

# SATURATION EFFECTS IN NEUTRON TAGGED DIS AT SMALL-X

XXV DAE-BRNS High Energy Physics Symposium  
IISER Mohali (Dec 12-16, 2022)

Arjun Kumar (IIT Delhi)

A.Kumar arXiv: 2208.14200

A.Kumar, T.Toll PRD105 (2022) 114045



विज्ञान एवं प्रौद्योगिकी विभाग  
DEPARTMENT OF  
**SCIENCE & TECHNOLOGY**

सत्यमेव जयते





# OUTLINE

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## ► Introduction to TDIS

## ► Semi-Inclusive $\gamma^* p$ cross section with leading neutrons

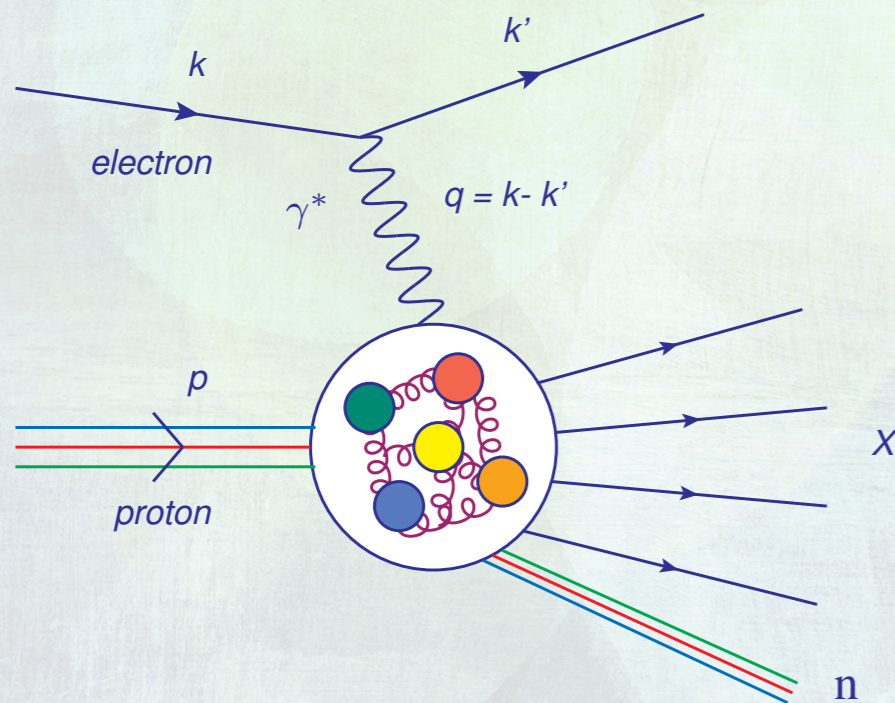
$$(e + p \rightarrow e' + X + n)$$

- ❖ Longitudinal structure and saturation

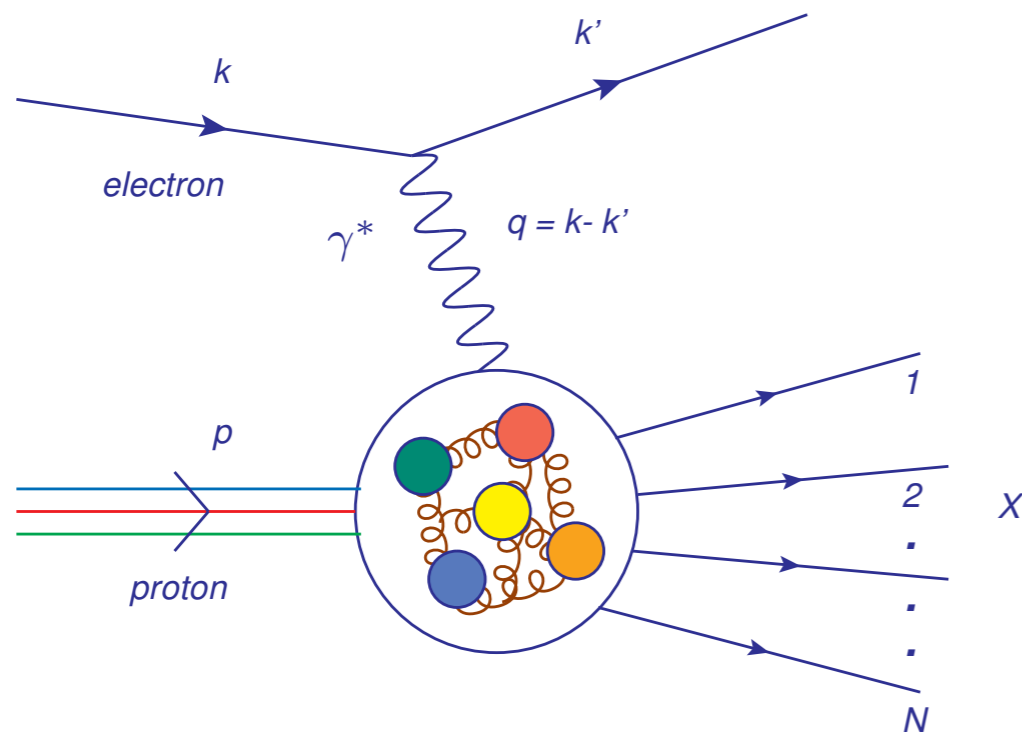
## ► Exclusive $J/\psi$ photo production with leading neutrons

$$(e + p \rightarrow e' + J/\psi + \pi + n)$$

- ❖ Higher sensitivity to saturation

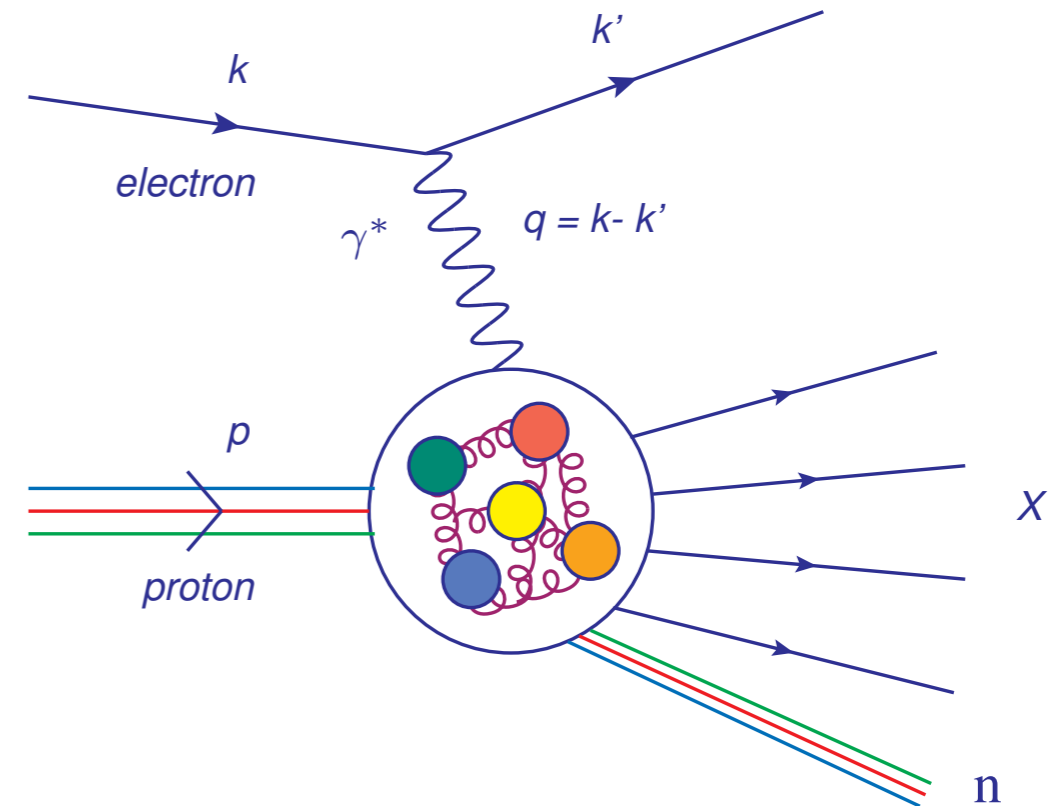


# INCLUSIVE DIS



- ❖ Detect the scattered electron
- ❖ Probe partonic structure of the targets
- ❖ Kinematic variables :  $x_{Bj}$ ,  $Q^2$ ,  $W$
- ❖ Precise measurements of the structure functions from HERA ( $F_2$ ,  $F_L$ )

# TAGGED DIS (TDIS)

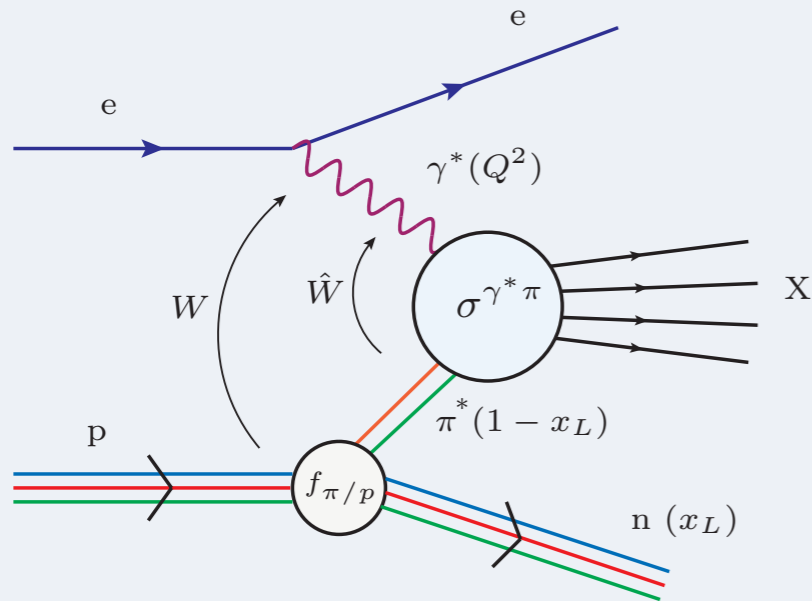


- ❖ Detect the scattered electron and the outgoing target nucleon
- ❖ Probe partonic structure of the “effective” targets not readily found
- ❖ Kinematic variables :  $x_{Bj}$ ,  $Q^2$ ,  $W$  and

$x_L \rightarrow$  momentum fraction carried by outgoing nucleon

$t \rightarrow$  four-momentum transfer squared at the nucleon vertex

# LEADING NEUTRONS (LN)



HI EPJC 74 (2014), 2915

❖ Forward neutrons:

$$\eta > 7.9, 0.1 < x_L < 0.94$$

$$0 < p_T < 0.6$$

❖ Kinematic variables:

$$\hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} = \frac{Q^2 + m_f^2}{(1 - x_L)W^2 + Q^2}$$

$$t \simeq -\frac{p_T^2}{x_L} - (1 - x_L) \left( \frac{m_n^2}{x_L} - m_p^2 \right)$$

❖ LN Structure function  $F_2^{LN}$ :

HI EPJC 68 (2010), 381

$$\frac{d^4\sigma^{ep \rightarrow eXn}}{dx dQ^2 dx_L dt} = \frac{4\pi\alpha_{EM}^2}{xQ^4} \left( 1 - y + \frac{y^2}{2} \right) F_2^{LN(4)}(x, Q^2, x_L, t)$$

❖ In terms of  $\gamma^*p$  cross section:

$$F_2^{LN}(x, Q^2, x_L) = \frac{Q^2}{4\pi^2\alpha_{EM}} \frac{d\sigma^{\gamma^*p \rightarrow Xn}}{dx_L}$$

J.D. Sullivan PRD 5 (1972), 1732

❖ In One Pion Exchange (OPE) approximation:

$$\frac{d^2\sigma(W, Q^2, x_L, t)}{dx_L dt} = f_{\pi/p}(x_L, t) \sigma^{\gamma^*\pi^*}(\hat{W}^2, Q^2)$$

$f_{\pi/p}(x_L, t)$  is pion splitting function,

$\sigma^{\gamma^*\pi^*}(\hat{W}^2, Q^2)$  is virtual photon-virtual pion cross section

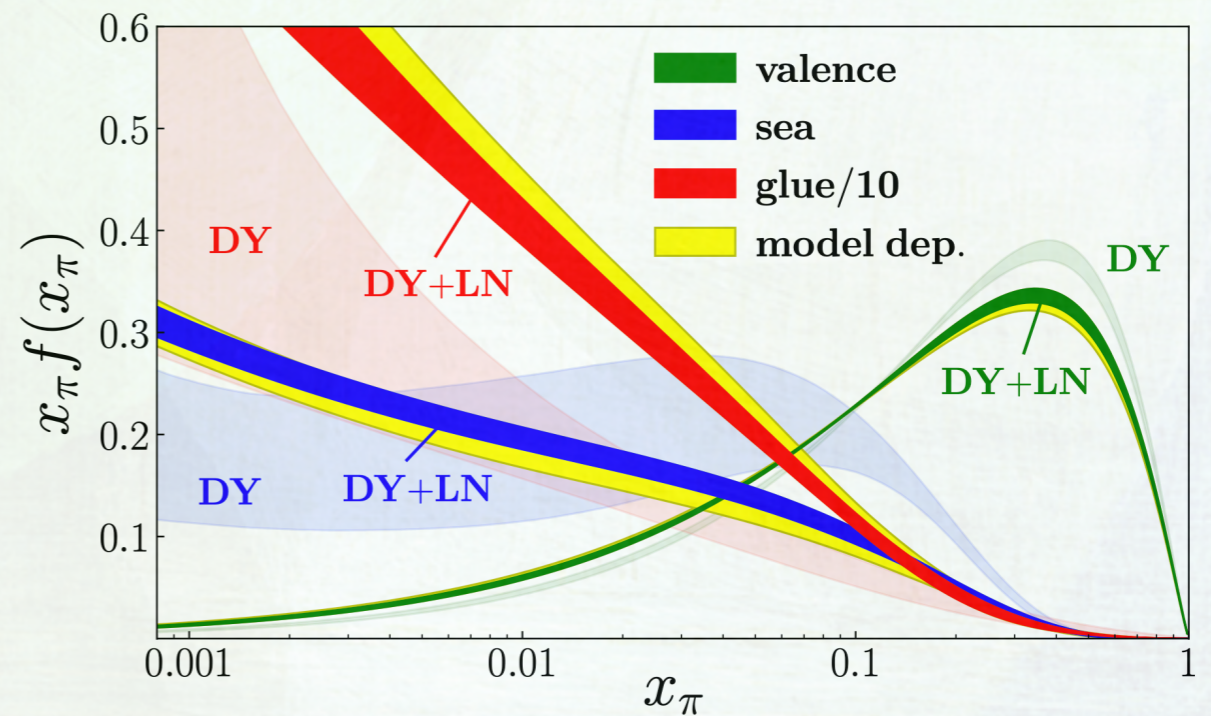
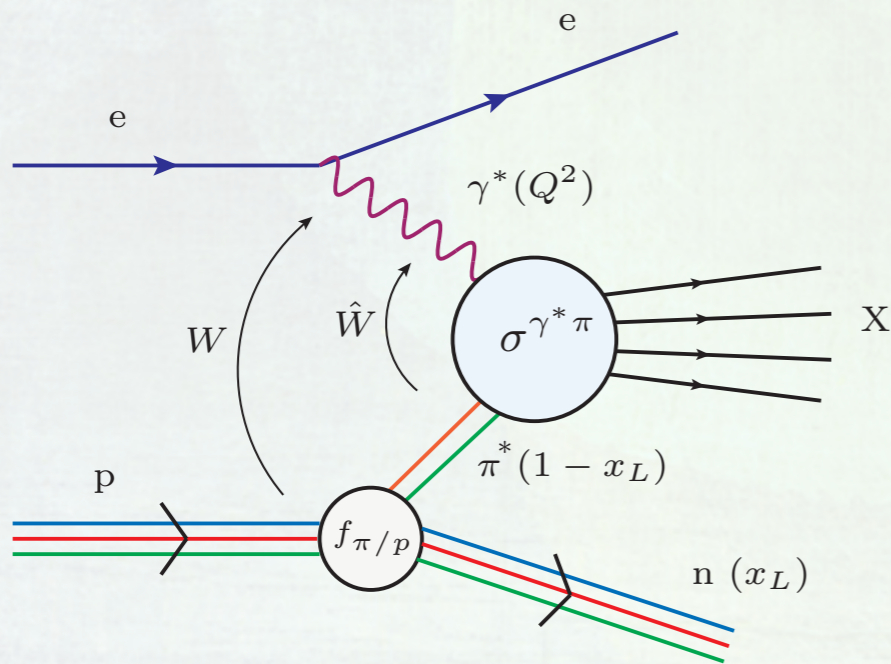
❖ OPE allows to extract the pion structure function  $F_2^\pi$ ,

$$F_2^{LN}(W, Q^2, x_L) = \Gamma(x_L, Q^2) F_2^\pi(W, Q^2, x_L)$$

$\Gamma(x_L, Q^2)$  is t-integrated flux of pions from proton



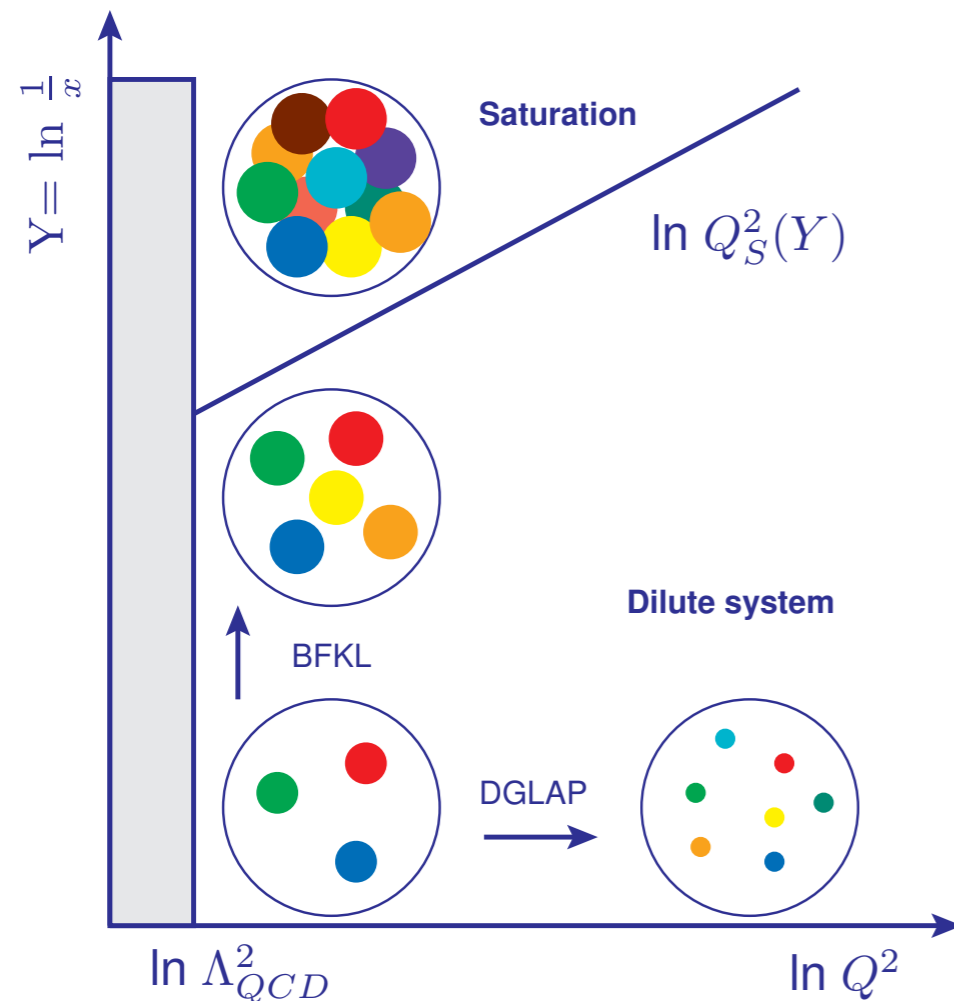
# SEMI-INCLUSIVE MEASUREMENTS WITH LEADING NEUTRONS



Barry et al PRL 121 (2018), 152001

- ★ Information on gluonic structure of pions in one pion exchange approximation
- ★ Saturation effects and Geometric Scaling
- ★ Feynman- $x_L$  spectra and Feynman scaling

# QCD PHASE SPACE DIAGRAM



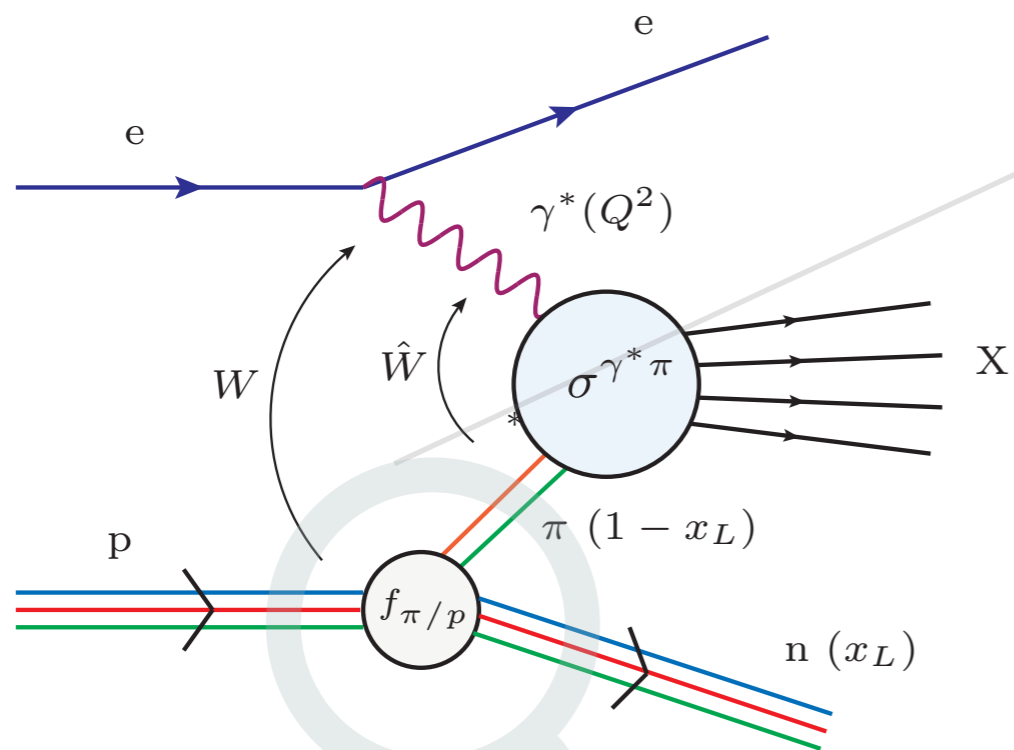
- ❖ Bjorken limit:  $Q^2 \rightarrow \infty, s \rightarrow \infty; x = \text{fixed}$ 
  - Phase space density (#partons/Area/ $Q^2$ ) decreases
  - Target becomes dilute
- ❖ High energy limit:  $x \rightarrow 0, s \rightarrow \infty; Q^2 = \text{fixed}$ 
  - Phase space density (#partons/Area/ $Q^2$ ) increases
  - Target becomes dense
- ❖ DGLAP & BFKL evolution equations violate unitarity
- ❖ High energy limit of QCD suggests presence of multi-body dynamics and non-linear physics

**Gluon Saturation:** The GLR-MQ equation [Gribov, Levin. & Ryskin, Mueller & Qiu 1986](#)

$$\frac{\partial^2 xg(x, Q^2)}{\partial \ln Q^2 \partial \ln(1/x)} = \frac{3\alpha_s}{\pi} xg(x, Q^2) - \frac{81\alpha_s^2}{16Q^2 R^2} (xg(x, Q^2))^2$$



# PION FLUX FROM PROTON



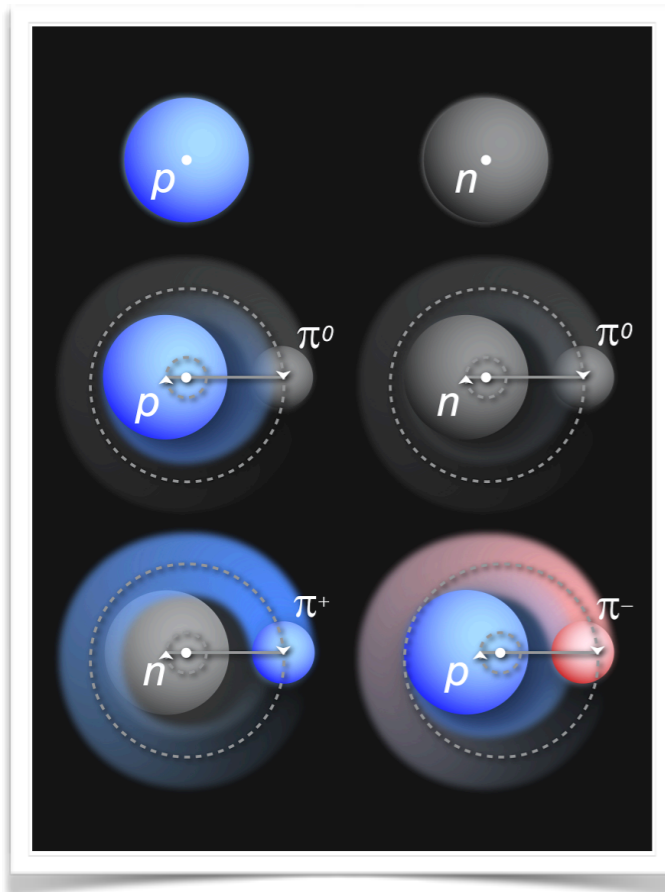
# PION FLUX FROM PROTON

Chiral approach:  $a=0.24, b=0.12$   
 Thomas, Melnitchouk & Steffens,  
 PRL85 (2000) 2892

- ❖ Proton as a superposition of states in meson-cloud models,

$$|p\rangle \rightarrow \sqrt{1-a-b} |p_0\rangle + \sqrt{a} \left( -\sqrt{\frac{1}{3}} |p_0 \pi^0\rangle + \sqrt{\frac{2}{3}} |n_0 \pi^+\rangle \right) + \sqrt{b} \left( -\sqrt{\frac{1}{2}} |\Delta_0^{++} \pi^-\rangle - \sqrt{\frac{1}{3}} |\Delta_0^+ \pi^0\rangle + \sqrt{\frac{1}{6}} |\Delta_0^0 \pi^+\rangle \right)$$

- ❖ Pion flux from proton is well known & can be calculated using chiral effective theory
- ❖ Previously used to explain hadron-hadron interactions at LHC



- ❖ We use the following flux factor: [Carvalho et al PLB 752 \(2016\) 76](#)

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi} \frac{2g_{p\pi p}^2}{4\pi} \frac{|t|}{(m_\pi^2 + |t|)^2} (1 - x_L)^{1-2\alpha(t)} [F(x_L, t)]^2$$

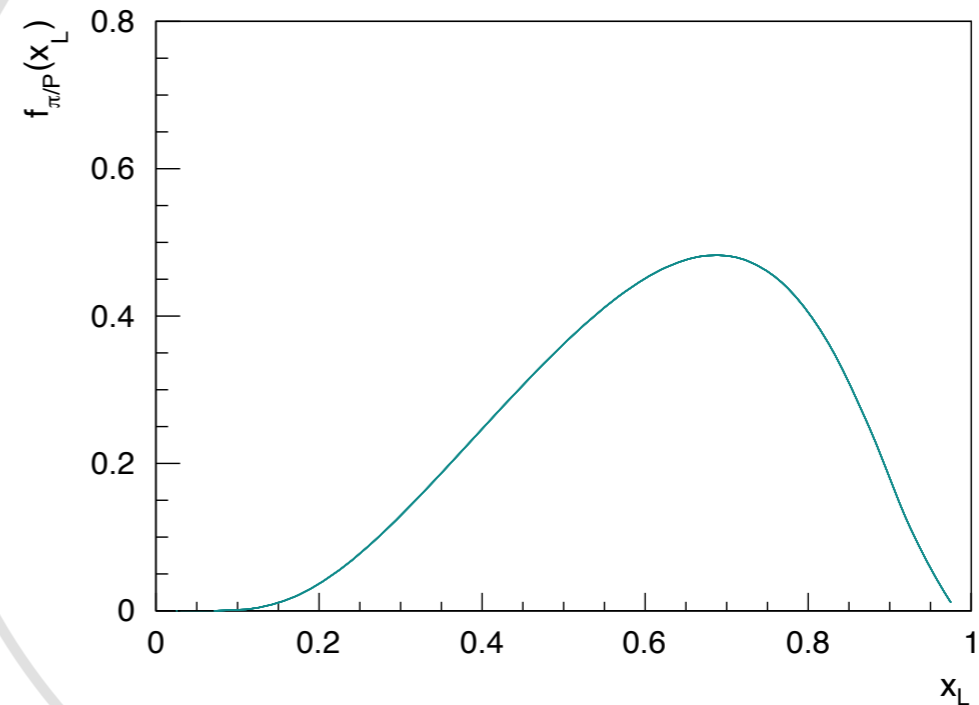
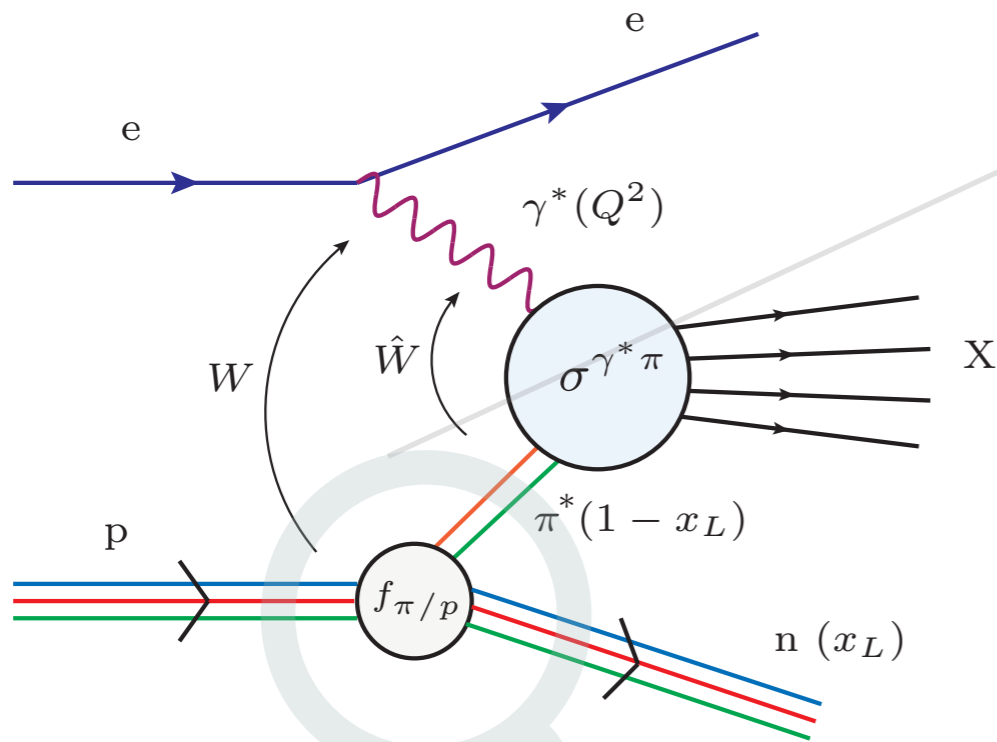
where the form factor is given by:

$$F(x_L, t) = \exp \left[ -R^2 \frac{|t| + m_\pi^2}{(1 - x_L)} \right], \alpha(t) = 0$$

- ❖ Used by H1 and ZEUS for the data analysis [HI EPJC 68 \(2010\), 381](#)



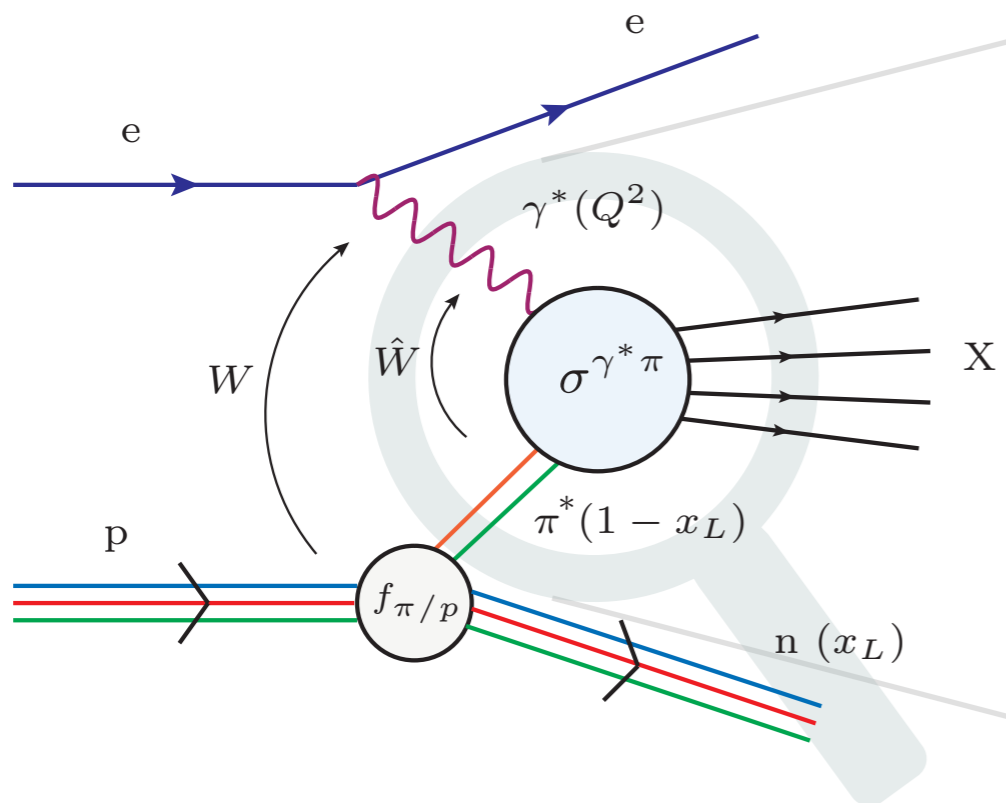
# PION FLUX FROM PROTON



❖ The pion flux peaks at  $x_L \sim 0.7$ , which suggests that the LN spectra should also peak at high-  $x_L$  values

# VIRTUAL PION-PHOTON CROSS SECTION

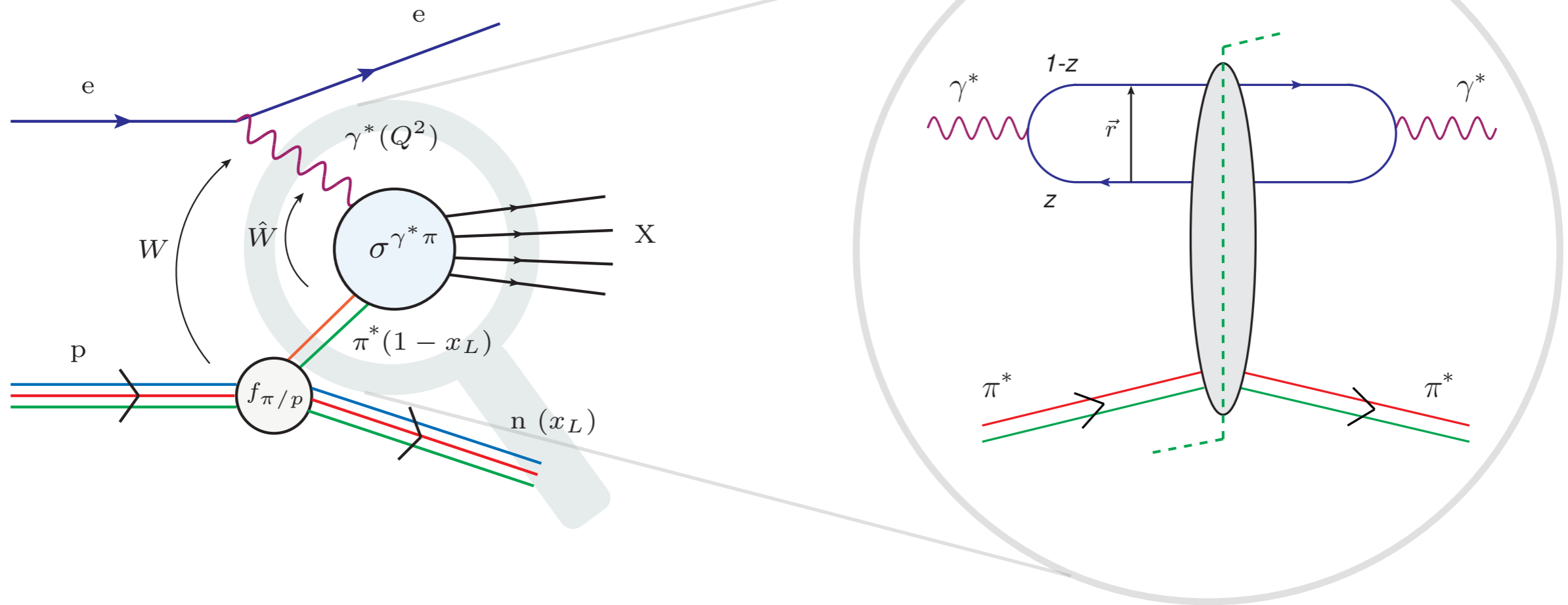
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- ❖ For total virtual photon- virtual pion cross section
  - use dipole framework (natural in target rest frame)

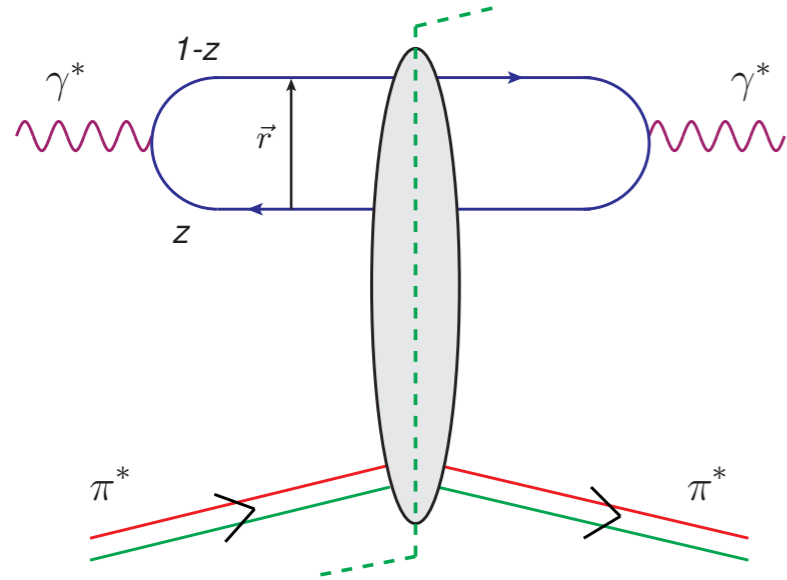


# VIRTUAL PION-PHOTON CROSS SECTION IN DIPOLE MODEL



- ❖ For total virtual photon- virtual pion cross section
  - use dipole framework (natural in target rest frame)

# VIRTUAL PION-PHOTON CROSS SECTION IN DIPOLE MODEL



Various stages of semi-inclusive scattering of photons on pions in dipole model:

1.  $\gamma^* \rightarrow q\bar{q}$  splitting (*QED*)
2. Dipole  $\rightarrow \pi^*$  scattering (*model + QCD*)
3. Dipole  $\rightarrow \gamma^*$  (*QED*)

$$\sigma_{L,T}^{\gamma^*\pi^*}(\hat{x}, Q^2) = \text{Im } \mathcal{A}(\hat{x}, Q^2, \Delta = 0) = \int d^2\mathbf{b} \int d^2\mathbf{r} \int \frac{dz}{4\pi} |\Psi_{L,T}^f(\mathbf{r}, z, Q^2)|^2 \frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x})$$

❖ Two phenomenological parameterisations:

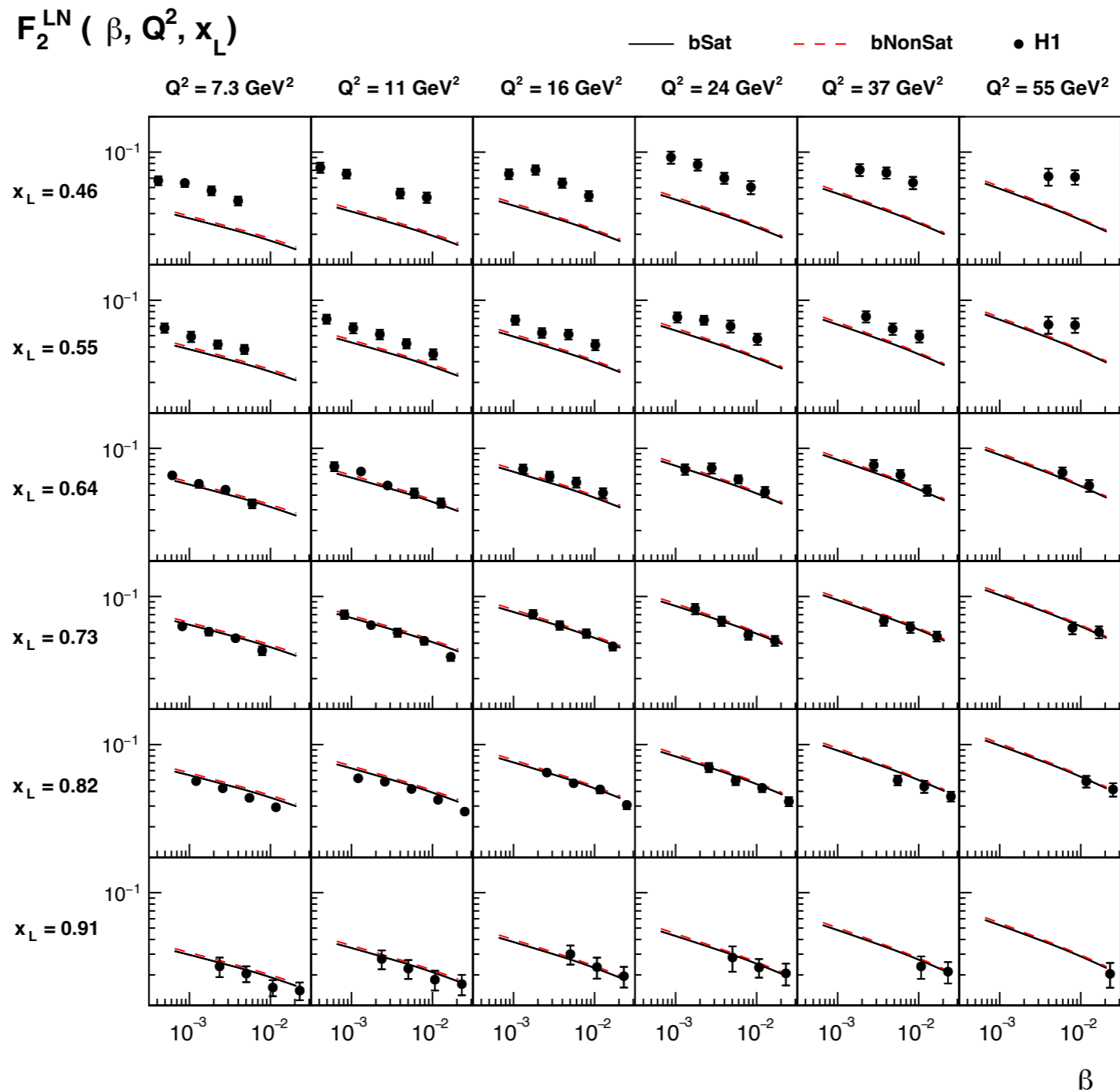
$$\text{bSat} : \frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x}) = 2 \left[ 1 - \exp\left( -\frac{\pi^2}{2N_C} r^2 \alpha_s(\mu^2) \hat{x} g(\hat{x}, \mu^2) T_\pi(\mathbf{b}) \right) \right]$$

$$\text{bNonSat} : \frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x}) = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) \hat{x} g(\hat{x}, \mu^2) T_\pi(\mathbf{b}) \text{ with}$$

with  $T_\pi(\mathbf{b}) = \frac{1}{2\pi B_\pi} e^{-\frac{b^2}{2B_\pi}}$  and  $\hat{x} g(\hat{x}, \mu_0^2) = A_g \hat{x}^{-\lambda_g} (1 - \hat{x})^6$  and  $\mu^2 = \mu_0^2 + \frac{C}{r^2}$ , the parameters  $A_g, \lambda_g, C$  are fitted to

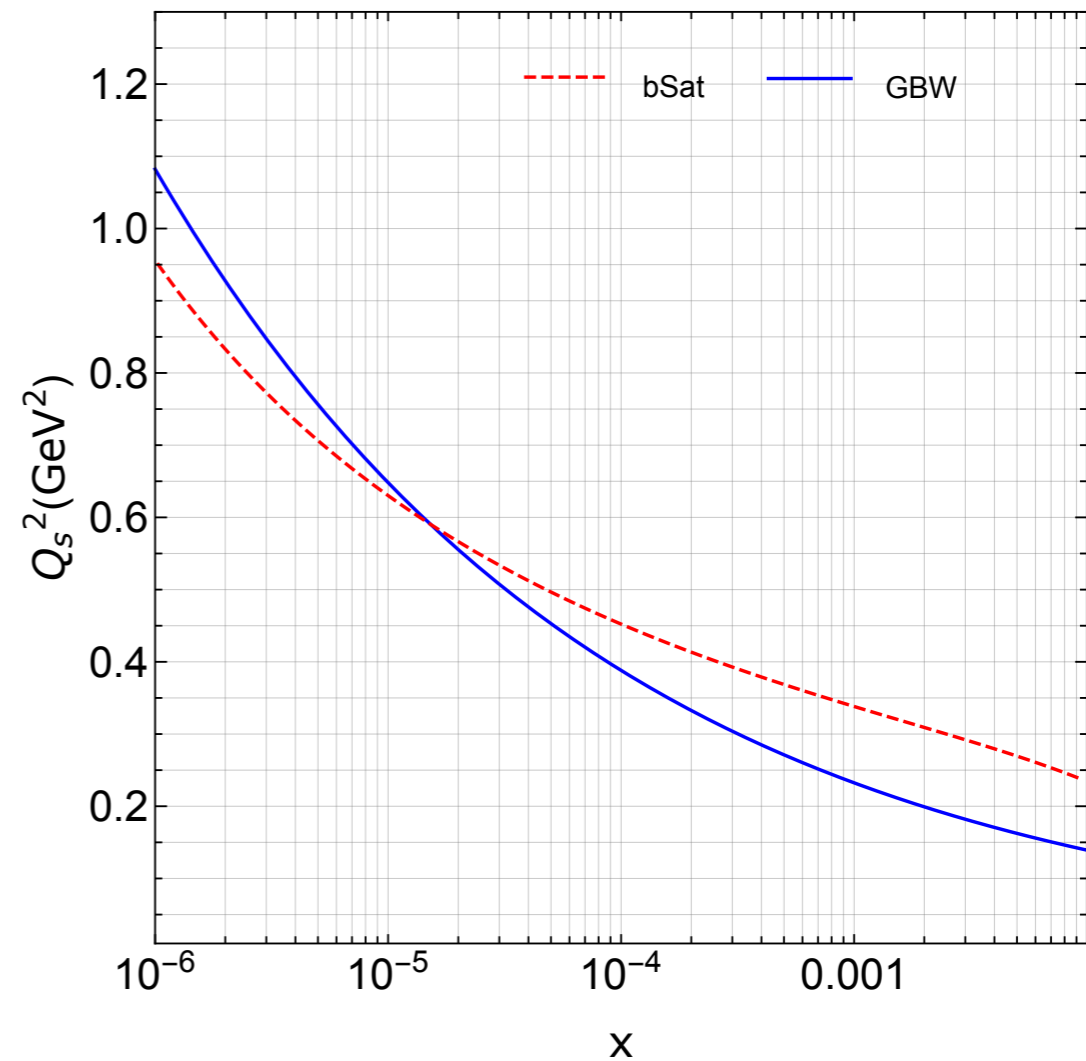
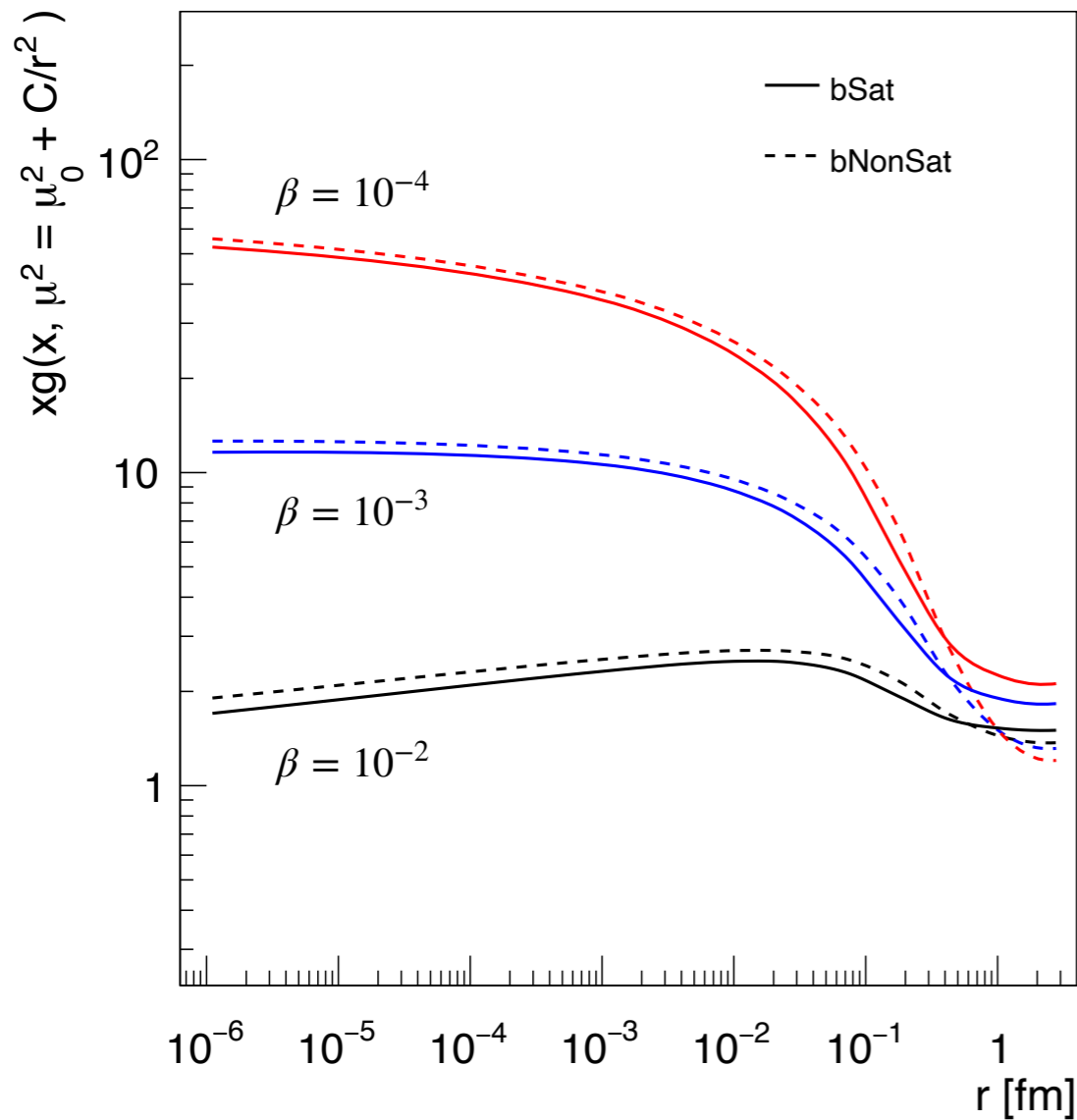
leading neutron structure function HERA data.

# LN STRUCTURE FUNCTION $F_2^{LN}$



| Fit | Model   | N  | $x_{Lmin}$ | $x_{Lmax}$ | $m_l$  | $m_c$  | $\mu_0$ | C                 | $A_g$             | $\lambda_g$          | $\chi^2/N_{dof}$  |
|-----|---------|----|------------|------------|--------|--------|---------|-------------------|-------------------|----------------------|-------------------|
| 1   | bSat    | 51 | 0.6        | 1.0        | 0.0300 | 1.3528 | 1.1     | $1.453 \pm 0.024$ | $1.208 \pm 0.012$ | $0.0600 \pm 0.0380$  | $58.75/48 = 1.22$ |
| 2   | bNonSat | 51 | 0.6        | 1.0        | 0.1516 | 1.3504 | 1.1     | $3.683 \pm 0.436$ | $1.799 \pm 0.710$ | $-0.0477 \pm 0.0038$ | $57.61/48 = 1.20$ |

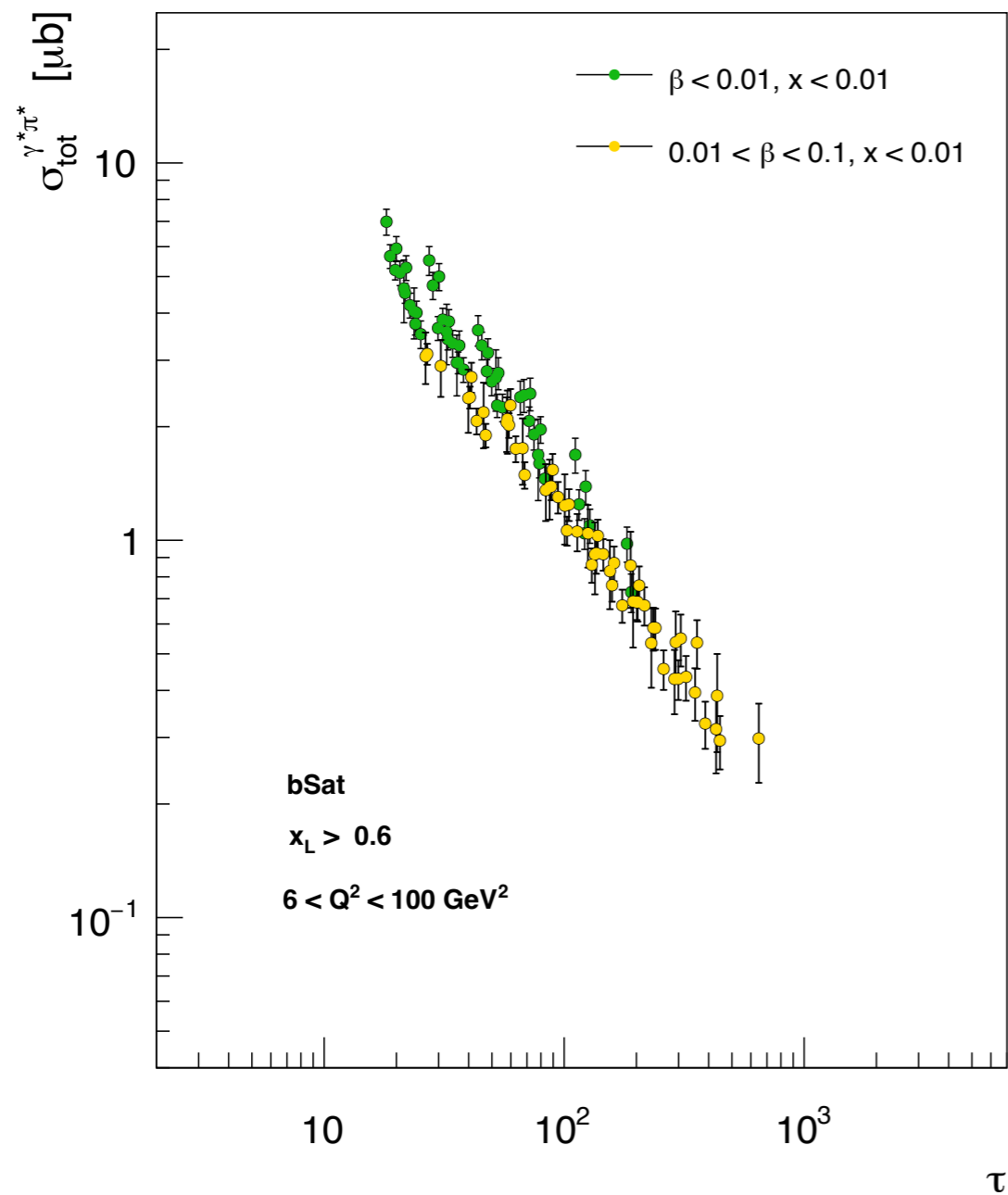
# GLUON DENSITY & SATURATION SCALE



- ❖ The initial condition effects are washed out in the evolution and gluon density is same for both the saturated and non-saturated dipole models.
- ❖ Saturation scale increases with decreasing  $x$  and reaches  $Q_s^2 \sim 1 \text{ GeV}^2$  for  $x \sim 10^{-6}$

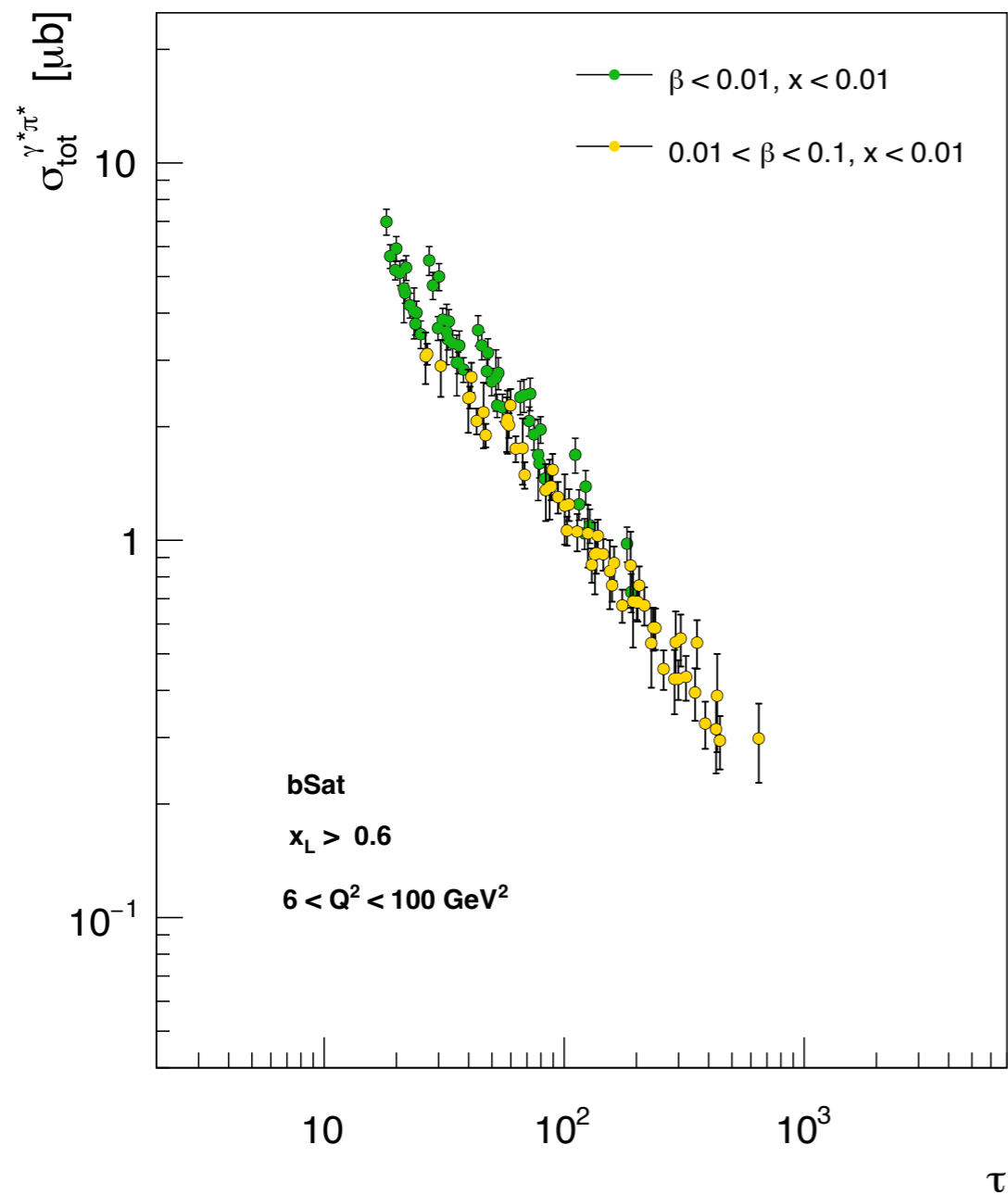


# GEOMETRIC SCALING





- ❖ The total cross section shows geometric scaling when plotted against  $\tau = \frac{Q^2}{Q_s^2(\beta)}$
- ❖ Can we say that the data shows saturation?
- ❖ Emergence of a scale  $Q_s^2(\beta)$  in data

# GEOMETRIC SCALING

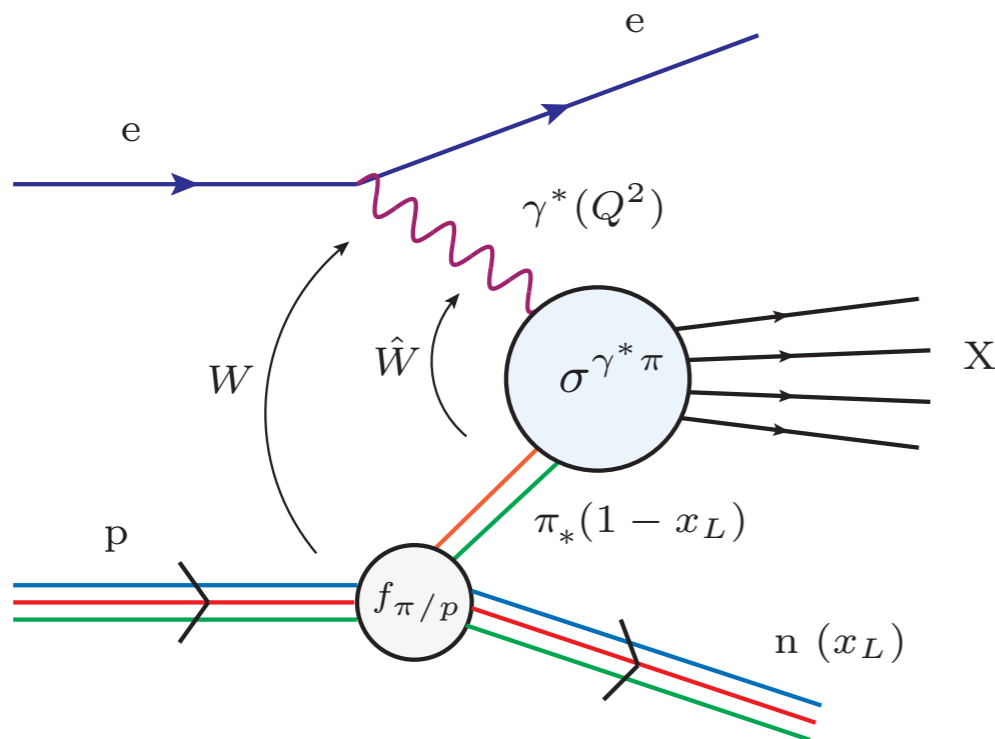


❖ The total cross section shows geometric scaling when plotted against  $\tau = \frac{Q^2}{Q_s^2(\beta)}$

❖ Can we say that the data shows saturation? 

❖ Emergence of a scale  $Q_s^2(\beta)$  in data 

# SEMI-INCLUSIVE PROCESS WITH LEADING NEUTRONS



## READ MORE HERE

[A.Kumar arXiv: 2208.14200](#)

[A.Kumar, T.Toll PRD 105 \(2022\) 114045](#)

## WHAT WE HAVE LEARNT

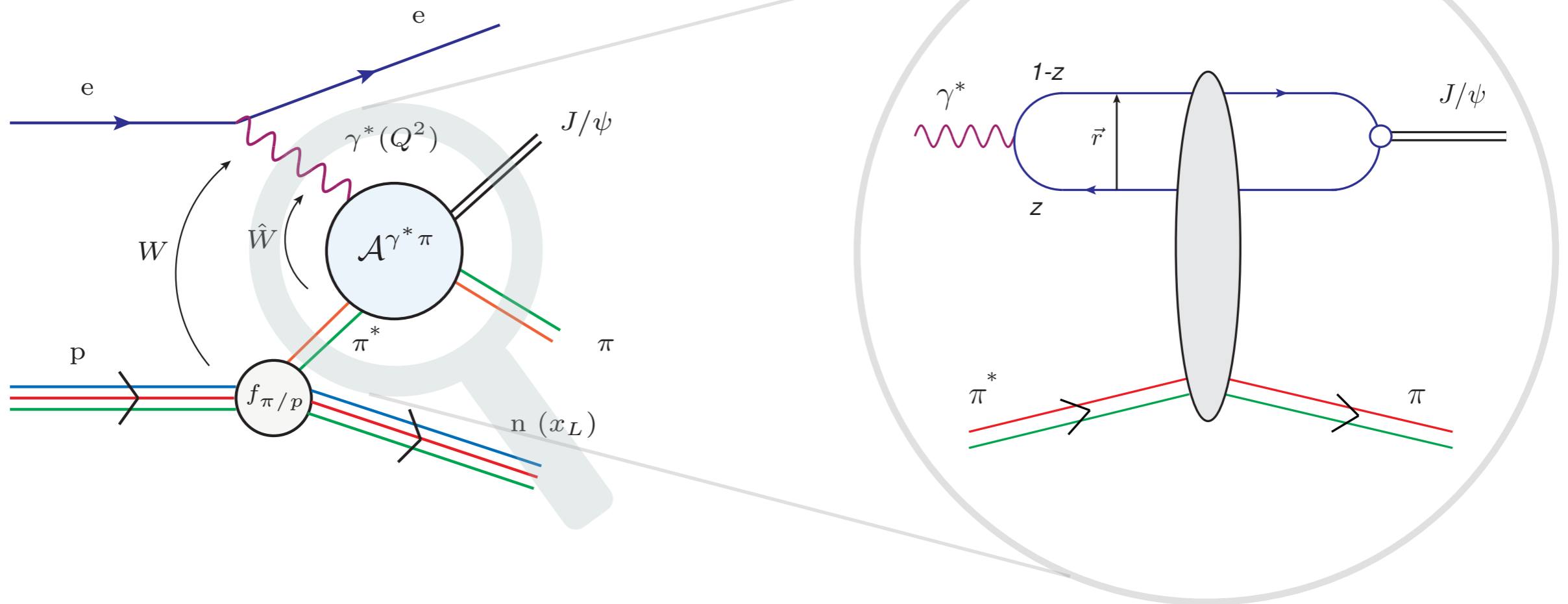
- ❖ Access to the structure of nearly on-shell pions through TDIS
- ❖ Dipole model phenomenology could successfully describe the leading neutron data
- ❖ Constrain the gluon distribution of pions at small- $x$  in the dipole framework
- ❖ The total  $\gamma^*\pi^*$  cross section shows geometric scaling behaviour
- ❖ **No hints of saturation effects**

## NOT COVERED HERE

- ❖ Feynman spectra of leading neutrons
- ❖ Feynman scaling and its link with saturation
- ❖ Universality of the pion and proton structure at small- $x$

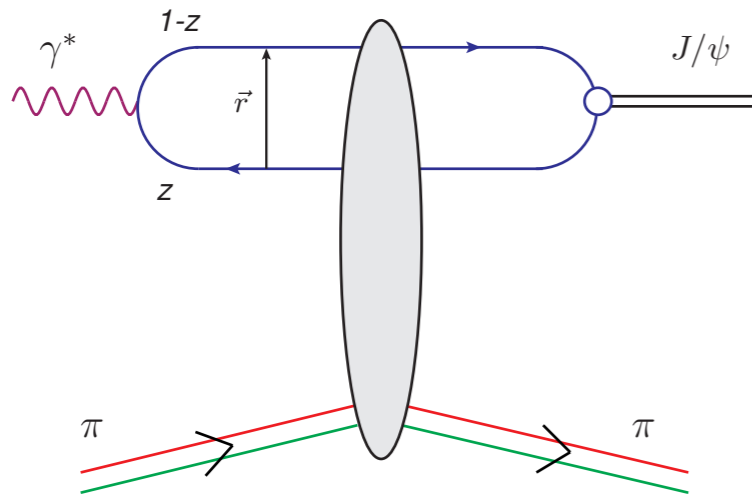


# EXCLUSIVE $J/\psi$ PRODUCTION WITH LEADING NEUTRONS



- ❖ Exclusive measurement :  $e + p \rightarrow e' + J/\psi + \pi + n$
- ❖ Experimental signature : Rapidity gap between  $J/\psi$  and the pion
- ❖ Highly sensitive to saturation

# EXCLUSIVE $J/\psi$ PRODUCTION WITH LEADING NEUTRONS



Various stages of exclusive  $J/\psi$  production in the dipole model:

1.  $\gamma^* \rightarrow q\bar{q}$  splitting (*QED*)
2. Dipole  $\rightarrow \pi^*$  scattering (*model + QCD*)
3. Dipole  $\rightarrow V.M$  (*model*)

$$\mathcal{A}_{T,L}^{\gamma^* \pi^* \rightarrow J/\psi \pi}(\hat{x}, Q^2, \Delta) \sim \int d^2\mathbf{b} \int d^2\mathbf{r} \int dz (\Psi^* \Psi)_{L,T}(\mathbf{r}, z, Q^2) e^{i\mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x})$$

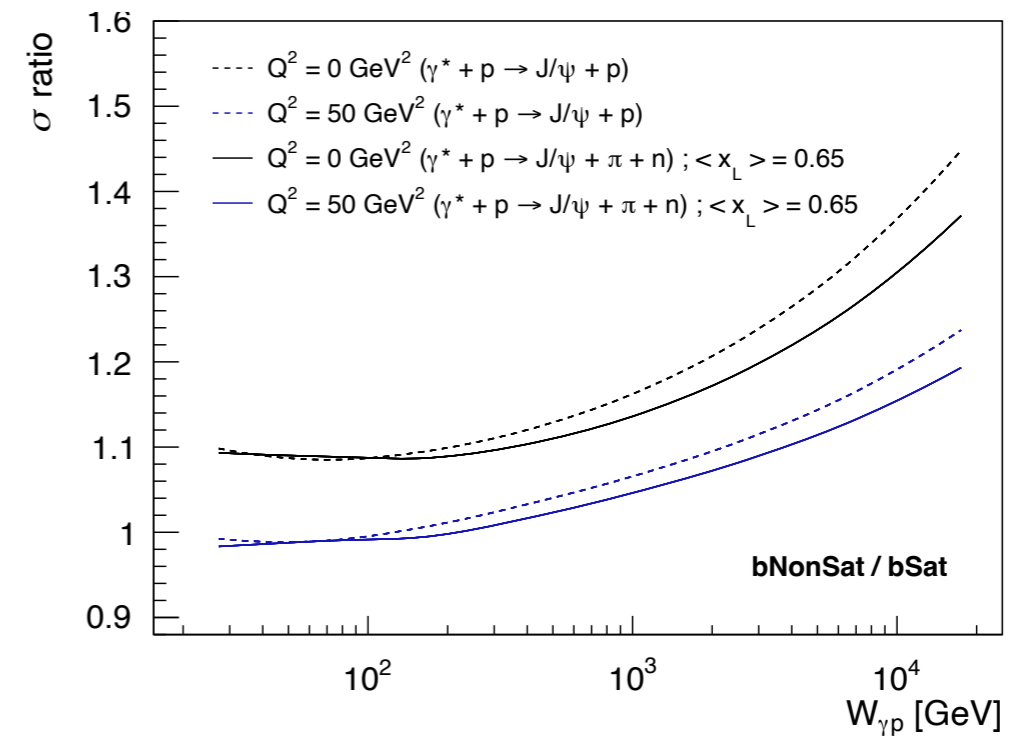
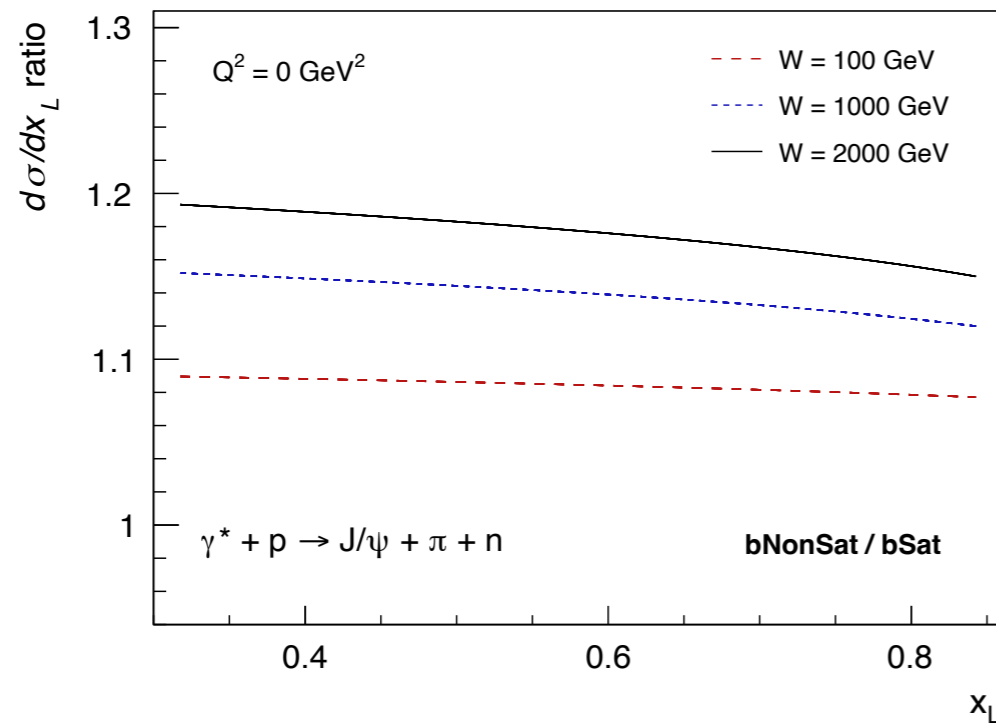
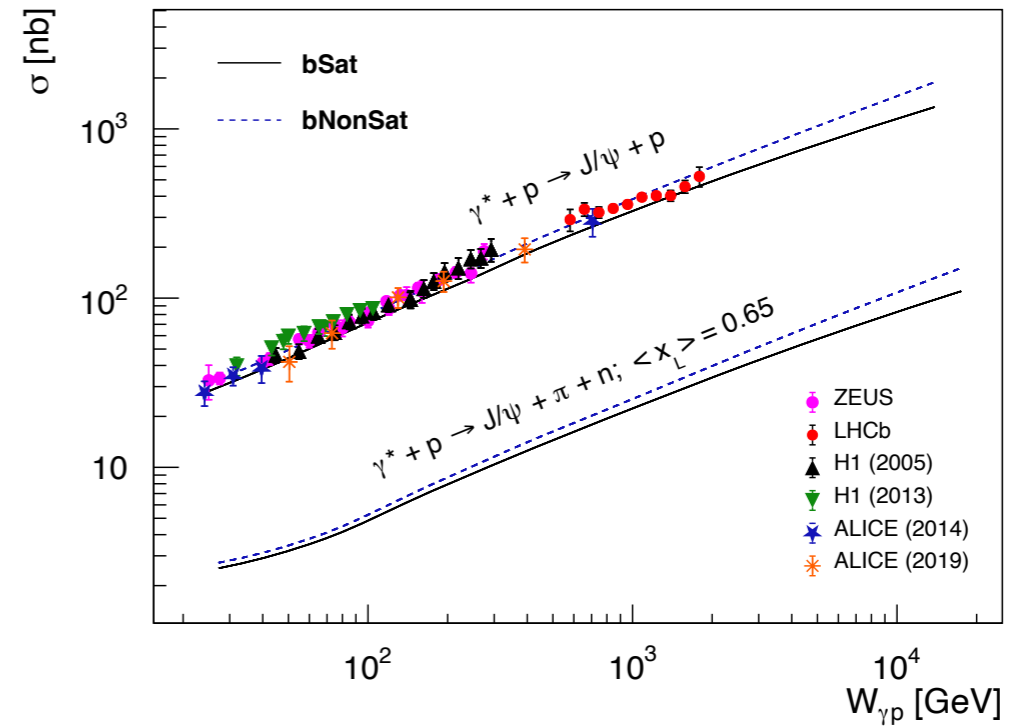
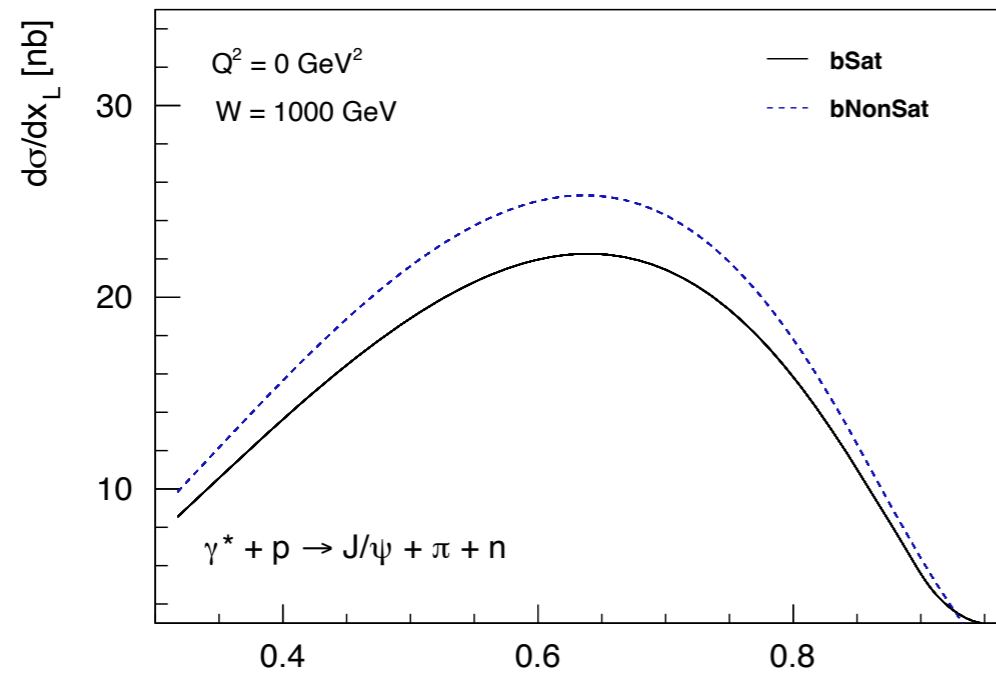
❖ Same dipole cross-section fitted from semi-inclusive data (bSat and bNonSat)

❖ The differential cross section is

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow V p}}{dt} = \frac{1}{16\pi} \left| \mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p} \right|^2 \sim [xg(x, Q^2)]^2$$

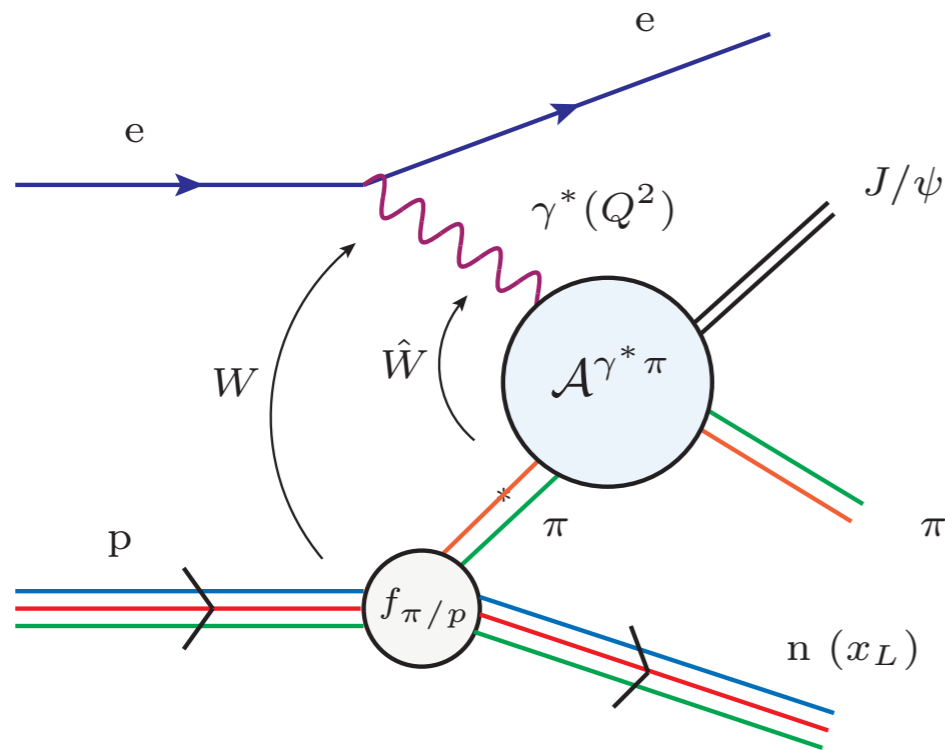
❖ Exclusive cross sections are highly sensitive to saturation

# EXCLUSIVE $J/\Psi$ PRODUCTION WITH LEADING NEUTRONS





# EXCLUSIVE MEASUREMENTS WITH LEADING NEUTRONS



## WHAT WE HAVE LEARNT

- ❖ Dipole model provides a unified framework to study inclusive and exclusive events
- ❖ The total exclusive  $J/\psi$  cross section is sensitive to saturation at high energies
- ❖ Feynman spectra of leading neutrons could be used to look for saturation effects at high energies
- ❖ Usual exclusive events with proton are more sensitive to saturation
- ❖ Probe for proton fluctuation into pion and neutron

## READ MORE HERE

[A.Kumar, T.Toll PRD 105 \(2022\) 114045](#)

More on exclusive physics and proton structure: review talk by T.Toll on 12/12/22

## NOT COVERED HERE

- ❖ Transverse gluon distribution of pions
- ❖ Gluon radius of pions and t-spectrum

# SUMMARY AND OUTLOOK

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- ❖ Investigated virtual photon scattering with the pion cloud of proton in  $ep$  scattering using dipole model
  - Semi-inclusive measurement:
    - probe the longitudinal structure function of pions
    - Both the saturated and non-saturated model describes the data
  - Exclusive measurement:
    - Sensitive to non-linear or saturation effects but less so than that of protons
    - Direct evidence of a proton fluctuation into pion and neutron
- ❖ Future experiments such as EIC and LHeC can measure the exclusively produces vector meson production in  $ep$  scattering with leading neutron events [More on EIC physics: A.Deshpande talk on 16/12/22](#)
- ❖ Measurement of photo-nuclear cross sections in UPC's at LHC and RHIC are good tests for our models and saturation physics.

THANK YOU

# QCD WITH ELECTRON ION COLLIDER (QEIC)-II

International workshop on

## QCD with Electron Ion Collider (QEIC) II

Indian Institute of Technology Delhi  
Dec 18-20, 2022

**Organising Committee :**  
Abhay Deshpande (Stony Brook/CFNS)  
Abhishek Muralidhar Iyer (IIT Delhi)  
Asmita Mukherjee (IIT Bombay)  
Bedangadas Mohanty (NISER)  
Suddha Shankar Dasgupta (NISER)  
Tobias Toll (IIT Delhi)

**Venue :**  
**Lecture Hall Complex**  
**LH-527**

QEIC II is the second in the series of workshops which aims to organise the Indian high energy and nuclear physics community around the EIC. The meeting aims at solidifying and enhancing the participation and contributions of the Indian high energy and nuclear physics community in the EIC project.

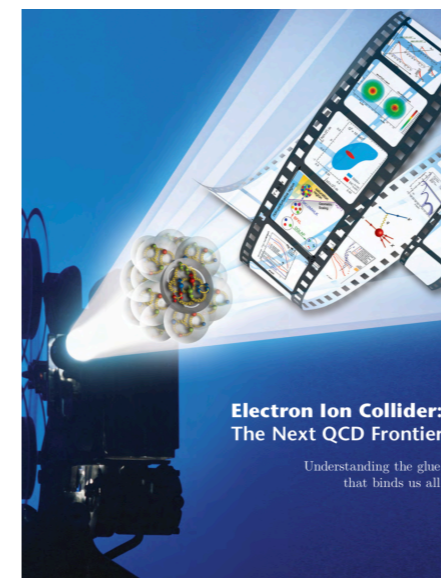


<https://indico.cern.ch/event/1196913/>

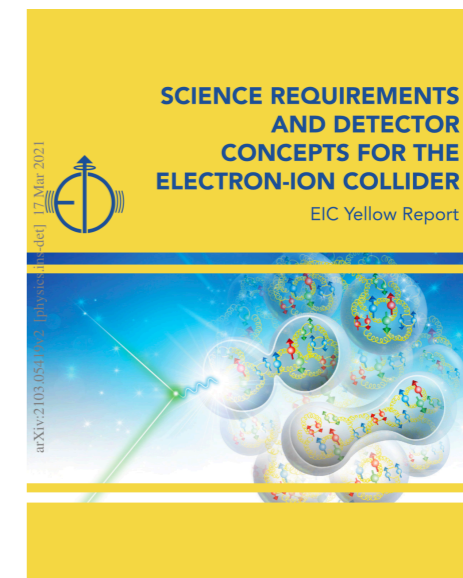


Center for Frontiers  
in Nuclear Science

- ➔ Inclusive measurement :  
*Longitudinal information ( $xP$ )*
- ➔ Semi-Inclusive measurement :  
*Transverse momentum information ( $k_T$ )*
- ➔ Exclusive measurement :  
*Spatial structure ( $b_T$ ) & Saturation*



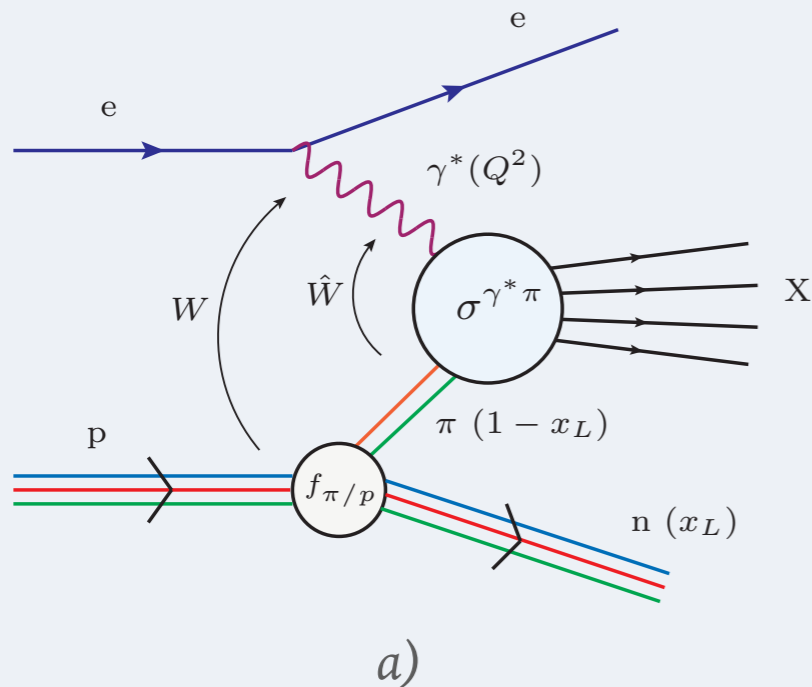
arXiv: 1212.1701



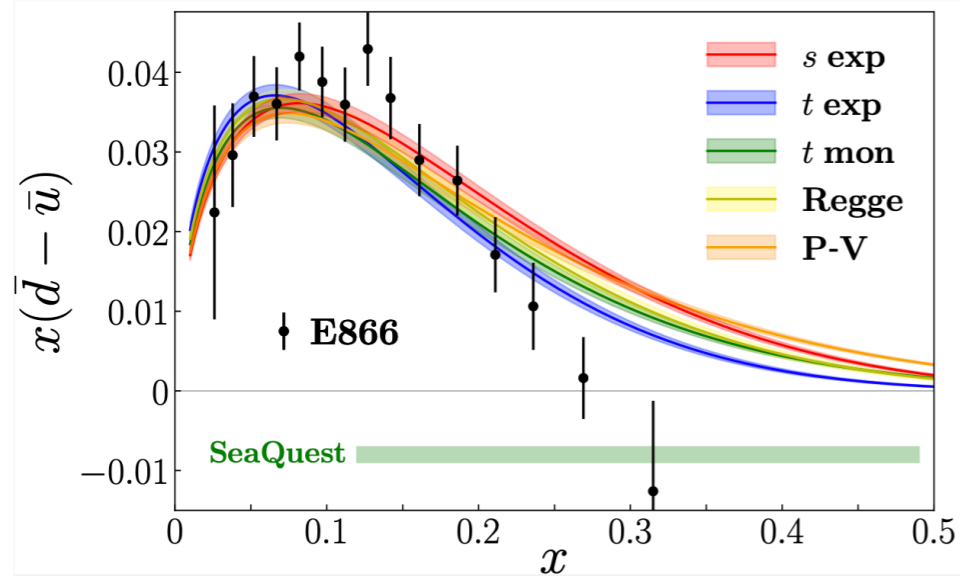
arXiv: 2103.05419

**BACKUP**

# FLAVOUR ASYMMETRY AND PION CLOUD



- ❖ Access to the structure function of “pion” via Sullivan process [J.D. Sullivan PRD 5 \(1972\), 1732](#)
- ❖ How does gluon density behave in the pion? Is there any universal behaviour at small-x ?
- ❖ How are gluons distributed inside pions? What is the gluon radius of pion?
- ❖ Sensitivity to the saturation effects
- ❖ Feynman scaling and its link with saturation
  - bCGC model [Carvalho, Gonçalves, Spiering, Navarra PLB 752 \(2016\) 76](#)
- ❖ Evidence of the pion cloud of the proton



Barry et al PRL 121, 152001

- Pions are main building blocks of nuclear matter
- Pion cloud models explain the light-quark asymmetry in the nucleon sea
- Pions are the Yukawa particles of the nuclear force (no evidence of excess)



# PION FLUX MODELS

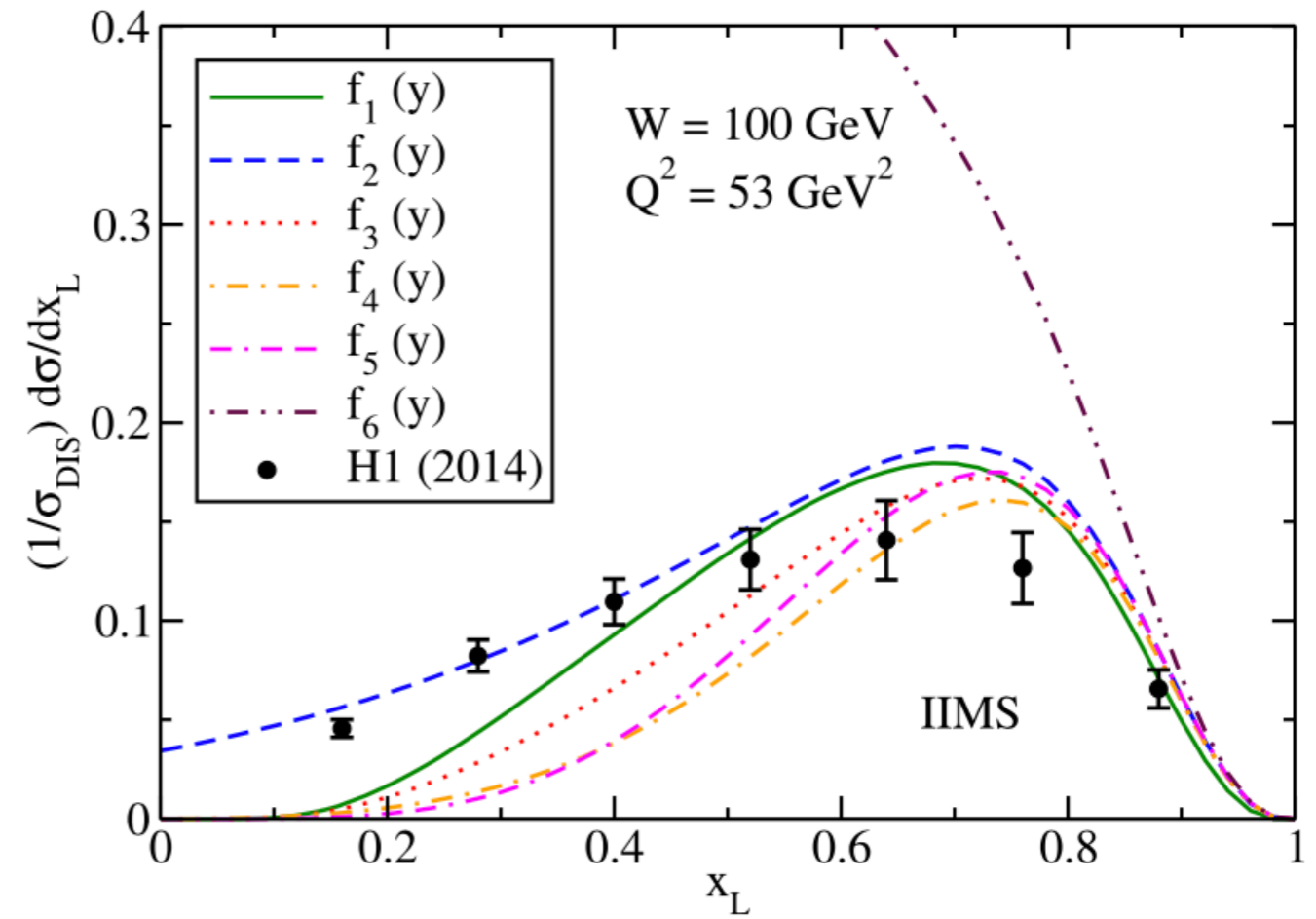
$$F_1(x_L, t) = \exp \left[ R^2 \frac{(t - m_\pi^2)}{(1 - x_L)} \right], \quad \alpha(t) = 0$$

$$F_2(x_L, t) = 1, \quad \alpha(t) = \alpha(t)_\pi$$

$$F_3(x_L, t) = \exp \left[ b(t - m_\pi^2) \right], \quad \alpha(t) = \alpha(t)_\pi$$

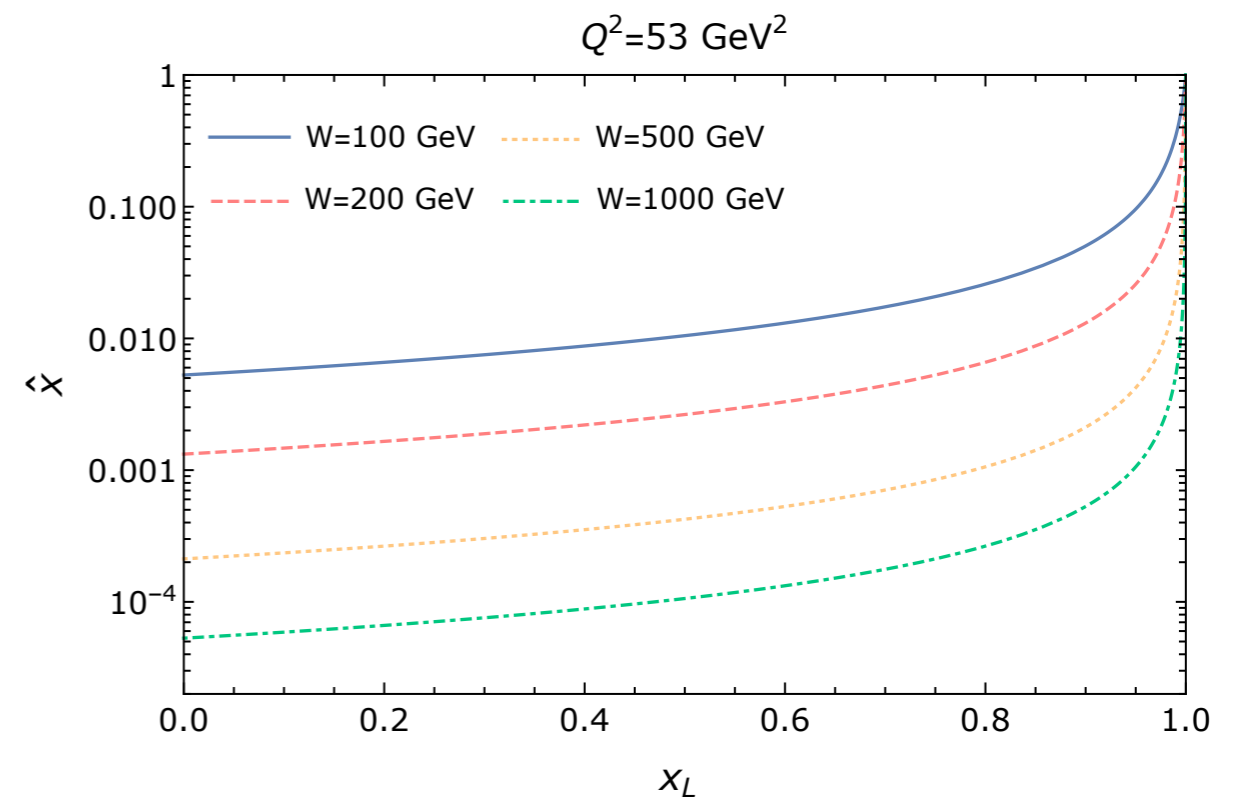
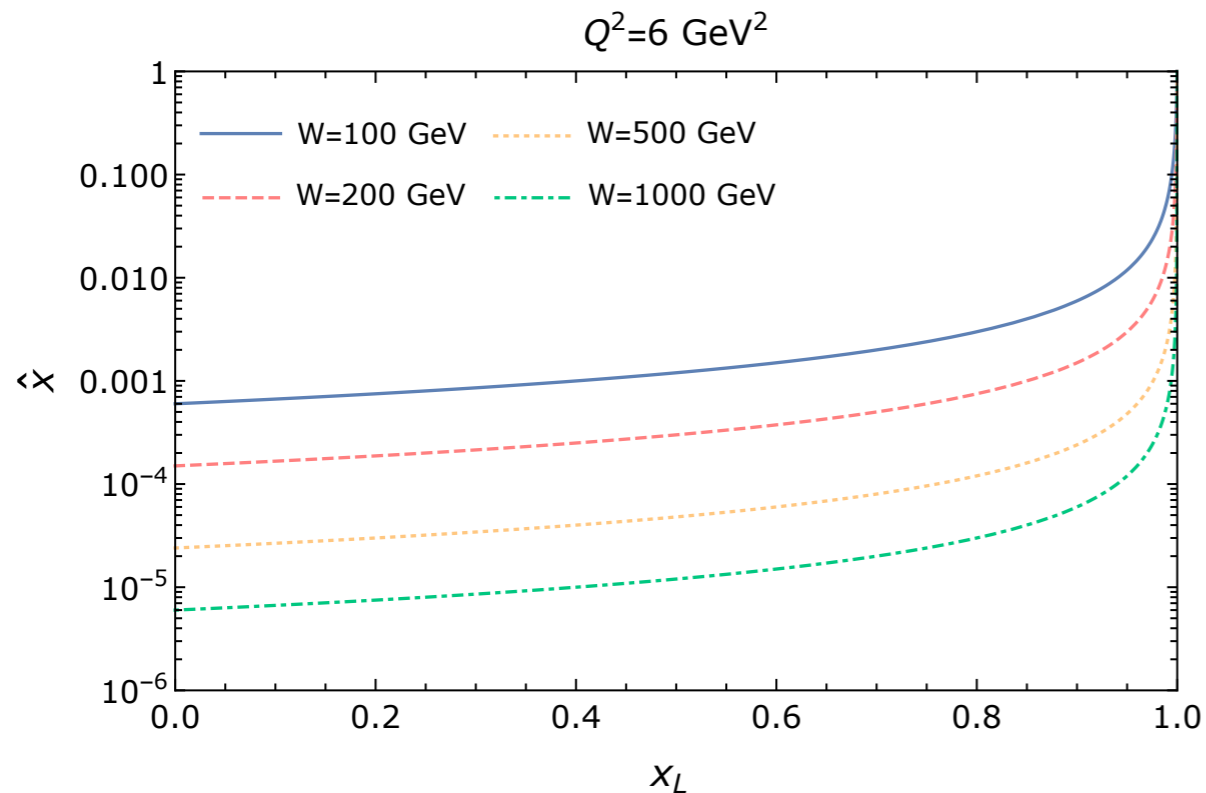
$$F_4(x_L, t) = \frac{\Lambda_m^2 - m_\pi^2}{\Lambda_m^2 - t}, \quad \alpha(t) = 0$$

$$F_5(x_L, t) = \left[ \frac{\Lambda_d^2 - m_\pi^2}{\Lambda_d^2 - t} \right]^2, \quad \alpha(t) = 0$$

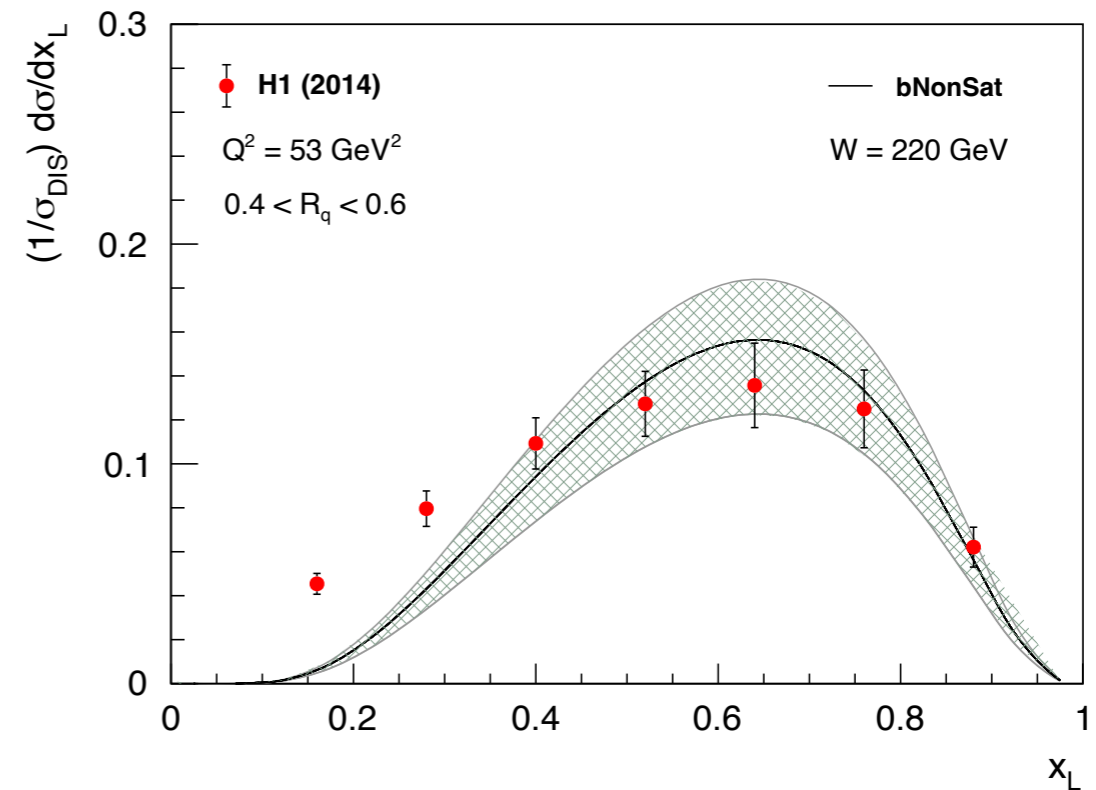
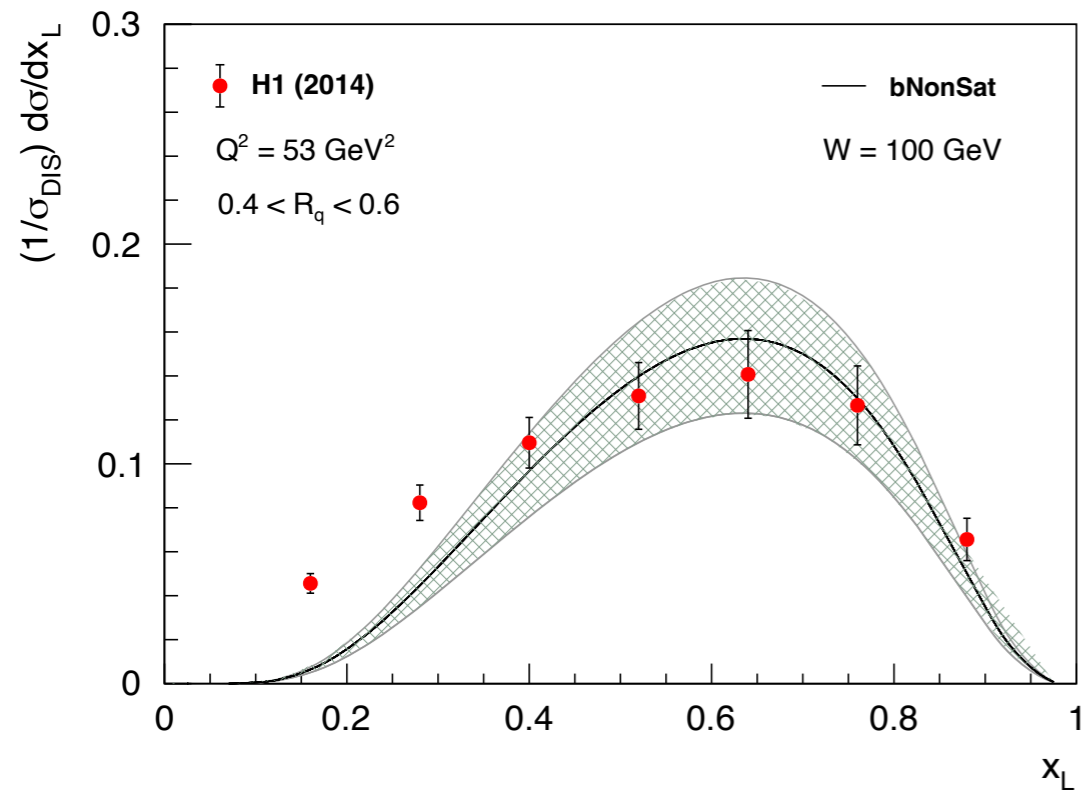
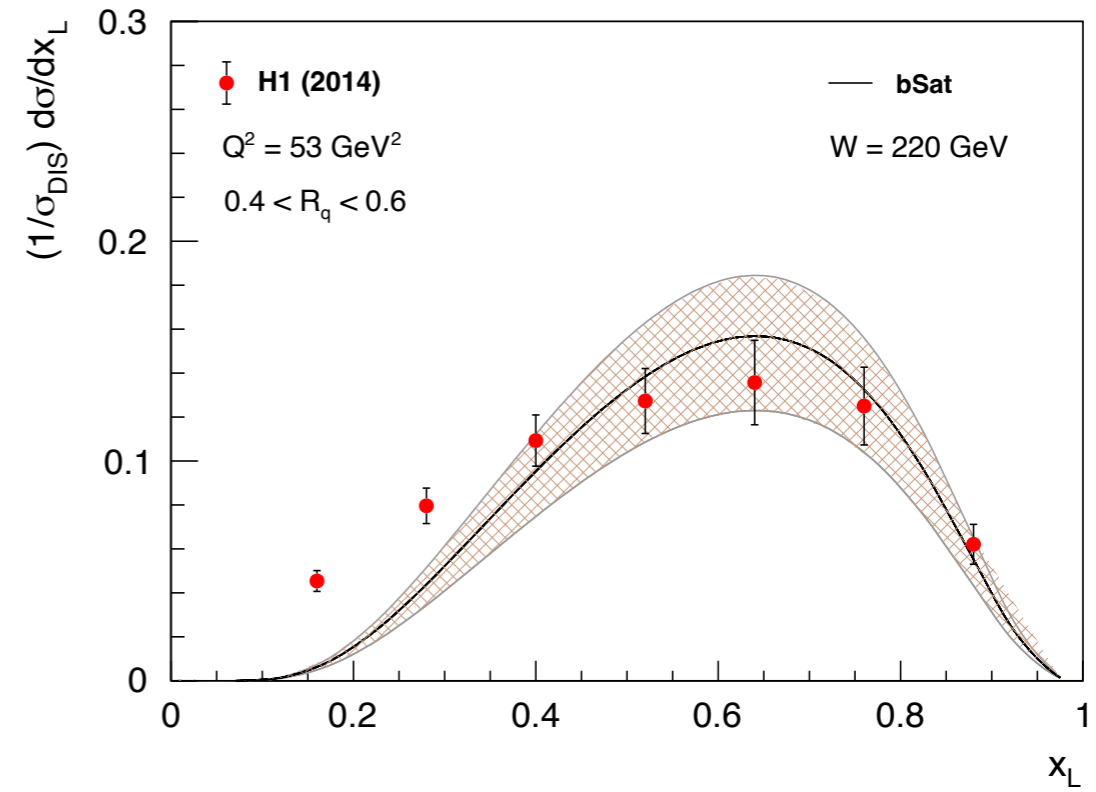
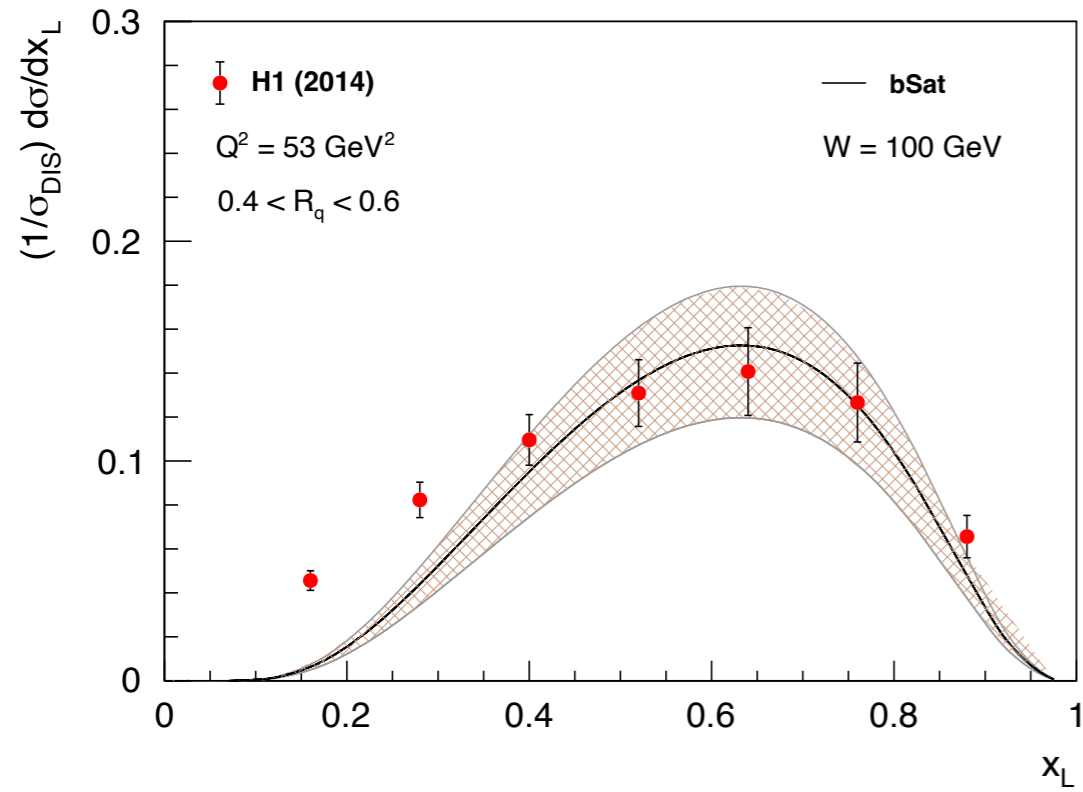


# LN AS A PROBE FOR SMALL-X PHYSICS

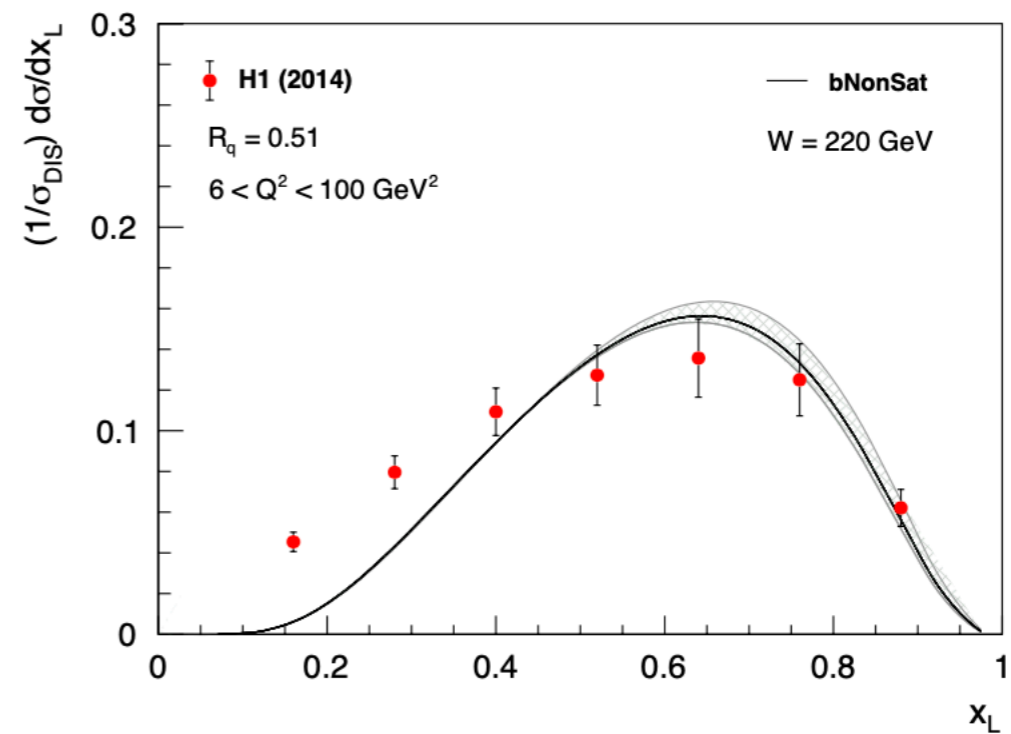
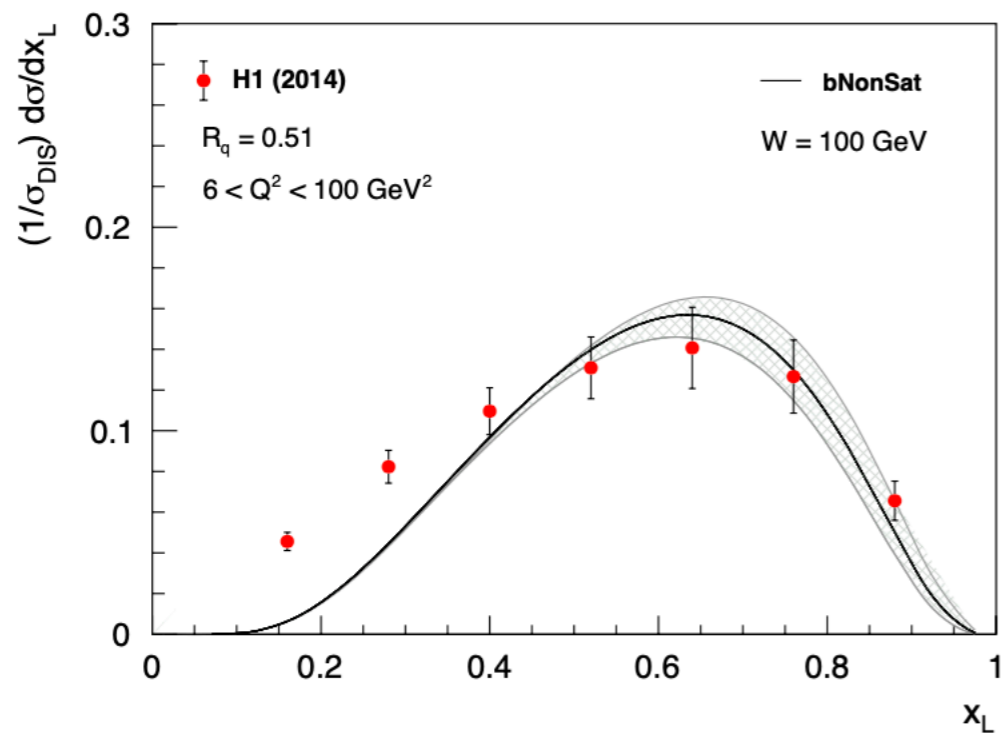
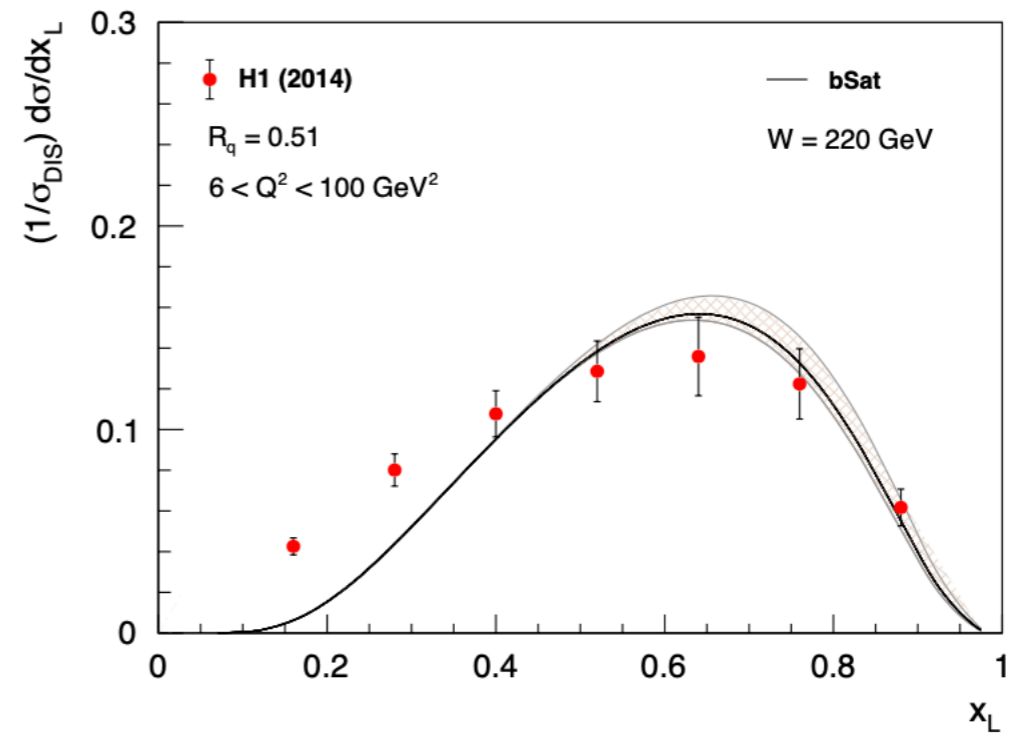
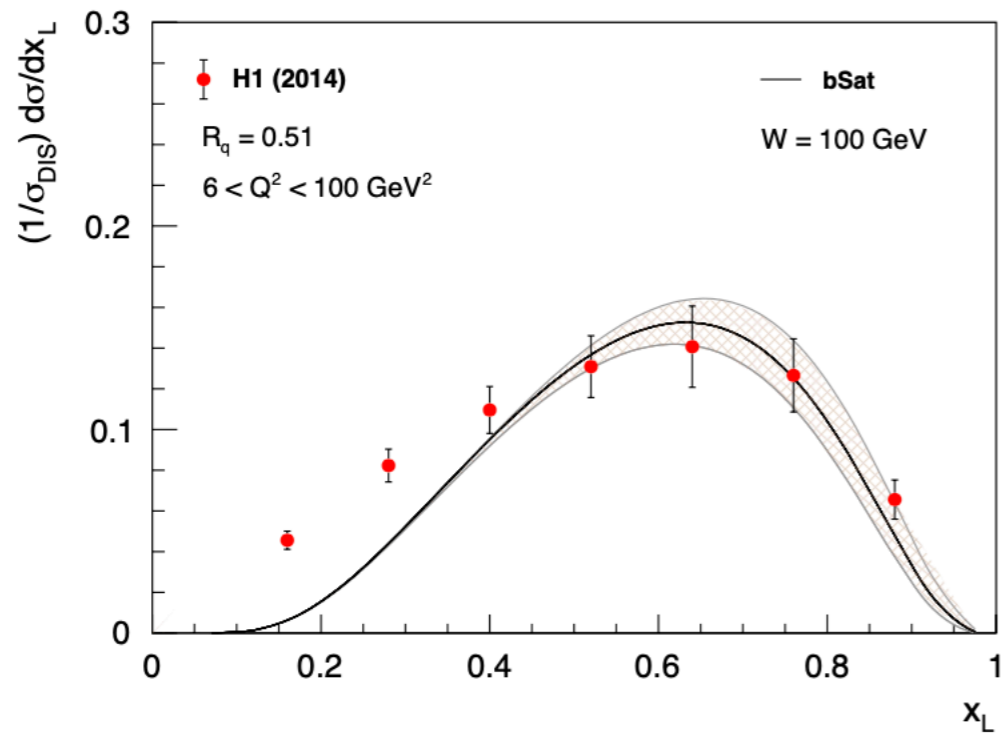
- ❖ The x value probed in such a process is  $\hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} = \frac{Q^2 + m_f^2}{(1 - x_L)W^2 + Q^2}$
- ❖ LN production is low x physics
- ❖ In principle, we could use the color dipole framework to investigate the pion properties at small-x
- ❖ Use the dipole model to calculate the pion structure function  $F_2^\pi$  and the leading neutron structure function  $F_2^{LN}$  to compare with the HERA Data



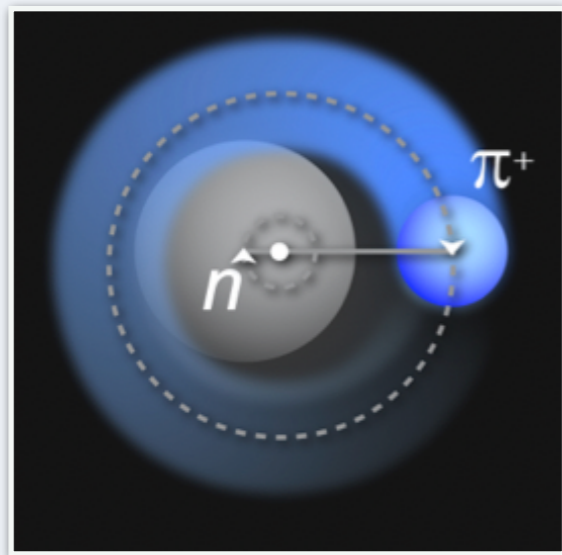
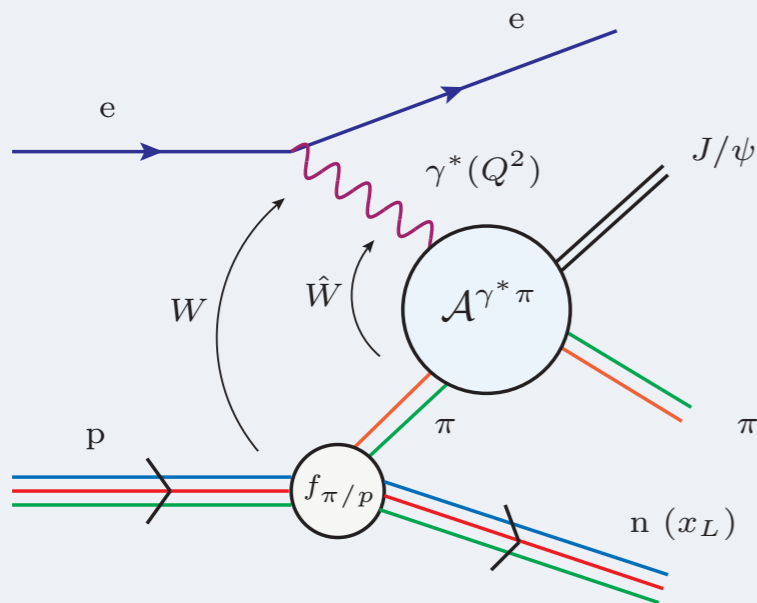
# FEYNMAN-X SPECTRA



# $Q^2$ SCALING IN FEYNMAN-X SPECTRA



# PROBING THE GLUON DISTRIBUTION



$$\sigma_{total} = \sigma_{yukawa} + \sigma_{fluctuations}$$

- ❖ The transverse profile of the virtual pion is,

$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} dz \rho_{\pi^*}(b, z)$$

where the radial part of the virtual pion wave function is given by Yukawa theory:

$$\rho_{\pi^*}(b, z) = \frac{m_{\pi}^2}{4\pi} \frac{e^{-m_{\pi} \sqrt{b^2 + z^2}}}{\sqrt{b^2 + z^2}}$$

- ❖ We assume that the real pion, as for the proton, is described by a Gaussian profile:

$$T_{\pi}(b) = \frac{1}{2\pi B_{\pi}} e^{-\frac{b^2}{2B_{\pi}}}$$

- ❖ At small  $|t'|$ , the dipole cannot resolve the pion and interacts with the whole cloud and on increasing the resolution (*increasing*  $|t'|$ ) the dipole interacts with the pion
- ❖ The transverse position of the pion inside the virtual pion cloud fluctuates event by event



# MASS RADIUS OF PION USING GENERALISED DISTRIBUTION AMPLITUDES

Kumano, Song, Teryaev PRD 97 (2018), 014020

## Hadron tomography by generalized distribution amplitudes in pion-pair production process $\gamma^*\gamma \rightarrow \pi^0\pi^0$ and gravitational form factors for pion

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(Dated: January 10, 2019)

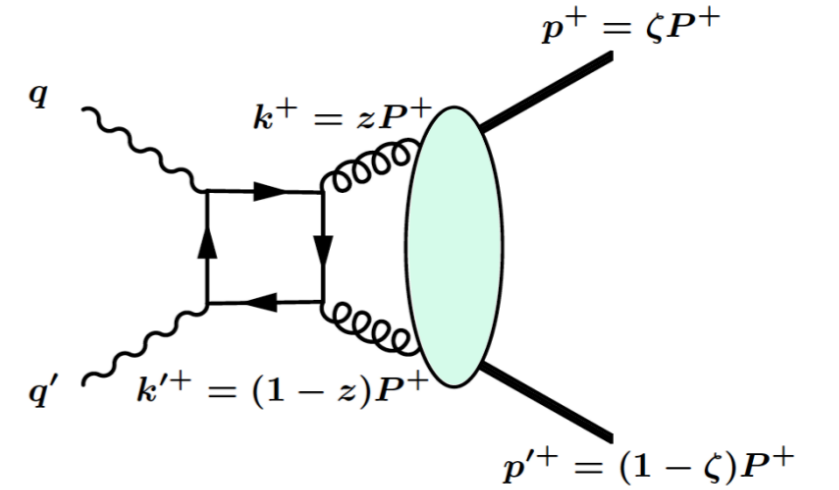


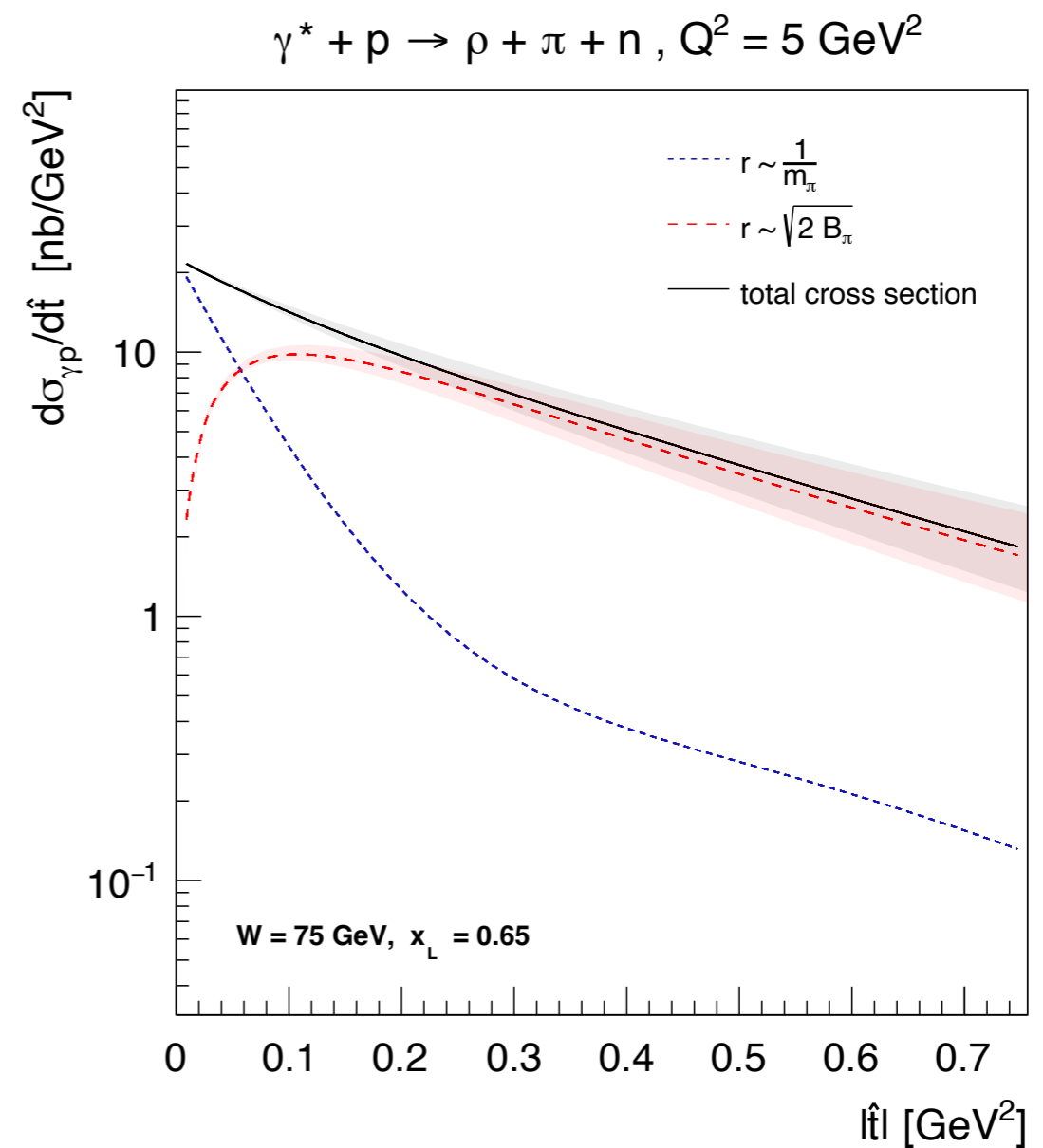
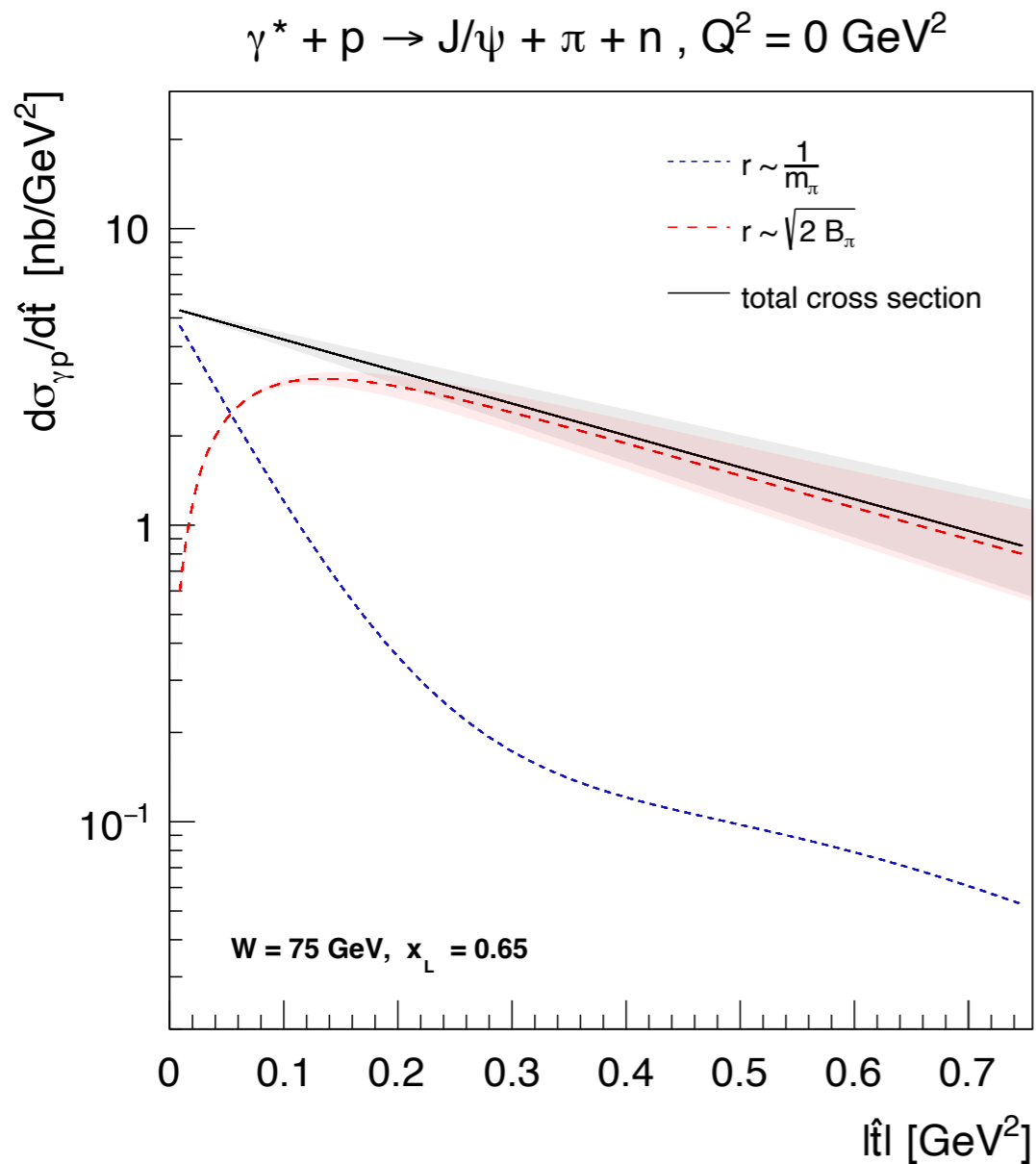
FIG. 4. Contribution to the two-photon cross section from the gluon GDA.

Hadron tomography can be investigated by three-dimensional structure functions such as generalized parton distributions (GPDs), transverse-momentum-dependent parton distributions, and generalized distribution amplitudes (GDAs). Here, we extract the GDAs, which are  $s$ - $t$  crossed quantities of the GPDs, from cross-section measurements of hadron-pair production process  $\gamma^*\gamma \rightarrow \pi^0\pi^0$  at KEKB. This work is the first attempt to obtain the GDAs from the actual experimental data. The GDAs are expressed by a number of parameters and they are determined from the data of  $\gamma^*\gamma \rightarrow \pi^0\pi^0$  by including intermediate scalar- and tensor-meson contributions to the cross section. Our results indicate that the dependence of parton-momentum fraction  $z$  in the GDAs is close to the

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm},$$

$$\sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}.$$

# PROBING THE GLUON DISTRIBUTION



- ❖ The cross section have two slopes due to interaction with different size scales at low  $|t'|$  and moderate  $|t'|$
- ❖ H1 data on exclusive  $\rho$  photo production with leading neutrons exhibits these two slopes in the differential distribution

# GOOD-WALKER FORMALISM

- Coherent cross-section probes **average  $\mathbf{b}$  dependence**  $\langle N(\mathbf{b}, \mathbf{r}, \mathbf{x}) \rangle_{\Omega}$  of dipole amplitude which provides the information about target geometry

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow V p}}{dt} = \frac{1}{16\pi} \left| \langle \mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p} \rangle_{\Omega} \right|^2$$

- Incoherent cross-section** : target dissociates ( $\mathbf{f} \neq \mathbf{i}$ ) Good, Walker 1960, Miettinen, Pumplin 1978

$$\begin{aligned} \sigma_{incoherent} &\sim \sum_{\mathbf{f} \neq \mathbf{i}} | \langle \mathbf{f} | \mathcal{A} | \mathbf{i} \rangle |^2 \\ &= \sum_{\mathbf{f}} \langle \mathbf{i} | \mathcal{A}^\dagger | \mathbf{f} \rangle \langle \mathbf{f} | \mathcal{A} | \mathbf{i} \rangle - \langle \mathbf{i} | \mathcal{A} | \mathbf{i} \rangle^\dagger \langle \mathbf{i} | \mathcal{A} | \mathbf{i} \rangle \\ &= \langle |\mathcal{A}|^2 \rangle_{\Omega} - \left| \langle \mathcal{A} \rangle_{\Omega} \right|^2 \end{aligned}$$

$$\frac{d\sigma_{total}}{dt} = \frac{1}{16\pi} \langle |\mathcal{A}|^2 \rangle_{\Omega}$$

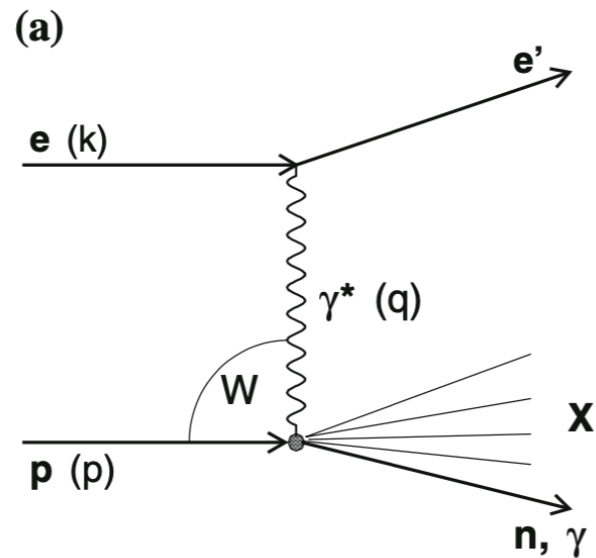
$$\frac{d\sigma_{coherent}}{dt} = \frac{1}{16\pi} \left| \langle \mathcal{A} \rangle_{\Omega} \right|^2$$

- Incoherent cross-section is the **variance** of amplitude which controls the amount of event-by-event fluctuations in target configurations

# FEYNMAN-X SPECTRA AT SMALL $x_L$

HI EPJC 74 (2014), 2915

Standard fragmentation (DJANGO)



One-pion approximation (RAPGAP)

