Recent updates on finite temperature and density QCD from lattice

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The Institute of Mathematical Sciences

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QCD interactions account for the origin of 99% of visible matter



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QCD phase diagram holds key to many fundamental questions



[Image Courtesy: www.cern.ch]

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Lattice QCD shows a crossover transition at small densities



[[]Image Courtesy: www.cern.ch]

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- Since m_{u,d} ≪ Λ_{QCD}, 2+1 flavor QCD respects U_L(2) × U_R(2) chiral symmetry to a good extent.
- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \rightarrow SU_V(2)$
- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.
 [HotOCD collab. 18. F. Burger et al. 18. Budgest-Wuppertal collab. 20]
- The singlet part U_A(1) is anomalous yet can affect the order of the chiral phase transition as m_{u,d} → 0.

[Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

 Do singlet and non-singlet chiral symmetries gets restored simultaneously?

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 U_A(1) is not an exact symmetry→ what observables to look for? Degeneracy of the 2-point (integrated) correlation

functions [Shuryak, 94]

$$\chi_\sigma = \chi_\delta + 2\chi_{
m disc}$$
 , $\chi_\eta = \chi_\pi - 2\chi_{
m 5,disc}$.

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The singlet and non-singlet parts mix strongly

• When chiral symmetry is restored

[L. Giusti, G. C. Rossi, M. Testa, 04, HotQCD 1205.3535]

$$\chi_\pi-\chi_\eta=2\,\,\chi_{5,{\it disc}}=\chi_\pi-\chi_\delta=2\,\,\chi_{\it disc}$$
 .





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The singlet and non-singlet parts mix strongly



Renormalized $U_A(1)$ breaking observable in the chiral limit

• For 2 + 1 flavor QCD a fit to diff. masses gives $\frac{\chi_{\pi} - \chi_{\delta}}{\tau^2}|_{m_l=0} = 156(13)$. [O. Kaczmarek, L. Mazur, S. S. 21]



- At $T < T_c$, the pion quasi-particle has a dispersion relation $\omega_{\mathbf{p}} = u(T)\sqrt{\mathbf{p}^2 + m_{\pi}^2}$.
- Using spectral function corresponding to the correlator of $\bar{\psi}\gamma_0\gamma_5\frac{\tau}{2}\psi$ one gets the pion velocity u from its residue considering no thermal modification. [D. T. Son & M. Stephanov, 98]

• Including thermal effects $ho(\omega, T) =$

$$\frac{f_{\pi}^2 M_{\pi} \Gamma(T)}{2u\pi} \left(\frac{1}{(\omega - M_{\pi} u)^2 + \Gamma(t)^2} - \frac{1}{(\omega + M_{\pi} u)^2 + \Gamma(t)^2} \right) + \dots$$

The *u* calculated for 2 flavor lattice QCD is consistent with the χ_{PT} ansatz if thermal width Γ = 30 MeV. Thermal modification by 16% for the chiral condensate at T = 128 MeV. [M. Co., H. Meyer et. al 22]
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- Role of anomalous part of chiral symmetry not well understood!
- Important to study hadron screening masses at finite *T*, baryon density. [See lattice studies by Thakkar, Hegde 22, Swansea Collab. 21]
- Fate of the non-Goldstone σ mode and $U_A(1)$ effects crucial to locate the critical end point.

[Image Courtesy: www.cern.ch]

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- At finite baryon densities conventional Monte-Carlo techniques suffer from sign problem!
- A practical way to circumvent: via Taylor series expansion around $\mu_B = 0$ [Bi-Swansea 02, Gavai, Gupta, 02].

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \frac{1}{2} \left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0, T) + \frac{1}{4!} \left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \dots$$

• For $\mu_B/T > 2$ the convergence is slow [HorQCD coll. 17]. Can be improved using new re-summation schemes (exp. resummation of the series, phase re-weighting..)to calculate thermodynamic quantities at $\mu_B/T = 3$.

[S. Mondal, P. Hegde, S. Mukherjee, 21, Budapest-Wuppertal Coll. 21, HotQCD coll. 22] [For a new resummation method see talk by S. Mitra, Wed 17:00 WG5]

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Method 2: Analytic continuation from Im. μ_B

[Fig. from Budapest-Wuppertal Collaboration, 22].



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Degrees of freedom and interactions



[Image Courtesy: www.cern.ch]

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Screening correlators in the deconfined phase

• Screening masses for the meson operator $J = \bar{\psi} \Gamma \psi$ can be derived from spatial correlators [De Tar & Kogut, 87].,

$$C(z) = \int_0^\beta d au \int d ext{x} d ext{y} \, \left< J(extbf{x}) J(0) \right> \sim \mathrm{e}^{-\mathrm{M}_{\mathrm{scr}} \mathbf{z}} + ..$$

- Has a more complicated relation to the spectral function at finite momenta. The screening mass are related to the meson excitations in the plasma.
- When there are well-defined bound state peaks in the spectral function the M_{scr} is simply the pole mass of the corresponding meson channel. At high T, the $M_{scr} = 2\sqrt{m_q^2 + (\pi T)^2}$.

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Screening correlators in the deconfined phase

• For light quarks vector and pseudo-sc. scr. masses are degenerate in NLO perturbation theory \rightarrow significant non-pert. effect even at 10 GeV! [Fig. from L. Guisti et. al., 21].



What happens to bottom quark screening states?



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When do scalar bottomonia melt?



- In non. relativistic limit $M \gg \pi T$, the heavy quarks undergo Langevin dynamics $\frac{d\mathbf{p}}{dt} = -\frac{\kappa \mathbf{p}}{2MT} + \zeta$
- momentum diffusion coeff. $\tau_{\text{heavy}} = \frac{M}{T}D = \frac{2MT}{\kappa}$
- Can be calculated from gluonic color electrical correlator in the limit M → ∞
- In perturbation theory NLO corrections to κ in α_s is twice order of magnitude as LO → need non-perturbative lattice techniques to calculate. [S. Caron-Huot et. al., 09]
- Recently O(M/T) corrections have been included $\kappa \sim \kappa_E + \frac{2}{3}\kappa_B \langle v^2 \rangle$



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I κ_B/T^3 , Banerjee et al., '22 I κ_B/T^3 , Brambilla et al., '22 I $\kappa_B/(T^3 Z_K^2)$, Altenkort et al.

1.5

 T/T_c

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[Bouttefeux & Laine, 20, D. Banerjee, S. Datta, M. Laine, 22, , Fig. courtesy L. Altenkort et. al. 22]





[Fig. from V. Leino, INT Workshop 22-3, Ref. N. Brambilla et al. 20, D. Banerjee, S. Datta, R. Gavai & P. Majumdar, 22]

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Photon production rate in Quark-Gluon plasma from lattice

- Measure of how-strongly interacting the QGP phase is!
- Calculated from vector current spectral function reconstructed from the correlation function calculated from lattice. In general

$$\label{eq:rho} \begin{split} \rho_V &= 2\rho_T + \lambda\rho_L \quad \text{[HotQCD Coll. 20, Mainz coll. 21]} \\ \text{[See talk by D. Bala, Thu 11:30 WG5]} \end{split}$$



Photon production rate in Quark-Gluon plasma from lattice



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Summary

• Lattice QCD results continue to throw in many surprises.

- Recent results gives us new insights on the thermal modifications of light hadrons in the chiral symmetry broken phase. The mechanism how chiral symmetry and deconfinement happens is not yet understood.
- Also shows that the deconfined quark-gluon plasma phase is non-perturbative even at $T \sim 1 \text{ GeV} \rightarrow \text{implications}$ for transport properties, thermalization?
- Need significant research for developing new lattice techniques to address fundamental questions in finite density QCD, non-equilibrium quantum field theories.

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