THE THREE LOOP FOUR PARTON SCATTERING IN QCD

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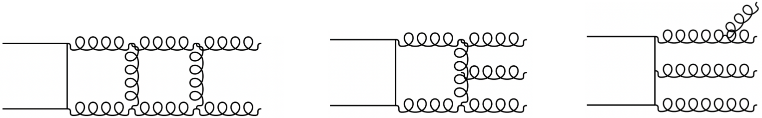
- $\sigma_{NNLO} \sim$



- 2 loop master integrals $2 \to 2$ processes

  [Anastasiou,Gehrmann,Oleari,Remiddi,Tausk '00]

- 2 loop amplitudes for all of the partonic channels $2 \to 2$ processes

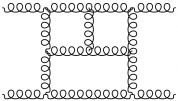
  [Bern,Dixon,Kosower'20; Bern,Dixon,Freitas'03; Anastasiou,Glover,Oleari,Yeomans'03, Ahmed,Henn,Mistlberger '19]

- IR Subtractions schemes for differential cross sections @ NNLO

  [Local (Antennas,ColorfulNNLO,Local analytic,Geometric, Nested subtractions,Stripper), Slicing (qT,N-jettiness), ...]

- Differential cross sections for di-jet productions

  [Currie,Gehrmann-De Ridder,Gehrmann,Huss,Mo,Pires '17,'19,'22; Czakon,Hameren,Mitov,Poncelet '19]



- 3-loop reduced integrand in $N = 4$ SYM, $N = 8$ SUGRA...

  [Bern,Carrasco,Dixon,Herrmann,Johansson,Kosower,Litsey,Roiban, Stankowicz,Trnka '07,'08,'12]

- 3-loop amplitudes in $N = 4$ SYM, $N = 8$ SUGRA

  [Henn, Mistlberger '16 '19]

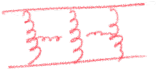
- Results for 3-loop master integrals for massless $2 \to 2$ processes

  [Henn, Mistlberger, Smirnov, Wasser '20]

- Simplest three loop QCD amplitude : $q\bar{q} \to \gamma\gamma$

  [Caola,Manteuffel,Tancredi '20]

- Rich IR singularity structure → presence of four particle correlation



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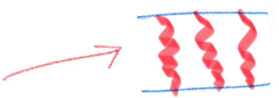
possibility of breaking the usual factorisation structure for di-jet @ N^3LO

[Catani,de Florian,Rodrigo ’12; Forshaw,Seymour,Siodmok ’12; Becher,Neubert,Shao ’21]

- Virtual ingredients to di-jet production @ N^3LO

- Much more control to perform very complex Integral By Parts (IBP) reductions



- Unambiguous picture for reggeisaton @ NNLL

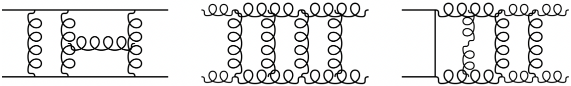
⇒ High energy *factorisation* structure → Extraction of 3 loop gluon Regge Trajectory

FOUR PARTON SCATTERING AT THREE LOOPS

We consider the *analytic* computations of all channels of massless four parton scattering in full QCD

- $q(p_1) + q(p_2) \to q(p_3) + q(p_4)$
- $g(p_1) + g(p_2) \to g(p_3) + g(p_4)$
- $q(p_1) + \bar{q}(p_2) \to g(p_3) + g(p_4)$

Caola, Chakraborty, Gambuti, Manteuffel, Tancredi :
2108.00055 (*JHEP'21*),
2112.11097 (*Editors' Suggestions PRL'22*),
2207.03503 (*JHEP'22*)



$\Rightarrow p_i^2 = 0$

$\Rightarrow s=(p_1+p_2)^2,\ t=(p_1+p_3)^2,\ u=(p_2+p_3)^2$

$\Rightarrow x = -t/s$

$\to s > 0,\ t < 0,\ u < 0,\ 0 < x < 1$

BOTTLENECK FOR QCD COMPUTATIONS

- Large number of Feyn. diagrams @ three loop

  $\Rightarrow gg \to gg \sim 50\,\text{k}, \quad q\bar{q} \to gg \sim 15\,\text{k}, \quad q\bar{q} \to q^{'}\bar{q}^{'} \sim 4\,\text{k}$

- Staggering number of Feyn. integrals to perform IBP reductions

  $\Rightarrow gg \to gg \sim 10^7\,, \quad q\bar{q} \to gg \sim 10^7, \quad q\bar{q} \to q^{'}\bar{q}^{'} \sim 10^6$

- Big intermediate file sizes

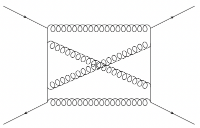
  $\Rightarrow gg \to gg \sim 250\,\text{GB}\,, \quad q\bar{q} \to gg \sim 40\,\text{GB}, \quad q\bar{q} \to q^{'}\bar{q}^{'} \sim 4\,\text{GB}$

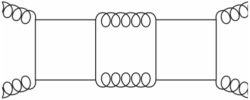
- Huge reductions in the sizes of the final results of the Helicity amplitudes

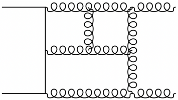
  $\Rightarrow$ $\sim$1 MB for each partonic channels

COLOR BASIS @ PARTONIC CHANNELS

$$\mathcal{A} = \sum_i^n \mathcal{A}^i \, \mathcal{C}_i \quad \longrightarrow \quad \text{Color Basis}$$







$$\mathcal{C}_1 = \delta_{i_1 i_4}\delta_{i_2 i_3} \; , \qquad \mathcal{C}_2 = \delta_{i_1 i_2}\delta_{i_3 i_4}$$

$$\begin{aligned}
\mathcal{C}_1 &= \mathrm{Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}] + \mathrm{Tr}[T^{a_1}T^{a_4}T^{a_3}T^{a_2}],\\
\mathcal{C}_2 &= \mathrm{Tr}[T^{a_1}T^{a_2}T^{a_4}T^{a_3}] + \mathrm{Tr}[T^{a_1}T^{a_3}T^{a_4}T^{a_2}],\\
\mathcal{C}_3 &= \mathrm{Tr}[T^{a_1}T^{a_3}T^{a_2}T^{a_4}] + \mathrm{Tr}[T^{a_1}T^{a_4}T^{a_2}T^{a_3}],\\
\mathcal{C}_4 &= \mathrm{Tr}[T^{a_1}T^{a_2}]\mathrm{Tr}[T^{a_3}T^{a_4}],\\
\mathcal{C}_5 &= \mathrm{Tr}[T^{a_1}T^{a_3}]\mathrm{Tr}[T^{a_2}T^{a_4}],\\
\mathcal{C}_6 &= \mathrm{Tr}[T^{a_1}T^{a_4}]\mathrm{Tr}[T^{a_2}T^{a_3}].
\end{aligned}$$

$$\mathcal{C}_1 = (T^{a_3}T^{a_4})_{i_2 i_1}, \, \mathcal{C}_2 = (T^{a_4}T^{a_3})_{i_2 i_1}, \, \mathcal{C}_3 = \delta^{a_3 a_4} \, \delta_{i_1 i_2}$$

- Identifying the independent *tensor* structures through the Lorentz and gauge invariance $\Longrightarrow$ $\boldsymbol{\mathcal{A}} = \sum_{i=1}^{N(4)} \bar{\mathcal{F}}_i \, \bar{T}_i \rightarrow$ d = 4

$$\downarrow$$

$$\text{d} = 4 - 2\epsilon$$

- Extract Form factors by defining the suitable *projectors* acting onto Feynman diagrams

$$\Longrightarrow \bar{\mathcal{F}}_i = \sum_{pol} P_i \cdot \boldsymbol{\mathcal{A}} \quad P_i = \sum_{i=1}^{N(4)} (\bar{T}^\dagger \bar{T})^{-1}_{ik} \, \bar{T}^\dagger_k$$

- Choosing linearly independent projectors (P_i) @ d $\rightarrow$ 4

$\Rightarrow$ Big reduction @ projectors in ’tHV scheme compare to CDR!

- Free of spurious poles in d $\rightarrow$ 4 !

[Peraro,Tancredi ’19,’20]

⇒ Projectors for $q\bar{q} \to q^{'}\bar{q}^{'}$

$$T_1 = \bar{u}(p_2)\,\gamma_\alpha\,u(p_1) \times \bar{u}(p_4)\,\gamma^\alpha\,u(p_3)\;, \quad T_2 = \bar{u}(p_2)\,\not{p}_3\,u(p_1) \times \bar{u}(p_4)\,\not{p}_2\,u(p_3)$$

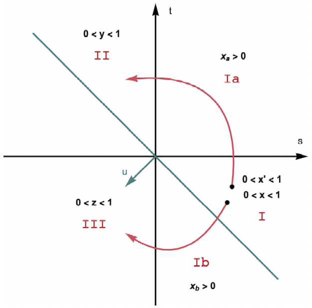
⇒ Projectors for $q\bar{q} \to gg$

$$\begin{aligned}
\mathcal{T}_1 =& \bar{u}(p_2)\not{\epsilon}_3 u(p_1)\epsilon_4\cdot p_2 \\
\mathcal{T}_2 =& \bar{u}(p_2)\not{\epsilon}_4 u(p_1)\epsilon_3\cdot p_1 \\
\mathcal{T}_3 =& \bar{u}(p_2)\not{p}_3 u(p_1)\epsilon_3\cdot p_1\epsilon_4\cdot p_2 \\
\mathcal{T}_4 =& \bar{u}(p_2)\not{p}_3 u(p_1)\epsilon_3\cdot\epsilon_4
\end{aligned}$$

⇒ Projectors for $gg \to gg$

$$\begin{aligned}
&T_1 = \epsilon_1\cdot p_3\;\epsilon_2\cdot p_1\;\epsilon_3\cdot p_1\;\epsilon_4\cdot p_2\;, \\
&T_2 = \epsilon_1\cdot p_3\;\epsilon_2\cdot p_1\;\epsilon_3\cdot\epsilon_4, \quad T_3 = \epsilon_1\cdot p_3\;\epsilon_3\cdot p_1\;\epsilon_2\cdot\epsilon_4, \\
&T_4 = \epsilon_1\cdot p_3\;\epsilon_4\cdot p_2\;\epsilon_2\cdot\epsilon_3, \quad T_5 = \epsilon_2\cdot p_1\;\epsilon_3\cdot p_1\;\epsilon_1\cdot\epsilon_4, \\
&T_6 = \epsilon_2\cdot p_1\;\epsilon_4\cdot p_2\;\epsilon_1\cdot\epsilon_3, \quad T_7 = \epsilon_3\cdot p_1\;\epsilon_4\cdot p_2\;\epsilon_1\cdot\epsilon_2, \\
&T_8 = \epsilon_1\cdot\epsilon_2\;\epsilon_3\cdot\epsilon_4 + \epsilon_1\cdot\epsilon_4\;\epsilon_2\cdot\epsilon_3 + \epsilon_1\cdot\epsilon_3\;\epsilon_2\cdot\epsilon_4\;.
\end{aligned}$$

- $s + t + u = 0$ → No Euclidean region for physical phase space



[Smirnov '99; Smirnov, Veretin '00; Tausk '00, Anastasiou, Gehrmann, Oleari, Remiddi '00]

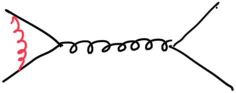
- Branch cut at x → 0, 1 → non trivial analytic continuation for *Logs*

  ⇒ Obtaining all of the relevant crossed channel of the partonic amplitude : $qq' \to q'q$ from $q\bar{q} \to q'\bar{q}'$ and $qg \to qg$ from $q\bar{q} \to gg$

INFRARED SINGULARITY STRUCTURE

Factorisation of IR singularities aft. UV subtraction

$$\mathcal{A}^{fin} = \mathcal{Z}_{IR}^{-1}\ \mathcal{A}^{ren}$$



Iterative solution of $\mathcal{Z}$

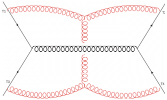
$$\mathcal{Z} = \mathbb{P}\exp\left[\int_{\mu}^{\infty} \frac{\mathrm{d}\mu'}{\mu'}\Gamma(\{p\},\mu')\right]$$

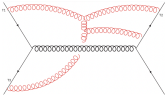
Different color correlation pattern for multiparton scattering :
$\rightarrow$ Dipole and Quadrupole

$$\Gamma = \Gamma_{\text{dipole}}\ +\ \Delta_{quad}^{loops\geq 3}\ + \ldots$$

$$\Gamma_{\text{dipole}} = \sum_{1\leq i<j\leq 4} T_i^a\ T_j^a\ \gamma^{\text{cusp}}\ \log\left(\frac{\mu^2}{-s_{ij}}\right) + \sum_{i=1}^{4} \gamma_i^{col}$$

$\Delta_4^{(3)} \rightarrow$





$\Downarrow$ four external partons correlated || $\Downarrow$ three external partons correlated

[Catani '98; Sterman,Yeomans '04; Aybat,Dixon,Sterman '06; Becher,Neubert '09,12; Gardi,Magnea '09, Almelid,Duhr,Gardi '16]

- Quadrupole correlations pattern :

$$\Delta_4^{(3)} = 128\, f_{abe} f_{cde} \left[\mathbf{T}_1^a \mathbf{T}_2^c \mathbf{T}_3^b \mathbf{T}_4^d\, D_1(x) - \mathbf{T}_4^a \mathbf{T}_1^b \mathbf{T}_2^c \mathbf{T}_3^d\, D_2(x)\right]$$

$$-16\, f_{abe} f_{cde}\, (\zeta_5 + 2\zeta_2\zeta_3)\, \sum_{i=1}^{4} \sum_{\substack{1\leq j<k\leq 4 \\ j,k\neq i}} \{\mathbf{T}_i^a\, \mathbf{T}_i^d\} \mathbf{T}_j^b\, \mathbf{T}_k^c$$

[Almelid,Duhr,Gardi '16]

- Contributes uniformly to the Transcendental weight

- Depends only on gluons interactions
  $\rightarrow$ Does not depend explicitly on matter content

  $\Rightarrow$ Universal for the gauge theories having gluon

  $\rightarrow$ confirmed for N=4 [Henn, Mistlberger '16]

- Absent for QED, due to abelian nature of photon

- μ independent

- Depends only on the *logs* of conformal cross-ratio

$$\frac{(-s_{ij})\,(-s_{kl})}{(-s_{ik})\,(-s_{jl})}$$

NEW RESUTLS @ THREE LOOPS

- The full *analytic* computations of all of the partonic channel in QCD :
  $q\bar{q} \to q^{'}\bar{q}^{'}, \;\; q\bar{q} \to q\bar{q}$ (+ relevant crossings)

  [Caola,Chakraborty,Gambuti,Manteuffel,Tancredi JHEP'21]

  $gg \to gg$

  [Caola,Chakraborty,Gambuti,Manteuffel,Tancredi *Editors' Suggestions* PRL '22]

  $q\bar{q} \to gg$ (+ relevant crossings)

  [Caola,Chakraborty,Gambuti,Manteuffel,Tancredi JHEP'22]

  $gg \to g\gamma$ and $q\bar{q} \to g\gamma$ (+ relevant crossings)

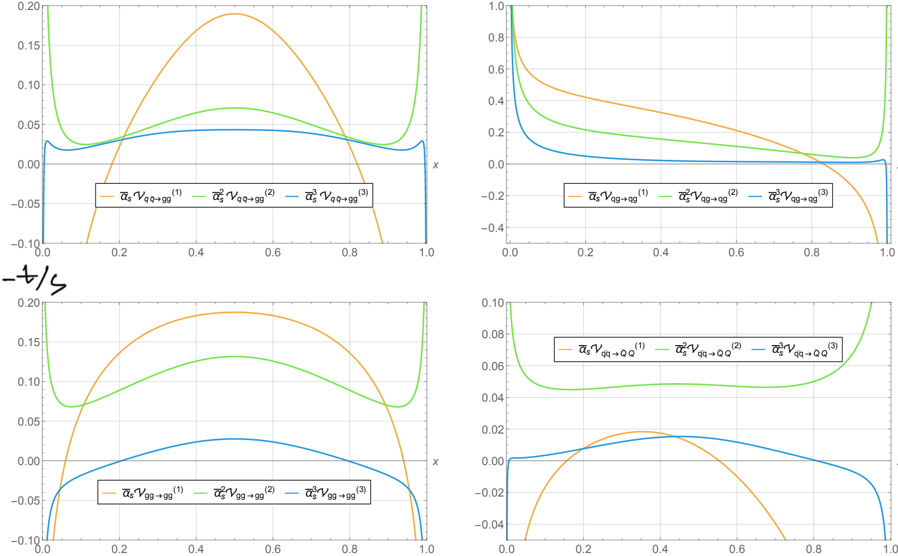
  [: to appear]

- Our calculations confirm the Wilson line predicted [Almelid,Duhr,Gardi '16] quadrupole IR structure in QCD

NUMERICAL RESUTLS @ THREE LOOPS

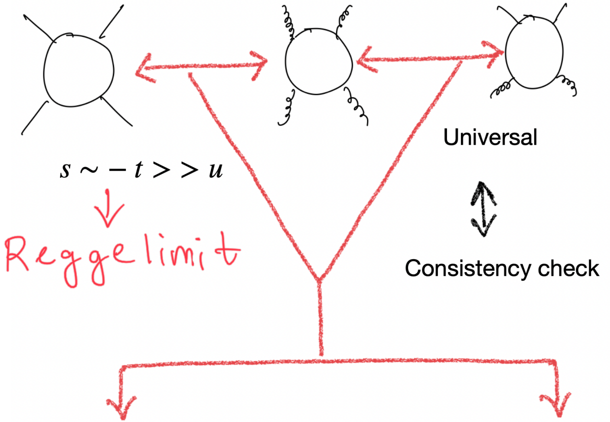
$$\frac{\langle A | A \rangle}{\langle A_0 | A_0 \rangle} \sim \mathcal{V}^{(0)} + \alpha_s \mathcal{V}^{(1)} + \alpha_s^2 \mathcal{V}^2 + \alpha_s^3 \mathcal{V}^3 + \ldots$$

$x \to -t/s$



NEW RESULTS @ HIGH ENERGY LIMIT

$$\mathcal{M}_{ij\to ij} = Z_i\, Z_j e^{L\mathbf{T}_t^2 \tau_g} \sum_{n=0}^{3} a_s^n \sum_{k=0}^{n} L^k \mathcal{O}_k^{(n)} \mathcal{M}_{ij\to ij}^{(0)}$$



$\mathcal{O}_k \to C_{q/g}$ 2-loop ϵ^2

Impact Factor

$\tau_g \to$ 3-loop gluon Regge Trajectory

[Caola, Chakraborty, Gambuti, Manteuffel, Tancredi]

THREE LOOP GLUON REGGE TRAJECTORY

$$\tau_g^3 = K_3 + N_c^2\left(16\zeta_5 + \frac{40\zeta_2\zeta_3}{3} - \frac{77\zeta_4}{3} - \frac{6664\zeta_3}{27} - \frac{3196\zeta_2}{81} + \frac{297029}{1458}\right) + \frac{n_f}{N_c}\left(-4\zeta_4 - \frac{76\zeta_3}{9} + \frac{1711}{108}\right) + N_c n_f\left(\frac{412\zeta_2}{81} + \frac{2\zeta_4}{3} + \frac{632\zeta_3}{9} - \frac{171449}{2916}\right) + n_f^2\left(\frac{928}{729} - \frac{128\zeta_3}{27}\right)$$

[Caola,Chakraborty,Gambuti,Manteuffel,Tancredi] [Falcioni,Gardi,Maher,Milloy,Vernazza]

$\Rightarrow$ K_3 represents the pole part of trajectory $\rightarrow$ expressed in cusp anomalous dimensions

- Only leading N_c contributes at $n_f = 0$ to the finite part

[Del Duca,Marzucca,Verbeek]

$\Rightarrow$ Maximally non-Abelian

$\Rightarrow$ The quark regge $\rightarrow$ $\frac{C_F}{C_A}$ gluon regge trajectory

upto two loop

$\xrightarrow{???}$ Eikonal object interpreted though Wilson line correlator

SUMMARY

- The first analytic computations of all 3-loop four-parton amplitudes in full QCD
- Verification of the Wilson line predicted Soft anomalous dimensions structure for three-loop multi-parton QCD amplitudes
- Factorisation behavior of four parton scattering at high energy
- Extraction of the 3-loop gluon Regge Trajectory in full QCD