Strong Field Electrodynamics

Ulrik I. Uggerhøj
on behalf of CERN NA63

Strong fields

Strong – compared to what?

relativistic (c) quantum ($\hbar$) field for electrons (m, e)

- The critical field:

$$\mathcal{E}_0 = \frac{m^2 c^3}{e\hbar} = 1.323285 \cdot 10^{16}\mathrm{V/cm} \quad B_0 = 4.414005 \cdot 10^{9}\mathrm{T}$$

Cartoon view of the quantum vacuum where particle-antiparticle pairs are continuously created and annihilated.

Advances in QED with intense background fields

a review paper on laser-electron interaction

arXiv:2203.00019v1

A. Fedotov[a], A. Ilderton[b], F. Karbstein[c,d,e], B. King[f], D. Seipt[c,d], H. Taya[g], G. Torgrimsson[h,i]

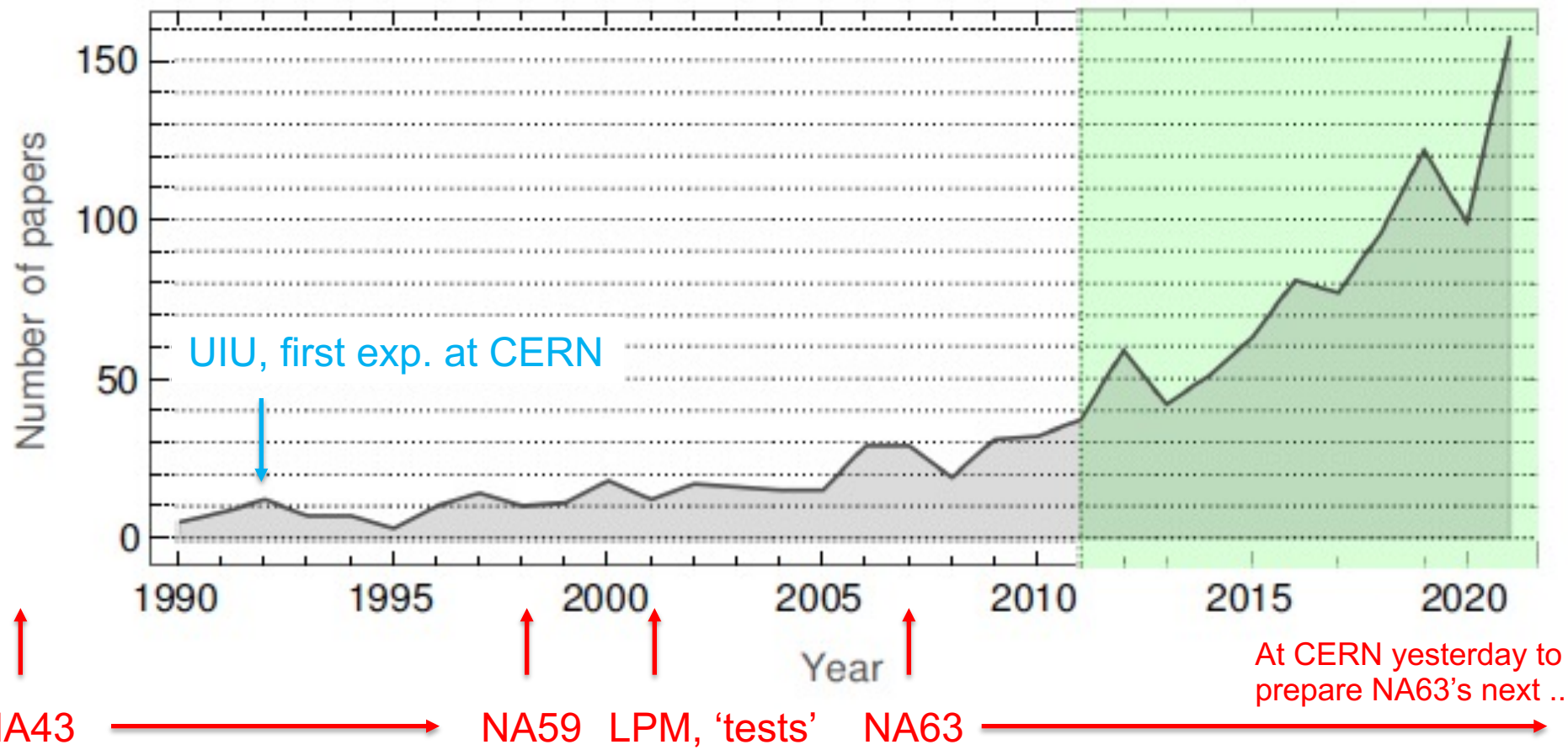
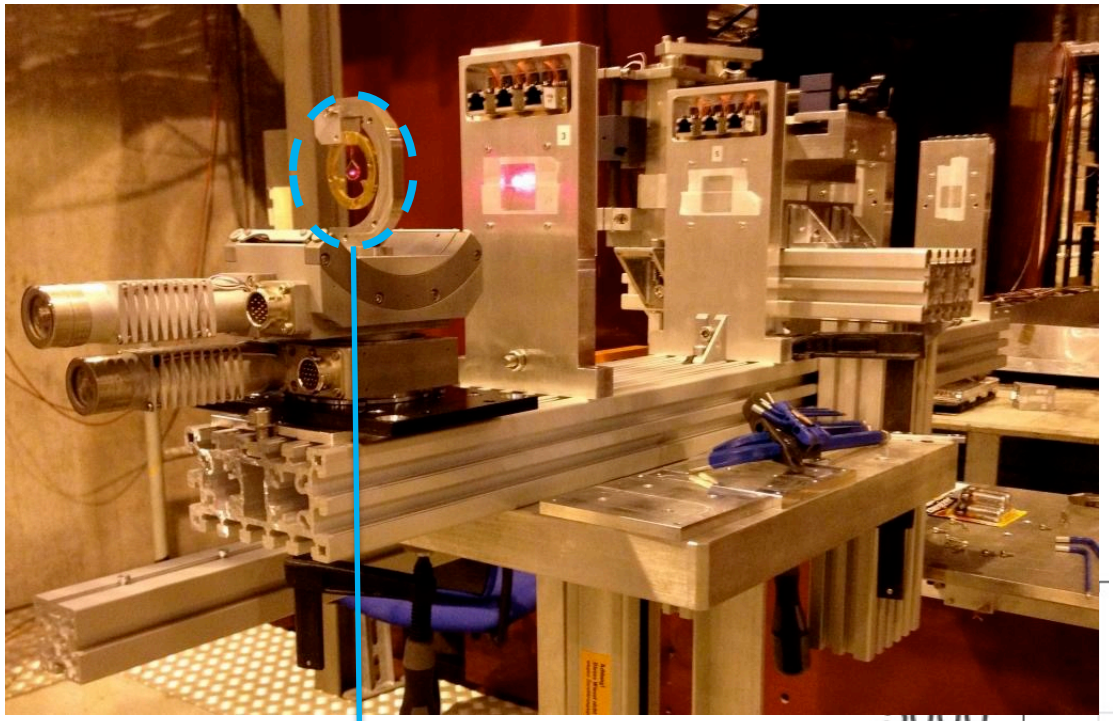
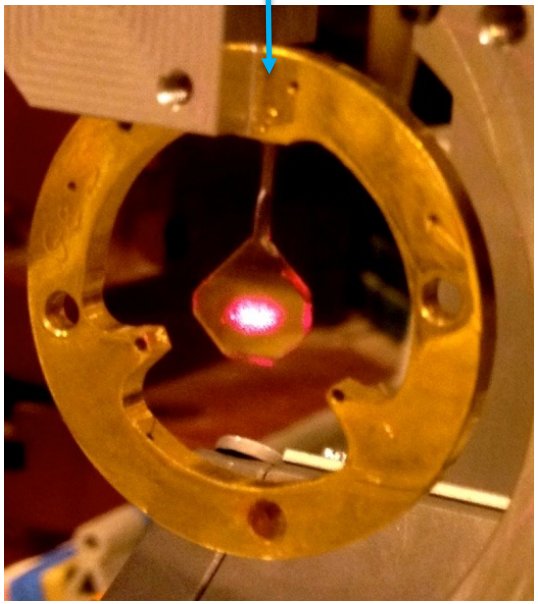
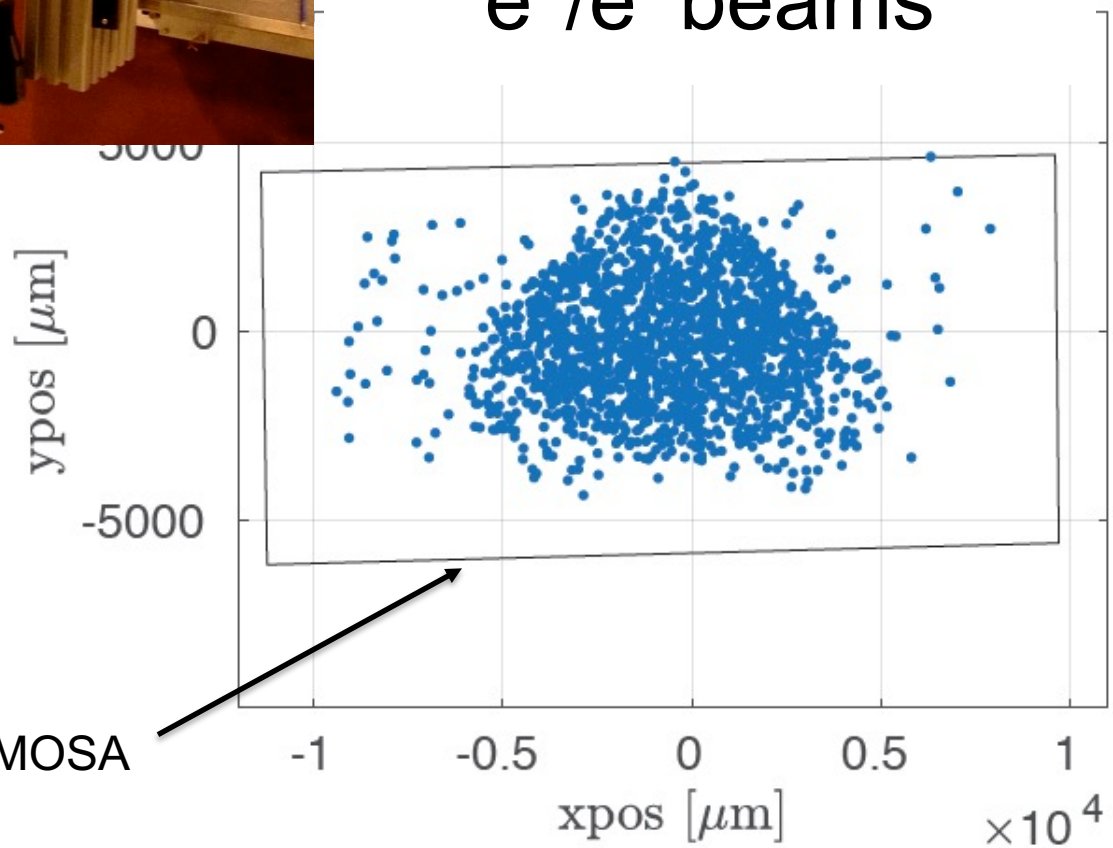


Figure 1: Indicative bibliometric search using NASA-ADS, for at least one of the following terms occurring in the abstract: "strong field QED", "nonlinear QED", "nonlinear Compton", "nonlinear Breit-Wheeler", "locally constant field", "Schwinger effect", "Schwinger pair". The shaded region is the last decade, on which the current review is focussed.

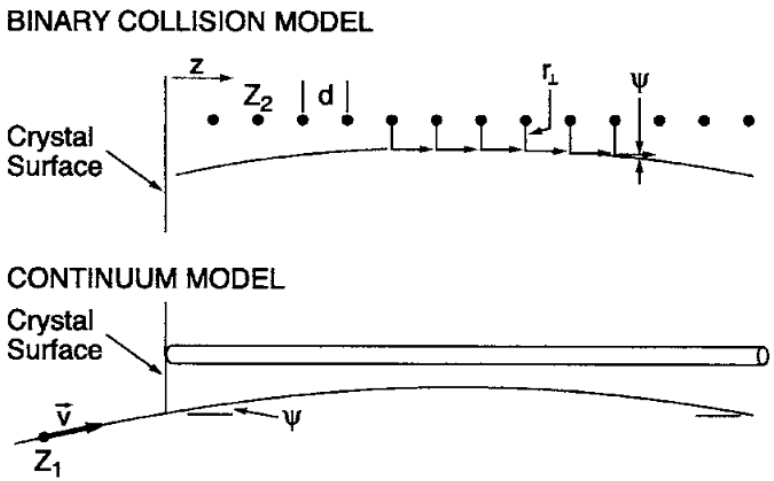
CERN NA63
SPS – H2, H4
Strong field effects by means of crystals in GeV e^+/e^- beams

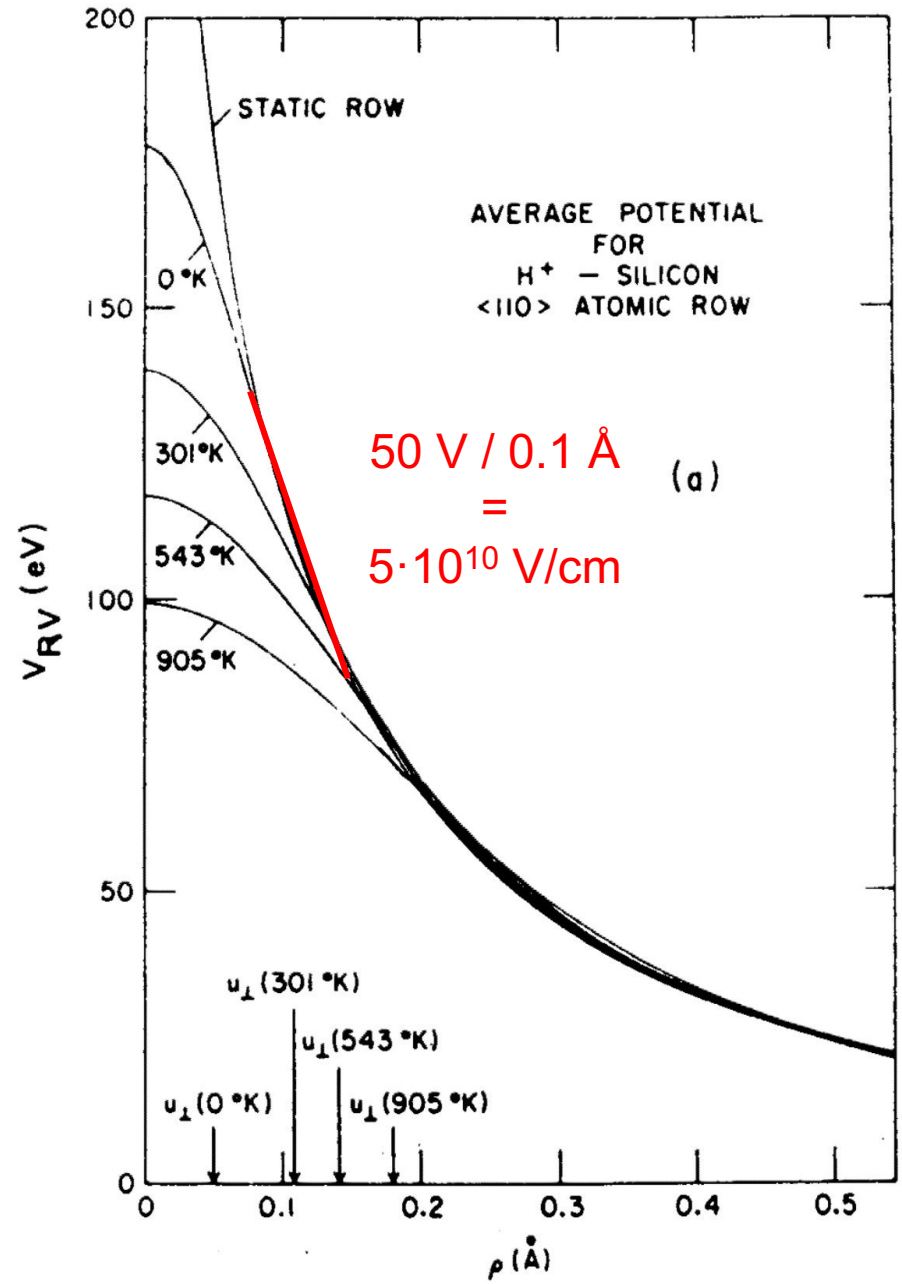






Crystals as a source of strong fields





Extremely strong electric fields

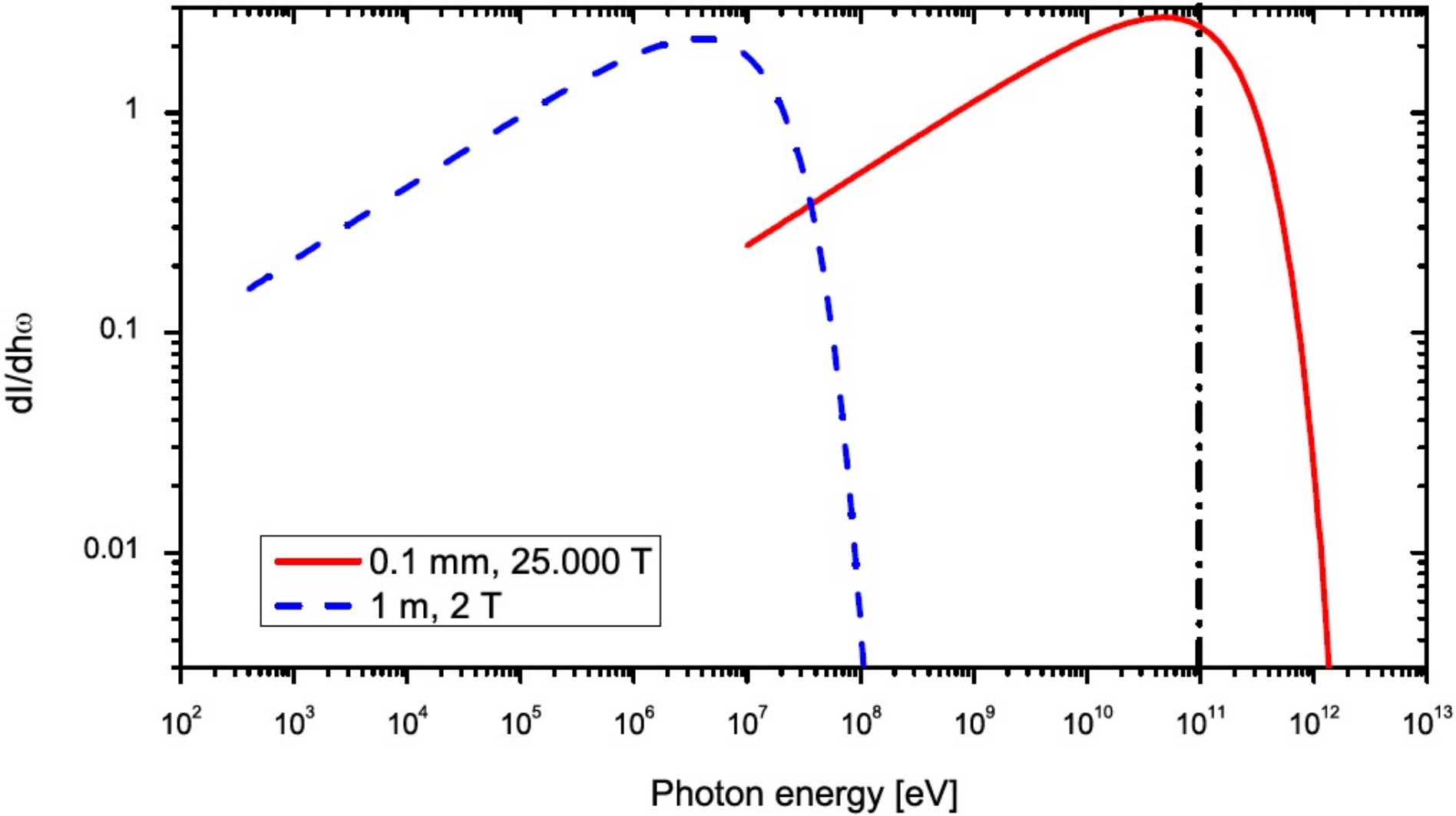
10^{10}-10^{11} V/cm

Relativistic invariant: $$\chi = \frac{\gamma \mathcal{E}}{\mathcal{E}_0}$$

$$\mathcal{E}_0 = mc^2 / e\lambda\!\!\!^{-}_c$$

$=1.32 \cdot 10^{16}$ V/cm

Radiation emission



Schwinger's condition for classical synchrotron radiation, 1949:

$$\frac{E}{mc^2} \ll \frac{mc^2}{(e\hbar/mc)H}, \qquad \chi = \frac{\gamma\mathcal{E}}{\mathcal{E}_0} \ll 1$$

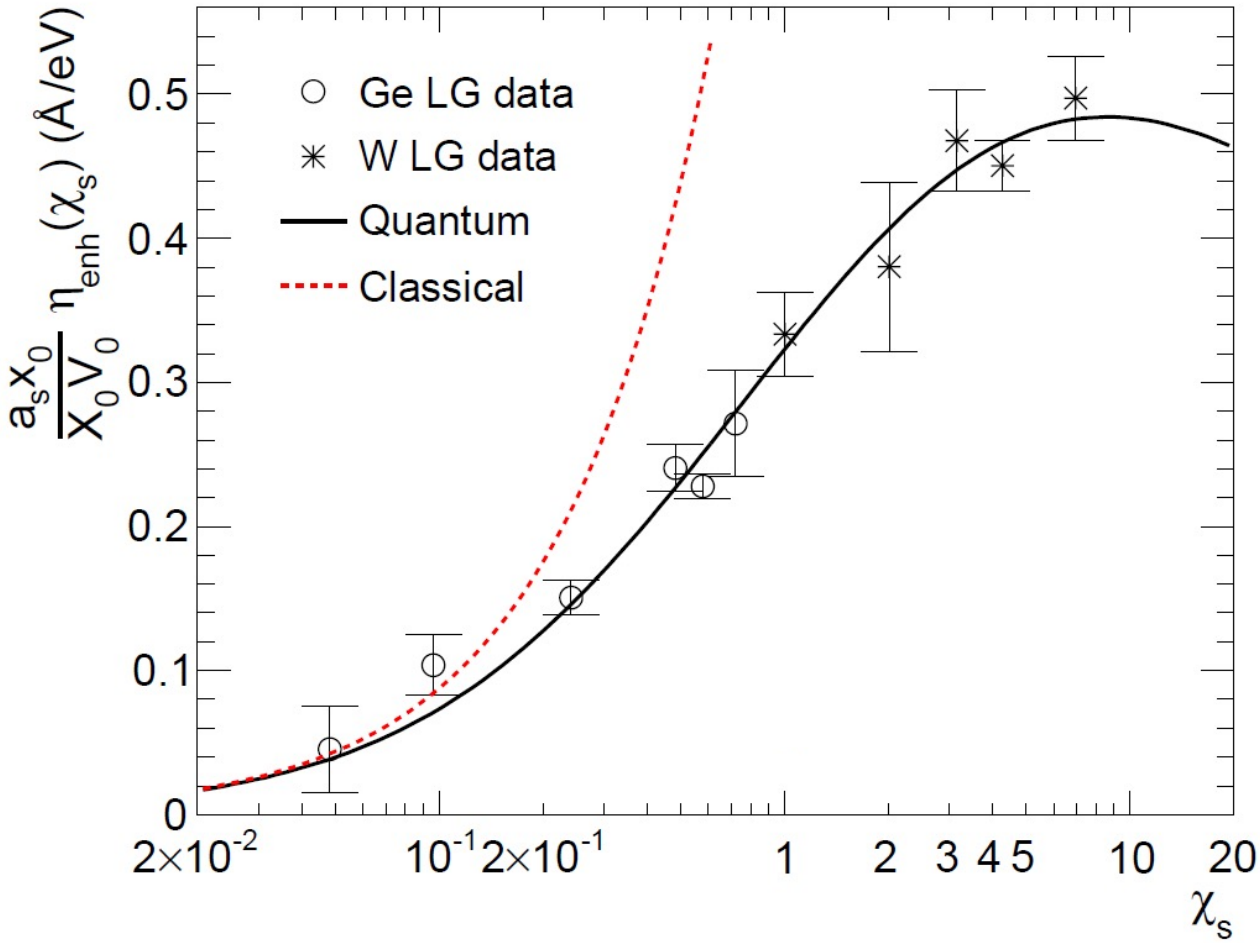
PHYSICAL REVIEW D **86**, 072001 (2012)

Experimental investigations of synchrotron radiation at the onset of the quantum regime

K. K. Andersen,[1] J. Esberg,[1] H. Knudsen,[1] H. D. Thomsen,[1] U. I. Uggerhøj,[1] P. Sona,[2] A. Mangiarotti,[3] T. J. Ketel,[4] A. Dizdar,[5] and S. Ballestrero[6]

(CERN NA63)

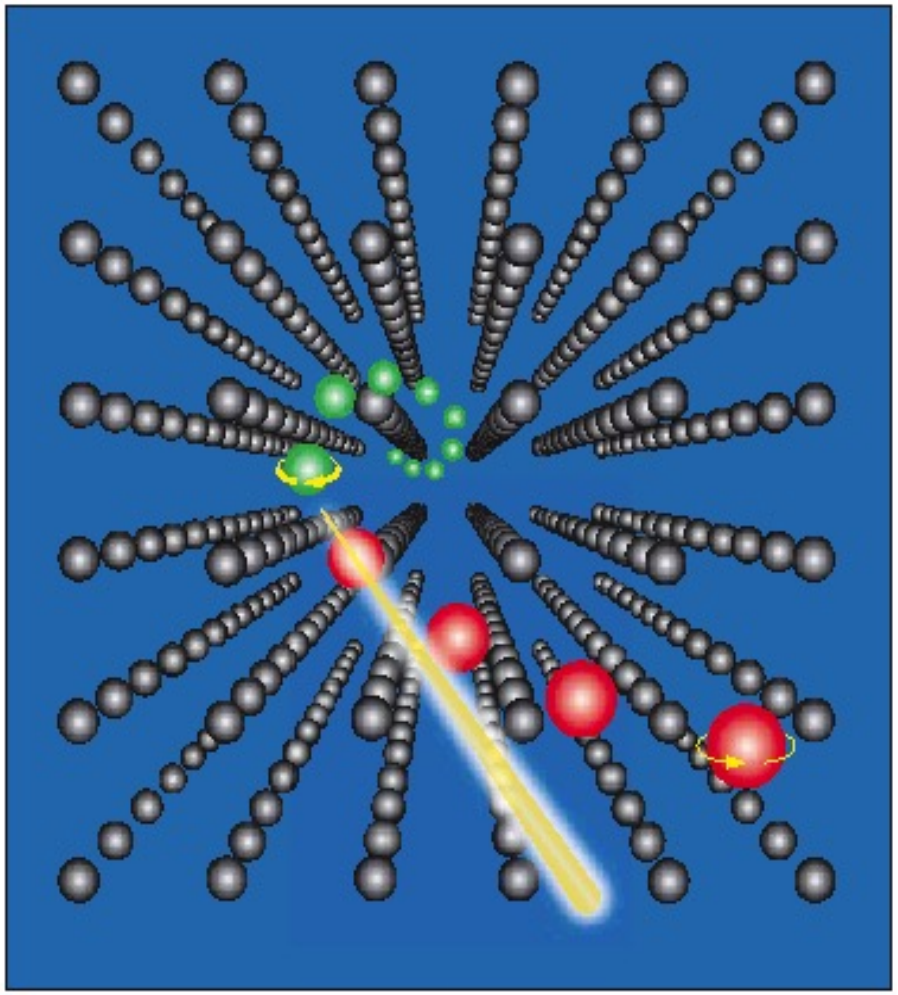
$$\frac{I_e}{I_{\mathrm{cl}}} = (1 + 4.8(1+\chi)\ln(1+1.7\chi) + 2.44\chi^2)^{-2/3}.$$



Classical -> Quantum synchrotron in strong fields

This suppression 'saves' the luminosity at CLIC: same suppression of beamstrahlung!

Spin-flip in strong field



$$B = \gamma \beta \mathcal{E}_{\mathrm{lab}}$$

$$W_{\mathrm{mag}} = -\overline{\mu} \cdot \overline{B}$$

$$\Delta W = e\hbar B / mc$$

$$\mathcal{E}_0 = m^2 c^3 / e\hbar$$

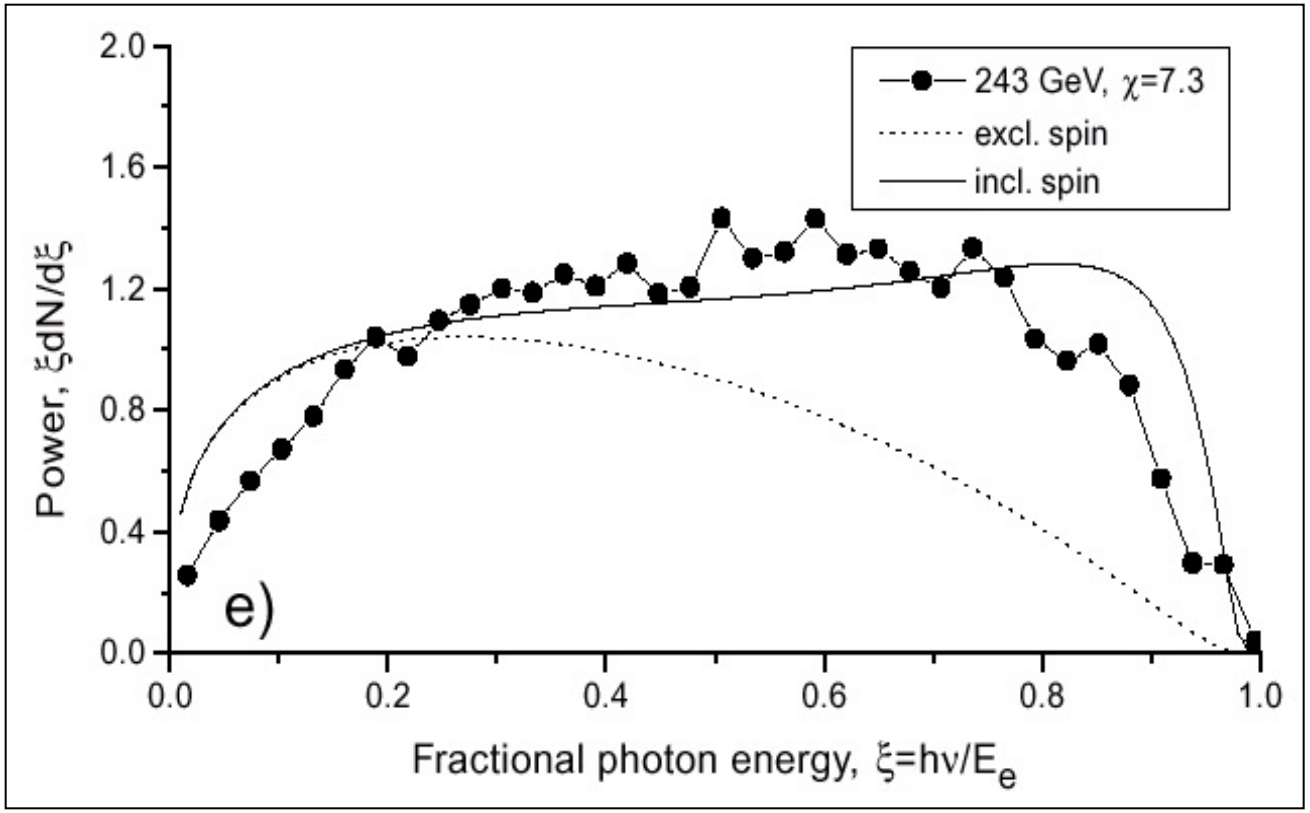
$$\boxed{\Delta W = \gamma^2 \beta \frac{\mathcal{E}}{\mathcal{E}_0} mc^2}$$

Equals incoming energy if

$$\chi = \gamma \mathcal{E} / \mathcal{E}_0$$

is one

Spin-flip



$$\chi = \gamma \mathcal{E} / \mathcal{E}_0$$

$$\Delta W = \gamma^2 \beta \frac{\mathcal{E}}{\mathcal{E}_0} mc^2$$

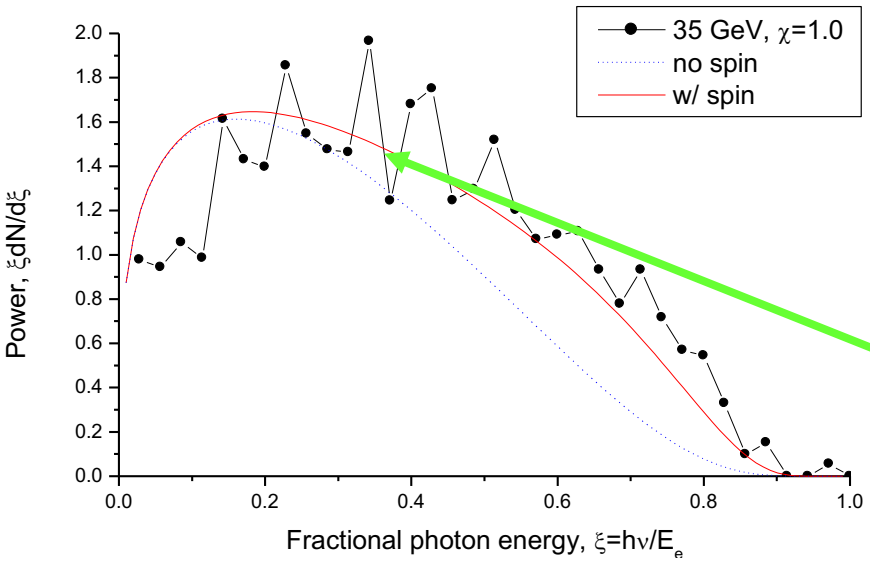
$$\tau = \frac{8\hbar}{5\sqrt{3}\,\alpha m}\left(\frac{B_0}{B}\right)^3 \frac{1}{\gamma^2} = \frac{8\hbar}{5\sqrt{3}\,\alpha m}\frac{\gamma}{\chi^3}$$

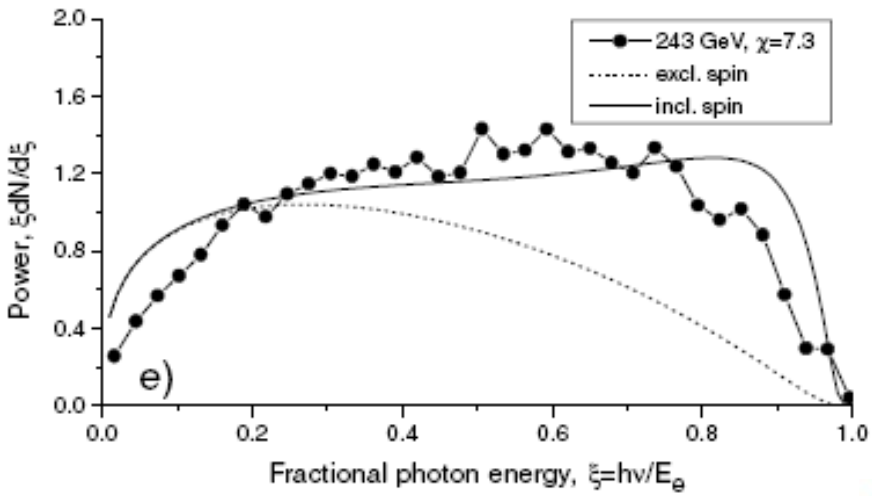
100 GeV
$\chi = 1$
10 μm

'Polarization time/length'

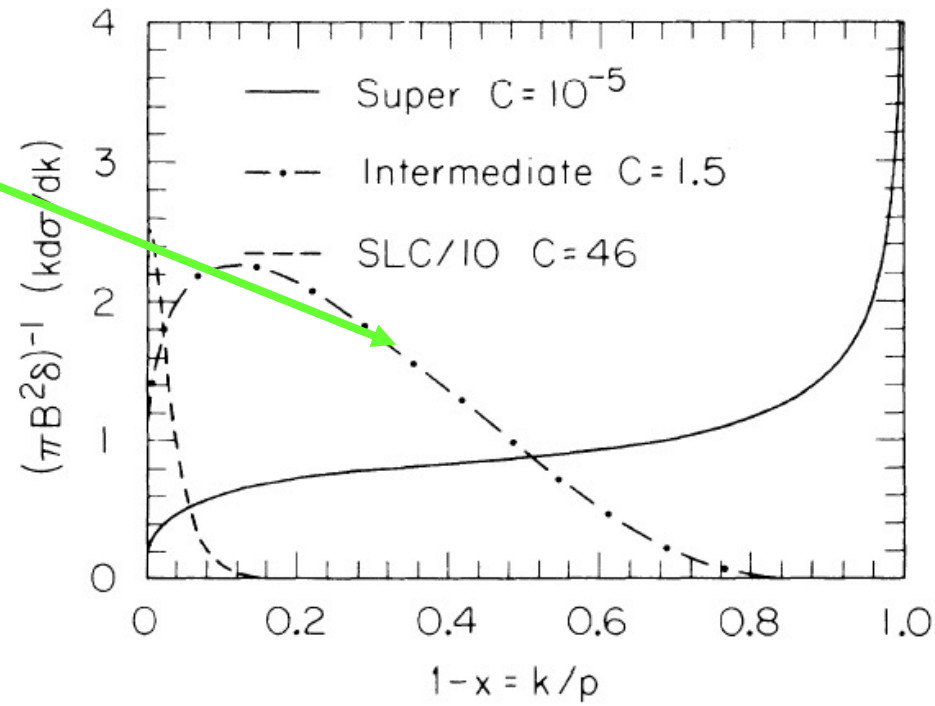
1 GeV
1 T field
7.3 AU

Spin contr. to beamstrahlung





Blankenbecler and Drell, "Quantum treatment of beamstrahlung", PRD **36**, 277 (1987)



Spin-flip process significant!

in CLIC $\langle \Upsilon \rangle \approx 4$

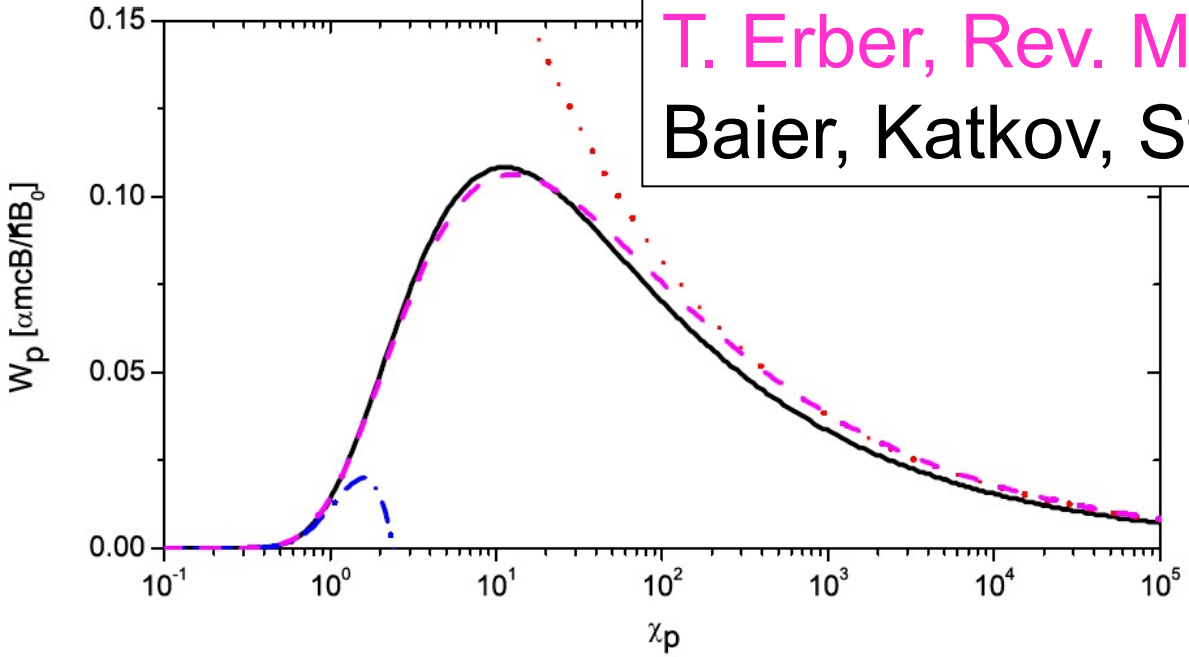
Strong field processes in beam-beam interactions at the Compact Linear Collider

J. Esberg,[1,2] U. I. Uggerhøj,[1] B. Dalena,[2] and D. Schulte[2]

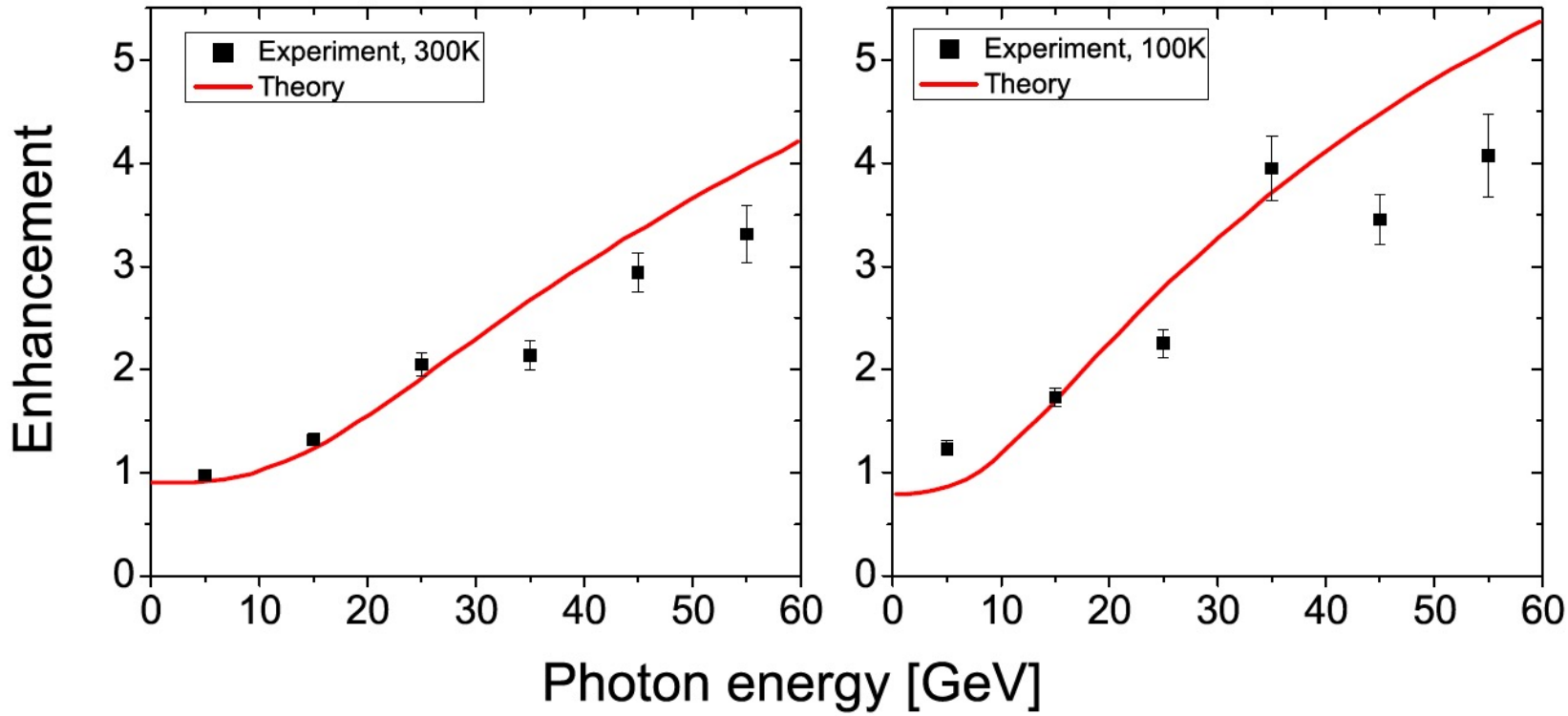
DOI: 10.1103/PhysRevSTAB.17.051003

T. Erber, Rev. Mod. Phys. 1969

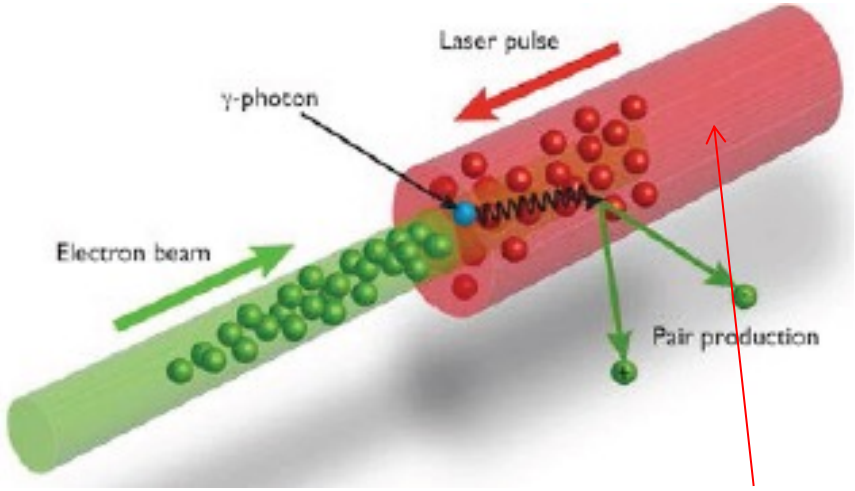
Baier, Katkov, Strakhovenko, 1990s



Strong field pair production



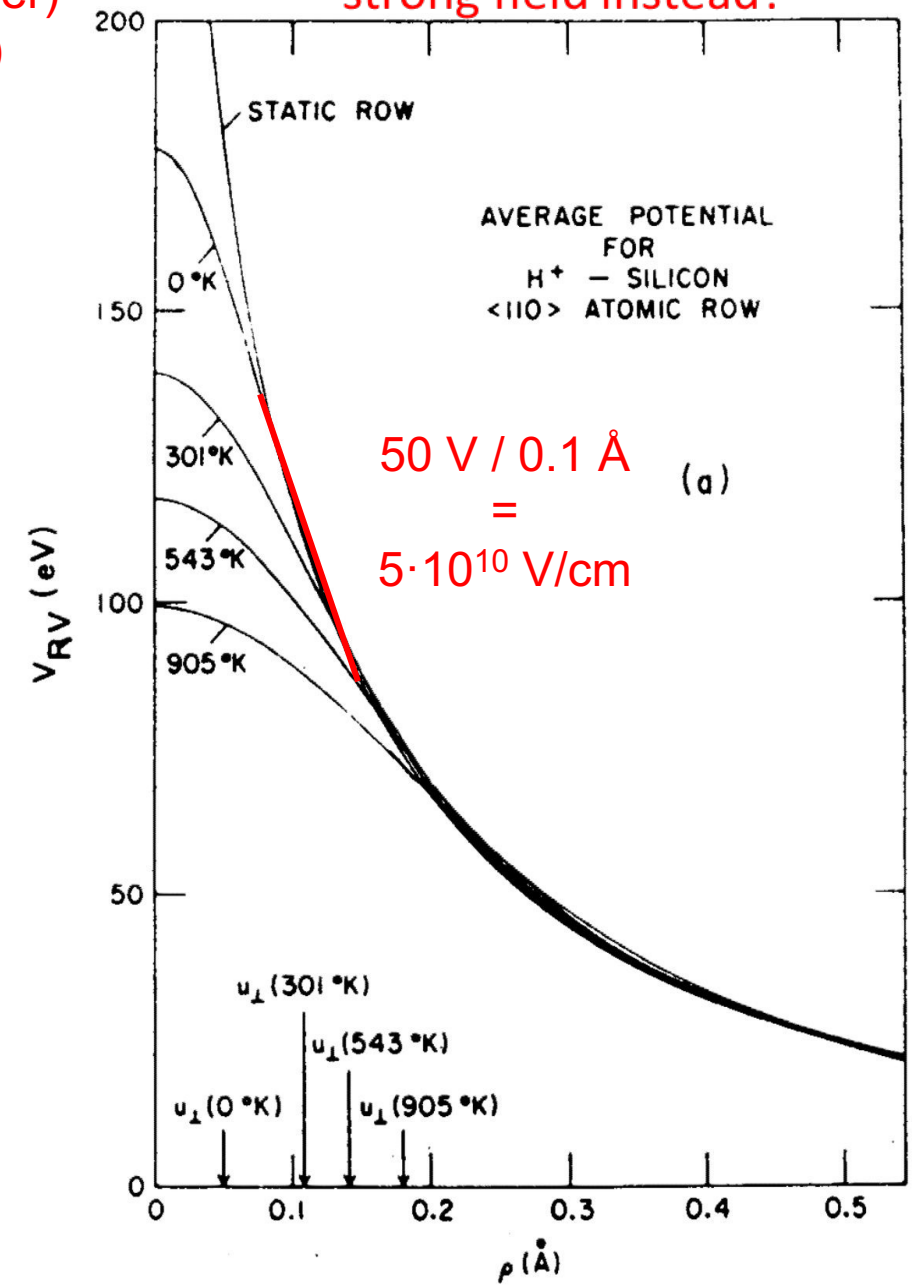
SLAC E-144 (mid-90's, 1 TW laser)
SLAC E-320 (almost ready to run, 10 TW laser)
LUXE @ DESY (ready in 2023, 30 TW laser)



Crystals:

Replace laser-pulse by virtual photons

Why not use a constant strong field instead?



Extremely strong electric fields

10^{10}-10^{11} V/cm

$$\chi = \gamma \mathcal{E} / \mathcal{E}_0$$

Classical Radiation Reaction

Jackson 1975 p. 786-798

$m\dot{\mathbf{v}} = \mathbf{F}_{\text{ext}}$ N2

$P(t) = \frac{2}{3}\frac{e^2}{c^3}(\dot{\mathbf{v}})^2$ Larmor

$m\dot{\mathbf{v}} = \mathbf{F}_{\text{ext}} + \mathbf{F}_{\text{rad}}$ $\mathbf{F}_{\text{rad}}$ "must" vanish if $\dot{\mathbf{v}} = 0$ (no radiation)

$$m(\dot{\mathbf{v}} - \tau\ddot{\mathbf{v}}) = \mathbf{F}_{\text{ext}}$$

Lorentz-Abraham-Dirac (LAD) equation

$$\mathbf{F}_{\text{rad}} = \frac{2}{3}\frac{e^2}{c^3}\ddot{\mathbf{v}} = m\tau\ddot{\mathbf{v}} \qquad \tau = \frac{2}{3}\frac{e^2}{mc^3}$$

No field, solution to LAD eq.:
(runaway – energy conservation)

$$a(t) = a_0 e^{t/\tau},$$

$$\tau = 6 \times 10^{-24}\text{s}.$$

Possible remedy: 'Landau-Lifshitz equation'

Step-fct. field, solution to LAD eq.:
(pre-acceleration - causality)

Classical Electrodynamics

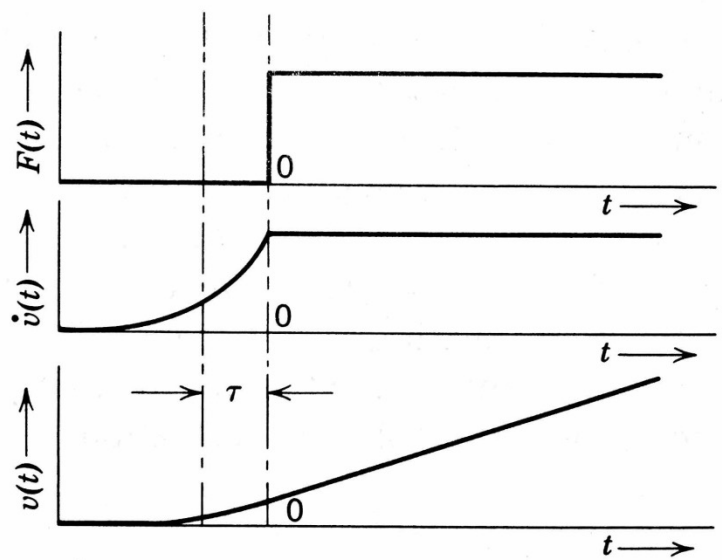
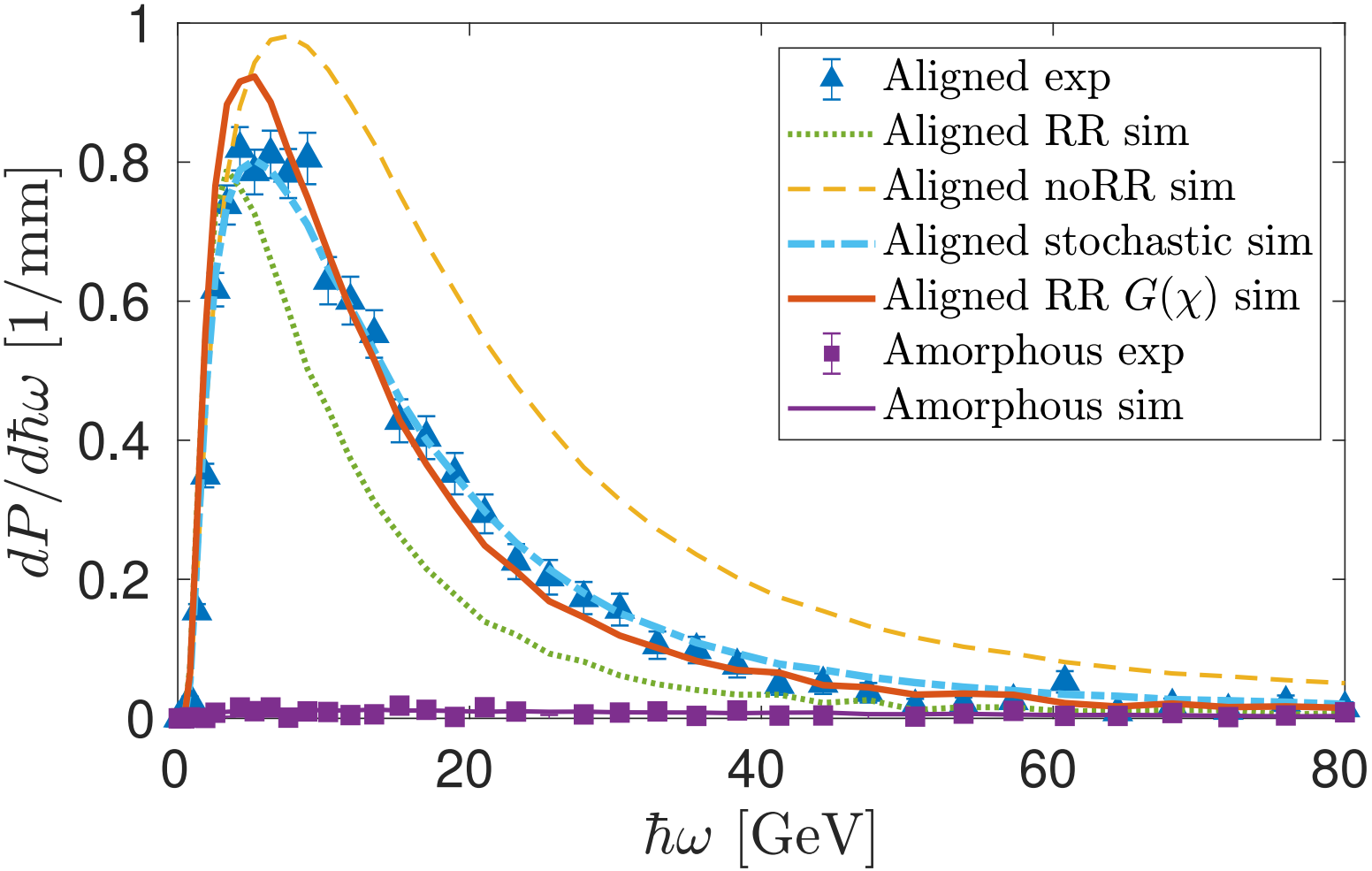


Fig. 17.1 "Preacceleration" of charged particle.

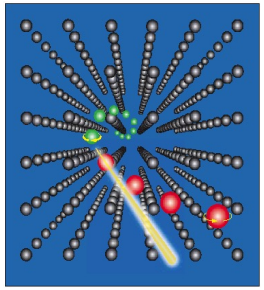
Radiation reaction in diamond



An example from a total of 22 experimental comparisons with theory reported in PRD.

Figure 2: Radiation power spectra obtained for 80 GeV (right) electrons traversing a 1.5 mm (top) thick diamond crystal aligned to the ⟨100⟩ axis, and the corresponding amorphous spectra. This spectrum has angular cuts, meaning that only particles with entry angle less than ψ_1 with respect to the crystal axis are included, where ψ_1 is the Lindhard critical angle with $\psi_1 \approx 35 \times 10^{-6}$ for 80 GeV electrons.

CERN NA63
2007-2022



<u>Crystalline targets (strong fields)</u>

- Quantum suppressed synchrotron radiation
- Spin-flip process
- Pair production
- Trident production
- Classical radiation reaction

- Bent crystals for beam extraction
- Crystalline undulator

<u>Amorphous targets</u>

- Direct measurement of the formation length
- King-Perkins-Chudakov effect (ionization suppression in vicinity of pair creation)
- LPM effect (multiple scattering suppression)
- Low-Z LPM effect
- TSF effect, logarithmic thickness dependence of rad. intensity in plural scattering
- Electromagnetically-induced nuclear-charge pickup

Scientific investigations in the framework of NA63

- Direct measurement of the Chudakov effect: PRL **100**, 164802 (2008); NIMB **269**, 1919 (2011)
- LPM effect: NIMB **266**, 5013 (2008); NIMB **269**, 1977 (2011); NIMB **289** 5-17 (2012); PRD **88**, 072007 (2013)
- Macroscopic formation length: PLB **672**, 323 (2009); PRL **108**, 071802 (2012); NIMB **315**, 278 (2013); PLB **732**, 309-314 (2014)
- Beamstrahlung in strong fields: JPCS **198**, 012007 (2009); PRST-AB **17**, 051003 (2014)
- Strong field trident production: PRD **82**, 072002 (2010)
- Logarithmic thickness dep. of radiation: PRD **81**, 052003 (2010)
- Quantum synchrotron radiation: PRD **86**, 072001 (2012)
- Strong field vacuum birefringence: PRD **88**, 053009 (2013)
- Quantum/classical Radiation Reaction: PLB **765**, 1-5 (2016); Nat. Comm. **82**, art. 795 (2018); PRR **1**, 033014 (2019); PRL 124, 044801 (2020); PRD **102**, 052004 (2020)

Significant damping in strong fields

quantum nonlinearity/strong field parameter χ

$$\chi^2 = (mF_{\mu\nu}u^\nu)^2/\mathcal{E}_0^2$$

$$\chi \simeq \gamma\mathcal{E}_\perp/\mathcal{E}_0$$

A 'specialty' of NA63 (and NA43) to address strong fields

ratio of damping force to external force

$$\eta = \alpha\gamma\chi = \alpha\gamma^2\mathcal{E}_\perp/\mathcal{E}_0$$

$$\alpha = e^2/\hbar c \simeq 1/137$$

classical for:

$\chi \ll 1$ which means: $\gamma \gg 1$

for significant damping

- Landau-Lifshitz equation, "Reduction of order", valid when $\chi\alpha \ll 1$

experiment: $\chi < 0.1$

Crystal	Energy	ψ_1	Θ_B
C $\langle 100 \rangle$	40 GeV	50 μrad	175 μrad
	80 GeV	35 μrad	
Si (110)	50 GeV	23 μrad	45 μrad

Substitution method takes account of quantum recoil:

$$\omega \rightarrow \omega^* = \omega/(1 - \hbar\omega/E)$$

Correction for quantum suppression of synchrotron radiation:

$$G(\chi) = \left[1 + 4.8(1+\chi)\ln(1+1.7\chi) + 2.44\chi^2\right]^{-2/3}$$

confirmed by:

PHYSICAL REVIEW D **86**, 072001 (2012)

Experimental investigations of synchrotron radiation at the onset of the quantum regime

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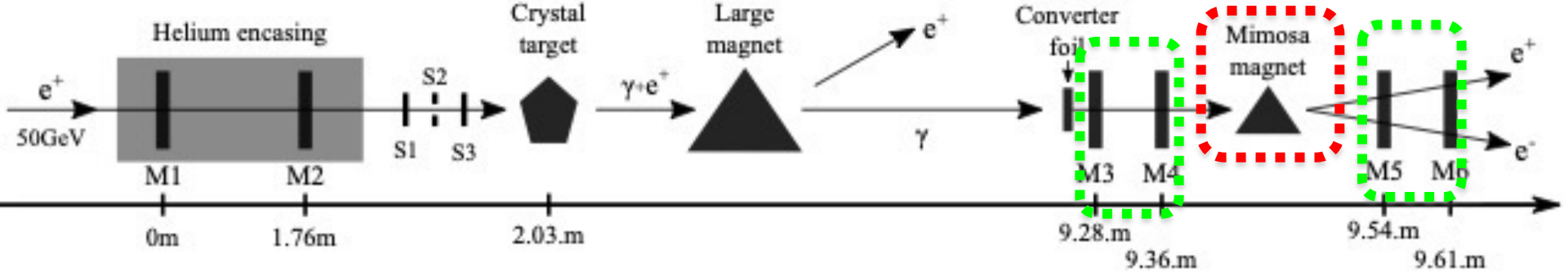
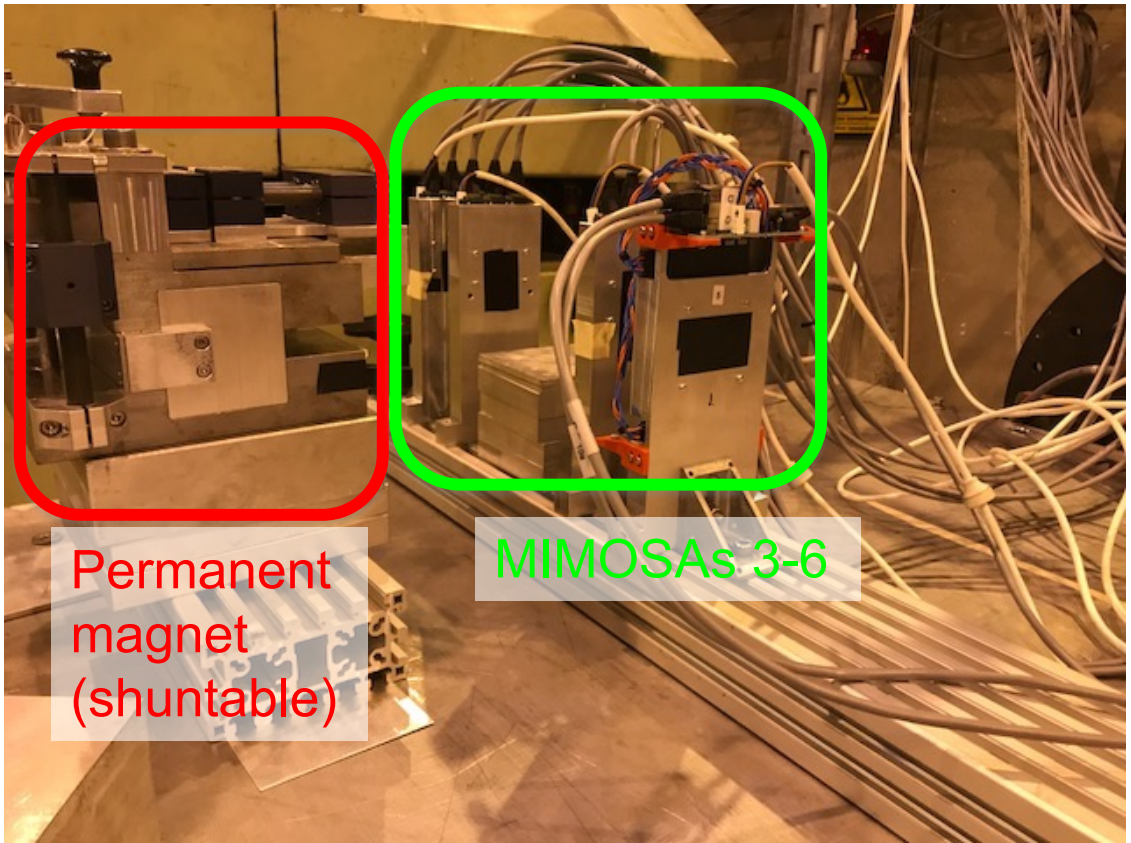
(CERN NA63)

Overview of the experiment



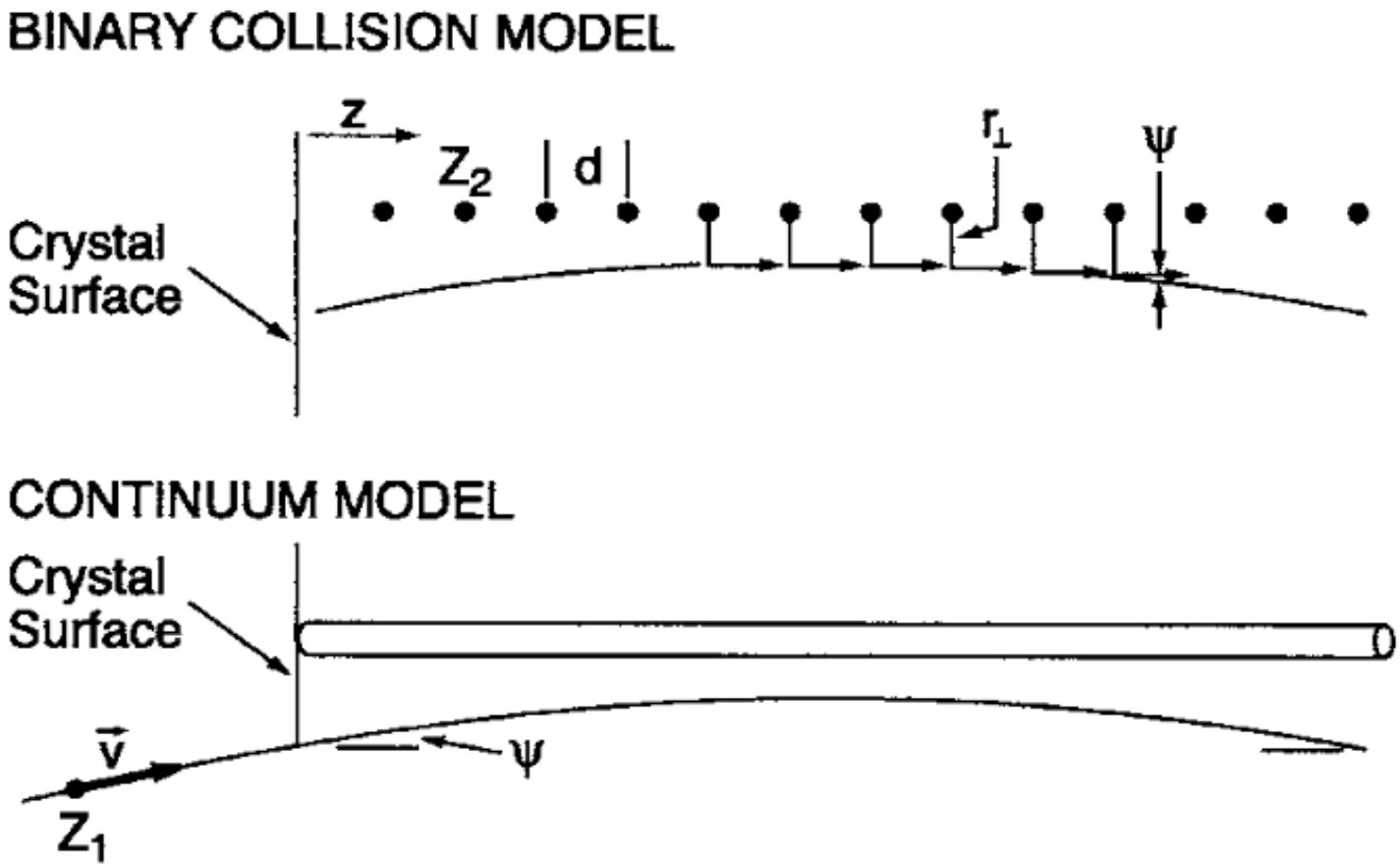
- In RR regime, naturally many photons are emitted per incoming charge
- Sufficiently thin converter foil is required to convert a single photon per event



Crystals as a source of strong fields

$\mathcal{E}_{1s}/\mathcal{E}_0 = \alpha^3 Z^3$

$\mathcal{E}_0 = mc^2/e\lambda\!\!\!^{-}_c$ =1.32·10^{16} V/cm



Example of results, silicon (2017 data)

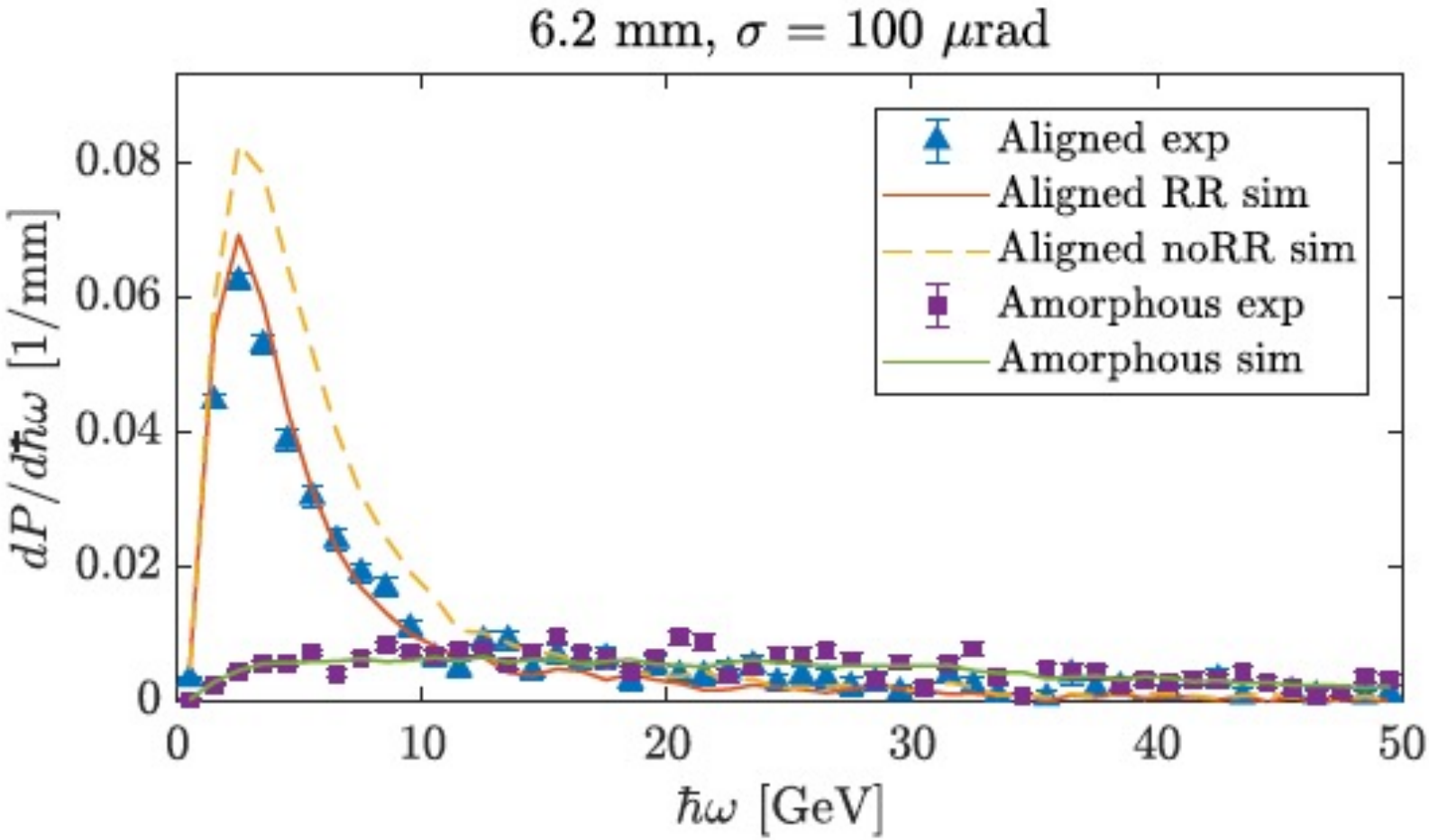


Figure 4: Radiation power spectra obtained for 50 GeV positrons passing 1.1, 2.0, 4.2 and 6.2 mm thick silicon crystals aligned to the (110) plane, and the corresponding amorphous spectra. These spectra has angular cuts, meaning that only particles with entry angle between $\pm$ 30 μrad with respect to the crystal planes are included.

MIMOSA-26 detectors

(M. Winter, Strasbourg)
Vertex detectors for CLIC (?)

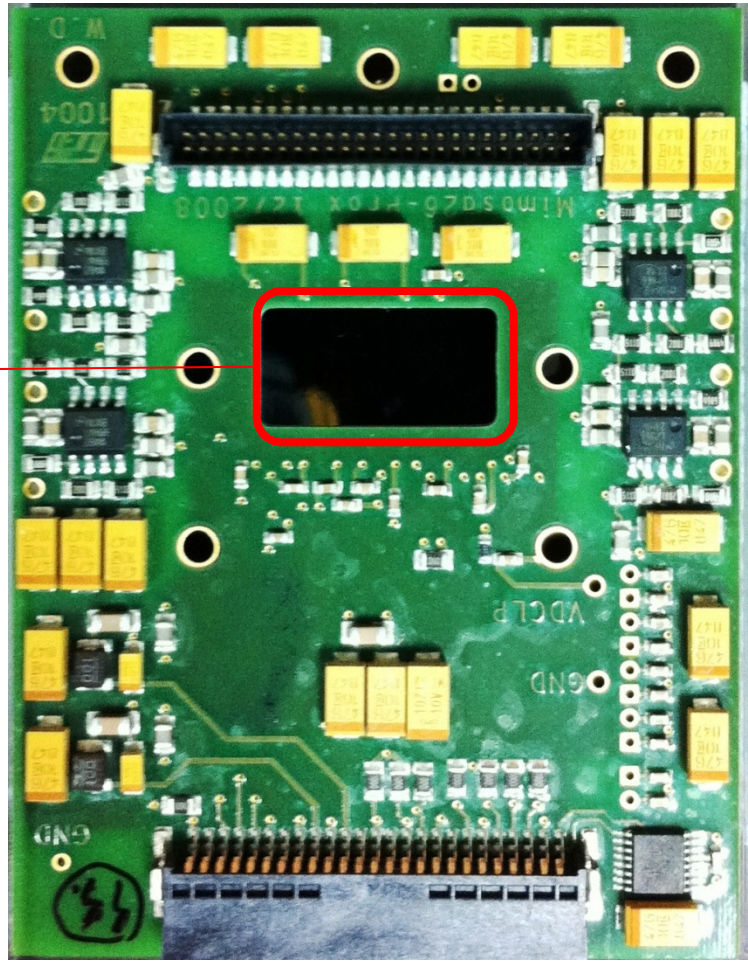
CMOS-based position sensitive detectors

1152 columns of
576 pixels, $\simeq$ 18.4 μm pitch

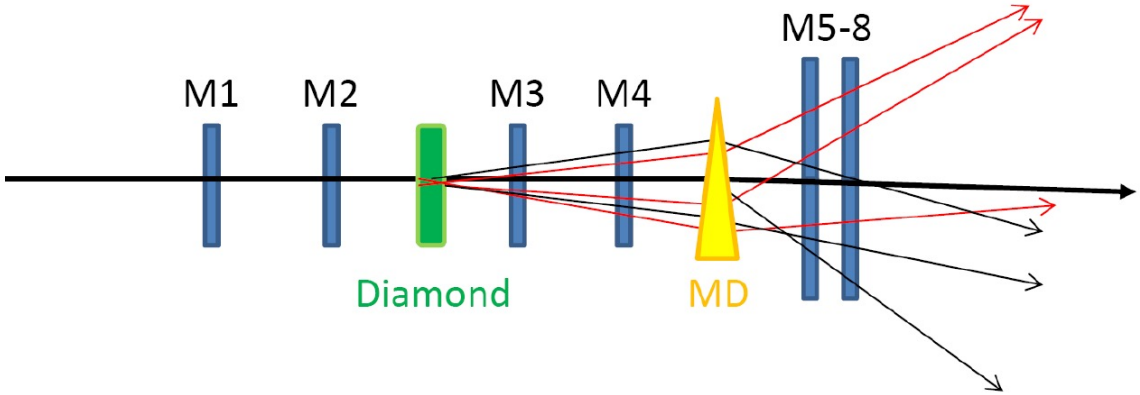
true multi-hit capability

$\Delta t / X_0 \simeq 0.05\%$

1×2 cm^2

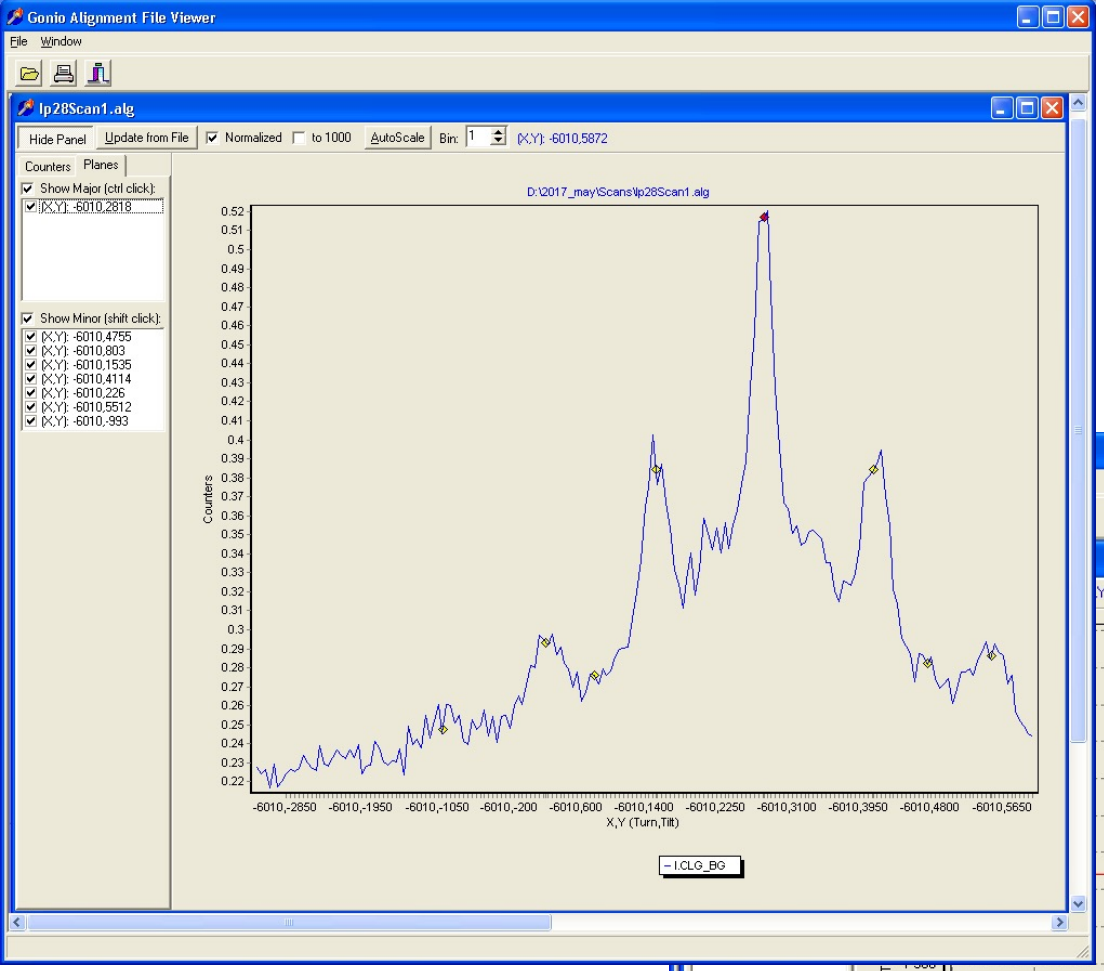


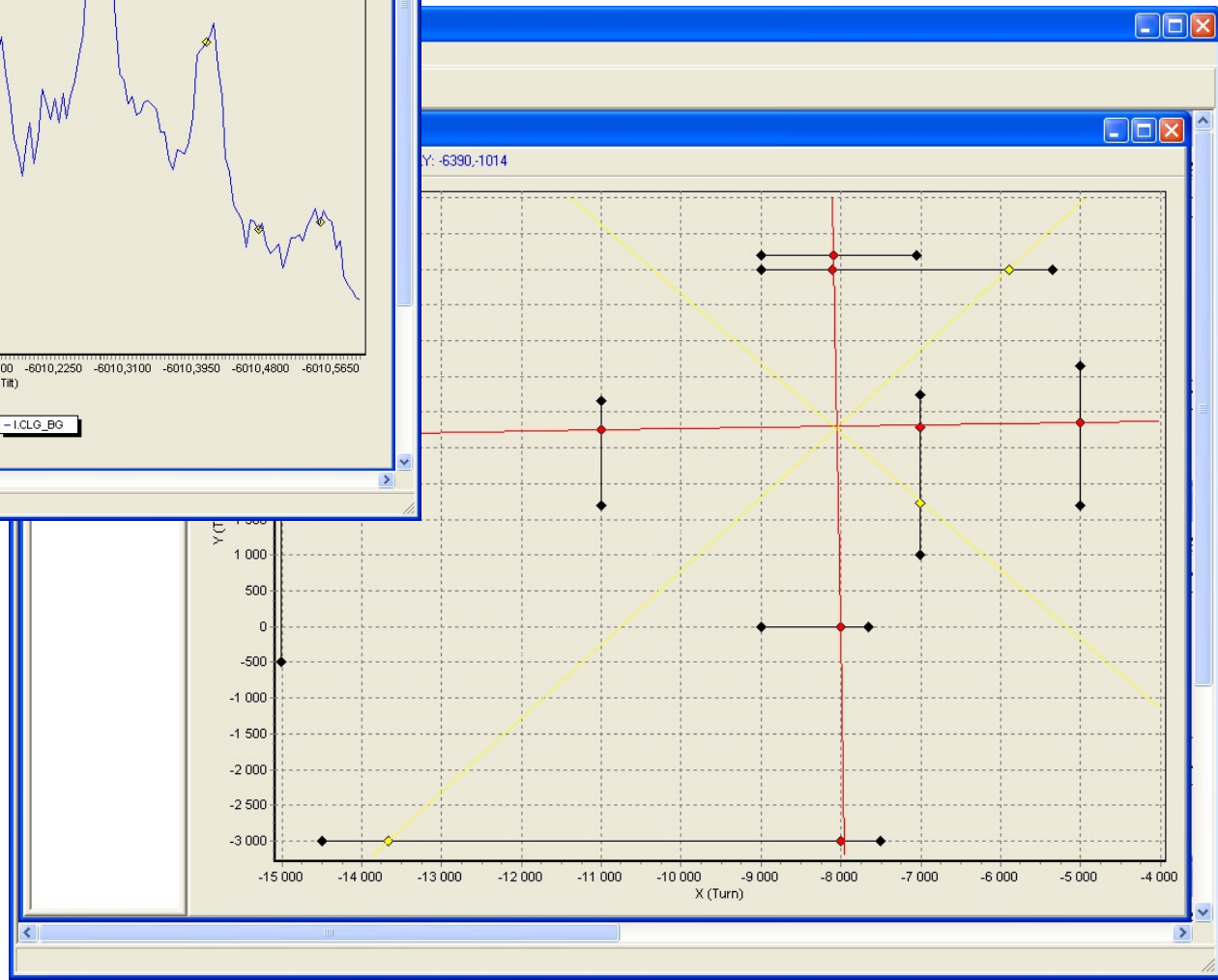
10 k frames/s, resolution 3.5 μm



Crystal planar alignment

Lindhard critical angle @ 50 GeV:
23 microrad





Crystals prealigned with x-rays

2017 data (planar case)

What is classical radiation reaction?

- Landau-Lifshitz equation, "Reduction of order": $\chi\alpha \ll 1$

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu + \frac{e^2}{m^2}F^{\mu\nu}F_{\nu\alpha}u^\alpha + \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

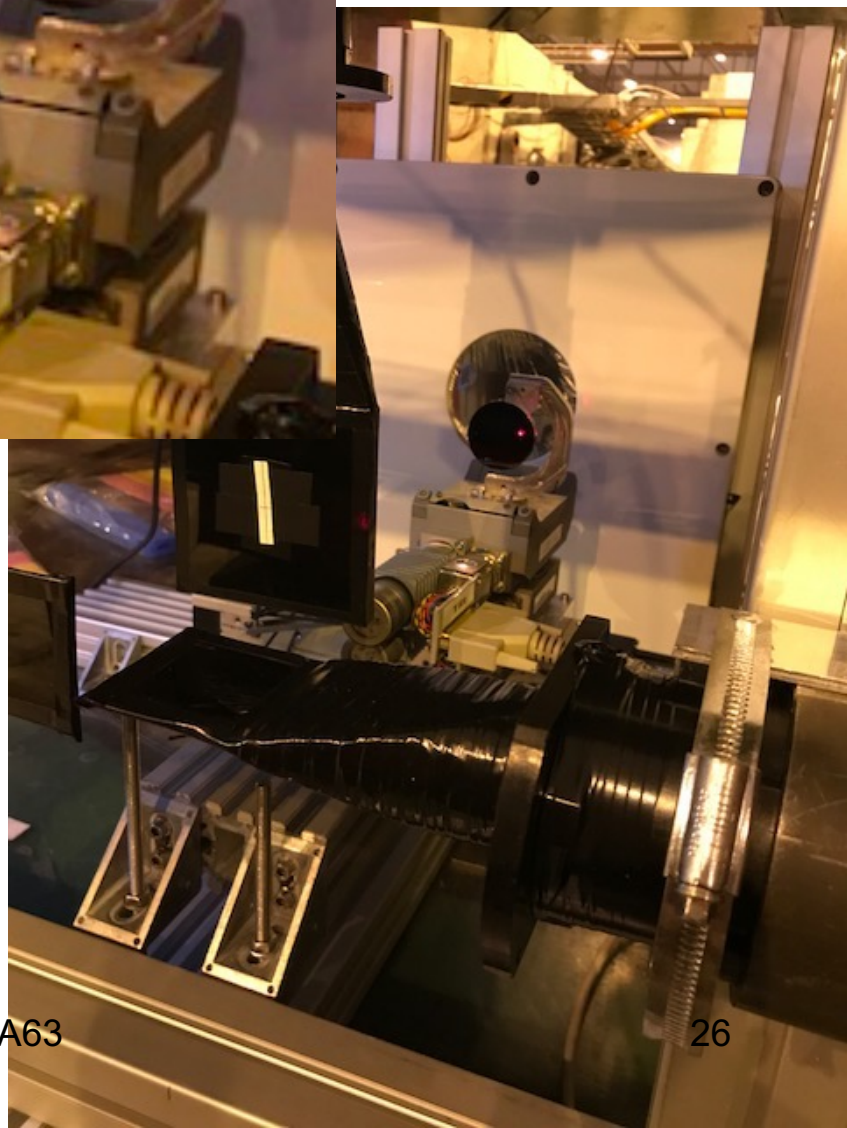
or in 3-vector notation:

$$\boldsymbol{f} = \frac{2e^3}{3m}\gamma\left\{\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\boldsymbol{E} + \boldsymbol{v}\times\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\boldsymbol{H}\right\} + \frac{2e^4}{3m^2}\left\{\boldsymbol{E}\times\boldsymbol{H} + \boldsymbol{H}\times(\boldsymbol{H}\times\boldsymbol{v}) + \boldsymbol{E}(\boldsymbol{v}\cdot\boldsymbol{E})\right\} - \frac{2e^4}{3m^2}\gamma^2\boldsymbol{v}\left\{(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{H})^2 - (\boldsymbol{E}\cdot\boldsymbol{v})^2\right\}$$

In the case of a time-independent electric field as found in a crystal this reduces to

$$\boldsymbol{f} = \underbrace{\frac{2e^3}{3m}\gamma\left\{(\boldsymbol{v}\cdot\nabla)\boldsymbol{E}\right\}}_{\text{Schott}} + \frac{2e^4}{3m^2}\left\{\boldsymbol{E}(\boldsymbol{v}\cdot\boldsymbol{E})\right\} - \frac{2e^4}{3m^2}\gamma^2\boldsymbol{v}\left\{(\boldsymbol{E})^2 - (\boldsymbol{E}\cdot\boldsymbol{v})^2\right\}$$

Detectors and crystal

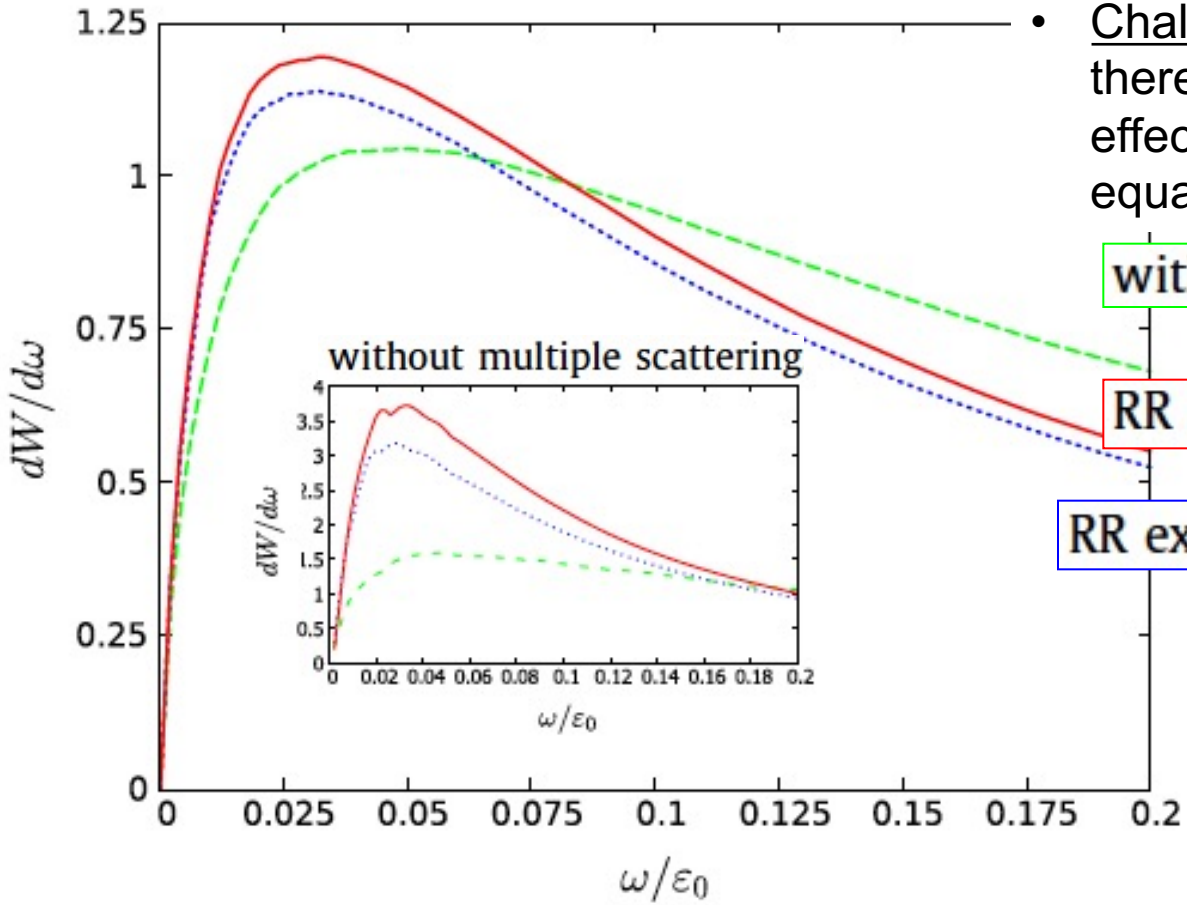


Investigation of classical radiation reaction with aligned crystals

A. Di Piazza [a,*], Tobias N. Wistisen [b], Ulrik I. Uggerhøj [b]

[a] Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117, Germany
[b] Department of Physics and Astronomy, Aarhus University, 8000 Aarhus, Denmark

Physics Letters B 765 (2017) 1–5



- Challenging, but the only place where there would be a chance to see the effect of the derivative term in the LL equation.

without RR

RR including the derivative term

RR excluding the derivative term

Derivative term not accessible in laser interactions

In a purely electric field (in the lab frame), ´Landau-Lifshitz' equation :

$$f = \frac{2e^3}{3m}\gamma\left\{(\boldsymbol{v}\cdot\nabla)\boldsymbol{E}\right\} + \frac{2e^4}{3m^2}\left\{\boldsymbol{E}(\boldsymbol{v}\cdot\boldsymbol{E})\right\} - \frac{2e^4}{3m^2}\gamma^2\boldsymbol{v}\left\{(\boldsymbol{E})^2 - (\boldsymbol{E}\cdot\boldsymbol{v})^2\right\}$$

Crystal	d_c	E	Cut	$\overline{\chi}$	$\%E_{\mathrm{LL}}$	$\%E_{\mathrm{LL,G}(\chi)}$
C ⟨100⟩	1.0 mm	40 GeV	No cut	0.0285	47.7%	20.2%
			$2\psi_1 < \psi < 5\psi_1$	0.0274	50.0%	24.0%
			$\psi_1 > \psi$	0.0311	40.8%	8.8%
		80 GeV	No cut	0.0479	59.7%	25.1%
			$2\psi_1 < \psi < 4\psi_1$	0.0470	58.3%	22.3%
			$\psi_1 > \psi$	0.0537	50.6%	6.9%
	1.5 mm	40 GeV	No cut	0.0258	46.4%	20.1%
			$2\psi_1 < \psi < 4\psi_1$	0.0253	48.1%	22.8%
			$\psi_1 > \psi$	0.0278	39.7%	8.9%
		80 GeV	No cut	0.0418	58.3%	25.1%
			$2\psi_1 < \psi < 4\psi_1$	0.0415	56.9%	22.6%
			$\psi_1 > \psi$	0.0576	49.2%	7.0%
Si (110)	1.1 mm	50 GeV	No cut	0.0155	33.5%	25.9%
			$\psi < 30\mu$rad	0.0140	16.1%	5.7%
	2.0 mm		No cut	0.0154	32.8%	24.7%
			$\psi < 30\mu$rad	0.0130	16.2%	6.38%
	4.2 mm		No cut	0.0141	31.8%	24.9%
			$\psi < 30\mu$rad	0.0123	16.7%	7.4%
	6.2 mm		No cut	0.0139	28.9%	21.5%
			$\psi < 30\mu$rad	0.0113	16.3%	7.1%

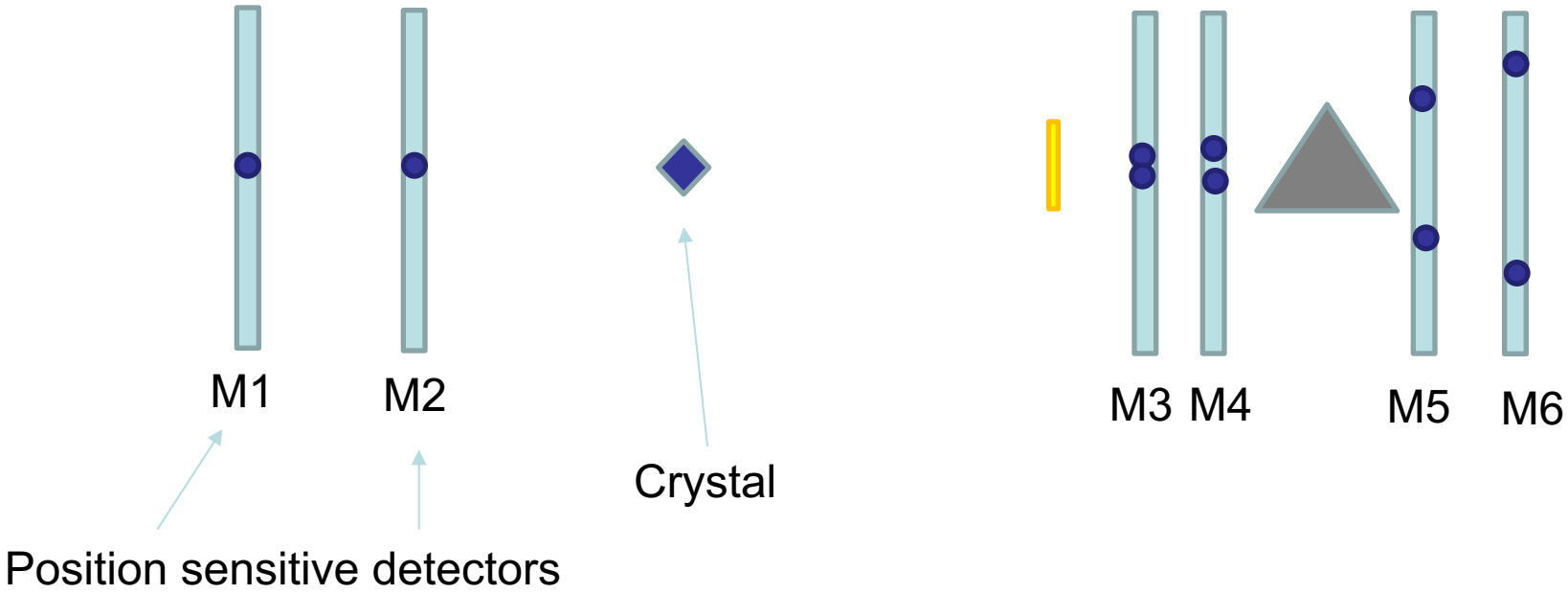
ratio of damping force to external force

$$\eta = \alpha\gamma\chi = \alpha\gamma^2\mathcal{E}_\perp/\mathcal{E}_0$$

This number shows a compromise: with increase of chi the damping becomes more significant, but the validity of the LL becomes more questionable: the fractional difference between energy lost according to the (Lorentz-force with LL damping) trajectory and energy lost according to the full spectrum increases.

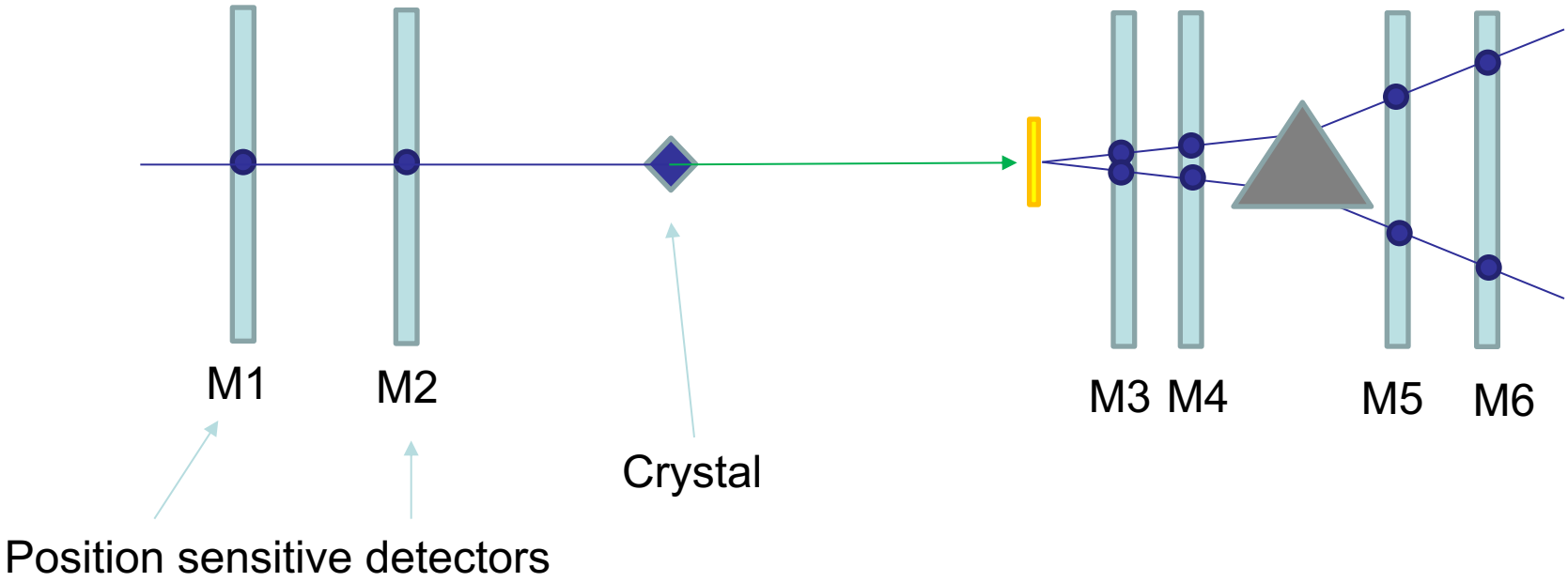
The experimental setup

- How does this setup measure photon energies?
- All you know is the position where some charged particles hit the detector



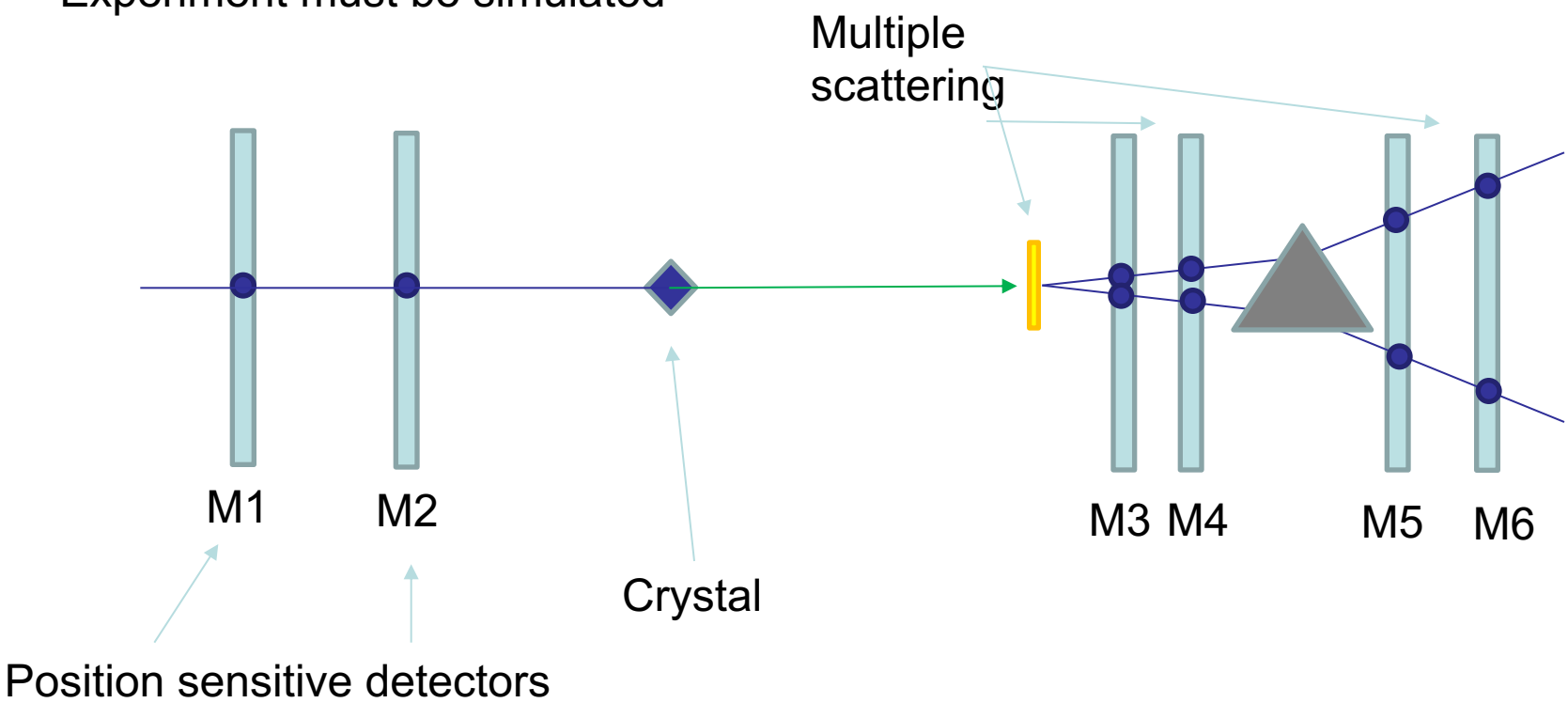
Designing the experiment.

- How does this setup measure photon energies?
- All you know is the position where some charged particles hit the detector



Designing the experiment.

- How does this setup measure photon energies?
- All you know is the position where some charged particles hit the detector
- Experiment must be simulated



Xtras

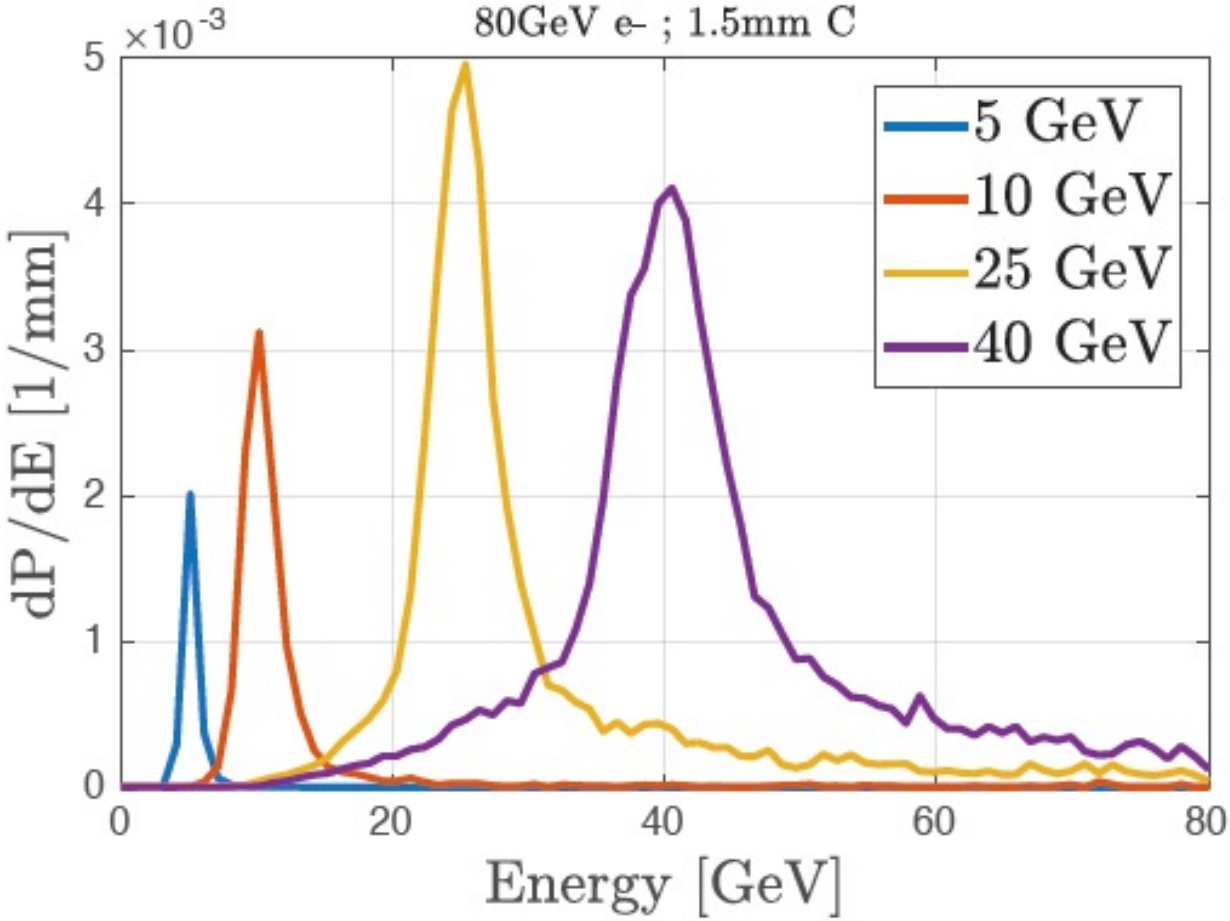


Figure 4.8: Simulations of the experiment assuming a monochromatic light source at 5 GeV (blue), 10 GeV (orange), 25 GeV (yellow) and 40 GeV (purple).