Effect of inhomogeneities on GW observables

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Inhomogeneities in the Universe

- The FLRW model of the Universe $\xrightarrow[\text{on the}]{\textit{based}}$ Cosmological Principle. CP $\rightarrow$ universe is spatially homogeneous and isotropic. Surveys have revealed that inhomogeneities are important on length scales even as large as 500 h^{-1} Mpc. [Wiegand, 2014]

- Backreaction approach $\rightarrow$ such an approach considers the universe to be only statistically homogeneous and isotropic at very large scales.

- At scales much larger than the scale of inhomogeneites, the global domain is assumed to be homogeneous.

Buchert's Approach

Irrotational fluid motion with dust. Flow is geodesic. Choose a flow-orthogonal coordinate system $x^\mu = (t, X^k)$, vanishing 3- velocity and comoving coordinates.

The metric -

$$ds^2 = -dt^2 + g_{ij} dX^i dX^j \tag{1}$$

Proposed by Buchert - simplify the problem and consider only scalar quantities and average them. [Buchert, 2000]

Buchert's approach - he decomposed Einstein's equations into a set of dynamical equations for scalar quantities.

The volume average of a scalar quantity $\langle f\rangle_{\mathcal{D}}(t)$ over the rest mass preserving domain $\mathcal{D}$, is given by [Wiegand(2010)]

$$\langle f\rangle_{\mathcal{D}}(t) = \frac{1}{|\mathcal{D}|_g}\int_{\mathcal{D}} f d\mu_g \tag{2}$$

where $|\mathcal{D}|_g = \int_{\mathcal{D}} d\mu_g$ is the volume of the domain and $d\mu_g = \sqrt{^{(3)}g(t, X^1, X^2, X^3)}dX^1dX^2dX^3$. The volume scale factor $a_{\mathcal{D}}$ is defined as :

$$a_{\mathcal{D}}(t) = \left(\frac{|\mathcal{D}|_g}{|\mathcal{D}_0|_g}\right)^{1/3} \tag{3}$$

where $|\mathcal{D}_0|_g$ is the volume of the domain at a reference time t_0 which we may take as the present time.

Using Buchert's averaging scheme, we can derive from Einstein's equations, the following equations governing evolution,

averaged Raychaudhuri equation,

$$3\frac{\ddot{a}_D}{a_D} = -4\pi G\langle\rho\rangle_D + Q_D + \Lambda\,, \tag{4}$$

averaged Hamiltonian constraint,

$$3H_D^2 = 8\pi G\langle\rho\rangle_D - \frac{1}{2}\langle R\rangle_D - \frac{1}{2}Q_D + \Lambda\,, \tag{5}$$

averaged continuity equation,

$$\partial_t\langle\rho\rangle_D + 3H_D\langle\rho\rangle_D = 0\,, \tag{6}$$

where Q_D is called the backreaction term,

$$Q_D := \frac{2}{3}\left(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\right) - 2\langle\sigma^2\rangle_D\,, \tag{7}$$

where θ is the local expansion rate and $\sigma^2 := \frac{1}{2}\sigma^i_j\sigma^j_i$ is the shear-scalar.

- The scalar functions are domain dependent.
- Global domain $\xrightarrow[\text{into}]{\textit{partitioned}}$ non interacting subregions $\mathcal{F}_I$ $\xrightarrow[\text{consists of}]{\textit{themselves}}$ elementary space entities $\mathcal{F}_I^{(\alpha)}$.
- Mathematically, $\mathcal{D} = \cup_I \mathcal{F}_I$, where $\mathcal{F}_I = \cup_\alpha \mathcal{F}_I^{(\alpha)}$.
- Average of the scalar function f on the domain $\mathcal{D}$ can be constructed by summing all the averages f on the subregions $\mathcal{F}_I$ as,

$$\langle f \rangle_{\mathcal{D}} = \sum_I |\mathcal{D}|_g^{-1} \sum_\alpha \int_{\mathcal{F}_I^{(\alpha)}} f d\mu_g = \sum_I \lambda_I \langle f \rangle_{\mathcal{F}_I} \tag{8}$$

where $\lambda_I = |\mathcal{F}_I|_g / |\mathcal{D}|_g$, is the volume fraction of the subregion $\mathcal{F}_I$.

Toy Model

The model considered consists → ensemble of 2 types of FLRW regions → empty underdense FLRW region and an overdense region described by a closed, dust-only FLRW model.[Koksbang, 2019]

Toy Model

$$t = t_0 \frac{\phi - \sin\phi}{\phi_0 - \sin\phi_0}$$
$$a_o = \frac{f_o^{1/3}}{2}(1 - \cos\phi) \tag{9}$$
$$a_u = \frac{f_u^{1/3}(\phi_0 - \sin\phi_0)}{\pi t_0} t^{\beta}$$

where f_0 and $f_u = 1 - f_o$ are the volume fractions of the over- and under-dense regions and β is a parameter which controls the evolution of a_u with time. Here we are taking, $2/3 < \beta \leq 1$. ϕ is the development angle of the overdense region and $\phi_0 = 3\pi/2$.

Toy model setup

$o \to$ overdense and $u \to$ underdense.

$$a_D = \left(\frac{a_u^3 + a_o^3}{a_{u,0}^3 + a_{o,0}^3} \right)^{1/3} \tag{10}$$

$$H_D = H_u \frac{a_u^3}{a_u^3 + a_o^3} + H_o \frac{a_o^3}{a_u^3 + a_o^3} \tag{11}$$

using (8),

$$\langle \rho \rangle_D \simeq \langle \rho \rangle_{2region} = \frac{\langle \rho \rangle_o a_o^3 + \langle \rho \rangle_u a_u^3}{a_o^3 + a_u^3} = \frac{\langle \rho \rangle_o a_o^3}{a_o^3 + a_u^3} \tag{12}$$

Covariant Scheme: we need to relate this theoretically calculated quantity with some observable in a sensible manner.

$$1 + z = \frac{1}{a_D}$$
$$H_D \frac{d}{dz}\left((1+z)^2 H_D \frac{dD_A}{dz}\right) = -4\pi G \langle \rho \rangle_D D_A \tag{13}$$

$D_A \rightarrow$ Angular Diameter Distance

Angular Diameter Distance - The angular diameter distance is defined as the ratio of an object's physical transverse size to its angular size (in radians). It is used to convert angular separations in telescope images into proper separations at the source.

Plots of Distances

D_H = Hubble Distance = $3000\ h^{-1}\ Mpc(h = 0.7)$

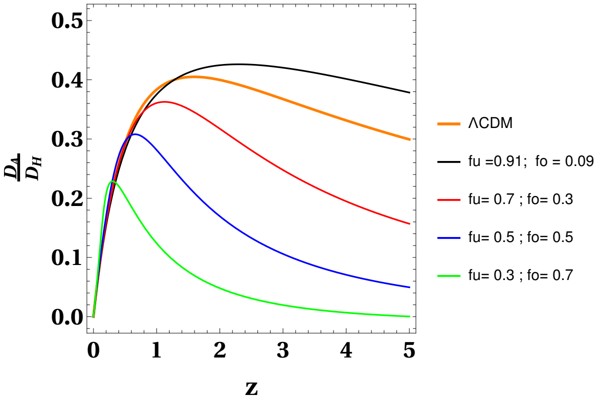

Figure 1: Plot of the ratio of angular diameter distance D_A to the present Hubble length-scale D_H w.r.t. redshift, for the ΛCDM case with different combinations of the fractions fu and fo for $\beta = 1$. For the combination $fu = 0.845$ and $fo = 0.155$, with $\beta = 1$, our model $\simeq$ ΛCDM model.

If we consider the whole universe as our global domain, then an N-body simulation [Wiegand(2010)] of large scale structure formation indicates that value of $(f_{o0}, f_{u0}) = (0.91, 0.09)$, (0→present time).

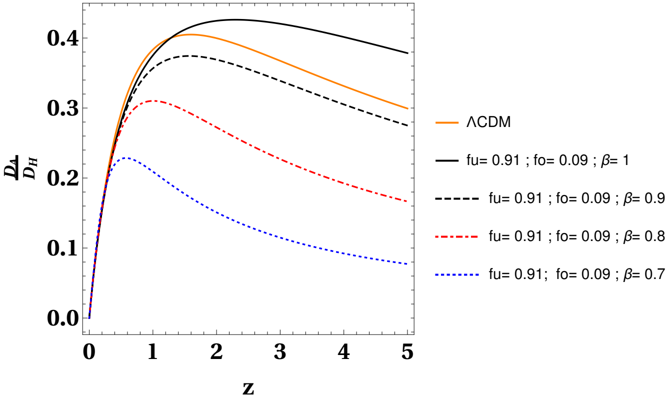

Figure 2: Plot of the ratio of angular diameter distance D_A to the present Hubble length-scale D_H w.r.t. redshift, for the ΛCDM case with different values of the parameter β, with the combination of the fractions $fu = 0.91$ and $fo = 0.09$. For $\beta = 0.92$, our model $\simeq$ ΛCDM.

Redshift dependent part of GW amplitude

$D_L = (1+z)^2 D_A$, and $(1+z) = \frac{1}{a_D}$.

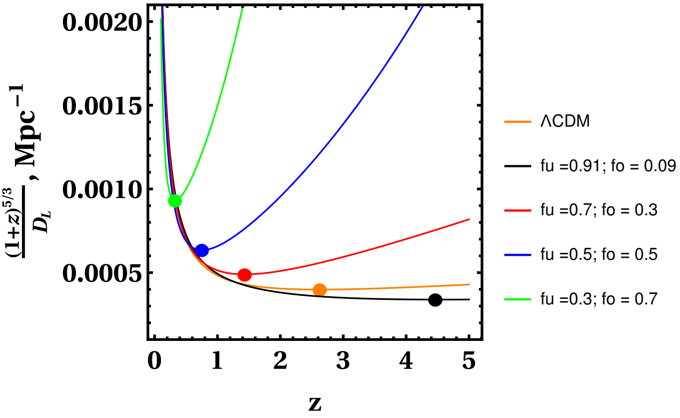

Figure 3: Plot of the redshift dependent part of the gravitational wave amplitude for various combinations of the volume fractions *fo* and *fu*, with $\beta = 1$. Position of the minima in curves have been denoted by dots.

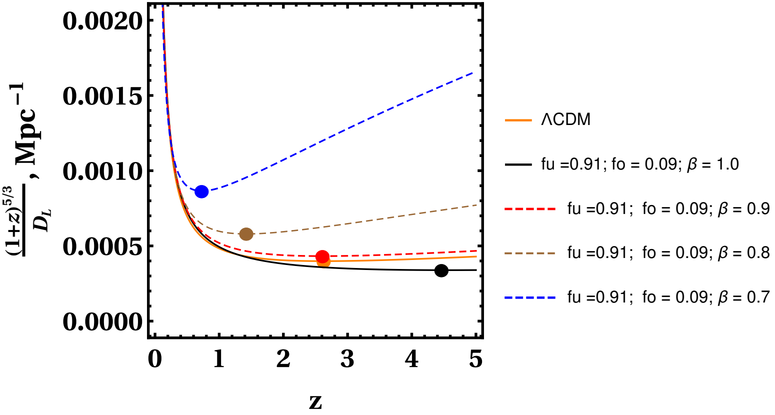

Figure 4: Plot of redshift dependent part of the gravitational wave amplitude w.r.t. redshift z, for the ΛCDM model and for our model for different values of the parameter β, for the combination of fractions $fu = 0.91$ and $fo = 0.09$. Position of the minima in curves have been denoted by dots. The shifting of the minima points is clearly visualized.

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Conclusions -

- We studied the impact of the inhomogeneities present in the underlying space on the amplitude of GWs - the amplitude of GWs comes out to be different for regions of spacetime with different fractions of f_u & f_o.
- The amplitude of GWs are also affected by the parameter β which governs the time evolution of the scale factor of the underdense region.

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