XXV DAE - HEP SYMPOSIUM 2022

$D_q^* \to D_q\gamma$: Probing the inner structure of D-meson

(Based on arXiv: 2301:XXX)

In collaboration with Prof. Namit Mahajan

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PHYSICAL RESEARCH LABORATORY

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Outline

- Motivation
- Introduction
- Light Cone Sum Rules in a Nutshell
- $D_q^* D_q \gamma$ coupling in LCSR
- Results
- Summary and discussion

Motivation

- Distribution amplitudes (DAs) are very crucial universal non-perturbative input for theoretical computations.
- DAs for heavy meson case are modelled using the heavy quark expansion. No precise form is known so far.
[Grozin and Neubert, PRD 55 (1997) 272-290]
- Exclusive decay of B-meson indicates that the first inverse moment of these DAs is a very important parameter.

$$\lambda_M^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega} \phi_+^M(\omega, \mu)$$

- For B-meson case, this parameter is constrained experimentally using $B^- \to \ell^- \nu_\ell \gamma$ decays (provides only the lower limit).
- Using $B^- \to \ell^- \nu_\ell \gamma$ and QCD sum rules, the value ranges between (0.45 ± 0.15) GeV. However, non-leptonic decays demands $\lambda_B \approx 0.2$ GeV.
[Lee and Neubert, PRD 72, 094028(2005), Baneke et. al, Eur.Phys.J.C 71 (2011) 1818]
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Our Objective

To estimate the inverse moment of the D-meson distribution amplitude using the experimental input on the branching ratio of radiative D_q^* decays.

$D_q^* \to D_q\gamma$ Decays: An Introduction

Coupling

- The amplitude for $D_q^* \to D_q\gamma$ $(q = u, d, s)$ is:

$$\mathcal{M}(D_q^* \to D_q(p)\gamma(k)) = e g_{D_q} \epsilon_{\mu\nu\rho\sigma} k^\rho \epsilon_\gamma^\sigma v^\nu \epsilon_{D_q^*}^\mu$$

$\frac{e g_{D_q}}{2}$ is the transition magnetic moment.

- The decay width: $\Gamma(D_q^*(p') \to D_q(p)\gamma(k)) = \frac{\alpha_{em}}{3} |g_{D_q}|^2 |\vec{k}|^3$

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Experimental Data

Channel	Branching Ratio	Decay widths	g_{D_q}
$D^{*+} \to D^+\gamma$ $(q = d)$	(1.6 ± 0.4) %	(83.4 ± 1.8) KeV	0.47
$D^{*0} \to D^0\gamma$ $(q = u)$	(35.3 ± 0.9) %	< 2.1 MeV	< 10.98
$D_s^{*+} \to D_s^+\gamma$ $(q = s)$	(93.5 ± 0.7) %	< 1.9 MeV	< 16.27

[PDG]

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Answer is **YES**!! We will do this using the method of **Light Cone Sum Rules.**

Light Cone Sum Rules in a Nutshell

Basic Idea

To calculate the hadronic objects of interest using the analytic properties of the correlation function involved.

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Matching the two gives estimates for the hadronic objects

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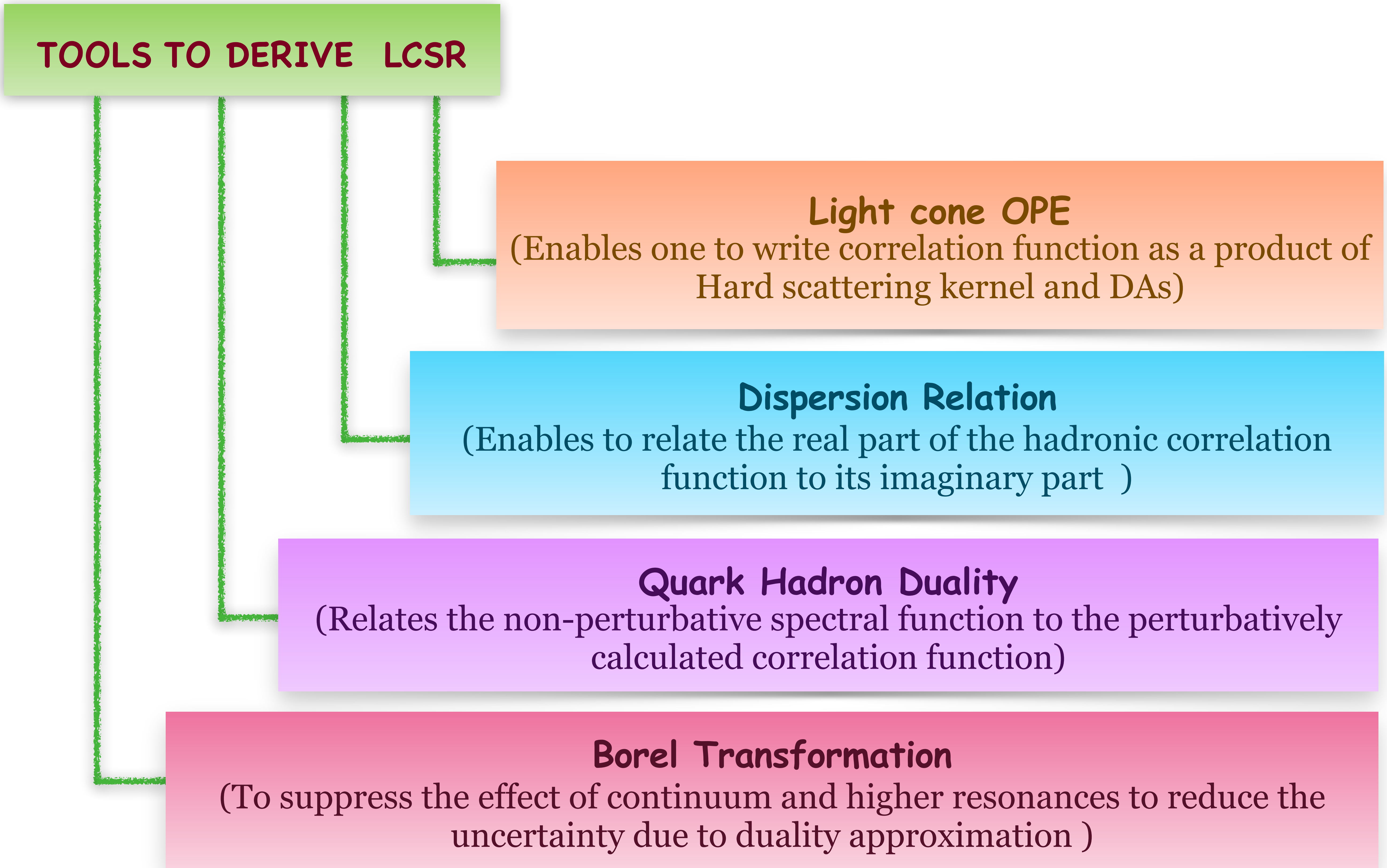
Quark Hadron Duality
(Relates the non-perturbative spectral function to the perturbatively calculated correlation function)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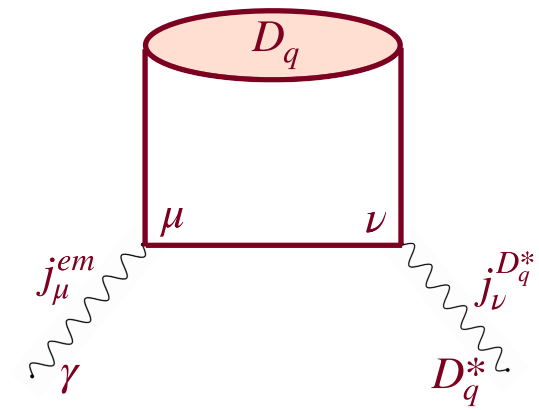
Quark Hadron Duality
(Relates the non-perturbative spectral function to the perturbatively calculated correlation function)
Borel Transformation
(To suppress the effect of continuum and higher resonances to reduce the uncertainty due to duality approximation)

Coupling in LCSR

- The correlation function involved:

$$T_{\mu\nu} = -ie\int d^4x e^{ik.x}\langle D_q(p)|T\left\{j_\mu^{em}(x)j_\nu^{D_q^*}(0)\right\}|0\rangle$$



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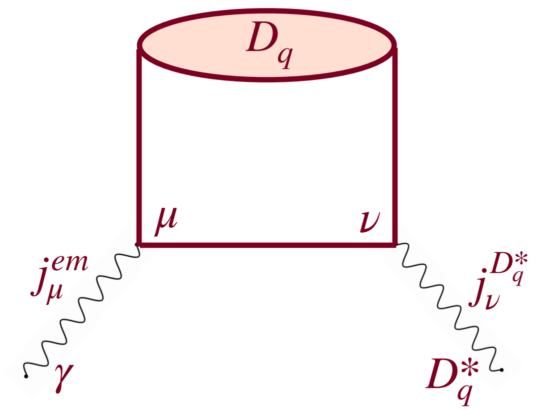

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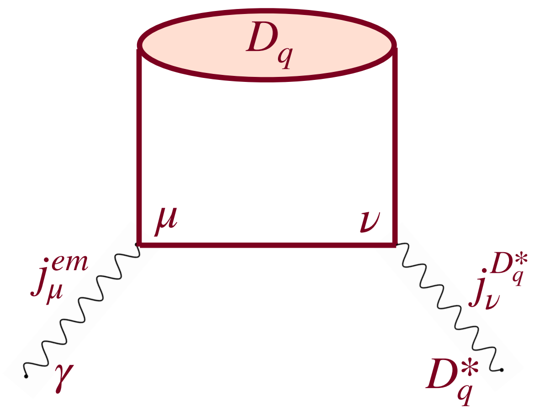

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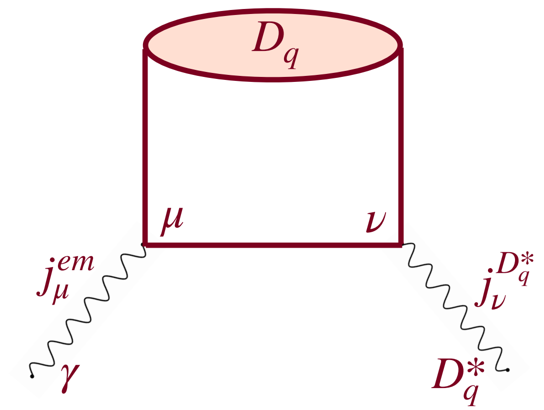

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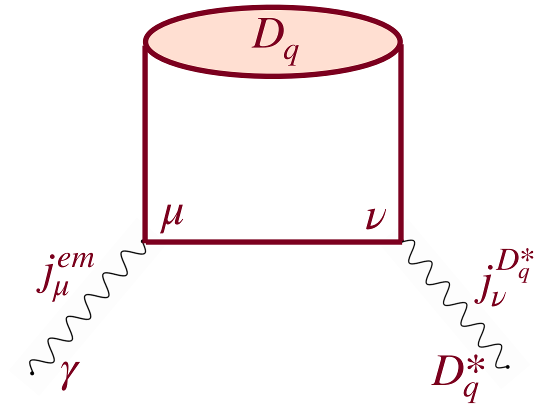

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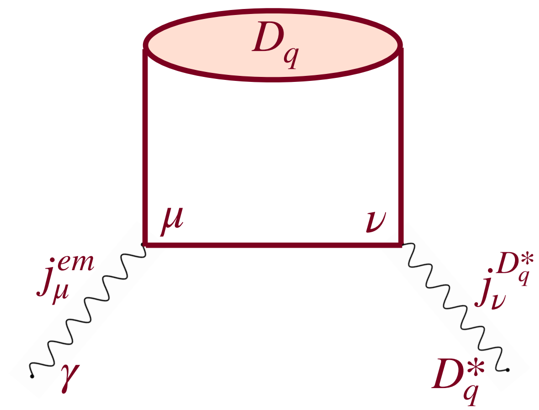

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Momentum of the D-meson carried by the light quark

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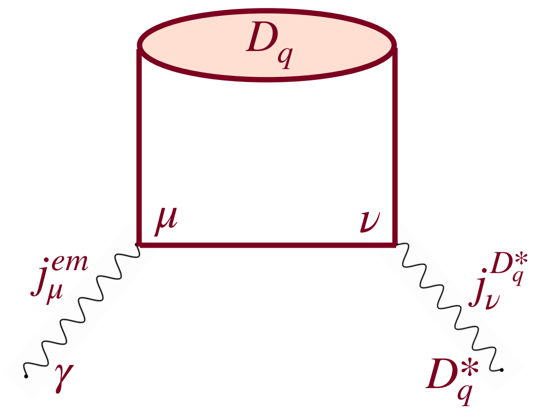

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$$T^{QCD}(s,Q^2) = ef_{D_q}m_{D_q}\int_0^\infty dw\left\{\phi_+^D(w)\left[\frac{Q_C}{(k-wv)^2-m_c^2}+\frac{Q_q}{(k+wv)^2-m_q^2}\right]+\psi_\pm^D(w)\left[\frac{Q_C m_c}{\left((k-wv)^2-m_c^2\right)^2}+\frac{Q_q m_q}{\left((k+wv)^2-m_q^2\right)^2}\right]\right\}$$

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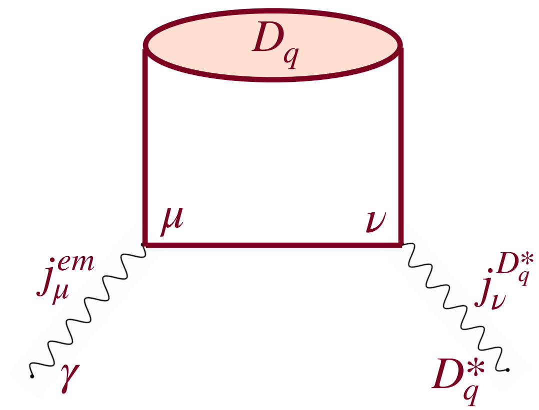

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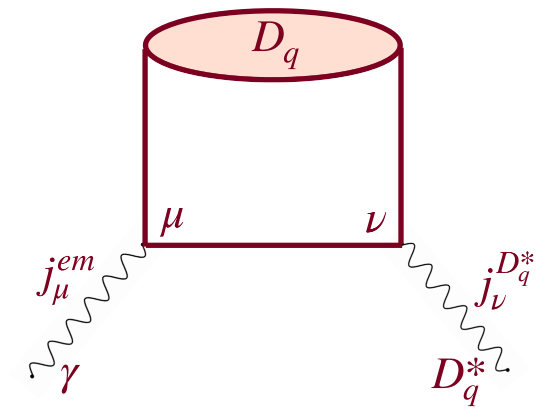

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→ Exponential Model

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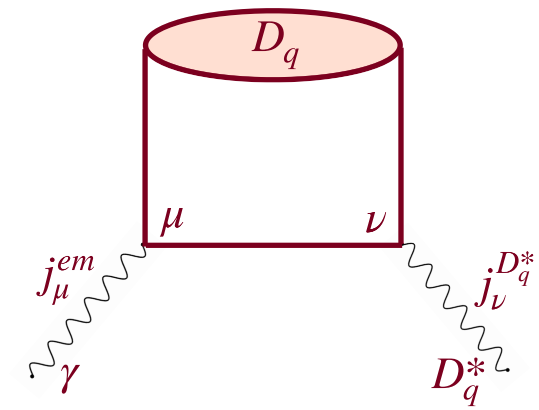

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[Grozin and Neubert, PRD 55 (1997) 272-290]

$\lambda_D = \omega_0$ in the exponential model $\Longrightarrow$ our objective is to find out ω_0

Dispersion Relation

- Using unitarity:

$$T_{\mu\nu}^{had}(p,k) = ie\langle D_q(p)\,|\,T\left\{j_\mu^{em}(x)j_\nu^{D_q^*}(0)\right\}|\,0\rangle \sim \langle D_q(p)\,|\,j_\mu^{em}\,|\,D_q^*(p+k)\rangle\langle D_q^*(p+k)\,|\,j_\nu^{D_q^*}(0)\,|\,0\rangle + \sum_n \langle D_q(p)\,|\,j_\mu^{em}\,|\,n\rangle\langle n\,|\,j_\nu^{D_q^*}(0)\,|\,0\rangle$$

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Contribution from higher resonance and continuum

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- Finally performing the Borel Transformation:

$$\mathscr{B}_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{\exp(-m^2/M^2)}{M^{2(k-1)}}$$

The final SUM RULE

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Results

- The analytic results for g_{D_q} is consistent with the heavy quark and chiral symmetry prediction, according to which $g_{D_q} \sim \frac{Q_c}{m_c}$.

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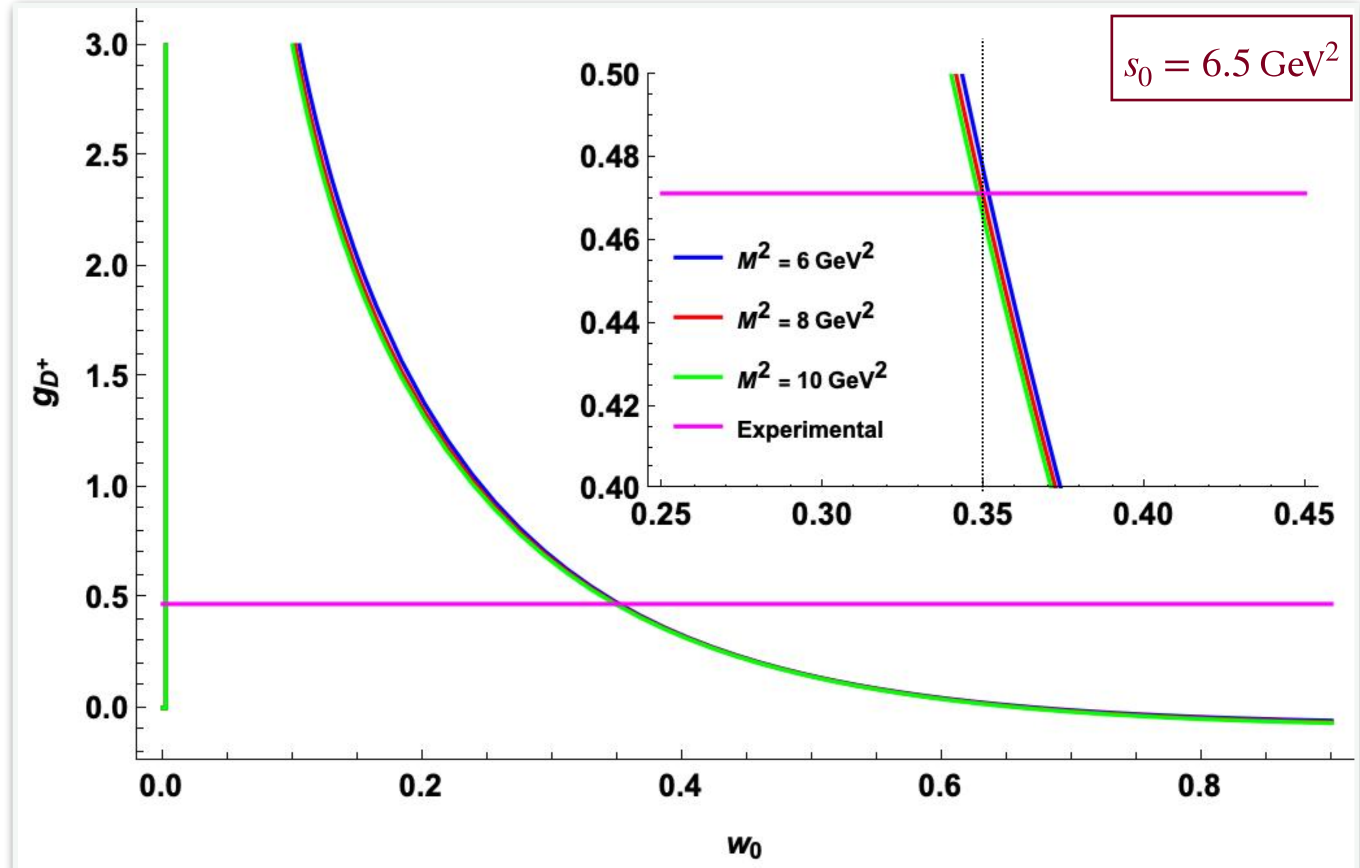

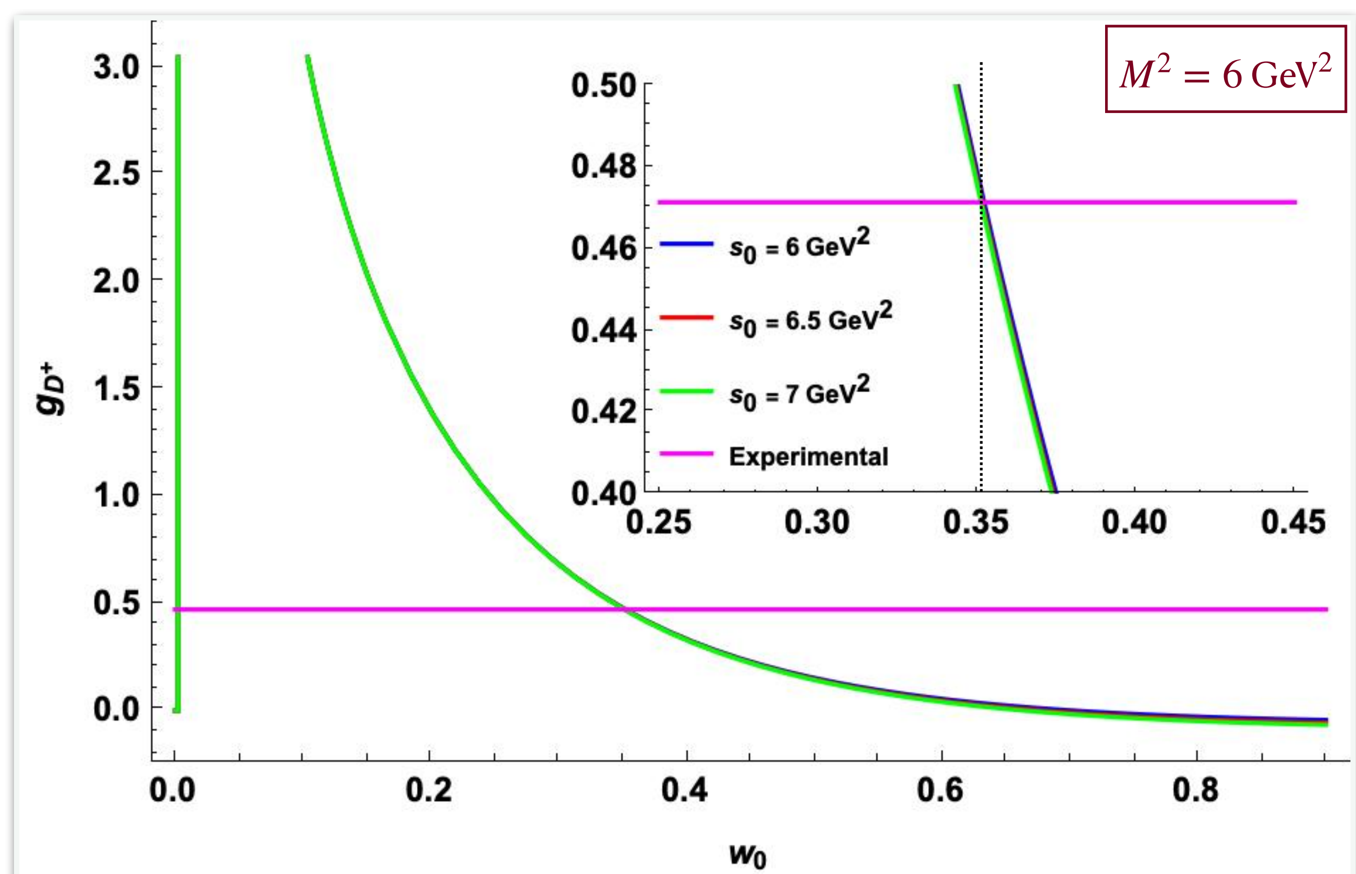

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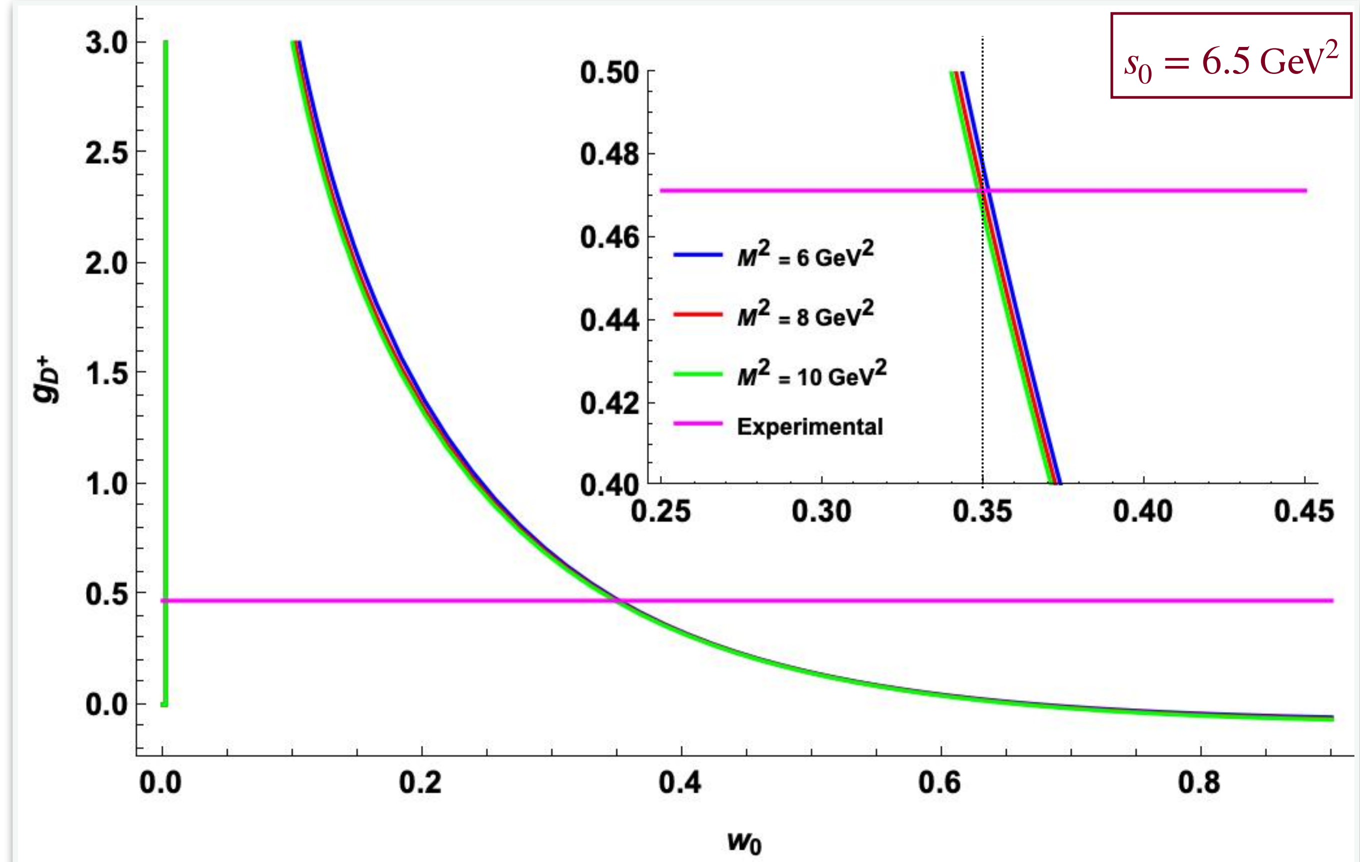

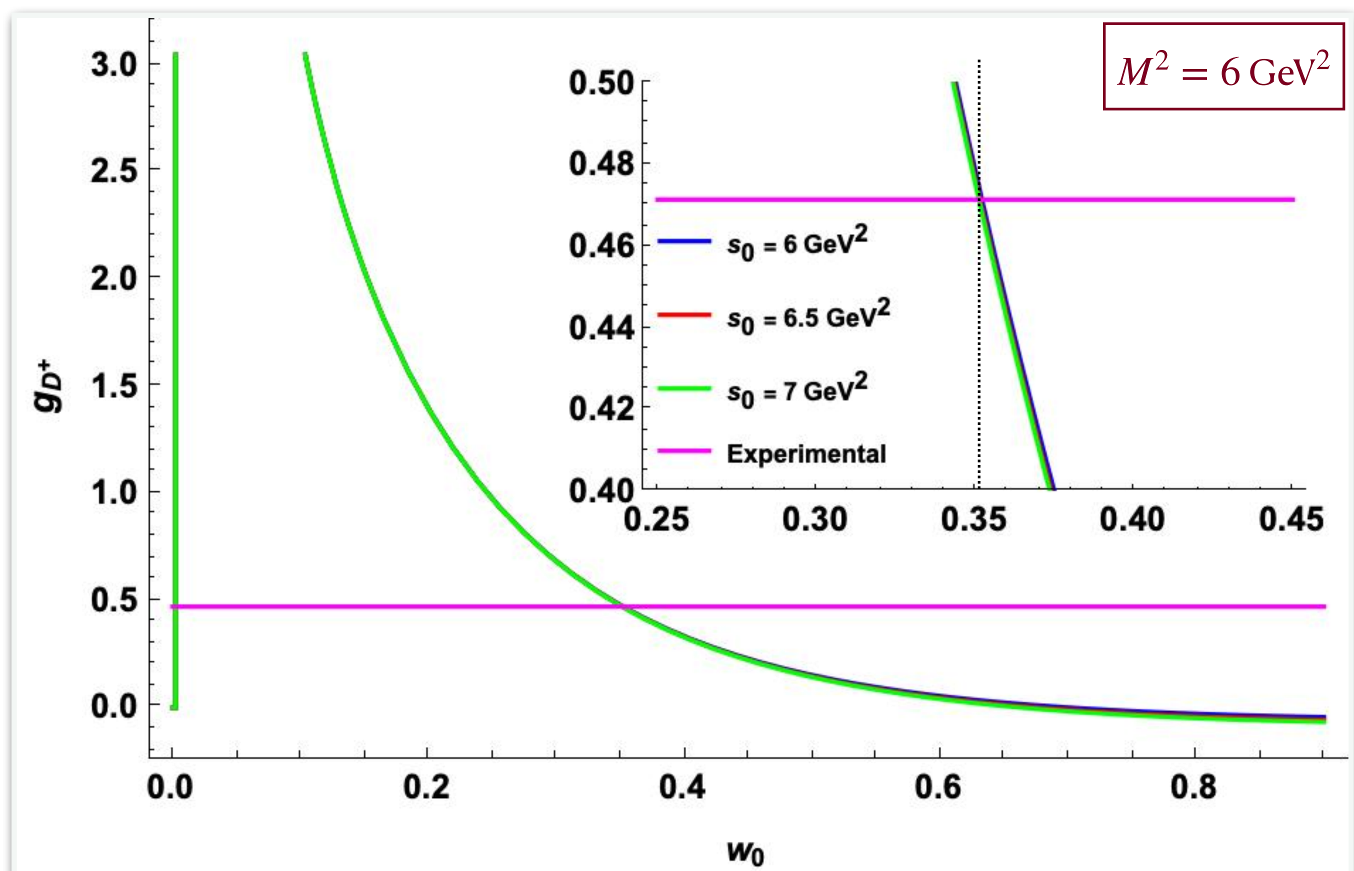

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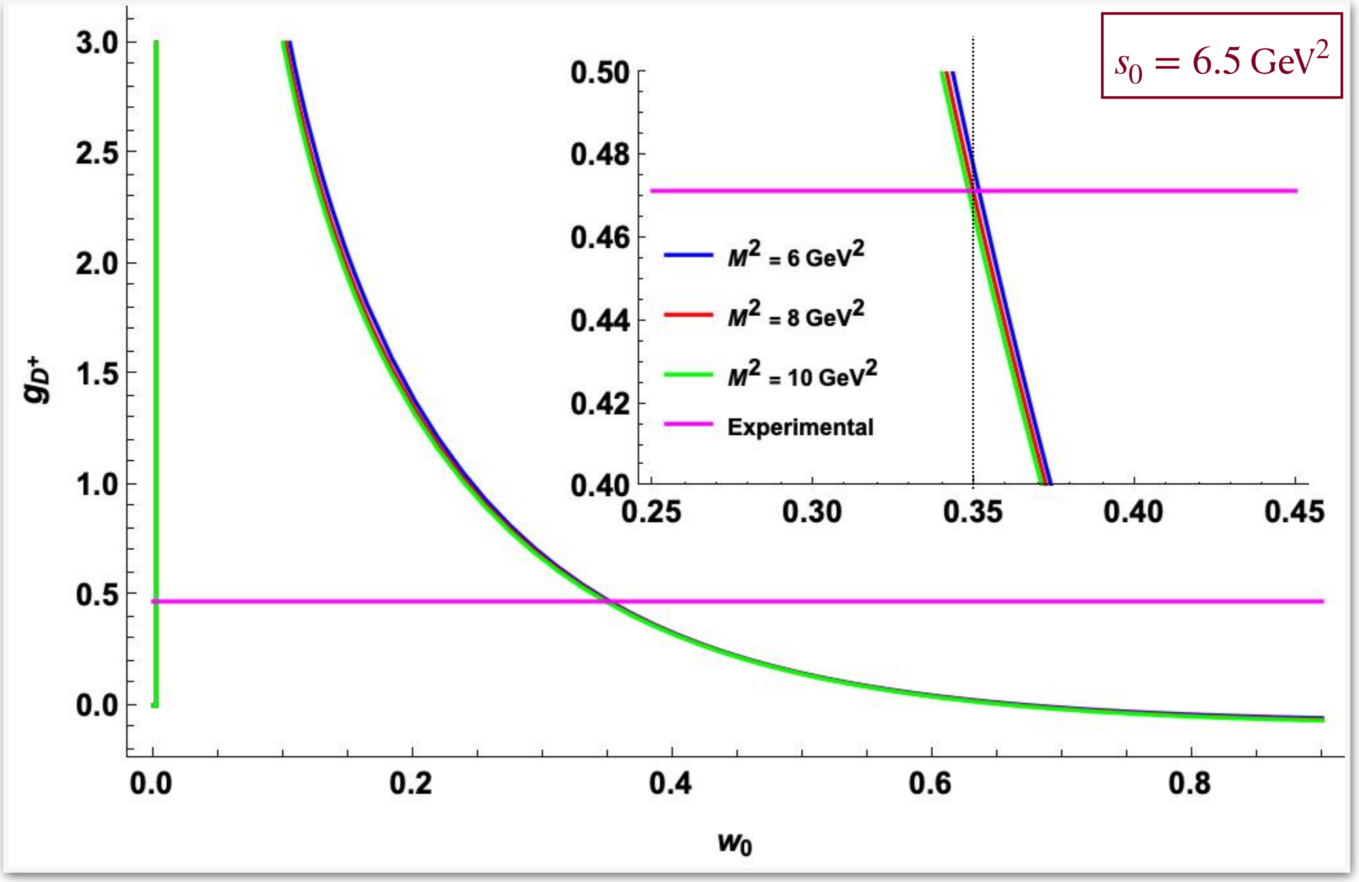

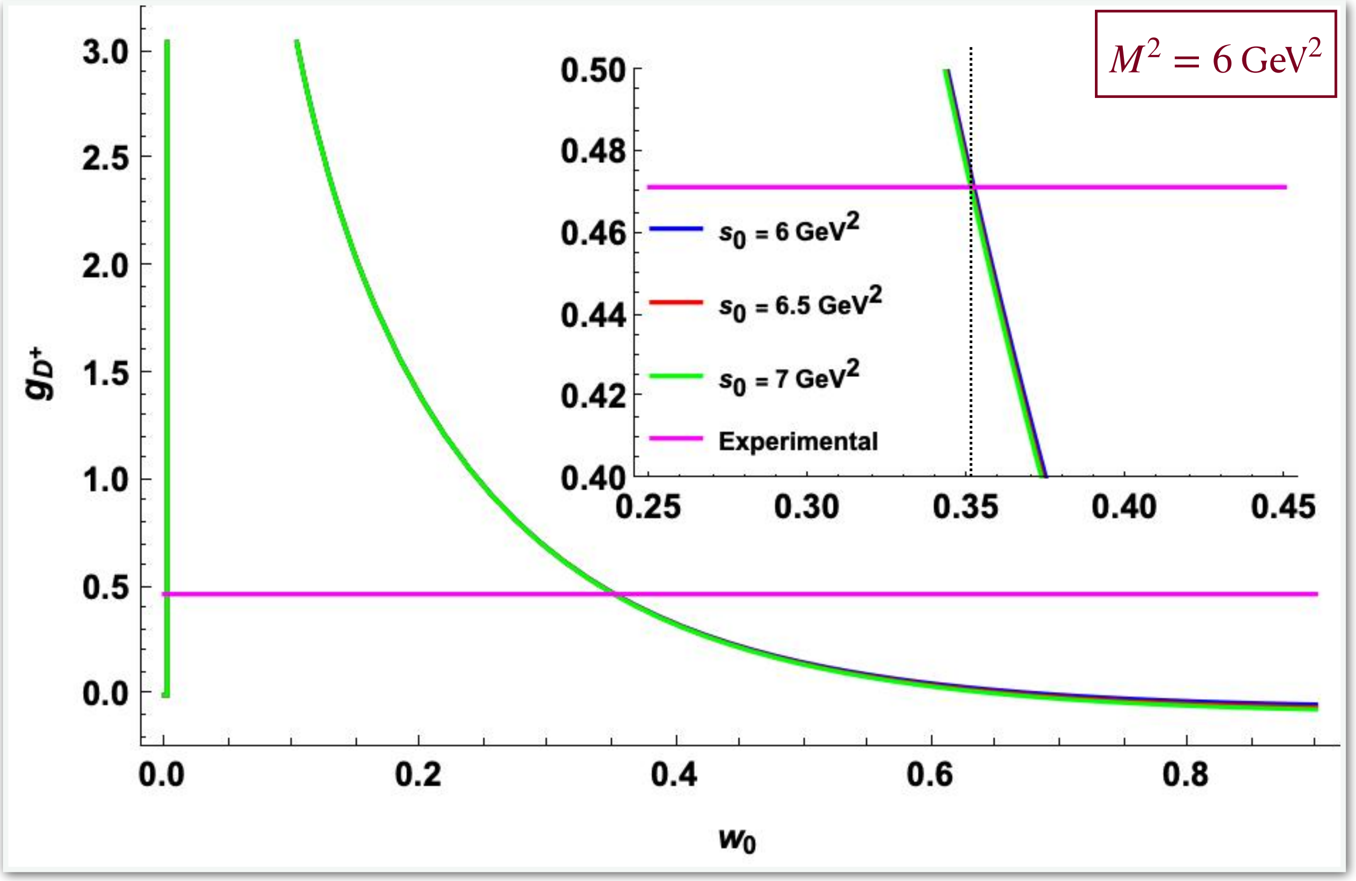

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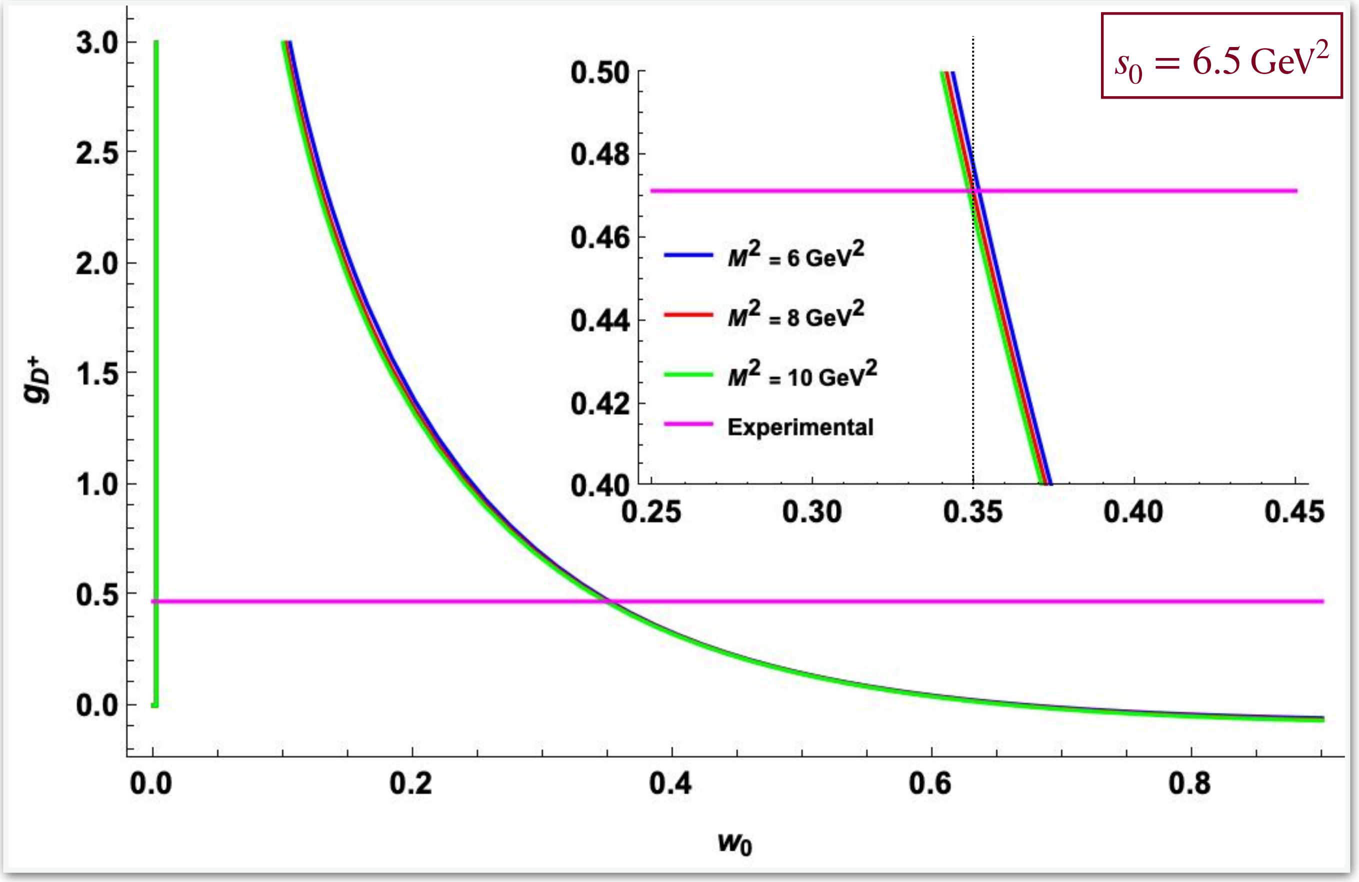

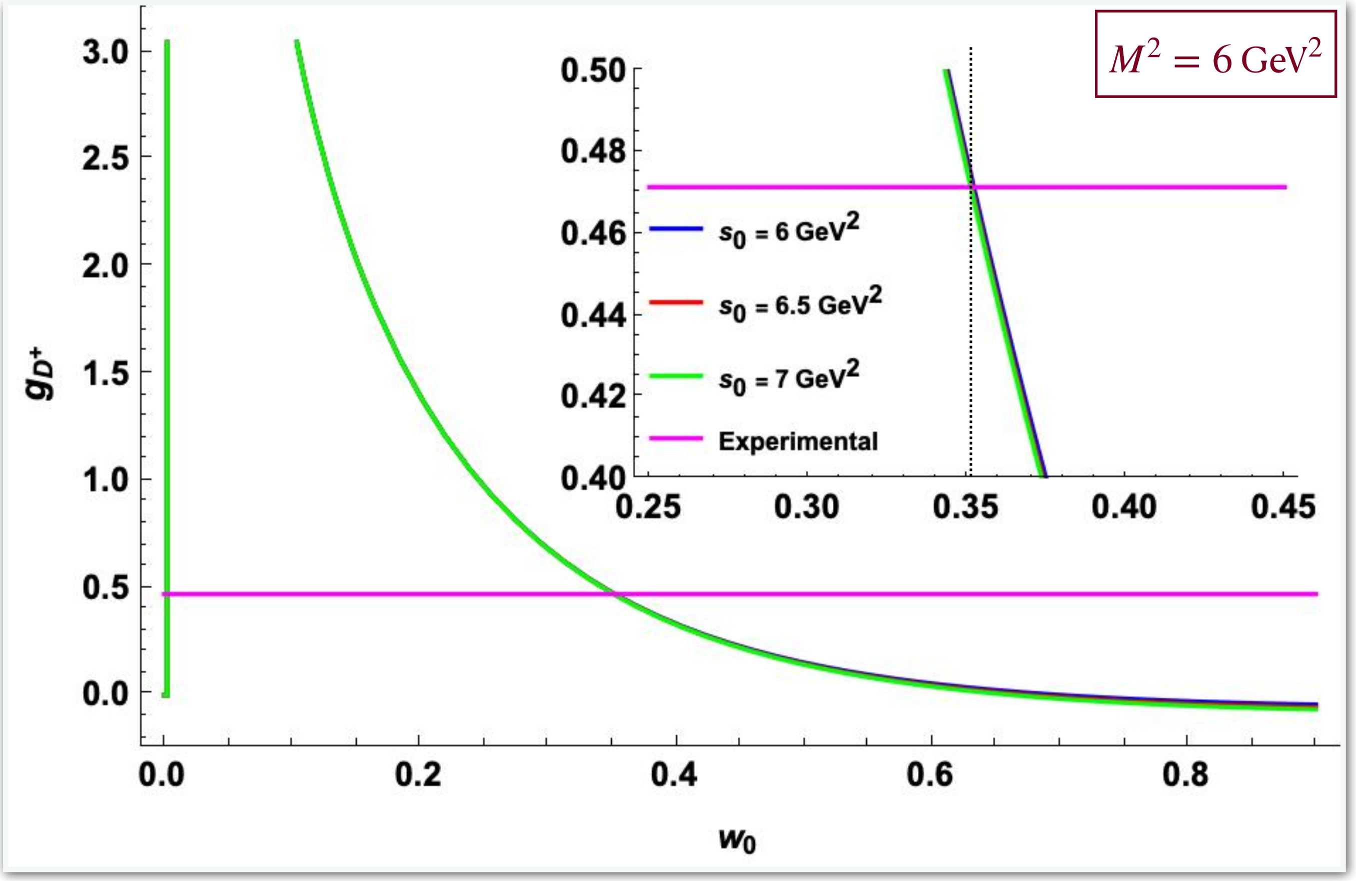

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- Proper estimates of the total decay width of the vector heavy mesons and their radiative decay channels are required at the experiments.


Thank you !!


Any
questions