viable mCP parameter space

Results for some astrophysical black hole and possible future bounds on mCP

Blackhole	Mass ($M_\odot$)	$f_{\rm mCP}\rho_o$ (GeV/cm^3)	ϵ	$m_{\rm mCP}$ (eV)	χ_d	$\frac{\Delta r_{\rm ph.}}{r^{\rm Sch.}_{\rm ph.}}$	$\frac{\Delta r_{\rm sh.}}{r^{\rm Sch.}_{\rm sh.}}$
M87*	6.5×10^9	10^6	10^{-17} 10^{-18}	2.5×10^{-14} 2×10^{-15}	0.12 0.19	1.5% 2.4%	−2.1% −3.4%
Sgr A*	4.3×10^6	10^9	10^{-17} 10^{-18}	10^{-12} 1.5×10^{-13}	0.08 0.03	0.90% 0.39%	−1.3% −0.58%
NGC104	2.3×10^3	10^{12}	10^{-17} 10^{-18}	6.5×10^{-11} 7.5×10^{-12}	0.02 0.01	0.21% 0.15%	−0.31% −0.23%

(a)

Blackhole	Mass ($M_\odot$)	$f_{\rm mCP}\dot{M}_{ac}$ ($M_\odot$/Year)	ϵ	$m_{\rm mCP}$ (eV)	χ_d	$\frac{\Delta r_{\rm ph.}}{r^{\rm Sch.}_{\rm ph.}}$	$\frac{\Delta r_{\rm sh.}}{r^{\rm Sch.}_{\rm sh.}}$
M87*	6.5×10^9	1	10^{-17} 10^{-18}	2.5×10^{-14} 2×10^{-15}	3.16 4.93	5.7% 9.9%	−35% −71%
Sgr A*	4.3×10^6	10^{-3}	10^{-17} 10^{-18}	10^{-12} 1.5×10^{-13}	4.51 2.00	8.8% 3.4%	−59% −20%
NGC104	2.3×10^3	10^{-6}	10^{-17} 10^{-18}	6.5×10^{-11} 7.5×10^{-12}	3.73 2.8	7% 5.0%	−44% −30%

(b)

Table 1: Variations in the photon sphere and shadow radii for a few candidate galactic black holes. **(a)** constant mass density **(b)** radially in-falling DM plasma.

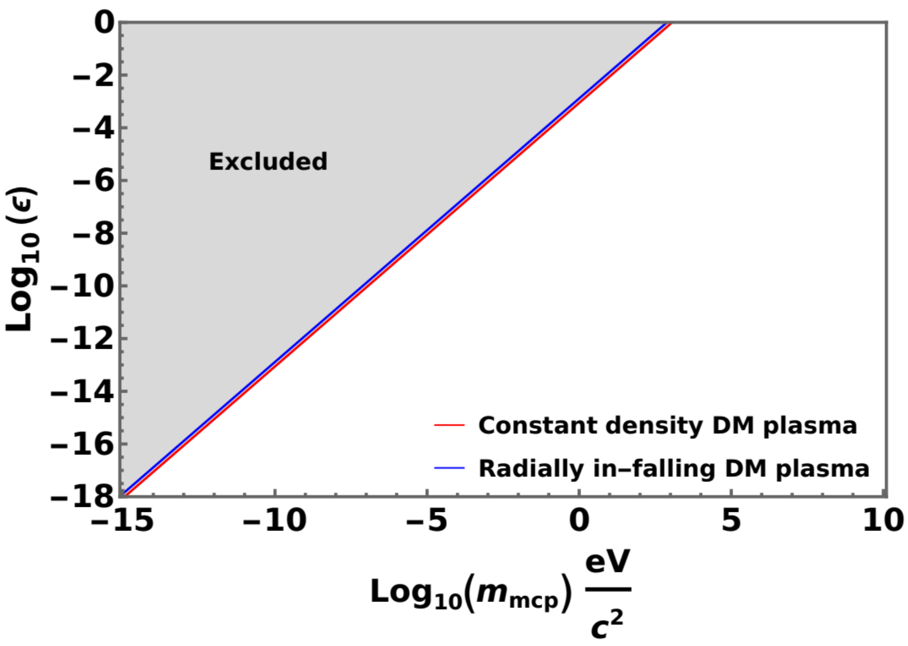


Fig. 1: Regions of the mCP parameter space that are excluded if one requires that $\Delta r_{\rm sh.}/r^{\rm Sch.}_{\rm sh.} < 20\%$ for M87* black hole.

Summary

- We found that the **photon sphere radius increases** and the **shadow radius decreases** as the dark matter plasma effect becomes more pronounced, as a function of the millicharged particle mass and charge.
- In the **constant mass density case**, it is found that the relative increase in the photon sphere radius can be as large as ~ 100%, and relative decrease in shadow radius can be ~25%.
- For **radially in-falling** dark matter plasma upto ~ 11% relative increase in the photons sphere radius and upto ~ 100% relative decrease in the shadow radius can be observed.
- We have also **speculated possible exclusion limits on the millicharged particle parameter space** $(\epsilon - m_{\rm mcp.})$ based on future precise observations of black hole shadow.

Thank You

Millicharged particle

- In a simplest model, with a dark *U(1)* gauge group, the Lagrangian will contain the **kinetic mixing term**,

$$\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - \frac{\chi}{2}F^{\mu\nu}\tilde{F}_{\mu\nu} \, .$$

- Scalar field in a dark sector has the coupling with dark gauge field,

$$\mathcal{L}^{s} \supset \left(\tilde{D}_\mu\tilde{\phi}\right)^\dagger \left(\tilde{D}^\mu\tilde{\phi}\right) - m_{\tilde{\phi}}^2|\tilde{\phi}|^2 \quad \text{where, } \tilde{D}_\mu = \partial_\mu + i\tilde{q}\tilde{A}_\mu \, .$$

- We require the **canonical normalization** of the gauge field kinetic terms, which is done by field redefinition,

$$\tilde{A}_\mu \to \tilde{A}_\mu - \chi A_\mu \, .$$

- The dark sector scalar field develops a **coupling with visible sector physical boson**,

$$\mathcal{L}^{s} \supset \left(D_\mu\tilde{\phi} - i\chi\tilde{q}A_\mu\tilde{\phi}\right)^\dagger \left(D^\mu\tilde{\phi} - i\chi\tilde{q}A^\mu\tilde{\phi}\right) - m_{\tilde{\phi}}^2|\tilde{\phi}|^2 \, .$$

- Therefore, the dark sector particle will be **effectively seen as a fractionally charged particle**, by the visible sector photon, with electromagnetic charge,

$$\epsilon = \frac{\chi\tilde{q}}{q_e} \, . \quad \text{where } q_e \text{ is the charge of an electron.}$$

B.Holdom (1986)

Constraint on millicharged particle

- We have obtained constraint on mass fraction ($f_{\rm mCP}$) of mCP DM to total DM by investigating **bullet clusters**.
- The subcluster will lose mass, when after collision the particles have velocity greater than escape velocity of cluster, which will happen for the angles,

$$\frac{v_{\rm esc}}{v} < \cos\theta < \sqrt{1-\left(\frac{v_{\rm esc}}{v}\right)^2} \, .$$

v : velocity of main cluster particles in the rest frame of subcluster.

$v_{\rm esc}$: escape velocity for a particle in subcluster.

θ : scattering angle in rest frame of subcluster.

- The differential cross section for mCP-mCP scattering is,

$$\frac{d\sigma}{d\Omega} = \frac{\epsilon^4\alpha^2\hbar^2c^2}{4m_{\rm mCP}^2 v^4 \sin^4(\theta/2)} \, ,$$

- Using above equation, we get the probability that a particle in subcluster will get knocked out,

$$\mathcal{P} \equiv f_{\rm mCP}\frac{\Sigma_s}{m_{\rm mCP}}\int_{\theta_{\rm min}}^{\theta_{\rm max}} d\Omega\frac{d\sigma}{d\Omega} = \frac{2\pi f_{\rm mCP}\Sigma_s\epsilon^4\alpha^2\hbar^2c^2}{m_{\rm mCP}^3 v^4}\left(\frac{1}{1-\cos\theta_{\rm min}} - \frac{1}{1-\cos\theta_{\rm max}}\right) .$$

Σ_s : DM surface mass density of subcluster

Feng (2009)

- Taking the data from the Bullet clusters, and assuming the mass lost in the collision is less than 25%, we obtain the relation,

$$f_{\rm mCP}\,\epsilon^4\alpha^2 \lesssim 10^{-33}\left(\frac{m_{\rm mCP}}{\rm eV}\right)^3 \, .$$

$\Sigma_s \simeq 0.3\,{\rm g/cm^2}$, $v \simeq 4800\,{\rm Km/s}$, $v_{\rm esc} \simeq 1200\,{\rm Km/s}$.

Markevitch et al. (2004)

- For example $\epsilon \sim 10^{-14}$ and $m_{\rm mCP} \sim 10^{-10}{\rm eV}$ it is found that $f_{\rm mCP} \leq 0.2\%$. Conservatively we take $f_{\rm mCP} = 0.1\%$.

Case I : Constant mass density dark matter plasma

- We take the mcP DM distribution to have constant mass density,

$$\tilde{\rho}(r) = \begin{cases} f_{\mathrm{mCP}}\, \rho_0 & R_{\mathrm{Sch.}} < r < \kappa R_{\mathrm{Sch.}}\ , \\ 0 & \kappa R_{\mathrm{Sch.}} \le r\ . \end{cases}$$

f_{mCP} : Mass fraction of mCP DM to total DM

ρ_0 : Total DM density

- g_{tt} component of the metric is ,

$$f(r) = \begin{cases} 1 - \frac{R_{\mathrm{Sch.}}}{r} - \frac{8\pi G \rho_0}{3c^2 r}\left(r^3 - R_{\mathrm{Sch.}}^3\right) & R_{\mathrm{Sch.}} < r < \kappa R_{\mathrm{Sch.}}\ , \\ 1 - \frac{R_{\mathrm{Sch.}}}{r} - \frac{8\pi G \rho_0 R_{\mathrm{Sch.}}^3}{3rc^2}\left(\kappa^3 - 1\right) & \kappa R_{\mathrm{Sch.}} \le r\ . \end{cases}$$

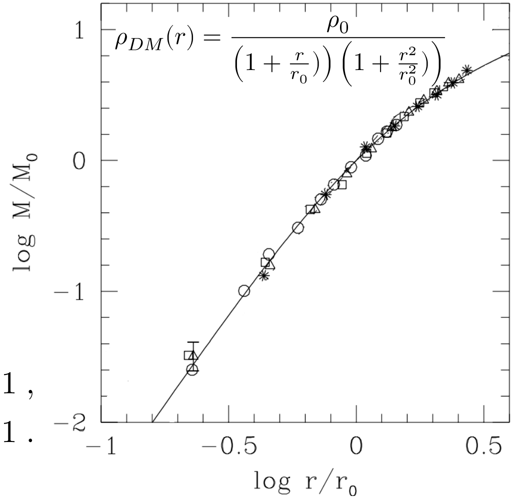
$$\chi_g \equiv \frac{8\pi \rho_0 G R_{\mathrm{Sch.}}^2}{3c^2}$$

$$\chi_d \equiv \frac{\epsilon^2 q_e^2 f_{\mathrm{mCP}} \rho_0}{\eta \epsilon_o m_{\tilde{p}}^2 \left(\frac{2\pi c}{\lambda_{\mathrm{EM}}}\right)^2}$$

- The photon sphere radius will be ,

$$\frac{d}{dr}\left[\frac{r^2}{1 - \frac{R_{\mathrm{Sch.}}}{r} - \chi_g\left(\left(\frac{r}{R_{\mathrm{Sch.}}}\right)^2 - \left(\frac{R_{\mathrm{Sch.}}}{r}\right)\right)} - \chi_d\, r^2\right]\Bigg|_{r = r_{\mathrm{ph.}}} = 0\ .$$

$$\left.\frac{r_{\mathrm{ph.}}}{R_{\mathrm{Sch.}}}\right|_{\chi_g \ll \chi_d} = \begin{cases} \frac{3 + \sqrt{9 - 8\chi_d} - 4\chi_d}{4(1 - \chi_d)} & \chi_d \neq 1\ , \\ 2 & \chi_d = 1\ . \end{cases}$$



A. Burket (1995)

- Shadow radius is given by,

$$r_{\mathrm{sh.}} = \sqrt{\left(1 - \frac{R_{\mathrm{Sch.}}}{r_{\mathrm{obs.}}} - \chi_g \frac{R_{\mathrm{Sch.}}}{r_{\mathrm{obs.}}}\left(\kappa^3 - 1\right)\right)\left(\frac{r_{\mathrm{ph.}}^2}{1 - \frac{R_{\mathrm{Sch.}}}{r_{\mathrm{ph.}}} - \chi_g\left(\left(\frac{r_{\mathrm{ph.}}}{R_{\mathrm{Sch.}}}\right)^2 - \left(\frac{R_{\mathrm{Sch.}}}{r_{\mathrm{ph.}}}\right)\right)} - \chi_d r_{\mathrm{ph.}}^2\right)}\ .$$

$$\left.\frac{r_{\mathrm{sh.}}}{R_{\mathrm{Sch.}}}\right|_{\chi_g \ll \chi_g} = \frac{1}{4}\sqrt{\frac{-18(3 + \sqrt{9 - 8\chi_d}) + 8(9 + 2\sqrt{9 - 8\chi_d} - 2\chi_d)\chi_d}{\chi_d - 1}}\left(\sqrt{1 - \frac{R_{\mathrm{Sch.}}}{r_{\mathrm{obs.}}}}\right)\ , \quad \chi_d < 1\ .$$

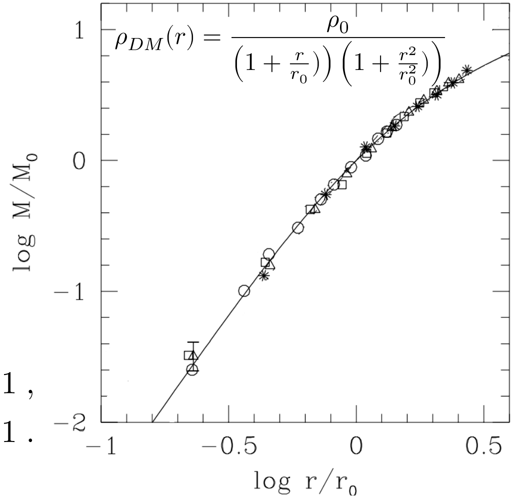
Case I : Constant mass density dark matter plasma

- We take the mcP DM distribution to have constant mass density,

$$\tilde{\rho}(r) = \begin{cases} f_{\rm mCP}\,\rho_0 & R_{\rm Sch.} < r < \kappa R_{\rm Sch.}\,, \\ 0 & \kappa R_{\rm Sch.} \leq r\,. \end{cases}$$

$f_{\rm mCP}$: Mass fraction of mCP DM to total DM

ρ_0 : Total DM density



A. Burket (1995)

- g_{tt} component of the metric is ,

$$f(r) = \begin{cases} 1 - \frac{R_{\rm Sch.}}{r} - \frac{8\pi G\rho_0}{3c^2 r}\left(r^3 - R_{\rm Sch.}^3\right) & R_{\rm Sch.} < r < \kappa R_{\rm Sch.}\,, \\ 1 - \frac{R_{\rm Sch.}}{r} - \frac{8\pi G\rho_0 R_{\rm Sch.}^3}{3rc^2}\left(\kappa^3 - 1\right) & \kappa R_{\rm Sch.} \leq r\,. \end{cases}$$

$$\chi_g \equiv \frac{8\pi\rho_0 G R_{\rm Sch.}^2}{3c^2}$$

$$\chi_d \equiv \frac{\epsilon^2 q_e^2 f_{\rm mCP}\rho_0}{\eta\epsilon_o m_{\tilde{p}}^2\left(\frac{2\pi c}{\lambda_{\rm EM}}\right)^2}$$

- The photon sphere radius will be ,

$$\frac{d}{dr}\left[\frac{r^2}{1 - \frac{R_{\rm Sch.}}{r} - \chi_g\left(\left(\frac{r}{R_{\rm Sch.}}\right)^2 - \left(\frac{R_{\rm Sch.}}{r}\right)\right)} - \chi_d\, r^2\right]\Bigg|_{r=r_{\rm ph.}} = 0\,.$$

$$\frac{r_{\rm ph.}}{R_{\rm Sch.}}\Bigg|_{\chi_g \ll \chi_d} = \begin{cases} \frac{3+\sqrt{9-8\chi_d}-4\chi_d}{4(1-\chi_d)} & \chi_d \neq 1\,, \\ 2 & \chi_d = 1\,. \end{cases}$$

- Shadow radius is given by,

$$r_{\rm sh.} = \sqrt{\left(1 - \frac{R_{\rm Sch.}}{r_{\rm obs.}} - \chi_g\frac{R_{\rm Sch.}}{r_{\rm obs.}}\left(\kappa^3 - 1\right)\right)\left(\frac{r_{\rm ph.}^2}{1 - \frac{R_{\rm Sch.}}{r_{\rm ph.}} - \chi_g\left(\left(\frac{r_{\rm ph.}}{R_{\rm Sch.}}\right)^2 - \left(\frac{R_{\rm Sch.}}{r_{\rm ph.}}\right)\right)} - \chi_d r_{\rm ph.}^2\right)}\,.$$

$$\frac{r_{\rm sh.}}{R_{\rm Sch.}}\Bigg|_{\chi_g \ll \chi_g} = \frac{1}{4}\sqrt{\frac{-18(3+\sqrt{9-8\chi_d}) + 8(9+2\sqrt{9-8\chi_d} - 2\chi_d)\chi_d}{\chi_d - 1}}\left(\sqrt{1 - \frac{R_{\rm Sch.}}{r_{\rm obs.}}}\right)\,, \quad \chi_d < 1\,.$$

Case II : Radially in-falling dark matter plasma

- We take the steady radial accretion of mcP DM. It will have a distribution ,

$$\tilde{\rho}(r) = \begin{cases} \frac{f_{\rm mCP}\dot{M}_{ac}}{4\pi c\sqrt{R_{\rm Sch.}r^3}} & R_{\rm Sch.} < r < \kappa R_{\rm Sch.} \ , \\ 0 & \kappa R_{\rm Sch.} \le r \ . \end{cases}$$

$f_{\rm mCP}$: Mass fraction of mCP DM to total DM
$\dot{M}_{\rm ac}$: Acceration rate of the DM

- g_{tt} component of the metric is given by ,

$$f(r) = \begin{cases} 1 - \frac{R_{\rm Sch.}}{r} - \frac{4G\dot{M}_{ac}}{3c^3 R_{\rm Sch.}^{\frac{1}{2}} r}\left(r^{\frac{3}{2}} - R_{\rm Sch.}^{\frac{3}{2}}\right) & R_{\rm Sch.} < r < \kappa R_{\rm Sch.} \ , \\ 1 - \frac{R_{\rm Sch.}}{r} - \frac{4G\dot{M}_{ac}R_{\rm Sch.}}{3c^3 r}\left(\kappa^{\frac{3}{2}} - 1\right) & \kappa R_{\rm Sch.} \le r \ . \end{cases}$$

$$\chi_g \equiv \frac{4G\dot{M}_{ac}}{3c^3} \ ,$$
$$\chi_d \equiv \frac{\epsilon^2 q_e^2 f_{\rm mCP}\dot{M}_{ac}}{4\pi\epsilon_o c m_{\tilde{e}} m_{\tilde{p}} R_{\rm Sch.}^2 E^2} \ .$$

- The photon sphere radius is ,

$$\frac{d}{dr}\left[\frac{r^2}{1 - \frac{R_{\rm Sch.}}{r} - \chi_g\left(\left(\frac{r}{R_{\rm Sch.}}\right)^{\frac{1}{2}} - \left(\frac{R_{\rm Sch.}}{r}\right)\right)} - \chi_d r^2\left(\frac{R_{\rm Sch.}}{r}\right)^{\frac{3}{2}}\right]\Bigg|_{r=r_{\rm ph.}} = 0 \ .$$

$$\frac{r_{\rm ph.}}{R_{\rm Sch.}} \simeq \frac{3}{2} + \frac{\chi_d}{18\sqrt{6}} + \frac{3}{16}\left(-8 + 3\sqrt{6}\right)\chi_g + O(\chi_{g,d}^2) \ .$$

- Shadow radius will be,

$$r_{\rm sh.} = \sqrt{\left(1 - \frac{R_{\rm Sch.}}{r_{\rm obs.}} - \chi_g\frac{R_{\rm Sch.}}{r_{\rm obs.}}\left(\kappa^{\frac{3}{2}} - 1\right)\right)\left(\frac{r_{\rm ph.}^2}{1 - \frac{R_{\rm Sch.}}{r_{\rm ph.}} - \chi_g\left(\left(\frac{r_{\rm ph.}}{R_{\rm Sch.}}\right)^{\frac{1}{2}} - \left(\frac{R_{\rm Sch.}}{r_{\rm ph.}}\right)\right)} - \chi_d r_{\rm ph.}^2\left(\frac{R_{\rm Sch.}}{r_{\rm ph.}}\right)^{\frac{3}{2}}\right)} \ .$$

$$\frac{r_{\rm sh.}}{R_{\rm Sch.}} \simeq \frac{3\sqrt{3}}{2}\left(1 - \frac{R_{\rm Sch.}}{r_{\rm obs.}}\right)^{\frac{1}{2}} - \frac{\chi_d}{3\sqrt{2}}\left(1 - \frac{R_{\rm Sch.}}{r_{\rm obs.}}\right)^{\frac{1}{2}} - \frac{3\sqrt{3}\chi_g}{8}\left(3\sqrt{6}\left(1 - \frac{r_{\rm obs.}}{R_{\rm Sch.}}\right) + 4\frac{r_{\rm obs.}}{R_{\rm Sch.}} - 6 + 2\kappa^{\frac{3}{2}}\right)\left(\frac{R_{\rm Sch.}^2}{r_{\rm obs.}(r_{\rm obs.} - R_{\rm Sch.})}\right)^{\frac{1}{2}} + \mathcal{O}(\chi_{g,d}^2) \ .$$