Reconstructing Cosmology
using
Principal Component Analysis

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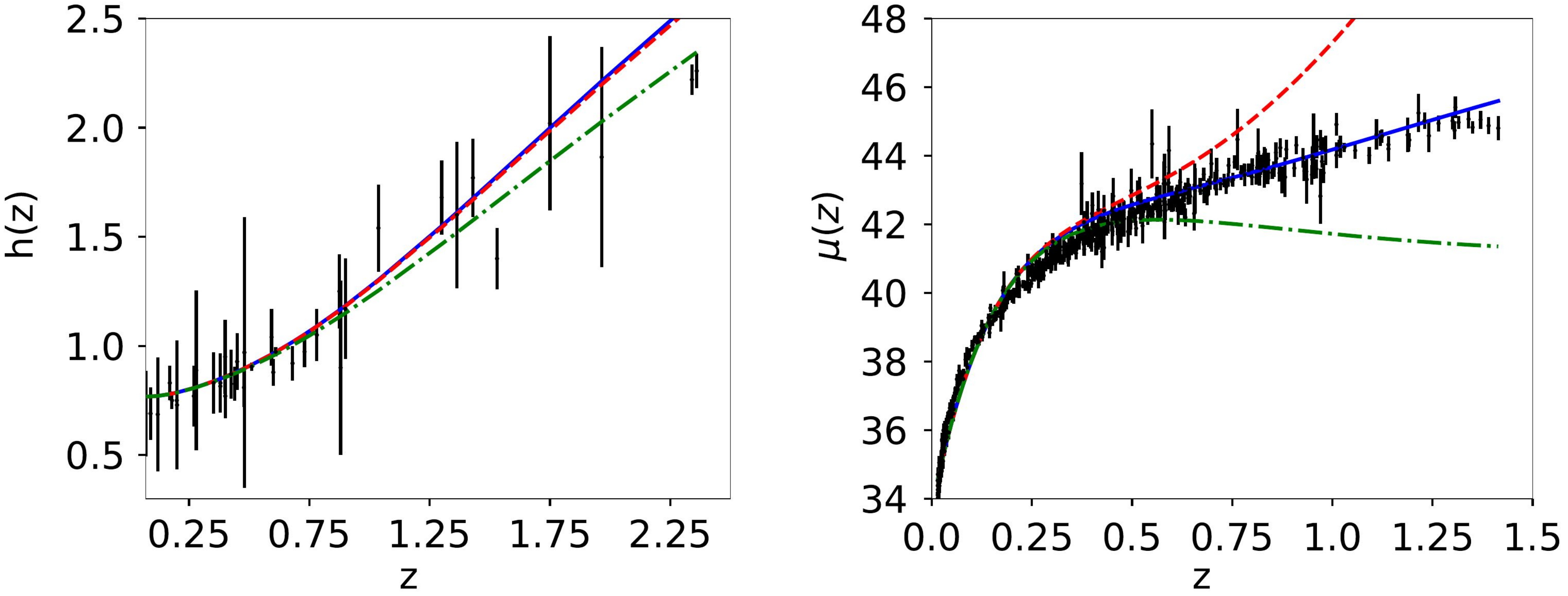
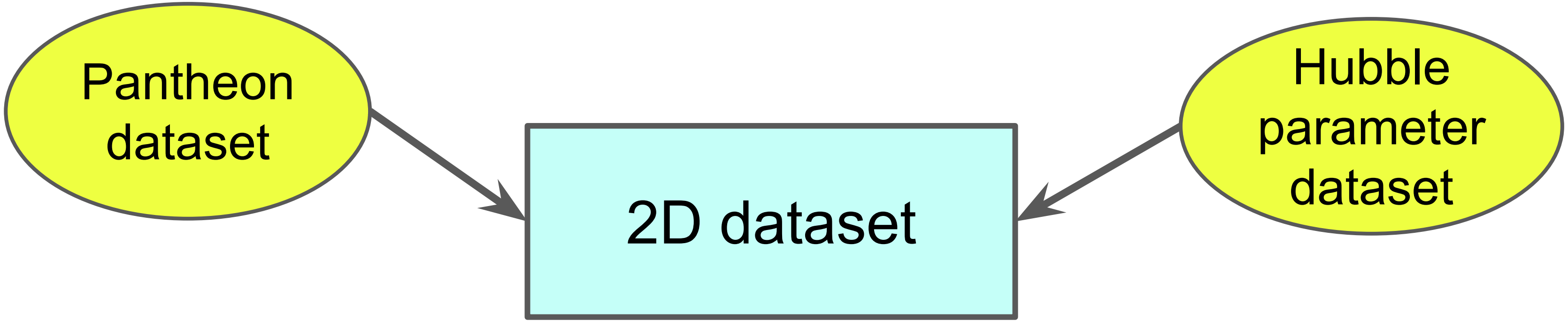
The output of the PCA variant



Content:

- Observational datasets being used and requirements of our methodology
- Principal Component Analysis (PCA) a general introduction
- Reconstruction of late-time cosmology using PCA
- Summary and Conclusion

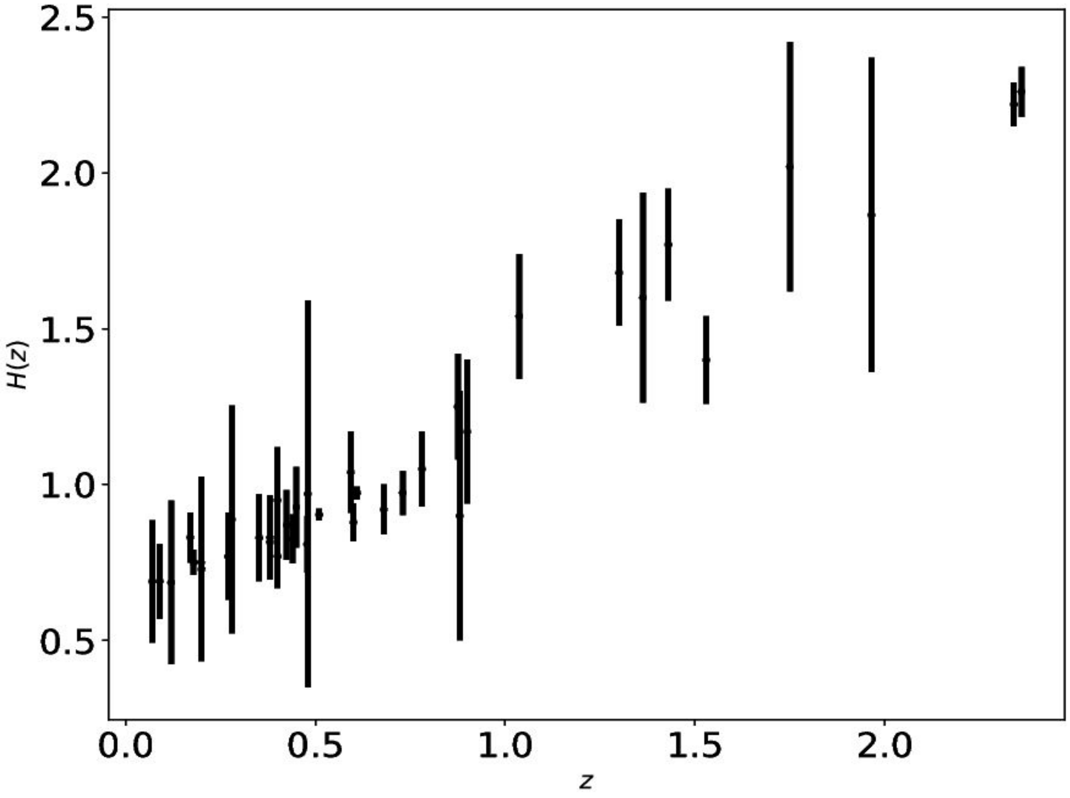
Observational datasets



z	m	σ_m
0.01	0.139	0.198
.	.	.
.	.	.

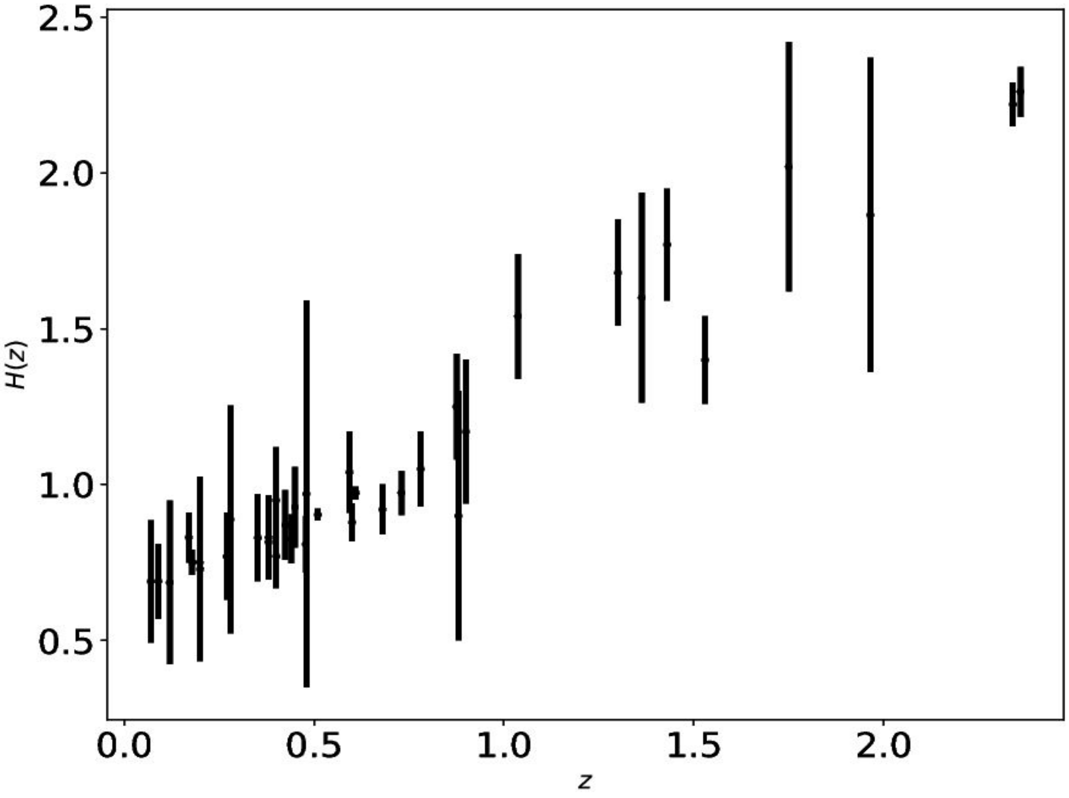
z	h	σ_h
0.07	0.69	0.196
.	.	.
.	.	.

Ways of reconstruction from data



- Reconstruction in model dependent parametric manner
- Reconstruction in model independent non-parametric manner
- Reconstruction through Principal Component Analysis(PCA)

Ways of reconstruction using PCA



- Using Fisher Matrix approach
- Using different realisation from dataset in hand
- Algorithm of PCA + Correlation coefficient calculation (CCC)

Algorithm of reconstruction from PCA

1) Expressing the dependent variable in a polynomial

Independent Unit basis

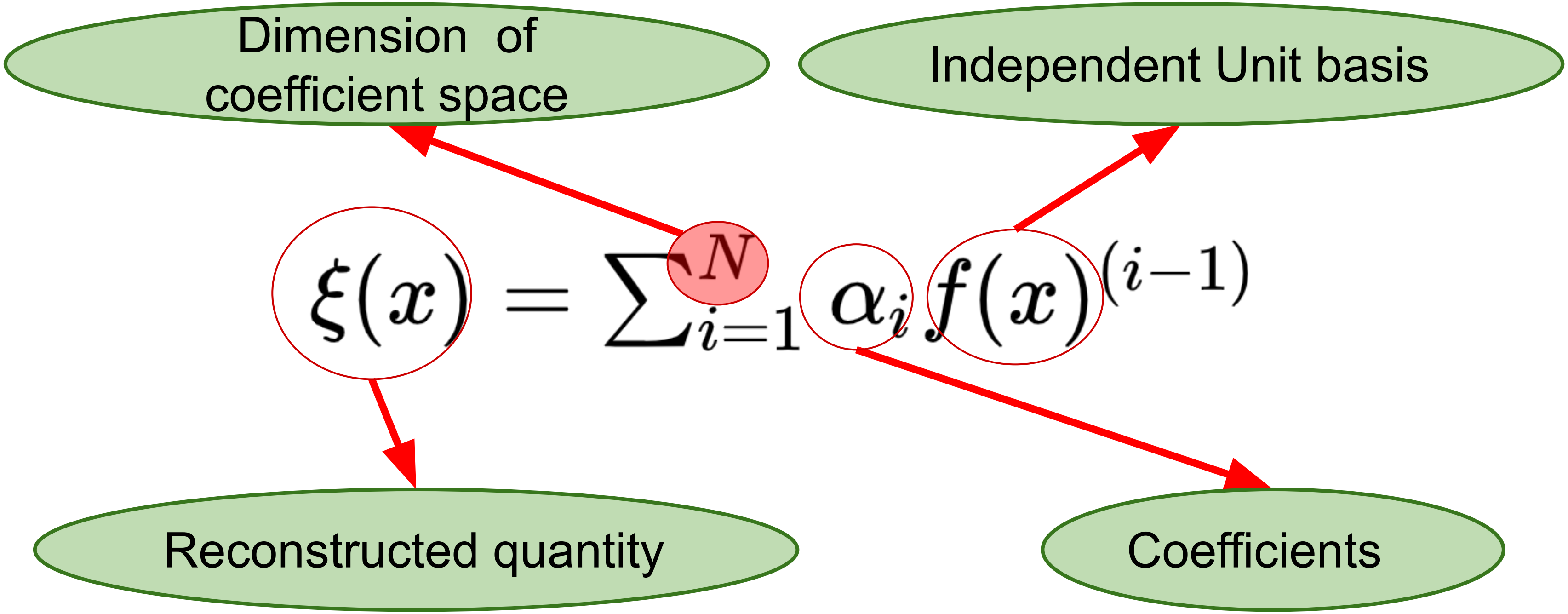
$$\xi(x) = \sum_{i=1}^{N} \alpha_i f(x)^{(i-1)}$$

Reconstructed quantity

Coefficients

Algorithm of reconstruction from PCA

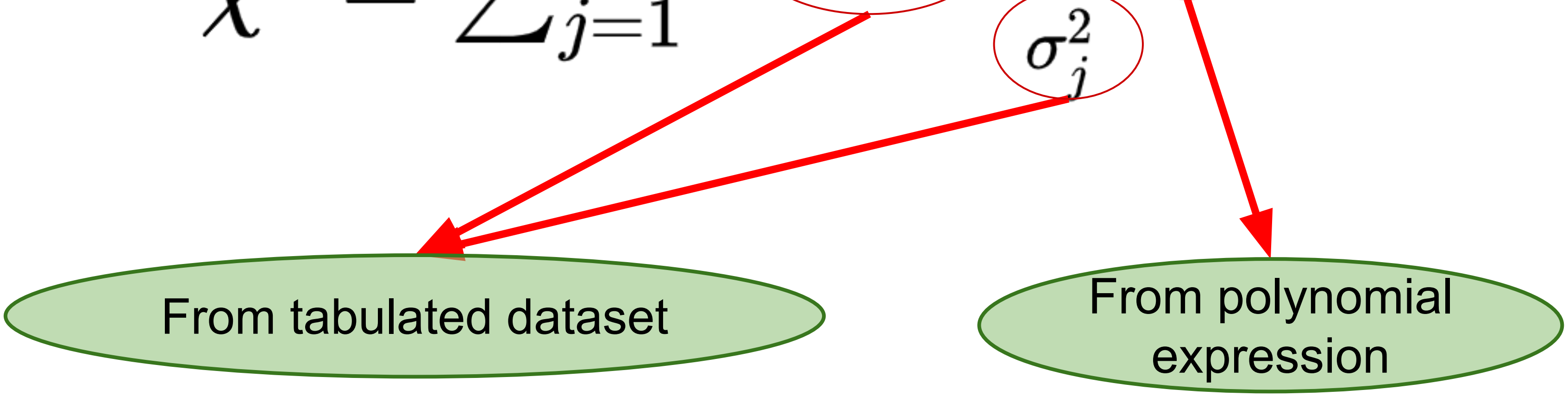
1) Expressing the dependent variable in a polynomial



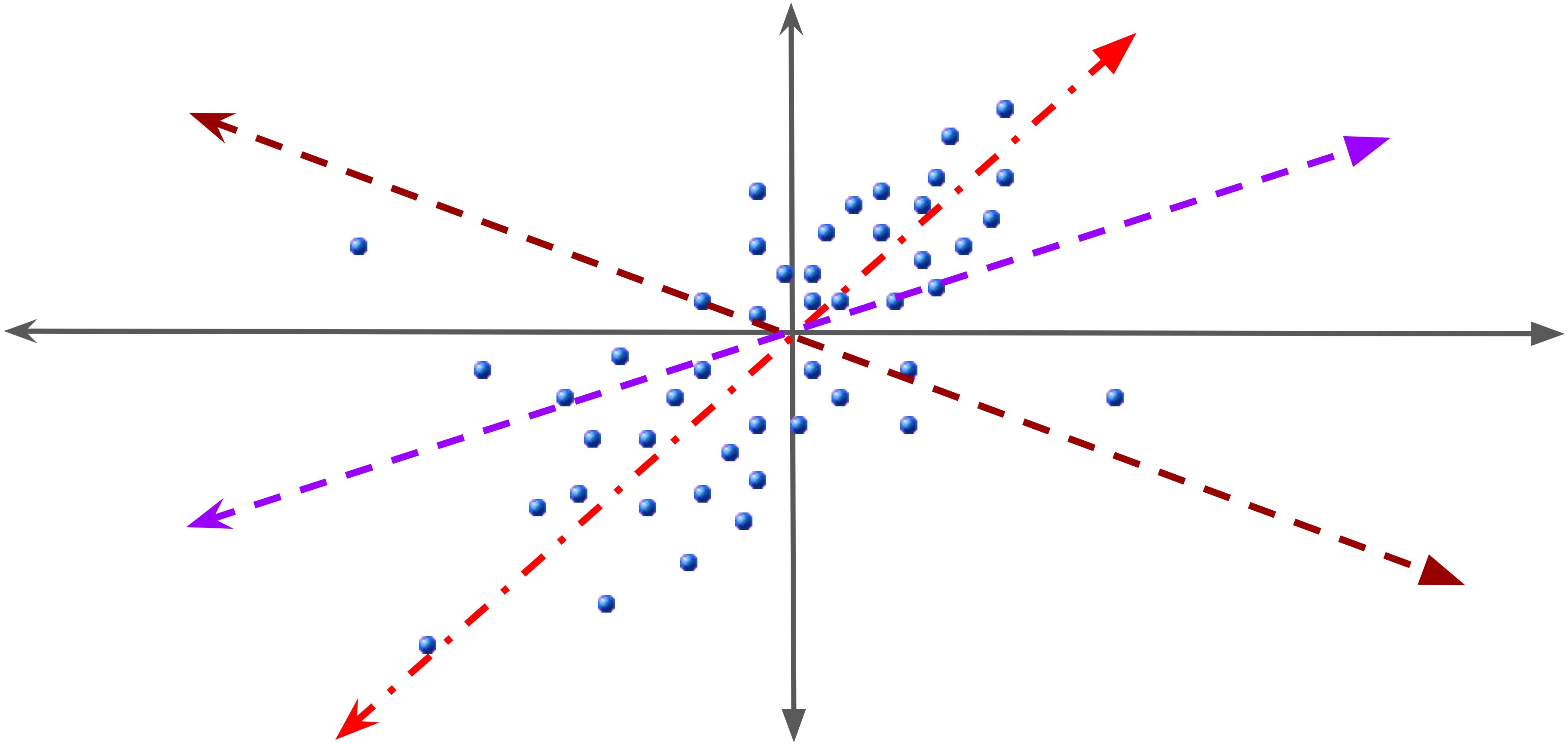
Algorithm of reconstruction from PCA

2) Division of N-dim coefficient space and the calculation of data-matrix of PCA

$$\chi^2 = \sum_{j=1}^{k} \frac{\left(\xi(x)_{data} - \xi(\{b_i\},x)\right)^2}{\sigma_j^2}$$



Algorithm of reconstruction from PCA



Algorithm of reconstruction from PCA

3) PCA data-matrix

$$\mathbf{Y} = \begin{pmatrix} b_1^{(1)} & b_2^{(1)} & \cdots & b_n^{(1)} \\ b_1^{(2)} & b_2^{(2)} & \cdots & b_n^{(2)} \\ b_1^{(3)} & b_2^{(3)} & \cdots & b_n^{(3)} \\ \vdots & \vdots & \ddots & \vdots \\ b_1^{(N)} & b_2^{(N)} & \cdots & b_n^{(N)} \end{pmatrix}$$

N —-> Number of dimension of coefficient space

n —–-> Number of patches created

Algorithm of reconstruction from PCA

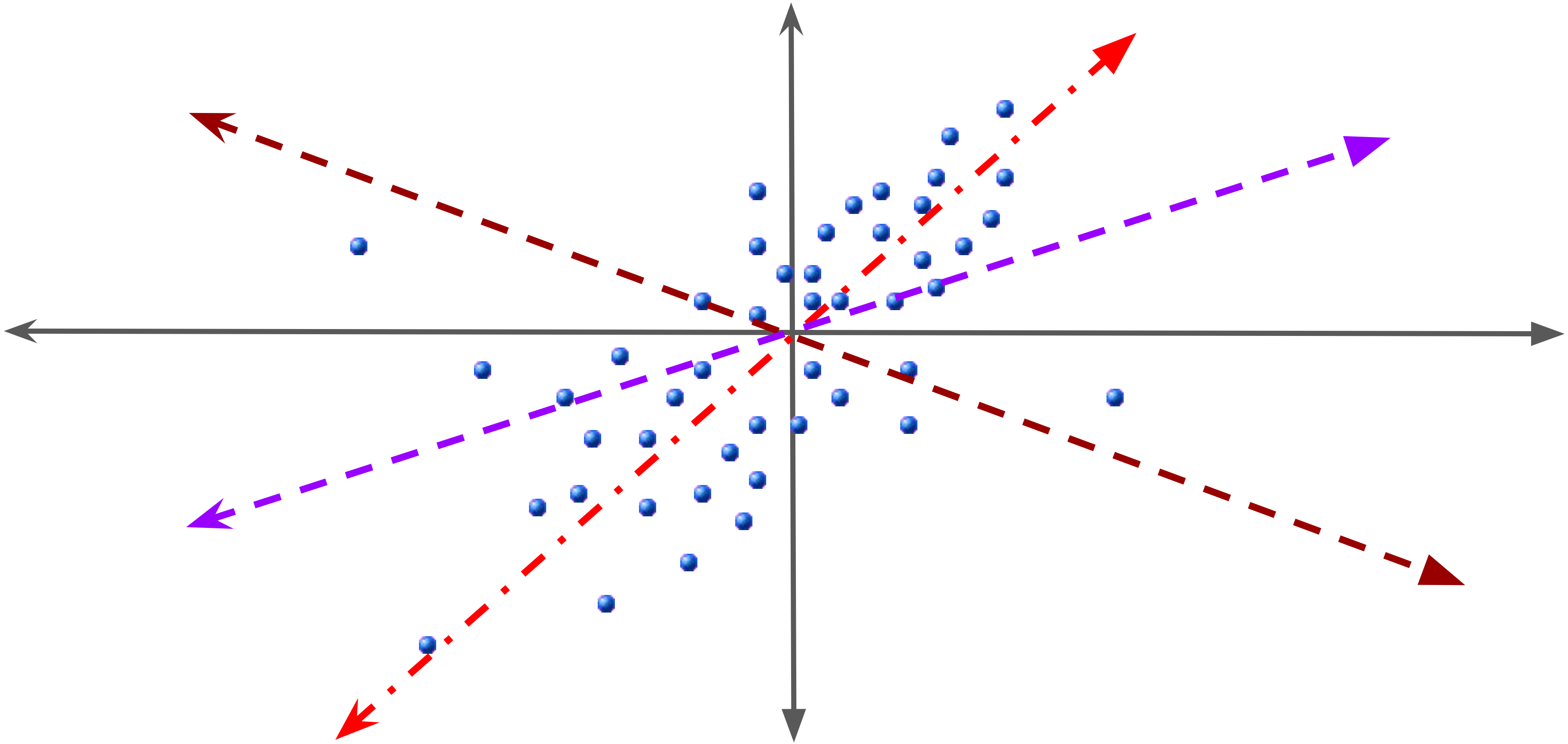
4) Covariance matrix $$\mathbf{C} = \frac{1}{n}\mathbf{Y}\mathbf{Y}^T$$

$$\mathcal{C} = \begin{matrix} \sigma_{b_1}^2 & Cov(b_1, b_2) & Cov(b_1, b_3) \\ Cov(b_2, b_1) & \sigma_{b_2}^2 & Cov(b_2, b_3) \\ Cov(b_3, b_1) & Cov(b_3, b_2) & \sigma_{b_3}^2 \end{matrix}$$

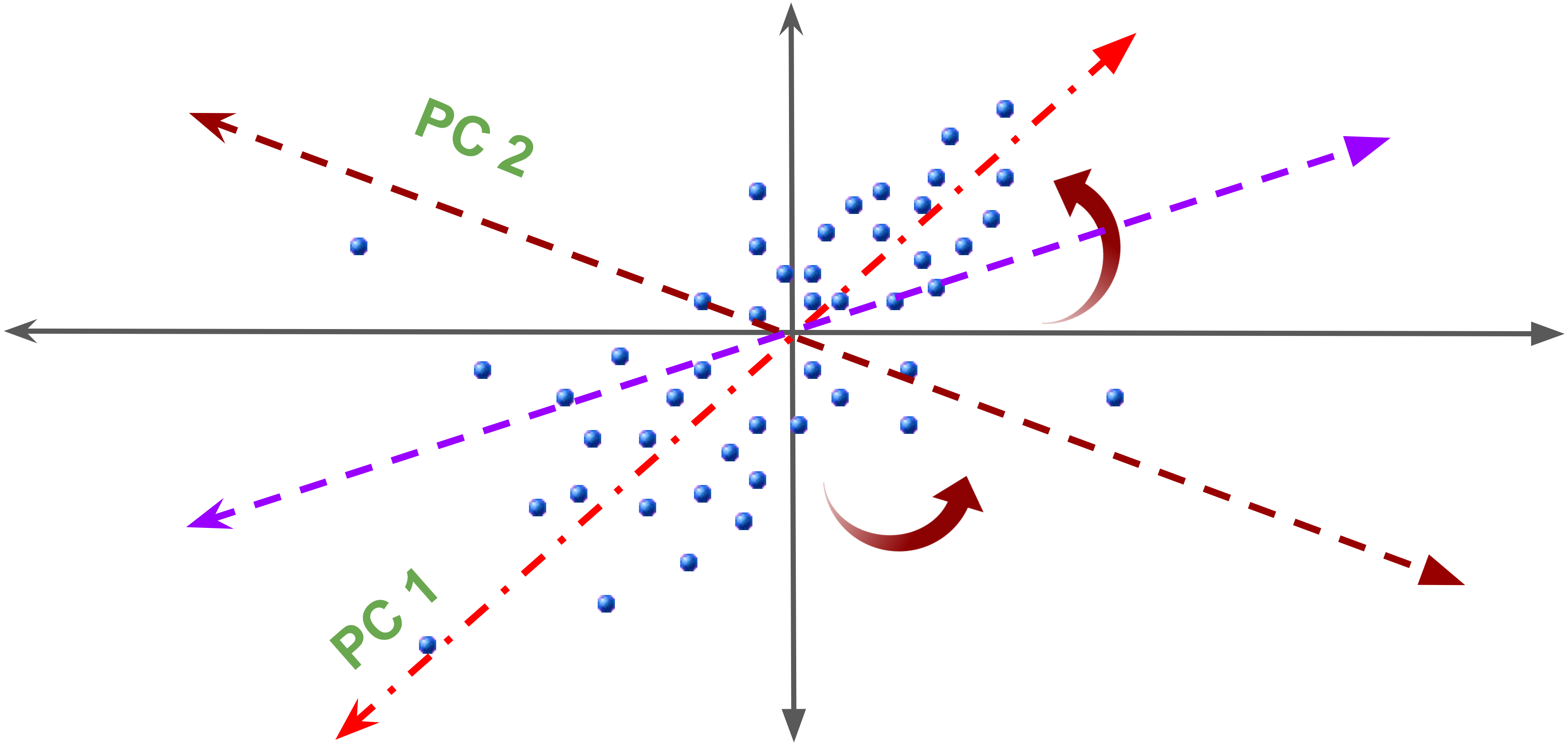
From noise

From signal

Algorithm of reconstruction from PCA

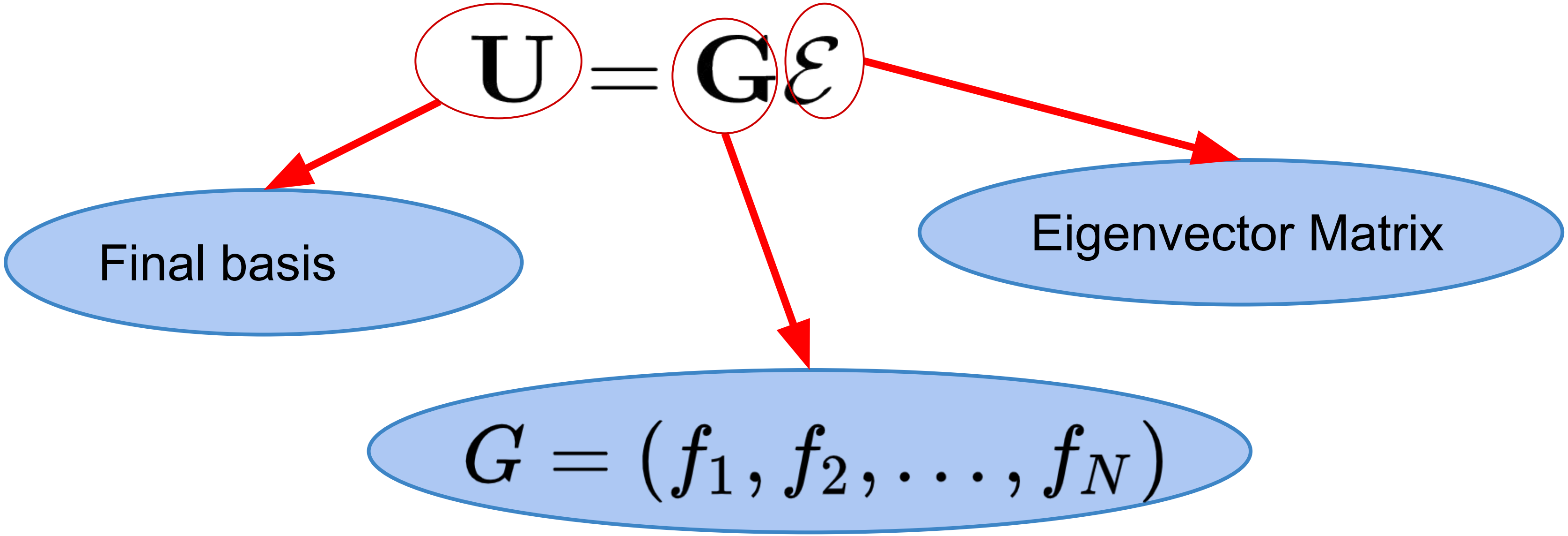


Algorithm of reconstruction from PCA



Algorithm of reconstruction from PCA

5) Diagonalisation of the covariance matrix and finding of Eigenfunctions



Algorithm of reconstruction from PCA

6) Reduction of dimension and the final form

Reduced final dimension

Final basis

$$\xi(x) = \sum_{i=1}^{M} \beta_i u(x)^{(i-1)}$$

Final coefficients

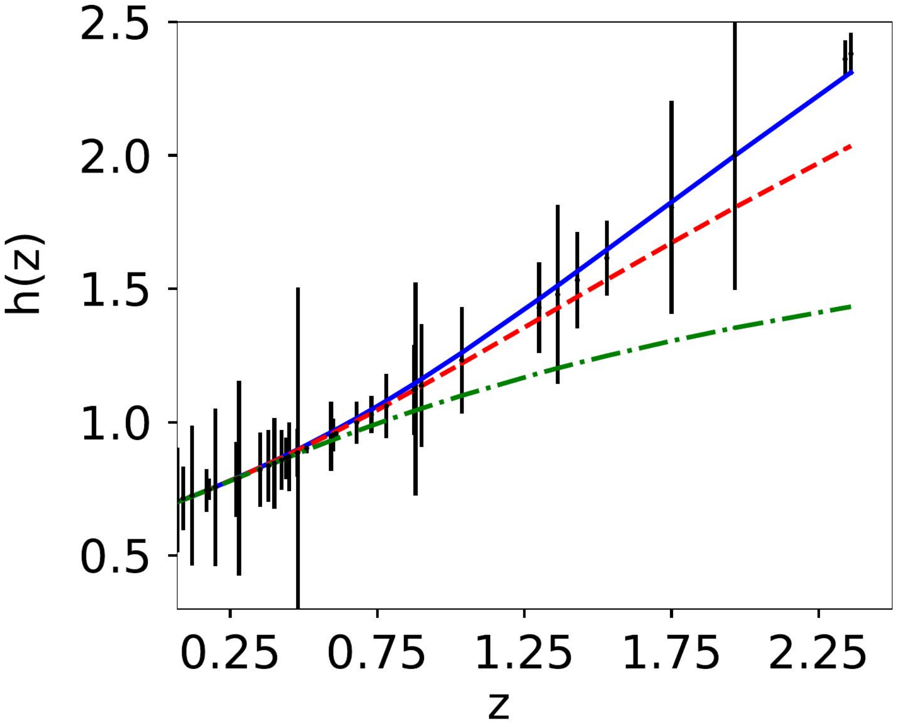
Reconstruction of w(z) derived approach

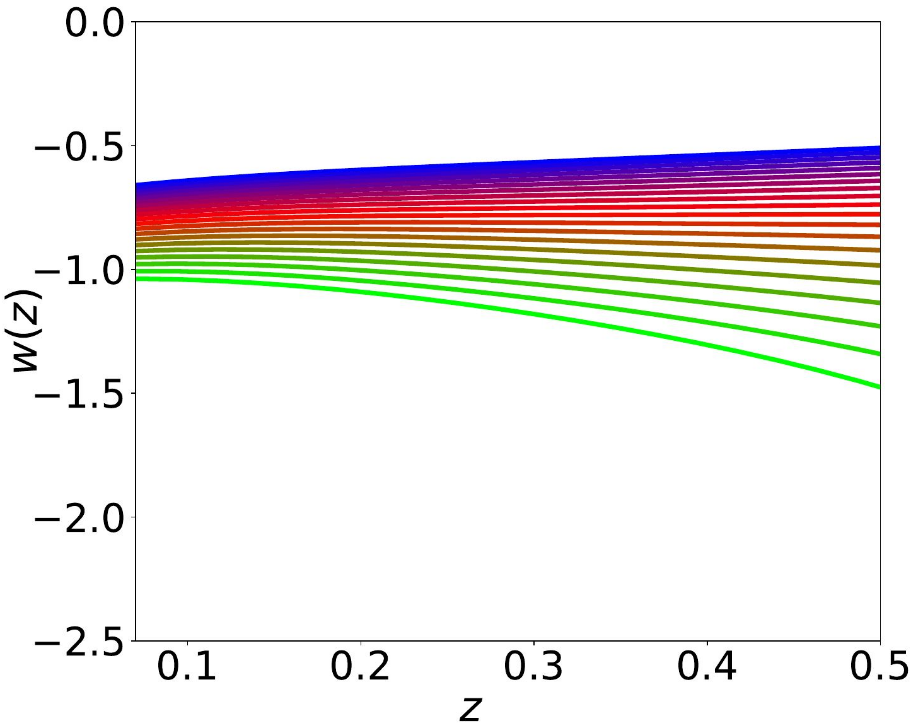
$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_x e^{3\int_0^z \frac{1+w(z')}{1+z'}dz'}\right]$$

From PCA

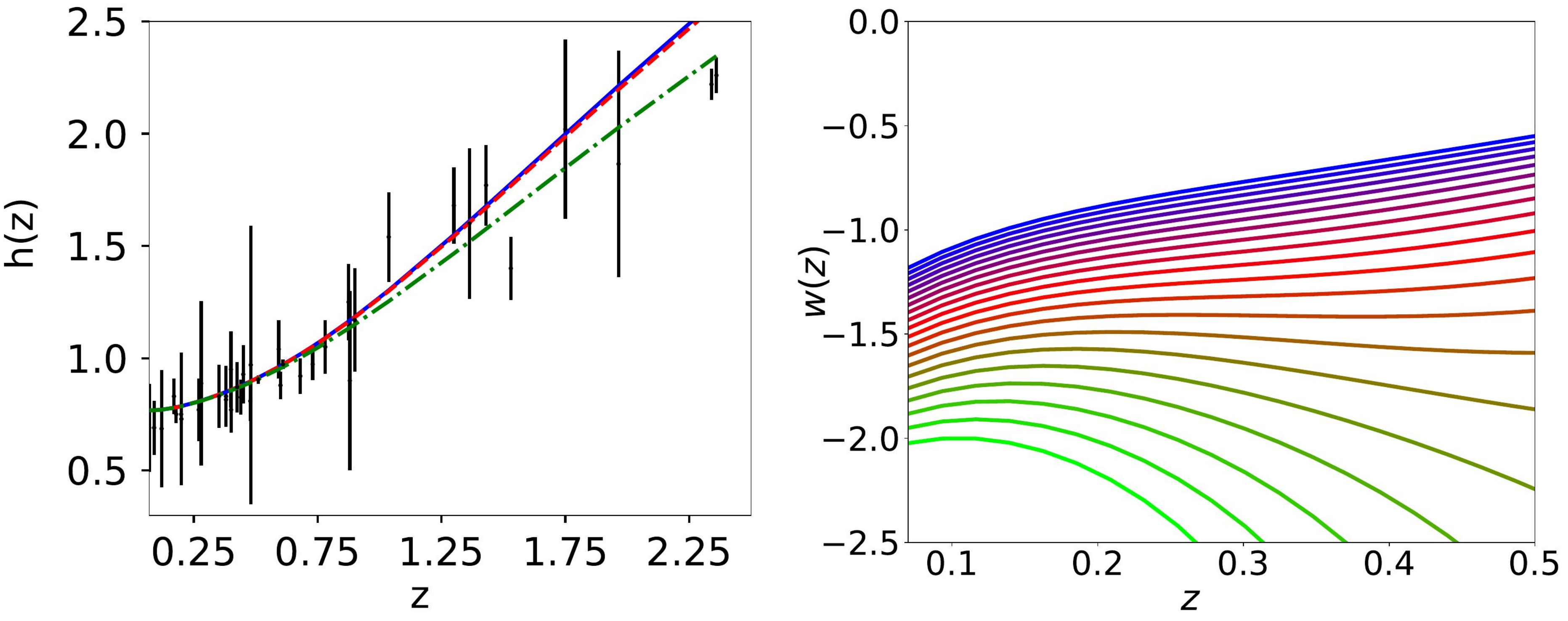
$$w(z) = \frac{3H^2 - 2(1+z)HH'}{3H_0^2(1+z)^3\Omega_M - 3H^2}.$$

Results(mock) [basis: (1-a)] Hz





Results(real) [basis: (1-a)] Hz



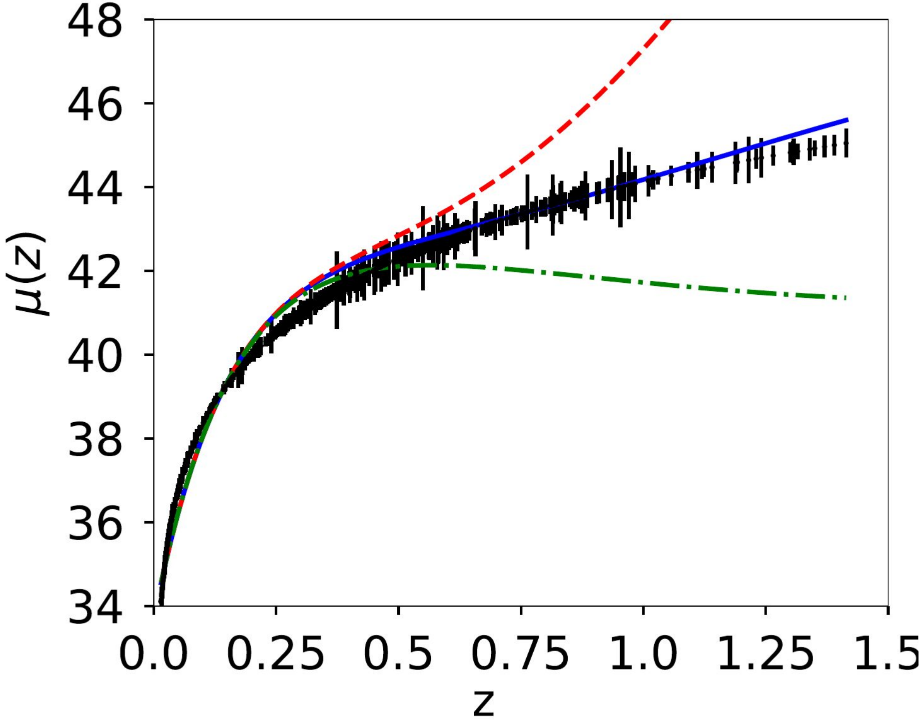
Reconstruction of w(z) derived approach

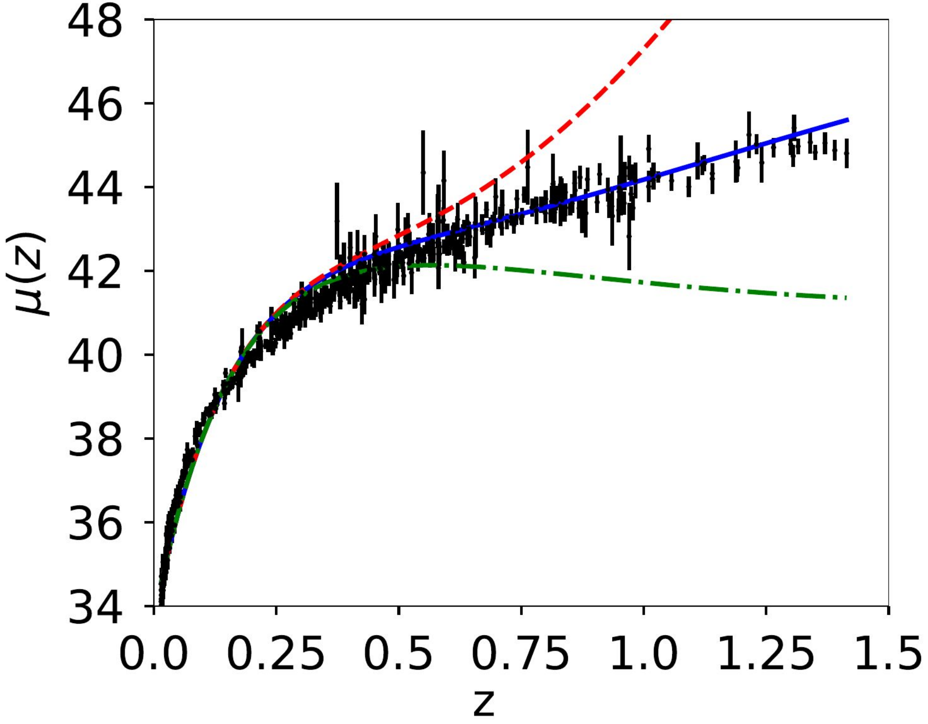
$$\mu(z) = 5\log\left(\frac{d_L}{1Mpc}\right) + 25$$

$$\mu \longrightarrow d_L \longrightarrow w$$

$$d_L(z) = \frac{c}{H_0}(1+z)\int_0^z \left(\Omega_m(1+z)^3 + \Omega_x e^{3\int_0^z \frac{(1+w(z'))dz'}{(1+z')}}\right)^{-1/2} dz$$

Results(derived approach) [basis: (1-a)] SNIa





Summary and Conclusion :

- **PCA + CCC** gives a script to reconstruct a quantity in a non-parametric way.
- We need only the 2D dataset as input to reconstruct the quantity. **Choice of basis function and number of PCs comes intrinsically from the algorithm.**
- **PCA + CCC** implies a slowly varying dark-energy equation of state parameter.

References :

- **“Reconstruction of late-time cosmology using Principal Component Analysis**”, Ranbir Sharma, Anakan Mukherjee, H K Jassal, *EPJP (2022)137:219*
- **“Inference of model parameter from Principal Component Analysis**”, Ranbir Sharma, H K Jassal, *arxiv::2211.13608*

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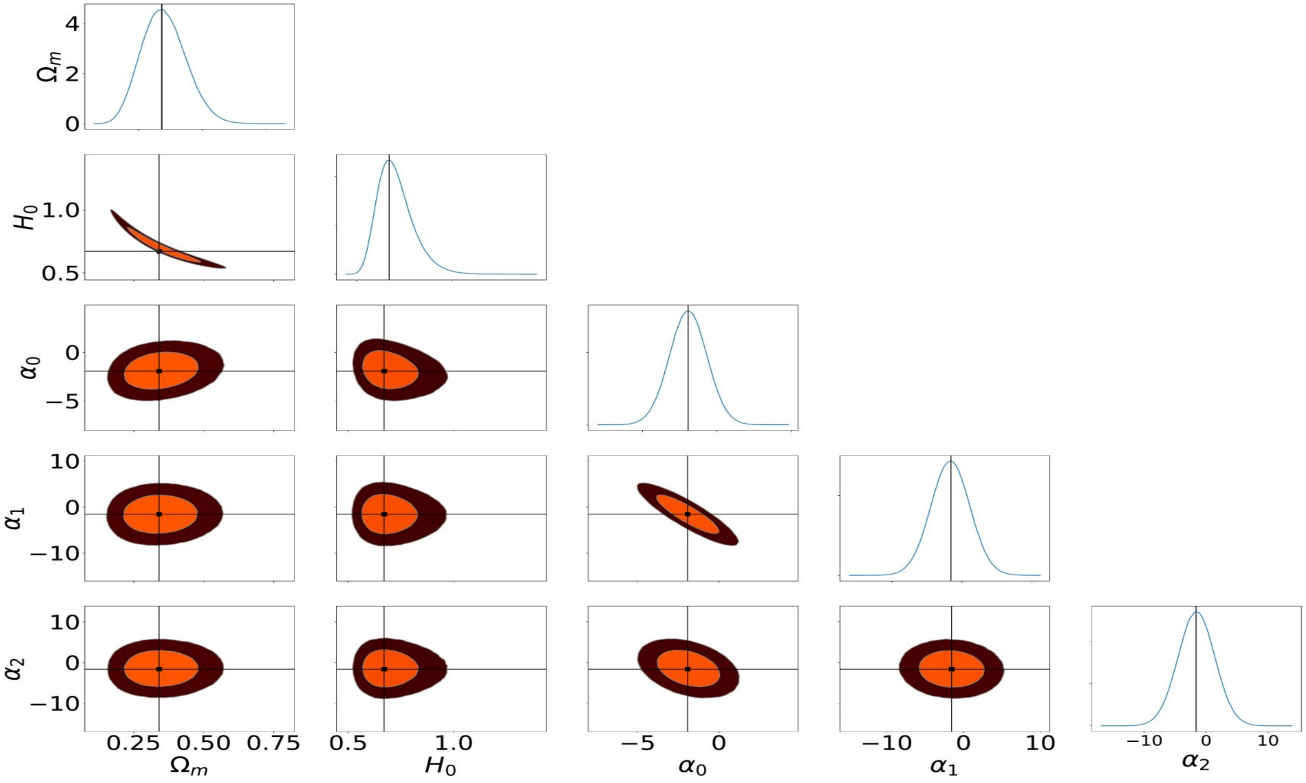
Inference of model's parameters from PCA



Error in PCA

- By masking the under-fitting points of the PCA-data-matrix we re-create the cov-matrix

- EigenValues of the cov-matrix gives the error information of the reconstruction

$$\sigma(\xi(x)) = \left[\sum_{i=1}^{M} \sigma^2(\beta_i) e_i^2(x)\right]^{\frac{1}{2}}$$

Correlation coefficient calculation (CCC)

Pearson CC → L → $$\rho = \frac{\mathrm{Cov}(A,B)}{\sigma_A \sigma_B}$$

Spearman CC → NL → $$r = \frac{\mathrm{Cov}(r_A, r_B)}{\sigma_{r_A} \sigma_{r_B}}$$

Kendall CC → NL → $$\tau = \frac{\text{Actual Score}}{\text{Maximum Possible Score}}$$

Inference of model's parameters from PCA

$$\chi^2 = \sum_{j=1}^{k} \frac{(\xi(x)_{PCA} - \xi(\{b_i\},x))^2}{\sigma_j^2}$$

From PCA + CCC

Error function from PCA

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_x e^{3\int_0^z \frac{1+w(z')}{1+z'}dz'}\right]$$

Inference of model's parameters from PCA

$$\chi^2 = \sum_{j=1}^{k} \frac{\left((\xi(x))_{PCA} - \xi(\{b_i\}, x) \right)^2}{\sigma_j^2}$$

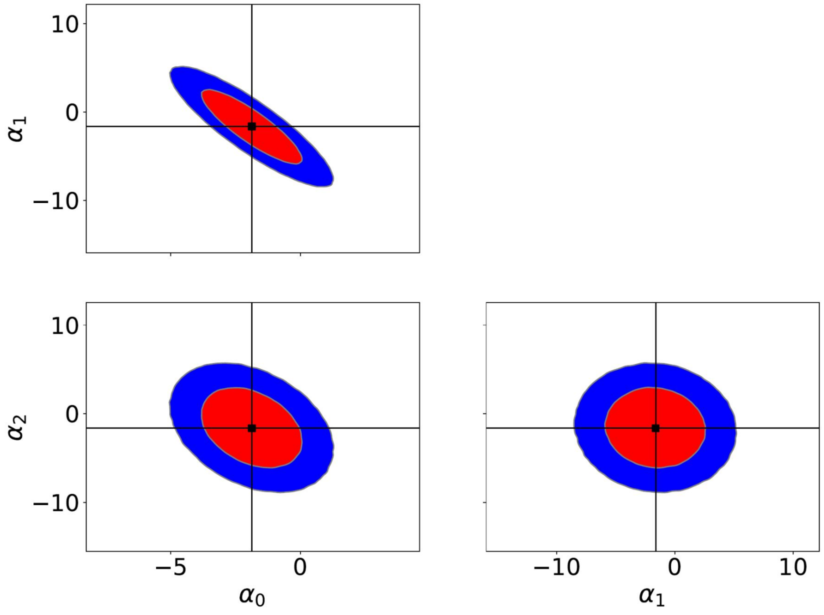
From PCA
+ CCC

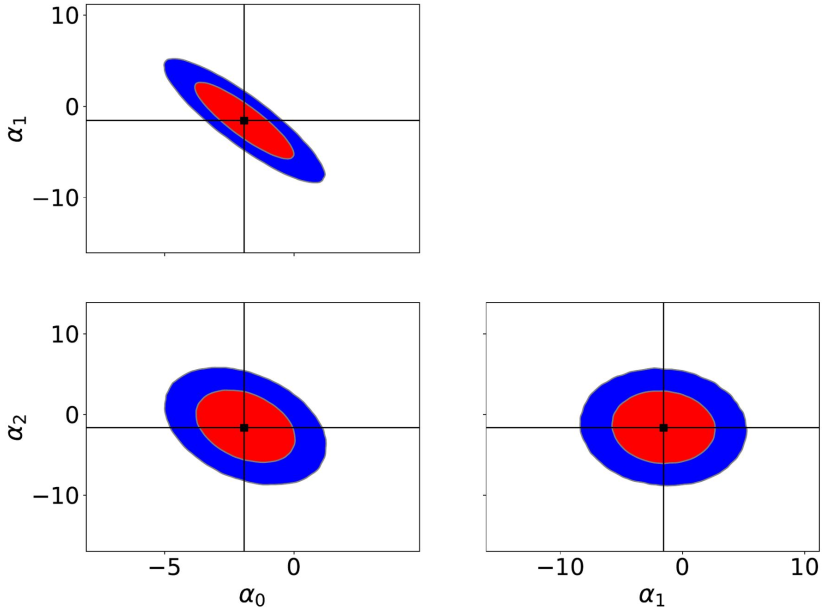
Er

$$w(z) = \sum_{i=1}^{m} \beta_i \Gamma(z)^{i-1}$$

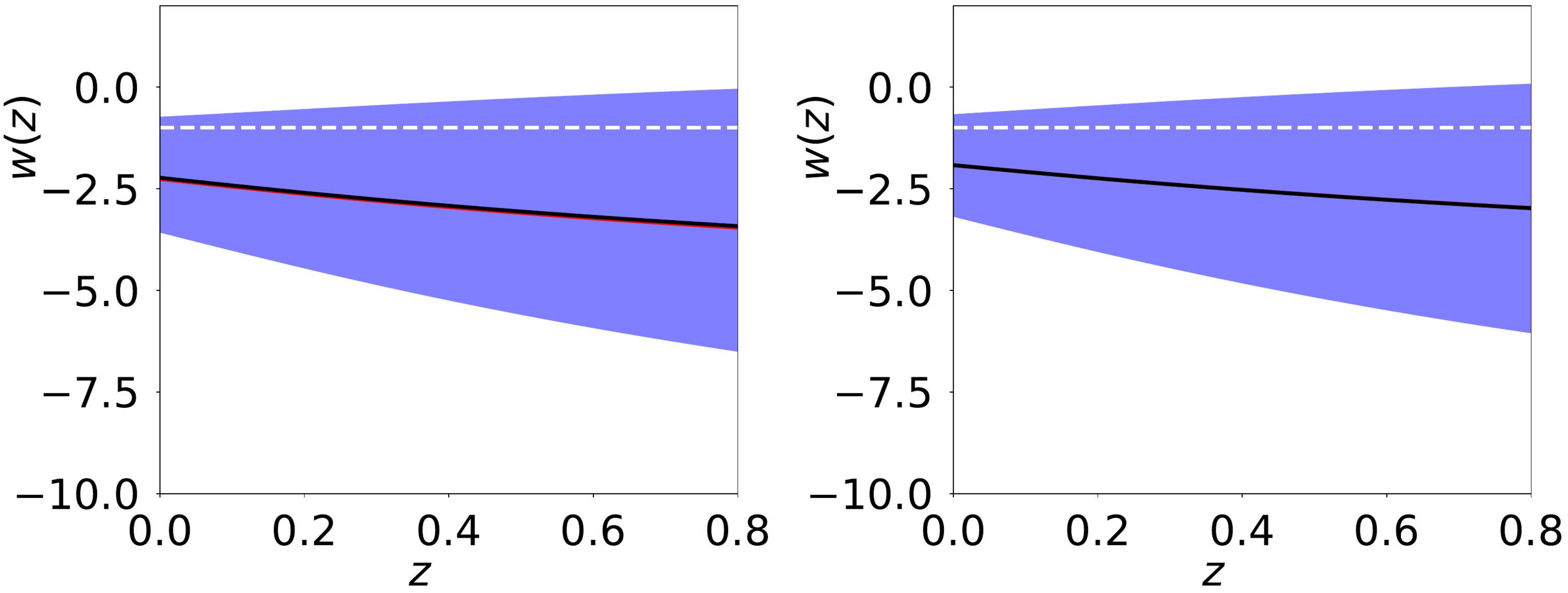
$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_x e^{3 \int_0^z \frac{1+w(z')}{1+z'} dz'} \right]$$

Inference of model's parameters from PCA

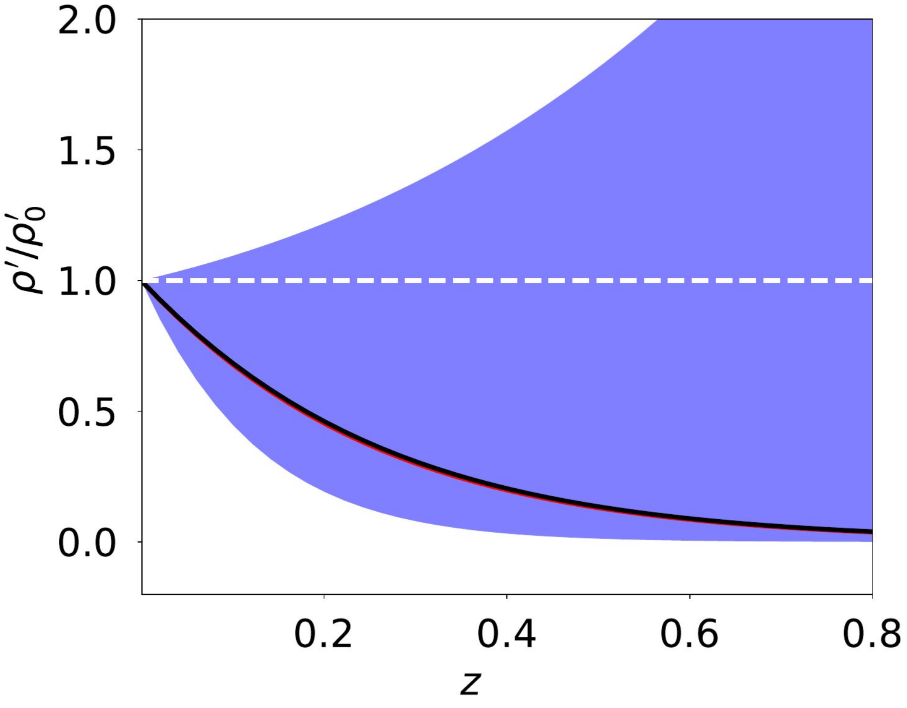


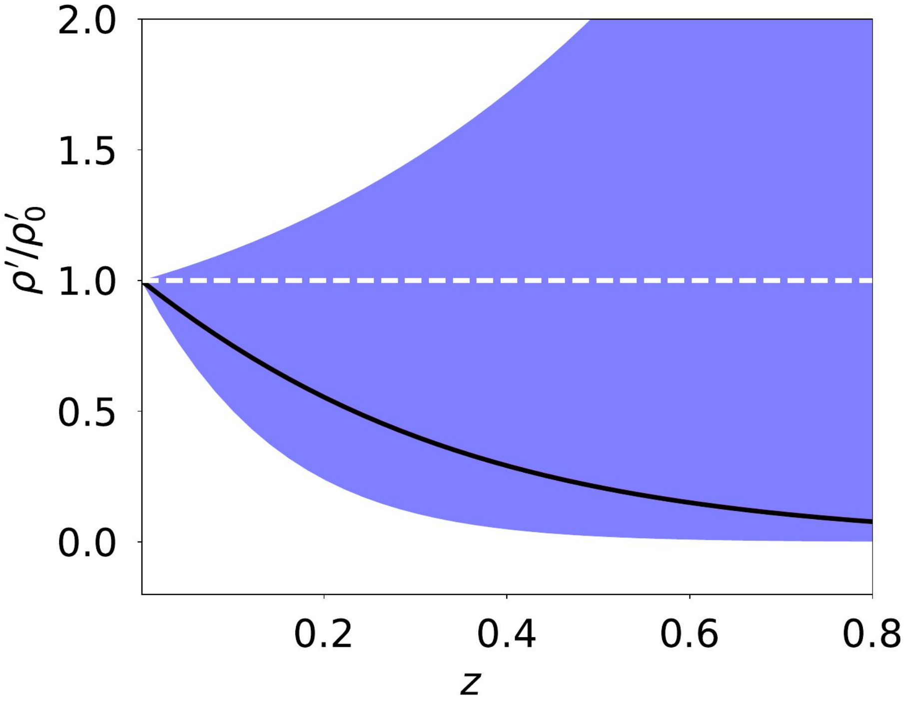


Inference of model's parameters from PCA

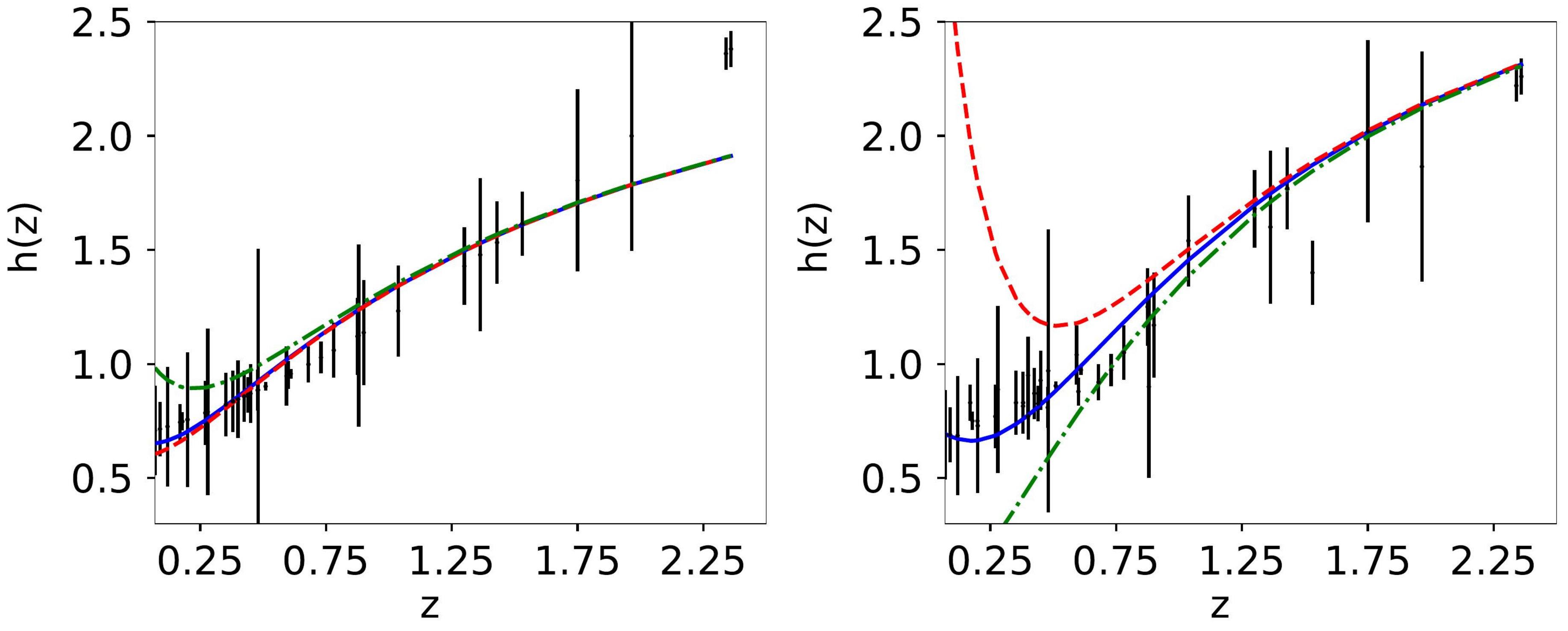


Inference of model's parameters from PCA

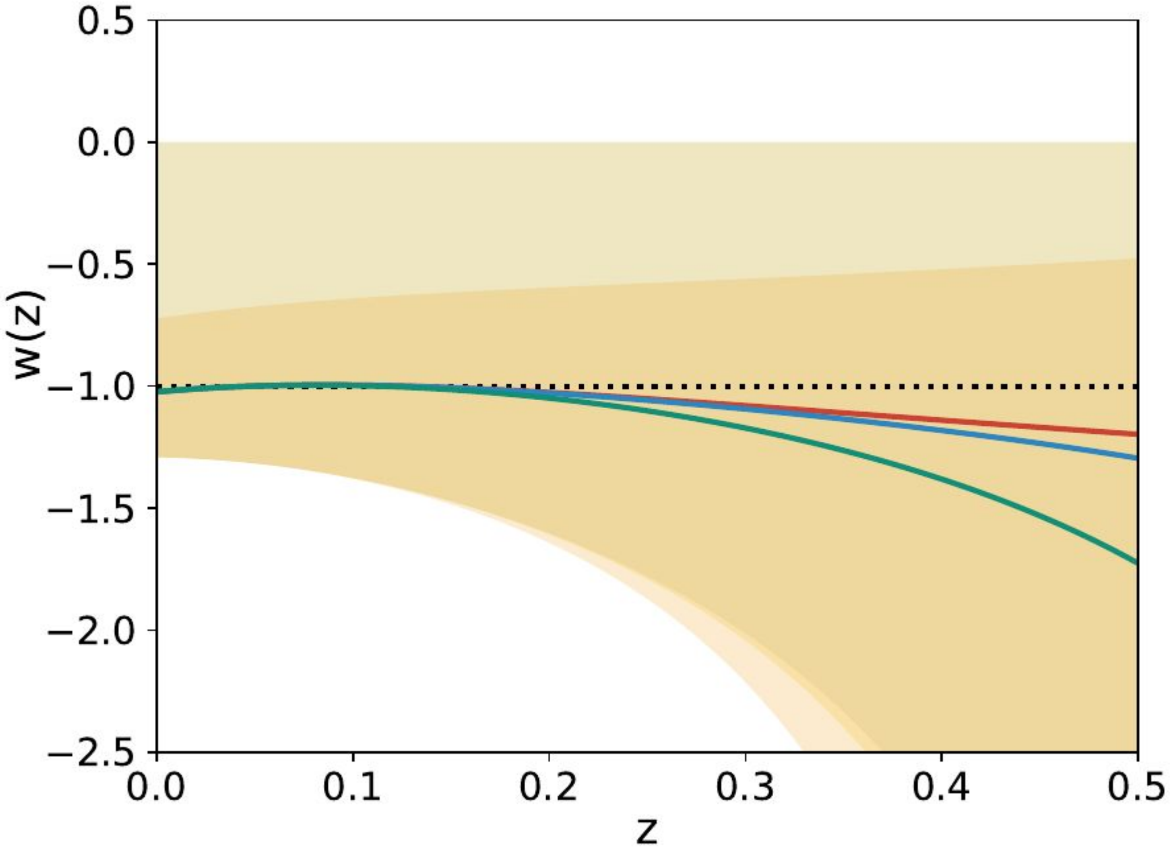




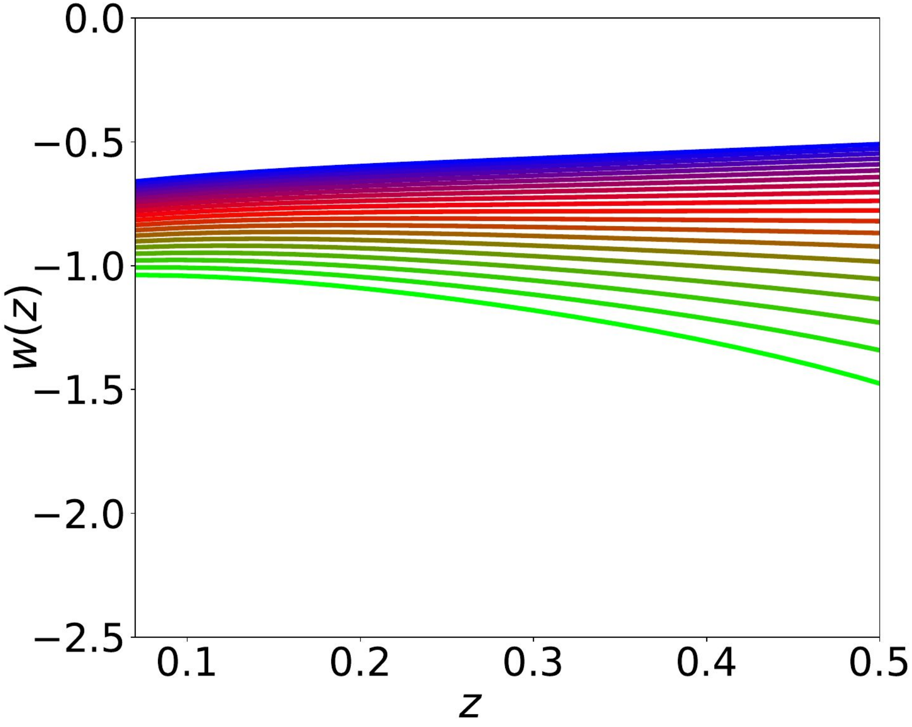
Results(derived approach) [basis: a] Hz

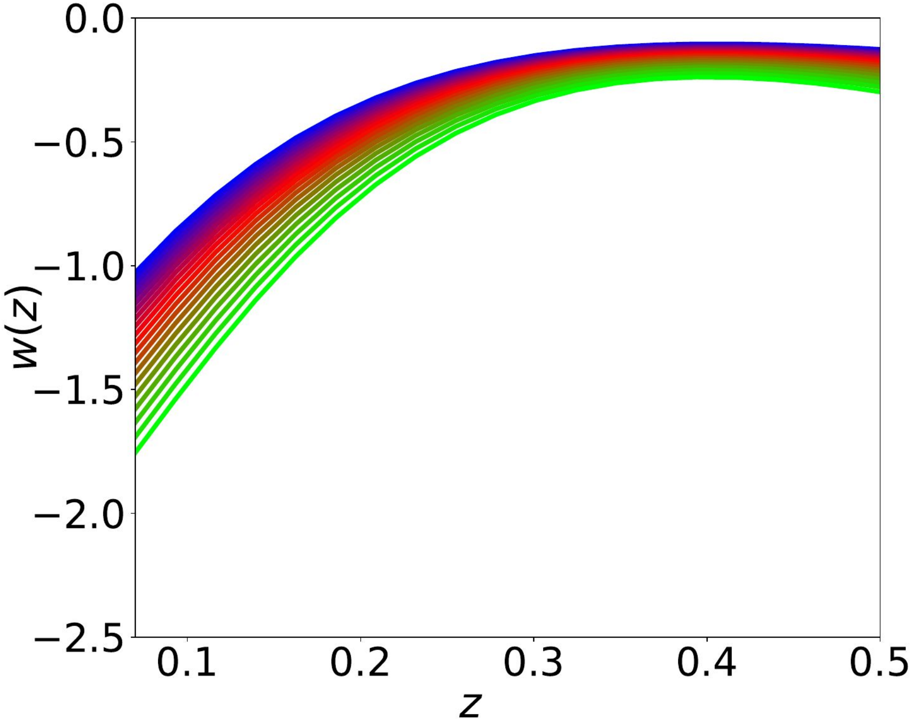


Results: w(z) from Hz simulated data

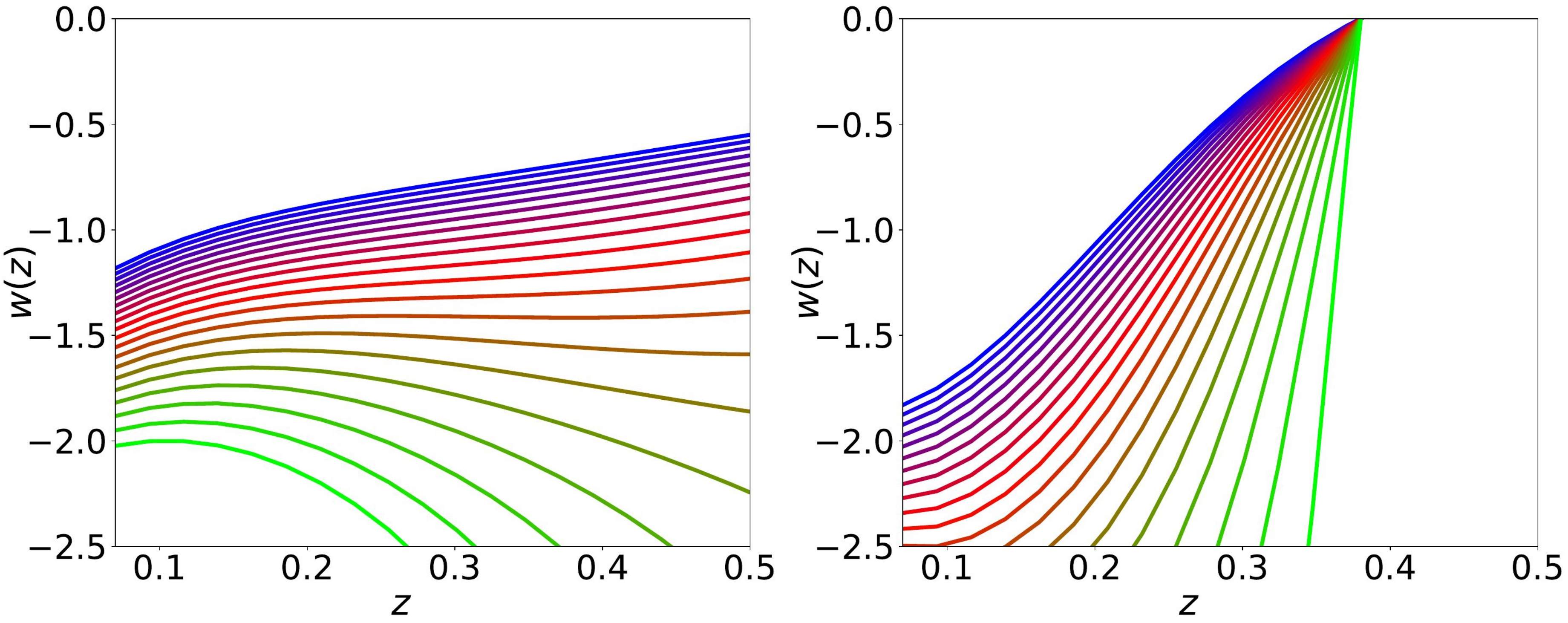


Results: w(z) from Hz simulated data

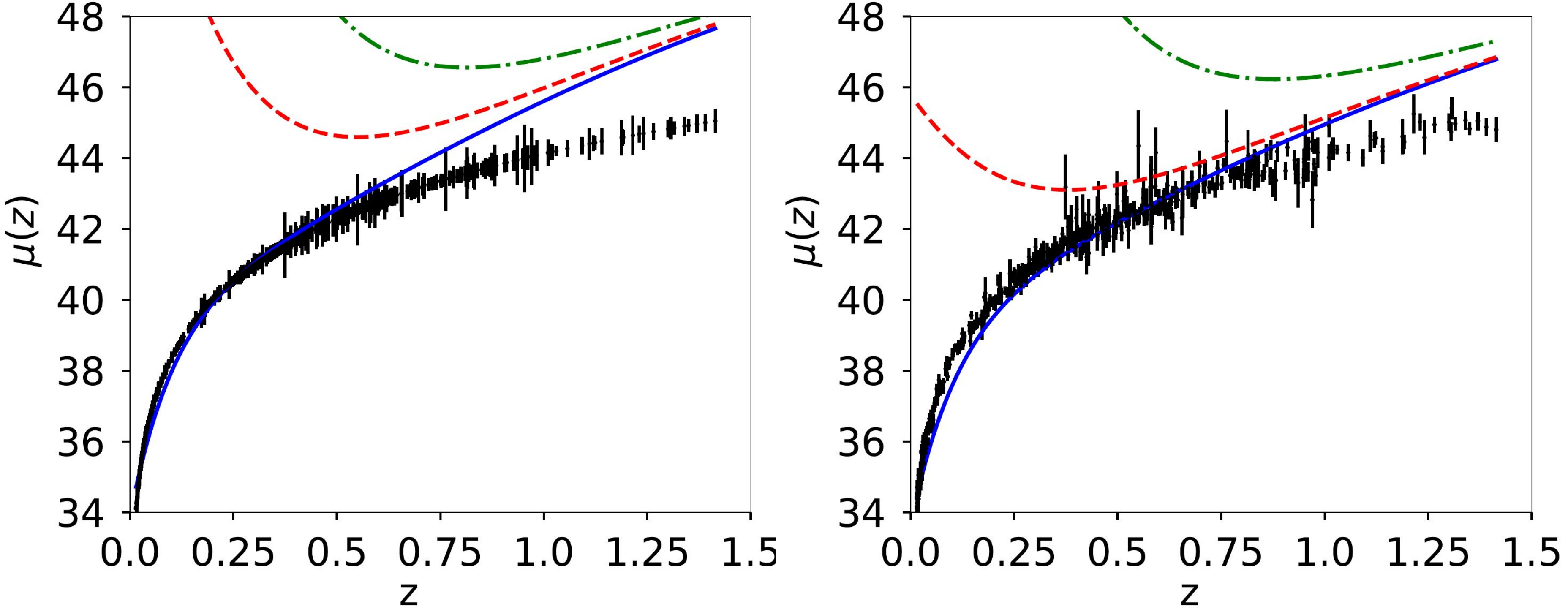




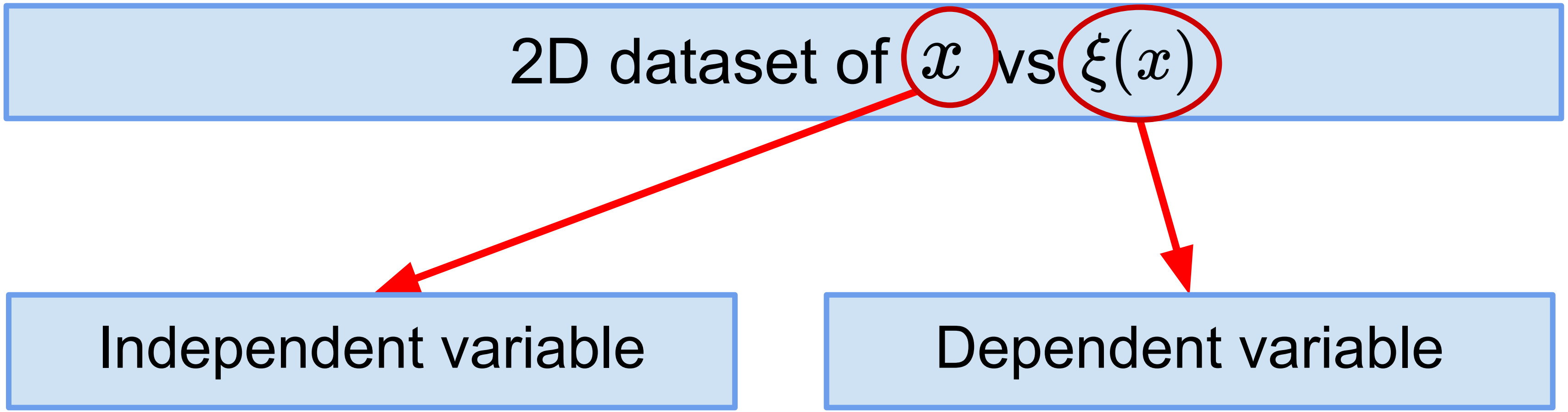
Results: w(z) from Hz real data



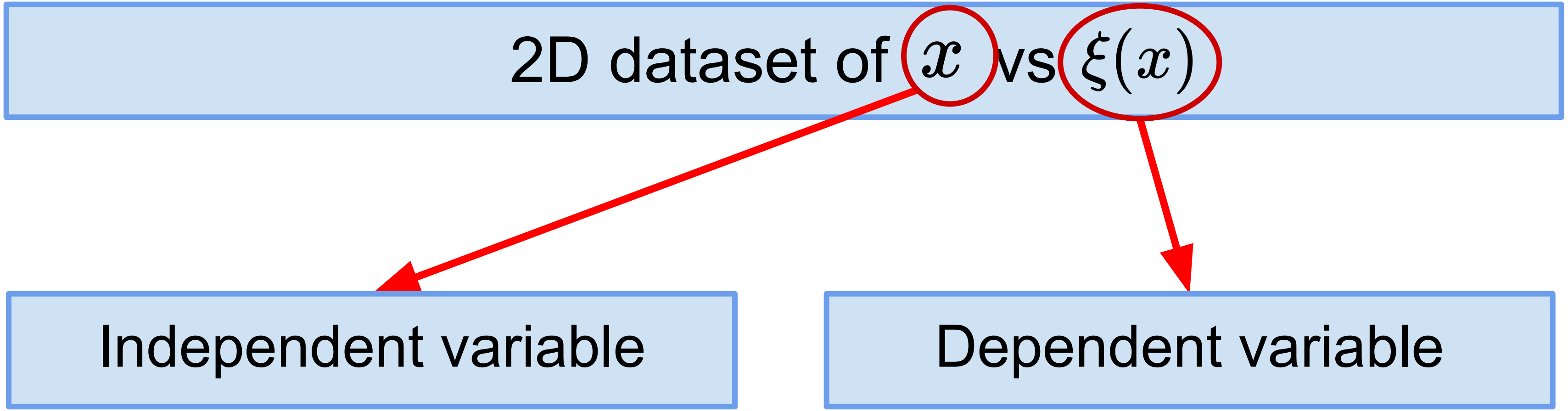
Results(derived approach) [basis: a] SNIa



Requirements of reconstruction



Requirements of reconstruction



Data should have **lesser** non-linear correlation than the linear