Collinear Functions for QCD Resummations

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Outline

- Introduction

  Transverse Momentum Resummation

- Computational Framework

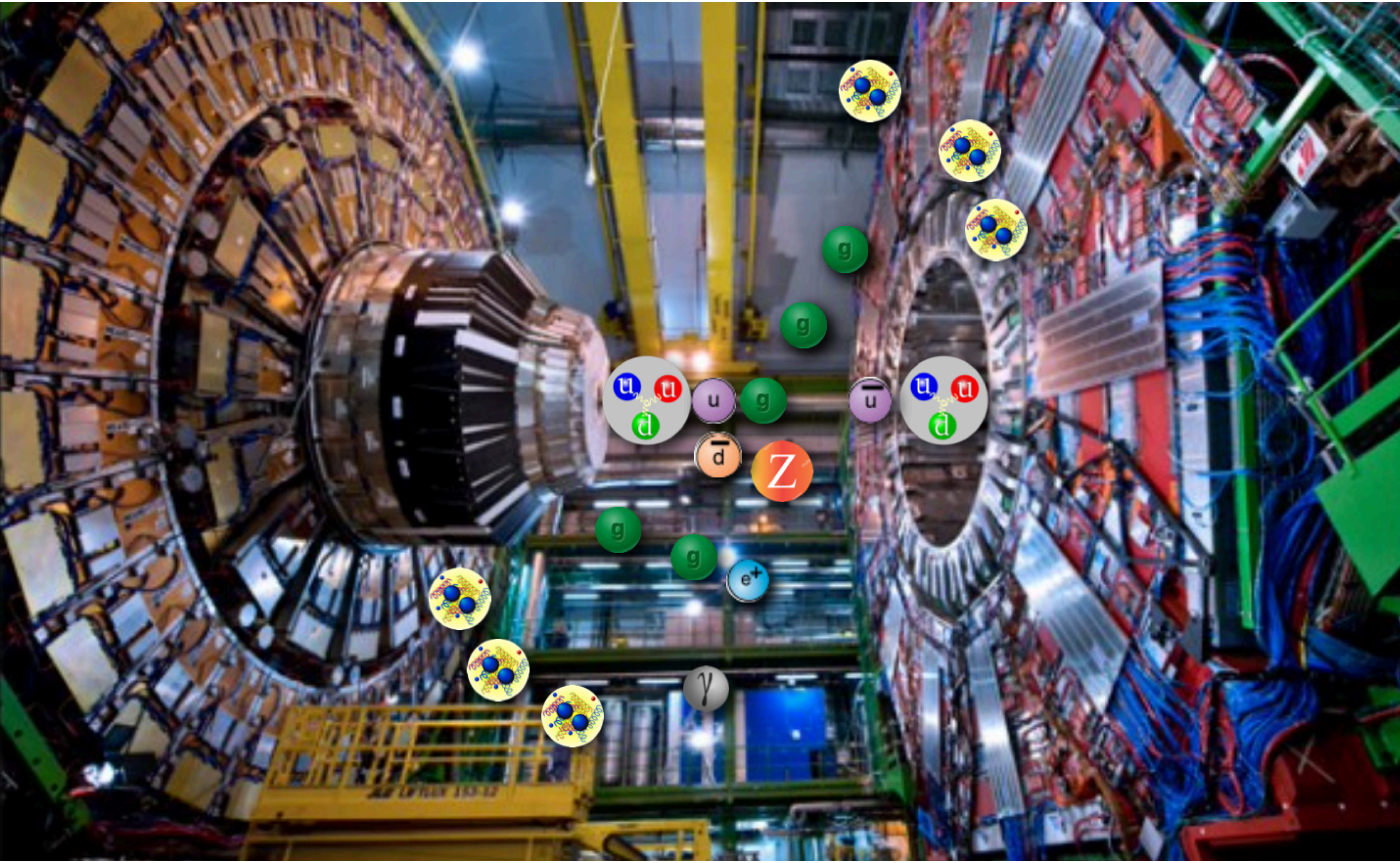
  Collinear Factorisation → Time-Like (TL), Space-Like (SL)

  Collinear Functions → Differential, Integrated

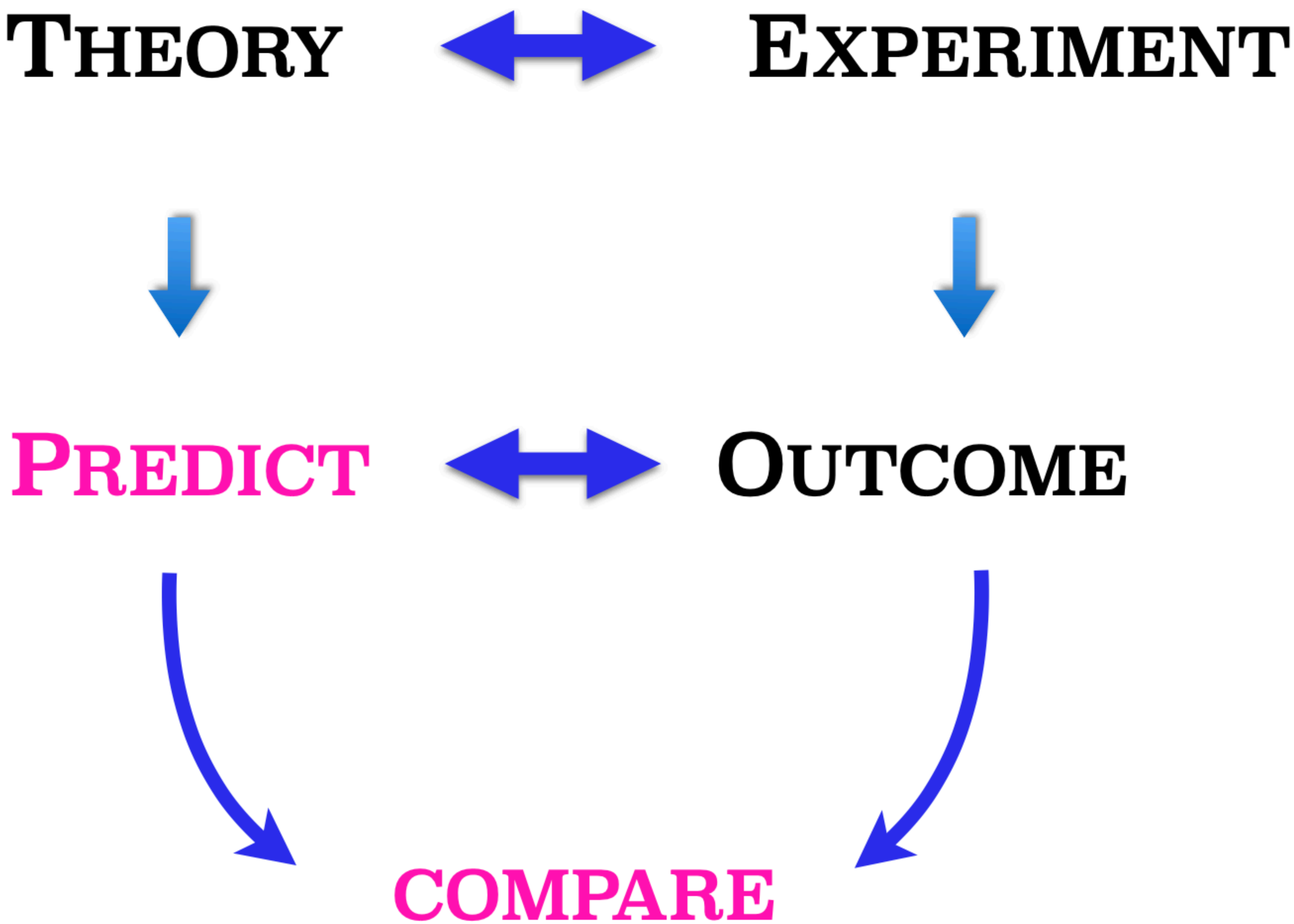
- Perturbative Results up to NNLO

- Summary & Outlook

The Large Hadron Collider



Thumb Rule for a Theoretical Physicist

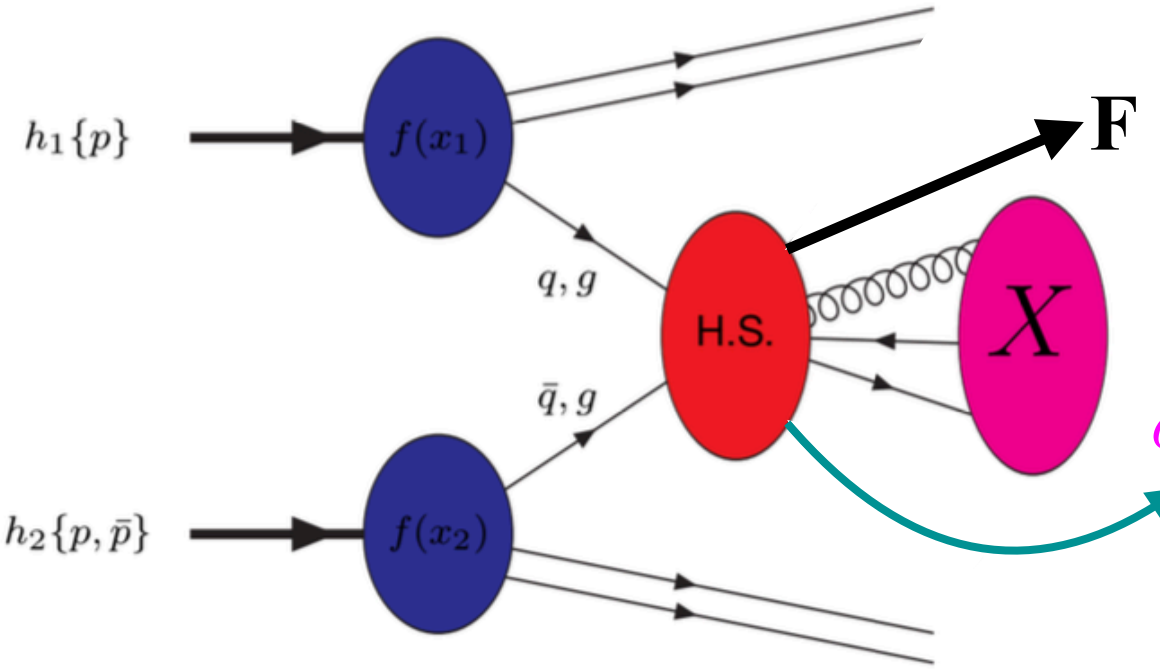


QCD Factorisation: Short Range and Long Range

$$h_1(p_1) + h_2(p_2) \to F(\{q_i\}) + X$$

additional radiation

$$c(x_1 p_1) + \bar{c}(x_2 p_2) \to F(q \equiv \sum_i q_i)\Big|\, q_\mu q^\mu = M^2 = Q^2$$



$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_S \hat{\sigma}_{ij}^{(1)} + \ldots$$

LO NLO

Long distance (non-perturbative)

$$\sigma_{h_1,h_2} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$

Short distance (perturbative)

Differential Cross Section: General Structure

- The differential cross section in a generic variable ω can be theoretically separated into following two parts

$$\frac{d\sigma_F}{d\omega} = \frac{d\sigma_F^{\text{sing.}}}{d\omega} (\equiv [d\sigma_F]) + \frac{d\sigma_F^{\text{reg.}}}{d\omega}$$

$(\mathbf{q_T}, \tau_0)$

process independent

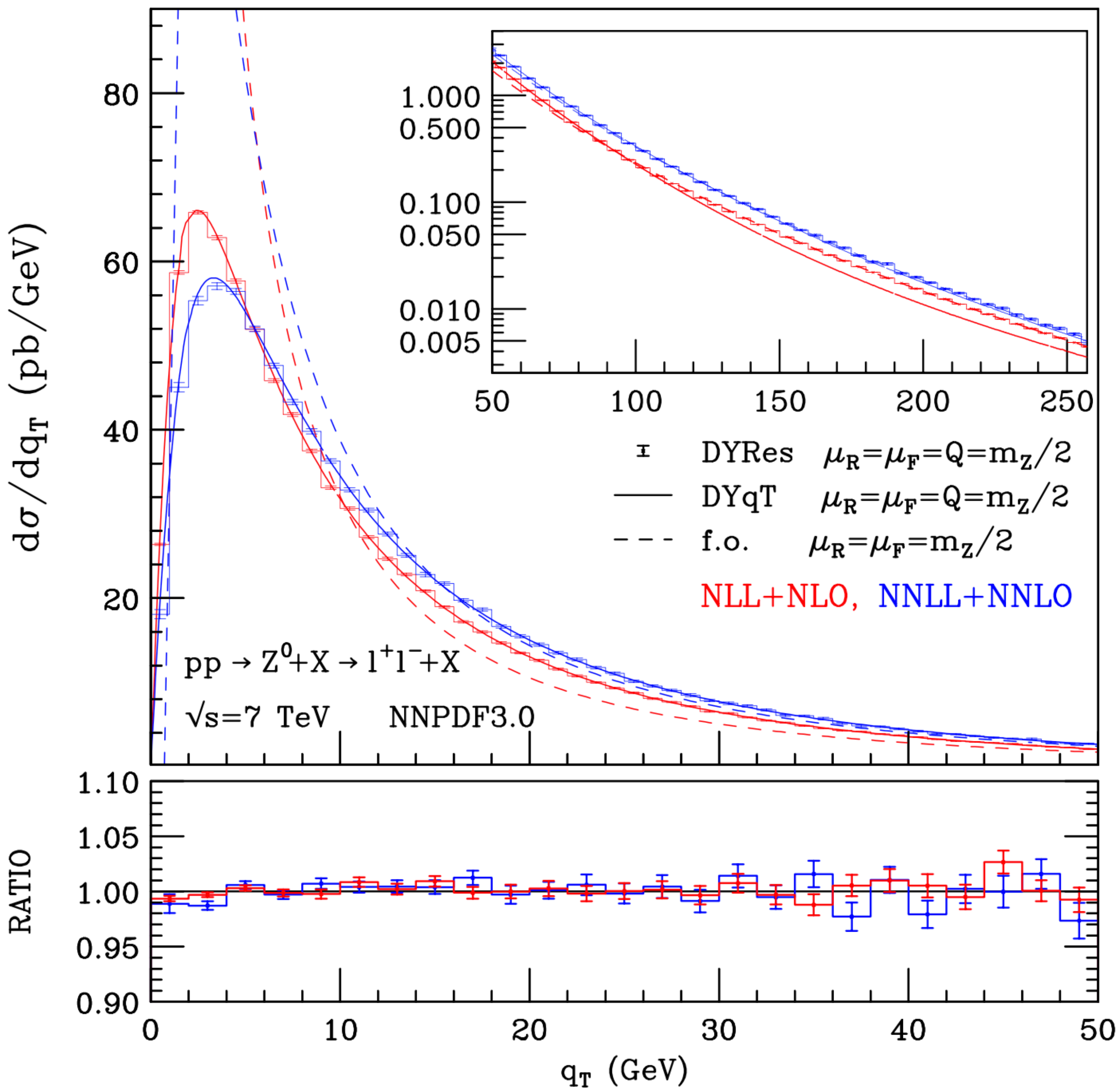
$$\left\{ \left[\frac{\ln^n \omega}{\omega} \right]_+ , \delta(\omega) \right\}$$

process dependent

$$\lim_{\omega_0 \to 0} \int_0^{\omega_0} d\omega \frac{d\sigma_F^{\text{reg.}}}{d\omega} = 0$$

- Singular contributions are of Soft and Collinear origin. Hence, they have some degree of universality that leads to Resummation of these terms.

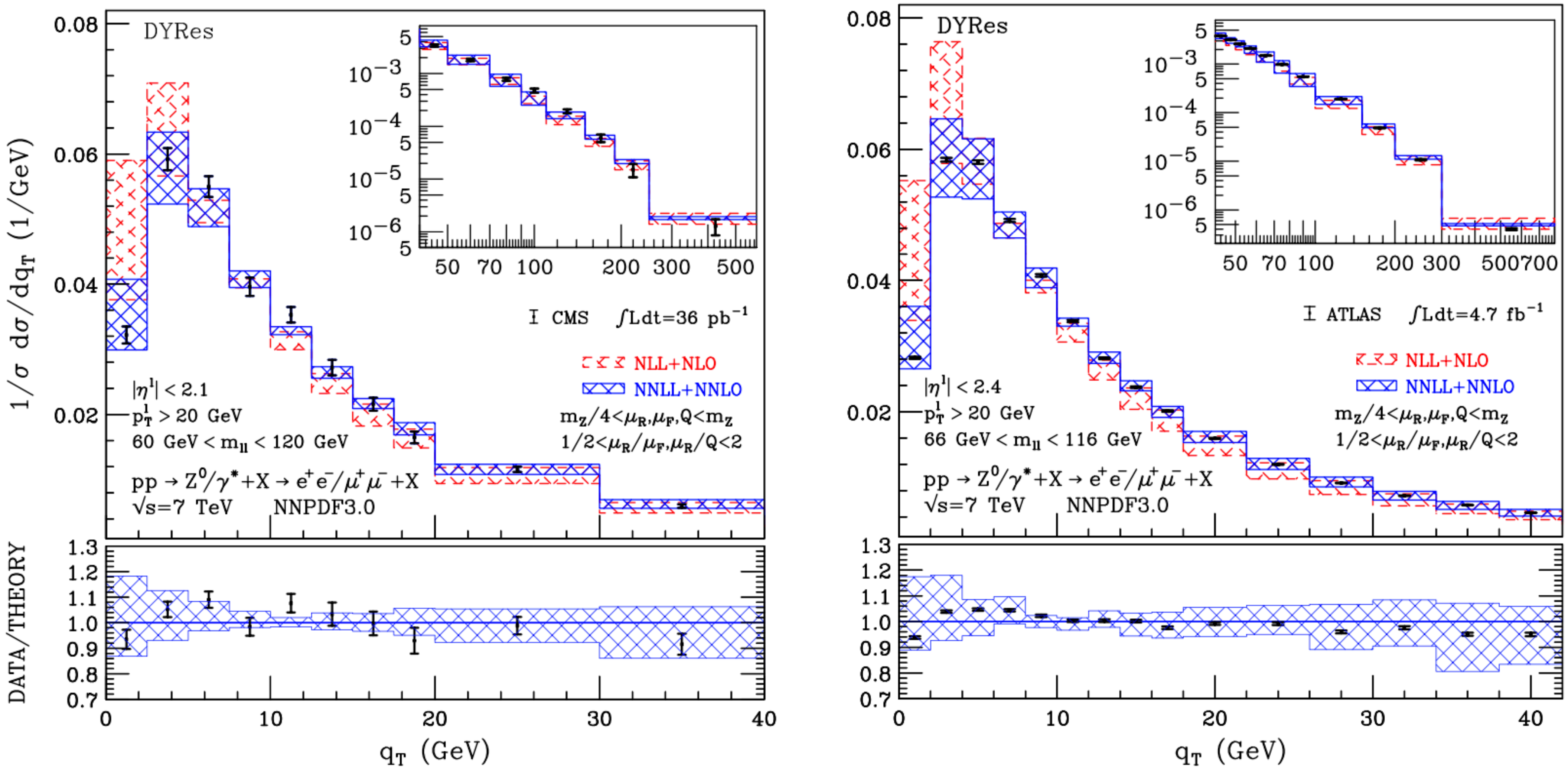
Why Resummation? Resummed Vs Fixed Order



- Low transverse momentum region: while the FO results are not reliable, the resummed results are smooth.
- High transverse momentum region: significant contributions are from the regular parts, making resummation ineffective.

[1] Catani, de Florian, Ferrera, Grazzini (1507.06937)

Resummed Prediction Vs Data



- Bulk of the events are produced in the low transverse momentum region.
- Theory is consistent with the data within the uncertainties.
- Overlap between two successive theory predictions showing perturbative convergence.
- From NLL+NLO to NNLL+NNLO: improvement in both uncertainties and data agreement.

Transverse Momentum Resummation

- Using the formalism of qT-resummation [1-2], singular part of the differential cross section in conjugate space has the following structure [2]

LO contribution

Sudakov form factor

$$[d\sigma_F] = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c},F}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}.\mathbf{q_T}} S_c(M,b)$$

$(p_1 + p_2)^2$

$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}\left(x_1/z_1, b_0^2/b^2\right) f_{a_2/h_2}\left(x_2/z_2, b_0^2/b^2\right)$$

hard collinear factor

parton densities

$b_0 = 2e^{-\gamma_E}$

[1] Collins, Soper, Sterman (1985)

[2] Catani, Cieri, de Florian, Ferrera, Grazzini (1311.1654)

Hard Collinear Factor: Structure

- For the quark anti-quark annihilation channel, the hard collinear factor has the following form

$$\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} = H_c^F\left(x_1p_1, x_2p_2; \Omega; \alpha_s(M^2)\right) C_{ca_1}\left(z_1; \alpha_s\left(b_0^2/b^2\right)\right) C_{\bar{c}a_2}\left(z_2; \alpha_s\left(b_0^2/b^2\right)\right)$$

process dependent hard function

specify the system F

collinear functions

- For the gluon gluon fusion channel, it has the form given by

$$\left[H^F C_1 C_2\right]_{gg;a_1a_2} = H^F_{\mu_1\nu_1,\mu_2\nu_2}\left(x_1p_1, x_2p_2; \Omega; \alpha_s(M^2)\right)$$
$$\times C^{\mu_1\nu_1}_{ga_1}\left(z_1; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)\right) C^{\mu_2\nu_2}_{ga_2}\left(z_2; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)\right)$$

where [1]

$$C^{\mu\nu}_{ga}\left(z; p_1, p_2, \mathbf{b}; \alpha_s\right) = d^{\mu\nu}(p_1,p_2) C_{ga}(z,\alpha_s) + D^{\mu\nu}(p_1,p_2;\mathbf{b}) G_{ga}(z;\alpha_s)$$

Azimuthal independent part

Azimuthal correlated part

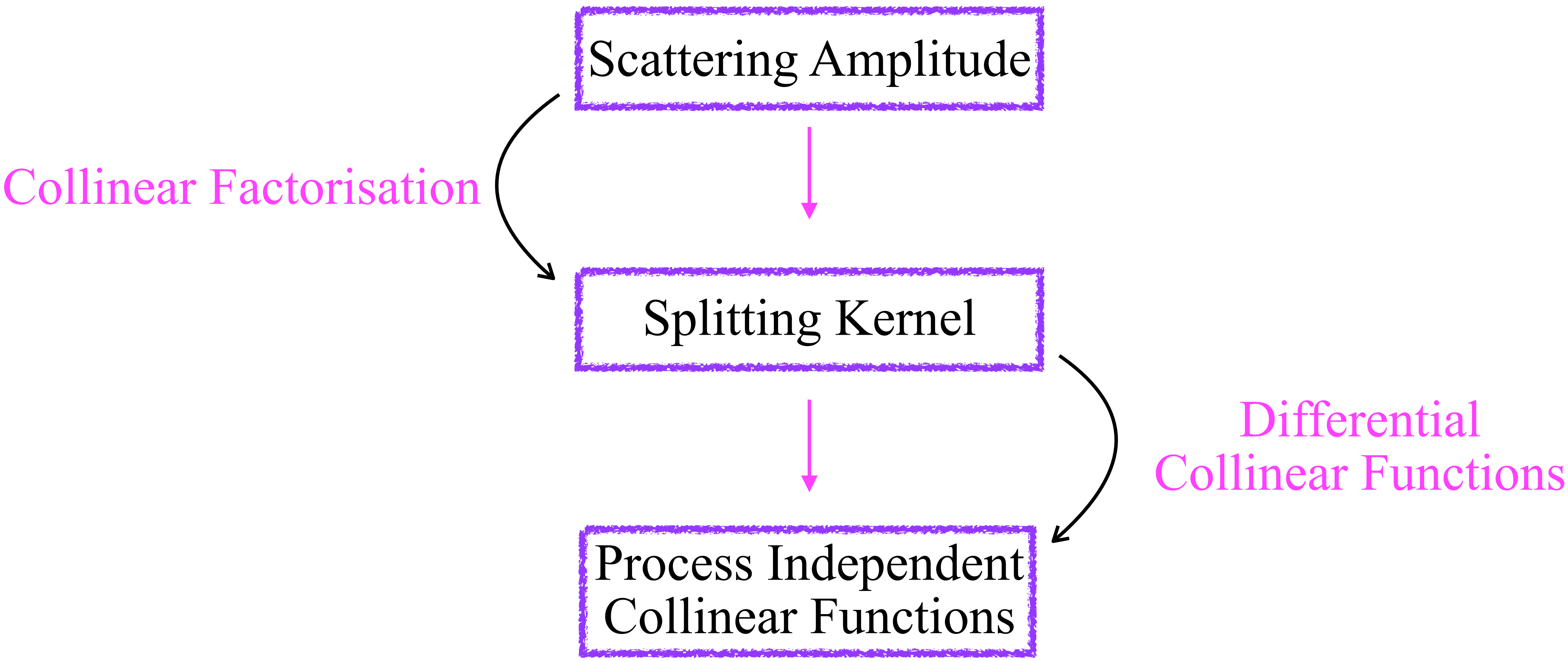
$$d^{\mu\nu}(p_1,p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1.p_2}$$

$$D^{\mu\nu}(p_1,p_2;\mathbf{b}) = d^{\mu\nu}(p_1,p_2) - 2\frac{b^\mu b^\nu}{\mathbf{b}^2} \quad \Big| \quad b^\mu = (0, \mathbf{b}, 0)$$

the indices μ and ν are the Lorentz indices of the gluon in hard scattering amplitude and its conjugate amplitude, respectively.

[1] Catani, Grazzini (1011.3918)

Goal: Collinear Functions



Collinear Factorisation of Matrix Element

- The collinear factorisation of hard scattering matrix element having N collinear partons in its most general form is given by [1]

$$|\mathcal{M}\left(\{q_i\};k_1,\ldots,k_N\right)|^2 = \langle\mathcal{M}\big(\{q_i\};\tilde{k}\big)|\,\boldsymbol{\mathcal{P}}\big(\{q_i\};k_1,\ldots,k_N;n\big)\,|\mathcal{M}\big(\{q_i\};\tilde{k}\big)\rangle + \ldots$$

auxiliary vector (n); reduced ME; non-singular terms ($\ldots$); non-coll. partons ($\{q_i\}$); coll. limit of $\sum_{i=1}^{N} k_i$ ($\tilde{k}$); splitting kernel ($\boldsymbol{\mathcal{P}}$)

- In general the collinear splitting kernel depends on the momenta and quantum numbers (colour) of non-collinear partons in addition to its dependence on the collinear partons.

- The TL collinear region is defined by

$$\{k_i^0\} > 0$$

- The splitting kernel is PI and this property of factorisation is called strict collinear factorisation.

- The SL collinear region is defined by

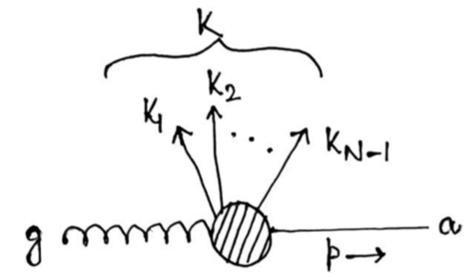
$$k_i^0 < 0$$

- Strict collinear factorisation is instead violated in SL collinear region.

[1] Catani, de Florian, Rodrigo (1112.4405)

Differential Collinear Function: TL Region

- We define the differential collinear functions for the gluon in the TL region as follows [1]



$$\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n) = \sum_{N=2}^{+\infty} \left[\prod_{m=1}^{N-1} \int \frac{d^d k_m}{(2\pi)^{d-1}} \delta_+\left(k_m^2\right)\right] \delta^{(d)}\left(k - \sum_{i=1}^{N-1} k_i\right) \times \sum_{a_1,\dots,a_{N-1}} \frac{\widetilde{\mathcal{P}}^{\mu\nu}_{g\to a_1\dots a_N}\left(k_1,\dots,k_N;n\right)}{\mathrm{SF}\left(a_1,\dots,a_{N-1}\right)}\Bigg|_{\substack{k_N=p\\ a_N=a}}$$

Bose symmetry factor

$$\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n) = d^{\mu\nu}(p;n)\,\mathcal{F}^{\mathrm{TL}}_{ga,\,\mathrm{az.in.}}(p,k;n) + D^{\mu\nu}(p,n;\mathbf{k_T},\epsilon)\,\mathcal{F}^{\mathrm{TL}}_{ga,\,\mathrm{corr.}}(p,k;n)$$

$$d^{\mu\nu}(p;n) = -g^{\mu\nu} + \frac{p^\mu n^\nu + n^\mu p^\nu}{np} - \frac{n^2 p^\mu p^\nu}{(np)^2}$$

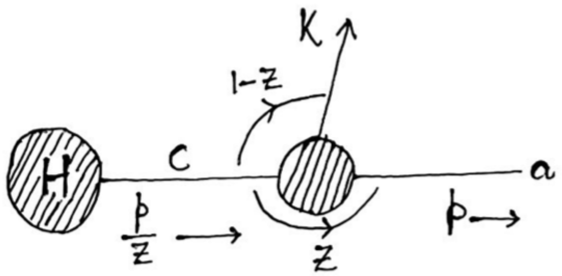
$$D^{\mu\nu}(p,n;\mathbf{k_T},\epsilon) = d^{\mu\nu}(p;n) - (d-2)\frac{k_T^\mu k_T^\nu}{\mathbf{k_T}^2}$$

and for the quark

$$\mathcal{F}_{ca}^{\mathrm{TL}}(p,k;n) = \sum_{N=2}^{+\infty} \left[\prod_{m=1}^{N-1} \int \frac{d^d k_m}{(2\pi)^{d-1}} \delta_+\left(k_m^2\right)\right] \delta^{(d)}\left(k - \sum_{i=1}^{N-1} k_i\right) \sum_{a_1,\dots,a_{N-1}} \frac{\mathcal{P}_{c\to a_1\dots a_N}\left(k_1,\dots,k_N;n\right)}{\mathrm{SF}\left(a_1,\dots,a_{N-1}\right)}\Bigg|_{\substack{k_N=p\\ a_N=a}}, \qquad c = q,\bar{q}$$

Integrated Collinear Function: TL Region

- We define the transverse momentum dependent collinear functions for the gluon case as follows



$$F_{ga}^{\mathrm{TL}\,\mu\nu}(z;p/z,\mathbf{q_T};n) = \delta(1-z)\,\delta^{(d-2)}(\mathbf{q_T})\;\delta_{ga}\;d^{\mu\nu}(p;n) + \int d^dk\;\delta^{(d-2)}(\mathbf{k_T}+\mathbf{q_T})\;\delta\left(\frac{k^+}{p^+}-\frac{1-z}{z}\right)\;\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n)$$

phase space in coll. limit

- For the quark case, it is given by

$$F_{ca}^{\mathrm{TL}}\left(z;\mathbf{q_T}^2,\frac{n^2\mathbf{q_T}^2}{(2np/z)^2}\right) = \delta(1-z)\,\delta^{(d-2)}(\mathbf{q_T})\;\delta_{ca} + \int d^dk\;\delta^{(d-2)}(\mathbf{k_T}+\mathbf{q_T})\;\delta\left(\frac{k^+}{p^+}-\frac{1-z}{z}\right)\;\mathcal{F}_{ca}^{\mathrm{TL}}(p,k;n)\;,\;c=q,\bar{q}$$

Renormalisation Procedure

- QCD strong coupling renormalisation

$$\alpha_S^u \mu_0^{2\epsilon} S_\epsilon = \alpha_S\left(b_0^2/b^2\right)\left(\frac{b_0^2}{b^2}\right)^{\epsilon}\left[1 - \frac{\alpha_S\left(b_0^2/b^2\right)}{\pi}\frac{\beta_0}{\epsilon} + \mathcal{O}\left(\alpha_S^2\left(b_0^2/b^2\right)\right)\right]$$

- Infrared factorisation in b-space

$$F_{ca}\left(z; \frac{n^2 b_0^2}{(2zpn)^2 b^2}; \alpha_S; \epsilon\right) = \mathcal{Z}_c\left(\frac{n^2 b_0^2}{(2zpn)^2 b^2}; \alpha_S; \epsilon\right)\sum_b \tilde{C}_{cb}\left(z; \frac{n^2 b_0^2}{(2zpn)^2 b^2}; \alpha_S; \epsilon\right) \otimes \Gamma_{ba}\left(z; \alpha_S; \epsilon\right)$$

AP splitting functions

$$\Rightarrow \tilde{C}_{cb}\left(z; \frac{n^2 b_0^2}{(2zpn)^2 b^2}; \alpha_S; \epsilon = 0\right) = C_{cb}\left(z; \alpha_S\right) \Rightarrow \Gamma_{ij}\left(z; \alpha_S\right) = \delta_{ij}\delta(1-z) - \frac{\alpha_S}{\pi}\frac{P_{ij}^{(0)}(z)}{\epsilon} + \mathcal{O}\left(\alpha_S^2\right)$$

- Infrared factorisation factors for both gluon and quark are as follows

$$\mathcal{Z}_g = 1 + \frac{\alpha_S}{\pi}\left[\frac{C_A}{2}\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\left(\frac{n^2 b_0^2}{(2znp)^2 b^2}\right)\right) + \frac{\beta_0}{\epsilon} - C_A\frac{\pi^2}{24}\right] + \mathcal{O}(\alpha_S^2)$$

$$\mathcal{Z}_q = 1 + \frac{\alpha_S}{\pi}\left[\frac{C_F}{2}\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\left(\frac{n^2 b_0^2}{(2znp)^2 b^2}\right)\right) + \frac{3}{4}\frac{C_F}{\epsilon} - C_F\frac{\pi^2}{24}\right] + \mathcal{O}(\alpha_S^2)$$

Results at NNLO: SL Case

- Azimuthally correlated TMD collinear functions are obtained as follows

$$G_{gq}^{(2)} = C_F^2 \left\{ -\frac{1-z}{2} + \frac{5}{4}\ln(z) - \frac{1}{4}\ln^2(z) - \frac{1-z}{2z}\Big[\ln(1-z) + \ln^2(1-z)\Big] \right\}
+ C_F n_f \left\{ -\frac{1-z}{3z}\left[\frac{2}{3} + \ln(1-z)\right] \right\} + C_A C_F \left\{ -\frac{11}{18z} + \frac{10}{9} - \frac{z}{2} - \ln(z)\left[\frac{1}{z} + \frac{5}{2}\right]
+ \frac{1}{2}\ln^2(z) + \frac{1-z}{z}\left[\frac{5}{6}\ln(1-z) + \frac{1}{2}\ln^2(1-z) + \mathrm{Li}_2(z) - \frac{\pi^2}{6}\right] \right\},$$

$$G_{gg}^{(2)} = C_F n_f \left\{ \frac{(1-z)^3}{2z} - \frac{1}{4}\ln^2(z) \right\} + C_A n_f \left\{ -\frac{17}{36z} + \frac{4}{9} + \frac{z}{12} + \frac{z^2}{36} - \frac{1}{6}\ln(z) \right\}
+ C_A^2 \left\{ -\frac{37}{36z} + \frac{31}{18} - \frac{13z}{12} + \frac{11z^2}{36} - \ln(z)\left[\frac{1}{z} + \frac{19}{12}\right] + \frac{1}{2}\ln^2(z) + \frac{1-z}{z}\left[\mathrm{Li}_2(z) - \frac{\pi^2}{6}\right] \right\}$$

Results at NNLO: TL Case

- Azimuthally correlated TMD collinear functions in the TL region are obtained as follows

$$
\begin{aligned}
G_{gq}^{TL(2)} =& C_F\Bigg\{ -\frac{1}{8}+\frac{3z}{4}-\frac{5z^2}{8}-\ln(z)\left[\frac{1}{4}+\frac{3z}{8}-\frac{z^2}{4}\right]+z(1-z)\Bigg[\frac{1}{4}\ln(1-z) \\
&+\frac{3}{8}\ln^2(z)-\frac{3}{2}\ln(z)\ln(1-z)+\frac{1}{4}\ln^2(1-z)-\mathrm{Li}_2(z)-\frac{\pi^2}{12}\Bigg]\Bigg\} \\
&+N_f\left\{z(1-z)\left[\frac{1}{9}-\frac{1}{6}\ln(z)+\frac{1}{6}\ln(1-z)\right]\right\}+C_A\Bigg\{\ln(z)\left[\frac{1}{4}+\frac{13z}{6}-\frac{17z^2}{12}\right] \\
&+\ln^2(z)\left[\frac{3z}{4}+\frac{z^2}{2}\right]+z(1-z)\Bigg[-\frac{25}{36}-\frac{5}{12}\ln(1-z)+\frac{1}{2}\ln(z)\ln(1-z) \\
&-\frac{1}{4}\ln^2(1-z)+\frac{1}{2}\mathrm{Li}_2(z)+\frac{\pi^2}{6}\Bigg]\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
G_{gg}^{TL(2)} =& C_F n_f\left\{\frac{1}{18z}+\frac{1}{2}+z-\frac{14z^2}{9}+\ln(z)\left[-\frac{1}{3z}+1+\frac{3z}{2}\right]+\frac{3z}{4}\ln^2(z)\right\} \\
&+C_A n_f\left\{-\frac{1}{36z}-\frac{1}{12}-\frac{4z}{9}+\frac{17z^2}{36}-\frac{z}{6}\ln(z)\right\}+C_A^2\Bigg\{-\frac{1}{36z}-\frac{5}{12}-\frac{20z}{9}+\frac{11z^2}{4} \\
&+\ln(z)\left[\frac{1}{3z}-1-\frac{67z}{12}+z^2\right]+z(1-z)\Big[\ln(z)\ln(1-z)-\mathrm{Li}_2(1-z)\Big] \\
&-\ln^2(z)\left[3z-\frac{z^2}{2}\right]\Bigg\}.
\end{aligned}
$$

Summary & Outlook

- I presented our method to compute both SL and TL collinear functions for QCD resummations using respective splitting kernels for the scattering amplitude.

- To compute these functions, we defined a differential version at the intermediate level and integrate them using proper observable definition to obtain collinear functions for transverse momentum resummation.

- For the azimuthally independent collinear functions, we have presented results up to NLO and for the azimuthally correlated case, we have results up to NNLO in perturbation theory.

Summary & Outlook

- NNLO results for the azimuthally independent collinear functions are under completion.
- Soft functions at NNLO are also under completion.
- Our formalism can be extended and applied to other observables such as jet mass distributions, energy-energy correlation functions etc.

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Thank you for your attention